Mathematics for Machine Learning

Homework IV

Due 11/11/2017

1. If $A_n \subset A_{n-1} \subset ... \subset A_1$, show that $\cap_{i=1}^{n} A_i = A_n$, $\cup_{i=1}^{n} A_i = A_1$

2. If X is the number of successes in n Bernoulli trials, find the probability that $X \geq 3$ and $X \geq 1$.

3. Let C_1 be an unbiased coin, and C_2 a biased coin with probability of heads $\frac{3}{4}$. At time $t = 0$, C_1 is tossed. If the result is heads, then C_1 is tossed at time $t = 1$. If the result is tails, C_2 is tossed at $t = 1$. Then, the process is repeated at time $t = 2, 3, ...$. In general, if heads appears at $t = n$, then C_1 is tossed at $t = n + 1$. If tails appears at $t = n$, then C_2 is tossed at $t = n + 1$.

   (a) Find y_n= the probability that the toss at $t = n$ will be head. Hint set a difference equation:

   $$y_n = a_1 y_{t-1} + ... + a_n y_{t-n}$$

4. Consider a sequence of Bernoulli Trials. Let R be the number of times that a head is followed immediately by a tail. For example, if $\omega = HHTHT$, then $R(\omega) = 2$ since a head is followed by a tail at trials 2 and 3, and also at trials 4 and 5. Find the probability function of R.

5. An employer is about to hire one new employee from a group of N candidates, whose future potential can be rated on a scale form 1 to N. the employer proceeds using the following rules:

   (a) Each candidate is seen in succession and a decision is made whether to hire the candidate.

   (b) Having rejected $m-1$ candidates ($m > 1$), the employer can hire the m^{th} only if the m^{th} candidate is better than the previous $m - 1$.

   Suppose a candidate is hired on thei^{th} trial. What is the probability that the best candidate was hired?