Source localization based on SVD without a priori knowledge

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Abstract

A weighted least squares (WLS) estimation procedure based on singular value decomposition (SVD) is proposed for the source localization problem under an additive measurement error model. In practical situation, the respective sensor reliability may differ. The WLS solves the problem by assigning different weights to the sensors. However, the existing WLS-based methods require a priori information, such as variance of measurement noise or the initial point of optimization. This can be a problem in an environment where the measurement noise variance cannot be accurately estimated or the initial point is not given.

Although a priori information is not required and can be implemented in real-time processing, since the maximum likelihood (ML) is a Taylor-series based iterative method, it requires a more computational time and resources. Therefore, we have proposed a new analytical algorithm that needs no a priori knowledge of noise statistics or initial information and requires less computational time. We have adopted SVD and estimated the weight using the inverse of the difference between the estimate and the measurement. The proposed method has been found to be more accurate than the existing LS-based methods such as BLUE-LSC, MDS.

Keywords

Singular Value Decomposition, Weighted least squares, Singular vector, Source, Mean square error, Difference

1. Introduction

Source localization is a technique that finds a geometrical point of intersection by using the measurements from each receiver, such as the time difference of arrival (TDOA), the time of arrival (TOA) or the angle of arrival (AOA). Localizing point sources using passive, stationary sensors is of considerable interest and has been a repeated theme of research in radar, sonar, global positioning systems, video conferencing and telecommunication areas. One commonly used location-bearing parameter is the TOA, that is, the one-way signal propagation or round trip time between the source and sensor. For two-dimensional localization, each noise-free TOA provides a circle centered at the sensor where the source must lie. By using $M \geq 3$ sensors, the source location can be uniquely determined by the intersection of circles.

In this paper, we have developed a new source position location algorithm using weighted least squares (WLS) based on Singular Value Decomposition (SVD) (abbreviated as WLS_SVD), for solving the circular equations. The existing WLS-based methods need a priori knowledge, such as the measurement noise variance, to combine the data of the respective sensor or sufficiently precise initial point in the procedure determining the Lagrange coefficient [1]-[3]. However, the statistics for measurement noise or initial information cannot be known generally in a real environment [1],[4]. Although a priori information is not required, since Taylor-series based iterative method is used, it has a disadvantage in terms of the computational time. Therefore, we have suggested a closed-form source localization method that does not require a priori information or an initial optimization point.

There are several existing data fusion methods for Multi-sensor localization. Largely, they can be divided into two categories: the maximum likelihood (ML)-based linearized iterative method and the least squares (LS)-based analytical method. The non-linear method is to solve the non-linear equations relating these TOA measurements directly, but it is computationally intensive. Apart from the direct methodology, another common technique that avoids solving the nonlinear equations is to linearize them, and the solution is then found, iteratively. However, ML-based linearized iterative approach requires an initial estimate and cannot guarantee convergence to the correct solution. On the other hand, the LS-based method has low computational time and ensures global convergence. Torrieri [5] derived a principal algorithm and analyzed the hyperbolic location systems and direction-finding location systems, and used distance and angle information for ML. Weylin [6] adopted the range difference information of a $T - R^n$ multistatic radar system transmitting a new grouped waveform of FMCW signal. Foy [7] adopted an iterative least square (ILS) method, which used a combination of range and angular information, and showed that the Taylor-series method works for a variety of problems, including the mixed-measurement mode. In all the three methods, the target position was estimated by using an iterative method.

Furthermore, Manolakis [8] used LS to estimate the three-dimensional target position based on range measurements from three stations, Farina [9],[10] combined the information on each Bistatic sonar pair using the Weighted Sum where the measurement is fused into the estimate by using the measurement noise variance of each receiver. Kadar [11] reported target positioning performance using the angle of arrival (AOA) measurement and its mean square error (MSE) variation according to several receiver deployments.

This paper is organized as follows. In Section 2, we have explained the distributed source positioning system using multiple sensors and the existing data fusion methods. In Section 3, we have dealt with the proposed algorithm that does not require a priori knowledge or the initial estimate. In Section 4, we have discussed the experimental result of evaluating the estimation performance of the WLS_SVD algorithm by comparing it with the BLUE-LSC [12], the MDS [13] and the ML algorithm [5]. Lastly, Section 5 presents the conclusions and the directions for future work.

2. Multi-sensor source localization modeling

In this section, we have explained the source localization system architecture.

As we have TOA measurement, we can attain Eq. (1) from the *i*th sensor, as shown in Figure 1.

$$\sqrt{(\hat{x}-x_i)^2+(\hat{y}-y_i)^2}+n_i=r_i \tag{1}$$

where $\hat{x}, \hat{y}$: source coordinates

x_i, y_i : the *i*th sensor coordinates

r_i : distance between the source and the *i*th sensor

n_i : Gaussian noise with $N(0, \sigma_i^2)$

Squaring both sides of (1) yields

$$r_i^2 = R^2 - 2\hat{x}x_i - 2\hat{y}y_i + (x_i^2+y_i^2) \tag{2}$$

$$\Rightarrow \; x_i\hat{x}+y_i\hat{y}-0.5R^2=\frac{1}{2}\left(x_i^2+y_i^2-r_i^2\right)$$

where $R=\sqrt{\hat{x}^2+\hat{y}^2}$, and the reference sensor is located in the origin (0,0).

$$\mathbf{A\theta}=\mathbf{b} \tag{3}$$

where

$$\mathbf{A}=\begin{bmatrix} x_1 & y_1 \\ \vdots & \vdots \\ x_N & y_N \end{bmatrix}, \quad \boldsymbol{\theta}=\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}, \quad \text{and} \quad \mathbf{b}=\frac{1}{2}\begin{bmatrix} x_1^2+y_1^2+R^2-r_1^2 \\ \vdots \\ x_N^2+y_N^2+R^2-r_N^2 \end{bmatrix} \tag{4}$$

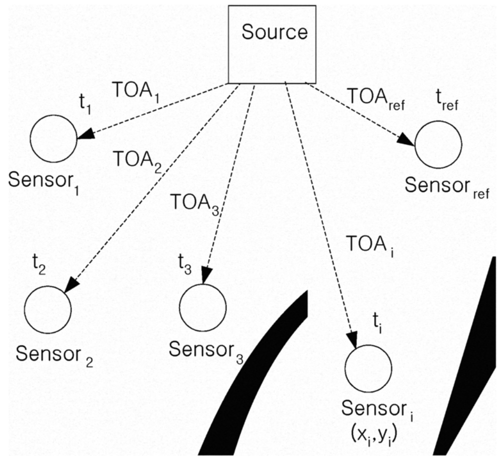

Figure 1. Multi-sensor localization systems modeling

Equation (3) solves the problem where $\mathbf{A\theta}=\mathbf{b}$, which is an overdetermined case. As $\mathbf{A}$ is a non-square matrix, we can find an approximate solution using the pseudo-inverse, and the solution can be represented as (5) [14].

$$\boldsymbol{\theta}_{LS}=(\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b} \tag{5}$$

Usually, to attain a more accurate location performance, weighted version of (5) is adopted as follows:

$$\boldsymbol{\theta}_{WLS}=(\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{b} \tag{6}$$

The methods of assigning the weight to the standard LS have been studied. However, these methods require a priori knowledge, e.g., the variance of the measurement noise. Additionally, Lagrange multiplier method requires the initial point for the optimization of cost function that consists of the constraint and the squared norm of error that needs to be minimized. In the existing methods, the weight is determined by using the inverse of the noise measurement variance and the distance between the source and sensor. In a real environment, we may not accurately know the noise variance and the initial guess point. Even if this is possible, the environment may be in a nonstationary state or the computational complexity of the iterative method may be high. Accordingly, it can be stated that the method that does not require the measurement noise variance or the initial point is more practical. Although a method that does not require a priori information exists, it requires a high computational complexity in the implementation and the lower and upper bounds of the unknown coordinates [15]. Therefore, we have proposed a simple analytical method that does not require a priori information and high computational complexity. In the following section, we will study the source localization method using WLS that does not require a priori knowledge or the initial point for optimization because of its explicit property.

3. Proposed method

In this section, we have explained the proposed WLS_SVD. In solving $\mathbf{A}\boldsymbol{\theta} = \mathbf{b}$, $\mathbf{A}$ can be decomposed by SVD as follows:

$$\mathbf{A} = \begin{bmatrix}\mathbf{u}_1 & \mathbf{u}_2 & \cdots & \mathbf{u}_N\end{bmatrix}\begin{bmatrix}\sigma_1 & 0 \\ 0 & \sigma_2 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0\end{bmatrix}\begin{bmatrix}\mathbf{v}_1^T \\ \mathbf{v}_2^T\end{bmatrix} \tag{7}$$

where $\sigma_1 > \sigma_2 > 0$.

The measurement vector should exist in the column space of **A** and the left singular vector should span the column space. Since the left singular vectors are mutually orthonormal and form the basis of the column space of **A**, the measurement vector can be represented as:

$$\mathbf{b} = \alpha_1\mathbf{u}_1 + \alpha_2\mathbf{u}_2 + \cdots + \alpha_N\mathbf{u}_N \tag{8}$$

where $\alpha_i = \mathbf{b}^T\mathbf{u}_i$, $\mathbf{u}_i$ is the *i*th left singular vector of **A**.

Clearly, the rank of **A** is two if the columns of **A** are selected to be orthogonal. Accordingly, we take two left singular vectors corresponding to the largest two singular values, and reconstruct the noise-removed measurement $\hat{\mathbf{b}}$.

$$\hat{\mathbf{b}} = \alpha_1\mathbf{u}_1 + \alpha_2\mathbf{u}_2 \tag{9}$$

Estimating the respective measurements for the plural measurements and then averaging them results in the improvement of the accuracy. However, by considering the increase of the computational time by the augmentation of the measurements, we estimated the measurement vector using two measurements.

The source localization accuracy is deteriorated by the noise that exists in the measurement vector. The LS estimate is the point at which the sum of LS residual becomes minimum. As can be seen from Figure 2, when $|n_B|$ is larger than $|n_A|$, the accuracy of the estimated position, where n_B is added, is deteriorated compared to the case where n_A is added. If the noise amount is larger, then the source localization performance becomes worse. Therefore, we determined the weight inversely proportional to the noise amount of **b**. We considered the absolute value of the difference between the noisy measurement (**b**) and the noise-removed measurement ($\hat{\mathbf{b}}$) as the noise amount. The estimate of the position is given as follows:

$$\boldsymbol{\theta}_{WLS} = (\mathbf{A}^T\mathbf{W}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{W}\mathbf{b} \tag{10}$$

$$\mathbf{W} = \mathrm{diag}\left(\frac{1}{\left|\mathbf{b}(1) - \hat{\mathbf{b}}(1)\right|}, \cdots, \frac{1}{\left|\mathbf{b}(N) - \hat{\mathbf{b}}(N)\right|}\right) \tag{11}$$

where $\mathbf{b}(i)$ and $\hat{\mathbf{b}}(i)$ represent the *i*th element of **b** and $\hat{\mathbf{b}}$, respectively.

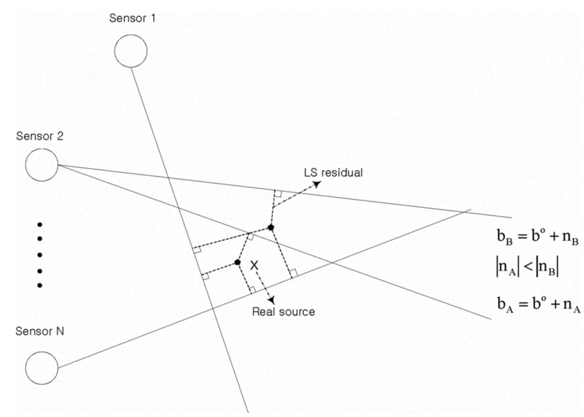

Figure 2. The effect of the measurement noise on the source localization performance

4. Simulation Results

In this section, we have compared the performance of several source localization algorithms. In this simulation, we performed a Monte-Carlo simulation of 1000 trials. The number of sensors varied from 6 to 14 at increments of two to observe the performance variation according to the number of sensors. The radius of the sensor network varied from 4 m to 10 m with 2 m intervals to demonstrate the performance variation. The sensors were located on the circle whose radius was 4, 6, 8, and 10 m with the gap of 360°/(*T*-1) where *T* denotes the total number of sensors. The measurement noise was Gaussian. The standard deviation (std.) of the sensors was different for each sensor to model the quality difference of the sensors. The std. of the *i*th sensor measurement noise was std.$\times\sqrt{i}$, where std. is varied from -60 dB to -20 dB at an interval of 5 dB. Single and omni-direction source was assumed and the state was considered as stationary. The source was located at [-10, -5] m. For this position, the mean square error (MSE) was calculated.

Figure 3 compares the MSE of each position estimation method under various range noise settings. Here, the receivers are arranged in the shape of a pentagon and the distance between the reference and sensor is 10 m.

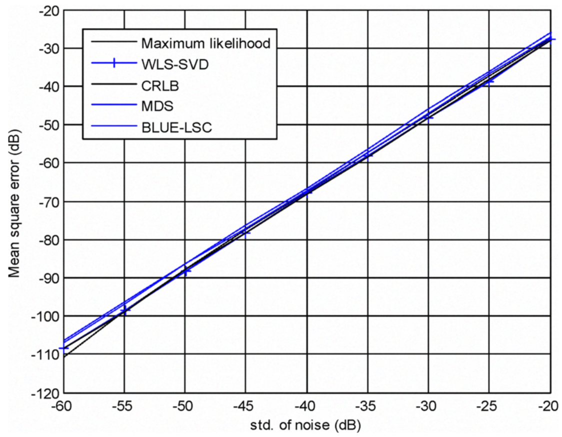

Figure 3. Comparison of MSE of respective algorithm to various noise at [x, y] = [-10, -5] m

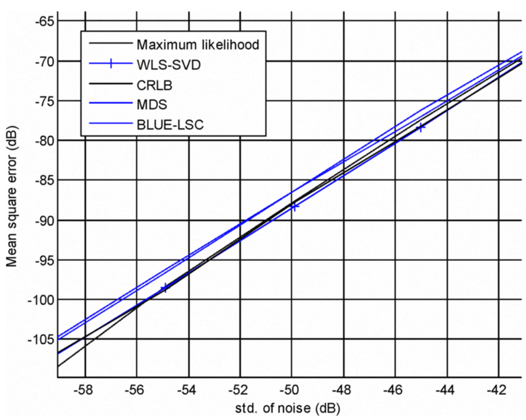

Figure 4. Expansion of part (from -58 dB to -42 dB) of Figure 3

Figure 4 is the enlarged result of Figure 3 between -58 dB and -42 dB. The WLS_SVD performance was found to be better than the existing methods, and approximated the Cramer-Rao lower bound (CRLB) and the ML. In general, the LS-based method was observed to be inferior to the ML in localization performance. However, since the LS-based method has the advantage compared to ML, with respect to the computational time and implementation simplicity, the LS-based method is used as the sub-optimal method.

Figure 5 shows the variation pattern of the MSE, with respect to the radius of the sensor network. Here, the noise std. is 0.2 $m \times [1, \sqrt{2}, \ldots, \sqrt{5}]$. The MSE of all the methods decreased as the radius of the sensor network increased [16]. It has been shown that the MSE of the proposed method is smaller than that of the existing LS-based methods.

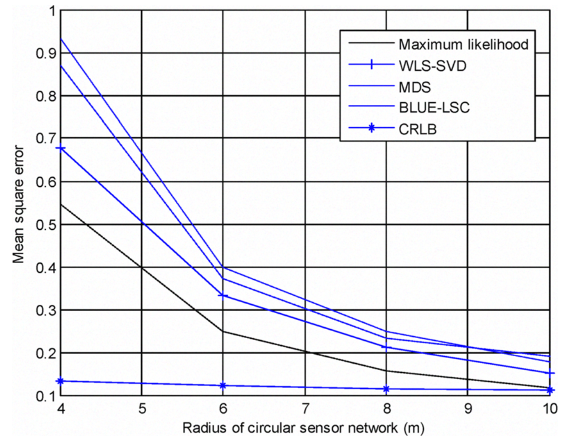

Figure 5. MSE to the radius of the circular sensor network at [x, y] =[-10, -5] m

Figure 6 represents the MSE according to the number of sensors where the ith sensor noise is 0.2 $m \times \sqrt{i}$ and the radius of sensor network is 10 m. It can be seen that the MSE is reduced as the number of sensors increases. The proposed methods are more efficient than the existing LS-based methods.

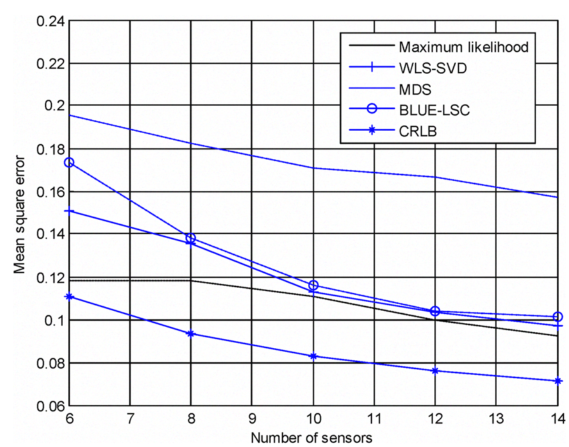

Figure 6. MSE to the number of sensors at [x, y] = [-10, -5] m

Table 1 shows the comparison result of each algorithm with respect to its computational time. The WLS-SVD is superior to the other existing algorithms and the ML is the worst in the computational time.

Table 1. Comparison of each algorithm in the computational time

	BLUE-LSC	ML	MDS	WLS-SVD
Computation time (sec)	1.5691×10^{-6}	6.3835×10^{-4}	1.4598×10^{-4}	4.0766×10^{-7}

5. Conclusion

In this paper, we proposed the WLS using the SVD based on Gaussian distribution. The MSE using the proposed method that uses the TOA measurement was smaller than that of the existing LS-based methods under various noise environments. An experiment was conducted by varying the number of receivers and the radius of the sensor network to determine whether they affect the source positioning performance. It was shown that the MSE obtained using the proposed method decreased as the radius of the sensor network increased and the number of receivers increased. Creating a table of the MSE in each case would provide a useful reference for optimal sensor deployment.

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