Black Holes in modified gravity theories

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Introduction

Einstein's General Relativity (GR) has revolutionized our understanding of gravity, marking significant advancements from Newtonian gravitational framework and leading to groundbreaking discoveries in astrophysics and cosmology. However, it has mostly been tested in its weak field regime. With the birth of gravitational wave (GW) astronomy we have the opportunity to study the gravitational interaction in its strong-field regime. In order to use observations by present and future GW detectors like LIGO, Virgo, LISA and the Einstein Telescope, to test general relativity in the strong-field regime, going beyond null-tests, we need to know which deviations we may expect. In particular, since most of the observed signal are produced by binary black hole coalescence processes, we need to study how these sources behave when general relativity is modified.
In addition, GR encounters issues when confronted with the strong-field scenarios, which further motivates the investigation of modifications to the theory.

This work deals with scalar-Gauss-Bonnet gravity (sGB). This is a modification of General Relativity that involves the inclusion within the Einstein-Hilbert action of quadratic curvature terms, specifically the Gauss-Bonnet invariant, coupled to a scalar field. These additions represent arguably the simplest extension of GR in the large-curvature regime [2] in which BH solutions are modified. Indeed, within these theories, no-hair theorems that characterize GR black holes do not apply[1]. Specifically, for sGB gravity, depending on the coupling function to the Gauss-Bonnet invariant, the solutions to the equations of motion can eventually admit BHs with non-trivial scalar hair [5], [1]. Furthermore, it is possible to maintain both hairy BHs and GR solutions simultaneously, through a mechanism known as *Spontaneous BH Scalarization*.

This thesis aims to address two issues that arise in this framework. The first one is that typically spontaneously scalarized sGB black holes have a minimum mass, a feature that is still not fully understood. The other one is that physical solutions should be stable under radial perturbations. Through numerical investigation, the objective is to identify coupling functions capable of inducing spontaneous scalarization while ensuring stability against radial perturbations down to infinitesimal black hole masses, or, alternatively, to extend the range of mass in which these features are satisfied. Furthermore, the study will explore $\mathbb{Z}_2$-symmetry-violating coupling functions[2], acknowledging that adherence to this symmetry, which is usually considered in the existing literature, is not mandatory for inducing spontaneous scalarization, unless following an Effective Field Theory (EFT) approach.

[1] No-hair theorems state that GR black hole solutions manifest only three independent degrees of freedom, namely their mass, angular momentum and electric charge. These are sufficient to describe those kind of solutions completely.

[2] These include all coupling functions that are not necessarily symmetric under the transformation $\phi \to -\phi$, where ϕ represents the scalar field.

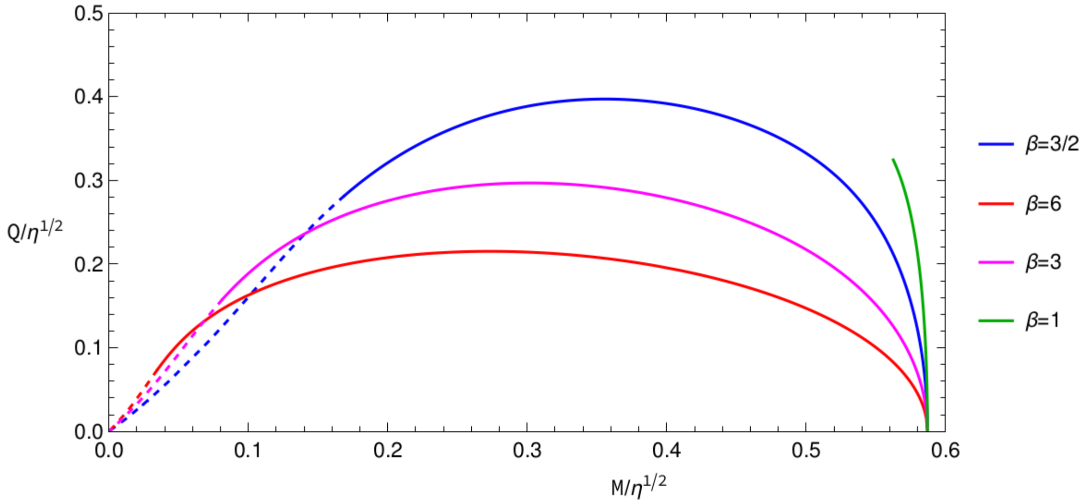

Figure 1: Dimensionless Scalar charge $Q/\eta^{1/2}$ as a function of the normalized black hole mass $\hat{M} = M/\eta^{1/2}$ for exponential coupling functions. η represents the coupling constant and has dimension of a mass squared. Dashed lines are used to indicate the unstable solutions, while straight lines refer to stable ones. $\beta = 1$ is below the critical value β_{crit} and thus its branch breaks down at a finite mass value, due to restrictions coming from the regularity condition for the scalar field at the horizon.

Thesis Results

Exponential coupling function In [4] it is demonstrated that a coupling function of the form $f(\phi) = \frac{\eta}{8\beta}\left(1 - e^{-\beta\phi^2}\right)$ can lead to scalarized BH[3] branches with no minimum mass limit if $\beta > \beta_{crit} \simeq 1.165625$[4]. However, no stability analysis has been provided for general value of $\beta > \beta_{crit}$, therefore we investigate this affair in this research. Notably, we find that as the β parameter increases, the range of masses of stable scalarized solutions expands: see Fig. 1 for the results.

In order to obtain a relation between the threshold value of mass below which scalarized solutions are unstable and the corresponding β value of the coupling, we proceed analyzing the effective potential felt by the perturbation of the scalar field. We saw that below a certain mass $\hat{M}_{thr}$[5] the effective potential shows singularities near the event horizon. Actually, the whole theory is no more hyperbolic for $\hat{M} < \hat{M}_{thr}$. In Fig. 2 we show the dependence of $\hat{M}$ with respect to β. We perform a best fit of data and obtain that the behavior is well described by a function $f(\beta) = a + b/\beta$ with a compatible with 0.

[3] Scalarized BH is defined as black hole solution that admits a non-trivial scalar field $\phi(r)$ which vanishes for $r \to \infty$.

[4] There is a factor 2 of difference between our formalism and the one used in [4]

[5] $\hat{M} = M/\eta^{1/2}$ is the dimensionless mass used for the analysis.

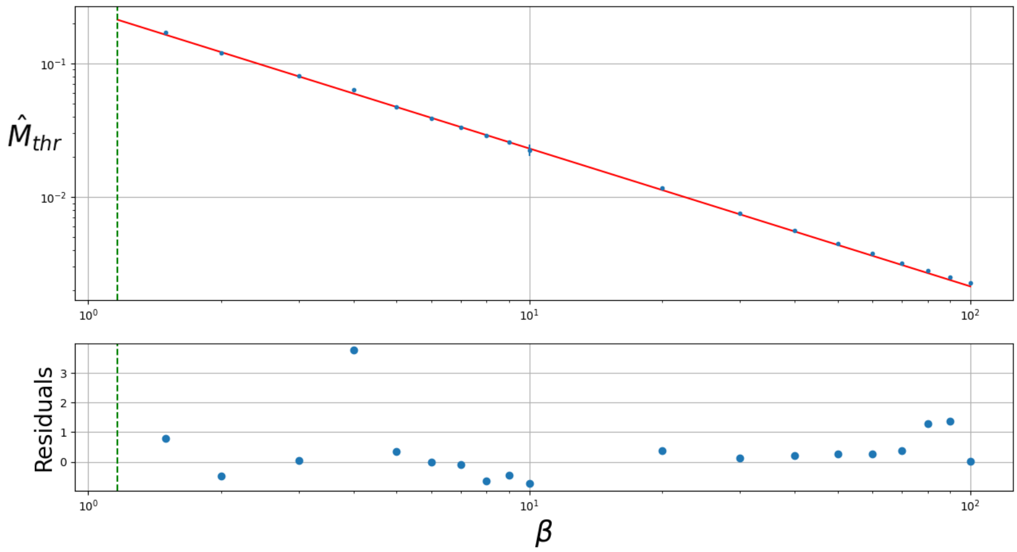

Figure 2: Mass threshold for hyperbolicity $\hat{M}_{thr}$ as a function of the β parameter for exponential coupling functions. The best-fit line in red is obtained using function $a + b/\beta$.

$\mathbb{Z}_2$**-symmetry-violating coupling function** In literature coupling functions symmetric under $\phi \to -\phi$ transformations are usually considered. However, this assumption is not necessary for the onset of scalarization. Consequently, we analyze a function of the type $f(\phi) = \frac{\eta}{8}\phi^2 + \frac{\lambda}{12}\phi^3$. The outcome shows a behavior similar to the quartic coupling function[6], as shown in Fig. 3. Spontaneous scalarization does emerge, as expected, and for sufficiently negative λ values scalarized solutions are also stable, namely $\lambda \lesssim -0.7$.

In Fig. 3 are also illustrated the scalarized solutions of coupling functions that do not include the quadratic term in their expressions, namely $f(\phi) = \frac{\eta}{16}\phi^4$ and $f(\phi) = \frac{\eta}{12}\phi^3$. Both branches are completely unstable, but in these cases Schwarzschild solutions remain stable for the whole range of possible masses $\hat{M}$.

Lorentzian coupling function Lastly, we try to identify all the features that a coupling function must satisfy to provide a *complete scalarized branch*, i.e. non-trivial scalar hairy solutions for the whole range of mass $\hat{M}$, from the bifurcation point[6] to $\hat{M} = 0$.

Lorentzian function $f(\phi) = \frac{\eta}{8}\beta^2\left(1 - \frac{\beta^2}{\phi^2+\beta^2}\right)$ is a good candidate.

As illustrated in Fig. 4, changing the value of β it is possible to enlarge the range of mass for stable scalarized branches. Nevertheless, even if infinitesimal $\hat{M}$s can be reached, diversely from exponential case, it is impossible to go any further, because of regularity condition towards the horizon. Thus, an additional requested feature is that the first derivative of the coupling function must vanish for $\phi \to \infty$ at least as rapidly as the exponential function.

[6]The $\hat{M}$ value that determines the lower limit for Schwarzschild stable solutions.

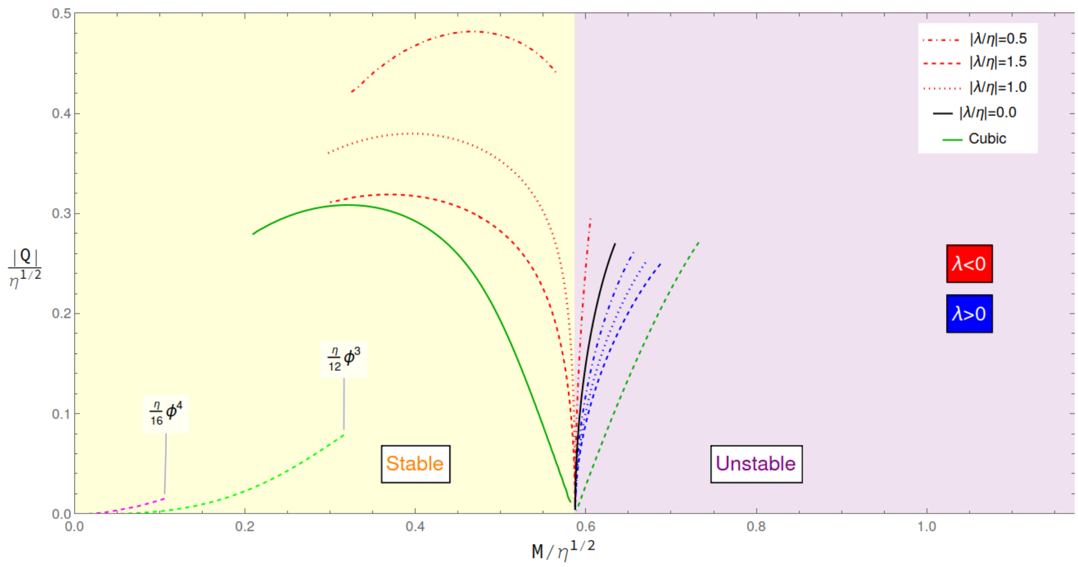

Figure 3: Dimensionless scalar charge as a function of the black hole mass $\hat{M}$ for different polynomial coupling functions. The purple region includes unstable scalarized solutions; the yellow region hosts stable ones. Dark-green branches are for $\mathbb{Z}_2$-symmetry violating coupling function. Quartic couplings $\frac{\eta}{8}\phi^2 + \frac{\lambda}{16}\phi^4$ show different phenomenology depending on the sign of the coupling constant λ, as studied in [6]. Thus, couplings with $\lambda < 0$ are reported in red and the $\lambda > 0$ ones in blue. Lastly, light-green and magenta dashed lines represent respectively purely cubic and quartic couplings. These are both unstable branches.

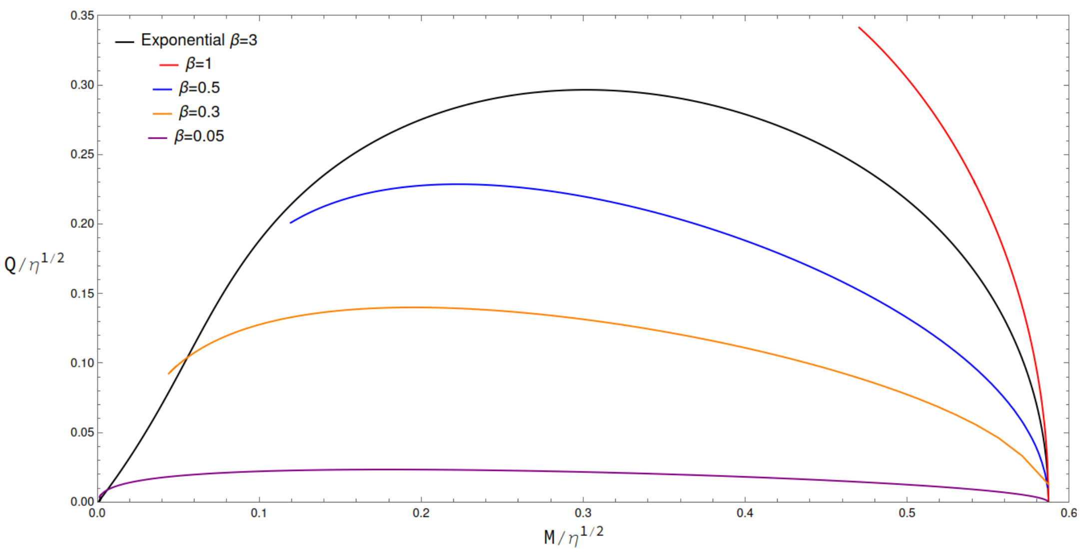

Figure 4: Dimensionless scalar charge against normalized mass for Lorentzian coupling function with different values of β. The curve in black is referred to the exponential coupling with $\beta = 3$ as a comparison with the Lorentzian case.

Conclusions

This research contributes to the investigation of modified gravity theories, focusing on delving deeper into one of the most promising among them, sGB gravity. It aims to advance the exploration of physical alternative solutions to those proposed by GR, that can be tested by present and upcoming GW detectors. Furthermore, it introduces original aspects that have not been previously considered in the existing literature.

Specifically, this work addresses the spontaneous scalarization phenomenon within sGB gravity. It is delineated how different kinds of coupling functions can lead to stable scalarized solutions, even in cases where $\mathbb{Z}_2$-symmetry is not observed.
We outline the conditions required for the coupling function to manifest *complete scalarized branches*, from the bifurcation point to vanishing value of dimensionless mass.
Furthermore, we highlight how stability can be achieved for small masses in the case of exponential coupling, and how the threshold mass $\hat{M}_{thr}$ depends on the parameters of the coupling function, completing the results of [4]. This is a favorable outcome for the theory. In fact, this framework permits to GR solutions to be still valid, consistently with current observations, but it also sets the stage for future investigations into stronger-field regimes (small masses/short radii), providing viable deviations.
The theory is not free from issues. In fact, for small masses it becomes ill-defined [3], showing singularities in the effective potential. This work illustrates that by considering a more general coupling function and varying its parameters, non-hyperbolicity for small masses can be reduced, but not completely avoided. This is a major drawback which might be due to the fact that the EFT expansion fails when the curvature is too large, and further corrections may be needed.

The analysis of the waveforms from LIGO, Virgo and future detector such as Einstein Telescope will be employed to test GR against a wide class of possible modifications [7], including those under the present investigation. Certain findings are already applicable for bounding our coupling constants [3].
It is an active area of research, not only from theoretical perspective, but also for imminent empirical analysis based on Gravitational Waves Physics.

References

[1] G Antoniou et al. "Evasion of no-hair theorems and novel black-hole solutions in Gauss-Bonnet theories". In: *Physical review letters* 120.13 (2018), p. 131102. URL: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.120.131102.

[2] Emanuele Berti et al. "Testing general relativity with present and future astrophysical observations". In: *Classical and Quantum Gravity* 32.24 (2015), p. 243001. URL: https://iopscience.iop.org/article/10.1088/0264-9381/32/24/243001.

[3] Jose Luis Blázquez-Salcedo et al. "Radial perturbations of the scalarized Einstein-Gauss-Bonnet black holes". In: *Physical Review D* 98.8 (2018), p. 084011. URL: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.98.084011.

[4] Pedro GS Fernandes et al. “Exploring the small mass limit of stationary black holes in theories with Gauss–Bonnet terms”. In: *Classical and Quantum Gravity* 39.23 (2022), p. 235015. URL: https://iopscience.iop.org/article/10.1088/1361-6382/aca010/meta.

[5] Hector O Silva et al. “Spontaneous scalarization of black holes and compact stars from a Gauss-Bonnet coupling”. In: *Physical review letters* 120.13 (2018), p. 131104. URL: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.120.131104.

[6] Hector O Silva et al. “Stability of scalarized black hole solutions in scalar-Gauss-Bonnet gravity”. In: *Physical Review D* 99.6 (2019), p. 064011. URL: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.99.064011.

[7] Helvi Witek et al. “Black holes and binary mergers in scalar Gauss-Bonnet gravity: scalar field dynamics”. In: *Physical Review D* 99.6 (2019), p. 064035. URL: https://journals.aps.org/prd/abstract/10.1103/PhysRevD.99.064035.