[3] H. Yesou, Y. Besnus, and J. Rolet, "Extraction of spectral information from landsat TM data and merger with SPOT panchromatic imagery—A contribution to the study of geological structures," *ISPRS J. Photogrammetry Remote Sens.*, vol. 48, no. 5, pp. 23–36, 1993.
[4] H. Li, B. S. Manjunathand, and S. K. Mitra, "Multisensor image fusion using the wavelet transform," *Graph. Models Image Process.*, vol. 57, no. 3, pp. 235–245, 1995.
[5] N. E. Huang, Z. Shen, S. R. Long, M. L. Wu, H. H. Shih, Z. Quanan, N. C. Yen, C. C. Tung, and H. H. Liu, "The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis," *Proc. Roy. Soc. A*, vol. 454, pp. 903–995, 1998.
[6] J. W. Tukey, *Exploratory Data Analysis*. Reading, MA: Addison-Wesley, 1977.
[7] M. Datig and T. Schlurmann, "Performance and limitations of the Hilbert-Huang Transformation (HHT) with an application to irregular water waves," *Ocean Eng.*, vol. 31, pp. 1783–1834, Oct. 2004.
[8] T. M. Rutkowski, R. Zdunek, and A. Cichocki, "Multichannel EEG brain activity pattern analysis in time-frequency domain with nonnegative matrix factorization support," in *Int. Congr. Series*, 2007, vol. 1301, pp. 266–269.
[9] R. R. Zhang, S. Ma, E. Safak, and S. Hartzell, "Hilbert-Huang transform analysis of dynamic and earthquake motion recordings," *J. Eng. Mech.*, vol. 129, pp. 861–875, 2003.
[10] D. P. Mandic, M. Golz, A. Kuh, D. Obradovic, and T. Tanaka, *Signal Processing Techniques for Knowledge Extraction and Information Fusion*. New York: Springer, 2008.
[11] D. Looney and D. P. Mandic, "A machine learning enhanced empirical mode decomposition," in *Proc. Int. Conf. Acoustics, Speech, Signal Processing (ICASSP)*, 2008, pp. 1897–1900.
[12] D. Looney, L. Li, T. Rutkowski, D. P. Mandic, and A. Cichocki, "Ocular artifacts removal from EEG using EMD," presented at the 1st Int. Conf. Cognitive Neurodynamics, Shanghai, China, Nov. 2007.
[13] H. Hariharan, A. Gribok, M. A. Abidi, and A. Koschan, "Image fusion and enhancement via empirical mode decomposition," *J. Pattern Recognit. Res. (JPRR)*, vol. 1, no. 1, pp. 16–32, 2006.
[14] X. Xu, H. Li, and A. N. Wang, "The application of BEMD to multispectral image fusion," in *Proc. Int. Conf. Wavelet Analysis Pattern Recognition (ICWAPR)*, 2007, vol. 1, pp. 448–452.
[15] M. U. B. Altaf, T. Gautama, T. Tanaka, and D. P. Mandic, "Rotation invariant complex empirical mode decomposition," in *Proc. Int. Conf. Acoustics, Speech, Signal Processing (ICASSP)*, 2007, vol. 3, pp. 1009–1012.
[16] G. Rilling, P. Flandrin, P. Goncalves, and J. M. Lilly, "Bivariate empirical mode decomposition," *IEEE Signal Process. Lett.*, vol. 14, pp. 936–939, 2007.
[17] Z. Wu and N. E. Huang, "Ensemble empirical mode decomposition: A noise-assisted data analysis method," Center for Ocean-Land-Atmosphere Studies, Tech. Rep. 193, 2004.
[18] T. Tanaka and D. P. Mandic, "Complex empirical mode decomposition," *IEEE Signal Process. Lett.*, vol. 14, pp. 101–104, Feb. 2007.

Semidefinite Programming Approach for Range-Difference Based Source Localization

Kenneth Wing Kin Lui, Frankie Kit Wing Chan, and H. C. So

***Abstract*—A common technique for passive source localization is to utilize the range-difference (RD) measurements between the source and several spatially separated sensors. The RD information defines a set of hyperbolic equations from which the source position can be calculated with the knowledge of the sensor positions. Under the standard assumption of Gaussian distributed RD measurement errors, it is well known that the maximum-likelihood (ML) position estimation is achieved by minimizing a multimodal cost function which corresponds to a difficult task. In this correspondence, we propose to approximate the nonconvex ML optimization by relaxing it to a convex optimization problem using semidefinite programming. A semidefinite relaxation RD-based positioning algorithm, which makes use of the admissible source position information, is proposed and its estimation performance is contrasted with the two-step weighted least squares method and nonlinear least squares estimator as well as Cramér–Rao lower bound.**

***Index Terms*—Range-difference measurements, semidefinite programming, source localization, time-delay estimation.**

I. Introduction

Source localization using measurements from an array of spatially separated sensors has received significant attention in the signal processing literature because of its important applications such as navigation [1], wireless communications [2], telecommunications [3], and sensor networks [4]. A common positioning approach is to use the time-difference-of-arrival (TDOA) measurements, that is, the differences in arrival times between pairs of sensor outputs which receive the radiated signal, assuming that the signal propagation speed is a known constant. Multiplying the TDOA by the propagation speed yields the range-difference (RD) and a hyperbola on which the source must lie can be formed. At least three sensors are needed to uniquely estimate the source position in the two-dimensional (2-D) plane while four or more are required for three-dimensional (3-D) localization, with the use of the knowledge of the sensor array geometry.

Basically, there are two approaches for source localization using the hyperbolic equations constructed from the RD measurements. The first approach is based on the nonlinear least-squares (NLS) framework [5]–[7] where Taylor-series expansion is utilized for linearization and the solution is solved in an iterative manner. Under the standard assumption that the RD measurements are Gaussian distributed, the global minimum of the multimodal NLS cost function corresponds to the maximum-likelihood (ML) position estimate. Although optimum estimation performance can be attained, it requires sufficiently precise initial estimates for the global solution, which indicates the difficulty of this approach because of the possibility of local convergence. The second approach is to reorganize the nonlinear equations into a set of linear equations [8]–[12] by squaring them and introducing an extra variable that is a function of the source position, so that global

Manuscript received August 15, 2008; revised November 02, 2008. First published December 09, 2008; current version published March 11, 2009. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Shahram Shahbazpanahi. The work described in this paper was supported by a grant from CityU (Project No. 7002132).

The authors are with the Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong (e-mail: hcso@ee.cityu.edu.hk).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org

Digital Object Identifier 10.1109/TSP.2008.2010599

convergence is ensured. In particular, this reorganization idea is first introduced in [8], [9] while later works of [10]–[12] utilize the relationship between the extra variable and the source position to achieve better estimation. Nevertheless, the reorganized linear equations are only valid at sufficiently small noise conditions for which the optimality of this approach holds [10].

In this correspondence, we consider the first approach by relaxing the nonconvex ML optimization to a convex optimization problem via semidefinite programming [13]–[15]. A semidefinite relaxation (SDR) RD-based positioning algorithm is proposed and this work can be considered as a follow-up of our previous paper [16] which tackles the range-based source localization using the SDR approach. It is worthy to point out that the SDR approach has been recently utilized for node localization in sensor networks [17]–[22], which addresses localization of multiple sources and generalizes the single source positioning problem in [16]. The work of [17] addresses the general graph realization problem of which sensor network localization is a special case. The upper and lower bounds on the corresponding SDR objective function are derived and performance improvements using regularization and gradient-descent technique are proposed. While [18] focuses on the ML and interval formulations for range-based node positioning. In [19], sensor network localization is modeled as the nearest Euclidean distance matrix completion problem and a robust prime-dual interior-point algorithm for performing SDR is suggested in [20]. Recently, Meng *et al.* [21] apply minimax approximation in the SDR formulation to produce a computationally efficient node positioning algorithm. Furthermore, localization with angle-of-arrival (AOA) measurements has been studied in [22]. Although it seems that the SDR methodology has been well applied in localization applications, most of the proposed methods deal with range [17]–[21] or AOA [22] measurements. To the best of our knowledge, SDR positioning with RD measurements has not been addressed to date in the literature.

The rest of the paper is organized as follows. In Section II, the basic SDR algorithm for RD-based positioning is first derived via approximation of the corresponding nonconvex ML optimization problem. Noting that the constraints in the basic algorithm are not tight as in the range-based scenarios, we devise additional constraints to remedy the SDR formulation. Their development is based on upper and lower bounds for the distances between the source and sensors as well as source location, assuming that the admissible source position is available. Simulation results are presented in Section III to evaluate the location estimation performance of the proposed estimator by comparing with the two-step weighted least squares (WLS) method [10], NLS estimator as well as Cramér–Rao lower bound (CRLB) [11]. Finally, conclusions are drawn in Section IV.

II. Algorithm Development

First of all, we would like to introduce the notation used in this paper. Bold upper case symbols denote matrices and bold lower case symbols denote vectors. The $\mathbf{0}_{m\times n}$ is the $m \times n$ zero matrix, $\mathbf{I}_m$ is the $m \times m$ identity matrix, ^{T} denotes transpose operator and $\|\mathbf{x}\|_2$ represents the 2-norm of a vector $\mathbf{x}$. For two symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A}-\mathbf{B}$ is positive semidefinite. Consider the RD-based localization problem in the 3-D space. Let $\mathbf{x}=[x,y,z]^T$ and $\mathbf{x}_i=[x_i,y_i,z_i]^T, i=1,2,\cdots,M$, be the unknown source position and known sensor positions with $M\geq 4$, respectively. Without loss of generality, let the first sensor be the reference. With known propagation speed, the TDOA information can be easily converted to RD measurements with respect to $\mathbf{x}_1$, denoted by d_{i1}, which are modeled as:

$$d_{i1}=\|\mathbf{x}-\mathbf{x}_i\|_2-\|\mathbf{x}-\mathbf{x}_1\|_2+q_{i1},\quad i=2,3,\cdots,M \tag{1}$$

The error in the RD q_{i1} is a zero-mean Gaussian process and the covariance of $\mathbf{q}=[q_{21},q_{31},\cdots,q_{M1}]^T$ is denoted by $\mathbf{\Sigma}=E\{\mathbf{q}\mathbf{q}^T\}$ where E is the expectation operator, is assumed known up to a scalar. It is noteworthy that in practice, we usually have $M(M-1)/2$ TDOA measurements among all pairs of sensors at the beginning and they will then be optimally processed [23]–[26] to produce $(M-1)$ TDOA estimates relative to the reference sensor which are more accurate. The task is to find $\mathbf{x}$ using the $(M-1)$ RD measurements $\{d_{i1}\}$.

Based on the above problem formulation, it is well known [5]–[7] that the ML estimate for $\mathbf{x}$ is achieved by minimizing the following NLS cost function:

$$\min_{\mathbf{x}}\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}\left(d_{i1}-\|\mathbf{x}-\mathbf{x}_i\|_2+\|\mathbf{x}-\mathbf{x}_1\|_2\right)\times\left(d_{j1}-\|\mathbf{x}-\mathbf{x}_j\|_2+\|\mathbf{x}-\mathbf{x}_1\|_2\right) \tag{2}$$

where

$$\mathbf{W}=\begin{bmatrix}0 & \mathbf{0}_{1\times(m-1)}\\ \mathbf{0}_{(m-1)\times 1} & \mathbf{\Sigma}^{-1}\end{bmatrix} \tag{3}$$

with w_{ij} denotes the (i,j) entry of the positive definite matrix $\mathbf{W}$. Now we transform (2) into a convex optimization as follows. Define $r_i=\|\mathbf{x}-\mathbf{x}_i\|_2$ as the distance between the source and the ith sensor and $\mathbf{r}=[r_1,r_2,\cdots,r_M]$. Expanding (2) and dropping the terms which have no effects on the minimization, ML estimation can be written as the following constrained optimization problem:

$$\begin{aligned}\min_{\mathbf{x},\mathbf{r}}&\left\{\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}(r_ir_j-2r_1r_j-2d_{i1}r_j)\right.\\&\left.+r_1\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}(d_{i1}+d_{j1})+r_1^2\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}\right\}\\ \text{s.t.}\quad & r_i=\|\mathbf{x}-\mathbf{x}_i\|_2.\end{aligned} \tag{4}$$

We further let $r_ir_j=r_{ij}$ be the (i,j) entry of $\mathbf{R}$ to re-express (4) as

$$\begin{aligned}\min_{\mathbf{x},\mathbf{r},\mathbf{R}}&\left\{\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}(r_{ij}-2r_{1j}-2d_{i1}r_j)\right.\\&\left.+r_1\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}(d_{i1}+d_{j1})+r_{11}\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}\right\}\\ \text{s.t.}\quad & \mathbf{R}=\mathbf{r}\mathbf{r}^T\\ & r_{ii}=\mathbf{x}^T\mathbf{x}+\mathbf{x}_i^T\mathbf{x}_i-2\mathbf{x}_i^T\mathbf{x},\quad i=1,2,\cdots,M\\ & r_i\geq 0,\quad i=1,2,\cdots,M\end{aligned} \tag{5}$$

so that the objective function in (5) is linear. We further assume that all $\{d_{i1}\}$ are non-negative, which are valid when the sensor closest to the source is chosen as the reference. Since the sign of r_j must follow that of d_{i1} in the minimization process, dropping the last inequality constraint does not affect the ML optimization. We then introduce a dummy variable y of the form $y=\mathbf{x}^T\mathbf{x}$ to transform $r_{ii}=\mathbf{x}^T\mathbf{x}+\mathbf{x}_i^T\mathbf{x}_i-2\mathbf{x}_i^T\mathbf{x}$, $i=1,2,\cdots,M$, to linear and convex constraints. Using the SDR principle, we relax the constraints $\mathbf{R}=\mathbf{r}\mathbf{r}^T$ as $\mathbf{R}\succeq\mathbf{r}\mathbf{r}^T$ and $y=\mathbf{x}^T\mathbf{x}$ as $y\geq\mathbf{x}^T\mathbf{x}$, respectively, to approximate (5) as a convex optimization problem:

$$\begin{aligned}\min_{\mathbf{x},\mathbf{r},\mathbf{R},y}&\left\{\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}(r_{ij}-2r_{1j}-2d_{i1}r_j)\right.\\&\left.+r_1\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}(d_{i1}+d_{j1})+r_{11}\sum_{i=2}^{M}\sum_{j=2}^{M}w_{ij}\right\}\\ \text{s.t.}\quad & \begin{bmatrix}\mathbf{R} & \mathbf{r}\\ \mathbf{r}^T & 1\end{bmatrix}\succeq\mathbf{0}_{(m+1)\times(m+1)}\\ & \begin{bmatrix}y & \mathbf{x}^T\\ \mathbf{x} & \mathbf{I}_3\end{bmatrix}\succeq\mathbf{0}_{4\times 4}\\ & r_{ii}=y+\mathbf{x}_i^T\mathbf{x}_i-2\mathbf{x}_i^T\mathbf{x},\quad i=1,2,\cdots,M.\end{aligned} \tag{6}$$

Equations (2)–(6) are analogous to the SDR development using range measurements [16]–[18]. As demonstrated in [16], the range-based SDR algorithm can provide approximate ML estimation performance as the corresponding constraints are tight. However, it is not true in (6) and we illustrate that $\mathbf{R} \succeq \mathbf{r}\mathbf{r}^T$ is not tight as follows. From $\mathbf{R} \succeq \mathbf{r}\mathbf{r}^T$, it is clear that the elements of $\mathbf{R}$ and $\mathbf{r}$ are not bounded above and below, respectively. In minimizing the objective function of (6), the values of $\{r_{ij}\}$ tend to decrease while those of $\{r_j\}$ are trying to increase to improve the tightness of $\mathbf{R} \succeq \mathbf{r}\mathbf{r}^T$. Nevertheless, the signs of $\{r_{1j}\}$ are negative and that of r_1 is positive in the objective function. This means that in the minimization process, the values of $\{r_{1j}\}$ and r_1 attempt to increase and decrease, respectively, which deteriorate the tightness of the inequality constraint. As a result, (6) cannot provide a high-quality solution.

In order to remedy the SDR formulation of (6), we devise additional constraints for $\{r_{ij}\}$ and $\{r_j\}$ to enhance the tightness of $\mathbf{R} \succeq \mathbf{r}\mathbf{r}^T$ by exploiting the admissible physical region of the source, denoted by $\mathcal{P}$, and the sensor geometry. Note that $\mathcal{P}$ should be available in many application scenarios such as speaker localization and mobile positioning. First we consider to form a line whose endpoints are the reference sensor and the ith sensor. The plane which is perpendicular to this line and contains its midpoint will divide the space into two regions: one includes the ith sensor and the other contains the reference sensor, denoted by $\mathcal{R}_i$ and $\mathcal{R}_{1i}$, respectively. Noting that the first sensor is closest to $\mathbf{x}$, a more compact region in which the source must be located, denoted by $\mathcal{R}$, is determined as $\mathcal{R} = \mathcal{R}_{12} \bigcap \mathcal{R}_{13} \bigcap \cdots \bigcap \mathcal{R}_{1M} \bigcap \mathcal{P}$, where $\bigcap$ is intersection operator. Note that the development of $\mathcal{R}$ is a novel idea. Then the lower and upper bounds for r_i, denoted by l_i and u_i, can be computed from $\{\mathbf{x}_i\}$ and $\mathcal{R}$:

$$l_i = \min_{\mathbf{p}\in\mathcal{R}} \{\|\mathbf{p}-\mathbf{x}_i\|_2\}, \quad i = 1, 2, \cdots, M \tag{7}$$

and

$$u_i = \max_{\mathbf{p}\in\mathcal{R}} \{\|\mathbf{p}-\mathbf{x}_i\|_2\}, \quad i = 1, 2, \cdots, M. \tag{8}$$

The constraints deduced from l_i and u_i are then

$$r_i - l_i \geq 0, \quad u_i - r_i \geq 0, \quad i = 1, 2, \cdots, M. \tag{9}$$

Multiplying each of the constraints involving r_i by a constraint involving r_j, and noting that $r_i r_j = r_{ij}$, we obtain

$$\begin{aligned} u_i u_j - u_i r_j - r_i u_j + r_{ij} &\geq 0, \quad i,j = 1, 2, \cdots, M, \quad i > j \\ l_i l_j - l_i r_j - r_i l_j + r_{ij} &\geq 0, \quad i,j = 1, 2, \cdots, M, \quad i > j \\ -u_i l_j + u_i r_j + r_i l_j - r_{ij} &\geq 0, \quad i,j = 1, 2, \cdots, M, \quad i > j \end{aligned} \tag{10}$$

which are also linear and convex constraints like (9) but now they are able to relate with $\mathbf{R}$ and thus will be useful to make $\mathbf{R} \succeq \mathbf{r}\mathbf{r}^T$ tighter. It is noteworthy that this constraint generation method also appears in the reformulation-linearization technique (RLT) [27] which can be used for solving nonconvex optimization problems.

In a similar manner, we produce additional constraints to strengthen the tightness of $y \geq \mathbf{x}^T\mathbf{x}$ as follows. The lower and upper limits for $\mathbf{x}$, denoted by $\mathbf{x}_l$ and $\mathbf{x}_u$, are

$$\begin{aligned} \mathbf{x}_l &= \arg\min_{\mathbf{p}\in\mathcal{R}} \{\mathbf{p}\} \\ \mathbf{x}_u &= \arg\max_{\mathbf{p}\in\mathcal{R}} \{\mathbf{p}\}. \end{aligned} \tag{11}$$

Thus, we have

$$\mathbf{x} - \mathbf{x}_l \geq 0, \qquad \mathbf{x}_u - \mathbf{x} \geq 0. \tag{12}$$

Applying the RLT relaxation to (12) and with the use of $y = \mathbf{x}^T\mathbf{x}$ yields

$$\begin{aligned} \mathbf{x}_u^T\mathbf{x}_u - \mathbf{x}_u^T\mathbf{x} - \mathbf{x}^T\mathbf{x}_u + y &\geq 0 \\ \mathbf{x}_l^T\mathbf{x}_l - \mathbf{x}_l^T\mathbf{x} - \mathbf{x}^T\mathbf{x}_l + y &\geq 0 \\ -\mathbf{x}_u^T\mathbf{x}_l + \mathbf{x}_u^T\mathbf{x} + \mathbf{x}^T\mathbf{x}_l - y &\geq 0. \end{aligned} \tag{13}$$

To summarize, adding the constraints of (10) and (13) to (6) gives our finalized SDR algorithm. It is worthy to point out that we do not impose (9) and (12) in the optimization process because they will enforce a hard decision on the location estimate which can lead to an infeasible problem. Rather than giving hard decisions on r_i and $\mathbf{x}$, we impose constraints on their corresponding dummy variables, namely, r_{ij} and y, and this will provide a robust estimate even though when the feasible region $\mathcal{R}$ is wrongly determined.

III. Simulation Results

Computation simulation has been performed to evaluate the RD-based localization performance of the proposed SDR algorithm in the 3-D case by comparing with the two-step WLS technique [10] and NLS estimator of (2) as well as CRLB [11]. The initial position estimate in the NLS method is provided by [10]. We adopt a package for specifying and solving convex programs, namely, CVX to implement the SDR algorithm [28], [29]. The RD errors are correlated Gaussian processes and its covariance matrix is of the form [10], [11]

$$\mathbf{\Sigma} = \sigma^2 \begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 2 \end{bmatrix}$$

where we modify the value of σ^2 to achieve different noise conditions. All results are averages of 500 independent runs. For the sake of simplicity to illustrate the computations of $\{l_i\}$, $\{u_i\}$, $\mathbf{x}_l$ and $\mathbf{x}_u$, we consider that there are 8 sensors and they are mounted on the eight corners of a room with dimension 6 m $\times$ 4 m $\times$ 2 m. That is, their positions are (0,0,0)m, (6,0,0)m, (0,4,0)m, (6,4,0)m, (0,0,2)m, (6,0,2)m, (0,4,2)m and (6,4,2)m, which are also the vertices of the rectangular prism $\mathcal{P}$. For this array geometry, $\mathcal{R}$ is the volume enclosed by $\{\mathbf{y}_i\}$, which are defined as

$$\mathbf{y}_i = \frac{\mathbf{x}_1 + \mathbf{x}_i}{2}, \quad i = 1, 2, \cdots, 8.$$

That is, $\mathcal{R}$ is also a rectangular prism with vertices $\{\mathbf{y}_i\}$. The bounds for the ranges, namely, l_i and u_i will be

$$\begin{aligned} l_i &= \min_{\mathbf{p}\in\{\mathbf{y}_1,\mathbf{y}_2,\cdots,\mathbf{y}_M\}} \{\|\mathbf{x}_i - \mathbf{p}\|_2\}, \\ u_i &= \max_{\mathbf{p}\in\{\mathbf{y}_1,\mathbf{y}_2,\cdots,\mathbf{y}_M\}} \{\|\mathbf{x}_i - \mathbf{p}\|_2\}, \qquad i = 1, 2, \cdots, 8. \end{aligned}$$

On the other hand, the bounds for $\mathbf{x}$, $\mathbf{x}_l$ and $\mathbf{x}_u$, are

$$\mathbf{x}_l = \arg\min_{\mathbf{p}\in\{\mathbf{y}_1,\mathbf{y}_2,\cdots,\mathbf{y}_M\}} \{\mathbf{p}\}, \quad \mathbf{x}_u = \arg\max_{\mathbf{p}\in\{\mathbf{y}_1,\mathbf{y}_2,\cdots,\mathbf{y}_M\}} \{\mathbf{p}\}.$$

Fig. 1 shows the mean square position error (MSPE) performance versus σ^2 when the source is located at (3.5,2.5,1.5)m. The MSPE is defined as $E[(x-\hat{x})^2 + (y-\hat{y})^2 + (z-\hat{z})^2]$, where $[\hat{x}, \hat{y}, \hat{z}]^T$ denotes an estimate of $\mathbf{x}$, and its unit is m^2, which becomes dBm^2 in dB scale. It is observed that the SDR algorithm is superior to the two-step WLS and NLS methods when $\sigma^2 \geq -35\,\mathrm{dBm}^2$ and $\sigma^2 \geq -25\,\mathrm{dBm}^2$, respectively, and is able to attain the CRLB for $\sigma^2 \geq -25\,\mathrm{dBm}^2$. On the other hand, the two-step WLS technique is advantageous only at very small noise conditions, namely, $\sigma^2 \leq -40\,\mathrm{dBm}^2$, because its derivation is based on a linearized version of (1) and the results also agree with the study in [10]. Since the initial estimate provided by the WLS algorithm is not sufficiently close to the global minimum at larger noise conditions, the NLS estimator is unable to attain optimum performance when $\sigma^2 > -30\,\mathrm{dBm}^2$. Note also that we have integrated the prior knowledge of the admissible source region in the SDR algorithm whereas it is difficult to utilize this information in the WLS and NLS schemes.

Fig. 2 shows the MSPEs versus the x-coordinate from -1 m to 7 m with $y = 2.5$ m and $z = 1.5$ m at a noise level of $\sigma^2 = -10$ dB. We see that the estimation performance of the proposed scheme can achieve the CRLB while the two-step WLS and NLS methods do not

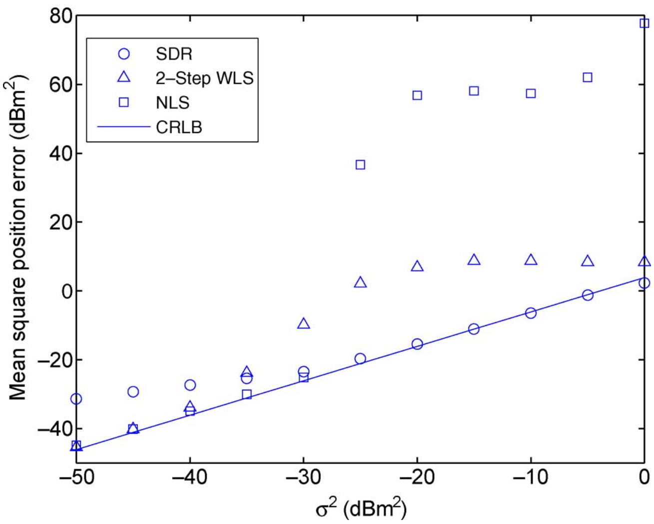

Fig. 1. Mean square position error versus σ^2 at $\mathbf{x} = (3.5, 2.5, 1.5)$ m.

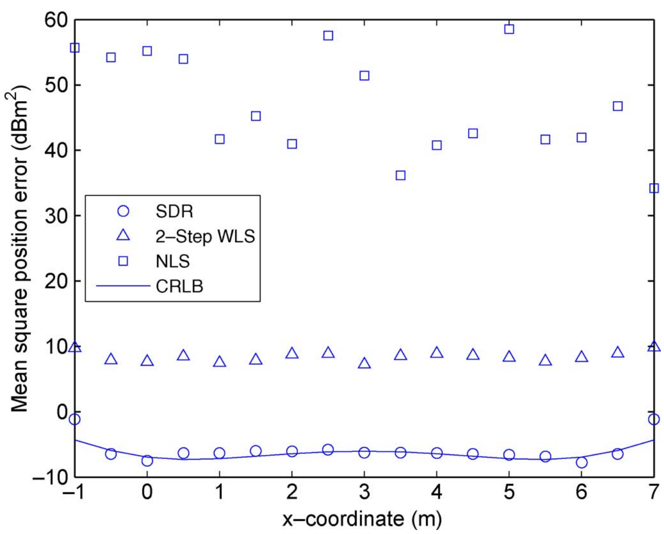

Fig. 2. Mean square position error versus x-coordinate at $\sigma^2 = -10$ dBm^2.

work satisfactorily. Furthermore, the robustness of the SDR algorithm is demonstrated via providing good accuracy even when the source is located outside the room, that is, $x < 0$ m and $x > 6$ m.

IV. Conclusion

The nonconvex maximum-likelihood estimation problem for range-difference (RD) based positioning has been approximated to a convex optimization problem using the semidefinite relaxation (SDR) technique. The admissible source position information is also utilized in developing tighter constraints for the proposed algorithm. When the RD errors are reasonably large, it is shown that the SDR algorithm is superior to the two-step weighted least squares method and its estimation performance can attain Cramér–Rao lower bound.

References

[1] J. B. Tsui, *Fundamentals of Global Positioning System Receivers*. New York: Wiley, 2000.

[2] J. J. Caffery, Jr., *Wireless Location in CDMA Cellular Radio Systems*. Boston, MA: Kluwer, 2000.

[3] *Acoustic Signal Processing for Telecommunication*, S. L. Gay and J. Benesty, Eds. Boston, MA: Kluwer Academic, 2000.

[4] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero, III, R. L. Moses, and N. S. Correal, "Locating the nodes: Cooperative localization in wireless sensor networks," *IEEE Signal Process. Mag.*, vol. 22, no. 4, pp. 54–69, Jul. 2005.

[5] D. J. Torrieri, "Statistical theory of passive location systems," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-20, no. 2, pp. 183–197, Mar. 1984.

[6] M. A. Spirito, "On the accuracy of cellular mobile station location estimation," *IEEE Trans. Veh. Technol.*, vol. 50, pp. 674–685, May 2001.

[7] C. Mensing and S. Plass, "Positioning algorithms for cellular networks using TDOA," in *Proc. Int. Conf. Acoustics, Speech, Signal Processing (ICASSP)*, Toulouse, France, May 2006, vol. IV, pp. 513–516.

[8] J. O. Smith and J. S. Abel, "Closed-form least-squares source location estimation from range-difference measurements," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 35, pp. 1661–1669, Dec. 1987.

[9] B. Friedlander, "A passive localization algorithm and its accuracy analysis," *IEEE J. Ocean. Eng.*, vol. 12, pp. 234–245, Jan. 1987.

[10] Y. T. Chan and K. C. Ho, "A simple and efficient estimator for hyperbolic location," *IEEE Trans. Signal Process.*, vol. 42, no. 8, pp. 1905–1915, Aug. 1994.

[11] Y. Huang, J. Benesty, G. W. Elko, and R. Mersereau, "Real-time passive source localization: A practical linear-correction least-squares approach," *IEEE Trans. Speech Audio Process.*, vol. 9, no. 8, pp. 943–956, Nov. 2001.

[12] K. W. Cheung, H. C. So, W.-K. Ma, and Y. T. Chan, "A constrained least squares approach to mobile positioning: Algorithms and optimality," *EURASIP J. Appl. Signal Process.*, vol. 2006, pp. 1–23, 2006, Article ID 20858.

[13] L. Vandenberghe and S. Boyd, "Semidefinite programming," *SIAM Rev.*, vol. 38, pp. 49–95, 1996.

[14] Y. E. Nesterov, "Semidefinite relaxation and nonconvex quadratic optimization," *Optim. Methods Softw.*, vol. 9, pp. 140–160, 1998.

[15] Y. Ye, "Approximating quadratic programming with bound and quadratic constraints," *Math. Program.*, vol. 84, pp. 219–226, 1999.

[16] K. W. Cheung, W.-K. Ma, and H. C. So, "Accurate approximation algorithm for TOA-based maximum likelihood mobile location using semidefinite programming," in *Proc. Int. Conf. Acoustics, Speech, Signal Processing (ICASSP)*, Montreal, QC, Canada, May 2004, vol. 2, pp. 145–148.

[17] P. Biswas, T.-C. Liang, K.-C. Toh, Y. Ye, and T.-C. Wang, "Semidefinite programming approaches for sensor network localization with noisy distance measurements," *IEEE Trans. Autom. Sci. Eng.*, vol. 3, no. 4, pp. 360–371, Oct. 2006.

[18] P. Biswas, T.-C. Liang, T.-C. Wang, and Y. Ye, "Semidefinite programming based algorithms for sensor network localization," *ACM Trans. Sensor Netw.*, vol. 2, no. 2, pp. 188–220, May 2006.

[19] Y. Ding, N. Krislock, J. Qian, and H. Wolkowicz, "Sensor network localization, Euclidean distance matrix completions, and graph realization," Univ. of Waterloo, Waterloo, ON, Canada, Tech. Rep. CORR 2006-23, 2006.

[20] N. Krislock, V. Piccialli, and H. H. Wolkowicz, "Robust semidefinite programming approaches for sensor network localization with anchors," Univ. of Waterloo, Waterloo, ON, Canada, Tech. Rep. CORR 2006-12, 2006.

[21] C. Meng, Z. Ding, and S. Dasgupta, "A semidefinite programming approach to source localization in wireless sensor networks," *IEEE Signal Process. Lett.*, vol. 15, pp. 253–256, 2008.

[22] P. Biswas, H. Aghajan, and Y. Ye, "Semidefinite programming algorithms for sensor network localization using angle of arrival information," in *Proc. 39th Annu. Asilomar Conf. Signals, Systems, Computers*, Pacific Grove, CA, Oct. 2005, pp. 220–224.

[23] R. O. Schmidt, "A new approach to geometry of range difference location," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 8, pp. 821–835, Nov. 1972.

[24] W. R. Hahn and S. A. Tretter, "Optimum processing for delay-vector estimation in passive signal arrays," *IEEE Trans. Inform. Theory*, vol. 19, pp. 608–614, Sep. 1973.

[25] R. O. Schmidt, "Least squares range difference location," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 32, pp. 234–242, Jan. 1996.

[26] H. C. So, Y. T. Chan, and F. K. W. Chan, "Closed-form formulae for optimum time-difference-of-arrival based localization," *IEEE Trans. Signal Process.*, vol. 56, no. 6, pp. 2614–2620, Jun. 2008.

[27] H. D. Sherali and W. P. Adams, *A Reformulation-Linearization Technique for Solving Discrete and Continuous Nonconvex Problems*. Boston, MA: Kluwer, 1998.

[28] M. Grant and S. Boyd, CVX: Matlab Software for Disciplined Convex Programming, Feb. 2008 [Online]. Available: http://stanford.edu/~boyd/cvx

[29] M. Grant and S. Boyd, "Graph implementations for nonsmooth convex programs," in *Recent Advances in Learning and Control (a Tribute to M. Vidyasagar)*, V. Blondel, S. Boyd, and H. Kimura, Eds. New York: Springer, 2008 [Online]. Available: http://stanford.edu/~boyd/graph_dcp.html, to be published