Laboratory Session : April 16, 2023

Exercises due on : April 30, 2023

Exercise 1 - Discrete random variable

- the probability distribution function of a discrete variable k is given by the *zero-truncated* Poisson distribution:

$$P(k) = \frac{\lambda^k e^{-\lambda}}{k!\,(1 - e^{-\lambda})} \qquad \text{for} \quad k = 1, 2, 3, \ldots$$

1) Write the R functions for the probability density and cumulative distribution functions, using the R naming convention.

- Assuming $\lambda = \mathbf{1.4}$,

2) Produce two plots showing the pdf and cdf, separately.

3) Compute the mean value and variance of the probability distribution using R.

4) Generate a sample of random numbers from this distribution and show them in an histogram. Evaluate the sample mean.

Exercise 2 - Continuous random variable

- The energy distribution of CR muons at sea level can be approximated as follows

$$p(E) = N \begin{cases} 1 & \text{for } E < E_0 \\ (E - E_0 + 1)^{-\gamma} & \text{for } E \geq E_0 \end{cases} \tag{1}$$

where $\mathrm{E}_0 = 7.25$ GeV and $\gamma = 2.7$.

a) Compute the normalisation factor N using R.

b) Plot the probability density function in R.

c) Plot the cumulative density function in R.

d) Compute the mean value using R

e) [`Optional`] Generate 10^6 random numbers from this distribution, show them in an histogram and superimpose the pdf (with a line or with a sufficient number of points).

Exercise 3

- Suppose that the average number of accidents at an intersection is two per day.

a) Using Markov's inequality, find a bound for the probability that at least five accidents will occur tomorrow.

b) Using Poisson random variables, calculate the probability that at least five accidents will occur tomorrow. Compare this value with the bound obtained in the previous point a).

c) Let the variance of the number of accidents be two per day. Using Chebyshev's inequality, find a bound on the probability that tomorrow at least five accidents will occur.

Exercise 4

The waiting period from the time a book is ordered until it is received is a random variable with mean seven days and standard deviation two days. If Helen wants to be 95% sure that she receives a book by certain date, how early should she order the book?

Exercise 5

An ordinary deck of 52 cards is divided randomly into 26 pairs. Using Chebyshev's inequality, find an upper bound for the probability that, at most, 10 pairs consist of a black and a red card.

Exercise 6

- In a stationary bus at the departure station, a passenger gets on the bus, on average every 30 seconds.

a) Compute the probability of getting more than 6 passenger after 2 minutes. Evaluate the probability of having less than 4 passenger after 3 minutes.

b) Simulate the distribution of the arrival time of the third passenger and superimpose the corresponding pdf.

c) Repeat the procedure of the point b) for the difference in arrival time between the fifth and the first passenger.