

My solutions to  
Deep Learning: Foundations and Concepts

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## 11 Structured Distributions

### 11.3

$$\begin{aligned} p(a = 0) &= \sum_{b,c} p(0, b, c) \\ &= p(0, 0, 0) + p(0, 0, 1) + p(0, 1, 0) + p(0, 1, 1) \\ &= 0.192 + 0.144 + 0.048 + 0.216 \\ &= 0.6 \end{aligned}$$

$$\begin{aligned} p(b = 0) &= \sum_{a,c} p(a, 0, c) \\ &= p(0, 0, 0) + p(0, 0, 1) + p(1, 0, 0) + p(1, 0, 1) \\ &= 0.192 + 0.144 + 0.192 + 0.064 \\ &= 0.592 \end{aligned}$$

$$\begin{aligned} p(c = 0) &= \sum_{a,b} p(a, b, 0) \\ &= p(0, 0, 0) + p(0, 1, 0) + p(1, 0, 0) + p(1, 1, 0) \\ &= 0.192 + 0.048 + 0.192 + 0.048 \\ &= 0.48 \end{aligned}$$

$$\begin{aligned} p(c = 1) &= 1 - p(c = 0) \\ &= 0.52 \end{aligned}$$

$$\begin{aligned} p(a = 0, b = 0) &= \sum_c p(0, 0, c) \\ &= p(0, 0, 0) + p(0, 0, 1) \\ &= 0.192 + 0.144 \end{aligned}$$

$$\begin{aligned}
&= 0.336 \\
p(a = 0, c = 0) &= \sum_b p(0, b, 0) \\
&= p(0, 0, 0) + p(0, 1, 0) \\
&= 0.192 + 0.048 \\
&= 0.24
\end{aligned}$$

$$\begin{aligned}
p(a = 0, c = 1) &= \sum_b p(0, b, 1) \\
&= p(0, 0, 1) + p(0, 1, 1) \\
&= 0.144 + 0.216 \\
&= 0.36
\end{aligned}$$

$$\begin{aligned}
p(a = 1, c = 0) &= \sum_b p(1, b, 0) \\
&= p(1, 0, 0) + p(1, 1, 0) \\
&= 0.192 + 0.048 \\
&= 0.24
\end{aligned}$$

$$\begin{aligned}
p(b = 0, c = 0) &= \sum_a p(a, 0, 0) \\
&= p(0, 0, 0) + p(1, 0, 0) \\
&= 0.192 + 0.192 \\
&= 0.384
\end{aligned}$$

$$\begin{aligned}
p(b = 0, c = 1) &= \sum_a p(a, 0, 1) \\
&= p(0, 0, 1) + p(1, 0, 1) \\
&= 0.144 + 0.064 \\
&= 0.208
\end{aligned}$$

$$\begin{aligned}
p(b = 1, c = 0) &= \sum_a p(a, 1, 0) \\
&= p(0, 1, 0) + p(1, 1, 0) \\
&= 0.048 + 0.048 \\
&= 0.096
\end{aligned}$$

$$p(a = 0|c = 0) = \frac{p(a = 0, c = 0)}{p(c = 0)}$$

$$\begin{aligned}
&= \frac{0.24}{0.48} \\
&= 0.5 \\
p(a = 1|c = 0) &= 1 - p(a = 0|c = 0) \\
&= 1 - 0.5 \\
&= 0.5 \\
p(a = 0|c = 1) &= \frac{p(a = 0, c = 1)}{p(c = 1)} \\
&= \frac{0.36}{0.52} \\
&= \frac{9}{13} \\
p(a = 1|c = 1) &= 1 - p(a = 0|c = 1) \\
&= 1 - \frac{9}{13} \\
&= \frac{4}{13} \\
p(b = 0|c = 0) &= \frac{p(b = 0, c = 0)}{p(c = 0)} \\
&= \frac{0.384}{0.48} \\
&= 0.8 \\
p(b = 1|c = 0) &= 1 - p(b = 0|c = 0) \\
&= 1 - 0.8 \\
&= 0.2 \\
p(b = 0|c = 1) &= \frac{p(b = 0, c = 1)}{p(c = 1)} \\
&= \frac{0.208}{0.52} \\
&= 0.4 \\
p(b = 1|c = 1) &= 1 - p(b = 0|c = 1) \\
&= 1 - 0.4 \\
&= 0.6 \\
\\
p(a = 0, b = 0|c = 0) &= \frac{p(0, 0, 0)}{p(c = 0)} \\
&= \frac{0.192}{0.48}
\end{aligned}$$

$$\begin{aligned}
&= 0.4 \\
&= 0.5 \cdot 0.8 \\
&= p(a = 0|c = 0)p(b = 0|c = 0) \\
p(a = 0, b = 1|c = 0) &= \frac{p(0, 1, 0)}{p(c = 0)} \\
&= \frac{0.048}{0.48} \\
&= 0.1 \\
&= 0.5 \cdot 0.2 \\
&= p(a = 0|c = 0)p(b = 1|c = 0) \\
p(a = 1, b = 0|c = 0) &= \frac{p(1, 0, 0)}{p(c = 0)} \\
&= \frac{0.192}{0.48} \\
&= 0.4 \\
&= 0.5 \cdot 0.8 \\
&= p(a = 1|c = 0)p(b = 0|c = 0) \\
p(a = 1, b = 1|c = 0) &= \frac{p(1, 1, 0)}{p(c = 0)} \\
&= \frac{0.048}{0.48} \\
&= 0.1 \\
&= 0.5 \cdot 0.2 \\
&= p(a = 1|c = 0)p(b = 1|c = 0)
\end{aligned}$$

$$\begin{aligned}
p(a = 0, b = 0|c = 1) &= \frac{p(0, 0, 1)}{p(c = 1)} \\
&= \frac{0.144}{0.52} \\
&= \frac{18}{65} \\
&= \frac{9}{13} \cdot 0.4 \\
&= p(a = 0|c = 1)p(b = 0|c = 1) \\
p(a = 0, b = 1|c = 1) &= \frac{p(0, 1, 1)}{p(c = 1)}
\end{aligned}$$

$$\begin{aligned}
&= \frac{0.216}{0.52} \\
&= \frac{27}{65} \\
&= \frac{9}{13} \cdot 0.6 \\
&= p(a = 0|c = 1)p(b = 1|c = 1) \\
p(a = 1, b = 0|c = 1) &= \frac{p(1, 0, 1)}{p(c = 1)} \\
&= \frac{0.064}{0.52} \\
&= \frac{8}{65} \\
&= \frac{4}{13} \cdot 0.4 \\
&= p(a = 1|c = 1)p(b = 0|c = 1) \\
p(a = 1, b = 1|c = 1) &= \frac{p(1, 1, 1)}{p(c = 1)} \\
&= \frac{0.096}{0.52} \\
&= \frac{12}{65} \\
&= \frac{4}{13} \cdot 0.6 \\
&= p(a = 1|c = 1)p(b = 1|c = 1)
\end{aligned}$$

## 11.4

$$\begin{aligned}
p(a = 1) &= 1 - p(a = 0) \\
&= 1 - 0.6 \\
&= 0.4
\end{aligned}$$

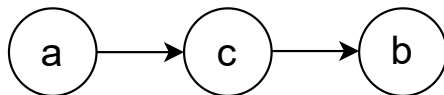
$$\begin{aligned}
p(c = 0|a = 0) &= \frac{p(a = 0, c = 0)}{p(a = 0)} \\
&= \frac{0.24}{0.6} \\
&= 0.4
\end{aligned}$$

$$p(c = 1|a = 0) = 1 - p(c = 0|a = 0)$$

$$\begin{aligned}
&= 1 - 0.4 \\
&= 0.6 \\
p(c = 0|a = 1) &= \frac{p(a = 1, c = 0)}{p(a = 1)} \\
&= \frac{0.24}{0.4} \\
&= 0.6 \\
p(c = 1|a = 1) &= 1 - p(c = 0|a = 1) \\
&= 1 - 0.6 \\
&= 0.4
\end{aligned}$$

$$\begin{aligned}
p(0, 0, 0) &= 0.192 \\
&= 0.6 \cdot 0.4 \cdot 0.8 \\
&= p(a = 0)p(c = 0|a = 0)p(b = 0|c = 0) \\
p(0, 0, 1) &= 0.144 \\
&= 0.6 \cdot 0.6 \cdot 0.4 \\
&= p(a = 0)p(c = 1|a = 0)p(b = 0|c = 1) \\
p(0, 1, 0) &= 0.048 \\
&= 0.6 \cdot 0.4 \cdot 0.2 \\
&= p(a = 0)p(c = 0|a = 0)p(b = 1|c = 0) \\
p(0, 1, 1) &= 0.216 \\
&= 0.6 \cdot 0.6 \cdot 0.6 \\
&= p(a = 0)p(c = 1|a = 0)p(b = 1|c = 1) \\
p(1, 0, 0) &= 0.192 \\
&= 0.4 \cdot 0.6 \cdot 0.8 \\
&= p(a = 1)p(c = 0|a = 1)p(b = 0|c = 0) \\
p(1, 0, 1) &= 0.064 \\
&= 0.4 \cdot 0.4 \cdot 0.4 \\
&= p(a = 1)p(c = 1|a = 1)p(b = 0|c = 1) \\
p(1, 1, 0) &= 0.048 \\
&= 0.4 \cdot 0.6 \cdot 0.2 \\
&= p(a = 1)p(c = 0|a = 1)p(b = 1|c = 0) \\
p(1, 1, 1) &= 0.096 \\
&= 0.4 \cdot 0.4 \cdot 0.6
\end{aligned}$$

$$= p(a = 1)p(c = 1|a = 1)p(b = 1|c = 1)$$



Directed graph corresponding to the factorization from 11.4.

## 11.11

$$\begin{aligned}
 p(a, b|d) &= \int p(a, b, c|d)dc \\
 &= \int p(a|d)p(b, c|d)dc && \text{by assumption } a \perp\!\!\!\perp b, c \mid d \\
 &= p(a|d) \int p(b, c|d)dc \\
 &= p(a|d)p(b|d)
 \end{aligned}$$