My solutions to

Deep Learning: Foundations and Concepts

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3 Standard Distributions

3.1

$$\begin{aligned}\sum p(x|\mu) &= \sum_{x\in\{0,1\}} \mu^x(1-\mu)^{1-x} \\ &= \mu^0(1-\mu)^{(1-0)} + \mu^1(1-\mu)^{1-1} \\ &= 1\cdot(1-\mu) + \mu\cdot 1 \\ &= 1-\mu+\mu \\ &= 1\end{aligned}$$

$$\begin{aligned}\mathbb{E}[x] &= \sum_{x\in\{0,1\}} x\mu^x(1-\mu)^{1-x} \\ &= 0 + 1\cdot\mu^1(1-\mu)^{1-1} \\ &= \mu\end{aligned}$$

$$\begin{aligned}\mathbb{V}[x] &= \mathbb{E}\left[x^2\right] - \mathbb{E}[x]^2 \\ &= \sum_{x\in\{0,1\}} x^2\mu^x(1-\mu)^{1-x} - \mu^2 \\ &= 0 + 1^2\cdot\mu^1(1-\mu)^{1-1} - \mu^2 \\ &= \mu - \mu^2 \\ &= \mu(1-\mu)\end{aligned}$$

$$\begin{aligned}H[x] &= \mathbb{E}\left[-\log_2 p\right] \\ &= \sum_{x\in\{0,1\}} -p(x)\log_2 p(x)\end{aligned}$$

$$
\begin{aligned}
&= \sum_{x\in\{0,1\}} -\mu^x(1-\mu)^{1-x}\log_2\left(\mu^x(1-\mu)^{1-x}\right) \\
&= \sum_{x\in\{0,1\}} -\mu^x(1-\mu)^{1-x}\left(x\log_2\mu + (1-x)\log_2(1-\mu)\right) \\
&= -\mu^0(1-\mu)^{1-0}\left(0\cdot\log_2\mu + (1-0)\log_2(1-\mu)\right) \\
&\quad - \mu^1(1-\mu)^{1-1}\left(1\cdot\log_2\mu + (1-1)\log_2(1-\mu)\right) \\
&= (1-\mu)\log_2(1-\mu) - \mu\log_2\mu
\end{aligned}
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