Chapter II
Measurements Used in Wireless Sensor Networks Localization

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ABSTRACT

Wireless sensor networks (WSN) localization relies on measurements. Availability of, and the information content in, these measurements depend on the network architecture, connectivity, node time synchronization and the signaling bandwidth between the sensor nodes. This chapter addresses wireless sensor networks measurements in a general framework based on a set of nodes, where each node either emits or receives signals. The emitted signal can for example be a radio, acoustic, seismic, infrared or sonic wave that is propagated in a certain media to the receiver. This general observation model does not make any difference between localization of sensor network nodes or unknown objects, or whether the nodes or objects are stationary or mobile. The information available for localization in wireless cellular networks (WCN) is in literature classified as direction of arrival (DOA), time of arrival (TOA), time difference of arrival (TDOA) and received signal strength (RSS). This chapter generalizes these concepts to the more general wireless sensor networks.

INTRODUCTION

Distributed sensor networks have been widely discussed for more than 30 years, but it is with more recent advances in hardware and processing capabilities that the vision of the wireless sensor networks can become a reality. In general, the ambition is to perform fusion of the information provided by the distributed sensors (Luo and Kay, 1989, Jayasimha et al., 1991, Wesson et al., 1981, Yemini, 1978, Akyildiz et al., 2002). One important wireless sensor networks research area concerns localization of objects moving within the coverage, or monitoring, area of the network. Applications include surveillance of both military and civilian areas and passages. Sensor networks are often deployed without knowledge of the exact sensor locations. Furthermore, sensor nodes might be mobile to some extent. Therefore, localization of individual nodes is also important. The scope of this chapter is measurements useful for all kinds of localization.

Wireless sensor network localization is in many ways similar to positioning in wireless cellular networks. This is particularly true when it comes to network-assisted positioning, where the network elements in the cellular network observe measurements related to the position of the mobile terminal. Some related surveys include (Caffery and Stuber, 1998, Zhao, 2002, Drane et al., 1998, Sun et al., 2005, Sayed et al., 2005, Pahlavan et al., 2002, Gustafsson and Gunnarsson, 2005, Deblauwe, 2008). In recent years, however, cellular network positioning has become more and more mobile-centric, where the mobiles perform the measurements, and the network either provides necessary information, or completes parts of the positioning calculations. Despite this fact, the generic measurements are still similar, which is evident after a direct comparison between this chapter and (Gustafsson and Gunnarsson, 2005). One can also consider wireless cellular networks as a special case of wireless sensor networks when it comes to localization in networks. Therefore, wireless cellular networks are used to some extent to exemplify wireless sensor network concepts.

The outline of the chapter is as follows. We start with explaining the main principles of WSN measurements in terms of a simple but generic transmitter-receiver model. This leads to a separation into waveform, time and power observations. The availability and limitations of these are discussed in terms of network architecture. Then more specific signal models are defined such as direction of arrival (DOA), time of arrival (TOA) and received signal strength (RSS), and properties in practice and typical performance values are discussed. Error modeling is discussed, and Gaussian mixture models (GMM) are considered as a generally applicable model. The potential estimation accuracy in terms of position root mean square error is discussed and related to fundamental and computable lower bounds, which can be used to evaluate network configuration. Aspects like routing protocols, information reporting protocols, signaling protocols, radio interface design, power consumption, etc. are all challenging parts of wireless sensor networks, but are not within the scope of this chapter.

GENERAL OBSERVATION MODELS

We start with a high level characterization of a wireless sensor network (WSN), and proceed with more detailed sensor models. An as generic vocabulary as possible will be used, and the particular application dependent notation will be introduced later on.

The WSN consists in general of nodes $i = 1, \ldots, M$ positioned at p^i_t at time t. A general observation model of a signal $s^j(t)$ emitted from node j and received at node i with an amplification $a^{ij}(t)$ and a delay τ_{ij} observed in noise $w^{ij}(t)$ is:

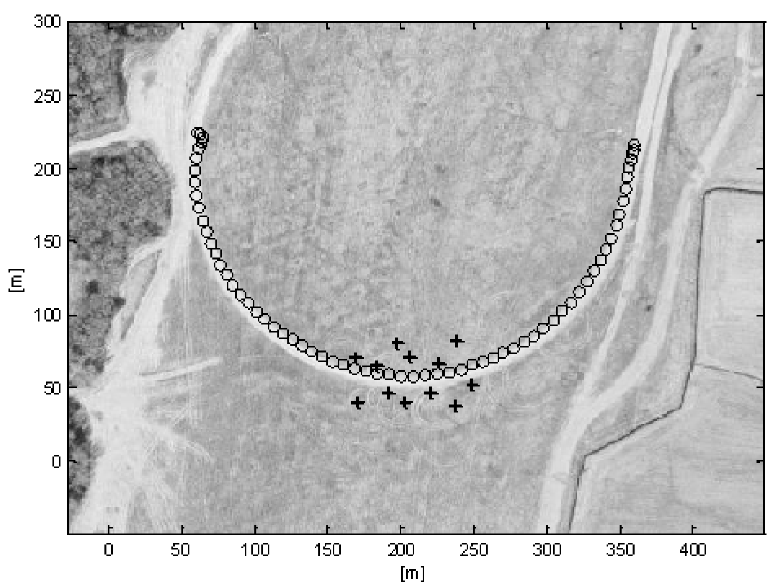
$$r^{ij}(t) = a^{ij}(t)s^j(t-\tau_{ij}) + w^{ij}(t) \tag{0.1}$$

A node can in this context be characterized with the following properties:

- The node either emits (target or emitter) a signal $s^j(t)$ or receives (sensor or receiver) a signal $r^j(t)$ at some frequency in some medium, or both. Examples include radio waves propagated with the speed of light, acoustic waves propagated with the speed of sound, seismic waves propagated in the earth or sonic waves propagated in water. The delay τ_{ij} depends on the propagation time among other things.
- A node is either a sensor node, which can act as both a target and a sensor, or an unknown object, which can only act as a target.
- The node can be known to be stationary or moving implying a constant or time-varying p^i_t.
- The node can have known or unknown position p^i_t at each time instant.
- One emitting and one receiving node can be anything between perfectly synchronized and completely unsynchronized. The same applies to two emitting nodes or two receiving nodes. The time index t refers to the time reference of the receiving node i, and synchronization errors between node i and j are included in τ_{ij}.
- The communication between two nodes (two emitters or two receivers or one of each) can be wired or wireless, and the bandwidth can be low or high.
- The emitted wave may propagate directly to the receiver (line of sight) or be subject to multipath effects. These effects have impact on both the amplification $a^{ij}(t)$ and the delay τ_{ij}.

These properties mean that the problems of determining the location of the sensor node itself, or of a different sensor node or an unknown object are equivalent and can be described with the same framework. Figure 1 shows an example of a WSN, where each sensor node consists of a microphone, a geophone and a magnetometer. The sensor nodes are stationary with known position. They have a fairly low bandwidth in their wireless communication links, and they are synchronized to an accuracy of a few milliseconds. The target node, on the other hand, is a moving vehicle with unknown position

Figure 1. The sensor nodes'+' in a WSN overlayed on an aerial photo and sampled positions 'o' of the unknown object (target node)



and on its way it is emitting acoustic and seismic waves. Depending on the type of vehicle, the range of these waves differs between some ten meters to some hundred meters.

The difference of a wireless sensor network and a sensor network in general is the assumption that at least two nodes exchange information using a wireless link and thus are subject to bandwidth limitations and non-trivial synchronization. Depending on sensor capabilities and the nodes' synchronization and communication capabilities, three different types of observations can be distinguished:

- **Waveform observations.** A highly capable sensor node is able to operate on the signal waveform $s^j(t)$, and this observation can be shared with other nodes if bandwidth allows. This is only meaningful when these nodes are synchronized at an accuracy comparable to the inverse waveform frequency. Sensors very close to each other (in the order of half a wavelength) can form a sensor array and correlate the phase of the emitted signal to get a direction of arrival estimates. For sensors further separated, there will be an integer problem in the ambiguity of the number of periods that may be resolved by merging other information.
- **Timing observations.** If a known or easily distinguished signature is embedded in the signal, the sensor can correlate the signal with the signature to accurately estimate time of arrival t-τ_{ij}. The timing estimation accuracy depends on the signature as well as the sensor capability. This is meaningful only if the sensor is synchronized either with the emitting node or another receiving node. In the former case, an absolute distance can be computed and in the latter case a relative distance follows. Geometrically, these two cases constrain the position to a circle or a hyberbolic function.
- **Power observations.** Another possibility is that the sensor estimates the received signal power (received signal strength - RSS) $||r^{ij}(t)||^2$. In essence, this means integrating the received signal power within a certain frequency band during an integration interval to estimate the received signal energy during the time interval. If the emitted power is known, RSS provides coarse range information. Otherwise, two or more sensors can compare their RSS observations to eliminate the unknown emitted power.

These three types are discussed and exemplified further below. However, first implications from the sensor network architecture and classification are addressed.

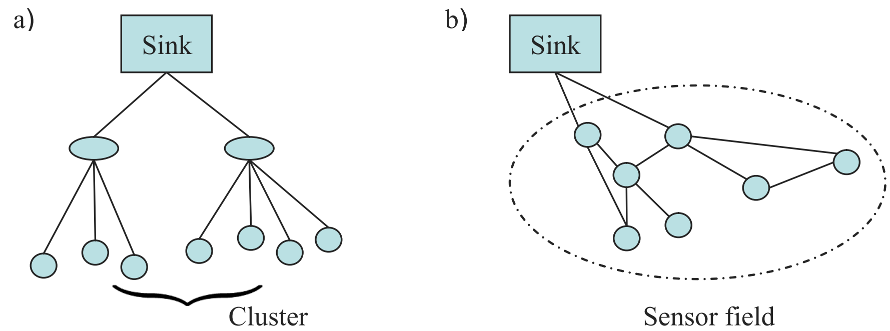
WIRELESS SENSOR NETWORK ARCHITECTURES AND CLASSIFICATIONS

The characteristics and behavior of a wireless sensor network is very much dependent on the architecture, organization and connectivity. A rather loosely connected ad-hoc network typically has different properties compared to a strictly hierarchical network. Handling and processing of time-related measurements are facilitated if the sensor nodes have a common time reference. The localization problem of unknown object positions is more challenging if the sensor node positions are unknown and time varying compared to if they are known and fixed.

Architecture and Connectivity

A network with a rather strict architecture will be referred to as a *hierarchical* network as illustrated by Figure 2a. Conversely, networks with a loose architecture as in Figure 2b are denoted *anarchical* (Wesson et al., 1981). The target node for the aggregated sensor information is referred to as the *sink*.

Figure 2. Two wireless sensor network architecture examples. (a) Hierarchical network with clusters of sensors associated to cluster heads. (b) Anarchical network with sensors loosely organized in a sensor field.



In a hierarchical network sensor nodes may be associated to more potent nodes known as *cluster heads* (C.-C. Shen et al., 2001) or *processing elements* (Jayasimha et al., 1991). Depending on the size of the sensor network, such clustering may be organized over multiple levels in a hierarchical fashion. The many low complexity, low power sensors with little processing power are placed in the bottom, while sensors with increasing complexity and processing power are found higher in the hierarchy. Information from child nodes may be aggregated, fused and refined by parent nodes before being passed upwards in the hierarchy. Furthermore, the cluster heads on the highest level distribute the information to the sink, where the final analysis and visualization take place. Connections are primarily established between child nodes and associated parent node, even though connections within the cluster is also possible. The strict hierarchy enables a prompt and reliable information transport. Wireless cellular networks are typically strictly hierarchical networks.

In an anarchical network, the sensors are loosely organized, and the sensor capability is typically more equal. The set of the sensors are popularly denoted the *sensor field* (Akyildiz et al., 2002), or *smart dust* (Kahn et al., 1999). The information may be aggregated and fused in some or all of the sensor nodes, before passing it on in the sensor field. Also the sink may be a node in the sensor field, but typically with somewhat better processing capabilities. Essentially all connections can be seen as volatile, and a connection between two nodes may be a multi-hop connection with several node to node links in sequence. Loosely organized networks with volatile communications links are often referred to as *mesh* networks (Akyildiz and Wang, 2005). Mechanisms such as self-configuration and self-organization (a new node finds its place in the network, and the network-wide connectivity establishes automatically), and self-healing (the connectivity is re-established after some disruptive event) are important components of the mesh network.

Sharing waveform observation between nodes is most probably only plausible in hierarchical networks. Distribution of timing and power observations should on the other hand be possible in any type of network, provided that the connection bandwidth between the nodes is sufficient.

Time Synchronization

In a distributed environment such as wireless sensor networks there may be no central synchronized time reference that regulates the activities of each node. Instead, each node manages its own time reference.

Most of the sensor measurements, both with respect to localization and relative distances, but also with respect to a general sensor observation, need to be associated with a common time reference in order to be properly fused together. If the time synchronization in the wireless sensor network is acceptable, then node-pairs can exchange information and measurements and readily use the information for localization. Waveform observations require the highest synchronization accuracy, timing observation intermediate accuracy and power observation the crudest accuracy. Synchronization inaccuracies can be included in the delay distribution in (0.1).

Network time synchronization protocols used over the fixed network are not applicable, due to the volatile and possibly time varying connectivity and the clock drift of the low complexity sensor devices. One tractable approach is to consider one node as the node with the time reference, and use adjustment mechanisms to gradually synchronize the sensor network. Such schemes are surveyed in (Jayasimha et al., 1991, Elson and Römer, 2003, Sivrikaya and Yener, 2004) and the results indicate that they provide accurate synchronization in the order of 10 μs. Recent advances provide algorithms that operate without a specific node as time reference (Solis et al., 2006, Schenato and Gamba, 2007) something that is very important for robustness reasons.

Sensor Node Location Information

Some nodes in the wireless sensor network may have known positions, either constant or time-varying. For example, the location information can be provided locally by some external location system such as GPS. Nodes with known position are referred to as *anchor* nodes, and nodes with unknown position consequently *non-anchor* nodes (Luo et al., 2006).

The wireless sensor network may consist of only non-anchor nodes. Then some sort of cooperative localization scheme (Patwari et al., 2005) is a necessity. The communication to support self-localization of sensor nodes can be either *interactive* or *non-interactive* (Luo et al., 2006). The interactive support means that all the nodes actively participate in the localization by signaling and sharing information (bidirectional connectivity), while in case of non-interactive support, most of the nodes passively observe other objects and beacons (unidirectional connectivity). These mechanisms may be supported by signals (e.g. pilot signals) broadcasted continuously or at least regularly from some nodes, which then act as *beacons* (Sun et al., 2005). Another way of exciting the self-localization mechanisms in non-interactive sensor networks is by observing the same mobile and unknown object both in time and in space and share the information with other nodes. This could be seen as a calibration phase.

In hierarchical networks, the existence of anchor nodes is quite plausible. Furthermore, an anchor node that acts as a beacon is denoted a *landmark* node (similar to a lighthouse) (Niculescu, 2004). Cluster head nodes become central and important, and this motivates that such nodes are relatively potent and deployed with care. This could include a determination of the exact node position. When some sensor locations are known as is likely in hierarchical networks, methods based on relative position measurements are available (N. Patwari et al., 2003). Other self-localization schemes include (Niculescu, 2004, Langendoen and Reijers, 2003, Savarese et al., 2001, Coates, 2004, Galstyan et al., 2004).

In some applications, the signal propagation speed is significantly higher than the velocity of an unknown object or a moving sensor node. This is the case for radio frequency signals. However, when the measurement signal is seismic or audible then the delay between signal emission to signal observation cannot be neglected.

SPECIFIC OBSERVATION MODELS

The notation assumes a two-dimensional position, $p^i_t = (X^i_t, Y^i_t)^T$, but can be extended to higher dimensions. Depending on whether one, two or three nodes have to collaborate to form an observation, the following notation will be used:

$$\begin{aligned} y_t^i &= h_{type}(p_t^i) + e_t^i \\ y_t^{ij} &= h_{type}(p_t^i, p_t^j) + e_t^{ij} \\ y_t^{ijk} &= h_{type}(p_t^i, p_t^j, p_t^k) + e_t^{ijk} \end{aligned} \tag{0.2}$$

Waveform Observations

Observing the full frequency band of a signal requires a quite capable sensor. For example, a message sent over radio is typically encoded to a base band signal, which then is modulated onto a carrier frequency of much higher frequency than the base band signal. The typical processing of the measurements includes correlation of waveform observations from different, spatially close, sensors. This is popularly referred to as sensor arrays. It could be instructive to consider observations from the different sensors as one sensor observation from a sensor array. As already mentioned, one application of the sensor array measurement at node i is *direction-of-arrival* estimation relative another node j.

Essentially all signal frequencies and media are possible. The nonlinear measurement function is given by

$$y_t^{ij} = h_{DOA}(p_t^i, p_t^j) + e_t^{ij} \tag{0.3}$$

Furthermore, the direction-of-arrival angle can be calculated as

$$h_{DOA} = \text{angle}(p_t^i - p_t^j) \tag{0.4}$$

Timing Observations

For signals with a known or detectable signature or fingerprint, timing information can be obtained by correlating the signal with the signature. The signature can be a pilot or a training sequence transmitted from a radio base station or node. Compared to a waveform observation, the signature is typically part of the base band signal. It can also be a detectable impulse in the signal, for example from a gun shot in an acoustic signal or from an explosion in a seismic signal. A similar signature can be the arrival of a message sent from a different location.

The accuracy depends mainly on the signature properties, but also on the processing capabilities of the sensor. One typical application of timing observations is *time-of-arrival* estimation

$$y_t^{ij} = h_{TOA}(p_t^i, p_t^j) + e_t^{ij} \tag{0.5}$$

If both nodes i and j have the same time reference, the time of signal transmission t^j and reception t^i combined can be related to the relative distance if the signal propagation speed of the medium v is known:

$$\left\| p_t^i - p_t^j \right\| = v(t^i - t^j) \tag{0.6}$$

In a message-oriented implementation, it could be easier to instead measure the round-trip time from time of transmission t^i_{tran} and time of response reception t^i_{resp}. If the processing time t^j_{proc} at the other node j is included in the response message, the following holds

$$\left\| p_t^i - p_t^j \right\| = \frac{v}{2}(t^i_{\text{resp}} - t^i_{\text{tran}} - t^j_{\text{proc}}) \tag{0.7}$$

The uncertainty in (0.6) is dominated by the difference in time references at the two locations, while the uncertainty in (0.7) depends on the time of arrival estimation of the response at node i and the accuracy of the processing time estimation at the other node j.

Hence, the nonlinear time-of-arrival measurement modeling can be summarized as

$$h_{TOA} = \left\| p_t^i - p_t^j \right\|. \tag{0.8}$$

As discussed for time-of-arrival measurements, a different or uncertain time reference can be troublesome. If the ambiguity in time-references t^j_{ref} and t^k_{ref} between two nodes j and k is known or can be resolved at some node or location, then *time-difference-of-arrival* measurements can become useful

$$y_t^{ijk} = h_{TDOA}(p_t^i, p^j, p^k) + e_t^{ijk} \tag{0.9}$$

A signal emitted (received) from node i and received (emitted in parallel) at nodes j and k at times t^{ij} and t^{ik}, respectively, leads to the following observation

$$\left\| p_t^i - p_t^j \right\| - \left\| p_t^i - p_t^k \right\| = \left(t^{ij} - t^{ik} + t^j_{ref} - t^k_{ref} \right) v \tag{0.10}$$

The uncertainty e_t^{ijk} depends both on the time-of-arrival estimation accuracies e_t^{ij} and e_t^{ik} as well as the time synchronization accuracy between the locations.

The nonlinear time-difference-of-arrival measurement is thus characterized by

$$h_{TDOA} = \left\| p_t^i - p_t^j \right\| - \left\| p_t^i - p_t^k \right\|. \tag{0.11}$$

If two of these positions are known, the position of the third is constrained to a hyperbolic function, whose asymptotes define a direction of arrival. This indicates a close relation of relative range information and angle information. Note the generality of this setup. Any of the three nodes can be the one to be located, and there is a duality between emitting and receiving. The distinguishing feature here is that exactly two nodes are synchronized, or the difference in time references is known somewhere in the sensor network.

Similar equations based upon relative distances also appear when considering measurements based on interferometrics (M. Maroti et. al, 2005). In such a case, two nodes A and B transmit pure sine waves at two close frequencies $f_A > f_B$, and two other nodes C and D measure and filter the total received signal power and estimate the absolute phase offset $\nu = \phi_A - \phi_B$. It has been shown that the relative phase offset

$\nu_C - \nu_D$ is proportional to a combination of relative distances, provided that the frequencies are close enough and the relative node distances are short enough, compared to the travel time of the signals:

$$y_t^{ABCD} = h_{IF}\left(p_t^A, p_t^B, p_t^C, p_t^D\right) + e_t^{ABCD} \tag{0.12}$$

$$h_{IF}\left(p_t^A, p_t^B, p_t^C, p_t^D\right) = \left\|p_t^A - p_t^D\right\| - \left\|p_t^B - p_t^D\right\| + \left\|p_t^B - p_t^C\right\| - \left\|p_t^A - p_t^C\right\| \tag{0.13}$$

Power Observations

Power observations are typically obtained by integrating the signal power over a certain time interval. The signal may be low-pass or band-pass filtered before calculating the signal power to capture the power in the frequency band where the signal is expected to reside. This means that the power signal sample interval is orders of magnitude longer than the signature sequences.

The observed power is related to the relative distance between two nodes

$$y_t^{ij} = h_{RSS}(\left\|p_t^i - p_t^j\right\|) + e_t^{ij} \tag{0.14}$$

Radio frequency signals, acoustic signals, seismic signals, radar signals etc have in common that the emitted signal strength $\overline{P}_0^i$ (bar denotes here and in the sequel power in linear scale) at node *i* approximately decays exponentially with distance in linear scale. For example, this is observed for radio signals in e.g. (Okumura et al., 1968, Hata, 1980). Hence, the received signal strength $\overline{P}^{ij}$ at node *j* can be modeled as

$$\overline{P}^{ij} = \overline{P}_0^i \left\|p_t^i - p_t^j\right\|^{-n_{ij}}. \tag{0.15}$$

The corresponding relation in logarithmic scale is

$$P^{ij} = P_0^i - n_{ij}\log(\left\|p_t^i - p_t^j\right\|) \tag{0.16}$$

where ‘log’ denotes the natural logarithm. It is a matter of taste whether a model in linear or in logarithmic scale is selected. One aspect is that the uncertainty e^{ij}_t with good approximation can be modeled as Gaussian when the received signal strength measurement equation is in logarithmic scale (Okumura et al., 1968, Hata, 1980). Equations (0.15) and (0.16) yield two alternative non-linear functions

$$\begin{aligned} h_{RSS,log} &= P_0^i - n_{ij}\log(\left\|p_t^i - p_t^j\right\|) \\ h_{RSS,lin} &= \overline{P}_0^i \left\|p_t^i - p_t^j\right\|^{-n_{ij}}. \end{aligned} \tag{0.17}$$

Figure 3 illustrates acoustic and seismic sensor signals, whose propagation can be described with (0.17) at good accuracy. Furthermore, Figure 4 shows WiMax radio signal RSS measurements as a function of logarithmic distance. Again, the observation model in (0.17) fits data well.

Scanning radar was mentioned as a means of estimating directions above. It is also an example of power measurements where the received radar signal power is determined. In this case, the transmitted power is known, the distance is round-trip, but the uncertainty comes from the propagation properties and the radar cross section of the object.

In wireless cellular networks, received signal strength (or quality) measurements are integral parts of the mobility management, which aims at ensuring that the mobiles are connected to the most favorable base station over time. The pilot signal strengths are estimated, and it is possible in some standards to inform the mobiles about the pilot transmission power.

Observations Related to Digital Maps

The power observations model in the previous section only models the exponential decay with distance of the power. The main reason for deviations from this model is due to diffraction and reflection phenomena. If a reliable signal propagation tool is available, or if some effort is spent scanning the observation area, this information could be aggregated into a database, which can be seen as a digital map with a measurement model

$$y_t^i = h_{MAP}(p_t^i) + e_t^i \tag{0.18}$$

The observed or predicted values associated with a specific position p^i and different nodes j can be seen as a fingerprint for this position. The map information could be received signal strength measurements at all possible locations in the area as discussed for indoor localization in (Chen and Kobayashi, 2002, Kaemarungsi and Krishnamurthy, 2004). In this case, the nonlinear measurement function depends on at least two node positions $h_{MAP}^j(p_t^i, p_t^j)$. As another example, an object could travel along the trajectory in Figure 2 during a measurement campaign, while all sensor nodes measure the received signals as is emitted from the object. This information could be stored in a database and subsequent sensor observations can be correlated to the map information.

Figure 3. Received sensor energy in log scale Pij versus log range $\log(\| p_t^i - p_t^j \|)$, together with a fitted linear relation as modeled in (0.14). The estimated path loss exponent nij is 2.3 and 2.6 respectively, and the model error standard deviation 0.56 and 0.60 respectively. If a dB scale is used, then these values scale by a factor of 10/log(10), which yields model error standard deviations of 2.4 and 2.6 dB.

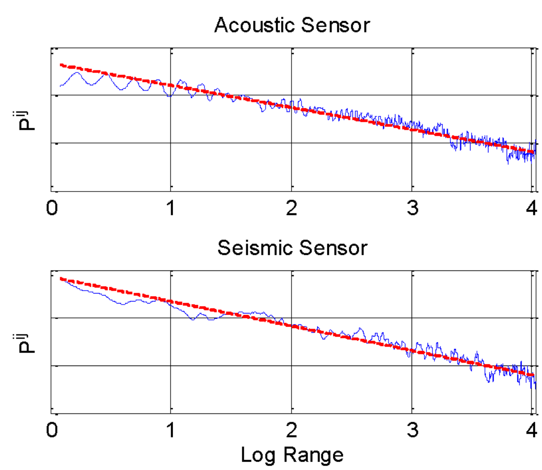
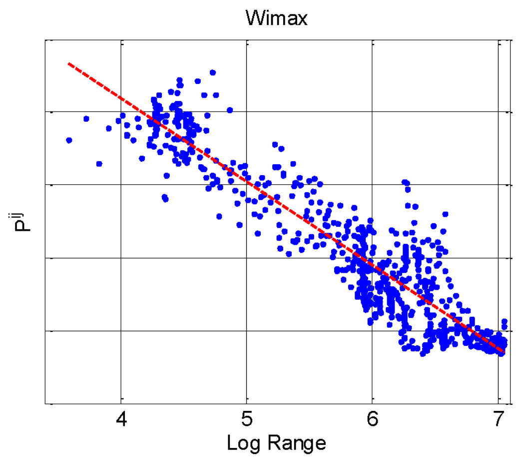


Figure 4. Radio signal received signal strength in log scale versus log range, together with a fitted linear relation as modeled in (0.14). The estimated path loss exponent n^{ij} is 2.3, and the estimated error distribution is Gaussian with standard deviation 0.70 (corresponding to 3 dB). Data from a Wimax radio network deployment in Brussels kindly provided by Mussa Bshara.



It could also be a digital map of walls in an area, and the distance to the walls in different directions are measured at node *i*. Similarly, it could be a digital road map (Gustafsson et al., 2002), where the measurement is the distance to the road which should be zero for road-bound mobility. Another example is a map of sea depths (bathymetry map), and the corresponding measurement is a depth measurement (Karlsson and Gustafsson, 2003). This means that the nonlinear measurement function depends only on one node position $h_{MAP}(p_t^i)$.

Position and Attitude Observations

Sensor observations for self-localization include local sensors measurements directly related to the position of the same node. A direct position estimate can simply be expressed as

$$y_t^i = p_t^i + e_t^i \tag{0.19}$$

and may be available from an external system such as the Global Positioning System. Typical GPS accuracy without differential support is in the order of 5-10 m. However, this could be significantly improved with differential support of some kind. The non linear measurement equation for position estimates is thus

$$h_{POS} = p_t^i \tag{0.20}$$

Note that GPS positioning can be incorporated either as a position estimate with estimated error statistics, or as separate timing estimates relative to the GPS satellites.

It is also possible that inertial measurement units (IMU) are available at certain nodes, providing attitude and acceleration information. Such measurements can be modeled as

$$y_t^i = h_{IMU}(\dot{p}_t^i, \ddot{p}_t^i) + e_t^i \tag{0.21}$$

Measurement Error Modeling

In all specific measurement models, the measurement noise is additive. A first and convenient approximation is that the measurement is unbiased and the noise is white and Gaussian with a standard deviation σ_e. Appropriate accuracy levels depend on both the type of measurement, as well as the network architecture and classification as discussed in the previous sections.

Waveform Observations

Geometrically, the spatial resolution of the intersection of two perfectly complementing AOA measurements is limited to $2d_A \sin(\alpha/2)$, where α is the angular resolution and d_A is the distance between the antennas. For $\alpha = 30°$, this means 36% of d_A. Approximately 5º - 10º can be considered appropriate.

Timing Observations

One key contributor to the measurement noise is the time reference inaccuracy between two nodes. Very exact time reference can be obtained by using a GPS receiver. Otherwise, one has to rely on different distributed time synchronization mechanisms. Furthermore, the timing determination accuracy depends on the resolution of the reference signal. In GSM, one training sequence bit corresponds to 554 m, and the timing accuracy is a fraction in the same order. In WCDMA the accuracy may be down to 0.5 chip of the scrambling code corresponding to an accuracy of 20 m. For radio frequency signals, the time reference accuracy needs to be less than 1 μs, but for audible signals can be far cruder since it is the accuracy relative the travel time of the signal that matters.

Power Observations

The received signal strength model describes signal propagation in a desert environment without interfering objects. The latter may result in diffraction and reflections, depending on the relation between the signal wavelength and the size of the objects. This shadow fading gives an additive component with 4-12 dB standard deviations. As has been discussed above, the accuracy of the power measurements can be improved by the use of well-predicted or measured power level maps. In such a case, the error standard deviation can be less than 3 dB.

Error Distributions and Correlations in More Detail

A generally applicable assumption is that the measurement error is Gaussian with a probability density function

$$p_E(e_t) = N(0, \sigma) \tag{0.22}$$

This may be valid, provided that the measurement is based on a signal that has traveled along the line-of-sight (LoS) between the nodes. In a non-LoS situation, the measurement will get a positive bias μ and probably another (larger) variance σ_{NLOS}

$$p_E(e_t) = N(\mu, \sigma_{NLOS}) \tag{0.23}$$

The problem here is that we cannot easily detect NLOS. One solution is to use robust algorithms. Another approach is to model the error distribution with a mixture, for instance the two-mode Gaussian mixture model (GMM)

$$p_E(e_t) = \alpha N(0, \sigma_{LOS}) + (1-\alpha) N(\mu_{NLOS}, \sigma_{NLOS}) \tag{0.24}$$

where (α, μ_{NLOS}, σ_{LOS}, σ_{NLOS}) are free parameters in the distribution. Here, e_t falls in the LOS distribution with probability α and the NLOS distribution with probability $1 - \alpha$. Algorithms based on this distribution will automatically be more robust than algorithms that do not model NLOS.

Furthermore, the errors may be correlated over time and space. For example, the diffraction and reflection phenomena that cause most of the line-of-sight errors for power measurements are strongly correlated to the terrain. One modeling approach (M. Gudmundson 1991) considers distance dependent measurement errors and is based on the introduction of a decorrelation distance d_{dc} after which the error correlation has dropped to e^{-1}. As an approximation, the correlation is only based on the position of one node i, typically the one of an unknown object

$$corr\left(e(p^i), e(p^i + \Delta)\right) = e^{-\|\Delta\|/d_{dc}} \tag{0.25}$$

Sensor Observations in Summary

Table 1 summarizes the discussed sensor observations. Note that the quality of the sensor information depends not only on the noise variance, but also on the size and variation in $h(p)$. The sensor information included in the measurements is further discussed in the next section.

FUNDAMENTAL PERFORMANCE BOUNDS

Consider now the set of all available measurements as a vector y, and all positions to be estimated as a vector p. The total signal model can then be written

$$y = h(p) + e \tag{0.26}$$

This is the basic measurement relation for localization services.

Firstly, a nonlinear least squares (NLS) optimization routine can be applied to get a position estimate $\hat{p}_{NLS}$. Secondly, if the noise covariance R=cov(e) is known, the weighted nonlinear least squares (WNLS) estimate $\hat{p}_{WNLS}$ can be computed. Finally, if the noise distribution p_E is known, the maximum likelihood (ML) approach applies, which gives $\hat{p}_{ML}$. In general, the position estimate can be obtained by minimizing an optimization criterion

Table 1. Mathematical notation of available sensor observations in wireless sensor networks described by the non-linear location dependency h_{type}, such that $y = h_{type} + e$

Type of Measurement	Nonlinear Measurements	Accuracy
Direction of arrival	$h_{DOA} = \text{angle}(p_t^i - p_t^j)$	5º - 10º
Time of arrival	$h_{TOA} = \lVert p_t^i - p_t^j \rVert$	5 – 100 m
Time difference of arrival	$h_{TDOA} = \lVert p_t^i - p_t^j \rVert - \lVert p_t^i - p_t^k \rVert$	10 – 60 m
Interferometrics	$h_{IF} = \lVert p_t^A - p_t^D \rVert - \lVert p_t^B - p_t^D \rVert + \lVert p_t^B - p_t^C \rVert - \lVert p_t^A - p_t^C \rVert$	0.1 – 1 m
Received signal strength	$h_{RSS,log} = P_0^i - n_{ij} \log(\lVert p_t^i - p_t^j \rVert)$ $h_{RSS,lin} = \overline{P}_0^i \lVert p_t^i - p_t^j \rVert^{-n_{ij}}$	4 – 12 dB
Digital map information	$h_{MAP}^j(p_t^i, p_t^j)$	(RSS MAP 3dB)
Position estimates Inertial sensors	$h_{POS} = p_t^i$ $h_{INS}(\dot{p}_t^i, \ddot{p}_t^i)$	5 – 20 m (GPS)

$$\hat{p} = \arg\min_p V(p) \tag{0.27}$$

where the different optimization routines are obtained by the following criteria

$$\begin{aligned} V_{NLS}(p) &= \left(y - h(p)\right)^T \left(y - h(p)\right) \\ V_{WNLS}(p) &= \left(y - h(p)\right)^T R^{-1} \left(y - h(p)\right) \\ V_{ML}(p) &= \log p_E \left(y - h(p)\right) \end{aligned} \tag{0.28}$$

The estimates come with an estimation uncertainty, whose second order moment is represented by a covariance matrix *P*. Increasing the amount of considered knowledge in the estimation step, improves the estimation accuracy and reduces the estimation error covariance. In general

$$P_{NLS} > P_{WNLS} > P_{ML} \tag{0.29}$$

For the user, the position root mean square error (RMSE) in meters is a useful measure. It includes estimation errors both due to the covariance and bias errors. RMSE can be lower bounded by the covariance matrix using the inequality

$$\text{RMSE} = \sqrt{\mathrm{E}\left[(X^o - \hat{X})^2 + (Y^o - \hat{Y})^2\right]} \geq \sqrt{\text{tr}\,\text{Cov}(\hat{p})} \tag{0.30}$$

where p^o denotes the true position. It should be noted that the covariance matrix may not be feasible to compute, because it is either not straightforward to express, or it may need an excessive number of computations.

A more challenging question is how small the covariance, and thus the RMSE, can be for a given sensor network constellation and measurement characteristics. Such a bound is provided by the Cramer-Rao Lower Bound (CRLB) for unbiased estimators. This bound is expressed in terms of the Fisher

Information Matrix (FIM) (Kay, 1993). CRLB has been analyzed thoroughly in the literature, primarily for DOA, TOA and TDOA (Botteron et al., 2004b, Botteron et al., 2004a, Botteron et al., 2002, Botteron et al., 2001, Patwari et al., 2003, Koorapaty et al., 1998), but also for RSS (Weiss, 2003, Koorapaty, 2004) and with specific attention to the impact from non-line-of-sight (Qi and Kobayashi, 2002b, Qi and Kobayashi, 2002a).

The 2 × 2 Fisher Information Matrix *J(p)* is defined as

$$\begin{aligned} J(p) &= \mathrm{E}\left(\nabla_p^T \log p_E(y-h(p))\nabla_p \log p_E(y-h(p))\right) \\ \nabla_p \log p_E(y-h(p)) &= \begin{pmatrix} \dfrac{\mathrm{d}\log p_E(y-h(p))}{\mathrm{d}X} & \dfrac{\mathrm{d}\log p_E(y-h(p))}{\mathrm{d}Y} \end{pmatrix} \end{aligned} \tag{0.31}$$

where $p=(X,Y)^T$ the two-dimensional position vector and $p_E(y\text{-}h(p))$ the likelihood given the error distribution p_E of the measurement uncertainty.

In case of Gaussian measurement error distributions $p_E(e)= N(0,R(p))$, the FIM equals

$$\begin{aligned} J(p) &= H^T(p)R(p)^{-1}H(p) \\ H(p) &= \nabla_p h(p) \end{aligned} \tag{0.32}$$

Table 2 summarizes the involved expressions for *h(p)* and its gradient for range and direction measurements with focus on location *i*.

In the general case with non-Gaussian error distributions, numerical methods are needed to evaluate the CRLB. The larger the gradient *H(p)*, or the smaller the measurement error, the more information is provided from the measurement, and the smaller potential estimation error.

Information is additive, so if two measurements are independent, the corresponding information matrices can be added. This is easily seen for instance from (0.32) for $H^T = (H_1^T, H_2^T)$ and R being block

Table 2. Analytical expressions related to some measurements related to the location i *and corresponding gradients, where* $\tilde{X}^{ij} = X^i - X^j$, $\tilde{Y}^{ij} = Y^i - Y^j$ $D^{ij} = \sqrt{(\tilde{X}^{ij})^2 + (\tilde{Y}^{ij})^2}$ $\varphi^{ij} = \mathrm{angle}\left(p_t^i - p_t^j\right)$.

Method	$h(p)$	$H(p) = \dfrac{\partial h(p)}{\partial p}$
DOA	φ^{ij}	$\left(\dfrac{-\tilde{Y}^{ij}}{(D^{ij})^2}, \dfrac{\tilde{X}^{ij}}{(D^{ij})^2}\right)$
TOA	D^{ij}	$\left(\dfrac{\tilde{X}^{ij}}{D^{ij}}, \dfrac{\tilde{Y}^{ij}}{D^{ij}}\right)$
TDOA	$D^{ij} - D^{ik}$	$\left(\dfrac{\tilde{X}^{ij}}{D^{ij}} - \dfrac{\tilde{X}^{ik}}{D^{ik}}, \dfrac{\tilde{Y}^{ij}}{D^{ij}} - \dfrac{\tilde{Y}^{ik}}{D^{ik}}\right)$
RSS	$P_0^i - n_{ij}\log(\lVert p^i - p^j \rVert)$ $\bar{P}_0^i \lVert p_t^i - p_t^j \rVert^{-n_{ij}}$	$\left(\dfrac{-n_{ij}\tilde{X}^{ij}}{(D^{ij})^2}, \dfrac{-n_{ij}\tilde{Y}^{ij}}{(D^{ij})^2}\right)$ $\left(\dfrac{-\bar{P}_0^i n_{ij}\tilde{X}^{ij}}{(D^{ij})^{n_{ij}+2}}, \dfrac{-\bar{P}_0^i n_{ij}\tilde{Y}^{ij}}{(D^{ij})^{n_{ij}+2}}\right)$

diagonal, in which case we can write $J = J_1 + J_2$. Plausible approximate scalar information measures are the trace of the FIM and the smallest eigenvalue of FIM

$$J_{\mathrm{tr}}(p) \triangleq \mathrm{tr}\ J(p), \quad J_{\min}(p) \triangleq \min \mathrm{eig}\ J(p) \tag{0.33}$$

The former information measure is additive as FIM itself, while the latter is an under-estimation of the information useful when reasoning about whether the available information is sufficient or not. Note that in the Gaussian case with a diagonal measurement error covariance matrix, the trace of FIM is the squared gradient magnitude.

The Cramer-Rao Lower Bound is given by

$$\mathrm{Cov}(\hat{p}) = \mathrm{E}(p^o - \hat{p})(p^o - \hat{p})^T \geq J^{-1}(p^o) \tag{0.34}$$

The CRLB holds for any unbiased estimate of $\hat{p}_t$. The lower bound may not be an attainable bound. It is known that asymptotically in the information, the ML estimate is $\hat{p} \sim \mathbb{N}(p^o, J^{-1}(p^o))$ (Lehmann, 1991) and thus reaches this bound, but this may not hold for finite amount of inaccurate data.

The right hand side of (0.34) gives an idea of how suitable a given sensor node configuration is for localization e.g. of an unknown object. It can also be used for *deployment design*, e.g. where to place the sensor nodes in order to enable pre-determined localization accuracy, given the sensitivity of the sensors. However, it should always be kept in mind though that this lower bound is quite conservative and relies on many assumptions. In practice, the root mean square error (RMSE) is perhaps of more importance. This can be interpreted as the achieved position error in meters. The CRLB implies the following bound:

$$\mathrm{RMSE} = \sqrt{\mathrm{E}\left[(X^o - \hat{X})^2 + (Y^o - \hat{Y})^2\right]} \geq \sqrt{\mathrm{tr}\,\mathrm{Cov}(\hat{p})} \geq \sqrt{\mathrm{tr}\ J^{-1}(p^o)} \tag{0.35}$$

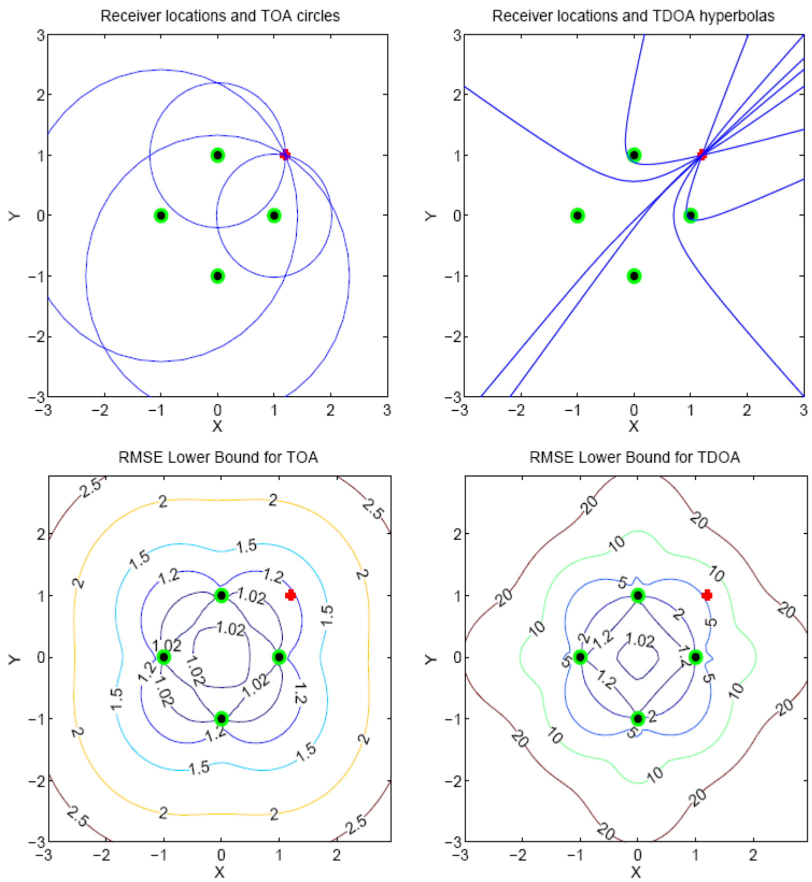
The first inequality becomes an equality for unbiased estimates. If RMSE requirements are specified, it is possible to make the sensor network denser until (0.35) indicates that the amount of information provided from the sensor nodes is enough.

A Geometric Example

Consider the scenario in Figure 5, where four sensor nodes are placed in the positions (-1,0), (1,0), (0,-1) and (0,1), respectively. Each node measures the arrival time of a transmitted signal from an unknown position (either a sensor node or an unknown object), using accurate and synchronized clocks. If the transmitter is also synchronized, the signal propagation time can be computed, which leads to a TOA measurement. Propagation time corresponds to a distance, which leads to the distance circles around each receiver in Figure 5 (upper left). If the transmitter is unsynchronized, each pair of receivers can compute a time-difference of arrival TDOA. This leads to a hyperbolic function where the transmitter can be located (Fang, 1990, Spirito and Mattioli, 1998).

The RMSE lower bounds from (0.35) for the TOA and TDOA measurements in this example are plotted in Figure 5 (lower plots). They indicate estimation accuracy limits spatially depending on the actual position of a sensor node or an unknown object. The level curves are scaled by σ_e, so a range error with standard deviation of 100 meter will in the most favorable position lead to a position estimation error of 100 meter. A bit counterintuitive, TDOA and TOA give the same performance close to the

Figure 5. First row: Example scenario with four sensor node placed in a square, and there is one transmitter at (1.2,1). With TOA measurements, each receiver measurement constrains the transmitter position to a circle, while with TDOA measurements, each pair of receiver measurements constrains the transmitter position to a hyperbola. Second row: RMSE lower bound implied by the CRLB for TOA and TDOA, respectively, measurements. The unit is scaled to the measurement standard deviation.



origin. To explain this, note that if all signals arrive simultaneously to all receivers, the transmission time does not add any information.

SUMMARY

This chapter addresses generic models of sensor observations in a wireless sensor network (WSN) by dividing these into detailed waveform observations, less detailed timing observations and much less detailed power observations. This view is well established in wireless cellular networks (WCN), though a more general framework was presented that besides radio waves also covers WSN with acoustic, seismic, sonic, infrared waves in different media as air, water and ground. A careful investigation associates these observation types to the more traditional measurement related to range, relative range and angle between nodes. A WSN is characterized by its architecture, connectivity, interactivity, the degree of synchronization and the available bandwidth in the wireless links, and these are also the main factors for which sensor observations that are plausible and can be shared between nodes and the kind of information that can be computed from the received signals.

A framework for analyzing the information content of various observations was also presented, that can be used to derive localization bounds for integrating any combination of observations in the network.

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