My solutions to

Deep Learning: Foundations and Concepts

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14 Sampling

14.1

$$
\begin{aligned}
\mathbb{E}\left[\frac{1}{L}\sum_{l=1}^{L} f\left(Z_l\right)\right] &= \frac{1}{L}\sum_{l=1}^{L}\mathbb{E}\left[f\left(Z_l\right)\right] && \text{lin. of exp.}\\
&= \frac{1}{L}\sum_{l=1}^{L}\mathbb{E}\left[f(Z)\right] && \left(Z_l\right)_{1\leqslant l\leqslant L} \text{ identically dist.}\\
&= \frac{1}{L}\cdot L\cdot\mathbb{E}\left[f(Z)\right]\\
&= \mathbb{E}\left[f(Z)\right]
\end{aligned}
$$

14.4

$$
\begin{aligned}
F(y) &:= \int_{-\infty}^{y}\frac{1}{\pi}\frac{1}{1+\hat{y}^2}d\hat{y}\\
&= \frac{1}{\pi}\left[\arctan y\right]_{-\infty}^{y}\\
&= \frac{1}{\pi}\left(\arctan y-\left(-\frac{\pi}{2}\right)\right)\\
&= \frac{\arctan y}{\pi}+\frac{1}{2}
\end{aligned}
$$

with

$$
F^{-1}(z) = \tan\left(\pi\left(z-0.5\right)\right) : (0,1) \longrightarrow \mathbb{R}
$$

So $Y := F^{-1}(Z) = \tan(\pi(Z-0.5)) \sim \text{Cauchy}(0,1)$.