

My solutions to  
Deep Learning: Foundations and Concepts

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### 3 Standard Distributions

#### 3.1

$$\begin{aligned}\sum p_{X;\mu} &= \sum_{x \in \{0,1\}} \mu^x (1-\mu)^{1-x} \\ &= \mu^0 (1-\mu)^{(1-0)} + \mu^1 (1-\mu)^{1-1} \\ &= 1 \cdot (1-\mu) + \mu \cdot 1 \\ &= 1 - \mu + \mu \\ &= 1\end{aligned}$$

$$\begin{aligned}\mathbb{E}[X] &= \sum_{x \in \{0,1\}} x \mu^x (1-\mu)^{1-x} \\ &= 0 + 1 \cdot \mu^1 (1-\mu)^{1-1} \\ &= \mu\end{aligned}$$

$$\begin{aligned}\mathbb{V}[X] &= \mathbb{E}[x^2] - \mathbb{E}[x]^2 \\ &= \sum_{x \in \{0,1\}} x^2 \mu^x (1-\mu)^{1-x} - \mu^2 \\ &= 0 + 1^2 \cdot \mu^1 (1-\mu)^{1-1} - \mu^2 \\ &= \mu - \mu^2 \\ &= \mu(1-\mu)\end{aligned}$$

$$\begin{aligned}\mathbb{H}[x] &= \mathbb{E}[-\log_2 p] \\ &= \sum_{x \in \{0,1\}} -p(x) \log_2 p(x)\end{aligned}$$

$$\begin{aligned}
&= \sum_{x \in \{0,1\}} -\mu^x (1-\mu)^{1-x} \log_2 (\mu^x (1-\mu)^{1-x}) \\
&= \sum_{x \in \{0,1\}} -\mu^x (1-\mu)^{1-x} (x \log_2 \mu + (1-x) \log_2 (1-\mu)) \\
&= -\mu^0 (1-\mu)^{1-0} (0 \cdot \log_2 \mu + (1-0) \log_2 (1-\mu)) \\
&\quad - \mu^1 (1-\mu)^{1-1} (1 \cdot \log_2 \mu + (1-1) \log_2 (1-\mu)) \\
&= (1-\mu) \log_2 (1-\mu) - \mu \log_2 \mu
\end{aligned}$$

### 3.2

$$\begin{aligned}
\sum p_{X;\mu} &= \sum_{\{-1,1\}} \left( \frac{1-\mu}{2} \right)^{\frac{1-x}{2}} \left( \frac{1+\mu}{2} \right)^{\frac{1+x}{2}} \\
&= \left( \frac{1-\mu}{2} \right)^{\frac{1-(-1)}{2}} \left( \frac{1+\mu}{2} \right)^{\frac{1+(-1)}{2}} + \left( \frac{1-\mu}{2} \right)^{\frac{1-1}{2}} \left( \frac{1+\mu}{2} \right)^{\frac{1+1}{2}} \\
&= \frac{1-\mu}{2} \cdot 1 + 1 \cdot \frac{1+\mu}{2} \\
&= \frac{1-\mu+1+\mu}{2} \\
&= 1
\end{aligned}$$

#### Mean

$$\begin{aligned}
\mathbb{E}[X] &= \sum_{\{-1,1\}} x \left( \frac{1-\mu}{2} \right)^{\frac{1-x}{2}} \left( \frac{1+\mu}{2} \right)^{\frac{1+x}{2}} \\
&= -1 \cdot \frac{1-\mu}{2} + 1 \cdot \frac{1+\mu}{2} \\
&= \frac{-1+\mu+1+\mu}{2} \\
&= \mu
\end{aligned}$$

#### Variance

$$\begin{aligned}
\mathbb{E}[X^2] &= \sum_{\{-1,1\}} x^2 \left( \frac{1-\mu}{2} \right)^{\frac{1-x}{2}} \left( \frac{1+\mu}{2} \right)^{\frac{1+x}{2}} \\
&= 1 \cdot \frac{1-\mu}{2} + 1 \cdot \frac{1+\mu}{2} \\
&= 1
\end{aligned}$$

$$\begin{aligned}\mathbb{V}[X] &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\ &= 1 - \mu^2\end{aligned}$$

## Entropy

$$\begin{aligned}\mathbb{H}[X] &= \mathbb{E}\left[\log \frac{1}{p_{X;\mu}}\right] \\ &= \sum_{x \in \{-1,1\}} \left(\log \frac{1}{p_{X;\mu}(x)}\right) p_{X;\mu}(x) \\ &= \left(\log \frac{2}{1-\mu}\right) \frac{1-\mu}{2} + \left(\log \frac{2}{1+\mu}\right) \frac{1+\mu}{2} \\ &= (\log(2) - \log(1-\mu)) \frac{1-\mu}{2} + (\log(2) - \log(1+\mu)) \frac{1+\mu}{2} \\ &= \log(2) \frac{1-\mu+1+\mu}{2} - \log(1-\mu) \frac{1-\mu}{2} - \log(1+\mu) \frac{1+\mu}{2} \\ &= \log(2) - \log(1-\mu) \frac{1-\mu}{2} - \log(1+\mu) \frac{1+\mu}{2}\end{aligned}$$