

Suppose you have already administered Y_1, Y_2, \dots, Y_{j-1} to a student. You want to select the j th item to minimize the expected Shannon entropy $ESH\left(\pi_{\{1, \dots, j-1, Y_j\}}\right)$. The expected Shannon entropy for dichotomous items is calculated by the following steps:

$\pi_n(\theta|Y_1, Y_2, \dots, Y_{j-1}) = \frac{P(Y_1, Y_2, \dots, Y_{j-1}|\theta)\pi_0(\theta)}{\int P(Y_1, Y_2, \dots, Y_{j-1}|\theta)\pi_0(\theta)d\theta}$, where $\pi_0(\theta)$ is the prior of theta distribution you desire.

$$\pi_{\{1, \dots, j-1, Y_j=1\}}(\theta) = \frac{P(Y_j = 1|\theta)\pi_n(\theta|Y_1, Y_2, \dots, Y_{j-1})}{\int P(Y_j = 1|\theta)\pi_n(\theta|Y_1, Y_2, \dots, Y_{j-1})d\theta}$$

$$P(Y_j = 1|Y_1, Y_2, \dots, Y_{j-1}) = \int P(Y_j = 1|\theta)\pi_n(\theta|Y_1, Y_2, \dots, Y_{j-1})d\theta$$

$$\pi_{\{1, \dots, j-1, Y_j=0\}}(\theta) = \frac{P(Y_j = 0|\theta)\pi_n(\theta|Y_1, Y_2, \dots, Y_{j-1})}{\int P(Y_j = 0|\theta)\pi_n(\theta|Y_1, Y_2, \dots, Y_{j-1})d\theta}$$

$$P(Y_j = 0|Y_1, Y_2, \dots, Y_{j-1}) = \int P(Y_j = 0|\theta)\pi_n(\theta|Y_1, Y_2, \dots, Y_{j-1})d\theta$$

$$\begin{aligned} ESH\left(\pi_{\{1, \dots, j-1, Y_j\}}\right) &= \int -\log\left(\pi_{\{1, \dots, j-1, Y_j=1\}}(\theta)\right) * \pi_{\{1, \dots, j-1, Y_j=1\}}(\theta) * P(Y_j = 1|Y_1, Y_2, \dots, Y_{j-1}) d\theta \\ &+ \int -\log\left(\pi_{\{1, \dots, j-1, Y_j=0\}}(\theta)\right) * \pi_{\{1, \dots, j-1, Y_j=0\}}(\theta) * P(Y_j = 0|Y_1, Y_2, \dots, Y_{j-1}) d\theta \end{aligned}$$