CCP for Localization

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***Abstract*—(copy from DSP15) Locating a radiating source from range or range-difference measurements in a passive sensor network has recently attracted an increasing amount of research interest as it finds applications in a wide range of network-based wireless systems. Striking results for these localization problems have emerged using squared range (SR-LS) or squared range-difference (SRD-LS) least-squares (LS) approaches. In this paper, we present improved LS methods that demonstrate improved localization performance when compared with the best known results from the literature.**

***Index Terms*—least squares; non-convex; convex-concave procedure; CCP; source localization; range measurements.**

I. Introduction

1) my intro: Least squares based algorithms constitute an important class of solution techniques as they are geometrically meaningful and often provide low complexity solution procedures with competitive estimation accuracy [**?**]-[**?**]. On the other hand, the error measure in an LS formulation for the localization problem of interest is shown to be highly non-convex, possessing multiple local solutions with degraded performance. This non-convexity excludes many local methods that are iterative, hence extremely sensitive to where the iteration begins. In the case of source localization, this inherent feature of local methods is particular problematic because the source location is assumed to be entirely unknown and can appear practically anywhere. Several non-iterative *global* localization techniques has been known in the literature. One representative in the class of global localization methods is the convex-relaxation based algorithm for range measurements proposed in [8], where the LS model is relaxed to a semidefinite programming (SDP) problem which is known to be convex [13]. “However, as discussed in [3], the optimal solution of this relaxed SDP does not always satisfy the near-rank-1 constraints of acceptable solutions to the source localization problem.” Another representative in this class is reference [12], where localization problems for range as well as range difference measurements are addressed by developing solution methods for *squared* range LS (SR-LS) and *squared* range difference LS (SRD-LS) problems. “The SR-LS approaches are computationally simpler than iterative minimization algorithms but they provide less accurate solutions than those provided by ML approaches [3], because they are suboptimal in the ML sense.”

In this paper we exploit the special structure of the cost function of an unconstrained LS formulation for the localization problem. In particular we present it as a difference of convex (DC) programming problem and solve it by applying a penalty convex-concave procedure (CCP), which is an effective heuristic method to deal with this class of problems. “The basic CCP requries a feasible initial point $\boldsymbol{x}_0$ to start the procedure. By introducing additinoal slack variables, a penalty CCP has been developed to accept nonfeasible initial points”. We focus on the LS formulation since it is known to be the maximum-likelihood estimator in the case of Gaussian white measurement noise and therefore would provide most accurate solution. Numerical results are presented for performance evaluation and comparisons.

II. Source Localization From Range Measurements

A. Problem Statement

The source localization problem considered here involves a given array of m sensors specified by $\{\boldsymbol{a}_1, \ldots, \boldsymbol{a}_m\}$ where $\boldsymbol{a}_i \in R^n$ contains the n coordinates of the ith sensor in space R^n. Each sensor measures its distance to a radiating source $\boldsymbol{x} \in R^n$. Throughout it is assumed that only noisy copies of the distance data are available, hence the *range measurements* obey the model

$$r_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| + \varepsilon_i, \quad i = 1, \ldots, m. \tag{1}$$

where ε_i denotes the unknown noise that has occurred when the ith sensor measures its distance to source $\boldsymbol{x}$. Let $\boldsymbol{r} = [r_1\ r_2 \ldots r_m]^T$ and $\boldsymbol{\varepsilon} = [\varepsilon_1\ \varepsilon_2 \ldots \varepsilon_m]^T$, the source localization problem can be stated as to estimate the exact source location $\boldsymbol{x}$ from the noisy range measurements $\boldsymbol{r}$.

B. LS Formulations and Review of Related Work

For the localization problem at hand, the range-based least squares (R-LS) estimate refers to the solution of the problem

$$\underset{\boldsymbol{x}}{\text{minimize}}\, f(\boldsymbol{x}) = \sum_{i=1}^{m} (r_i - \|\boldsymbol{x} - \boldsymbol{a}_i\|)^2 \tag{2}$$

Formulation (2) is connected to the maximum-likelihood (ML) location estimation that determines $\boldsymbol{x}$ by examining the probabilistic model of the error vector $\boldsymbol{\varepsilon}$. If $\boldsymbol{\varepsilon}$ obeys a Gaussian distribution with zero mean and covariance $\boldsymbol{\Sigma} = \text{diag}(\sigma_1^2, \ldots, \sigma_m^2)$, then the maximum likelihood (ML) location estimator in this case is known to be

$$\boldsymbol{x}_{ML} = \arg \min_{\boldsymbol{x} \in R^n} (\boldsymbol{r} - \boldsymbol{g})^T \Sigma^{-1} (\boldsymbol{r} - \boldsymbol{g}) \tag{3}$$

where $\boldsymbol{g} = [g_1 \ g_2 \dots \ g_m]^T$ with $g_i = \|\boldsymbol{x} - \boldsymbol{a}_i\|$. It follows immediately that the ML solution in (3) is identical to the R-LS solution of problem (2) when covariance $\boldsymbol{\Sigma}$ is proportional to the identity matrix, i.e., $\sigma_1^2 = \ldots = \sigma_m^2 = 1$.

There are many methods for continuous unconstrained optimization [?], however most of them are local methods in the sense they are sensitive to the choice of initial point, and give no guarantee to yield global solutions when applied to non-convex objective functions. Unfortunately, the objective function in (2) is highly non-convex, possessing many local minimizers even for small-scale systems.

Reference [?] addresses problem (2) by a convex relaxation technique where (2) is modified to a convex problem known as semidefinite programming [?]. A rather different approach is proposed in [?] where the localization problem (2) is tackled by developing techniques to address the *squared range based LS* (SR-LS) problem

$$\underset{\boldsymbol{x}}{\text{minimize}} \sum_{i=1}^{m} (\|\boldsymbol{x} - \boldsymbol{a}_i\|^2 - r_i^2)^2 \tag{4}$$

whose global solution can be computed by converting (4) into the class of generalized trust region subproblems (GTRS) [?], [?] and exploring its KKT conditions which are both necessary and sufficient optimality conditions. Although SR-LS solves (4) exactly, the produced solution remains to be an approximation of the original LS problem in (2) because it is no longer a ML solution.

(in this work...)

III. Convex-Concave Procedure

A. CCP

"The CCP refers to an effective heristic method to deal with a class of *nonconvex* problems of the form

$$\begin{aligned} &\underset{\boldsymbol{x}}{\text{minimize}} && f(\boldsymbol{x}) - g(\boldsymbol{x}) \\ &\text{subject to:} && f_i(\boldsymbol{x}) \le g_i(\boldsymbol{x}) \quad \text{for: } i = 1, 2, \ldots, m \end{aligned} \tag{5}$$

where $f(\boldsymbol{x}), g(\boldsymbol{x}), f_i(\boldsymbol{x}), g_i(\boldsymbol{x})$ for $i = 1, 2, \ldots, m$ are convex. The basic CCP algorithm is n iterative procedure including two key steps (in the k-th iteration):

(i) Convexify the objective function and constraints by replacing $g(\boldsymbol{x})$ and $g_i(\boldsymbol{x})$, respectively, with their affine approximations

$$\hat{g}(\boldsymbol{x}, \boldsymbol{x}_k) = g(\boldsymbol{x}_k) + \bigtriangledown g(\boldsymbol{x}_k)^T (\boldsymbol{x} - \boldsymbol{x}_k)$$
$$\hat{g}_i(\boldsymbol{x}, \boldsymbol{x}_k) = g_i(\boldsymbol{x}_k) + \bigtriangledown g_i(\boldsymbol{x}_k)^T (\boldsymbol{x} - \boldsymbol{x}_k) \text{ for: } i = 1, 2, \ldots, m$$

(ii) Solve the convex problem

$$\begin{aligned} &\underset{\boldsymbol{x}}{\text{minimize}} && f(\boldsymbol{x}) - g(\boldsymbol{x}) \\ &\text{subject to:} && f_i(\boldsymbol{x}) \le g_i(\boldsymbol{x}) \quad \text{for: } i = 1, 2, \ldots, m \end{aligned} \tag{6}$$

Because of the convexity of all the functions involved, it can be shown that the basic CCP is a descent algorithm and the iterates $\boldsymbol{x}_k$ converge to the critical point of the original problem. The basic CCP requires a *feasible* initial point $\boldsymbol{x}_0$ to start the procedure. By introducing additional slack variables, a penalty CCP has been developed to accept nonfeasible initial points.

[StB14] "variations and extensions of the on the convex-concave procedure, a method used to find a local solution to difference of convex programming problems. hard to solve, no polynomial time alg exist. CCP is a local heuristic, thus final solution often depends on the initial point x_0. It is typical to initialize the algorithm with several feasible x_0 and take as the final choise of x the final point found with the lowest objective value over the different runs. The initial point can be random or through a heuristic. In SQP the problem at each iteration is approximated by a quadratic program (convex quadratic objective and linear constraints. CCP is able to retain all information from the convex component of each term and only linearizes the concave portion." SQP is easier to solve at each step, but CCP allows / beneficial to take advantege of more information.

"Penalty CCP (first extension) removes the need for an initial feasible point. We relax our problem by adding slack variables to our constraints and penalizing the sum of the violations. By initially putting a low penalty on violations, we allow for constraints to be violated so that a region with lower objective value can be found. Thus this approach may be desirable even if a feasible initial point is known. .... Penalizing the sum of violations is equivalent to using the 1 norm and is well known to induce sparsity. Therefore, if we are unable to satisfy all constraints, the set of violated constraints should be small. The theory of exact penalty functions tells us that if τ_i is greater than the largest optimal dual variable associated with the inequalities in the convexified subproblem (4), then solutions to (4) are solutions of the relaxed convexified problem, and subject to some conditions on the constraints, if a feasible point exists, solutions to the relaxed problem are solutions to the convexified problem (e.g., $\sum_{i=1}^{m} s_i = 0$) [HM79, DPG89]." This algorithm is not a descent algorithm, but the objective value will converge, although the convergence may not be to a feasible point of the original problem.

Algorithm 3.1

"Provided τ_{max} is larger than the largest optimal dual variable in the unrelaxed subproblems (4) the value of τ_{max} will have no impact on the solution. This value is unlikely to be known; we therefore choose τ_{max} large. Observe that, for sufficiently large τ_{max}, if (1) is convex, and the constraint conditions are met, then penalty CCP is not a heuristic but will find an optimal solution."

IV. fitting the design problem to a CCP framework

A. Reformulation

B. Placing lower and upper bounds on the errors

C. algorithm based on CCP

V. Numerical Results

VI. Conclusion

The conclusion goes here.

ACKNOWLEDGMENT

The authors would like to thank...

REFERENCES

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.