RADAR CODE OPTIMIZATION FOR MOVING TARGET DETECTION

Mohammad Mahdi Naghsh[*+], *Mojtaba Soltanalian*[†], *Petre Stoica*[†], *and Mahmoud Modarres-Hashemi*[+]

[+]ECE Dept., Isfahan University of Technology, Isfahan, Iran
[†]Dept. of Information Technology, Uppsala University, Uppsala, Sweden

ABSTRACT

In this paper, we study the problem of optimal pulsed-radar transmit code design for detection of moving targets in the presence of clutter. To this end, a new data modeling is introduced and optimal detectors for known and unknown target Doppler shift are presented. In the case of unknown Doppler shift, the expressions of the optimal detector and its performance metrics are too complicated to be amenable for code design. Therefore, we consider a substitute metric whose maximization results in maximization of a lower bound on the J-divergence. For a known target Doppler shift, the metric directly determines the performance of the optimal detector. In order to solve the emerging highly non-convex code design problem, we devise a computational framework (which we call CADCODE) that lays the ground for an optimal code design via a cyclic minimization. As low peak-to-average-power ratio (PAR) codes are of interest in many applications, we apply the CADCODE framework to obtain such codes. Several numerical examples are provided to illustrate the performance of the proposed algorithm.

Index Terms- Clutter, target detection, radar code design

1. INTRODUCTION

Radars as well as many other active sensing systems face the simultaneous effects of signal-dependent and independent interferences. The signal-dependent interference, known as clutter, is the echo of the transmitted signals produced by uninteresting obstacles. On the other hand, the signal-independent interferences include various types of noise, jamming, and other unwanted emissions. Doppler shifts of the moving targets play an important role in distinguishing the targets from the background clutter. However, the target Doppler shift is usually unknown at the transmitter. Considering such an ambiguity along with the presence of clutter, and the practical implementation demands for low peak-to-average-power ratio (PAR) make the transmit code design a challenging task.

This work was supported in part by the European Research Council (ERC) under Grant #228044, and the Swedish Research Council.

* Please address all the correspondence to M. M. Naghsh, Phone: (+98) 311-3912450; Fax: (+98) 311-3912451; Email: mm_naghsh@ec.iut.ac.ir

The signal design for radar performance improvement has been an active area of research in the last decades; however, the majority of previous works have considered either stationary target or clutter-free scenarios. The effect of clutter has been considered in early studies for stationary targets, or targets with known Doppler shifts (see e.g. [1]). In [2], optimal energy spectral density of the transmit signal has been obtained for stationary targets. A related problem to that of [2] has been considered in [3] with PAR constraint. The unknown Doppler shift of the target has been taken into account in [4] with PAR constraint in the absence of clutter. In this paper, we study the radar signal design for detection of a moving target in the presence of clutter. To this end, optimal detectors for both cases where target Doppler shift is known and unknown are presented. For unknown Doppler shift, the expressions of the optimal detector and its performance metrics appear to be too complicated for utilization in code design. Therefore, we consider a metric whose maximization results in maximization of a lower bound on the J-divergence. For known target Doppler shifts, the metric directly determines the performance of the optimal detector. The raised optimization problem is highly non-convex, and to the best of our knowledge, designing codes for improving detection performance in such scenarios has not been addressed in the literature prior to this work. To tackle the problem, a computational framework based on a **C**yclic **A**lgorithm for **D**irect **CO**de **DE**sign (which we call CADCODE) is proposed to carry out the code design via a cyclic minimization. The PAR constraint is also taken into account in the design.

The rest of this paper is organized as follows. In Section 2, we present the data modeling and optimal detectors for known and unknown target Doppler shifts. The code optimization is discussed in Section 3. Section 4 considers the PAR constrained code design. Numerical examples are provided in Section 5. Finally, Section 6 concludes the paper.

2. DATA MODELING AND OPTIMAL DETECTOR

We consider a narrow-band pulsed-radar system using a train of pulses. The baseband transmit signal can be formulated as

$$s(t) = \sum_{n=0}^{N-1} a_n \, \phi(t - nT_{PRI}) \tag{1}$$

where $\phi(.)$ is the basic unit-energy transmit pulse, T_{PRI} is the pulse repetition interval, and $\{a_n\}_{n=0}^{N-1}$ are the weights that are to be optimally designed. At the transmitter, the baseband signal is modulated by a carrier frequency ω_c. The backscattered signal from a point-like moving target can be expressed as

$$r(t) = \alpha_t s(t-\tau) e^{j(\omega_c+\nu)(t-\tau)} + c(t) + w(t) \tag{2}$$

where α_t is the amplitude of the target echo (accounting for target reflectivity and channel effects), τ and ν denote the target delay and Doppler shift, respectively, $c(t)$ is the clutter component, and $w(t)$ represents the signal-independent interferences. We assume that both $c(t)$ and $w(t)$ are Gaussian random processes. In particular, we assume that the clutter component is the signal echo produced by many individual point scatterers (distributed across the delay and Doppler domains) which are statistically independent. Under such an assumption, $c(t)$ can be formulated as [1]

$$c(t) = \sum_{k=1}^{N_{\mathrm{ct}}} \sum_{l=1}^{N_{\mathrm{cd}}} \rho_{kl} s(t-\tau_k) e^{j(\omega_c+\omega_l)(t-\tau_k)} \tag{3}$$

where N_{ct} and N_{cd} are the number of clutter scatterers in the delay and Doppler domains, respectively, and ρ_{kl} is the amplitude of a specific clutter scatterer at time delay τ_k and Doppler shift ω_l (due to the internal clutter motion).

At the receiver, the matched filter $\phi^*(-t)$ is usually applied to the downconverted received signal (i.e. $r(t)e^{-j\omega_c t}$) and the output of the matched filter is then sampled at the time delays corresponding to the range-cell under test, i.e. $t = nT_{PRI} + \tau$ for $0 \le n \le N-1$. The discrete-time received signal $\mathbf{r}$ for the range-cell corresponding to the time delay τ can be written as

$$\mathbf{r} = \alpha\, \mathbf{a} \odot \mathbf{p} + \mathbf{a} \odot \mathbf{c} + \mathbf{w} \tag{4}$$

where $\alpha = \alpha_t e^{-j\omega_c \tau}$, $\mathbf{a} \triangleq [a_0 \quad a_1 \quad \dots \quad a_{N-1}]^T$ is the code vector (to be designed), $\mathbf{p} \triangleq [1 \; e^{j\omega} \quad \dots \quad e^{j(N-1)\omega}]^T$ with ω being the normalized Doppler shift of the target, $\mathbf{c}$ is the vector corresponding to the clutter component, the vector $\mathbf{w}$ represents the signal-independent interferences, and the symbol $\odot$ denotes Hadamard product (see [4] for a similar modeling and its derivation).

Using (4), the target detection problem can be cast as the following binary hypothesis test:

$$\begin{cases} \mathrm{H}_0: & \mathbf{r} = \mathbf{a} \odot \mathbf{c} + \mathbf{w} \\ \mathrm{H}_1: & \mathbf{r} = \alpha \mathbf{a} \odot \mathbf{p} + \mathbf{a} \odot \mathbf{c} + \mathbf{w} \end{cases} \tag{5}$$

Note that the covariance matrices of $\mathbf{c}$ and $\mathbf{w}$ (denoted by $\mathbf{C}$ and $\mathbf{M}$) can be assumed to be priori known (e.g. they can be obtained by using geographical, meteorological, or prescan information) [5]. For a known target Doppler shift and the target with Swerling-I model ($\alpha \sim \mathcal{CN}(0, \sigma^2)$), using the derivation in [6] in the case of (5) yields the following optimal detector:

$$|\mathbf{r}^H \left(\mathbf{M} + \mathbf{ACA}^H\right)^{-1} (\mathbf{a} \odot \mathbf{p})|^2 \underset{\mathrm{H}_1}{\overset{\mathrm{H}_0}{\lessgtr}} \eta \tag{6}$$

where η is the detection threshold and $\mathbf{A} \triangleq \mathrm{Diag}(\mathbf{a})$. The performance of the detector in (6) is a monotonically increasing function of the following criterion [6]:

$$\lambda = \sigma^2 (\mathbf{a} \odot \mathbf{p})^H \left(\mathbf{M} + \mathbf{ACA}^H\right)^{-1} (\mathbf{a} \odot \mathbf{p}). \tag{7}$$

The target Doppler shift ω is usually unknown at the transmitter. In such cases, the optimal detector for the detection problem in (5) with unknown ω can be obtained by considering the pdf of ω. The distribution of the vector $\mathbf{r}$ is no longer Gaussian and the optimal detector does not lead to a closed-form expression. The optimal detector for unknown ω can be obtained using the results of [6] as:

$$\int_\Omega \frac{f(\omega)}{1+\lambda}\, e^{\left(\frac{\sigma^2 |\mathbf{r}^H (\mathbf{M}+\mathbf{ACA}^H)^{-1} (\mathbf{a}\odot\mathbf{p})|^2}{1+\lambda}\right)} d\omega \underset{\mathrm{H}_1}{\overset{\mathrm{H}_0}{\lessgtr}} \eta' \tag{8}$$

where $\Omega = [\omega_l, \omega_u]$ denote the considered interval for the target Doppler shift ω and $f(\omega)$ denote the pdf of ω. It is worth mentioning that the pdf of ω and the values of ω_l and ω_u can be obtained in practice using a priori knowledge about the type of target as well as employing cognitive methods [5].

3. CODE OPTIMIZATION

The code design to improve detection performance of the system when the target Doppler shift ω is known can be dealt with by maximization of the following metric for a given ω:

$$\begin{aligned} &(\mathbf{a} \odot \mathbf{p})^H \left(\mathbf{M} + \mathbf{ACA}^H\right)^{-1} (\mathbf{a} \odot \mathbf{p}) \\ &= \mathrm{tr}\left\{\mathbf{A}^H (\mathbf{M} + \mathbf{ACA}^H)^{-1} \mathbf{App}^H\right\}. \end{aligned} \tag{9}$$

In cases where target Doppler shift is unknown, the expressions for optimal detector and its performance metrics are too complicated to be used for code design. In such a circumstance, we consider the following *average* metric:

$$\mathrm{tr}\left\{\mathbf{A}^H (\mathbf{M} + \mathbf{ACA}^H)^{-1} \mathbf{AW}\right\} \tag{10}$$

where $\mathbf{W} = \mathrm{E}\{\mathbf{pp}^H\}$. Interestingly, it can be shown that maximizing the above metric results in maximization of a lower bound on the J-divergence associated with the detection problem for unknown ω. More precisely, the J-divergence associated with the binary hypothesis test is given by

$$\mathcal{J} \;=\; \mathrm{E}\{\log(\mathcal{L})|H_1\} - \mathrm{E}\{\log(\mathcal{L})|H_0\} \tag{11}$$

where $\mathcal{L}$ represents the likelihood ratio of the problem. Therefore, for the detection problem in (5) with unknown ω, we obtain $\mathcal{J} = E\{\mathcal{J}|\omega\}$ where

$$\mathcal{J}|\omega = \frac{\lambda^2}{1+\lambda}. \tag{12}$$

Consequently, using Jensen inequality we conclude

$$\mathcal{J} = \mathrm{E}\left\{\frac{\lambda^2}{1+\lambda}\right\} \geq \frac{(\mathrm{E}\{\lambda\})^2}{1+\mathrm{E}\{\lambda\}}. \tag{13}$$

As the right-hand side of the above inequality is a monotonically increasing function of $\mathrm{E}\{\lambda\}$, maximization of the metric leads to maximization of a lower bound on J-divergence (see e.g. [2] for connections of J-divergence and the performance of a hypothesis test). Therefore, the code optimization problem under an energy constraint can be cast as the following problem:

$$\begin{aligned} \max_{\mathbf{A}} \quad & \mathrm{tr}\left\{\left(\mathbf{A}^{-1}\mathbf{M}\mathbf{A}^{-H}+\mathbf{C}\right)^{-1}\mathbf{W}\right\} \\ \text{subject to} \quad & \mathrm{tr}\left\{\mathbf{A}\mathbf{A}^H\right\} \leq e \end{aligned} \tag{14}$$

where e denotes the total transmit energy. In the following, we devise the CADCODE framework to tackle the non-convex problem (14).

We begin by noting that as $\mathbf{W} \succeq \mathbf{0}$ there must exist a full column-rank matrix $\mathbf{V} \in \mathbb{C}^{N\times\delta}$ such that $\mathbf{W} = \mathbf{V}\mathbf{V}^H$. As a result,

$$\begin{aligned} & \mathrm{tr}\left\{\mathbf{A}^H(\mathbf{M}+\mathbf{A}\mathbf{C}\mathbf{A}^H)^{-1}\mathbf{A}\mathbf{W}\right\} \\ & = \mathrm{tr}\left\{\mathbf{V}^H\mathbf{A}^H(\mathbf{M}+\mathbf{A}\mathbf{C}\mathbf{A}^H)^{-1}\mathbf{A}\mathbf{V}\right\}. \end{aligned} \tag{15}$$

Let $\mathbf{\Theta} \triangleq \theta\mathbf{I} - \mathbf{V}^H\mathbf{A}^H(\mathbf{M}+\mathbf{A}\mathbf{C}\mathbf{A}^H)^{-1}\mathbf{A}\mathbf{V}$, and

$$\theta \geq \frac{\lambda_{max}(\mathbf{W})}{\frac{1}{e}\lambda_{min}(\mathbf{M})+\lambda_{min}(\mathbf{C})} \tag{16}$$

which ensures $\mathbf{\Theta} \succ \mathbf{0}$. Note that the optimization problem (14) is equivalent to the minimization problem

$$\min_{\mathbf{A}} \quad \mathrm{tr}\{\mathbf{\Theta}\} \quad \text{subject to}\, \mathrm{tr}\{\mathbf{A}^H\mathbf{A}\} \leq e. \tag{17}$$

Now define

$$\mathbf{R} \triangleq \begin{bmatrix} \theta\mathbf{I} & \mathbf{V}^H\mathbf{A}^H \\ \mathbf{A}\mathbf{V} & \mathbf{M}+\mathbf{A}\mathbf{C}\mathbf{A}^H \end{bmatrix} \tag{18}$$

and observe that for $\mathbf{U} \triangleq [\mathbf{I}_\delta \;\; \mathbf{0}_{N\times\delta}]^T$ we have

$$\mathbf{U}^H\mathbf{R}^{-1}\mathbf{U} = \mathbf{\Theta}^{-1}. \tag{19}$$

To tackle (17) let $g(\mathbf{A},\mathbf{Y}) \triangleq \mathrm{tr}\{\mathbf{Y}^H\mathbf{R}\mathbf{Y}\}$ (with $\mathbf{Y}$ being an auxiliary variable), and consider the following minimization problem:

$$\begin{aligned} \min_{\mathbf{A},\mathbf{Y}} \quad & g(\mathbf{A},\mathbf{Y}) \\ \text{subject to} \quad & \mathbf{Y}^H\mathbf{U} = \mathbf{I} \\ & \mathrm{tr}\{\mathbf{A}^H\mathbf{A}\} \leq e. \end{aligned} \tag{20}$$

For fixed $\mathbf{A}$, the minimizer $\mathbf{Y}$ of (20) can be obtained using Result 35 in [7, p. 354] as

$$\mathbf{Y} = \mathbf{R}^{-1}\mathbf{U}(\mathbf{U}^H\mathbf{R}^{-1}\mathbf{U})^{-1}. \tag{21}$$

On the other hand, for fixed $\mathbf{Y}$, the minimization of $g(\mathbf{Y},\mathbf{A})$ w.r.t. $\mathbf{A}$ yields the following quadratically-constrained quadratic program (QCQP):

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^H\left((\mathbf{Y}_2\mathbf{Y}_2^H)\odot\mathbf{C}^T\right)\mathbf{a} + 2\Re(\mathbf{d}^H\mathbf{a}) \\ \text{subject to} \quad & \mathbf{a}^H\mathbf{a} \leq e \end{aligned} \tag{22}$$

where $\mathbf{Y} \triangleq [\mathbf{Y}_{1\ \delta\times\delta} \;\; \mathbf{Y}_{2\ N\times\delta}]^T$ and $\mathbf{d} \triangleq \mathrm{diag}(\mathbf{V}^*\mathbf{Y}_1^*\mathbf{Y}_2^T)$. Note that the positive semi-definiteness of $(\mathbf{Y}_2\mathbf{Y}_2^H)\odot\mathbf{C}^T$ guarantees the convexity of (22). The QCQP in (22) can be solved efficiently using the Lagrange multiplier method.

It is straightforward to verify that at the minimizer $\mathbf{Y}$ of (20), we have $g(\mathbf{Y},\mathbf{A}) = \mathrm{tr}\{\mathbf{\Theta}\}$. From this property, we conclude that each step of the cyclic minimization of (20) leads to a decrease of $\mathrm{tr}\{\mathbf{\Theta}\}$. Indeed, let $f(\mathbf{A}) = \mathrm{tr}\{\mathbf{\Theta}\}$ and note that

$$\begin{aligned} f\left(\mathbf{A}^{(k+1)}\right) = g\left(\mathbf{Y}^{(k+2)},\mathbf{A}^{(k+1)}\right) &\leq g\left(\mathbf{Y}^{(k+1)},\mathbf{A}^{(k+1)}\right) \\ &\leq g\left(\mathbf{Y}^{(k+1)},\mathbf{A}^{(k)}\right) = f\left(\mathbf{A}^{(k)}\right) \end{aligned}$$

where the superscript k denotes the iteration number. The first and the second inequality in (3) hold true due to the minimization of $g(\mathbf{A},\mathbf{Y})$ w.r.t. $\mathbf{Y}$ and $\mathbf{A}$, respectively. As a result, *CADCODE converges to a stationary point of (14).* It is worth noting that the minimization steps of CADCODE (which are summarized in Table 1) can be solved either analytically or using standard interior-point methods.

Table 1. CADCODE for Optimal Radar Code Design

Step 0: Initialize the code vector $\mathbf{a}$ using a random vector in $\mathbb{C}^N$, and form $\mathbf{R}$ as defined in (18).
Step 1: Compute $\mathbf{Y} = \mathbf{R}^{-1}\mathbf{U}(\mathbf{U}^H\mathbf{R}^{-1}\mathbf{U})^{-1}$.
Step 2: Solve the optimization problem (22) to obtain the code vector $\mathbf{a}$.
Step 3: Repeat steps 1 and 2 until a pre-defined stop criterion is satisfied, e.g. $\|\mathbf{a}^{(k+1)}-\mathbf{a}^{(k)}\| \leq \epsilon$ for some $\epsilon > 0$.

4. CONSTRAINED CODE DESIGN

In order to use the power resources efficiently and to avoid non-linear effects at the transmitter, unimodular or low PAR sequences are of practical interest in many applications. It is possible to use the CADCODE framework to deal with an arbitrary PAR constraint, viz. $\mathrm{PAR}(\mathbf{a}) = \frac{\max_m \{|a_m|^2\}}{\frac{1}{N}\|\mathbf{a}\|^2} \leq \zeta$.

In order to design low-PAR codes via the CADCODE framework, the minimization of $g(\mathbf{Y},\mathbf{A})$ in (20) w.r.t. a low-PAR code vector $\mathbf{a}$ can be accomplished using the optimization problem

$$\begin{aligned} \min_{\mathbf{a}} \quad & \mathbf{a}^H\left((\mathbf{Y}_2\mathbf{Y}_2^H)\odot\mathbf{C}^T\right)\mathbf{a} + 2\Re(\mathbf{d}^H\mathbf{a}) \\ \text{subject to} \quad & \max_{m=0,\cdots,N-1} \{|a_m|^2\} \leq \zeta \\ & \|\mathbf{a}\|^2 = N. \end{aligned} \tag{23}$$

We note that (23) is a non-convex QCQP with PAR constraint, which can be equivalently written as

$$\begin{aligned} \min_{\mathbf{a}} \quad & \widetilde{\mathbf{a}}^H \mathbf{J}\, \widetilde{\mathbf{a}} \\ \text{subject to} \quad & \max_{m=0,\cdots,N-1} \{|a_m|^2\} \leq \zeta \\ & \|\mathbf{a}\|^2 = N \end{aligned} \tag{24}$$

where $\widetilde{\mathbf{a}} = [\mathbf{a} \quad 1]^T$, and

$$\mathbf{J} = \begin{bmatrix} (\mathbf{Y}_2 \mathbf{Y}_2^H) \odot \mathbf{C}^T & \mathbf{d} \\ \mathbf{d}^H & 0 \end{bmatrix}.$$

For any $\lambda > \lambda_{max}(\mathbf{J})$ we can reformulate the latter problem as

$$\begin{aligned} \max_{\mathbf{a}} \quad & \widetilde{\mathbf{a}}^H \mathbf{K}\, \widetilde{\mathbf{a}} \\ \text{subject to} \quad & \max_{m=0,\cdots,N-1} \{|a_m|^2\} \leq \zeta, \\ & \|\mathbf{a}\|^2 = N \end{aligned} \tag{25}$$

with $\mathbf{K} = \lambda \mathbf{I}_{N+1} - \mathbf{J}$. Interestingly, derivation of the power-method like iterations in [8] can be extended to the case of PAR-constrained $\mathbf{a}$ in (25). More precisely, the code vector $\mathbf{a}$ of the $(l+1)^{th}$ iteration (denoted by $\mathbf{a}^{(l+1)}$) can be obtained from the last estimate of $\mathbf{a}$, i.e. $\mathbf{a}^{(l)}$, via solving the optimization problem

$$\begin{aligned} \max_{\mathbf{a}^{(l+1)}} \quad & \|\mathbf{a}^{(l+1)} - \widehat{\mathbf{a}}^{(l)}\| \\ \text{subject to} \quad & \max_{m=0,\cdots,N-1} \{|a_m^{(l+1)}|^2\} \leq \zeta \\ & \|\mathbf{a}^{(l+1)}\|^2 = N \end{aligned} \tag{26}$$

where $\widehat{\mathbf{a}}^{(l)}$ represents the vector containing the first N entries of $\mathbf{K}\, \widetilde{\mathbf{a}}^{(l)}$. The optimization problem (26) is a "nearest-vector" problem with PAR constraint. Such PAR constrained problems can be solved efficiently using a recursive algorithm proposed in [9]. Finally, we note that as a scaling does not affect the PAR metric, the low-PAR codes obtained by CADCODE framework can be scaled to fit any desired level of energy.

5. NUMERICAL EXAMPLES

Numerical results will be provided to examine the performance of the proposed method. We consider the performance of the uncoded system (with $\mathbf{a} = \sqrt{\frac{e}{N}}\mathbf{1}$) as the benchmark. In the numerical examples, we assume that the signal-independent interference can be modeled as a first-order auto-regressive process with a parameter equal to 0.5, as well as a white noise at the receiver with variance equal to 0.01. Furthermore, for clutter we let $C_{m,n} = \rho^{(m-n)^2},\ 1 \leq m, n \leq N$ with $\rho = 0.8$. The extension of the CADCODE framework to the case of unimodular code (i.e. with PAR=1) design is referred to as CADCODE-U.

An example of code design for a Doppler shift interval $\Omega = [\omega_l, \omega_u] = [-1, 1]$ is considered for code length $N = 16$. The results are depicted in Fig. 1. Fig. 1(a) shows the values of the obtained metric corresponding to the code obtained by CADCODE, CADCODE-U, and also the uncoded system. It can be observed from Fig. 1 that, as expected, a coded system employing CADCODE or CADCODE-U possesses larger values of metric than that of the the uncoded system. It is also practically seen that the performance obtained by randomly generated codes is similar to that of the all-one code used in the uncoded system. Moreover, Fig. 1 reveals that the quality of the codes obtained via constrained designs is very similar to that of unconstrained designs. However, there are minor degradations due to imposing the unimodularity constraint. We also observe the saturation phenomenon in Fig. 1(a). More precisely, for sufficiently large values of the transmit energy (i.e., e), the increase in the average metric is negligible. Note that, the value of the metric (for non-singular $\mathbf{C}$) asymptotically converges to:

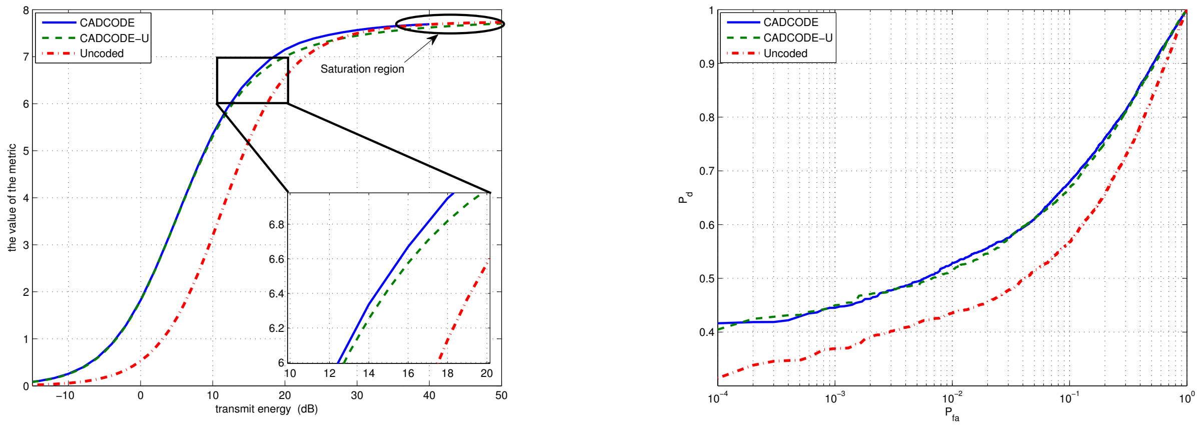
$$\lim_{e \to \infty} \operatorname{tr}\left\{ \left((\mathbf{A}^H \mathbf{M}^{-1} \mathbf{A})^{-1} + \mathbf{C} \right)^{-1} \mathbf{W} \right\} = \operatorname{tr}\{\mathbf{C}^{-1}\mathbf{W}\}.$$

The performance of the devised algorithms w.r.t. the receiver operating characteristic (ROC) of the optimal detector for unknown target Doppler shift is studied in Fig. 1(b). The detector is implemented via numerically evaluating the integral in (8). The simulated ROC is illustrated in Fig. 1(b) for $\sigma^2 = 3$ and $e = 10$ by considering 10000 sets of random generated data. Similar behaviors to that of Fig. 1 can be seen in this figure. Precisely, the performance of the system using the optimal codes outperforms that of the uncoded system. Minor differences exist in the ROC corresponding to CADCODE and CADCODE-U.

The effect of the code length N on the values of the metric is illustrated in Fig. 2 for a fixed transmit energy $e = 10$. It can be seen that as N grows large, the quality of the proposed coding schemes improves substantially. This is due to the fact that for a large N the code design problem has more degrees of freedom.

6. CONCLUDING REMARKS

The problem of optimal code design for moving target detection in the presence of clutter was considered. A new discrete-time formulation was introduced in (4) for moving target detection using pulsed-radars in the presence of clutter. To handle the unknown Doppler shift of the target, maximization of a metric was considered that leads to maximization of a lower bound on J-divergence. A Cyclic Algorithm for Direct COde DEsign, referred to as the CADCODE, was suggested to tackle the arising non-convex code optimization problem. In CADCODE, the code design objective function is iteratively minimized via a cyclic minimization of an auxiliary function of the code matrix. The convergence of CAD-



(a) (b)

Fig. 1. The design of optimal codes of length $N = 16$. (a) depicts the values of the metric for CADCODE and CADCODE-U methods as well as the uncoded system vs. the transmit energy. (b) plots the ROC of the optimal detector (unknown ω) for $\sigma^2 = 3$ and transmit energy$= 10$.

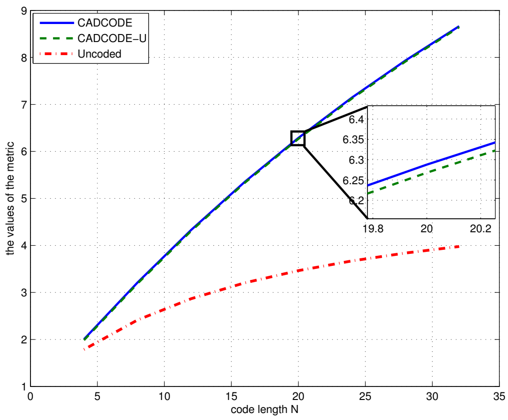


Fig. 2. The values of the metric associated with CADCODE, CADCODE-U, and the uncoded system vs. the code length N.

CODE was studied. Finally, the derivations of CADCODE was extended to tackle PAR constrained problems.

7. REFERENCES

[1] D. F. Delong and E. M. Hofstteter, "On the design of optimum radar waveforms for clutter rejection," *IEEE Trans. Inf. Theory*, vol. 13, pp. 454–463, Jul. 1967.

[2] S. M. Kay, "Waveform design for multistatic radar detection," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 45, pp. 1153–1165, Jul. 2009.

[3] P. Stoica, H. He, and J. Li, "Optimization of the receive filter and transmit sequence for active sensing," *IEEE Transactions on Signal Processing*, vol. 60, no. 4, pp. 1730 –1740, Apr. 2012.

[4] A. De Maio, Y. Huang, M. Piezzo, S. Zhang, and A. Farina, "Design of optimized radar codes with a peak to average power ratio constraint," *IEEE Trans. Signal Process.*, vol. 59, pp. 2683–2697, Jun. 2011.

[5] S. Haykin, "Cognitive radars," *IEEE Signal Processing Magazine*, vol. 23, no. 1, pp. 30–40, Jan. 2006.

[6] S. M. Kay, *Fundamentals of Statistical Signal Processing-Volume II: Detection Theory*, 1st ed. New Jersey: Prentice Hall, 1998.

[7] P. Stoica and R. Moses, *Spectral Analysis of Signals*. New Jersey: Prentice Hall, 2005. [Online]. Available: http://user.it.uu.se/ ps/ref.html

[8] M. Soltanalian and P. Stoica, "Designing optimized unimodular codes is not always hard," *IEEE Trans. Signal Process.*, Submitted, 2012.

[9] J. A. Tropp, I. S. Dhillon, R. W. Heath, and T. Strohmer, "Designing structured tight frames via an alternating projection method," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 188–208, Jan. 2005.