Spontaneous Scalarization of Black Holes in Scalar-Gauss-Bonnet Gravity and Stability Analysis under radial perturbations

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Introduction

Einstein's General Relativity (GR) has revolutionized our understanding of gravity, marking significant advancements from Newtonian gravitational framework and leading to groundbreaking discoveries in astrophysics and cosmology. Despite its remarkable success in explaining gravitational phenomena across a wide range of scales, GR encounters issues when confronted with the strong-field regimes, near compact objects like Black Holes (BH) and Neutron Stars (NS). These environments, characterized by high-curvature spacetime regions, demand a deeper understanding of gravity beyond the GR paradigm.

This work deals with scalar-Gauss-Bonnet gravity (sGB), a modified theory that extends GR by incorporating quadratic curvature terms into the Einstein-Hilbert action. These corrections are a promising example of extension of General Relativity in the strongfield/large-curvature regime [3]. They naturally arise in an effective field-theory framework, and also from compactifications of String-Theory solutions. Moreover, in these theories the no-hair theorems do not apply: black holes are different from those of GR. Specifically, for sGB gravity, depending on the coupling function to the Gauss-Bonnet invariant (which contains the quadratic corrections), the solutions of the equations of motion can exhibit different behaviours and eventually admit BHs with non-trivial scalar hair. Furthermore, it is possible to maintain hairy BHs and GR solutions simultaneously through a mechanism known as Spontaneous BH Scalarization.

This thesis aims to address two issues which arise in this framework. The first one is that lots of sGB theories have a minimum mass for scalarized solutions whose limit is caused by regularity conditions on the horizon; the other one is that physical solutions should be stable under radial perturbations. Through numerical investigation the objective is to identify coupling functions capable of inducing spontaneous scalarization while ensuring stability against radial perturbations down to infinitesimal black hole masses, or, alternatively, to extend the ranges of mass in which these features are satisfied. Furthermore, the study will explore Z2 -symmetry-violating coupling functions, acknowledging that adherence to this symmetry is not necessary unless following an Effective Field Theory approach.

Thesis work

Firstly a brief theoretical introduction to sGB gravity is provided, followed by a description of the numerical methods used to solve the field equations. The results of the numerical simulations are then presented, and finally the conclusions are drawn.

Spontaneous Scalarization

Differentiatig the sGB action with respect to the metric and to the scalar field yields the field equations [12]. These equations represent a generalization of the Einstein equations, incorporating an additional term in the stress-energy tensor due to the scalar field, along with a modification of the Klein-Gordon equation featuring the appearance of the Gauss-Bonnet invariant ($\mathscr{G}$). It is straightforward that assuming a coupling function $f(\phi)$ whose first derivative with respect to ϕ vanishes at a certain finite value of the scalar field, it is possible to derive GR equations. Indeed, the equations depend solely on $f'(\phi)$. Consequently, it may be necessary to consider $\mathbf{Z}_2$-symmetric coupling functions, thereby eliminating terms like $\phi\mathscr{G}$. In fact, while such couplings do yield solutions characterized by scalar hair, they necessarily exclude those consistent with GR [8]. Last condition that the coupling function must satisfy is that the product $f''(\phi)\mathscr{G}$ should be positive to evade no hair theorems, as shown in [11] and [1].

Quadratic coupling function meets all the requirements for spontaneous scalarization and it has been widely studied in literature [11], [5]. Therefore for this work we initially focus on this coupling type to refine our numerical methods drawing insights from existing research findings, being ready to move to more complex coupling functions lately. Every numerical analysis has been computed through WOLFRAM MATHEMATICA notebooks.

We operate within spherical coordinates and assume a spherically symmetric spacetime. The field equations, sourced from [9], are solved for both the metric functions and the scalar field.

To integrate the field equations it is necessary to impose boundary conditions at the event horizon r_h and at spatial infinity. The former are obtained by expanding the equations in terms of the radial coordinate around the horizon. The integration is computed from $r_h + \varepsilon$ up to a suitable finite value for infinity r_{inf}. Both ε and r_{inf} have been adjusted to ensure that they are sufficiently small/large, respectively, while checking for any changes in solutions depending on them. The boundary solutions at infinity are matched subsequently. The procedure is the following. We expand the equations in terms of the inverse of radial coordinate [7]. Subsequently, we determine the values of the black hole mass, scalar charge, and the scalar field at infinity ϕ_{inf} as integration constants by substituting the integration solutions into the expansion. For this purpose, we use the results of the integrations with only boundary consitions at horizon. Then, we examine the value of ϕ_{inf}. Given that tests in low-field regimes lean heavily towards General Relativity, it is preferable to have solutions where the scalar field approaches zero asymptotically. Therefore, employing a shooting method, for a fixed value of the coupling constant, we perform integration with various initial values of the scalar field at the horizon. Ultimately, we classify a solution as scalarized when ϕ_{inf} is null.

The shooting method cannot be performed for each value of the scalar field at the horizon, though. Indeed, to guarantee regularity of ϕ towards this region, it follows from the expansion of the equations of motion that the coupling function must satisfy a condition that imposes restrictions on the choice of boundary values [11]. For clarity, towards horizon the condition is $r_h^4 \geq 96(f'(\phi_{0,h}))^2$. This leads to a minimum mass for scalarized solutions.

Using this algorithm, we've identified the range of coupling constants wherein scalarized solutions exist, revealing a specific mass value at which the Schwarzschild solution starts coexisting with the scalarized one. We refer to this critical mass value as the "*bifurcation point*". Multiple bifurcation points may exist, depending on the number of nodes present in the scalar field function, but our focus lies solely on the fundamental branch, namely, the zero-node branch.

Stability Analysis

The stability of the solutions is assessed by studying the behaviour of the scalar field under radial perturbations. As shown in [9], we add radial perturbations to the metric and to the scalar field and expanding the equations of motion up to first order yields to the perturbed equations. Manipulating these equations allows us to derive a single differential equation for the perturbation of the scalar field, denoted as $\varphi_1(t,r)$. Notably, all coefficients in this equation solely depend on the unperturbed solutions. Thus, it is imperative to first integrate the background field equations and subsequently solve the perturbed equation. In order to study the mode stability of the background configuration, we decompose the perturbation as $\varphi_1(t,r) = \varphi(r)e^{-\omega t}$, so time dependence can be simplified.

Through a variable substitution, adopting *tortoise coordinates*, the equation for φ_1 can be cast into a Schrödinger-like form, as detailed in [4]. This also allows to highlight the effective potential to which the perturbation is subjected. The potential vanishes at both boundaries. To integrate we use then a Quasi-Normal-Mode approach, searching for solutions that are outgoing towards infinity and ingoing towards the event horizon. By following this assumptions, we derive the boundary conditions: the perturbation function must vanish towards both limits and its derivative should be constant.

Numerically, we conducted two different integrations of the perturbed equation: one originating from the horizon and the other from infinity, with both terminating at an intermediate distance x_m [4], [10]. The frequency of the waveform was specified as an external parameter. To assess stability, we exclusively examine purely imaginary positive frequencies (as in [10]). For the boundary conditions at the event horizon, owing to numerical challenges, we initially utilized the Schwarzschild tortoise coordinate to approximate the value of φ and its derivative at a finite distance sufficiently close to the horizon. The infinity limit has been determined in the same way as in the integration for the background.

After computing the two solutions, we verify their linear dependence by evaluating the Wronskian at the intermediate point x_m: a Wronskian value of zero indicates linear dependence between the solutions [2]. Notably, when employing tortoise coordinates, this result remains independent of the choice of x_m.

The same approach has been used to study the stability of Scwarzschild background solutions.

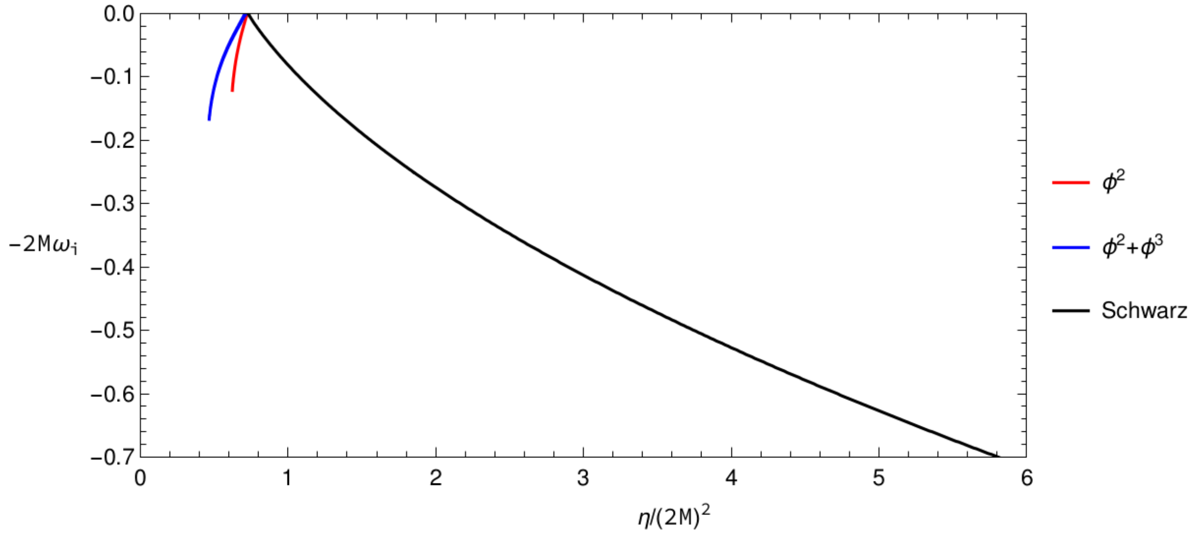

Figure 1: Unstable frequencies as function of the coupling constants η for quadratic coupling function and $\frac{\eta}{4}\left(\frac{1}{2}\phi^2+\frac{1}{3}\right)$. We appreciate how unstable Schwarzschild branch begins from the bifurcation point towards greater values of the coupling constant (lower masses).

For quadratic coupling function we derive the result that scalarized branches are fully unstable and from the bifurcation point onwards, also Schwarzschild soltion is (see Fig. 1 and 3). This suggests that we should extend the analysis to different coupling functions.

Results

Quartic coupling function As previously explored in [10], we introduce a quartic term to the quadratic coupling function. Our investigation reveals that certain combinations of the coupling constants give rise to stable scalarized solutions within the region where the GR solution is unstable (Fig. 3). However, there still exists a minimum mass limit, beyond which only the Schwarzschild solution persists, albeit in an unstable state.

Exponential coupling function In [6] it is demonstrated that a coupling function of the form $f(\phi)=\frac{\eta}{8\beta}\left(1-e^{-\beta\phi^2}\right)$ can lead to scalarized black holes branches with no minimum mass limit if $\beta>\beta_{crit}\simeq 1.165625$[1]. The study presented in [4] illustrates a particular case of this phenomenon. We confirm these findings and additionally investigate the stability of the solutions under radial perturbations. Notably, above β_{crit}, as the β parameter increases, the range of masses of stable scalarized solutions expands, even if the scalar charge gradually decreases. See Fig. 2 for the results. The relationship between β and the mass below which

[1]There is a factor 2 of difference between our formalism and the one used in [6]

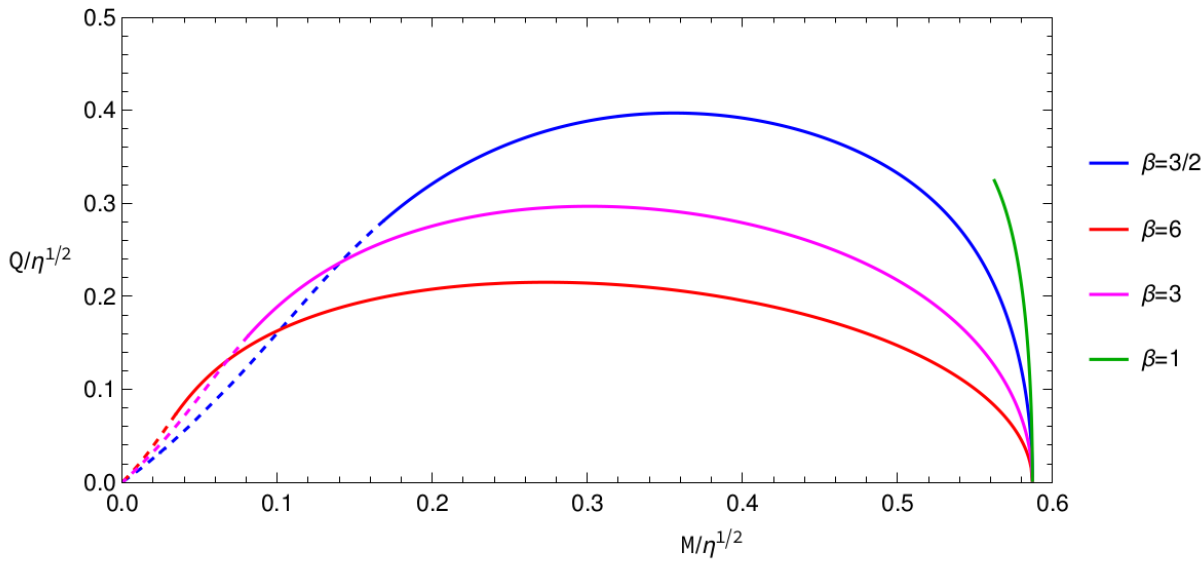

Figure 2: Scalar charge as a function of the black hole mass for exponential coupling functions. Dashed lines are used to indicate the unstable solutions, while straight lines represent stable ones. $\beta = 1$ is below the critical value β_{crit} and thus the shooting methods stops at finite mass.

instability arises appears to be linear and merits further investigation in the subsequent sections of my thesis.

Z_2-symmetry-violating coupling function We propose that coupling functions symmetric under $\phi \to -\phi$ transformations are required when considering an EFT approach, incorporating the exclusion of the linear term within a symmetry of the theory. However this assumption is not necessary for the onset of scalarization. The only mandatory feature is that the first derivative of $f(\phi)$ must vanish for some finite value of ϕ. Consequently, we analyzed a function of the type $f(\phi) = \frac{\eta}{8}\phi^2 + \frac{\lambda}{12}\phi^3$. This shows a behaviour similar to the quartic coupling function, as shown in Fig. 3. Spontaneous scalarization does emerge as expected and for negative λ values scalarized solutions are also stable.

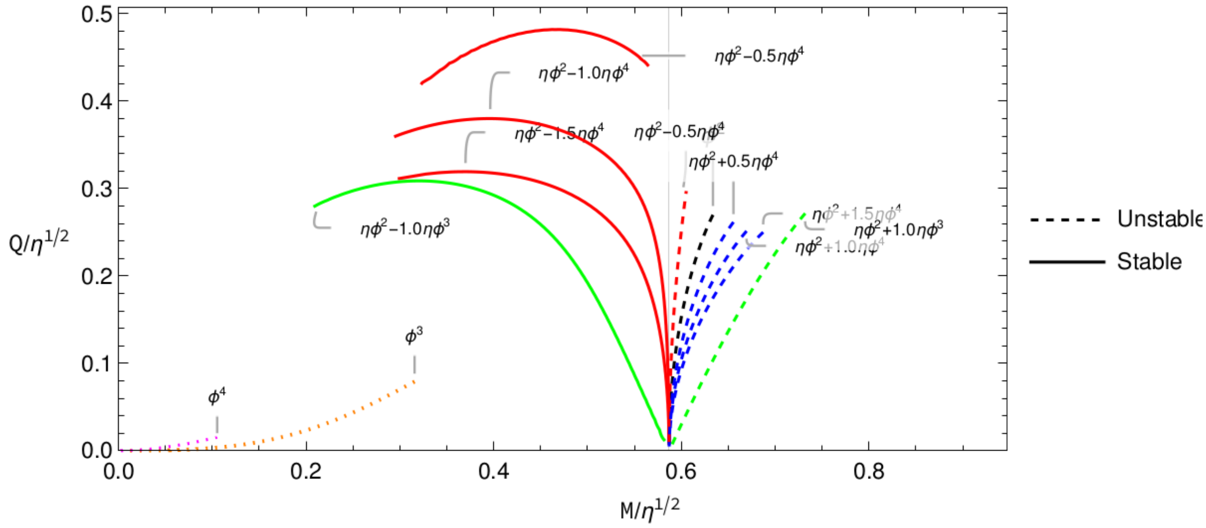

Figure 3: Scalar charge as a function of the black hole mass for different values of the coupling constant λ, for polynomial coupling functions. The bifurcation point is marked with a vertical dashed line in gray. Dashed lines are used to indicate the unstable solutions, while straight lines represent stable ones. Green branches are for $\mathbf{Z}_2$-symmetry violating coupling function. Quartic couplings $\eta\phi^2 + \lambda\phi^4$ show different phenomenology depending on the sign of the cooupling constant λ. Thus, in red we reported couplings with $\lambda < 0$ and in blue the $\lambda > 0$ ones. Lastly, black, orange and magenta dashed lines represent respectively purely quadratic, cubic and quartic couplings.

Conclusions

This research contributes to the investigation surrounding modified theories of gravity, focusing on delving deeper into one of the most promising among them, sGB gravity. It aims to advance the exploration of physical alternative solutions to those proposed by GR, and introduces original aspects that have not been previously considered in the existing literature, thereby providing justification for the assumptions made so far.

Specifically, we have shown that different kind of coupling functions can lead to stable scalarized solutions, even in cases where they do not observe $\mathbf{Z}_2$ symmetry.
Minimum mass limit for scalarized solutions appears to disappear for some coupling functions and it may be necessary to admit all terms of ϕ in the expansion of $f(\phi)$ around null scalar field to exhibit this feature. Our findings align with previous research, such as that presented in [10], which illustrated that the presence of at least two terms in the expansion can ensure a stable scalarized branch.
Furthermore, we highlight how stability can be achieved for small masses. This is a favorable outcome for the theory. In fact, within this framework, GR solutions are still valid, consistently with current observations, but it also sets the stage for future investigations into

stronger-field regimes. Here, deviations from GR expectations may offer opportunities for spontaneous scalarization to emerge as a viable alternative worth to be taken into account.

References

[1] G Antoniou, A Bakopoulos, and P Kanti. "Evasion of no-hair theorems and novel black-hole solutions in Gauss-Bonnet theories". In: *Physical review letters* 120.13 (2018), p. 131102.

[2] Georgios Antoniou, Caio FB Macedo, Ryan McManus, and Thomas P Sotiriou. "Stable spontaneously-scalarized black holes in generalized scalar-tensor theories". In: *Physical Review D* 106.2 (2022), p. 024029.

[3] Emanuele Berti et al. "Testing general relativity with present and future astrophysical observations". In: *Classical and Quantum Gravity* 32.24 (2015), p. 243001.

[4] Jose Luis Blázquez-Salcedo, Daniela D Doneva, Jutta Kunz, and Stoytcho S Yazadjiev. "Radial perturbations of the scalarized Einstein-Gauss-Bonnet black holes". In: *Physical Review D* 98.8 (2018), p. 084011.

[5] Daniela D Doneva and Stoytcho S Yazadjiev. "New Gauss-Bonnet black holes with curvature-induced scalarization in extended scalar-tensor theories". In: *Physical review letters* 120.13 (2018), p. 131103.

[6] Pedro GS Fernandes, David J Mulryne, and Jorge FM Delgado. "Exploring the small mass limit of stationary black holes in theories with Gauss–Bonnet terms". In: *Classical and Quantum Gravity* 39.23 (2022), p. 235015.

[7] Panagiota Kanti, NE Mavromatos, J Rizos, Kyriakos Tamvakis, and E Winstanley. "Dilatonic black holes in higher curvature string gravity. II. Linear stability". In: *Physical Review D* 57.10 (1998), p. 6255.

[8] Panagiota Kanti, Nick E Mavromatos, John Rizos, Kyriakos Tamvakis, and Elizabeth Winstanley. "Dilatonic black holes in higher curvature string gravity". In: *Physical Review D* 54.8 (1996), p. 5049.

[9] Caio FB Macedo, Jeremy Sakstein, Emanuele Berti, Leonardo Gualtieri, Hector O Silva, and Thomas P Sotiriou. "Self-interactions and spontaneous black hole scalarization". In: *Physical Review D* 99.10 (2019), p. 104041.

[10] Hector O Silva, Caio FB Macedo, Thomas P Sotiriou, Leonardo Gualtieri, Jeremy Sakstein, and Emanuele Berti. "Stability of scalarized black hole solutions in scalar-Gauss-Bonnet gravity". In: *Physical Review D* 99.6 (2019), p. 064011.

[11] Hector O Silva, Jeremy Sakstein, Leonardo Gualtieri, Thomas P Sotiriou, and Emanuele Berti. "Spontaneous scalarization of black holes and compact stars from a Gauss-Bonnet coupling". In: *Physical review letters* 120.13 (2018), p. 131104.

[12] Thomas P Sotiriou and Shuang-Yong Zhou. "Black hole hair in generalized scalar-tensor gravity: An explicit example". In: *Physical Review D* 90.12 (2014), p. 124063.