My solutions to

# Deep Learning: Foundations and Concepts

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## 4 Single-Layer Networks: Regression

### 4.3

$$
\begin{aligned}
2\sigma(2a) - 1 &= 2\frac{1}{1+e^{-2a}} - 1 \\
&= \frac{2}{1+e^{-2a}} - \frac{1+e^{-2a}}{1+e^{-2a}} \\
&= \frac{2-(1+e^{-2a})}{1+e^{-2a}} \\
&= \frac{1-e^{-2a}}{1+e^{-2a}} \\
&= \frac{e^a\left(1-e^{-2a}\right)}{e^a\left(1+e^{-2a}\right)} \\
&= \frac{e^a - e^{-2a+a}}{e^a + e^{-2a+a}} \\
&= \frac{e^a - e^{-a}}{e^a + e^{-a}} \\
&= \tanh(a)
\end{aligned}
$$

$$
\begin{aligned}
\implies 2\sigma(2a) &= \tanh(a) + 1 \\
\Leftrightarrow \sigma(2a) &= \frac{1}{2}\tanh(a) + \frac{1}{2} \\
\Leftrightarrow \sigma(a) &= \frac{1}{2}\tanh\left(\frac{a}{2}\right) + \frac{1}{2}
\end{aligned}
$$

$$
\implies y(x,w) = w_0 + \sum_{j=1}^{M} w_j \sigma\left(\frac{x-\mu_j}{s}\right)
$$

$$
\begin{aligned}
&= w_0 + \sum_{j=1}^{M} w_j \left( \frac{1}{2} \tanh\left( \frac{x - \mu_j}{2s} \right) + \frac{1}{2} \right) \\
&= \underbrace{w_0 + \sum_{j=1}^{M} \frac{w_j}{2}}_{:= u_0} + \sum_{j=1}^{M} \underbrace{\frac{w_j}{2}}_{:= u_j} \tanh\left( \frac{x - \mu_j}{2s} \right)
\end{aligned}
$$