My solutions to

Deep Learning: Foundations and Concepts

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2 Probabilities

2.1

$$\begin{aligned}
p(C=1|T=1) &= \frac{p(T=1|C=1)p(C=1)}{p(T=1)} && \text{Bayes' theorem} \\
&= \frac{p(T=1|C=1)p(C=1)}{p(T=1,C=0)+p(T=1,C=1)} && \text{sum rule} \\
&= \frac{p(T=1|C=1)p(C=1)}{p(T=1|C=0)p(C=0)+p(T=1|C=1)p(C=1)} && \text{product rule} \\
&= \frac{0.9 \cdot 0.001}{0.03 \cdot (1-0.001) + 0.9 \cdot 0.001} \\
&\approx 0.029
\end{aligned}$$

2.2

Let Y denote the yellow die, B the blue die, G the green die and R the red die. We consider throws of pairs of independent dice, i.e. $p(D_1, D_2) = p(D_1)p(D_2)$. Each die takes on a unique value in a given throw, such that e.g. $(G=5) := (G=5, B=x)$ and $(G=1, B=0)$ are mutually exclusive events, hence $p(G=5 \text{ or } (G=1, B=0)) = P(G=5) + P(G=1, B=0)$.

- $$\begin{aligned}
p(B>Y) &= p(B=4, Y=3) \\
&= p(B=4)p(Y=3) \\
&= \frac{4}{6} \cdot \frac{6}{6} = \frac{2}{3}
\end{aligned}$$
- $$\begin{aligned}
p(G>B) &= p(G=5 \text{ or } (G=1, B=0)) \\
&= p(G=5) + p(G=1)p(B=0) \\
&= \frac{3}{6} + \frac{3}{6} \cdot \frac{2}{6} = \frac{2}{3}
\end{aligned}$$

- $$\begin{aligned} p(R > G) &= p(R = 6 \text{ or } (R = 2, G = 1)) \\ &= p(R = 6) + p(R = 2)p(G = 1) \\ &= \frac{2}{6} + \frac{4}{6} \cdot \frac{3}{6} = \frac{2}{3} \end{aligned}$$
- $$\begin{aligned} p(Y > R) &= p(Y = 3, R = 2) \\ &= p(Y = 3)p(R = 2) \\ &= \frac{6}{6} \cdot \frac{4}{6} = \frac{2}{3} \end{aligned}$$