Mathematics for Machine Learning

Homework I

Due 11/04/2017

1. Suppose the coefficient matrix of a linear system of three equations in three variables has a pivot position in each column. Explain why the system has a unique solution.

2. The solutions (x, y) of a single linear equation $ax + by = c$ form a plane in $\mathbb{R}^2$ when a, b are not all zero. Program in Jupyter Notebook a program that plots a set of three linear equations whose graphs

   (a) Intersect in a single point.

   (b) They do no have intersection.

3. Determine h and k such that the solution set of the system (i) is empty, (ii) contains a unique solution, and (iii) contains infinitely many solutions.

   (a)
   $$\begin{aligned} x_1 + 3x_2 &= k \\ 4x_1 + hx_2 &= 8 \end{aligned}$$

   (b)
   $$\begin{aligned} -2x_1 + hx_2 &= 1 \\ 6x_1 + kx_2 &= 8 \end{aligned}$$

4. Consider the problem of determining whether the following system of equations is consistent for all b_1, b_2, b_3:

   $$\begin{aligned} w_{11}x_1 + w_{12}x_2 + w_{13}x_3 &= b_1 \\ w_{21}x_1 + w_{22}x_2 + w_{23}x_3 &= b_2 \\ w_{31}x_1 + w_{32}x_2 + w_{33}x_3 &= b_3 \end{aligned}$$

   (a) Define appropriate vectors and matrices, and restate the problem in terms of an augmented matrix.

   (b) Implement the Gauss-Jordan Algorithm in Jupyter Notebook for the problem.

5. Explain why a set $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3, \boldsymbol{v}_4\}$ in $\mathbb{R}^5$ must be linearly independent when $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ is linearly independent and $\boldsymbol{v}_4$ is not in $Span\,\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$.

6. Use the Gauss-Jordan algorithm to solve the inhomogeneous system for the linear regression presented at the class:

   (a) Generate a convenient set of linear related points in $\mathbb{R}^2$.

   (b) Use Gauss-Jordan to find variables a and b.