Abstract.

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Given $M+1$ sensors let $\boldsymbol{x} \in R^d$ denote unknown coordinates of the source, y_i - energy reading from the i^{th} sensor, $\boldsymbol{r}_i$ its coordinates, g_i - sensor gain factor, μ_i - mean of the square of the background noise, $i = 1, \dots, M$. Let $\boldsymbol{r}_0$ be a reference sensor. Referring to the eq. (17), ref [1] the energy ratio of the i^{th} and 0 sensor are:

$$k_i = \left(\frac{\frac{y_i - \mu_i}{g_i}}{\frac{y_0 - \mu_0}{g_0}} \right)^{-1/2} = \frac{\|\boldsymbol{x} - \boldsymbol{r}_i\|}{\|\boldsymbol{x} - \boldsymbol{r}_0\|} \tag{1}$$

The center $\boldsymbol{c}_i$ and the radius ρ_i of the hyper-sphere associated with the sensor i and sensor 0 (for the case $0 < k < 1$) are given by:

$$\boldsymbol{c}_i = \frac{\boldsymbol{r}_i - k_i^2 \cdot \boldsymbol{r}_0}{1 - k_i^2}, \rho_i = \frac{k_i \|\boldsymbol{r}_i - \boldsymbol{r}_0\|}{1 - k_i^2} \tag{2}$$

Eq.(18) ref [1] can be written as:

$$\|\boldsymbol{x} - \boldsymbol{c}_i\|^2 = \rho_i^2 \Leftrightarrow \|\boldsymbol{x}\|^2 - 2\boldsymbol{c}_i^T \boldsymbol{x} = \rho_i^2 - \|\boldsymbol{c}_i\|^2 \tag{3}$$

or

$$\|\boldsymbol{x}\|^2 + \boldsymbol{a}_i^T \boldsymbol{x} = b_i \tag{4}$$

where

$$\boldsymbol{a}_i = -2\boldsymbol{c}_i, b_i = \rho_i^2 - \|\boldsymbol{c}_i\|^2 \tag{5}$$

Therefore the unknown location of the source can be found via minimization of the criterion:

$$\text{minimize} \sum_{i=1}^{M} \left(\|\boldsymbol{x}\|^2 + \boldsymbol{a}_i^T \boldsymbol{x} - b_i \right)^2 \tag{6}$$

The problem above can be re-formulated as a constrained LS problem (with $\boldsymbol{y} = [\boldsymbol{x}^T \quad \|\boldsymbol{x}\|^2]^T$)

$$\begin{aligned} &\text{minimize}\; \|\boldsymbol{A}\boldsymbol{y} - \boldsymbol{b}\|^2 \\ &\text{subject to:} \begin{array}{l} \boldsymbol{y}^T \boldsymbol{C} \boldsymbol{y} + 2\boldsymbol{f}^T \boldsymbol{y} = 0 \\ y_{d+1} \geq 0 \end{array} \end{aligned} \tag{7}$$

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{a}_1^T & 1 \\ \vdots & \vdots \\ \boldsymbol{a}_M^T & 1 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} \rho_1^2 - \|\boldsymbol{c}_1\|^2 \\ \vdots \\ \rho_M^2 - \|\boldsymbol{c}_M\|^2 \end{pmatrix} \tag{8}$$

$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{I}_{d\times d} & \boldsymbol{0}_{d\times 1} \\ \boldsymbol{0}_{1\times d} & 0 \end{pmatrix}, \boldsymbol{f} = \begin{pmatrix} \boldsymbol{0} \\ -0.5 \end{pmatrix} \tag{9}$$

In ref [1] a separate case was considered that if $k_j = 1$, for some $1 \leq j \leq M$, then the solution of (17) in ref [1] forms a hyperplane between sensors $\boldsymbol{r}_j$ and $\boldsymbol{r}_0$. In this case formulation (7) is still valid with some changes to how matrix $\boldsymbol{A}$ is formed:

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{a}_1^T & 1 \\ \vdots & \vdots \\ \boldsymbol{a}_j^T & 0 \\ \vdots & \vdots \\ \boldsymbol{a}_M^T & 1 \end{pmatrix}. \tag{10}$$

References

1. X. Sheng and Y.-H. Hu, "ML Multiple-Source Localization Using Acoustic Energy Measurements with Wireless Sensor Networks," *IEEE Trans. on Signal Process.*, vol. 53, pp. No.1, pp. 44-53, Jan. 2005.