LEAST SQUARE SOLUTIONS OF ENERGY BASED ACOUSTIC SOURCE LOCALIZATION PROBLEMS

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ABSTRACT

In this study we compare the performance of five least-square based methods for the localization of a target using intensity measurements of randomly placed acoustic sensors. Specifically, we propose a novel quadratic-term elimination (QE) method that gives a closed form least square solution and empirically yields the highest localization accuracy. Applications of this method to the localization and tracking of moving vehicles in a wireless sensor field experiment showed promising results.

1. INTRODUCTION

Source localization using acoustic sensors is an important signal-processing task that has found broad range of applications. Most localization methods depend on three types of physical variables measured by or derived from sensor readings for localization: time delay of arrival (TDOA) [1], [2], [3], [4] direction of arrival (DOA) and received sensor signal strength (intensity) or power Recently, we proposed a robust energy-based localization (EBL) method to estimate target location using signal energy measurements at neighboring sensors [5], [6]. However, for real-time applications, a closed-form estimator is desirable. Most current research in this area focuses on the least squares algorithm and its variants. These least squares methods, mostly based on TDOA measurements, include the spherical intersection (SX) [7], plan intersection [8], spherical interpolation (SI) [9], hyperbolic intersection [10], linear intersection (LC) [11] and other modifications such as quadratic-correction SI [10], linear-correction SI [12], and Sphere Interpolation (SI) Method [13].

In this paper we first formulate the EBL localization problem as a constrained least square problem. Based on this formulation, we apply four least-squares-based algorithms, namely, LC, SX, SI, and the SM methods to solve this problem. We also propose an efficient, yet simple quadratic-term elimination (QE) algorithm and compare its performance to the four existing constrained least square algorithms using computer simulations.

The remaining of this paper is organized as follows. First, we formulate the EBL problem as a constrained least square problem, and present the LC, SX, SI, and SM algorithms as possible solutions. Then, we derive the QE method. Next we perform extensive computer simulation to compare their performance.

PROBLEM FORMULATION

Consider the problem of localizing a target using N acoustic sensors in a sensor field. The target (sound source) radiates acoustical signal $s(n)$ that is propagated through the free air along the ground surface. The acoustic energy measured at individual acoustic sensor i over a fixed time duration t is

$$y_i(t) = g_i \cdot \frac{s(t)}{\left\| r(t) - r_i \right\|^2} + \varepsilon_i(t) \qquad (1)?$$

$\varepsilon_i(t)$ can be approximated very well with a Gaussian random variable with $E\{\varepsilon_i(t)\} = \sigma_i^2$, and $\mathrm{Var}\{\varepsilon_i(t)\}$ can be determined using an empirically derived formula [14]. We also assume that g_i is known using sensor calibration. Replacing $\varepsilon_i(t)$ by σ_i^2, eq. (1) becomes

?
$$\frac{y_i(t) - \sigma_i^2}{g_i} \approx \frac{s(t)}{\left\| \underline{r}(t) - \underline{r}_i \right\|^2} \qquad (2)$$

The left hand side of equation (2) can be seen as the (normalized) acoustic energy that is inversely proportional to the square of the target-sensor distance. For the simplicity of notations, let us designate sensor node #0 as a reference node. Define

$$\kappa_i := \left(\frac{[y_i(t) - \sigma_i^2]/g_i(t)}{[y_0(t) - \sigma_0^2]/g_0(t)} \right)^{-1/2} = \frac{\left\| \underline{r}(t) - \underline{r}_i \right\|}{\left\| \underline{r}(t) - \underline{r}_0 \right\|}.$$

Then, *all* possible target locations $r(t)$ that yield the computed value of κ_i must lie on a hyper-sphere (a circle in 2D coordinates) specified by the following equation:

$$\| \underline{r}(t) - \underline{c}_i \| = \rho_i \qquad (3)$$

where the center $\underline{c}_i$ and the radius ρ_i are given by:

$$\underline{c}_i = \frac{\underline{r}_i - \kappa_i^2 \underline{r}_0}{1-\kappa_i^2}\,;\; ?_i = \frac{?_i \, \| \underline{r}_i - \underline{r}_0 \|}{1-?_i^2}$$

In equation (2), the variance of $\varepsilon_i(t)$ is not taken into account. The sensor location estimates $\underline{r}_i$, and $\underline{r}_0$, as well as the gain estimate g_i may also be inaccurate. We denote these interferences cumulatively as e_i:

$$\| \underline{r}(t) - \underline{c}_i \| = \rho_i + e_i$$

With the assumption that e_i is AWGN, the optimal estimate of $\underline{r}$(t) is the maximum likelihood estimate which seeks to minimize the following cost function:

$$J(\underline{r}) = \sum_i \left| \, \| \underline{r} - \underline{c}_i \| - \rho_i \right|^2 \qquad (4)$$

Note that a reference sensor node is not necessary for computing the energy ratio factor κ_i. Any pair of sensor nodes will suffice. Hence, there could be as many as $N(N-1)/2$ terms in equation (4) where N is the total number of sensors that detect the target acoustic energy.

Taking squares on both sides, eq. (3) becomes

$$2\underline{c}_i^T \underline{r} = \| \underline{r} \|^2 + \| \underline{c}_i \|^2 - \rho_i^2$$

Denote $\theta_i = \|\underline{c}_i\|^2 - \rho_i^2$, and consider the effect of perturbation, above equation takes the following form:

$$2\underline{c}_i^T \underline{r} - \|\underline{r}\|^2 = \begin{bmatrix} 2\underline{c}_i^T & -1 \end{bmatrix} \begin{bmatrix} \underline{r} \\ \|\underline{r}\|^2 \end{bmatrix} = \theta_i + e_i' \qquad (5)$$

Define a matrix $\mathbf{C}$ such that its i^{th} row is $2\underline{c}_i^T$, a vector $\underline{u}$ = [1 1 … 1]^T, and a vector $\underline{\theta}$ whose i^{th} element is θ_i. Then the unknown source location $\underline{r}$ can be solved from a non-linearly constrained, linear least square problem:

Find $\underline{r}$ and R_s that minimizes a quadratic cost function

$$\left\| \begin{bmatrix} \mathbf{C} & -\underline{u} \end{bmatrix} \begin{bmatrix} \underline{r} \\ R_s \end{bmatrix} - \underline{\theta} \right\|^2 = \| \mathbf{M}\underline{q} - \underline{\theta} \|^2$$

where $\underline{q} = [\underline{r}^T \;\; R_s]^T$, and $\mathbf{M} = \begin{bmatrix} \mathbf{C} & -\underline{u} \end{bmatrix}$, subject to the constraint $R_s = \| \underline{r} \|^2$ or equivalently,

$$\underline{q}^T \mathbf{\Sigma}\, \underline{q} = 2\underline{q}^T \underline{\pi} \qquad (6)$$

with $\mathbf{\Sigma} = diag(1,1,0)$ and $\underline{\pi} = [0\ 0\ 1/2]^T$.

To solve this constrained optimization problem, the Lagrange multiplier method can be used:

$$\begin{aligned} L(\underline{\theta},\lambda) &= (\mathbf{M}\underline{q} - \underline{\theta})^T(\mathbf{M}\underline{q} - \underline{\theta}) + \lambda(\underline{q}^T \Sigma \underline{q} - 2\underline{q}^T \underline{\pi}) \\ &= \underline{q}^T(\mathbf{M}^T\mathbf{M} + \lambda\Sigma)\underline{q} - \underline{q}^T(2\mathbf{M}^T\underline{\theta} + 2\lambda\underline{\pi}) + \underline{\theta}^T\underline{\theta} \end{aligned}$$

Set the gradient of L with respect to $\underline{q}$ to 0, we have

$$\begin{aligned} &\nabla_{\underline{q}} L(\underline{q},\lambda) = 2(\mathbf{M}^T\mathbf{M} + \lambda\Sigma)\underline{q} - (2\mathbf{M}^T\underline{\theta} + 2\lambda\underline{\pi}) = 0 \\ &\Rightarrow (\mathbf{M}^T\mathbf{M} + \lambda\Sigma)\underline{q} = \mathbf{M}^T\underline{\theta} + \lambda\underline{\pi} \end{aligned} \qquad (7)$$

The solution of $\underline{q}$ in eq. (7) as a function of λ and other parameters can be substituted into the constraint equation (6) and to solve for λ explicitly. This value of λ then can be substituted back to eq. (7) to give an explicit solution of $\underline{q}$. This leads to a nonlinear function in λ, and a nonlinear optimization package can be used to attend this goal. With λ solved, then $\underline{q}$ can be solved.

Linear Correction (LC) [12]

Let an initial estimate of $\underline{q}$ be denoted by $\underline{q}_0$ such that

$$\underline{q}_0 = \underline{q} + \Delta\underline{q}$$

Substitute $\underline{q}_0$ into eq. (7), and rearrange terms, we have

$$\lambda(\mathbf{\Sigma}\, \underline{q}_0 - \underline{\pi}) = \mathbf{M}^T\underline{\theta} - \mathbf{M}^T\mathbf{M}\underline{q}_0 \qquad (8)$$

Let $\underline{q}_{LS} = (\mathbf{M}^T\mathbf{M})^{-1}\mathbf{M}^T\underline{\theta} = \mathbf{M}^{\dagger}\underline{\theta}$

be the least square solution of the over-determined equation $\mathbf{M}\underline{q} = \underline{\theta}$, and $\mathbf{M}^{\dagger}$ is the Pennrose-Moore pseudo inverse of the matrix $\mathbf{M}$. Assume that $\underline{q}_0 = \underline{q}_{LS}$, after simplification and rearranging terms, one has the following linear-correction equation:

$$\underline{q}_{LC} = \left(\mathbf{I} - \lambda_0\left(\mathbf{M}^T\mathbf{M}\right)^{-1}\mathbf{S}\right)^{-1}\underline{q}_{LS} + \lambda_0\left(\mathbf{M}^T\mathbf{M}\right)^{-1}\underline{\pi}$$

where $$\lambda_0 = \frac{\underline{q}_0^T\mathbf{M}^T(\underline{\theta} - \mathbf{M}\underline{q}_0)}{\underline{q}_0^T\underline{\pi}}$$

Sphere intersection (SX) Method [7]

Suppose that $\| \underline{r} \|^2 = R_s$ is available, then $\underline{r}$ can be solved explicitly. Refer to eq. (5), one may formulate an over-determined linear system of equations:

$$\underbrace{\begin{bmatrix} 2\underline{c}_1^T \\ 2\underline{c}_2^T \\ \vdots \\ 2\underline{c}_M^T \end{bmatrix}}_{\mathbf{C}} \underline{r} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{\underline{u}} R_s + \underbrace{\begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_M \end{bmatrix}}_{\underline{\theta}} + \underbrace{\begin{bmatrix} e_1' \\ e_2' \\ \vdots \\ e_M' \end{bmatrix}}_{\underline{e}'}$$

or $$\mathbf{C}\,\underline{r} = R_s\,\underline{u} + \underline{\theta} + \underline{e}' \qquad (9)$$

The least square solution is

$$\underline{r} = \mathbf{C}^{\dagger}(\underline{u}\,R_s + \underline{\theta}). \qquad (10)$$

To derive the value of R_s, substitute above expression into the constraint that $\| \underline{r} \|^2 = R_s$. This leads to a quadratic equation in R_s:

$$f(R_s) = \| \mathbf{C}^{\dagger}\underline{u} \|^2 R_s^2 + (2\,(\mathbf{C}^{\dagger}\underline{\theta})^T(\mathbf{C}^{\dagger}\underline{u}) - 1)\, R_s + \| \mathbf{C}^{\dagger}\underline{\theta} \|^2 = 0$$

For this equation to have two real-valued roots, the coefficients must satisfy the relation

$$(2\,(\mathbf{C}^{\dagger}\underline{\theta})^T(\mathbf{C}^{\dagger}\underline{u}) - 1)^2 - 4\,\| \mathbf{C}^{\dagger}\underline{u} \|^2 \cdot \|\mathbf{C}^{\dagger}\underline{\theta}\|^2 \geq 0.$$

Or equivalently, $(\mathbf{C}^{\dagger}\underline{\theta})^T(\mathbf{C}^{\dagger}\underline{u}) \leq 1/4$.

Sphere Interpolation (SI) Method [13]

With the *SI* method, the constraint that $R_s = \| \underline{r} \|^2$ is discarded. Instead, the value of R_s is chosen such that $\| \underline{e}' \|^2 = \|\mathbf{C}\underline{r} - (R_s\, \underline{u} + \underline{\theta})\|^2$ is minimized. This requires that the vector $R_s\, \underline{u}+\underline{\theta}$ in eq. (9) lie within the subspace spanned by columns of the **C** matrix. Or, equivalent, be perpendicular to the orthogonal subspace of **C**. Define $\mathbf{P_C} = \mathbf{C}(\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T = \mathbf{C}\mathbf{C}^+$ as the *projection matrix* corresponding to the matrix **C**, and $\mathbf{P_C}^\perp = \mathbf{I} - \mathbf{P_C}$ to be its orthogonal complement. The goal is to find R_s such that $\| \mathbf{P_C}^\perp (R_s\, \underline{u} + \underline{\theta}) \|^2$ is minimized. The least square solution to this problem is

$$\hat{R}_s = -\frac{\left(\mathbf{P}_\mathbf{C}^\perp \underline{\theta}\right)^T \left(\mathbf{P}_\mathbf{C}^\perp \underline{u}\right)}{\left\|\mathbf{P}_\mathbf{C}^\perp \underline{u}\right\|} = -\frac{\underline{\theta}^T \mathbf{P}_\mathbf{C}^\perp \underline{u}}{\sqrt{\underline{u}^T \mathbf{P}_\mathbf{C}^\perp \underline{u}}}$$

Here the symmetry property, $(\mathbf{P_C}^\perp)^T = \mathbf{P_C}^\perp$, and the idem-potent property, $(\mathbf{P_C}^\perp)(\mathbf{P_C}^\perp) = \mathbf{P_C}^\perp$, of a projection matrix are used. Substitute Eq. (8) into eq.(10), an estimate of $\underline{r}$ can be found:

$$r_{SI} = \mathbf{C}^\dagger \left(\mathbf{I} - \frac{\underline{u} \cdot \underline{u}^T \mathbf{P}_\mathbf{C}^\perp}{\sqrt{\underline{u}^T \mathbf{P}_\mathbf{C}^\perp \underline{u}}} \right) \cdot \underline{\theta}$$

Subspace Minimization (SM) [9]

The difficulty in solving eq. (9) for $\underline{r}$ is the unknown quantity R_s on the right hand side of the equation. One way to eliminate R_s is to project the solution into a subspace that is perpendicular to the vector $\underline{u}$. This can be accomplished by pre-multiplying every term in eq. (9) by a projection matrix $\mathbf{P}_u^\perp := \mathbf{I} - \underline{u}\underline{u}^T / \underline{u}^T \underline{u}$. Since $\mathbf{P}_u^\perp \underline{u} = \underline{0}$, this leads to

$$\mathbf{P}_u^\perp \mathbf{C}\, \underline{r} = \mathbf{P}_u^\perp \underline{\theta} + \mathbf{P}_u^\perp\, \underline{e}' \qquad (11)$$

Finally, the target position $\underline{r}$ can be estimated from eq. (11) as:

$$\underline{r}_{SM} = (\mathbf{P}_u^\perp \mathbf{C})^\dagger \mathbf{P}_u^\perp \underline{\theta} = (\mathbf{C}^T \mathbf{P}_u^\perp \mathbf{C})^{-1} \mathbf{C}^T \mathbf{P}_u^\perp \underline{\theta}$$

Qquadratic-term elimination (QE) Method

Square both sides of Eq. (3), and rearrange terms, one has $\|\underline{r}\|^2 = 2\underline{c}_i^T \underline{r} + \rho_i^2 - \|\underline{c}_i\|^2$. Similarly, for sensor *j*, we have $\|\underline{r}\|^2 = 2\underline{c}_j^T \underline{r} + \rho_j^2 - \|\underline{c}_j\|^2$. Equating both equations, the quadratic term of the solution $\|\underline{r}\|^2$ can be eliminated. This leads to a hyper-plane equation:

$$\left(\underline{c}_i - \underline{c}_j\right)^T \underline{r} = \frac{1}{2}\left[\left(\|\underline{c}_i\|^2 - \rho_i^2\right) - \left(\|\underline{c}_j\|^2 - \rho_j^2\right)\right] \equiv \theta_i - \theta_j$$

This hyper-plane corresponds to the intersection of two hyper-spheres. Since parameters $\underline{c}_i$, $\underline{c}_j$, ρ_i, and ρ_j may contain errors, so does the corresponding hyper-plane equation.

By pairing all available hyper-spheres, say, *M* such hyper-planes may be obtained. Denote a $M \times 2$ ($M \times 3$ in 3D case) matrix **C** and a $M \times 1$ column vector Θ, such that each row of **C** contains a vector of $\underline{c}_i - \underline{c}_j$, and the corresponding entry of Θ contains $\theta_i - \theta_j$. Then the source location can be estimated by solving an *un-constrained Quadratic-term Elimination (QE) least square problem*:

Solve $\underline{r}$ such that $\|\mathbf{C}\, \underline{r} - \Theta\|^2$ is minimized.

The significance of the QE problem formulation is that a closed-form solution of $\underline{r}$ can be expressed in terms of the **C** matrix and the Θ vector explicitly. Specifically,

$$\underline{R}_{QE} = \mathbf{C}^\dagger \Theta$$

SIMULATION

We compare the proposed unconstrained least square QE method against four existing constrained least square methods, as well as a multi-resolution (MR) non-linear search method.

We performed computer simulations with one acoustic source and *N* acoustic sensors. The source (target) has a constant energy of 100. The target and the sensor nodes are randomly located in a 100m × 100m squared "region of interest". The locations of the sensors and of the target are independently and uniformly distributed in the region. The energy received by each sensor is pre-computed to be proportional to $1/\|\underline{r}(t) - r_i\|^2$. For each sensor and source location configuration, K=1000 independent trials of Monte-Carlo simulations were performed, and the RMS error of *K* estimation errors (bias) are computed.

Perturbation on localization parameters

We first examine the effect of various perturbations of the parameters used in EBL and the noise contamination on the performance of various algorithms. These factors include the variations (Δ) on sensor locations $\underline{r}_i$, gain calibration *g*, and the signal to noise ratio (SNR). A total of 7 parameter settings are considered in the simulations. Note the first parameter setting corresponds to the "control" case where no perturbation is involved. The number of sensors N = 20. For MR search algorithm, we use three levels of resolution (10m, 5m, 1m). Results are shown in table 2.

Table 2. RMS error (in meters) of simulations

	MR	QE	LC	SX	SI	SM
control	3.70	0.00	0.00	15.96	0.00	0.00
Δr = 0.5	3.81	0.90	0.36	15.85	0.36	0.37
Δr = 1	4.59	1.85	0.70	14.92	0.73	0.74
Δg = 0.5	10.86	11.02	24.21	15.70	24.19	23.51
Δg = 1	26.50	36.50	32.95	23.70	33.46	33.70
SNR = 20	25.49	38.01	36.39	880.04	41.97	39.03
SNR = 10	30.47	39.79	40.88	1610.0	45.50	45.29

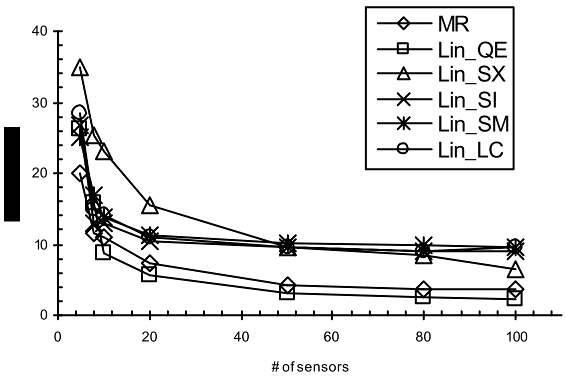

Figure 1. RMS error of localization error for different methods as a function of number of sensors

From table II, we observe that an increase in perturbation on the parameters ($\Delta\alpha$, Δr, and Δg) leads to significantly larger estimation error (Table 2). All five linear algorithms, except for SX, displayed similar robustness to perturbation and noise contamination. However, under the uncertainty of the sensor gain calibration or additive noise, the MR algorithm performs consistently better than any linear algorithm in that the RMS error is typically 10 meters smaller.

Sensor densities

We next investigated the impact on performance due to different sensor densities. We chose the parameter setting $\Delta\alpha = 0.5$, $\Delta r = 0.5$, $\Delta g = 0.2$, SNR = 60 dB to allow all localization parameters to subject to slight perturbation. The number of sensors to be tested are N = 5, 8, 10, 20, 50, 80, 100. The results are shown in Figure 1.

As observed in figure 1, when the number of sensors is increased, the accuracy of source localization also improves. In fact, QE performs slightly better than MR when there are more than 10 sensors used. For other linear methods, the reduction of RMS localization error is much slower.

CONCLUSION

In this paper we presented an un-constrained least-square solutions QE to the energy-based target localization problem. Compared to existing constrained least square solutions, the QE method yield superior performance and is a closed form solution. Due to space limitation, detailed algorithm derivation and additional simulation results are omitted in this manuscript, and can be found in [15] that has been submitted for publication.

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