Chapter X
Theory and Practice of Signal Strength-Based Localization in Indoor Environments

A. S. Krishnakumar
Avaya Labs Research, USA

P. Krishnan
Avaya Labs Research, USA

ABSTRACT

In this chapter, the authors concentrate on signal strength-based localization in indoor wireless networks, with emphasis on 802.11 networks. The authors briefly summarize some architectures and approaches researchers have taken to address this problem. They then present some insight into theoretical limits to location accuracy, and identify that the issues driving research work in this area will not only be location accuracy but other factors like deployment ease, management simplicity, adaptability, and cost of ownership and maintenance. With this insight, they present the LEASE architecture for localization that allows easy adaptability of localization models. The chapter discusses the use of Bayesian networks for localization and presents a zero-configuration Bayesian localization algorithm that simplifies the maintenance of the model. Although presented in the context of signal strength-based localization in indoor environments, the concepts are general enough to be applicable to sensor, ad hoc, mesh, and infrastructure-based deployments. They conclude with some open issues.

INTRODUCTION

Indoor wireless networks, especially 802.11-based wireless systems, are increasingly being deployed. Looking beyond simple untethered network access, services based on end-user location information provide compelling benefits, and in some cases satisfy regulatory concerns. Examples of such services include location-aware content delivery, emergency location, presence-enabled applications, services using location-based resource management, and location-based access control.

The techniques used for localization will depend on the constraints imposed on the problem, and the underlying technology being used. For example, base stations in outdoor wireless (e.g., cellular) networks are controlled by the service provider and have specialized hardware and software. The endpoints may have service provider-specific software as well. In contrast, indoor wireless networks are built using off-the-shelf components (e.g., access points) and endpoints that are more open (e.g., laptops). Clearly, these two environments present different constraints for localization, and hence the architecture and techniques needed for localization in these environments would differ. Sensor networks usually consist of low-power, low-bandwidth components and these impose additional constraints on the techniques employed.

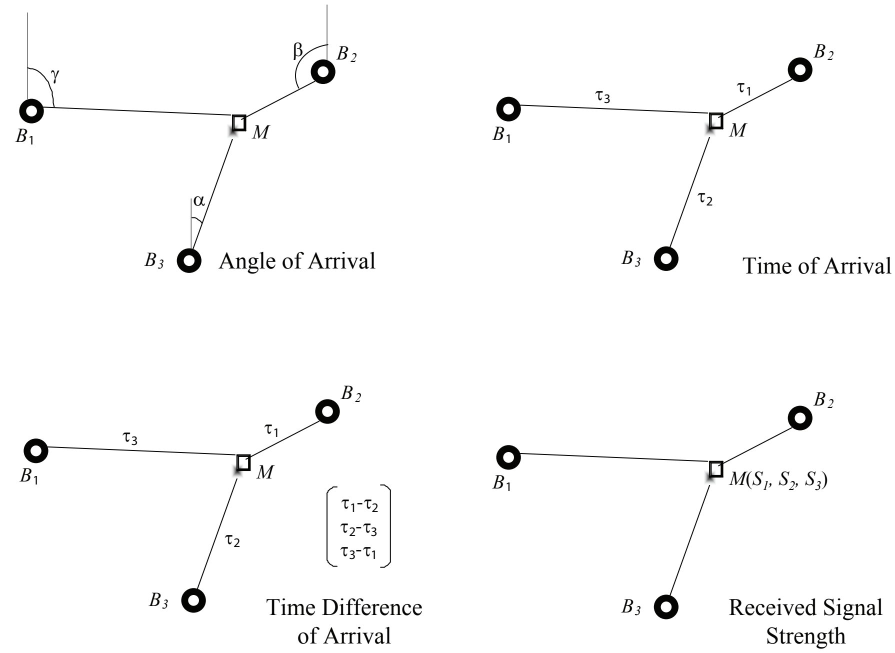
Localization is of value in both wired and wireless environments. In this chapter, we concentrate on wireless localization, and more specifically on indoor wireless environments. A commercially attractive option for localization in some scenarios is the Global Positioning System (GPS). Usually, GPS technology works well outdoors but has problems in indoor environments. Furthermore, GPS receivers form a closed platform and are co-resident with the device being located. In typical indoor environments, the devices used are general-purpose and do not necessarily have GPS receivers. Therefore, we look at localization aided by the wireless technology itself, namely the classes of techniques that can be used in radio networks. We then concentrate on localization in indoor wireless networks, specifically networks based on IEEE 802.11.

In radio networks, four classes of techniques have generally been used for localization, as depicted in Figure 1. These techniques are based on different features of the radio signal: angle of arrival, time of arrival, time difference of arrival, and received signal strength. The first is a technique using angles or an *angulation* technique. The other three are based on distances and thus are *lateration* techniques. With each technique, the location may be obtained directly by employing geometry, by using scene analysis techniques, or by probabilistic methods. These techniques have been used in many application contexts, e.g., navigation, radar, cellular communication systems, and robotics. An overview of the application of these techniques and others for indoor localization can be found in Hightower and Borriello (2001) and Pahlavan et al. (2002). An overview of localization in CDMA cellular systems is available in Caffrey and Stuber (1998).

Some of the techniques mentioned above require capabilities not typically found in off-the-shelf components used in indoor wireless communications. In general, time-of-arrival and time-difference-of-arrival techniques require an accurate time reference. This is usually available in systems such as cellular communications since an accurate time reference is needed for proper communication as well. Special equipment – such as multiple directional antennas or an antenna with a steerable beam – is needed to measure the angle of arrival. Other systems such as *Cricket* described in Priyantha et al. (2000) require special co-located radio and ultrasound transceivers.

Our emphasis in this chapter is on localization in indoor wireless networks, and specifically, techniques for localization in 802.11-based wireless networks. As mentioned earlier, the capabilities mentioned above are not available in typical off-the-shelf components used for indoor wireless networks. For example, with 802.11 systems the typical time reference available is of the order of 100ns[1], which is insufficient for accurate location based on time of arrival or time difference of arrival. The special requirements for localization via angle of arrival, time of arrival, and time difference of arrival methods make signal

Figure 1. Different techniques used for wireless location



strength-based approaches more attractive for indoor wireless localization. In signal strength-based localization, the received signal strength measurements from several transmitters are used as the basis for localization. The topic of signal strength-based localization has seen a lot of interesting work and many techniques have been proposed, experimentally verified (e.g., in Bahl and Padmanabhan (2000a), Krishnan et al. (2004), Ladd et al. (2002), Prasithsangaree et al. (2002), Roos et al. (2002), Saha et al. (2003), Youssef et al. (2003)), and analytically studied (e.g., in Elnahrawy et al. (2004), Krishnakumar and Krishnan (2005), Malaney (2004)).

The rest of the chapter is organized as follows. First, we provide an overview of the various techniques used for wireless localization. We then concentrate on signal strength-based localization, and discuss both experimental techniques and theoretical work in this area. Other issues that are important in practice, including deployment and maintenance cost, and research work addressing these concerns are considered next. We conclude with open issues.

LOCALIZATION IN WIRELESS NETWORKS: A BRIEF OVERVIEW

Localization is a much-studied topic in the context of radar, cellular WAN and many other technologies. Even if we restrict ourselves to indoor localization, there are many approaches to the problem, as surveyed in Hightower and Borriello (2001), Nerguizian et al. (2001) and Pahlavan et al. (2002). While most of this chapter concentrates on signal-strength based localization techniques, it is instructive to briefly understand systems based on other approaches: specifically, time of arrival, angle of arrival, time difference of arrival, and proximity and we present them in this section.

All the work described in subsequent sections focuses on localization in a plane. In many practical situations, one would like to have at least a quantized estimate of location in the third dimension. Although this issue has not been studied extensively, we outline some initial attempts at dealing with this in a later section.

Systems Using Time of Flight

Global Positioning Satellite (GPS) navigation system is a very well-known example of localization systems. This system uses the time-of-flight of radio signals to perform lateration. Its performance indoors can be problematic due to signal propagation issues. When used outdoors, GPS provides an accuracy of 1-5 meters (only with differential correction; see e.g., USDOT (2002) and Moore et al. (2002)) over 95% of the time. In this chapter sequel, we use "accuracy of x units (y %)" to indicate an accuracy of x units y % of the time. The time of flight approach is difficult to use in indoor wireless systems due to a lack of an accurate time reference in commercially available network interface cards. For example, as pointed out earlier, with 802.11 wireless systems, the typical time reference available is of the order of 100ns, which is insufficient for accurate location based on time of arrival or time difference of arrival. However, in Günther and Hoene (2005), the authors present a technique to increase the resolution by using multiple delay measurements and applying statistical techniques to improve the accuracy.

Systems Using Angle of Arrival

Obtaining angle-of-arrival information requires directional antennas or advanced signal processing. This is usually not available in typical off-the-shelf mobile terminals. The use of angle-of-arrival information indoors is also problematic due to the effects of multipath propagation. There may be situations where the signal that is registered is not the direct, line-of-sight signal, but a reflected one. This could introduce significant error in the location measurement. However, this technique can be and is used outdoors with satisfactory results. Some examples are emergency E911 service in North American wide-area cellular systems and VHF omnidirectional ranging used in flight navigation. An indoor positioning system using angle of arrival is described in Niculescu and Nath (2003). Based on simulations, they report positioning error as a fraction of communication range and as such it cannot be directly compared with experimental results reported for other methods.

Systems Using Time Difference of Arrival

A good example of a system using time-difference-of-arrival information is the Long Range Navigation (LORAN) system (see Sonnenberg (1988)) used by ships and aircraft. This was the main navigation system used by marine craft before the advent of GPS navigation systems. If the position of two LORAN transmitters is known, then the receiver is positioned somewhere on a hyperbolic curve between the transmitters where the time difference of received signals is constant. Using another pair of transmitters with at least one different transmitter, the position can be determined as the intersection of two hyperbolic curves. The Active Bat system described in Harter et al. (1999) employs time difference of arrival in a different fashion than above to compute ranges and perform multi-lateration. In this system, the entities to be tracked carry tags that emit an ultrasonic pulse to a grid of ceiling mounted receivers. This pulse is sent as a response to a request sent by a controller using radio frequency signals. When the controller sends a request to the tag, it simultaneously sends a reset signal to the ceiling receivers using a wired network. The ceiling receivers use the time between the reset signal from the controller and the ultrasound signal from the tag to compute a range value to the tag. These are then sent to the controller, which then performs multi-lateration. This system has an accuracy of 9cm (95%). The Cricket system described in Priyantha et al. (2000) is similar, but does not require ceiling-mounted receivers or a central controller. The mobile terminals act as receivers and also perform timing and computation functions. In their evaluation, the Cricket system estimates location within a 4ft x 4ft (1.2m x 1.2m) cell every time.

Systems Using Proximity

One of the earliest reported indoor location systems is the Active Badge system described in Want et al. (1992). This system used mobile diffuse infrared transmitters and a fixed grid of receivers. There is no angulation or lateration involved; location is determined via proximity. It locates objects at the granularity of the size of a typical room. Many commercially available avalanche transceivers (see e.g., EN282:1997 (1997)) used by backcountry skiers are also based on proximity information. Another example of a proximity-based location system is the RFID system, which is becoming increasingly popular. These systems use an RFID tag, active or passive, on the entity being tracked. An active tag can transmit its ID when in the vicinity of an RFID reader. When a passive tag is in the vicinity of reader, it reflects the incident RF energy with its own ID information superimposed on it. This information can be used to infer the location of the tagged terminal. These tags can be usually read from a range of 3ft (0.9m) for passive tags to 20ft (1.6m) for active ones.

SYSTEMS FOR AD HOC AND WIRELESS SENSOR NETWORKS

It is instructive to understand, at a high level, the general principles used in localization in wireless sensor networks. Wireless sensor networks are a special class of ad hoc networks and have been an active area of investigation and continue to be so. The interested reader is referred to the survey in Akyildiz et al. (2002) for an introduction. These networks typically operate in environments where there is no available infrastructure and it is desirable to minimize centralized processing and control. In these systems, all mobile terminals have similar sensors and processing capabilities. In such environments, localization techniques that do not depend on an infrastructure are needed. By exchanging information with their neighbors and engaging in distributed computing, the mobile terminals converge to an estimate of nearby objects' positions. These techniques use some combination of proximity, triangulation, and scene analysis.

There is a wealth of literature on localization in wireless sensor networks that is beyond the scope of this chapter. What follows is a brief introduction to this substantial literature.

Systems using proximity have been described in Bulusu et al. (2000). In that paper, it is assumed that there are a fixed number of reference nodes arranged in a regular mesh which transmit a periodic beacon signal with period T. These nodes have an overlapping coverage region. The localization technique is based on connectivity. The beacon transmissions of the reference nodes are synchronized such that there is exactly one beacon from each reference node in any time interval T. A node listens for a specified time interval t and computes a connectivity metric that is the ratio of the number of beacon transmissions heard from a reference node to the total number of beacons transmitted by that node in that interval. If this ratio exceeds a threshold, then that reference node is considered *connected* to the node being localized. The location of the node is then computed as the centroid of the locations of all the reference nodes that are connected to this node.

A more general method using connectivity-induced constraints is due to Doherty et al. (2001). This method removes the constraint of a regular mesh for reference nodes and introduces a more generic constraint model. As before, the locations of a few reference nodes are assumed to be known. If two nodes are in communication, the distance between them is constrained to be less than or equal to the radio range of the terminals. Given the known locations and the communication graph (i.e., who can hear whom), the problem of localization is converted to a feasibility problem of satisfying all the constraints imposed by the communication graph. This formulation requires centralized processing.

Unlike the systems considered above, the SpotON system described in Hightower et al. (2000) uses lateration based on measured distance between low-cost radio tags where the measurement is based on received signal strength information. The use of received signal strength for localization will be elaborated in the rest of the chapter. Some other examples of localization systems for wireless sensor networks may be found in Albowicz et al. (2001), He et al. (2003), and Savvides et al. (2001).

USING RECEIVED SIGNAL STRENGTH FOR LOCALIZATION

Our emphasis in this chapter, as described earlier, is signal strength-based localization. We now outline the general principles used in signal strength-based localization and categorize techniques to help in understanding them. It is to be noted that these principles apply as well to systems using signal strength for localization in wireless sensor networks. In this case, some of the sensor nodes are at known locations, thus taking on the role of access points in indoor WLAN environments. In the sequel, it is to be understood that wherever there is a reference to access points, it could be substituted by transmitters in a sensor network.

Please refer to Figure 2 for an explanation of terms used in this section. The access points (or sensor transceivers) are marked as AP_i. The profiled signal strength vector at location *i* is s_i whose components are the received signal strengths (a deterministic value or a random variable with a specified distribution) from the different access points. For example, $s_{i,1}$ is the received signal strength at location *i* from the access point AP_1.

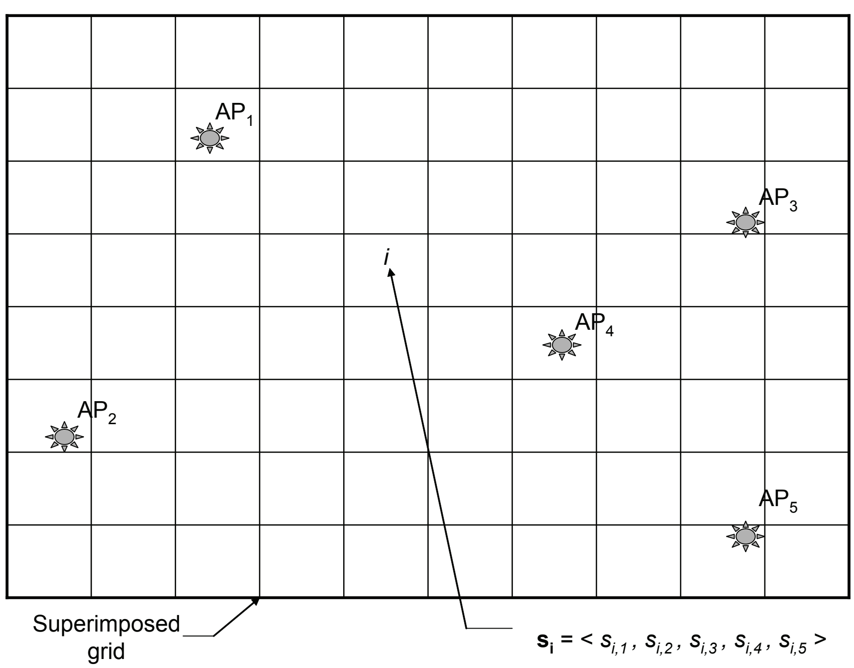
We classify localization techniques along two dimensions: one is based on the nature of the computational technique used and the other is based on where the measurements are collected. More specifically, computational techniques are classified as *deterministic* or *probabilistic* while measurements may be collected at the *client* or by the *infrastructure*. We elaborate on these classifications below.

Deterministic and Probabilistic Techniques

In one approach, the received signal strength measurements can be used to determine the range to the transmitter (based on propagation models) and then for localization via multi-lateration. Another approach is to build an *a priori* radio signal strength map of the region under consideration and determine location based on the measured signal strengths using some optimization criterion. For example, the minimum euclidean distance in the signal-strength vector space can be used as the optimization criterion as described in Bahl and Padmanabhan (2000a). We call such techniques *deterministic*. As described in Rappaport (1996), the received signal strength is a random process that may be modeled by a log-normal distribution. Based on this fact, many probabilistic approaches have been proposed. In these approaches, a signal strength probability distribution based on profiling measurements is determined on a lattice of locations superimposed upon the area under consideration. When attempting to determine the unknown location of a terminal, the received signal strength measurement is estimated using probabilistic methods such as maximum likelihood estimation. Instead of profiling, it is possible to postulate a propagation model and estimate the posterior distribution of the parameters and the location based on the measurements and a specified prior distribution. All such techniques may be termed *probabilistic*.

1. **Deterministic techniques:** One example of deterministic techniques is RADAR described in Bahl and Padmanabhan (2000a). Earlier, Christ and Godwin (1993) proposed a conceptually similar technique using custom hardware and non-802.11 radios. In RADAR, the area under consideration is divided into cells and a radio profile is obtained. The radio profile assigns a signal strength vector to each cell; the components of the vector are the measured received signal strengths from fixed radio access points. To determine the location, various flavors of nearest neighbor algorithms are employed. In Bahl and Padmanabhan (2000a), a multi-lateration approach is also described. In this approach, the profiling data is used to compute the parameters of a propagation model. The computed model is used to convert received signal strength measurements into range values. Subsequently, multi-lateration is used to estimate the location. The authors report an accuracy of 2.9-4.3m (50%).

Figure 2. Basic elements of analysis based on received signal strength



2. **Probabilistic techniques:** Several types of probabilistic techniques have been proposed in the literature. One class of techniques requires profiling; see for example, Ladd et al. (2002). In this approach, signal strength measurements are made for profiling purposes. However, each cell is assigned a probability distribution for the signal strength values instead of a single vector value. The measured signal strength value and the prior distributions are then used to compute a maximum likelihood estimate of the location. Other estimators could also be used, e.g., the mean value of a Bayesian posterior distribution (based on an assumed prior distribution for the location). Other probabilistic approaches may be found in Abnizova et al. (2001) and Myllymäki et al. (2001). A system using probabilistic techniques is the HORUS system described in Youssef and Agrawala (2005), Youssef et al. (2003), and Youssef and Agrawala (2004a), which has as its goals high accuracy and low computation. Location clustering is used to reduce computational burden. This system addresses the causes of wireless channel variations to improve accuracy. Signal strength distributions instead of single values are used in its radio map. The reported accuracy of the system in Youssef et al. (2003) is 7ft (2.1m) (90%) and 3.5ft (1.1m) (50%). In Youssef and Agrawala (2004b), an argument is provided to show that probabilistic techniques can provide better accuracy than deterministic ones.

Another class of probabilistic techniques does not require any prior profiling. In this case, measurements from multiple terminals are used to estimate all their locations simultaneously. This is done by assuming a parameterized propagation model and estimating both the parameters of the model and the locations of the terminals simultaneously. This technique has the added advantage that it automatically adapts to changing radio environments. Recently, the use of Bayesian networks to perform this computation has been reported in Madigan et al. (2005). An earlier example of the use of Bayesian networks for location is the Nibble system described in Castro et al. (2001). Nibble was used only to localize the radio terminal at room-level granularity as opposed to a more accurate estimate of the actual location described in Madigan et al. (2005). Also, Nibble locates terminals individually whereas the method in Madigan et al. (2005) estimates the location of multiple terminals simultaneously.

Client- vs. Infrastructure-Based Systems

Another way to group the techniques is based on where the signal strength measurements are made: at the terminal being located or at other terminals in the network, e.g. sensors, sniffers or access points. We call these client-based and infrastructure-based techniques respectively.

In this grouping, techniques such as RADAR and those described in Ladd et al. (2002), Myllymäki et al. (2001) will fall into the client-based group. An infrastructure-based system called LEASE was described in Krishnan et al. (2004) and further developed in Ganu et al. (2004), and is outlined in a later section. Note that most infrastructure-based techniques can be deployed as client-based techniques. Similarly, several client-based techniques can be adapted to be infrastructure-based. The main issue in such adaptations is the amount of data needed by the technique to build its models.

In Table 1, we present a 2 x 2 matrix that groups some of the reported techniques based on received signal strength measurements according to these two classifications: deterministic/probabilistic and client-based/infrastructure-based.

Table 1. Grouping of localization techniques

	Client-based	Infrastructure-based
Deterministic	Bahl and Padmanabhan (2000a) – *RADAR* Prasithsangaree et al. (2002)	Krishnan et al. (2004) – *LEASE*
Probabilistic	Youssef et al. (2003) Youssef and Agrawala (2004a) – *HORUS* Madigan et al. (2005) – *Bayesian Nets* Abnizova et al. (2001)	Madigan et al. (2005) – *Bayesian Nets* Castro et al. (2001) - *Nibble*

LOCALIZATION ACCURACY

As pointed out earlier, the fact that received signal strength (RSS)-based techniques can be implemented using existing hardware (off-the-shelf) makes them extremely attractive, leading to significant research in RSS-based localization. Even though signal strength measurements are available for "free," they are not the easiest to use. Multi-path propagation in indoor environments makes working with signal strength measurements challenging. The terminals and associated network cards are also heterogeneous, adding to the complexity of the problem. There is also the issue of the correct metrics to use when analyzing location accuracy. For example, is distance error the correct metric? Or, is it a room level accuracy that is desired? Notwithstanding these issues, the median estimation error has traditionally been popular amongst researchers in experimental studies of localization.

Several techniques have been proposed and experimentally evaluated for localization using signal strength. For example, as mentioned earlier, the RADAR system from Bahl and Padmanabhan (2000a) exhibited a localization accuracy of 2.9-4.3m (50%) and Youssef et al. (2003) reported an accuracy of 7ft (2.1m) (90%). The accuracy of the LEASE system from Krishnan et al. (2004) was 7-15ft (2.1-4.5m) (50%). However, these studies were done in different experimental environments.

Recently, some interesting studies have compared various techniques on the same experimental test bed (see Elnahrawy et al. (2004) and references therein). A noteworthy observation was presented in Elnahrawy et al. (2004): over a range of algorithms, approaches, and environments, there appeared to be limits to achievable localization accuracy. In particular, a median localization error of 10ft (3m) and 97th percentile of 30ft (9.1m) was generally observed. More specifically, the focus in Elnahrawy et al. (2004) is on area-based algorithms that trade accuracy (the likelihood that an object is within an area) for precision (the size of the returned area). The authors present and evaluate three area-based algorithms, determining accuracy and precision at the distance and room level. Using data from two sites (a university building and an industrial setting), they show that a wide range of area-based algorithms have similar fundamental performance. They then compare against point-based algorithms – specifically choosing the algorithms from Bahl and Padmanabhan (2000a) and Roos et al. (2002) (with some possible variants) as representative of deterministic and probabilistic strategies respectively. A key result was that the proposed area-based and chosen point-based techniques had "striking similarity" in performance graphs (apart from some Bayesian techniques that had lesser localization accuracy).

To understand if there was something more fundamental that dictated the similarity in localization accuracy results, they studied uncertainty probability density functions along the x and y axes generated from their Bayesian network. The wide distributions observed, especially in the industrial data set, were indicative of a high degree of uncertainty, which they concluded was more fundamental rather than an artifact of the techniques alone. They also allude to results from Battiti et al. (2002) that found that a host of learning approaches had similar performance to maximum likelihood estimation (as in Roos et al. (2002)).

An interesting question raised by these studies is: "Is there any theoretical limit to the localization accuracy using techniques based on signal strength?" To understand this, we present an analytical framework that tries to ascertain the attainable accuracy of such techniques.

ANALYTICAL UNDERSTANDING OF SIGNAL STRENGTH-BASED LOCALIZATION

There is limited analytical work attempting to understand the fundamental issues governing localization accuracy. We present below a synopsis of technical work dealing with this issue.

In Youssef and Agrawala (2004b), the authors developed an analytical framework for calculating the average distance error and the probability of error in location. They showed that probabilistic decision techniques can provide more accurate localization than deterministic ones, since unlike deterministic techniques, probabilistic methods can take into account that the signal strength vector is not always symmetric and identical at all locations. In Krishnakumar and Krishnan (2005), the authors analyzed the fundamental limits of the accuracy of localization using signal strength measurements. The received signal strength is a stochastic variable due to the effect of multi-path propagation. The main intuition in Krishnakumar and Krishnan (2005) was to recognize that a variation in measured signal strength due to change in location is indistinguishable from a variation due to shadowing. Hence, any decision rule will map a set of locations in the neighborhood of a point (x,y) to the point (x,y). Intuitively, this is the *uncertainty* in the location estimate caused by signal variance. The main idea was to define a quantity that captured this uncertainty in localization and derive an expression for this quantity under very general assumptions by mapping uncertainty in signal strength space to location uncertainty in the (x,y) plane. Various features of uncertainty were then analyzed, and also specifically for a log-linear radio propagation model. The lower bound derived matched favorably with experimental work providing an analytical explanation for the observations in Elnahrawy et al. (2004). We now provide more details of this work.

Theoretical Accuracy Limits

Assume that a signal strength vector $\vec{s}$ of dimension n is used to locate a terminal. This vector can be determined, for example, from n access points, sniffers or sensors, depending on whether a client-based or infrastructure-based deployment is used. The received signal strength is a stochastic variable due to multi-path propagation. As explained in Rappaport (1996), the logarithm of the received signal strength can be modeled as a normal distribution around a mean value with variance σ_i^2, $1 \le i \le n$. The σ_i can be different from the shadowing variance if filtered signal strength measurements are used. A common method to deal with short-term variations in signal strength due to fast fading is to use the median of a few uncorrelated measurements. It is reasonable to assume that the individual components of the signal strength vector $\vec{s}$ are independent and that the mean signal strength at a location is a differentiable function over a region of interest.

The approach in Krishnakumar and Krishnan (2005), summarized in Figure 3, is to define an α-region in location space such that the total probability that the observed signal strength is due to an emitter located at some point in the region is α. (There may be more than one such region satisfying the condition.) As shown in the figure, let $\mathcal{T}$ be a mapping from location to mean signal strength. Restricting attention to the cases when $\mathcal{T}$ is one-to-one (e.g., when propagation loss is a monotonic function of distance from the emitter as with inverse exponential propagation functions) and when signal variance distribution is symmetric, we can compute the characteristics of the α-region in location space by mapping it to the signal space and then back to the location space.

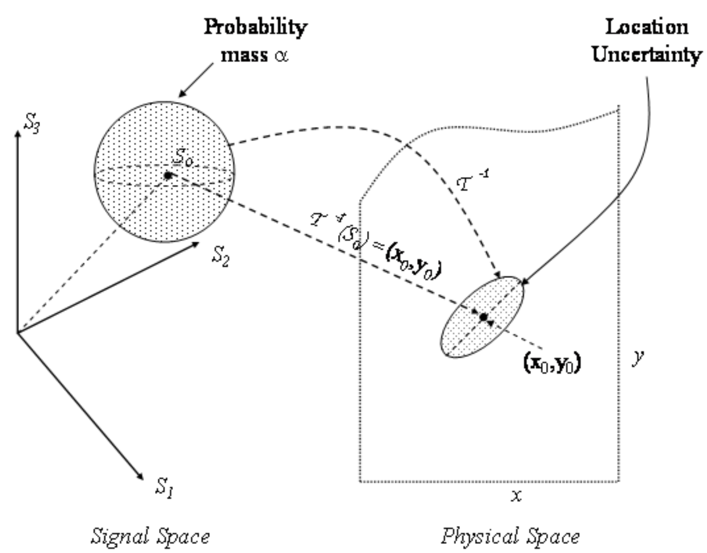
To do this, one first computes the characteristics of the hypervolume in signal strength space centered on the mean signal strength vector that encloses a probability mass of α. It is shown that the hypervolume is a hyperellipsoid with semi-axes $R_n\sigma_i$, where R_n is a scaling factor related to the confidence level α. This relationship is given by

$$\alpha = \frac{\Gamma\left(n/2, R_n^2/2\right)}{\Gamma\left(n/2\right)},$$

where Γ(· , ·) is the incomplete gamma function. Given α and n, the equation above can be used to compute R_n. (Details of this derivation appear in Krishnakumar and Krishnan (2005), Appendix II.)

Clearly, there is a mapping between location and mean signal strength. In practice, for signal strength estimation techniques to work, this mapping has some properties; e.g., each location maps to a mean signal strength vector. Using the nature of the region in signal strength space as derived earlier, the analysis then derives the structure of the uncertainty in location. In particular, it is shown that the uncertainty region in the (x, y) plane is an ellipse whose equation is $ax^2 + by^2 + cxy + d = 0$, where

Figure 3. Mapping uncertainty in signal strength space to uncertainty in location



$$a = \sum_{i=1}^{n} \frac{t_{i1}^2}{\sigma_i^2} \quad , \quad b = \sum_{i=1}^{n} \frac{t_{i2}^2}{\sigma_i^2}$$

$$c = \sum_{i-1}^{n} \frac{2t_{i1}t_{i2}}{\sigma_i^2} \quad , \quad d = -R_n^2$$

and $T = \{t_{i,j}\}$ is the Jacobian of the mapping $\mathcal{T}$ from location to mean signal strength measurements. The mapping T is an n x 2 matrix. In particular, $t_{i,1} = \partial s_i / \partial x$, and $t_{i,2} = \partial s_i / \partial y$, where s_i is the i^{th} component of the signal strength vector $\vec{s}$.

Properties of the Uncertainty Region

Several interesting properties of the uncertainty region can be determined once its structure is known. The semi-major axis, semi-minor axis and their geometric mean are quantities of interest. In Krishnakumar and Krishnan (2005), they are defined to be the upper, lower, and mean uncertainty, respectively, and shown to be bounded quantities. The maximum uncertainty over the convex hull bounded by the access points for various configurations was studied assuming a log-linear radio propagation model. Several interesting characteristics that were observed are summarized here. For example, the variation in uncertainty when the APs (transmitters) are at the vertices of an equilateral triangle or a regular square, both inscribed in a circle of radius 100 units, is shown in Figure 4. The figures show the contours of equal uncertainty on the surface and their projections on the (x, y)-plane.

There are several open problems in understanding and analytically determining the properties of uncertainty in the region of the convex hull bounded by the APs. For example, it appears that there is at least one location of locally minimum uncertainty in the convex hull of the AP locations. Is this always so, and is an analytical expression for the location and values of such minima obtainable? Is there a provable number of such minima in the convex hull region? What happens when the APs are not uniformly located? And so on.

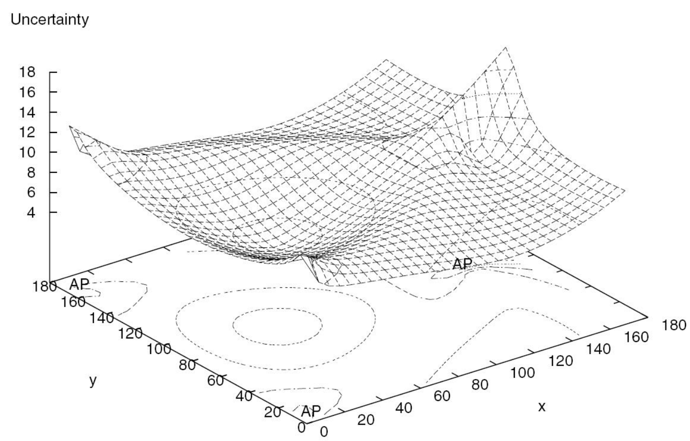
The expressions also led to other fundamental observations. For example, for confidence levels above 0.8 (i.e., $\alpha > 0.8$) while keeping all other quantities unchanged, the uncertainty increases disproportionately. Under simplifying assumptions, it is shown that uncertainty is proportional to the variance in signal strength. This dependence is important to understand because it is a factor that can be influenced by the localization algorithm. Several algorithms effectively reduce this variance to improve the localization performance. This may be achieved by using multiple samples as described in Bahl and Padmanabhan (2000a) and Krishnan et al. (2004), probabilistic techniques as described in Ladd et al. (2002) and Youssef et al. (2003), or autoregressive models as described in Youssef and Agrawala (2004a). The tradeoff between computational complexity and the method used for variance reduction must be understood when considering the use of a technique.

The deployment of APs or sniffers affects the attainable accuracy in determining location. It is, therefore, important to understand this relationship. The analysis example in Krishnakumar and Krishnan (2005) shows that if APs are added to the same area, the uncertainty decreases. However, if the area of coverage is increased while increasing the number of APs, the minimum uncertainty increases or remains stable in some cases. The evaluation of the variation of minimum uncertainty for different AP/sniffer placement strategies is an interesting open issue. Uncertainty is also related to the circular probability; the reader is referred to Krishnakumar and Krishnan (2005) for more details omitted from this summarization.

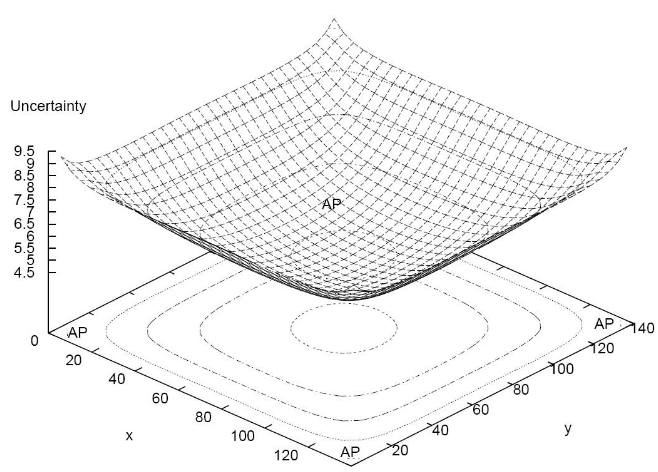
Experimental Validation

It is instructive to use actual measurements from an indoor 802.11b network to calculate the parameters needed to compute minimum uncertainty. This was done in Krishnakumar and Krishnan (2005), where the computed minimum uncertainty was compared to the reported median errors in the experimental literature. Details may be found in the original reference. Considering an 802.11b network where 3 APs are located at the vertices of an equilateral triangle of side 150 ft (45.7m), the minimum uncertainty location is at the centroid of the triangle and is computed to be approximately 4.5ft (1.4m), using non-preprocessed

Figure 4. Max. uncertainty (in ft) in the region enclosed by n APs; α = 0.75, and $\sigma_i = 0.707$, $\forall i$ (reprinted with permission from Krishnakumar, A.S., and Krishnan, P., On the Accuracy of Signal Strength-Based Location Estimation Techniques. Proceedings of the 2005 IEEE Infocom Conference, © 2005 IEEE).



n = 3



n = 4

signal measurements to compute variance. This computed minimum uncertainty compares favorably with the reported median error values in the experimental literature, e.g., RADAR (≈ 2.9m) (Bahl and Padmanabhan (2000a)), LEASE (2.1 – 4.5m) (Krishnan et al. (2004)), HORUS (≈ 1.1m) (Youssef et al. (2003)). Therefore, the uncertainty appears to be a realistic indicator of the median error. The lower value using the probabilistic technique in Youssef et al. (2003) could be attributed to the preprocessing of the measurements leading to a lower signal variance and to the smaller distance between the APs. This corroborates in a different way some of the observations in Elnahrawy et al. (2004).

Experimental results obtained from systems with widely varying parameters are difficult to compare. Elnahrawy et al. (2004) address this problem by comparing them on the same platform. An alternative is to evaluate the effect of the various algorithms on the raw parameters that feed into the uncertainty computation and determine how the lower bound will be affected as was done in Krishnakumar and Krishnan (2005).

An Analysis Using Cramér-Rao Bounds

Motivated by a security aspect of localization, a lower-bound on the variance of the position of a node in the 2-dimensional plane is derived in Malaney (2004). In the application considered by Malaney, terminals are equipped with a GPS system and report their location. However, the terminals may not be trusted to report accurate location information. The problem here is to verify if the supplied location information seems trustworthy, based on the network's own internal signal strength measurements. The problem becomes one of verifying rather than determining the location of a terminal.

In trying to assess if a claimed location by a terminal is indeed feasible given the network's signal strength measurements, Malaney used the Cramér-Rao bound to compute the lower bound on the localization variance. The location reported by the terminal, signal strength measurements made by the network, and the lower-bound on localization variance are then used to compute the confidence level of the location reported by the terminal. While the form of the analysis in Krishnakumar and Krishnan (2005) has parallels to the analysis in Malaney (2004) and Malaney's analysis effectively computes a lower-bound on the localization accuracy, the use of the technique in Malaney (2004) is for different purposes and the two analyses are not the same. Malaney's analysis technique is summarized here in the spirit of being another method to measure and use localization accuracy estimates that can also be used to evaluate localization estimators.

Specifically, in Malaney (2004), the author assumes the log-normal distribution for received signal strength. By then writing the expression for distribution of signal strength received at a point (x_0, y_0) from a node at position (x_i, y_i), Malaney derives an expression to calculate the terms of the Fisher information matrix. If b denotes $(10k/(\sigma \ln 10))^2$, where σ is the standard deviation of the shadowing (in dB), and k is the environment-dependent path-loss exponent, the Fisher matrix turns out to be:

$$\begin{pmatrix} b\sum_{i=1}^{n}\frac{\sin^2\varphi_i}{d_i^2} & \frac{b}{2}\sum_{i=1}^{n}\frac{\sin 2\varphi_i}{d_i^2} \\ \frac{b}{2}\sum_{i=1}^{n}\frac{\sin 2\varphi_i}{d_i^2} & b\sum_{i=1}^{n}\frac{\sin^2\varphi_i}{d_i^2} \end{pmatrix}$$

where d_i refers to the distance from the node of unknown position (x_0, y_0) to a node of known position (x_i, y_i), n is the number of dimensions in the signal vector, $\sin\varphi_i = (x_i - x_0)/d_i$ and $\cos\varphi_i = (y_i - y_0)/d_i$

Malaney then computes the inverse of the Fisher matrix, and the sum of the diagonal terms of this matrix provides the Cramér-Rao bound on the variance of the position of a node in the 2-dimensional plane. This quantity, σ_{CR}^2, is derived to be

$$\sigma_{CR}^2 = \frac{\sum_{i=1}^{n} \frac{1}{d_i^2}}{\left(\frac{10k}{\sigma \ln 10}\right)^2 \left(\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \frac{\sin^2(\varphi_i - \varphi_j)}{d_i^2 d_j^2}\right)}.$$

The estimate computed above is used to determine if a terminal is misrepresenting its location by verifying the best estimate of the actual position using the derived Cramér-Rao bound, and, more importantly, figuring out how many standard deviations the terminal is away from its claimed position. It is assumed that authorized nodes could pick up the signal strength readings from the terminal and these could be employed by the algorithm for detecting violations.

MANAGING THE COMPLEXITY OF DEPLOYMENT AND OPERATION

In addition to the aspects previously considered that focused on localization accuracy, there are other issues to be taken into account. One such issue is the effect of model variation with environmental changes and the need to adapt the models to preserve localization accuracy. Another is the cost and topology of deployment to obtain the best coverage at the least cost. The sections that follow expand upon these topics.

Adaptation to Environmental Changes

In this subsection, the cost and complexity of building and maintaining the model in changing environments is considered. Even in normal office environments, changing environmental, building, and occupancy conditions could have an effect on signal propagation models as observed in Bahl et al. (2000b). This variation could be due to environmental changes such as rearranged furniture, seasonal occupancy changes, or structural changes such as addition or removal of temporary walls. More rapid changes may be expected in dynamic environments such as an operational warehouse with moving forklifts. It is a challenge to keep the model adapted so that the results remain reasonably accurate. Moves, additions, and changes of transmitters may also require the model to be rebuilt. The models are difficult to maintain and update if purely static techniques are used. We discuss techniques to deal with this problem in this section. Additionally, as pointed out in Smailagic et al. (2001), profiling involves an upfront cost and effort to deployment, and adds to the complexity of maintaining the model. In this context, one seeks simple non-parametric models that can be built with little or no profiling and achieve localization accuracy comparable to techniques that profile the site extensively. This is discussed in the next subsection on zero profiling techniques.

The complexity of building and maintaining the model was identified in Bahl et al. (2000b) and Smailagic et al. (2001). In indoor environments, multi-path propagation is accommodated by using an appropriate exponent in the inverse power law propagation model. This exponent has to be determined experimentally. The exponent could change over time due to changes in the physical environment of the site. Even if a propagation model is not used explicitly, an equivalent underlying radio map has to be determined. This map is also time-variant as with the propagation model. Some techniques have been attempted to address this problem. In Bahl et al. (2000b), an appropriate model from a database of models was chosen based on the reference signal strength seen between access points. The model building problem was tackled to some extent in Smailagic et al. (2001) where a specific functional relationship between signal strength and distance was generated empirically for their site. In practice, the measured signal strength contours are usually anisotropic, unlike the circularly symmetric functions used in Smailagic et al. (2001). In Pandey et al. (2005), a client-assisted data collection scheme to address the issue of dynamically updating the information needed for the radio profile is presented. Their system uses sniffers and client software to build a database of signal strength maps that are continually updated.

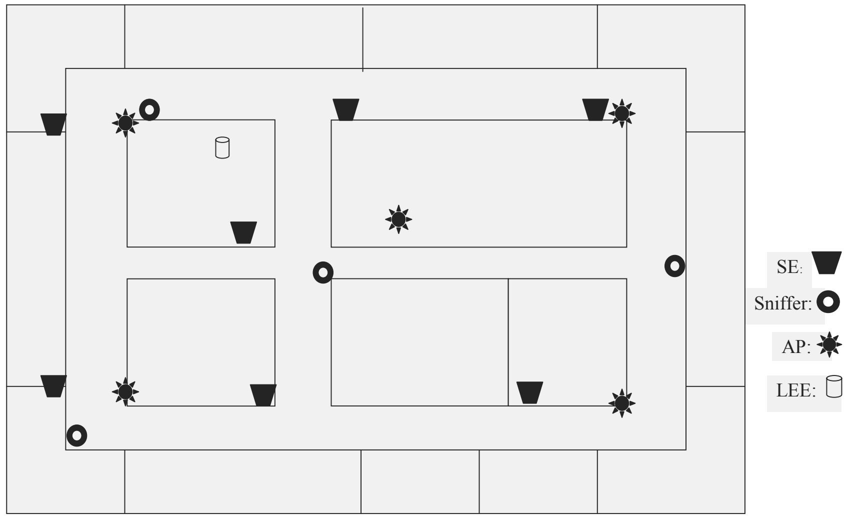
The LEASE system to address the model adaptation issue was presented in Krishnan et al. (2004). The LEASE system comprises three main components: stationary emitters (SEs), sniffers, and a location estimation engine (LEE). In Figure 5 we show a possible office site floor with some access points (APs/transmitters), SEs, and sniffers. The LEE can be located anywhere in the network.

The SEs in the LEASE system are standard, inexpensive wireless transmitters that emit a few packets occasionally. The SEs are of small form factor and usually battery-powered. The sniffers sniff on the wireless medium, cycling through a set of specified frequencies and listen for all communication from wireless clients and SEs. They record the received signal strength (RSS) from them. This information is sent to the LEE. The LEE also needs the coordinates of the SEs which could be broadcast by the SE. Using the SEs as "fixed points" of known location, a localization model is built by the LEE as needed, based on the RSS from the SEs. In a sensor network, different sensors could take on the roles of SEs or sniffers.

The LEE builds a model for each sniffer as follows. First, it smoothes the data points, e.g., using a generalized additive model (GAM) (see Hastie and Tibshirani (1990)). Second, a synthetic model is generated. The site is divided into small grids (e.g., grids of 3ft × 3ft (0.9m × 0.9m) cells). Using Akima splines (see Akima (1996a), Akima (1996b) and Akima (1970)) the smoothed values obtained from the GAM are interpolated to estimate the RSS at each grid center. The synthetic model for the specific sniffer is the generated RSS-grid information with an estimated RSS for each grid point. Repeating the above technique for each of the n deployed sniffers, gives a set of grids for the site, where each grid has an associated n-vector of estimated RSS. This n-vector corresponds to the profiled RSS from each AP as seen at each grid point, assuming the APs and sniffers are co-located. The LEE also uses absence of a signal in localization. This is useful since the absence narrows the search area by indicating that the point in question is far away from the sniffer. A variation of the nearest-neighbor algorithm is used for matching the received client RSS vector to the model.

One main result in Krishnan et al. (2004) was that for an office site of size 30,000 sq ft. (2,787 sq. m), with only 12 SEs, a median error of 15ft (4.5m) can be obtained. Increasing the number of SEs does reduce the error further; e.g., with 104 SEs, the median error was just 7ft (2.1m). Different sites also exhibited different accuracies. The authors also motivated a normalized error metric that takes into account the work done in building the model, the localization errors, how dynamic the signal environment is, etc. They showed that the LEASE technique is efficient in terms of the normalized error metric.

Figure 5. A possible office site with components of the LEASE system



Zero-Profiling Techniques

Localization systems that make use of an explicit propagation model, or a radio map of the region of coverage, require initial profiling. This can be a labor-intensive process. Although automated systems to perform this measurement have been reported in the literature (e.g., Hills and Schlegel (2004)), it still represents an additional cost for deployment. This brings up the feasibility of zero-profiling techniques. As the name implies, these techniques will require no profiling and can be deployed out-of-the-box. As such, they can be used to advantage for the following purposes: a) adaptation in dynamic environments, and b) lowering the cost of deployment. One such technique making use of Bayesian networks was described in Madigan et al. (2005). While this approach does not require profiling, it is computationally expensive and needs further work to be suitable for real-time operation. This remains an interesting area of research. We briefly describe the Bayesian approach below.

In Madigan et al. (2005), instead of trying to locate a single terminal, the model tries to *simultaneously* locate a set of terminals. By appropriately exploiting signal strength information from a collection of terminals, it is shown that the localization for the entire set can be improved. The model is particularly relevant as the number of wireless terminals increases. The methodology uses hierarchical Bayesian graphical models (see Gelman et al. (2003) and Spiegelhalter and Lauritzen (1990)) for wireless localization. The study demonstrates that a hierarchical Bayesian approach, incorporating physical knowledge about the nature of Wi-Fi signals, can provide accurate location estimates without *any* location information in the training data, leading to a truly adaptive, zero-profiling technique for localization.

A *graphical model* is a multivariate statistical model embodying a set of conditional independence relationships. A graph displays the independence relationships. The vertices of the graph correspond to random variables and the edges encode the relationships. In the Bayesian framework, model parameters are random variables and appear as vertices in the graph. When some variables are discrete and others continuous, or when some of the variables are latent or have missing values, a closed-form Bayesian analysis generally does not exist. Analysis then requires either analytic approximations of some kind or simulation methods. The Markov chain Monte Carlo (MCMC) simulation method as described in Spiegelhalter and Lauritzen (1990) can be used for this purpose.

We illustrate this formulation with an example. Figure 6 shows the Bayesian network graph for the problem under consideration. An arrow leading from *a* to *b* implies that *b* is dependent on *a*. In this model, X and Y are the location coordinates to be estimated. D_i is the range to the transmitter *i*, S_i is the received signal strength from transmitter *i*, b_{ij} is a coefficient in the propagation model for signal transmission from transmitter *i* and τ_i is the precision of the normal distribution used to model signal strength variation. Here, the observed quantities are the signal strengths and the quantities to be estimated are the position coordinates X and Y. In the process of estimation, the model variables b_{ij} and D_i are estimated as well. To complete the model, each node should be assigned a conditional density given its parents. As an example, in Figure 6 we could specify[2]:

$X \sim \text{uniform}(0,L)$
$Y \sim \text{uniform}(0,W)$
$S_i \sim N(b_{i0} + b_{i1} \log D_i , \tau_i)$, $i = 1,2,3$,
$b_{i0} \sim N(0,0.001)$, $i = 1,2,3$,
$b_{i1} \sim N(0,0.001)$, $i = 1,2,3$.

Here, L and W are the length and width of the rectangular area under consideration. The signal strength (measured on a logarithmic scale) decays approximately linearly with the logarithm of distance. The notation $N(\mu, \tau)$ indicates a normal distribution with mean μ and precision τ. If we have prior information about the values of b_{i0} and b_{i1}, they could be incorporated via the mean value.

The model shown in Figure 6 treats the model parameter set for each transmitter as independent from each other. While this is quite general, it can also be computationally expensive. Although the propagation model of each transmitter has different model parameters, we could model them all as stochastic variables that are identically distributed with a common mean and variance (or equivalently, precision). This is a hierarchical model and can simplify the computational burden without sacrificing predictive accuracy. The Bayesian network graph for the hierarchical model is shown in Figure 7. For additional details and experimental results, the reader is referred to Madigan et al. (2005).

Deployment for Coverage and Localization

The location of the transmitters and the powers at which they transmit are chosen to satisfy certain objectives in any deployment. Most common is a minimum-cost deployment that ensures signal strength above a certain threshold over the service area, i.e. coverage. Another criterion could be the minimization of communication cost as in Kasetkasem and Varshney (2001). Other considerations may come into play. For example, for adequate coverage it may be desired that at any given point in the coverage area at least two transmitters with adequate signal strength be visible. Maximum coverage at minimum cost has been the main metric in wireless network deployment. There is a wealth of literature on placement optimization for coverage in wireless sensor networks, see for example, Meguerdichian et al. (2001) and Zou and Chakrabarty (2004).

However, a deployment that satisfies this criterion may not be optimal for localization, and may in fact be inadequate. As an example, visibility of three transmitters at any point is required for localization. Consider a long, rectangular area to be covered, as in Figure 8. From a coverage perspective, a line of

Figure 6. Bayesian network graph for localization

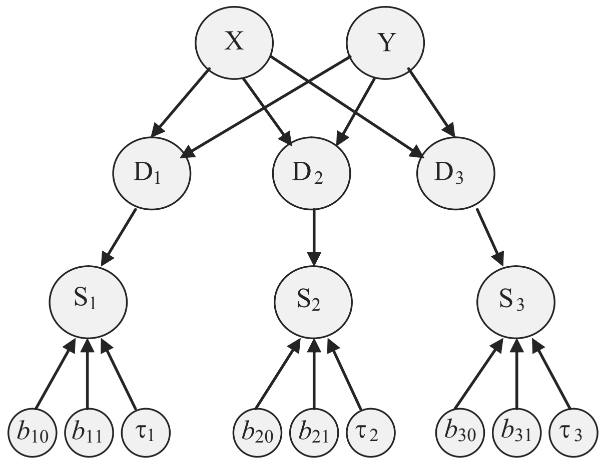
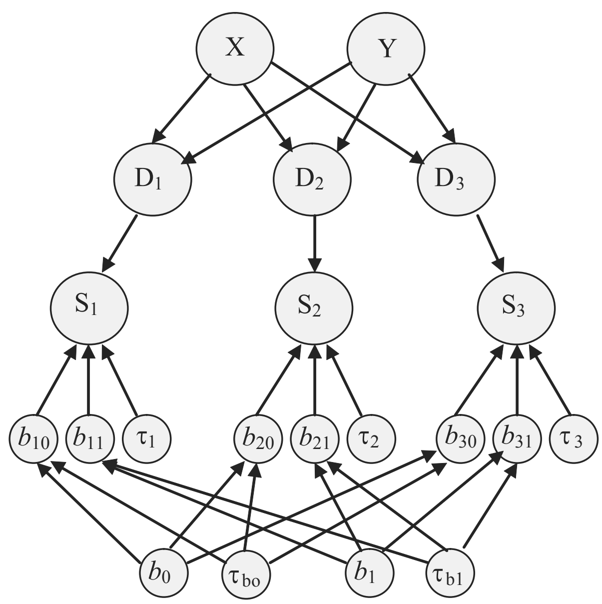


Figure 7. Graph of a hierarchical Bayesian network for the localization problem



transmitters running down the middle parallel to the long side would be optimal, and will minimize the amount of signal leaked to the outside. However, this is not a good configuration for localization, since there is no way to disambiguate locations that are symmetrically situated about this axis of transmitters. For example, the location of the two terminals shown in Figure 8 can not be disambiguated. Placement optimization for localization has not been studied as extensively as that for coverage. Joint optimization for coverage and localization is even less studied. An example can be found in Chakrabarty et al. (2002). This remains an interesting topic for further research.

OTHER ISSUES IN LOCALIZATION

Apart from the problem of deployment for coverage and localization described earlier, there are other issues in signal strength-based localization that have not been studied extensively and present opportunities for research. Three categories of issues are summarized below.

Dealing with the Third Dimension

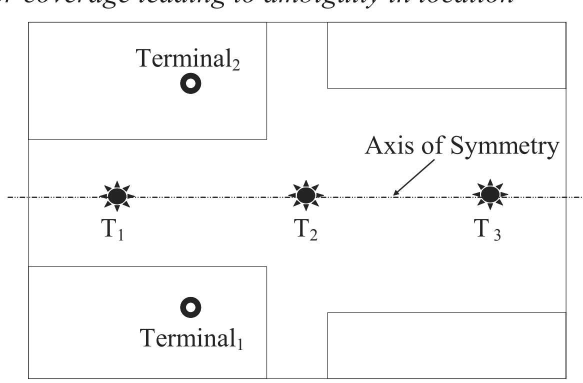
While most researchers have concentrated on locating a terminal on a building floor (i.e., two dimensional localization), there is in fact a third dimension that may be discrete, namely, which floor the terminal is on. One could call this "2.5-D" localization. The common assumption is that a floor attenuates a signal significantly and that the strongest signal will be from the floor on which the terminal resides. However, the problem is that the strongest signal may not be seen from a transmitter on the same floor. As an example, consider a transmitter at the bottom of an open stairwell and a terminal at the top of the stairwell.

A heuristic for floor estimation was described in Krishnan et al. (2004). The main idea was to take into account all visible transmitters (or sniffers that can see transmissions from a terminal). The heuristic uses modified majority logic to estimate the floor. The 2.5D localization problem is an interesting one that the reader may want to pursue.

Security Considerations

Security is a major concern in the deployment of wireless networks. While many of the security problems with WEP encryption etc. have been addressed by newer standards such as IEEE 802.11i (2005), there are a class of problems that arise when location is used for security purposes. For example, access control may be based on location. In these situations, the integrity of the data used to perform the estimation becomes very important. In the case of RSS-based methods, this means the integrity of RSS measurements. If client-based reporting is used, it is possible for a malicious terminal to report false values and thus spoof its location. This problem can be alleviated by using methods where the measurements are obtained by sniffers, sensors or similar components. Another approach to evaluating the integrity of client-reported data may be found in Malaney (2004).

Figure 8. Deployment for coverage leading to ambiguity in location



Even if such methods are used, a client terminal using directional antennas can throw off the localization algorithm. A malicious terminal could use a directional antenna to project itself into a secure space. While it can be easy to determine that a directional antenna is being used inside the region of coverage, such use is difficult to determine without additional equipment when the malicious terminal is located outside the coverage area. The use of a small number of reflector antennas located on the periphery of the building, facing outside, could address this issue. To our knowledge, this has not been investigated.

Management

While management is also a cost of ownership issue, there are some interesting questions in management that need to be noted. Does the architecture require additional components to be deployed (hardware and software)? If so, what is the method to manage these components? Does the scheme depend on fail-safe operation of these components? Is the security of the solution adequately addressed? We believe that several of these questions have not been adequately addressed in current work and represent an opportunity for future research in this area.

CONCLUSION

In this chapter, we have given a brief overview of indoor localization techniques. We focused in particular on received signal strength-based techniques for indoor wireless deployments using 802.11. Some of the techniques may be applied to other radio technologies also, e.g., Bluetooth. We summarized work that dealt with theoretical limits to accuracy of localization using received signal strength, including some experimental results. We presented techniques used to adapt to changing environments, and a zero-profiling Bayesian approach for localization. We have listed several other research issues in this area, including deployment, floor estimation, security and management.

This chapter is intended as an overview of signal strength-based localization, the research issues and some results obtained by various researchers. It is not intended to be a comprehensive survey of the vast literature in this area.

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ENDNOTES

[1] This applies to 802.11b/g. The chip rate used by 802.11b is 11 times the symbol rate due to the use of 11-chip Barker codes. Given that the symbol rate is 1 MHz, this leads to a chip rate of 11 MHz and hence a clock period of about 91ns. Hence the limitation. Since commercially available chips support both b and g, we use the 100ns figure.

[2] Although b_{i0} is non-zero, the distribution assumes a zero mean value indicating a total lack of knowledge about the location of the terminal. As the measurement data are incorporated and posterior distributions are calculated, the distribution will become centered on the actual mean value. If prior knowledge is available, as in tracking situations, it can be incorporated by using an appropriate mean value in the prior distribution for b_{i0}.