Model Predictive Control of Active Suspension System on a Quarter Car Model

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Introduction to suspension system

Suspension is the term given to the system of springs, shock absorbers and linkages that connects a vehicle to its wheels.

Two most important features are expected to be improved:

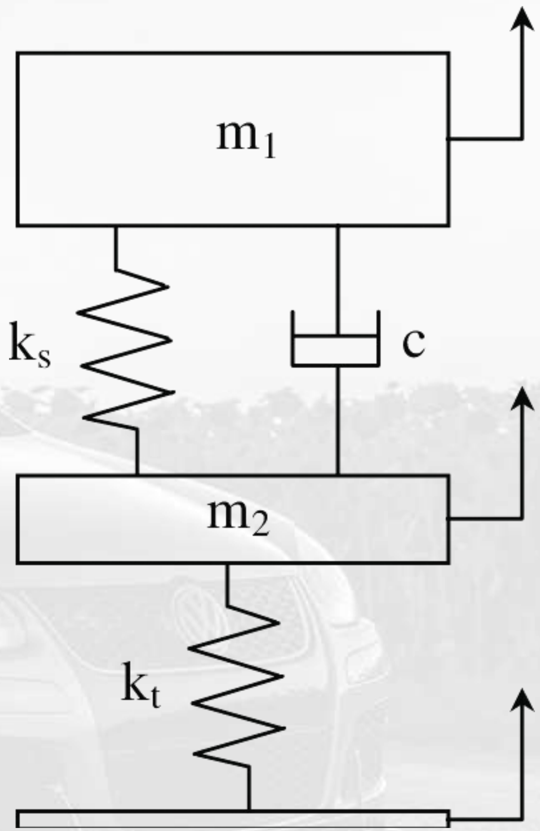
- **Disturbance absorbing** (i.e. passenger comfort).
- **Attenuation of the disturbance transfer to the road** (i.e. car handling).

Two type of suspension system

Passive suspensions:

- simple and cost-effective.

- They consist of a spring (KS) and a damper (CS) which is fixed parameters defined during the design phase.



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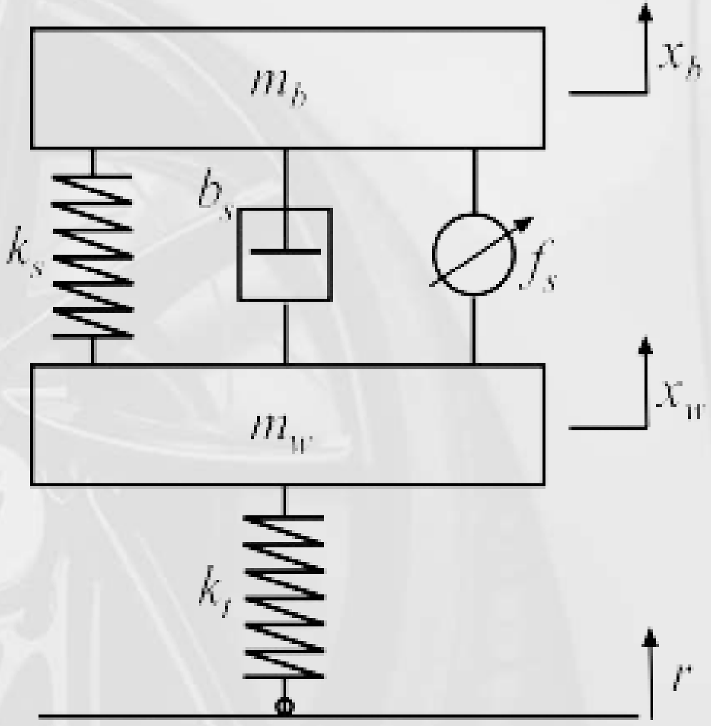
Two type of suspension system

Active suspension:

- Is an automotive technology that controls the vertical movement of the wheels with an onboard system.

- Two main functionalities:

  - isolate the vehicle body from external disturbance inputs.
  - maintain a firm contact between the road and the tyres.



ACTIVE SUSPENSION SYSTEM MODELS

Quarter Car Model:

Applying Newton's second law, the dynamic equations that govern the two-mass system are [1]:

$$\begin{cases} m_s\,\ddot{z}_s + c_s(\dot{z}_s - \dot{z}_{us}) + k_s(z_s - z_{us}) = -f_A \\ m_{us}\ddot{z}_{us} + c_s(\dot{z}_{us} - \dot{z}_s) + k_s(z_{us} - z_s) + c_{us}(\dot{z}_{us} - \dot{z}_0) + k_{us}(z_{us} - z_0) = f_A \end{cases} \quad (1)$$

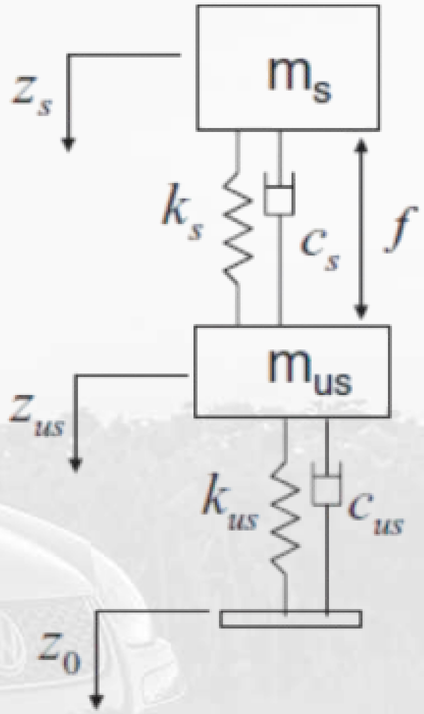


Fig. 1. Model of 2-DOF quarter car.

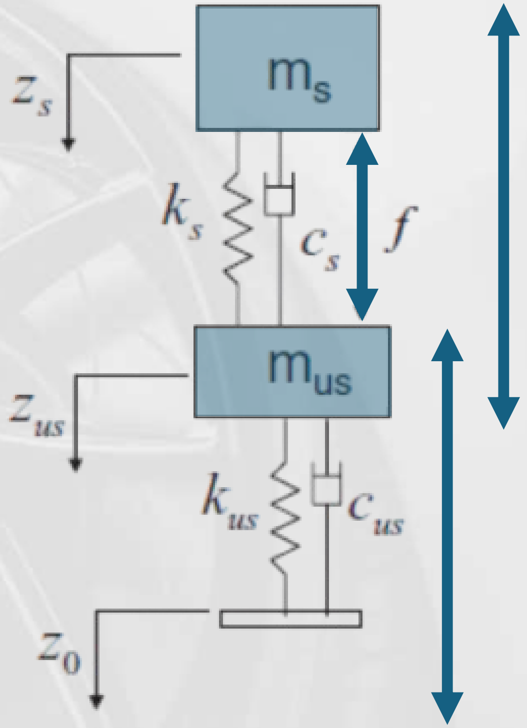
[1]:Yu, Shuyou, et al. "Model predictive control of magneto-rheological damper semi-active suspension with preview." 2020 Chinese Automation Congress (CAC). IEEE, 2020.

ACTIVE SUSPENSION SYSTEM MODELS

In order to represent the states: $x = \begin{bmatrix} z_s - z_{us} \\ \dot{z}_s \\ z_{us} - z_0 \\ \dot{z}_{us} \end{bmatrix}$

the input: $u = \begin{bmatrix} f_A \\ \dot{z}_0 \end{bmatrix}$

the output: $y = \begin{bmatrix} \ddot{z}_s \\ z_s - z_{us} \\ z_{us} - z_0 \end{bmatrix}$



ACTIVE SUSPENSION SYSTEM MODELS

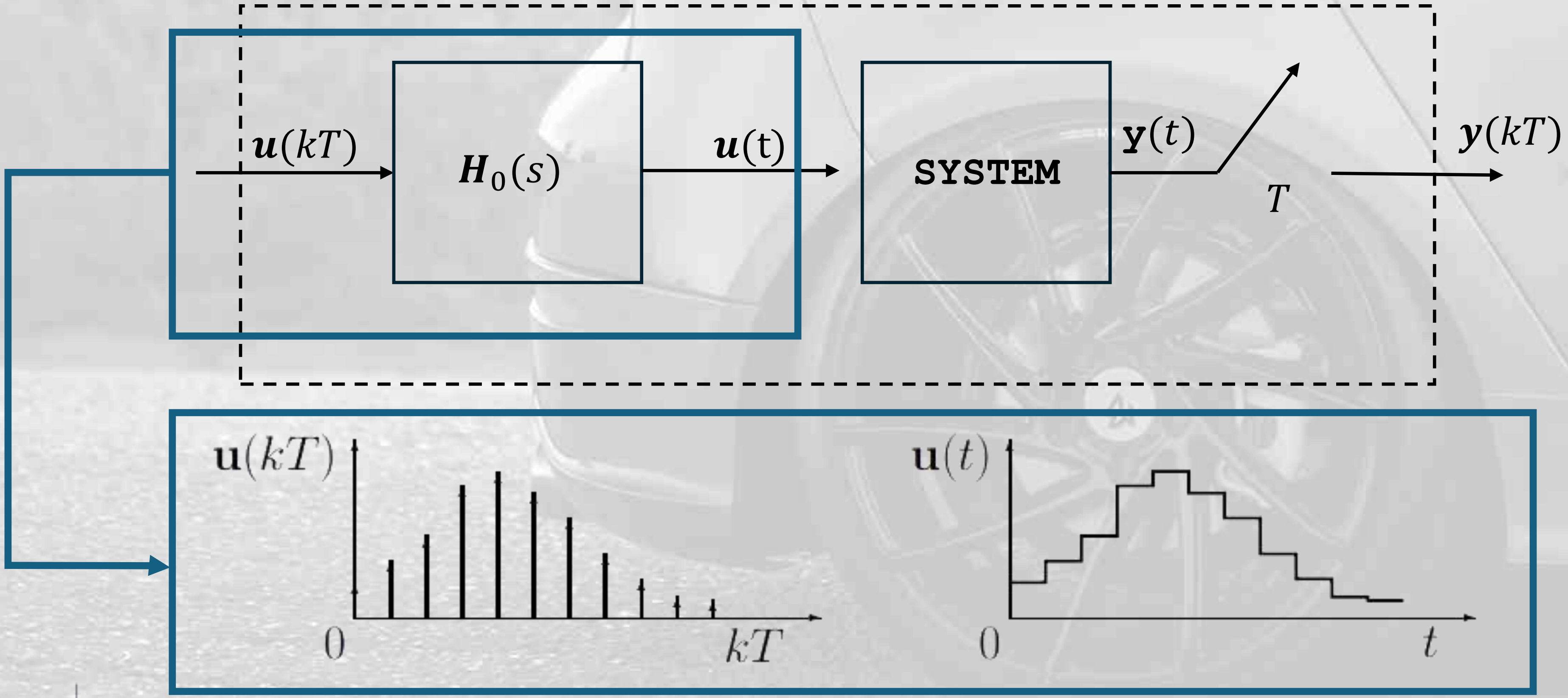
the linear state-space representation were derived as follows [1]:

$$A=\begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{k_{us}}{m_u} & -\frac{c_s}{m_u} \end{bmatrix};\quad B=\begin{bmatrix} 0 & 0 \\ \frac{1}{m_s} & 0 \\ 0 & -1 \\ -\frac{1}{m_u} & 0 \end{bmatrix};\quad C=\begin{bmatrix} -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};\ D=\begin{bmatrix} -\frac{1}{m_s} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix};$$

[1]:Yu, Shuyou, et al. "Model predictive control of magneto-rheological damper semi-active suspension with preview." 2020 Chinese Automation Congress (CAC). IEEE, 2020.

ACTIVE SUSPENSION SYSTEM MODELS

The discretization of the continuous state space matrices was set at a sampling time of 0.01s. The discretization method used was zero-order hold:



$$\boldsymbol{u}(t) = \boldsymbol{u}(kT) \quad \text{per} \quad kT \le t \le (k+1)T$$

ACTIVE SUSPENSION SYSTEM MODELS

Road preview disturbance model is described as [1]:

$$\dot{z}_{bump}(t) = \frac{A}{2}\left(1 - \cos\left(\frac{2\pi V(t-t_0)}{L}\right)\right); \qquad t_0 \leq t \leq \frac{L}{2}$$

A : bump height;
L : bump length;
V: speed of the car;
t_0 : starting time of the bump;
t : simulation time length;

[1]:Yu, Shuyou, et al. "Model predictive control of magneto-rheological damper semi-active suspension with preview." 2020 Chinese Automation Congress (CAC). IEEE, 2020.

ACTIVE SUSPENSION SYSTEM MODELS

Here the parameters for the model considered, based on data for the BMW 530i car presented in [2]:

Symbol	Variable name	Value	Unit
m_s	Sprung mass	395.3	kg
m_{us}	Unsprung mass	48.3	kg
k_s	Spring stiffness	30.01e3 N/m	N/m
c_s	Spring damping	1450	Ns/m
k_{us}	Tyre Stiffness	3.4e5	N/m
c_{us}	Tyre Damping	0	Ns/m
$f_{A,max}$	Maximum actuator force	±2500	N
$(z_{us} - z_0)_{max}$	Maximum tyre deflection	±0.0128	m
$(z_s - z_{us})_{max}$	Maximum suspension stroke	+0.09, -0.08	m

[2]:Van der Sande, T. P. J., et al. "Robust control of an electromagnetic active suspension system: Simulations and measurements." *Mechatronics* 23.2 (2013): 204-212.

ACTIVE SUSPENSION SYSTEM MODELS

As it can be seen in this table, no discomfort is caused when $\ddot{z}_s$ is less 0.315 m/s2.[3]

RMS of vertical acceleration $\ddot{z}_s$ (m/s2)	**comment**
< 0.315	Not uncomfortable
0.315-0.63	A little uncomfortable
0.5-1	Fairly uncomfortable
0.8-1.6	Uncomfortable
1.25-2.5	Very uncomfortable
>2	Extremely uncomfortable

[3] Standard, I., Mechanical vibration and shock - Evaluation of human exposure to whole body vibration - Part one *Journal of Magnetism and Magnetic Materials*, 1997.

MODEL PREDICTIVE CONTROL DESIGN

Control goal:

- The goal is to stabilize the discretized linear system as described in the equation *(1).*
- The equilibrium at origin represents that the car is moving with **zero** body vibration and **zero** tyre deflection.

MODEL PREDICTIVE CONTROL DESIGN

Cost function:

$$J(x_0, u_{\boldsymbol{N}}) = \ x_{k+T_p}^T Q_F x_{k+T_p} \sum_{i=k}^{k+T_c} u_i^T R u_i + \sum_{i=k}^{k+T_p-1} x_i^T Q x_i$$

The choices of T_c, Q and R were obtained following a tuning procedure which could ensure a balance between control action and settling-time.

$$T_c \ = 6; \qquad Q = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}; \qquad R = 1e^{-7}; \qquad Q_f = 0.5 \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

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MODEL PREDICTIVE CONTROL DESIGN

Constrains:

- $x(i) \;=\; A\,x(i-1) + \;B\,u(i-1)$
- $-2500N \leq \boldsymbol{u}(i) \;\leq +2500N$
- $$\begin{bmatrix} -0.08 \\ -0.163 \\ -0.0128 \\ -1.965 \end{bmatrix} \leq \underbrace{\begin{bmatrix} z_s - z_{us} \\ \dot{z}_s \\ z_{us} - z_0 \\ \dot{z}_{us} \end{bmatrix}}_{x(i)} (i) \leq \begin{bmatrix} 0.09 \\ 0.14 \\ 0.0128 \\ 2.78 \end{bmatrix}$$

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RESULTS

Comfort and safety analysis :

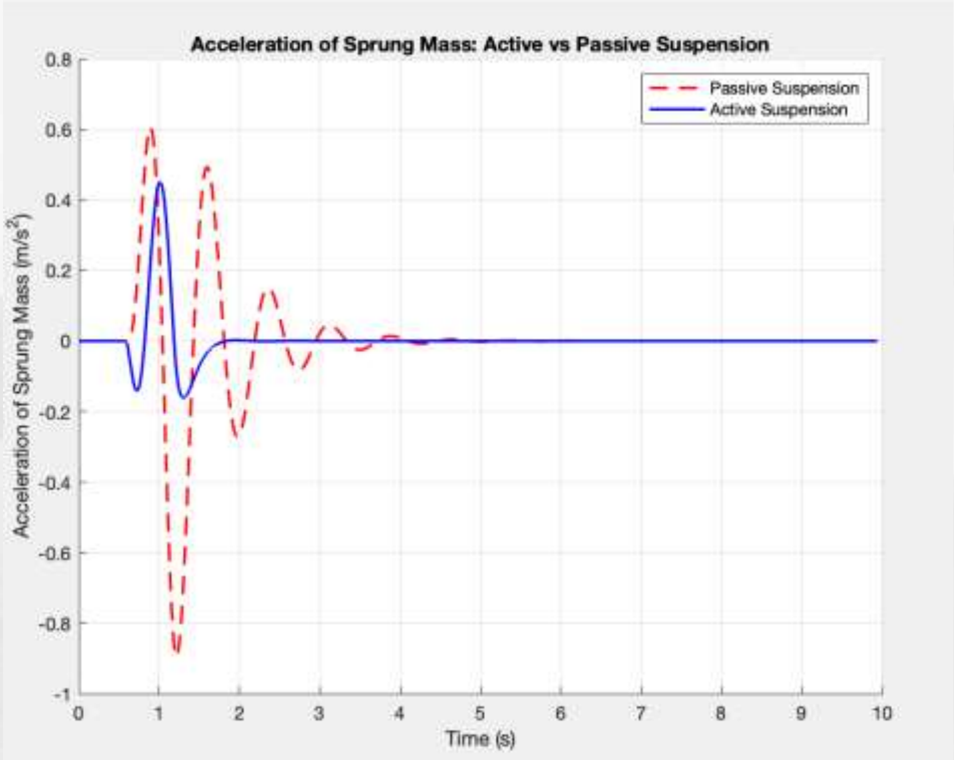
MPC controller performance has been analysed based on how well the comfort and safety objectives for the passengers are achieved at two speed scenarios:

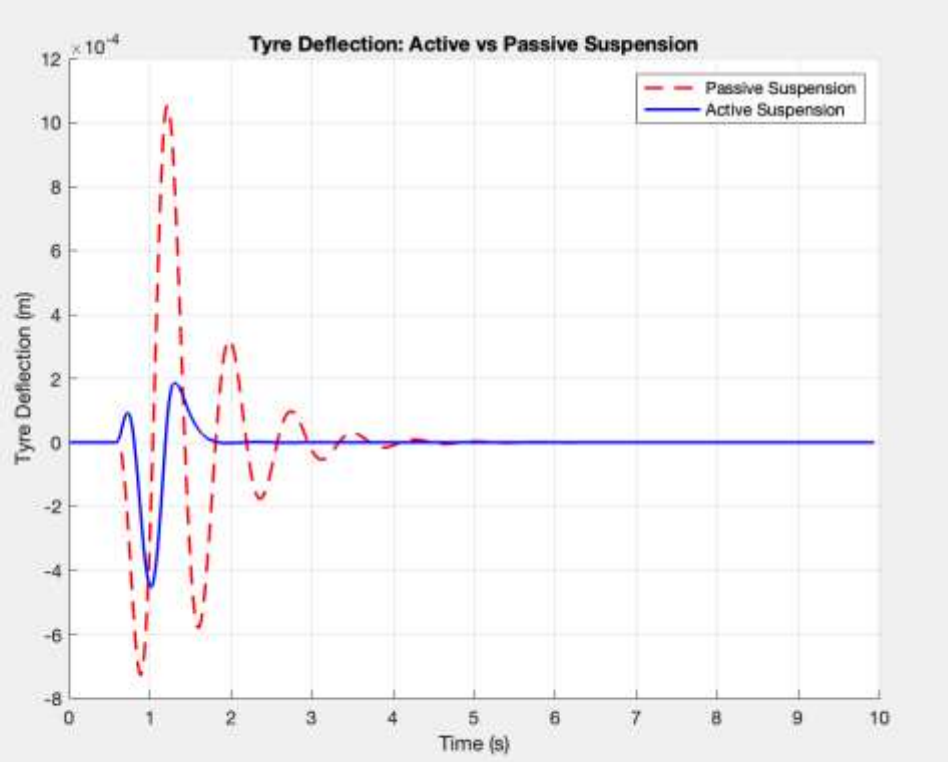
- **V = 30 km/h**;
- **V = 60 km/h**;

RESULTS

0.315-0.63	A little uncomfortable
0.8-1.6	Uncomfortable

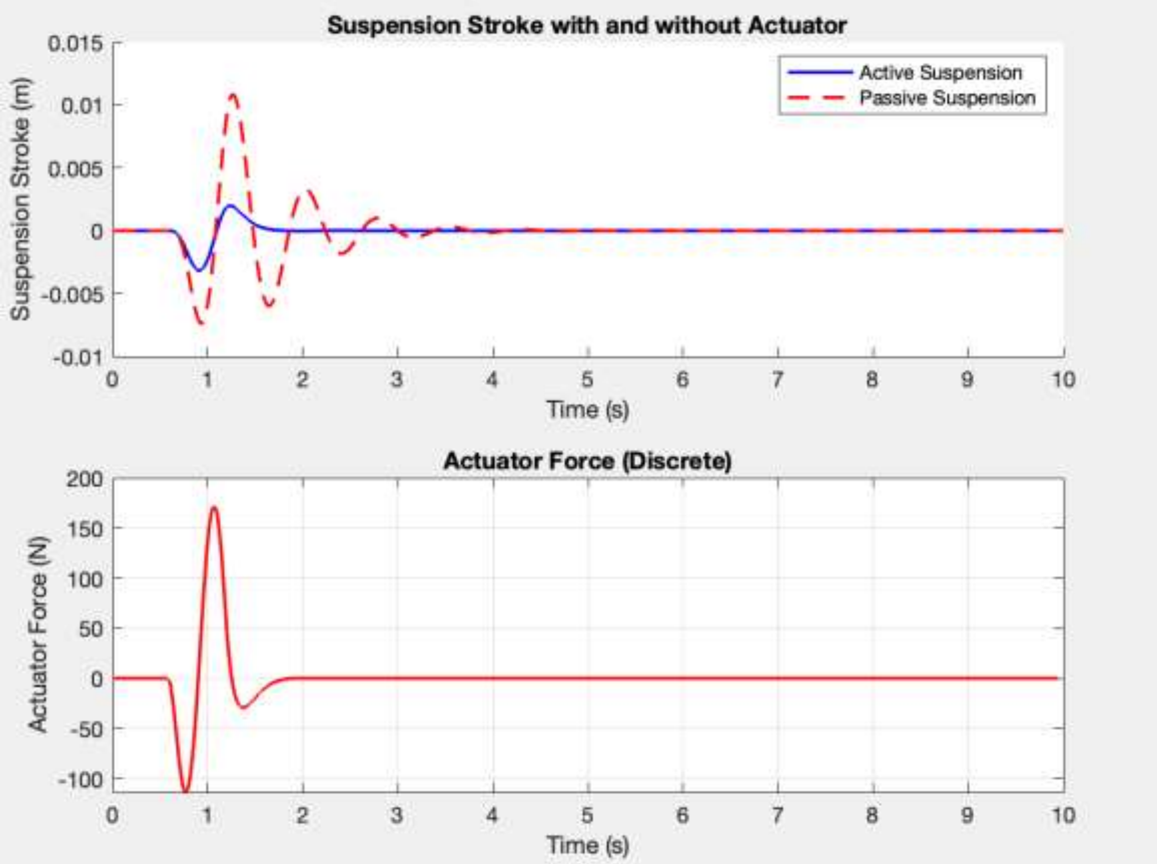
V = 30 km/h

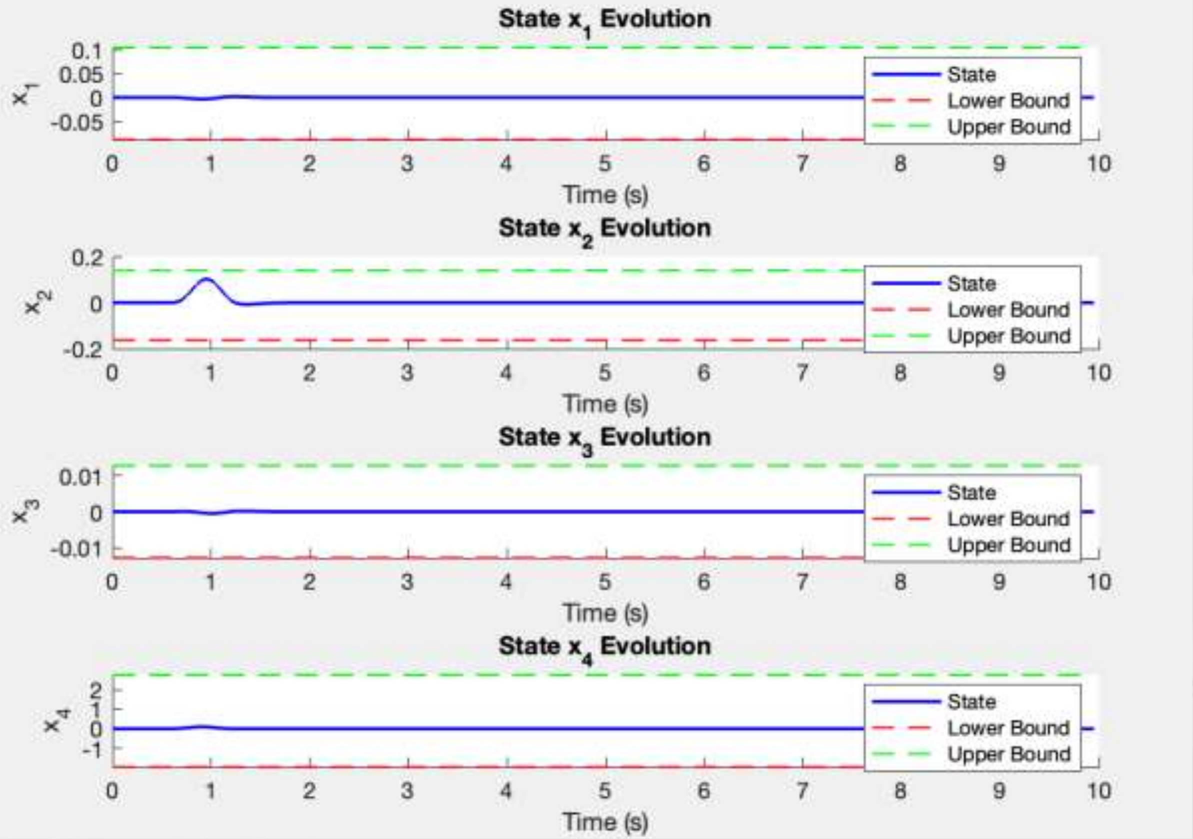




RESULTS

V = 30 km/h





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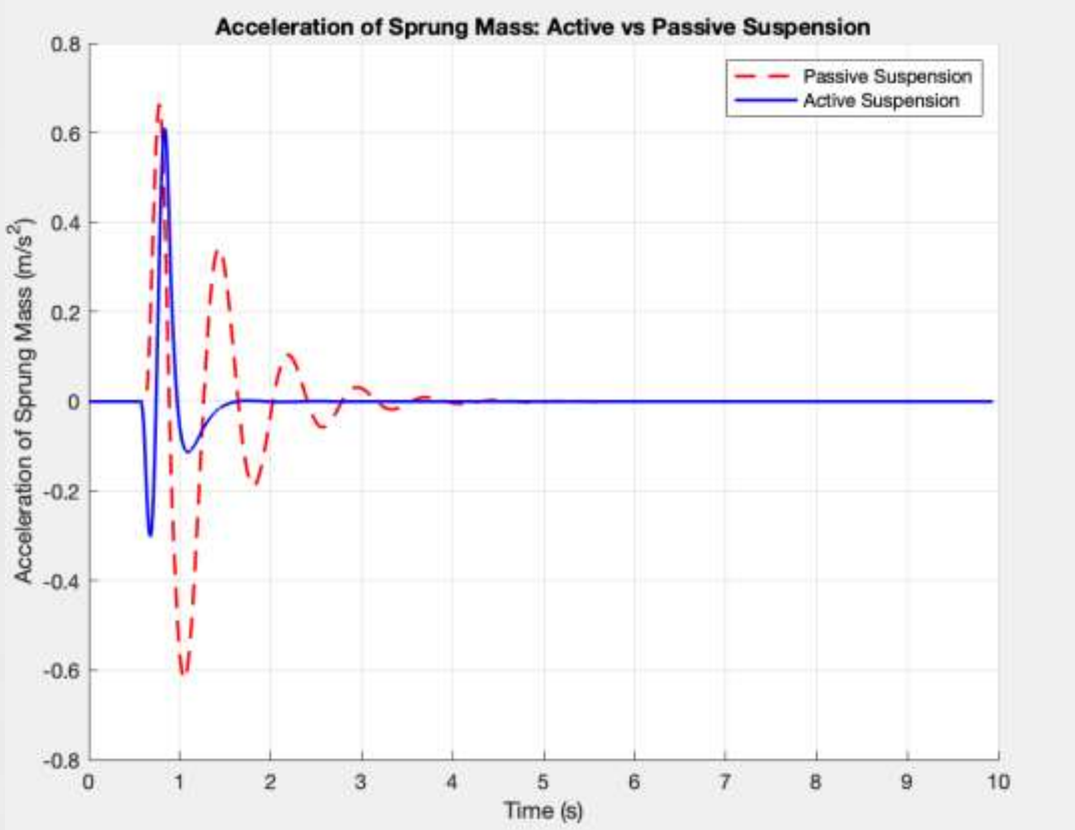
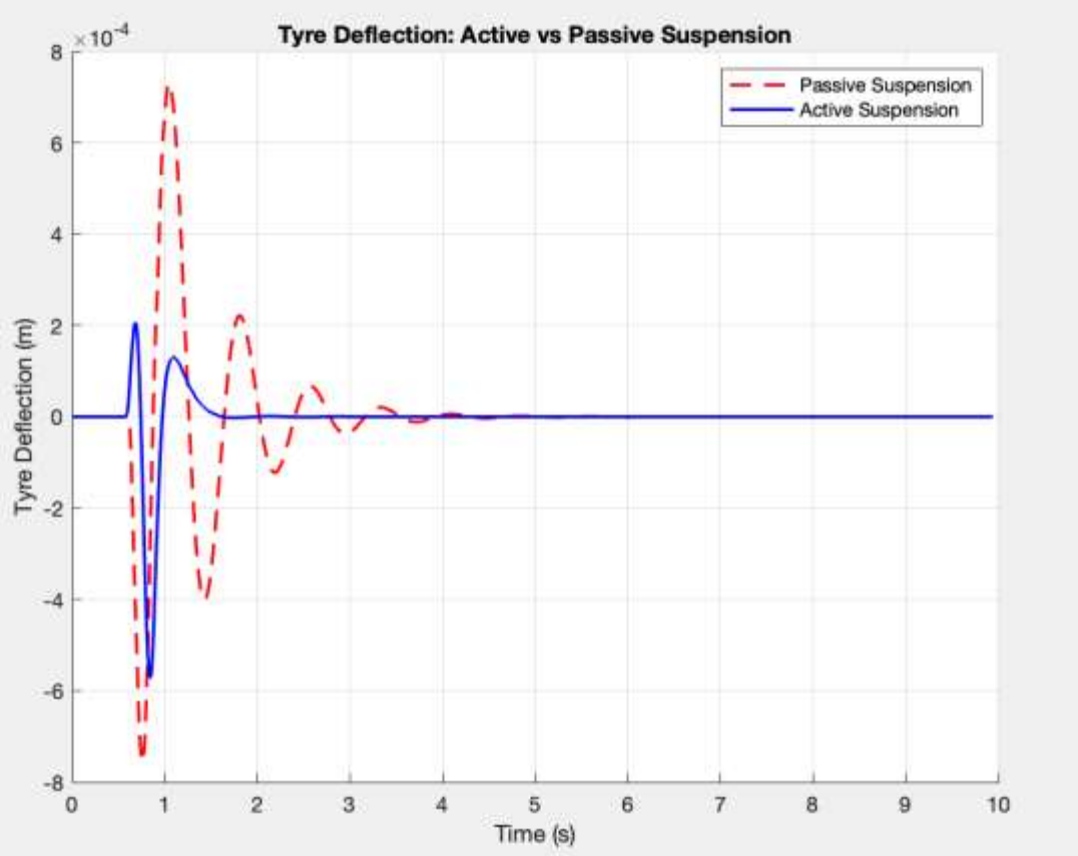
RESULTS

0.5-1	Fairly uncomfortable

V = 60 km/h





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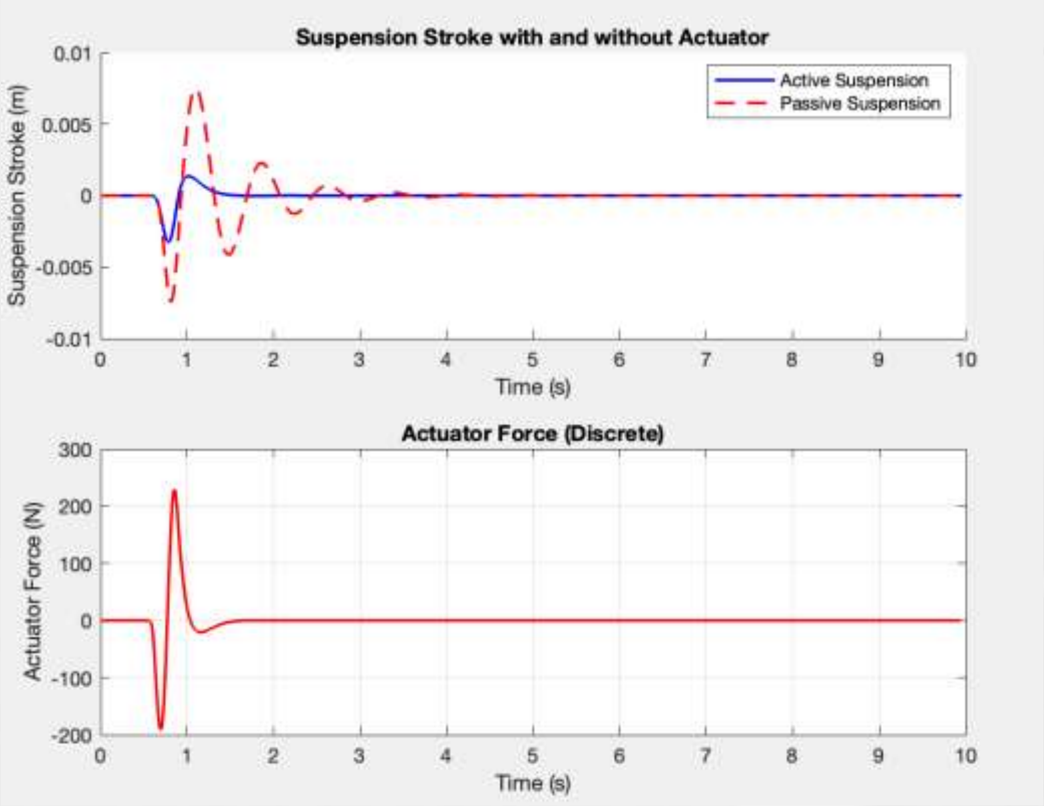
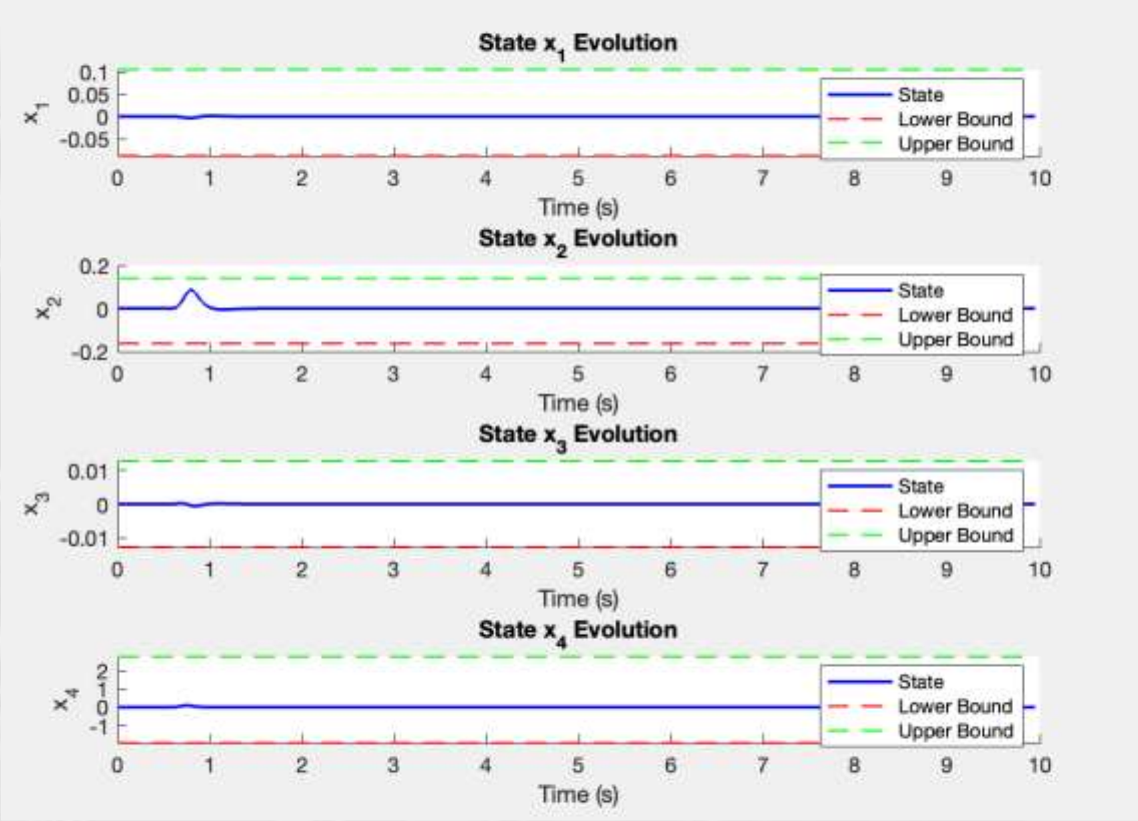
RESULTS

V = 60 km/h





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CONCLUSION

- At lower speeds, the MPC controller performs better;
- This is due to the fact that the controller has more freedom to act at lower speeds;
- The higher controller freedom is probably a consequence of the constraint on the state vector components;

THANKS!