

Mathematics for Machine Learning

Homework III

Due 11/11/2017

1. If a subspace S is contained in a subspace V , prove that S^\perp contains V^\perp .
2. Find $A^T A$ if the columns of A are unit vectors, all mutually perpendicular.
3. Suppose I give you four nonzero vectors $\mathbf{r}, \mathbf{n}, \mathbf{c}, \mathbf{l}$ in \mathbb{R}^2
 - (a) What are the conditions for those to be bases for the four fundamental sub-spaces

$$C(A^T), N(A), C(A), N(A^T)$$

of a 2 by 2 matrix.

- Take a look at the Strang for the definitions.

- (b) What is the possible matrix A ?
4. The points $P = (x, x, x)$ and $Q = (y, 3y, -1)$ are on two lines in space that do not meet. Choose x and y to minimize the squared distance $\|P - Q\|^2$
 - (a) What is the perpendicular line connecting the closest P and Q ?
5. Given the data set provided implement the Least Squared Error discussed in class and answer the following questions:
 - (a) What is the probability of error and the probability of correct classification by simply using the rules $\mathbf{w}^T \mathbf{x} > 0$ and $\mathbf{w}^T \mathbf{x} < 0$.
 - (b) Given that $A\mathbf{x} = \mathbf{b}$ —(noise e with mean zero). Get the error vectors and multiply by $(A^T A)^{-1} A^T$ to get $\hat{\mathbf{x}} - \mathbf{x}$, basically the difference between the estimation of the average error (Where $\hat{\mathbf{x}}$ is unbiased).
 - i. What is the error?
 - ii. Define what is an unbiased error.