An Efficient Constrained Weighted Least Squares Algorithm for Moving Source Location Using TDOA and FDOA Measurements

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***Abstract*—In this paper, by utilizing the time difference of arrival (TDOA) and frequency difference of arrival (FDOA) measurements of a signal received at a number of receivers, an efficient constrained weighted least-squares (CWLS) algorithm for estimating the position and velocity of a moving source is proposed, which exploits the known relation between the intermediate variable and the source location coordinates explicitly. On basis of Newton's method, a numerical iterative solution can be obtained allowing real-time implementation and ensuring global convergence. Simulation results show that the proposed estimator achieves remarkably better performance than the two-step weighted least squares (WLS) approach, which makes the Cramr-Rao lower bound (CRLB) at a sufficiently high noise level before the threshold effect occurs.**

***Index Terms*—Source location, constrained weighted least-squares (CWLS), time differences of arrival (TDOA), frequency difference of arrival (FDOA), Newton's method.**

I. INTRODUCTION

THE location of an emitting source, as the focus of considerable research efforts, has always drawn significant attention due to its various applications including radar, sonar, surveillance, navigation, wireless communication, and sensor networks [1].

Modern location system often uses mobile platforms as receivers, for example airplanes, satellites, unmanned aerial vehicles (UAVs). Mobile platforms have the advantage of creating frequency difference of arrival (FDOA) due to the relative motion between the source and receivers, which can be exploited to improve the location accuracy [1]. Recently, Ho et al. [2] performed a theoretical study on the moving source location problem. With the knowledge of the receivers' position and velocity at the time of observation as well as the signal propagation speed, the source location can be described by the highly nonlinear equations related to TDOA and FDOA measurements. They also proposed a novel two-step weighted least squares (WLS) approach for this problem. It avoids convergence and initialization problems in the Taylor-series iterative method [3], which can attain the Cramr-Rao lower bound (CRLB) only at high signal-to-noise ratios (SNR).

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Starting from the WLS function, this paper derives an efficient constrained weighted least-squares (CWLS) algorithm for moving source location based on Newton's iterative method. The developed estimator here is different from the CWLS algorithm in [4], [5]. It is much more computationally attractive than the one in [4], [5] because it needs no estimation of the Lagrange multipliers, which is computationally burdensome. Furthermore, [4], [5] only examines TDOA location of stationary sources with fixed receivers, while the proposed approach can not only estimate the position of the source from TDOA-only observations, but also estimate the position and velocity of the moving source from TDOA and FDOA measurements.

II. MOVING SOURCE LOCATION PROBLEM

A scenario of M moving receivers is considered in a three-dimensional (3-D) space to determine the moving source with unknown position $\mathbf{u} = [x, y, z]^T$ and velocity $\dot{\mathbf{u}} = [\dot{x}, \dot{y}, \dot{z}]^T$ using TDOA and FDOA measurements. The receiver positions $\mathbf{s}_i = [x_i, y_i, z_i]^T$ and velocities $\dot{\mathbf{s}}_i = [\dot{x}_i, \dot{y}_i, \dot{z}_i]^T, i = 1, 2, \cdots, M$ are assumed known, where $(*)^T$ denotes the transpose of $(*)$. The location problem requires at least 4 receivers to produce three TDOAs and FDOAs simultaneously. This paper focuses on the overdetermined scenario where the number of receivers is larger than 4 [2].

Let the true distance between the emitter and receiver i be

$$r_i = \|\mathbf{u} - \mathbf{s}_i\|_2, \tag{1}$$

where $\| * \|_2$ represents the 2-norm, $i = 1, 2, \cdots, M$, and

$$r_{i1} = r_i - r_1, \quad i = 2, \cdots, M, \tag{2}$$

where r_{i1} is the range difference.

The time derivative of (1) gives the relationship between the range rate and source location parameters:

$$\dot{r}_i = \frac{(\dot{\mathbf{u}} - \dot{\mathbf{s}}_i)^T (\mathbf{u} - \mathbf{s}_i)}{r_i}. \tag{3}$$

To make use of FDOAs, taking the time derivative of the true range difference r_{i1} in (2) gives its rate $\dot{r}_{i1}$ as

$$\dot{r}_{i1} = \dot{r}_i - \dot{r}_1, \quad i = 2, \cdots, M, \tag{4}$$

In practice, we assume that the TDOAs $\mathbf{T}_d = [t_{21}, t_{31}, \cdots, t_{M1}]^T$ and FDOAs $\mathbf{F}_d = [f_{21}, f_{31}, \cdots, f_{M1}]^T$ estimates were generated by additive noise. The range difference and its rate are respectively proportional to the TDOA and FDOA in a constant-velocity propagation medium. This implies that the measurements of range difference $\mathbf{d} = [d_{21}, d_{31}, \cdots, d_{M1}]^T$ and its rate $\dot{\mathbf{d}} = [\dot{d}_{21}, \dot{d}_{31}, \cdots, \dot{d}_{M1}]^T$

resulted from the noisy TDOAs and FDOAs can be expressed [1] as

$$\mathbf{d} = c \cdot \mathbf{T}_d = \mathbf{r} + \mathbf{e}, \tag{5}$$

$$\dot{\mathbf{d}} = c \cdot \mathbf{F}_d / f_0 = \dot{\mathbf{r}} + \dot{\mathbf{e}}, \tag{6}$$

where $\mathbf{e} = [e_{21}, e_{31}, \cdots, e_{M1}]^T$, $\dot{\mathbf{e}} = [\dot{e}_{21}, \dot{e}_{31}, \cdots, \dot{e}_{M1}]^T$ are corresponding measurement noise vectors, f_0 is the carrier frequency and c is the signal propagation speed.

III. CWLS Algorithm for TDOA and FDOA Location

The solution derivation begins with defining an auxiliary vector $\mathbf{\Theta} = [\mathbf{v}^T, r_1, \dot{\mathbf{v}}^T, \dot{r}_1]^T$, where $\mathbf{v} = \mathbf{u} - \mathbf{s}_1$, $\dot{\mathbf{v}} = \dot{\mathbf{u}} - \dot{\mathbf{s}}_1$. It contains the unknown source location parameters and two intermediate variables $r_1, \dot{r}_1$, which can be used to reorganize the corresponding TDOA and FDOA nonlinear equations into a set of pseudolinear ones so that a closed-form estimate of the unknowns can be generated [6].

The solution development begins with the relationship that the range difference is $r_i = r_{i1} + r_1$. Put the expressions of r_i and r_1 into $r_i = r_{i1} + r_1$, and then squared its both sides, the TDOA equation can be expressed as:

$$r_{i1}^2 + 2r_{i1}r_1 = (\mathbf{s}_i - \mathbf{s}_1)^T(\mathbf{s}_i - \mathbf{s}_1) - 2(\mathbf{s}_i - \mathbf{s}_1)^T(\mathbf{u} - \mathbf{s}_1). \tag{7}$$

To make use of FDOAs, taking the time derivative of (7):

$$\begin{aligned} 2(r_{i1}\dot{r}_{i1} + \dot{r}_{i1}r_1 + r_{i1}\dot{r}_1) &= 2(\dot{\mathbf{s}}_i - \dot{\mathbf{s}}_1)^T(\mathbf{s}_i - \mathbf{s}_1) \\ &- 2(\dot{\mathbf{s}}_i - \dot{\mathbf{s}}_1)^T(\mathbf{u} - \mathbf{s}_1) - 2(\mathbf{s}_i - \mathbf{s}_1)^T(\dot{\mathbf{u}} - \dot{\mathbf{s}}_1). \end{aligned} \tag{8}$$

Substituting $r_{i1} = d_{i1} - e_{i1}$ and $\dot{r}_{i1} = \dot{d}_{i1} - \dot{e}_{i1}$ into (7), (8), and neglecting the second-order error term, then

$$\begin{aligned} 2r_i e_{i1} &= d_{i1}^2 + 2d_{i1}r_1 \\ &- (\mathbf{s}_i - \mathbf{s}_1)^T(\mathbf{s}_i - \mathbf{s}_1) + 2(\mathbf{s}_i - \mathbf{s}_1)^T(\mathbf{u} - \mathbf{s}_1), \\ 2(\dot{r}_i e_{i1} + r_i \dot{e}_{i1}) &= 2(d_{i1}\dot{d}_{i1} + \dot{d}_{i1}r_1 + d_{i1}\dot{r}_1) \\ &- 2(\dot{\mathbf{s}}_i - \dot{\mathbf{s}}_1)^T(\mathbf{s}_i - \mathbf{s}_1) \\ &+ 2(\dot{\mathbf{s}}_i - \dot{\mathbf{s}}_1)^T(\mathbf{u} - \mathbf{s}_1) + 2(\mathbf{s}_i - \mathbf{s}_1)^T(\dot{\mathbf{u}} - \dot{\mathbf{s}}_1). \end{aligned} \tag{9}$$

Collecting all items in (9) for each $i, i = 1, 2, \cdots, M$ and forming

$$\mathbf{D}[\mathbf{e}^T, \dot{\mathbf{e}}^T]^T = \mathbf{A\Theta} - \mathbf{b}, \tag{10}$$

where

$$\mathbf{A} = 2\begin{bmatrix} (\mathbf{s}_2 - \mathbf{s}_1)^T & d_{21} & \mathbf{0}^T & 0 \\ \vdots & \vdots & \vdots & \vdots \\ (\mathbf{s}_M - \mathbf{s}_1)^T & d_{M1} & \mathbf{0}^T & 0 \\ (\dot{\mathbf{s}}_2 - \dot{\mathbf{s}}_1)^T & \dot{d}_{21} & (\mathbf{s}_2 - \mathbf{s}_1)^T & d_{21} \\ \vdots & \vdots & \vdots & \vdots \\ (\dot{\mathbf{s}}_M - \dot{\mathbf{s}}_1)^T & \dot{d}_{M1} & (\mathbf{s}_M - \mathbf{s}_1)^T & d_{M1} \end{bmatrix}, \tag{11}$$

$$\mathbf{b} = \begin{bmatrix} (\mathbf{s}_2 - \mathbf{s}_1)^T(\mathbf{s}_2 - \mathbf{s}_1) - d_{21}^2 \\ \vdots \\ (\mathbf{s}_M - \mathbf{s}_1)^T(\mathbf{s}_M - \mathbf{s}_1) - d_{M1}^2 \\ (2\dot{\mathbf{s}}_2 - \dot{\mathbf{s}}_1)^T(\mathbf{s}_2 - \mathbf{s}_1) - 2d_{21}\dot{d}_{21} \\ \vdots \\ (2\dot{\mathbf{s}}_M - \dot{\mathbf{s}}_1)^T(\mathbf{s}_M - \mathbf{s}_1) - 2d_{M1}\dot{d}_{M1} \end{bmatrix}, \tag{12}$$

$$\mathbf{D} = \begin{bmatrix} \mathbf{B} & \mathbf{O} \\ \dot{\mathbf{B}} & \mathbf{B} \end{bmatrix}, \tag{13}$$

where $\mathbf{B} = 2\mathrm{diag}\{r_2, r_3, \cdots, r_M\}$, $\dot{\mathbf{B}} = 2\mathrm{diag}\{\dot{r}_2, \dot{r}_3, \cdots, \dot{r}_M\}$, and $\mathbf{O}$ is a zero square matrix of appropriate size.

Although (10) is a linear equation in $\mathbf{\Theta}$, the intermediate variables r_1, $\dot{r}_1$ are, in fact, related to $\mathbf{v}$ and $\dot{\mathbf{v}}$.Taking this constraint into account is necessary. The weighted least-squares solution can be obtained by minimizing

$$J(\mathbf{\Theta}) = (\mathbf{A\Theta} - \mathbf{b})^T\mathbf{W}^{-1}(\mathbf{A\Theta} - \mathbf{b}) \quad \text{subject to} \quad r_1^2 = \mathbf{v}^T\mathbf{v}, \dot{r}_1 r_1 = \dot{\mathbf{v}}^T\mathbf{v}, \tag{14}$$

where $\mathbf{W}$ is a symmetric weighting matrix. For sufficiently small measurements errors, the optimum weighting matrix for the CWLS algorithm is found by using the best linear unbiased estimator (BLUE) as in [2]:

$$\mathbf{W}^{-1} = [(\mathbf{Dn})(\mathbf{Dn})^T]^{-1} = (\mathbf{DQD}^T)^{-1}, \tag{15}$$

where $\mathbf{Q}$ is the covariance matrix of $\mathbf{n} = [\mathbf{e}^T, \dot{\mathbf{e}}^T]^T$.

Dividing $\mathbf{A}$ as $[\mathbf{A}_{13}, \mathbf{A}_4, \mathbf{A}_{57}, \mathbf{A}_8]$, where $\mathbf{A}_{13}, \mathbf{A}_4, \mathbf{A}_{57}$, $\mathbf{A}_8$ denote the first three columns, the fourth column, the fifth to seventh columns and the last column of $\mathbf{A}$ respectively. Defining $\mathbf{x} = [\mathbf{v}^T, \dot{\mathbf{v}}^T]^T$, (14) can be rewritten as

$$J(\mathbf{\Theta}) = \mathbf{g}^T\mathbf{W}^{-1}\mathbf{g} = J(\mathbf{x}) \tag{16}$$

where $\mathbf{g} = \mathbf{A}_{13}\mathbf{v} + \mathbf{A}_4\sqrt{\mathbf{v}^T\mathbf{v}} + \mathbf{A}_{57}\dot{\mathbf{v}} + \mathbf{A}_8\dot{\mathbf{v}}^T\mathbf{v}/\sqrt{\mathbf{v}^T\mathbf{v}} - \mathbf{b}$.

To find the CWLS solution, we must minimize $J(\mathbf{x})$ with respect to $\mathbf{x}$ which is a nonlinear minimization problem. It is difficult to yield an explicit solution by minimizing (16) with analytical methods. Numerical methods can be applied to solve these nonlinear equations. When applied to finding the minimum of a function, Newton's method has order-two convergence if the function to be minimized is twice differentiable [6].

The Newton iteration is given by

$$\mathbf{x} = \mathbf{x}_0 - \mathbf{Hess}^{-1}\mathbf{J}_{\mathbf{x}}, \tag{17}$$

where $\mathbf{J}_{\mathbf{x}} = \partial J(\mathbf{x}_0)/\partial\mathbf{x}$, and $\mathbf{Hess} = \partial^2 J(\mathbf{x}_0)/\partial\mathbf{x}\partial\mathbf{x}^T$ are respectively the gradient and Hessian of $J(\mathbf{x})$ at $\mathbf{x}_0$. So the object function of the proposed algorithm is greatly different from the object function of the method in [Section III.B, 1], and the proposed algorithm in the paper is based on second-order Taylor-series expansion of, ignoring the terms higher than second.

The gradient and Hessian of $J(\mathbf{x})$ can be respectively expressed as

$$\frac{\partial J(\mathbf{x})}{\partial\mathbf{x}} = \frac{\partial\mathbf{g}^T\mathbf{W}^{-1}\mathbf{g}}{\partial\mathbf{x}} = \mathbf{h}(\mathbf{W}^{-1} + \mathbf{W}^{-T})\mathbf{g}, \tag{18}$$

$$\frac{\partial J^2(\mathbf{x})}{\partial\mathbf{x}\partial\mathbf{x}^T} = \mathbf{h}(\mathbf{W}^{-1} + \mathbf{W}^{-T})\mathbf{h}^T + \mathbf{\Psi\Upsilon}, \tag{19}$$

where

$$\begin{aligned} \mathbf{h} &= \frac{\partial\mathbf{g}^T}{\partial\mathbf{x}} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \end{bmatrix}, \\ \mathbf{\Psi} &= \mathbf{g}(\mathbf{W}^{-1} + \mathbf{W}^{-T}) \otimes \mathbf{I}_6, \\ \mathbf{\Upsilon} &= \frac{\partial}{\partial\mathbf{x}}\Big[\mathrm{vec}\Big(\frac{\partial\mathbf{g}^T}{\partial\mathbf{x}}\Big)\Big] = \begin{bmatrix} \mathbf{\Upsilon}_1 & \mathbf{\Upsilon}_2 \\ \mathbf{\Upsilon}_2 & \mathbf{O} \end{bmatrix}, \end{aligned} \tag{20}$$

where

$$\begin{aligned} \mathbf{h}_1 &= \mathbf{A}_{13}^T + (\mathbf{v}^T\mathbf{v})^{-1/2}\mathbf{v}\mathbf{A}_4^T \\ &+ (\mathbf{v}^T\mathbf{v})^{-1/2}\dot{\mathbf{v}}\mathbf{A}_8^T - (\dot{\mathbf{v}}^T\mathbf{v})(\mathbf{v}^T\mathbf{v})^{-3/2}\mathbf{v}\mathbf{A}_8^T, \end{aligned} \tag{21}$$

TABLE I
POSITIONS AND VELOCITIES OF RECEIVERS

Receiver no. i	x_i	y_i	z_i	$\dot{x}_i$	$\dot{y}_i$	$\dot{z}_i$
1	300	100	150	30	-20	20
2	400	150	100	-30	10	20
3	300	500	200	10	-20	10
4	350	200	100	10	20	30
5	-100	-100	-100	-20	10	10

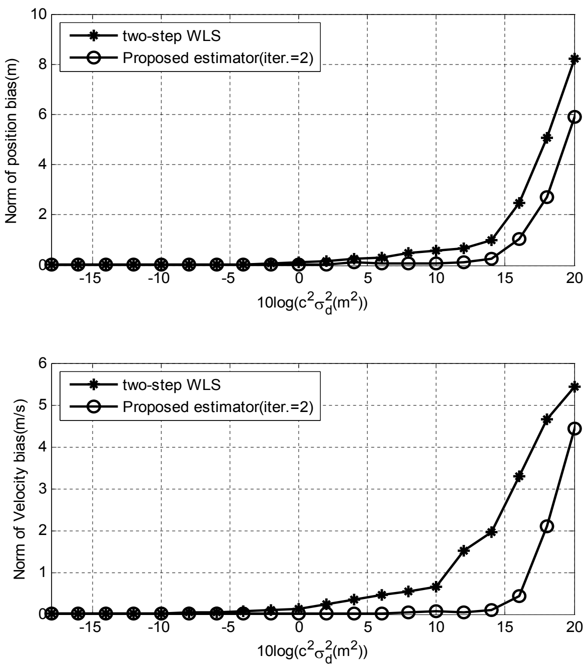

Fig. 1. Comparison of the estimation bias of the proposed estimator with the two-step WLS method versus measurement error for near-field source located at (285 325 275) m.

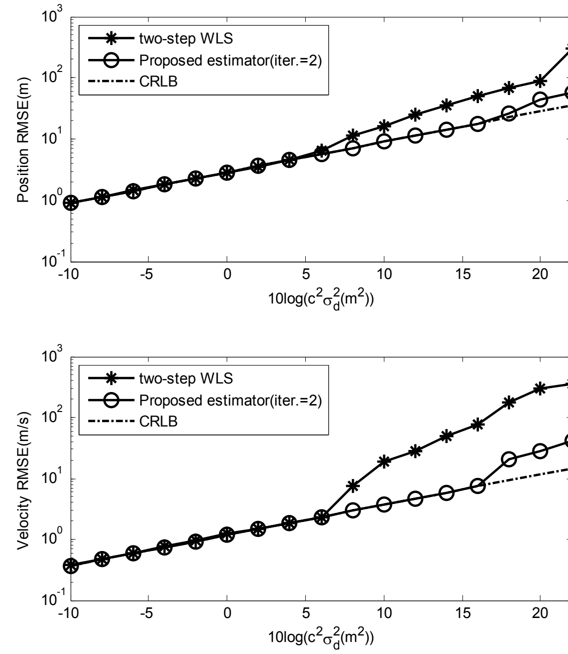

Fig. 2. Comparison of the RMSE of the proposed estimator with the two-step WLS method versus measurement error for near-field source located at (285 325 275) m.

$$\mathbf{h}_2 = \mathbf{A}_{57}^T + (\mathbf{v}^T\mathbf{v})^{-1/2}\mathbf{v}\mathbf{A}_8^T, \tag{22}$$

$$\begin{aligned}\boldsymbol{\Upsilon}_1 &= (\mathbf{v}^T\mathbf{v})^{-1/2}(\mathbf{I}_3 \otimes \mathbf{A}_4) - (\mathbf{v}^T\mathbf{v})^{-3/2}(\mathbf{v} \otimes \mathbf{A}_4)\mathbf{v}^T \\ &- (\mathbf{v}^T\mathbf{v})^{-3/2}(\dot{\mathbf{v}} \otimes \mathbf{A}_8)\mathbf{v}^T + (\dot{\mathbf{v}}^T\mathbf{v})(\mathbf{v}^T\mathbf{v})^{-5/2}(\mathbf{v} \otimes \mathbf{A}_8)\mathbf{v}^T \\ &- (\mathbf{v}^T\mathbf{v})^{-3/2}(\mathbf{v} \otimes \mathbf{A}_8)\dot{\mathbf{v}}^T - (\dot{\mathbf{v}}^T\mathbf{v})(\mathbf{v}^T\mathbf{v})^{-3/2}(\mathbf{I}_3 \otimes \mathbf{A}_8), \\ \boldsymbol{\Upsilon}_2 &= (\mathbf{v}^T\mathbf{v})^{-1/2}(\mathbf{I}_3 \otimes \mathbf{A}_8) - (\mathbf{v}^T\mathbf{v})^{-3/2}(\mathbf{v} \otimes \mathbf{A}_8)\mathbf{v}^T,\end{aligned} \tag{23}$$

and $\mathbf{O}$ is a zero square matrix of appropriate size.

A critical problem with Newton's method is that it cannot guarantee the convergence of the iteration. Simulations in section IV illustrate that iterating (17) twice is sufficient to obtain an accurate solution, and the non-convergence seldom arises if choosing the LS solution $\boldsymbol{\Theta} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b}$ as the initial estimation $\mathbf{x}_0$ in the case of small measurement noise. And in the case of large measurement noise level, the LS solution may be too noisy to be chosen as the initial estimation, so choosing the LCLS solution in [7] as the initial estimation can satisfy the demand.

IV. SIMULATIONS

This section presents a set of Monte Carlo simulations to corroborate the theoretical development and to evaluate the performance of the proposed algorithm by comparing the estimation bias, estimation accuracy and computational complexity of it with the two-step WLS method [2]. Two repetitions are applied for the two-step WLS method. In the following simulation scenarios, the unit for the positions is meters, and that for the velocities is meters per second.

An array of five receivers is chosen as the geometry in [2], the positions and velocities of receivers are listed in Table I. The TDOA and FDOA estimates are generated by adding the zero mean Gaussian noise to the true values. The covariance matrices of TDOA and FDOA are $\sigma_d^2\mathbf{R}$ and $0.1\sigma_d^2\mathbf{R}$, where $\mathbf{R}$ was set to 1 in the diagonal elements and 0.5 otherwise, and σ_d^2 is the variance of the measurement noise [2]. The TDOA and FDOA noises are uncorrelated. Thus, we have $\mathbf{Q} = \mathrm{diag}\{\sigma_d^2\mathbf{R}, 0.1\sigma_d^2\mathbf{R}\}$. The estimation accuracy is investigated for source as the measurement errors increase. The estimation bias and accuracy are investigated for source as measurement errors increases. The estimation bias in terms of norm of estimation bias is defined as $\mathrm{bias}(\mathbf{u}) = \|\frac{1}{K}\sum_{k=1}^{K}\mathbf{u}_k - \mathbf{u}^o\|_2$ for position and $\mathrm{bias}(\dot{\mathbf{u}}) = \|\frac{1}{K}\sum_{k=1}^{K}\dot{\mathbf{u}}_k - \dot{\mathbf{u}}^o\|_2$ for velocity, and the estimation accuracy in terms of the root mean squares error (RMSE) is defined as $\mathrm{RMSE}(\mathbf{u}) = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\|\mathbf{u}_k - \mathbf{u}^o\|_2}$ for position and $\mathrm{RMSE}(\dot{\mathbf{u}}) = \sqrt{\frac{1}{K}\sum_{k=1}^{K}\|\dot{\mathbf{u}}_k - \dot{\mathbf{u}}^o\|_2}$ for velocity, where $\mathbf{u}^o$ and $\dot{\mathbf{u}}^o$ express the true position and velocity of the source, and $\mathbf{u}_k$, $\dot{\mathbf{u}}_k$ denote the estimated source position and velocity at ensemble k, and K=10 000, which is the number of ensemble runs.

The first simulation is concerned with near-field source location. The true position and velocity of the source were $\mathbf{u} = [285, 325, 275]^T$ and $\dot{\mathbf{u}} = [-20, 15, 40]^T$.

In Fig. 1, the estimation results clearly demonstrate the biases of the proposed CWLS method are nonetheless significantly smaller than the two-step WLS method especially for higher measurement noise level.

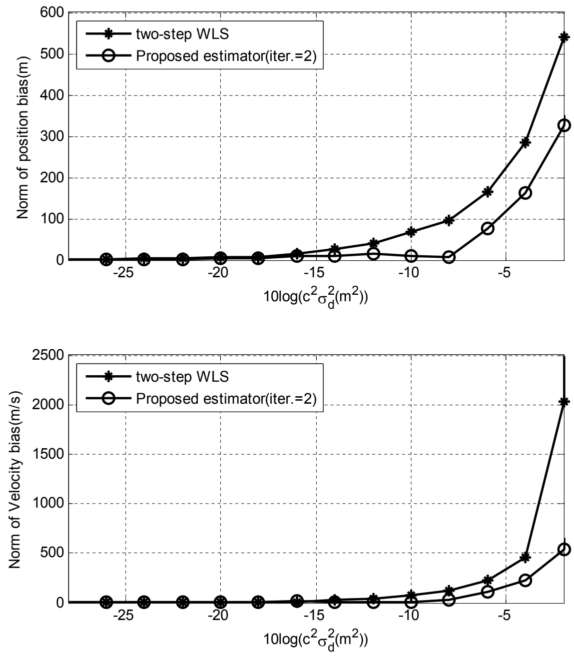

Fig. 3. Comparison of the estimation bias of the proposed estimator with the two-step WLS method versus measurement error for far-field source located at (2000 2500 3000) m.

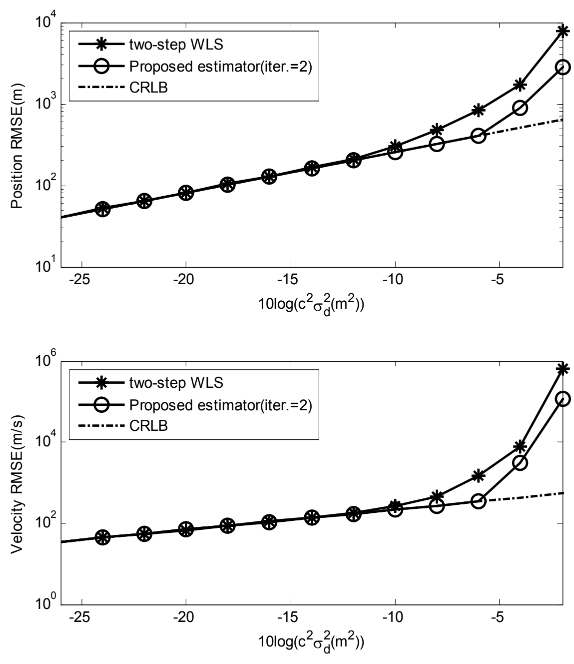

Fig. 4. Comparison of the RMSE of the proposed estimator with the two-step WLS method versus measurement error for far-field source located at (2000 2500 3000) m.

Fig. 2 shows the accuracy of position and velocity estimate of the proposed method in terms of RMSE as the noise power increases, compared with the TDOA/FDOA location algorithm two-step WLS, as well as CRLB. For the source position and velocity estimations the two-step WLS method departs precipitously from the CRLB at a noise power about 6 dB, while the proposed estimator gives inaccurate estimate at the noise power about 16 dB. The threshold effect of the proposed method occurs at a noise power that is about 10 dB, which is later than that of the two-step WLS method as the noise power increases. This appealing feature benefits the cases with large TDOA and FDOA measurement errors, in contrast to the two-step WLS approach.

Figs. 3 and 4 depicts the results for a far-field source at $\mathbf{u} = [2000, 2500, 3000]^T$ with velocity of $\dot{\mathbf{u}} = [-20, 15, 40]^T$. The proposed estimator is superior to the two-step WLS method in terms of estimation bias and RMSE for estimating the position and velocity of a far-field moving source.

The estimation bias and accuracy is two of the metrics to evaluate the performance of the proposed algorithm, and computational complexity is another important metric to evaluate the performance of algorithms. The price of the better performance for the developed approach is that its computational complexity is a bit larger than two-step WLS method. In the same simulation condition, the time of 10000 independent runs for two-step WLS algorithm and the proposed algorithm with two iterations are approximately 14.0315 s and 18.6228 s, respectively. It is obviously that the proposed algorithm has the same simplicity and can be easily implemented in a real-time system as the two-step WLS method.

V. Conclusion

In this paper, an efficient CWLS method for estimating the position and velocity of a moving source based on TDOA and FDOA measurements from an array of receivers is proposed. This method does not have convergence and initialization problems, as in the conventional linear iterative method. It has the same simplicity and can be easily implemented in a real-time system as the two-step WLS method, but the threshold effect of it occurs apparently later than the two-step weighted least square method, as the measurement noise increases. The accuracy of the estimate achieves the CRLB at a sufficiently high noise level before the threshold effect occurs. This feature is appealing for both moderate and high measurement noise levels in practice.

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