Chapter I
Introduction to Wireless Sensor Network Localization

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ABSTRACT

Localization is an important aspect in the field of wireless sensor networks that has attracted significant research interest recently. The interest in wireless sensor network localization is expected to grow further with the advances in the wireless communication techniques and the sensing techniques, and the consequent proliferation of wireless sensor network applications. This chapter provides an overview of various aspects involved in the design and implementation of wireless sensor network localization systems. These can be broadly classified into three categories: the measurement techniques in sensor network localization, sensor network localization theory and algorithms, and experimental study and applications of sensor network localization techniques. This chapter also gives a brief introduction to the other chapters in the book with a focus on explaining how these chapters are related to each other and how topics covered in each chapter fit into the architecture of this book and the big picture of wireless sensor network localization.

INTRODUCTION

Distributed sensor networks have been discussed for more than 30 years, but the vision of wireless sensor networks (WSNs) has been brought into reality only by the recent advances in wireless communications and electronics, which have enabled the development of low-cost, low-power and multi-functional sensors that are small in size and communicate over short distances. Today, cheap, smart sensors, networked through wireless links and deployed in large numbers, provide unprecedented opportunities for monitoring and controlling homes, cities, and the environment. In addition, networked sensors have a broad spectrum of applications in the defence area, generating new capabilities for reconnaissance and surveillance as well as other tactical applications (Chong & Kumar, 2003).

Localization (location estimation) capability is essential in most WSN applications. In environmental monitoring applications such as animal habitat monitoring, bush fire surveillance, water quality monitoring and precision agriculture, the measurement data are meaningless without an accurate knowledge of the location from where the data are obtained. Moreover, the availability of location information may enable a myriad of applications such as inventory management, intrusion detection, road traffic monitoring, health monitoring, reconnaissance and surveillance.

WSN localization techniques are used to estimate the locations of the sensors with initially unknown positions in a network using the available *a priori* knowledge of positions of a few specific sensors in the network and inter-sensor measurements such as distance, time difference of arrival, angle of arrival and connectivity. Sensors with the *a priori* known location information are called *anchors* and their locations can be obtained by using a global positioning system (GPS), or by installing anchors at points with known coordinates, etc. In applications requiring a global coordinate system, these anchors will determine the location of the sensor network in the global coordinate system. In applications where a local coordinate system suffices (e.g., in smart homes, hospitals or for inventory management where knowledge like in which room a sensor is located is sufficient), these anchors define the local coordinate system to which all other sensors are referred. Because of constraints on the cost and size of sensors, energy consumption, implementation environment (e.g., GPS is not accessible in some environments) and the deployment of sensors (e.g., sensors may be randomly scattered in the region), most sensors do not know their own locations. These sensors with unknown location information are called *non-anchor* nodes and their coordinates need to be estimated using a sensor network localization algorithm. In some other applications, e.g., for geographic routing in WSN, where there are no anchor nodes and also knowledge of the physical location of a sensor is unnecessary, people are more interested in knowing the position of a sensor *relative* to other sensors. In that case, sensor localization algorithms can be used to estimate the relative positions of sensors using inter-sensor measurements. The obtained estimated locations are usually a reflected, rotated and translated version of their global coordinates.

In this chapter, we provide an overview of various aspects of WSN localization with a focus on the techniques covered in the other chapters of this book. These chapters can be broadly classified into three categories: the *measurement techniques* in sensor network localization, sensor network *localization theory and algorithms*, and *experimental study and applications of sensor network localization* techniques.

The rest of the chapter is organized as follows. In Section MEASUREMENT TECHNIQUES, measurement techniques in WSN localization and the basic principle of localization using these measurements are discussed. These measurements include *angle-of-arrival (AOA) measurements*, *distance related measurements* and *received signal strength (RSS) profiling techniques*. Distance related measurements are further classified into *one-way propagation time* and *roundtrip propagation time* measurements, the *lighthouse approach* to distance measurements, *RSS-based distance measurements*, *time-difference-of-arrival (TDOA)* measurements and *connectivity* measurements. In Section LOCALIZATION THEORY AND ALGORITHMS, fundamental theory underpinning WSN localization algorithms and some fundamental problems in WSN localization are discussed with a focus on the use of graph theory in WSN localization. Later in this section, a set of major localization algorithms are discussed. Section EXPERI-

MENTAL STUDIES AND APPLICATIONS OF WSN LOCALIZATION discusses implementation of WSN localization techniques and their use in a number of areas, e.g., intelligent transportation and WSN routing. The aim of each of these three later sections is to provide an overall review of its topic and to give brief introduction of the relevant chapters of the book.

MEASUREMENT TECHNIQUES

WSN localization relies on measurements. There are many factors that affect the choice of the algorithm to be used for a specific application and the accuracy of the estimated locations, to name but a few, the network architecture, the average node degree (i.e., the average number of neighbours per sensor), the geometric shape of the network area and the distribution of sensors in that area, sensor time synchronization and the signalling bandwidth among the sensors. However, it is the type of measurements employed and the corresponding precision that fundamentally determine the estimation accuracy of a localization system and the localization algorithm being implemented by this system. Measurements also determine the type of algorithm that can be used by a particular localization system.

In a typical WSN localization system, the available measurements can often be related to the coordinates of sensors using the following generic formula:

$$\boldsymbol{Y} = \boldsymbol{h}(\boldsymbol{X}) + \boldsymbol{e}$$

where $\boldsymbol{Y}$ is the vector of all measurements, $\boldsymbol{X}$ contains the true coordinate vectors of sensors whose locations are to be estimated and $\boldsymbol{e}$ is the vector of measurement errors. If the distribution of measurement errors f_e is known, the estimated locations of sensors can be obtained using the maximum likelihood approach by minimizing an optimization criterion:

$$\hat{\boldsymbol{X}} = \arg\min\left(\log f_e\left(\boldsymbol{Y} - \boldsymbol{h}\left(\hat{\boldsymbol{X}}\right)\right)\right)$$

A particular cost function related to this optimization criterion is the *Fisher Information Matrix*

$$\boldsymbol{J}(\boldsymbol{X}) = E\left(\nabla_X^T \log f_e\left(\boldsymbol{Y} - \boldsymbol{h}(\boldsymbol{X})\right)\nabla_X \log f_e\left(\boldsymbol{Y} - \boldsymbol{h}(\boldsymbol{X})\right)\right)$$

where $\nabla_X \log f_e\left(\boldsymbol{Y} - \boldsymbol{h}(\boldsymbol{X})\right)$ is the partial derivative of $\log f_e\left(\boldsymbol{Y} - \boldsymbol{h}(\boldsymbol{X})\right)$ with respect to X evaluated at $\boldsymbol{X}$.

A common technique that has been widely used to evaluate the location accuracy that can be expected from measurements is the Cramer-Rao bound. The Cramer-Rao lower bound is given by

$$Cov\left(\hat{\boldsymbol{X}}\right) = E\left(\boldsymbol{X} - \hat{\boldsymbol{X}}\right)\left(\boldsymbol{X} - \hat{\boldsymbol{X}}\right)^T \geq \boldsymbol{J}^{-1}(\boldsymbol{X})$$

The Cramer-Rao bound is valid for any unbiased estimator of sensor locations and gives the best performance that can be achieved by an unbiased location estimator. Therefore it is a valuable tool for analysing the information content of various measurements. ***Chapter II - Measurements Used in Wireless Sensor Networks Localization*** features a thorough discussion on this topic. It establishes a common framework for analysing the information content of various measurements, which can be used to derive localization bounds for integration of any combination of measurements in the network.

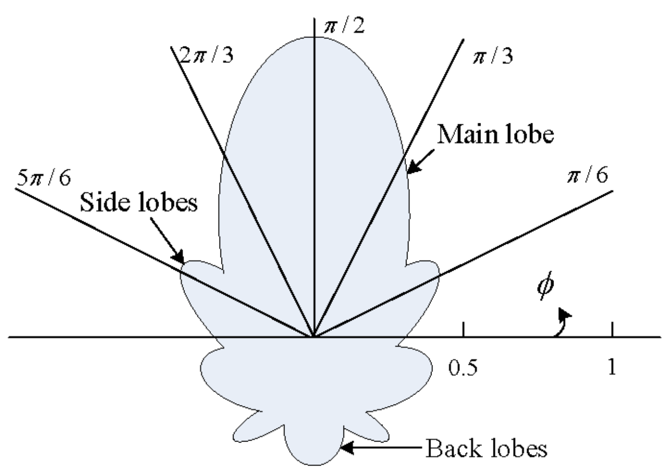
Measurement techniques in WSN localization can be broadly classified into three categories: *AOA measurements*, *distance related measurements* and *RSS profiling techniques*. Next, we introduce these three categories in more detail.

Angle-of-Arrival Measurements

The *AOA* measurements are also known as the *bearing* measurements or the *direction of arrival* measurements. The AOA measurements can usually be obtained from two categories of techniques: those making use of the receiver antenna's amplitude response and those making use of the receiver antenna's phase response. In addition to the directivity of the antenna (Cheng, 1989), the accuracy of AOA measurements are affected by other environmental factors like shadowing and multipath, and the later effect may make the transmitter look like located at a different direction of the receiver.

The first category of AOA measurements is widely known as *beamforming* and it is based on the anisotropy in the reception pattern (Cheng, 1989) of an antenna. The size of the measurement unit can be comparatively small with regards to the wavelength of the signals. Figure 1 shows the beam pattern of a typical anisotropic antenna. When the beam of the receiver antenna is rotated electronically or mechanically, the direction corresponding to the maximum signal strength is taken as the direction of the transmitter. The accuracy of the measurements is determined by the sensitivity of the receiver and the beam width. Using a rotating beam has the potential problem that the receiver cannot differentiate the signal strength variation caused by the varying amplitude of the transmitted signal and the signal strength variation caused by the anisotropy in the reception pattern. This problem can be dealt with by using a second non-rotating and omnidirectional antenna at the receiver. The impact of varying signal strength can be largely removed by normalizing the signal strength received by the rotating anisotropic antenna with respect to the signal strength received by the non-rotating omnidirectional antenna. Alternatively, one may also use multiple stationary antennas with known, anisotropic antenna patterns to overcome the difficulty caused by the varying signal strength problem. Comparing the signal strength received from each antenna at the same time, together with the knowledge of their antenna patterns, leads to an estimate of the transmitter direction, even when the signal strength changes (Koks, 2005).

Figure 1. The horizontal antenna pattern of a typical anisotropic antenna in polar coordinates



The other category of AOA measurement techniques is widely known as *phase interferometry* and it derives the AOA measurements from the measurements of the phase differences in the arrival of a wave front (Rappaport, Reed, & Woerner, 1996). A large receiver antenna (relative to the wavelength of the transmitter signal) or an antenna array is typically required when using this technique. Figure 2 shows an antenna array of N elements. The adjacent antennas are separated by a fixed distance d. For a transmitter far away from the antenna array, its distance to the k^{th} antenna can be approximated by

$$R_k \approx R_0 - kd\cos\theta \tag{1}$$

where R_0 is the distance between the transmitter and the 0^{th} antenna and θ is the direction of the transmitter viewed from the antenna array. The transmitter signal received by the adjacent antennas will have a phase difference of $2\pi \frac{d\cos\theta}{\lambda}$ with λ being the wavelength of the transmitter signal. Therefore the AOA of the transmitter with respect to the antenna array can be derived from the measurements of the phase differences. The accuracy of the AOA measurements obtained using this approach is usually not affected by high signal-to-noise-ratio (SNR) but this approach may fail in the presence of strong co-channel interference and/or multipath signals (Rappaport, Reed, & Woerner, 1996).

The accuracy of AOA measurements is limited by the directivity of the antenna and the measurements are further complicated by the presence of *shadowing* and *multipath* in the measurement environment. A major challenge in AOA measurements is therefore the accurate estimation of AOA in the presence of multipath and shadowing. AOA measurements rely on a direct *line-of-sight (LOS)* path between the transmitter and the receiver. A multipath component from the transmitter signal may appear as a signal coming from an entirely different direction and consequently causes a very large error in the AOA measurement.

Multipath problems in AOA measurements have been usually addressed using *maximum likelihood (ML)* algorithms (Rappaport, Reed, & Woerner, 1996). Depending on the assumptions being made about the statistical characteristics of the transmitter signals, i.e., whether the structure of the transmitter signal is known or unknown to the receiver, these ML algorithms can be further classified into deterministic (Agee, 1991; Halder, Viberg, & Kailath, 1993; Jian, Halder, Stoica, & Viberg, 1995) and stochastic (Biedka, Reed, & Woerner, 1996; Bliss & Forsythe, 2000; Ziskind & Wax, 1988) ML algorithms.

Yet another class of AOA estimation techniques, which relies on the presence of a multi-antenna array that is composed of, say, N antennas at the receiver, is based on the so-called *subspace-based algorithms* (Paulraj, Roy, & Kailath, 1986; Roy & Kailath, 1989; Schmidt, 1986; Tayem & Kwon, 2004). The most well known methods in this category are *MUSIC (multiple signal classification)* and *ESPRIT (estimation of signal parameters by rotational invariance techniques)* (Paulraj et al., 1986; Roy & Kailath, 1989). The measured transmitter signal received at the N antennas of the receiver antenna array is considered as a vector in N dimensional space. A correlation matrix is formed utilizing the N signals received at the antennas of the receiver antenna array. By using an eigen-decomposition of the correlation matrix, the vector space is separated into signal and noise subspaces. Then the MUSIC algorithm searches for nulls in the magnitude squared of the projection of the direction vector onto the noise subspace. The nulls are a function of angle-of-arrival, from which AOA can be estimated. Other techniques that have been developed based on the MUSIC algorithms include *Root-MUSIC* (Barabell, 1983), a polynomial rooting version of MUSIC which improves the resolution capabilities of MUSIC, *WMUSIC* (Kaveh & Bassias, 1990), a weighted norm version of MUSIC which also gives an extension in the resolution capabilities to the original MUSIC. ESPRIT (Paulraj et al., 1986; Roy & Kailath, 1989) is based on the estimation of signal parameters via rotational invariance techniques. It uses two displaced subarrays of matched sensor doublets to exploit an underlying rotational invariance among signal subspaces for such an array. A comprehensive experimental evaluation of MUSIC, Root-MUSIC, WMUSIC, Min-Norm (Kumaresan & Tufts, 1983) and ESPRIT algorithms can be found in (Klukas & Fattouche, 1998). A significant number of AOA measurement techniques have been developed which are based on MUSIC and ESPRIT, to cite but two, see e.g., (Klukas & Fattouche, 1998; Paulraj et al., 1986). Readers may refer to (Schell & Gardner, 1993) for a detailed discussion on AOA measurement techniques.

Chapter III - Overview of RF Localization Sensing Techniques and TOA-Based Positioning for WSNs provides further discussion on AOA measurements using antenna arrays, and gives the Cramer-Rao lower bound on AOA estimation error. The lower bound is determined by the SNR of the received signal from the transmitter, the carrier frequency of the transmitter and the number of antenna elements of the antenna array.

In $\Re^2$, AOA measurements from a minimum of two receivers can be used to estimate the location of the transmitter. However in the presence of measurement errors, more than two AOA measurements will be needed for accurate location estimate. In the presence of measurement errors, AOA measurements from more than two receivers will not intersect at the same point. This is illustrated in Figure 3.

Denote by $\boldsymbol{X}_t = [x_t, y_t]^T$ the true coordinate vector of the transmitter whose location is to be estimated from AOA measurements $\alpha = [\alpha_1, \ldots, \alpha_N]^T$, where N is the total number of receivers. Let $\boldsymbol{X}_i = [x_i, y_i]^T$ be the known coordinate vector of the i^{th} receiver associated with the i^{th} AOA measurement α_i. Denote by $\theta(\boldsymbol{X}_t) = [\theta_1(\boldsymbol{X}_t), \ldots, \theta_N(\boldsymbol{X}_t)]$ the AOA vector of the transmitter located at x_t from the receiver locations, i.e., $\theta_i(\boldsymbol{X}_t)$ $(i \in \{1, \ldots, N\})$ is related to x_t and x_i by

$$\tan\theta_i(\boldsymbol{X}_t) = \frac{y_t - y_i}{x_t - x_i} \tag{2}$$

Figure 2. An illustration of AOA measurements using an antenna array of N antennas

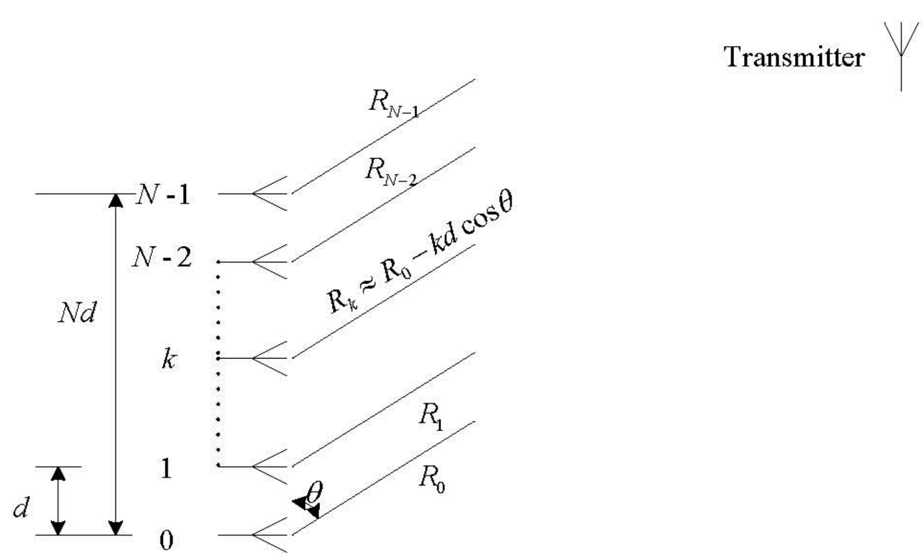
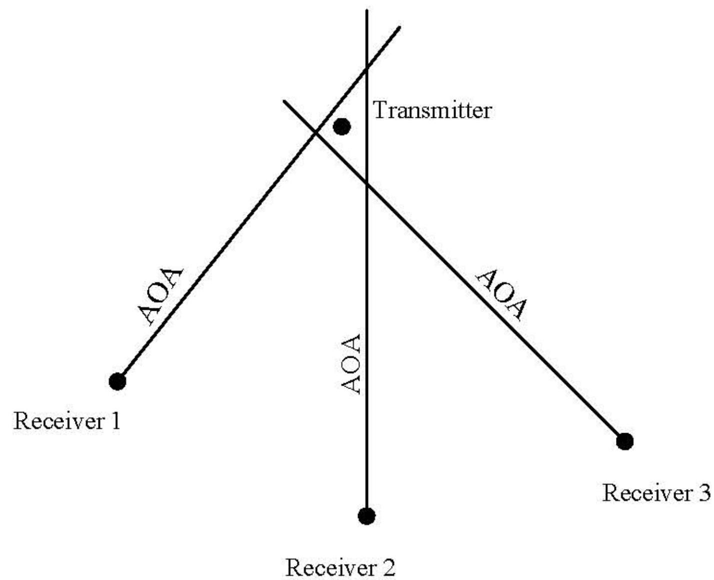


Figure 3. In the presence of measurement errors, AOA measurements from three receivers will not intersect at the same point



In the presence of measurement errors, the measured AOA vector $\boldsymbol{\alpha}$ consists of the true bearing vector corrupted by noise $\boldsymbol{e} = [e_1, \ldots, e_N]^T$, which is usually assumed to be additive zero mean Gaussian noise with covariance matrix $\boldsymbol{S} = diag\{\sigma_1, \ldots, \sigma_N\}$, i.e.,

$$\alpha = \theta(\boldsymbol{X}_t) + \boldsymbol{e} \tag{3}$$

The transmitter location can then be estimated using an ML estimator as follows:

$$\hat{\boldsymbol{X}}_t = \arg\min\left[\theta(\hat{\boldsymbol{X}}_t) - \alpha\right]^T \boldsymbol{S}^{-1}\left[\theta(\hat{\boldsymbol{X}}_t) - \alpha\right] \tag{4}$$

When the receivers are identical and much closer to each other than to the transmitter, the variances of AOA measurement errors can be considered as equal, i.e., $\sigma_1^{\ 2} = \cdots = \sigma_N^{\ 2} = \sigma^2$. The nonlinear optimization problem in Equation (4) can be solved by a Newton-Gauss iteration (Gavish & Weiss, 1992; Torrieri, 1984), which requires an initial estimate of the transmitter location close to its true location. If additional information, such as the measurement errors being small or rough estimates of the distances between the transmitter and the receivers, is available *a priori*, techniques like the Stanfield approach (Stanfield, 1947) can be used to simplify the optimization problem in Equation (4) and an analytical solution to $\hat{\boldsymbol{X}}_t$ can be obtained directly. We refer the readers to (Gavish & Weiss, 1992; Torrieri, 1984) for more detailed discussions on this topic.

Distance Related Measurements

Measurements that can be classified into the category of distance related measurements include *propagation time based measurements*, i.e., *one-way propagation time measurements*, *roundtrip propagation* *time measurements* and *TDOA* measurements; *RSS based measurements*; and *connectivity measurements*. Another interesting approach to distance measurements, which does not fall into any of the above categories, is the *lighthouse approach* (Romer, 2003).

One-Way Propagation Time Measurements

The principle of *one-way propagation time measurements* is straightforward: measuring the difference between the sending time of a signal at the transmitter and the receiving time of the signal at the receiver. Given this time difference measurement and the propagation speed of the signal in the media, the distance between the transmitter and the receiver can be obtained. Time delay measurement is a relatively mature field. The most widely used method for obtaining time delay measurement is the *generalized cross-correlation method* (Carter, 1981, 1993; Knapp & Carter, 1976).

A major challenge in the implementation of one-way propagation time measurements is that it requires the local time at the transmitter and the local time at the receiver to be accurately synchronized. Any difference between the two local times will become the bias in the one-way propagation measurement. At the speed of light, a very small synchronization error of 1ns will translate into a distance measurement error of 0.3m. The accurate synchronization requirement may add to the cost of sensors, by demanding a highly accurate clock, or increase the complexity of the sensor network, by demanding a sophisticated synchronization algorithm. This disadvantage makes one-way propagation time measurements a less attractive option in WSNs.

In addition to using an accurate clock for each sensor or using a sophisticated synchronization algorithm, an interesting approach has been proposed in the literature which overcomes the synchronization problem (Priyantha, Chakraborty, & Balakrishnan, 2000) based on the observation that the speed of sound in the air is much smaller than the speed of light or radio-frequency (RF) signal in the air. A combination of RF and ultrasound hardware is used in the technique. On each transmission, a transmitter sends an RF signal and an ultrasonic pulse at the same time. The RF signal will arrive at the receiver earlier than the ultrasonic pulse. When the receiver receives the RF signal, it turns on its ultrasonic receiver and listens for the ultrasonic pulse. The time difference between the receipt of the RF signal and the receipt of the ultrasonic signal is used as an estimate of the one-way acoustic propagation time. This method gives fairly accurate distance estimate at the cost of additional hardware and complexity of the system because ultrasonic reception suffers from severe multipath effects caused by reflections from walls and other objects. This method is referred to as *time-difference-of-arrival (TDOA)* measurement, i.e., measurement of the difference between the arrival times of RF signal and ultrasonic signal, in some papers as well as some chapters in this book. However it should be noted that it is different from the TDOA measurements discussed later in this chapter and in most papers on geolocation.

Roundtrip Propagation Time Measurements

Roundtrip propagation time measurements measure the difference between the time when a signal is sent by a sensor and the time when the signal returned by a second sensor comes back to the original sensor. Since the same local clock is used to compute the roundtrip propagation time, there is no synchronization problem. The major error source in roundtrip propagation time measurements is the delay required for handling the signal in the second sensor. This internal delay is either known via *a priori* calibration, or measured and sent to the first sensor to be subtracted. A technique that can be used to overcome the above internal delay problem involves the cooperation of the two sensors in the measurements. First sensor A sends a signal to sensor B at sensor A's local time t_{A1}, the signal arrives at sensor B at sensor B's local time t_{B1}. After some delay, sensor B sends a signal to sensor A at sensor B's local time t_{B2}, together with the time difference $t_{B2} - t_{B1}$. The signal arrives at sensor A at sensor A's local time t_{A2}. Then sensor A is able to compute the round-trip-time using $(t_{A2} - t_{A1}) - (t_{B2} - t_{B1})$. Because the computation only needs the difference between two local time measurements at sensor A and the difference between two local time measurements at sensor B, no synchronization problem exists. The internal delay in the second sensor B is also removed in the round-trip time measurements. A detailed discussion on circuitry design for roundtrip propagation time measurements can be found in (McCrady, Doyle, Forstrom, Dempsey, & Martorana, 2000).

In addition to the synchronization error, the accuracy of both one-way and roundtrip propagation time measurements is affected by noise, signal bandwidth, non-line-of-sight (NLOS) and multipath. Recently, ultra-wide band (UWB) signals have started to be used for accurate propagation time measurements (Gezici et al., 2005; Lee & Scholtz, 2002). A UWB signal is a signal whose bandwidth to centre frequency ratio is larger than 0.2 or a signal with a total bandwidth of more than 500 *MHz*. In principle, UWB can achieve higher accuracy because its bandwidth is very large and therefore its pulse has a very short duration. This feature makes fine time resolution of UWB signals and easy separation of multipath signals possible.

Chapter III - Overview of RF Localization Sensing Techniques and TOA-Based Positioning for WSNs first discusses time of arrival (TOA) measurement techniques and challenges in the measurements. The chapter then focuses on the identification of NLOS conditions in TOA measurements and techniques that can be used to mitigate the performance impact of NLOS conditions.

Chapter IV - RF Ranging Methods and Performance Limits for Sensor Localization gives a detailed discussion on the impacts of various factors, including noise, clock synchronization, signal bandwidth and multipath, on the accuracy of propagation time measurements. The chapter also features a discussion on the characteristics of some deployed systems.

In $\mathfrak{R}^2$, measured distances from a non-anchor node to three non-collinear anchors determine three circles whose centres are at the three anchors and radii are the associated measured distances respectively. When there is no measurement error, the three circles intersect at a single point which is the location of the non-anchor node. In the presence of measurement errors, the three circles do not intersect at a single point. A large number of approaches have been developed to estimate the location of the non-anchor node in such noisy cases. Assuming the measurement errors are additive zero mean Gaussian noises, for a non-anchor node at unknown location X_t with noise-contaminated distance measurements $\tilde{\boldsymbol{d}} = \left[\tilde{d}_1, \ldots, \tilde{d}_N\right]^T$ to N anchors at known locations $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_N$, an ML formulation of the location estimation problem is given by

$$\hat{\boldsymbol{X}}_t = \arg\min \left[\boldsymbol{d}\left(\hat{\boldsymbol{X}}_t\right) - \tilde{\boldsymbol{d}}\right]^T \boldsymbol{S}^{-1} \left[\boldsymbol{d}\left(\hat{\boldsymbol{X}}_t\right) - \tilde{\boldsymbol{d}}\right] \tag{5}$$

where $\boldsymbol{d}\left(\hat{\boldsymbol{X}}_t\right) = \left[\|\hat{\boldsymbol{X}}_t - \boldsymbol{X}_1\|, \ldots, \|\hat{\boldsymbol{X}}_t - \boldsymbol{X}_N\|\right]^T$ and $\boldsymbol{S}$ is the covariance matrix of the distance measurement errors. This minimization problem can be solved using ML techniques similar to those discussed in the previous section.

In real applications the situation is much more complicated. Some challenges that can be encountered in distance-based localization include: the distance measurement error may be neither additive nor Gaussian noises; the measured distances may be biased; a non-anchor node may have to derive its location from the estimated locations (containing errors) of its neighbouring non-anchor nodes instead of anchors; if a non-anchor node is a neighbour of a set of nodes which are almost collinear, the non-anchor node may not be able to uniquely determine its location estimate; the network topology may be irregular, not to mention the challenge of designing a computationally efficient localization algorithm for large scale networks. It is these challenges that make distance-based localization problem both challenging and intriguing. The other chapters of this book explore various aspects of distance-based localization problems and lead readers to establish a solid understanding in both distance-based localization and localization using other types of measurements.

Time-Difference-of-Arrival Measurements

Time-difference-of-arrival (TDOA) measurements measure the difference between the arrival times of a transmitter signal at two receivers respectively. In $\Re^2$, denote the coordinates of the two receivers by $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$, and the coordinates of the transmitter by $\boldsymbol{X}_t$. The measured TDOA Δt_{ij} is related to the locations of the two receivers by

$$\Delta t_{ij} = t_i - t_j = \frac{1}{c}\left(\| \boldsymbol{X}_t - \boldsymbol{X}_i \| - \| \boldsymbol{X}_t - \boldsymbol{X}_j \|\right) \tag{6}$$

where t_i and t_j are the arrival times of the transmitter signal at receivers i and j respectively and c is the propagation speed of the transmitter signal. Assuming the receiver locations are known and the two receivers are perfectly synchronized, Equation (6) defines one branch of a hyperbola on which the transmitter must lie. The foci of the hyperbola are at the locations of the receivers i and j. In a system of N receivers, there are N–1 linearly independent TDOA measurements, hence N–1 linearly independent equations like (6). In $\Re^2$, TDOA measurements from a *minimum* of three receivers are required to uniquely determine the location of the transmitter. This is illustrated in Figure 4.

The accuracy of TDOA measurements is affected by the synchronization error between receivers and multipath. The accuracy and temporal resolution capabilities of TDOA measurements will improve when the separation between receivers increases because this increases differences between times of arrival. Readers are referred to (C. K. Chen & Gardner, 1992; Rappaport, Reed, & Woerner, 1996; Schell & Gardner, 1993) for more detailed discussion.

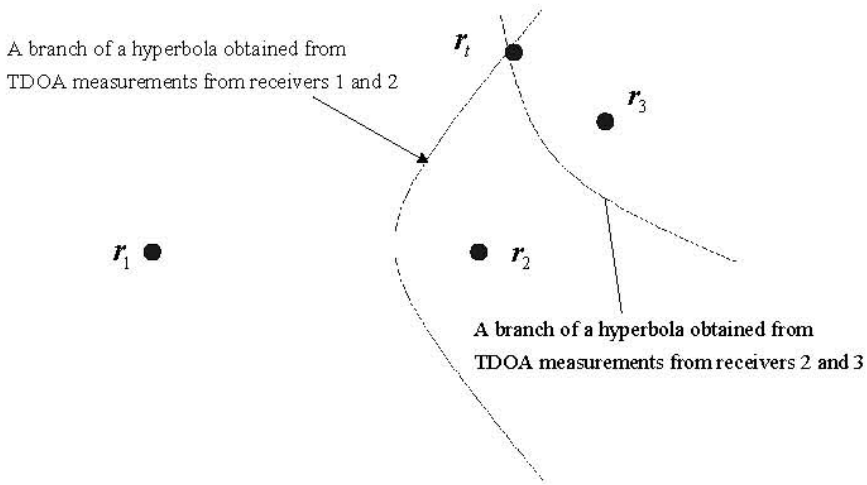
In the presence of measurement errors and assuming that the errors are in the form of additive zero mean Gaussian noise, in a system of N receivers, the TDOA equations can be written compactly in matrix form as

$$\Delta\tilde{\boldsymbol{t}} = \Delta\boldsymbol{X} + \boldsymbol{e} \tag{7}$$

where $\Delta\tilde{\boldsymbol{t}} = \left[\Delta\tilde{t}_{21}, \Delta\tilde{t}_{31}, \ldots, \Delta\tilde{t}_{N1}\right]^T$, $\Delta\boldsymbol{X} = \frac{1}{c}\left[\|\boldsymbol{X}_t - \boldsymbol{X}_1\| - \|\boldsymbol{X}_t - \boldsymbol{X}_2\|, \ldots, \|\boldsymbol{X}_t - \boldsymbol{X}_1\| - \|\boldsymbol{X}_t - \boldsymbol{X}_N\|\right]^T$ and $\boldsymbol{e} = [e_{21}, \ldots, e_{N1}]$ with e_{j1} being the measurement error of $\Delta\tilde{t}_{j1}$. Defining $\boldsymbol{f}\left(\hat{\boldsymbol{X}}\right) = \frac{1}{c}\left[\|\hat{\boldsymbol{X}} - \boldsymbol{X}_1\| - \|\hat{\boldsymbol{X}} - \boldsymbol{X}_2\|, \ldots, \|\hat{\boldsymbol{X}} - \boldsymbol{X}_1\| - \|\hat{\boldsymbol{X}} - \boldsymbol{X}_N\|\right]^T$, an ML formulation of the location estimation problem using TDOA measurements is:

$$\hat{\boldsymbol{X}}_t = \arg\min\left[\Delta\tilde{\boldsymbol{t}} - \boldsymbol{f}\left(\hat{\boldsymbol{X}}\right)\right]^T \boldsymbol{S}^{-1}\left[\Delta\tilde{\boldsymbol{t}} - \boldsymbol{f}\left(\hat{\boldsymbol{X}}\right)\right] \tag{8}$$

Figure 4. Two intersecting branches of two hyperbolas obtained by TDOA measurements from three receivers uniquely determine the location of the transmitter



where $\boldsymbol{S}$ is the covariance matrix of TDOA measurement errors. Equation (8) however is in a very complicated form. In order to obtain a reasonably simple estimator, $f(\boldsymbol{X})$ can be linearized around a reference point $\boldsymbol{X}_0$ using Taylor series:

$$\boldsymbol{f}(\boldsymbol{X}) \approx \boldsymbol{f}(\boldsymbol{X}_0) + \boldsymbol{f}'(\boldsymbol{X}_0)(\boldsymbol{X} - \boldsymbol{X}_0) \tag{9}$$

where $f'(\boldsymbol{X}_0)$ is the partial derivative of $f(\boldsymbol{X})$ with respect to $\boldsymbol{X}$ evaluated at $\boldsymbol{X}_0$. A recursive solution to the maximum likelihood estimator can then be obtained (Torrieri, 1984):

$$\hat{\boldsymbol{X}}_{t,k+1} = \hat{\boldsymbol{X}}_{t,k} + \left(\boldsymbol{f}'\left(\hat{\boldsymbol{X}}_{t,k}\right)^T \boldsymbol{S}^{-1} \boldsymbol{f}'\left(\hat{\boldsymbol{X}}_{t,k}\right)\right)^{-1} \boldsymbol{f}'\left(\hat{\boldsymbol{X}}_{t,k}\right)^T \boldsymbol{S}^{-1} \left(\Delta\tilde{\boldsymbol{t}} - \boldsymbol{f}\left(\hat{\boldsymbol{X}}_{t,k}\right)\right) \tag{10}$$

This method obviously relies on a good initial guess of the transmitter location. Furthermore, the method can result in significant location estimation errors in some situations due to geometric delusion of precision (GDOP) effects. GDOP describes situation in which a relatively small measurement error can cause a large location estimation error because the transmitter is located on a portion of the hyperbola far away from the receivers (Bancroft, 1985; Rappaport, Reed, & Woerner, 1996). There are many other approaches presented in the literature on TDOA based location estimation and we refer readers to (Abel, 1990; Chan & Ho, 1994; Crippen & Havel, 1988; Dogancay, 2005; B. T. Fang, 1990; Smith & Abel, 1987)

Received Signal Strength Measurements

Received signal strength (RSS) measurements estimate the distances between neighbouring sensors from the received signal strength measurements between the two sensors (Bergamo & Mazzini, 2002; Elnahrawy, Li, & Martin, 2004; Madigan et al., 2005; Niculescu & Nath, 2003; Patwari et al., 2005). Most wireless devices have the capability of measuring the received signal strength.

The wireless signal strength received by a sensor from another sensor is a monotonically decreasing function of their distance. This relationship between the received signal strength and distance is popularly modelled by the following log-normal model:

$$P_r(d)[dBm] = P_0(d_0)[dBm] - 10n_p \log_{10}\left(\frac{d}{d_0}\right) + X_\sigma \tag{11}$$

where $P_0(d_0)[dBm]$ is a reference power in dB milliwatts at a reference distance d_0 from the transmitter, n_p is the path loss exponent that measures the rate at which the received signal strength decreases with distance, and X_σ is a zero mean Gaussian distributed random variable with standard deviation σ and it accounts for the random effect caused by shadowing. Both n_p and σ are environment dependent. The path loss exponent n_p is typically assumed to be a constant however some measurement studies suggest the parameter is more accurately modelled by a Gaussian random variable or different path loss exponent should be used for a receiver in the far-field region of the transmitter or in the near-field region of the transmitter. Given the model and model parameters, which are obtained via *a priori* measurements, the inter-sensor distances can be estimated from the RSS measurements. Localization algorithms can then be applied to these distance measurements to obtain estimated locations of sensors.

Chapter V - Calibration and Measurement of Signal Strength for Sensor Localization features a thorough discussion on a number of practical issues involved in the use of RSS measurements for distance estimation. The chapter focuses on device effects and modelling problems which are important for the implementation of RSS-based distance estimation but are not well covered in the literature. These include transceiver device manufacturing variations, battery effects on transmit power, nonlinearities in the circuit, and path loss model parameter estimation. Measurement methodologies are presented to characterize these effects for wireless sensors and suggestions are made to limit impact of these effects.

Note that in addition to the log-normal model many other models have also been proposed in the literature which can better describe the wireless signal propagation characteristic for signals within a specific frequency spectrum in a specific environment, for example Longley-Rice model, Durkin's model, Okumuran model, Hata model and wideband PCS microcell model for outdoor environments, and Ericsson multiple breakpoint model, attenuation factor model and the combined use of site specific propagation models and graphical information system databases for radio signal prediction in indoor environments (Rappaport, 2001).

Yet another interesting technique to estimate the distance between an optical receiver and an optical transmitter is the *lighthouse approach* reported in (Romer, 2003). The lighthouse approach estimates the distance between an optical receiver and a transmitter of a parallel rotating optical beam by measuring the time duration that the receiver dwells in the beam. A parallel optical beam is a beam whose beam width is constant with respect to the distance from the rotational axis of the beam. It is the characteristic of the parallel beam that the time the optical receiver dwells in the beam is inversely proportional to the distance between the optical receiver and the rotational axis of the beam enables the distance measurements. A major advantage of the lighthouse approach is the optical receiver can be of a very small size and low cost, thus making the idea of "smart dust" possible. However the transmitter may be large and expensive. The approach also requires a direct LOS between the optical receiver and the transmitter.

Connectivity Measurements

Connectivity measurements are possibly the simplest measurements. In connectivity measurements, a sensor measures which sensors are in its transmission range. Such measurements can be interpreted as binary distance measurements, i.e., either another particular sensor is within the transmission range of a given sensor or it is outside the transmission range of that sensor.

A sensor being in the transmission range of another sensor defines a proximity constraint between these two sensors, which can be exploited for localization. In its simplest form, a non-anchor sensor being a neighbour of three anchors means the non-anchor sensor is very close to the three anchors and many algorithms then use the centroid of the three anchors as the estimated location of the non-anchor sensor. In the later section, we shall give a more detailed discussion of connectivity-based localization algorithms in large scale networks.

RSS Profiling Measurements

Above, we have mentioned some techniques to estimate the distances between sensors from RSS measurements. Localization algorithms can then be applied to these distance measurements to obtain estimated locations of sensors. The implementation of such localization techniques however faces two major challenges: first the wireless environments, especially indoor wireless environments, are very complicated. It is often difficult to determine the best model for RSS-based distance estimation. Second, the determination of model parameters is also a difficult task. Such difficulties can be overcome using another category of localization techniques, namely the *RSS profiling-based localization techniques* (Bahl & Padmanabhan, 2000; Krishnan, Krishnakumar, Ju, Mallows, & Gamt, 2004; Prasithsangaree, Krishnamurthy, & Chrysanthis, 2002; Ray, Lai, & Paschalidis, 2005; Roos, Myllymaki, & Tirri, 2002), which estimate sensor location from RSS measurements directly.

The RSS profiling-based localization techniques works by first constructing a form of map of the signal strength behaviour of anchor nodes in the coverage area. The map is obtained either offline by *a priori* measurements or online using sniffing devices (Krishnan et al., 2004) deployed at known locations. The RSS profiling-based localization techniques have been mainly used for location estimation in wireless local area networks (WLANs), but they would appear to be attractive also for WSNs.

In RSS profiling-based localization systems, in addition to anchor nodes (e.g., access points in WLANs) and non-anchor nodes, a large number of sample points, e.g., sniffing devices or *a priori* chosen locations at which the RSS measurements from anchors are to be obtained before the localization of non-anchor nodes starts, are distributed throughout the coverage area of the sensor network. At each sample point, a vector of signal strengths is obtained, with the k^{th} entry corresponding to the signal strength received from the k^{th} anchor at the sample point. Of course, many entries of the signal strength vector may be zero or very small, corresponding to anchor nodes at larger distances (relative to the transmission range) from the sample point. The collection of all these vectors provides (by extrapolation in the vicinity of the sample points) a RSS map of the whole region. The collection constitutes the RSS map, and it is unique with respect to the anchor locations and the environment. The model is stored in a central location. By referring to the RSS map, a non-anchor node can estimate its location using the RSS measurements from anchors by either choosing the location of the sample point, whose signal strength vector is the closest match of that of the non-anchor node, to be its location, or derive its estimated location from the locations of a set of sample points whose signal strength vectors better match that of the non-anchor node than other sample points.

In this section, a number of measurement techniques and the basic principles of location estimation using these measurements are discussed. Which measurement technique to use for location estimation will depend on the requirements of the specific application on localization accuracy, cost and complexity of localization algorithms. Typically, localization algorithms based on AOA and propagation time measurements are able to achieve better accuracy than localization algorithms based on RSS measurements. However, that improved accuracy is achieved at the expense of higher equipment cost. Also the high nonlinearity and complexity in the observation model, i.e., the equation relating the coordinates of sensors to measurements, of AOA and TDOA measurements make them a less attractive option than distance measurements for location estimation in large scale multi-hop wireless sensor networks.

SENSOR NETWORK LOCALIZATION THEORY AND ALGORITHMS

In this section, we give a brief introduction to some fundamental theories in sensor network localization and major sensor network localization algorithms as well as introducing the relevant chapters of the book.

Graph Theory and its Applications in Sensor Network Localization

The task of WSN localization algorithms is to estimate the locations of sensors with initially unknown location information, i.e., the non-anchors, by using *a priori* knowledge of the locations of a few sensors, i.e., anchors, and inter-sensor measurements such as distance, AOA, TDOA and connectivity. A fundamental question in sensor network localization is whether a solution to the localization problem is unique. The network, with the given set of anchors, non-anchors and inter-sensor measurements, is said to be *uniquely localizable* if there is a unique set of locations consistent with the given data. Graph theory has been found to be particularly useful for solving the above problem of unique localization. Graph theory also forms the basis of many localization algorithms, especially for the category of distance-based localization problem, noting that it has been used to study the localization problem using other types of measurements, e.g., TDOA and AOA measurements, as well.

The task of distance-based localization problem is to estimate the locations of non-anchors using the known locations of anchors and inter-sensor distance measurements. A graphical model for distance-based localization problem can be built by representing each sensor in the network uniquely with a vertex and vice versa. An edge exists between two vertices if the distance between the corresponding sensors is known. Note that there is always an edge between two vertices representing two anchors as the distance between two anchors can be obtained from their known locations. The obtained graph $G(V,E)$ with V being the set of vertices and E being the set of edges is called the *underlying graph* of the sensor network. Details of graph theoretical representations of WSNs and their use in localization can be found in ***Chapter 6- Graph Theoretic Techniques in the Analysis of Uniquely Localizable Sensor Networks.***

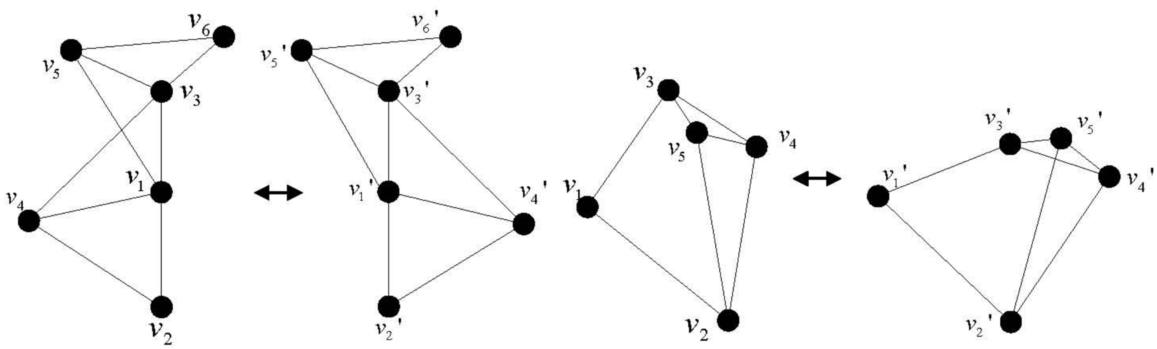
In rigid graph theory, a mapping $p : V \to \Re^d$ ($d \in \{2,3\}$), assigning a location in $\Re^d$ to each vertex of graph $G = (V, E)$, is called a $d-$dimensional *representation* of G. With this definition the localization problem can be seen as finding the *correct representation* of the underlying graph of the WSN that is consistent with the given data. Given a graph $G = (V, E)$ and a representation p of it, the pair (G, p) is called a *framework*. A particular graph property associated with unique localizability of sensor networks is *global rigidity*: A framework (G, p) is called *globally rigid* if every framework (G, p_1) satisfying $\|p_1(i) - p_1(j)\| = \|p(i) - p(j)\|$ for any vertex pair $i, j \in V$, which are connected by an edge in E, also satisfies the same equality for any other vertex pairs that are not connected by an edge. A relaxed form of global rigidity is *rigidity*: A framework (G, p) is *rigid* if there exists a sufficiently small positive constant e_p such that every framework (G, p_1) satisfying $\|p_1(i) - p(i)\| < \varepsilon_p$ for all $i \in V$ and $\|p_1(i) - p_1(j)\| = \|p(i) - p(j)\|$ for any vertex pair $i, j \in V$, which are connected by an edge in E, satisfies $\|p_1(i) - p_1(j)\| = \|p(i) - p(j)\|$ for any other vertex pairs that are not connected by a single edge as well. If the framework (G, p) formed by the underlying graph G of a WSN and its correct representation p is not rigid, there are an infinite number of solutions to the localization problem that are consistent with the given data.

If the framework (G, p) formed by the underlying graph G of a WSN and its correct representation p is globally rigid, the sensor network with at least three non-collinear anchors in $\Re^2$ or four non-coplanar anchors in $\Re^3$ is uniquely localizable. If a framework (G, p) is rigid but not globally rigid, there exist two types of discontinuous deformations that can prevent finding a unique representation of G consistent with the information of anchor node positions and distance measurements: *flip ambiguities* and *discontinuous flex ambiguities*. In *flip ambiguities* in $\Re^d$ ($d \in \{2,3\}$), a vertex (sensor) v has a set of neighbours which span a $(d-1)$-dimensional subspace, e.g., v has only d neighbours, in $\Re^2$ v has a set of neighbours located on a line, or in $\Re^3$ v has a set of neighbours located on a plane, which leads to the possibility of the neighbours forming a mirror through which v can be reflected. In *discontinuous flex ambiguities* in $\Re^d$ ($d \in \{2,3\}$), the removal of an edge or a set of edges allows the remaining part of the graph to be flexed to a different realization (which cannot be obtained from the original realization by translation, rotation or reflection) such that the removed edge can be reinserted with the same length. Figure 5 shows an example of flip ambiguity and discontinuous flex ambiguity in $\Re^2$. Note that in Figure 5.(a) and 5.(b), both the figure on the left side and the figure on the right side satisfy the same set of distance constraints but the locations of vertices are different, which means the associated sensor network is not uniquely localizable.

Using graph theory, we can identify necessary conditions as well as sufficient conditions that need to be satisfied by the underlying graph of a sensor network in order for the network to be uniquely localizable. ***Chapter VI*** gives a detailed overview of this topic, providing various results in graph theory to characterize uniquely localizable networks in two dimensions. Conditions required for the sensor network to be uniquely localizable are discussed and techniques to test the unique localizability are introduced. While the focus of the chapter is 2-dimensional distance-based localization, the authors also consider sensor networks with mixed distance and AOA measurements as well as unique localizability of 3-dimensional networks.

Note that the unique localizability conditions mentioned above are independent of the specific localization algorithm being used. Furthermore, the above discussion has been carried out without considering measurement errors. The problem becomes more complicated when the effects of measurement errors are considered. For example, it has become a common knowledge that in $\Re^2$ in the presence of measurement errors, a non-anchor node connected to a set of two or more anchors which are exactly or almost collinear, the non-anchor node is *likely* to have *flip ambiguity* problem. However we are yet to establish an accurate knowledge in the area, i.e., given the measurement error distribution and anchor locations, how to compute the probability that the non-anchor's location estimation be contaminated by

Figure 5. An illustration of the flip and discontinuous flex ambiguity in 2D: (a) Flip ambiguity: The neighbours of vertex v_4, v_1, v_2 and v_3 are on the same line. Vertex v_4 can be reflected across the line on which vertices v_1, v_2 and v_3 locate to a new position without violating the distance constraints. (b) Discontinuous flex ambiguity: Removing the edge between v_3 and v_4, the vertices v_1, v_2, v_3 and v_4 can be moved continuously to other positions while maintaining the length of the edges between them. When these vertices move to positions such that the edge between v_3 and v_4 can be reinserted with the same length, we obtain a new graph. Both the graph on the left side and the graph on the right side satisfy the same set of distance constraints.



flip ambiguity error? The problem is further complicated in a large scale network where the non-anchor node may have to rely on the inaccurate location estimates of its non-anchor neighbours to estimate its own location. Therefore the analysis on unique localizability can be used to label those sensors with large errors in their location estimates so that those errors do not propagate to the rest of the network.

It is worth noting that flip ambiguity and discontinuous flex ambiguity problems do not necessarily occur in every sensor network. The probability of occurrence of ambiguities is generally smaller in dense networks where the average number of neighbours per node is high. However when such ambiguities occur, they generally cause a large error in the location estimate of a non-anchor node. This error may further propagate to other non-anchor nodes when they use the estimated location of the non-anchor node to determine their own locations. Therefore the performance impact of flip ambiguity and discontinuous ambiguity on sensor network localization may be significant. This has been validated by a number of analytical and simulation studies including some of our own work.

Graph theory has also been used to characterize large scale networks in which the design of an efficient localization algorithm is possible. The computational complexity of localization algorithms is an important consideration in the localization of large scale networks and the computational complexity of distance-based localization algorithms in large scale networks has been investigated in the literature (Aspnes et al., 2006; Eren et al., 2004; Saxe, 1979). In general, the computational complexity of localization algorithms is exponential in the number of sensor nodes (Saxe, 1979). Nevertheless, there is a category of networks where the design of efficient localization algorithms is possible. Specifically, if the underlying graph of the network is a *bilateration*, *trilateration* or *quadrilateration* graph, it is possible to design localization algorithms whose computational complexity is polynomial (and on occasions linear) in the number of sensor nodes (Aspnes et al., 2006; Cao, Anderson, & Morse, 2005; Eren et al., 2004).

A graph $G = (V,E)$ is called a *bilateration grap*h if there exists an ordering of vertices $v_1, v_2, \ldots, v_{|V|}$, termed *bilaterative ordering*, such that (i) the edges (v_1, v_2), (v_1, v_3), (v_2, v_3) are all in E, (ii) each vertex v_i for $i = 4,5,\ldots,|V|-1$ is connected to (at least) two of the vertices in $v_1, v_2, \ldots, v_{i-1}$, and (iii) the vertex $v_{|V|}$ is connected to (at least) three of the vertices $v_1, v_2, \ldots, v_{|V|-1}$. The symbol $|V|$ denotes the cardinality of set V. If the underlying graph of a network is a bilateration graph, an efficient sequential localization algorithm can be designed for the network (J. Fang, Cao, Morse, & Anderson, 2006). The concepts of *trilateration graphs* and *quadrilateration graphs* are defined analogously. Note that trilateration and quadrilateration graphs are necessarily bilateration graphs as well. We refer readers to the above reference and ***Chapter VII - Sequential Localization with Inaccurate Measurements*** for more detailed discussions on this topic. ***Chapter VII*** further presents an efficient sequential algorithm for estimating sensor locations using inaccurate distance measurements. The algorithm is based on the above graph theory concepts; the authors have further developed existing work by demonstrating that it is possible to design a computationally efficient sequential localization algorithm for networks whose underlying graphs are not necessarily bilateration graphs.

Sensor Network Localization Algorithms

Centralized vs. Distributed Localization

Based on the approach of processing the individual inter-sensor measurement data, localization algorithms can be broadly classified into two categories: *centralized algorithms* and *distributed algorithms*. In centralized algorithms, all the individual inter-sensor measurements are sent to a single central processor where the estimated locations of non-anchor nodes are computed; while in distributed algorithms each node (or a group of nodes in close proximity to each other) estimate its (their) own location(s) using inter-sensor measurements and the location information collected from its (their) neighbours. Major approaches for designing centralized algorithms include *multidimensional scaling (MDS)*, *linear programming* and *stochastic optimization* approaches. Some well-known distributed localization algorithms include the "*DV-hop*" and "*DV-distance*" algorithms (Niculescu & Nath, 2001), a number of other algorithms based on the above two algorithms (Chris Savarese & Rabaey, 2002; C. Savarese, Rabaey, & Beutel, 2001), and the *nonparametric belief propagation* algorithms (Ihler, Fisher, Moses, & Willsky, 2005) and its variants (Fox, Hightower, Lin, Schulz, & Borriello, 2003). The "*sweep*" category of *sequential algorithms* reported in ***Chapter VII*** also represents a promising direction in the development of distributed algorithms, which may offer an optimum balance between localization accuracy and computational efficiency in large scale sensor networks.

Centralized and distributed distance-based localization algorithms can be compared from several perspectives, including location estimation accuracy, implementation and computational complexities, and energy consumption.

Distributed localization algorithms are generally considered to be more computationally efficient and easier to implement in large scale networks. However in certain networks where centralized information architecture already exists, such as road traffic monitoring and control, environmental monitoring, health monitoring, and precision agriculture monitoring networks, the measurement data of all the nodes in the network need to be collected and sent to a central processor unit. In such a network the individual sensors may be of limited computational capability; it is convenient to piggyback localization related measurements to other measurement data and send them together to the central processing unit. Therefore a centralized localization algorithm appears to be a natural choice for such networks with existing centralized information architecture.

In terms of location estimation accuracy, centralized algorithms are likely to provide more accurate location estimates than distributed algorithms. One of the reasons is the availability of global information in centralized algorithms. However centralized algorithms suffer from the scalability problem and generally are not feasible to be implemented for large scale sensor networks. Other disadvantages of centralized algorithms, as compared to distributed algorithms, are their requirement of higher computational complexity and lower reliability due to accumulated information inaccuracies/losses involved in multihop transmission from individual sensors to the centralized processor over a WSN.

On the other hand, distributed algorithms are more difficult to design because of the potentially complicated relationship between local behaviour and global behaviour. That is, algorithms that are locally optimal may not perform well globally. Optimal distribution of the computation of a centralized algorithm in a distributed implementation in general remains an open research problem. Error propagation is another potential problem in distributed algorithms. Moreover, distributed algorithms generally require multiple iterations to arrive at a stable solution. This may cause the localization process to take longer time than the acceptable in some cases.

From the perspective of energy consumption, the individual amounts of energy required for each type of operation in centralized and distributed localization algorithms in the specific hardware and the transmission range setting needs to be considered. Depending on the setting, the energy required for transmitting a single bit could be used to execute 1,000 to 2,000 instructions (Chen, Yao, & Hudson, 2002). Centralized algorithms in large networks require each sensor's measurements to be sent over multiple hops to a central processor, while distributed algorithms require only local information exchange between neighbouring nodes. Nevertheless, in distributed algorithms, many such local exchanges may be required, depending on the number of iterations needed to arrive at a stable solution. A comparison of the communication energy efficiencies of centralized and distributed algorithms is provided in (Rabbat & Nowak, 2004), where it is concluded that in general, if in a given sensor network and distributed algorithm, the average number of hops to the central processor exceeds the necessary number of iterations, then the distributed algorithm will be more energy-efficient than a typical centralized algorithm.

Finally it is worth noting that the separation between distributed localization algorithms and centralized localization algorithms can sometimes be blurred. Any algorithm for distributed localization can always be applied to centralized problems. Distributed versions of centralized algorithms can also be designed for certain applications. A typical way of designing distributed versions of centralized algorithms involves dividing the entire network into several overlapping regions; implementing centralized localization algorithms in each region; then stitching these local maps for each region together by using common nodes between overlapping regions to form a global map (Capkun, Hamdi, & Hubaux, 2001; Ji & Zha, 2004; Oh-Heum & Ha-Joo, 2008). Such techniques may offer an optimum tradeoff between the advantages and disadvantages of centralized and distributed algorithms discussed above. A particular example of such techniques is multidimensional scaling-based localization, which is discussed further in the next subsection.

In the rest of this section, we give a brief introduction to each major localization technique.

Multidimensional Scaling Algorithms

The *Multidimensional Scaling (MDS)* technique can find its basis in graph theory and was originally used in psychometrics and psychophysics. It is often used as part of exploratory data analysis or information visualization technique that displays the structure of distance-like data as a geometric picture. The typical goal of MDS is to create a configuration of points in one, two, or three dimensions, whose inter-point distances are "close" to the known (and possibly inaccurate) inter-point distances. Depending on the criteria used to define "close", many variants of the basic MDS exist. MDS has been applied in many fields, such as machine learning and computational chemistry. When used for localization, MDS utilizes connectivity or distance information between sensors for location estimation.

Typical procedure of MDS algorithms involves first computing the shortest paths (i.e., the least number of hops) between all pairs of nodes. If distances between all pairs of sensors along the shortest path connecting two nodes are known, the distance between the two nodes along the shortest path can be computed. This information is used to construct a distance matrix for MDS, where the entry (i, j) represents the distance along the shortest path between nodes i and j. If only connectivity information is available, the entry (i, j) then represents the least number of hops between nodes i and j. Then MDS is applied to the distance matrix and an approximate value of the relative coordinates of each node is obtained. Finally, the relative coordinates are transformed to the absolute coordinates by aligning the estimated relative coordinates of anchors with their absolute coordinates. The location estimates obtained using earlier steps can be refined using a least-squares (LS) minimization.

The basic form of MDS is a centralized localization technique and may only be used in a regular network where the distance between two nodes along the shortest path is close to their Euclidean distance. However several variants of the basic MDS algorithm are proposed which allow the implementation of MDS technique in distributed environment and in irregular networks.

Chapter VIII - MDS-Based Localization provides a more detailed discussion on MDS localization techniques and presents several network localization methods based on these techniques. The chapter first introduces the basics of MDS techniques, and then four algorithms based on MDS: *MDS-MAP(C), MDS-MAP(P), MDS-Hybrid* and *RangeQ-MDS*. MDS-MAP(C) is a centralized algorithm. MDS-MAP(P) is a variant of MDS-MAP(C) for implementation in distributed environment. It has better performance than MDS-MAP(C) in irregular networks. MDS-Hybrid considers relative location estimation in an environment without anchors. RangeQ-MDS uses a quantized RSS-based distance estimation technique to achieve more accurate localization than algorithms using binary measurements of connectivity only (i.e., two nodes are either connected or not connected).

Linear Programming Based Localization Techniques

Many distance-based or connectivity-based localization problems can be formulated as a convex optimization problem and solved using linear and semidefinite programming (SDP) techniques (Doherty, Pister, & El Ghaoui, 2001). Semidefinite programs are a generalization of the linear programs and have the following form

$$\begin{array}{ll} \text{Minimize} & \boldsymbol{c}^T \boldsymbol{X} \\ \text{Subject to} & \boldsymbol{F}(\boldsymbol{X}) = \boldsymbol{F}_0 + \boldsymbol{X}_1\boldsymbol{F}_1 + \cdots + \boldsymbol{X}_N\boldsymbol{F}_N \\ & \boldsymbol{AX} < \boldsymbol{B} \\ & \boldsymbol{F}_k = \boldsymbol{F}_k^T \end{array} \tag{12}$$

where $\boldsymbol{X} = [\boldsymbol{X}_1, \boldsymbol{X}_2, \ldots, \boldsymbol{X}_N]^T$ and $\boldsymbol{X}_k = [x_k, y_k]^T$ represents the coordinate vector of node k. The quantities $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{c}$ and $\boldsymbol{F_k}$ are all known. The inequality in (12) is known as a *linear matrix inequality (LMI).*

If only connectivity information is available, a connection between nodes i and j can be represented by a "radial constraint" on the node locations: $\left\|\boldsymbol{X}_i - \boldsymbol{X}_j\right\| \le R$ with R being the transmission range of wireless sensors. This constraint is a convex constraint and can be transformed into an LMI to be used in (12). A solution to the coordinates of the non-anchor nodes satisfying the "radial constraints" can be obtained by leaving the objective function $\boldsymbol{c}^T\boldsymbol{X}$ blank and solving the problem. Obviously there may be many possible coordinates of the non-anchor nodes satisfying the constraints, i.e., the solution may not be unique. If we set the entry of $\boldsymbol{c}$ corresponding to x_k (or y_k) to be 1 (or -1) and all other elements of $\boldsymbol{c}$ to be zero, the problem becomes a constrained maximization (or minimization) problem, which gives respectively the maximum (or minimum) value of x_k (or y_k) satisfying the constraints in (12). A rectangular box bounding the location estimates of the non-anchor node k can be obtained from these lower and upper bound on x_k and y_k. The detailed connectivity-based localization algorithm is reported in (Doherty et al., 2001).

The above SDP formulation of the connectivity-based localization problem can be readily extended to incorporate distance measurements (Doherty et al., 2001). In (Biswas & Ye, 2004) the distance-based localization problem is used in a quadratic form and solved using SDP. In (Liang, Wang, & Ye, 2004) gradient search is used to fine tune the initial estimated locations obtained using SDP and improves the accuracy of localization.

Note that different linear programming techniques have been used in various chapters of this book.

Stochastic Optimization Based Localization Techniques

The stochastic optimization approach provides an alternative formulation and solution of the distance-based localization problem using combinatorial optimization notions and tools. One of the most widely used tools in this approach is the *simulated annealing (SA)* technique (Kannan, Mao, & Vucetic, 2005).

SA is a technique for combinatorial optimization problems. The SA algorithm exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process) and the search for a minimum in a more general system. It is a generalization of the Monte Carlo method. It transforms a poor, unordered solution into a highly optimized, desirable solution. This principle of SA technique with an analogous set of "controlled cooling" operations was used in the combinatorial optimization problems, such as minimizing functions of multiple variables, to obtain a highly optimized, desirable solution (Kirkpatrick, Gelatt, & Vecchi, 1983). We refer the readers to (Kannan et al., 2005; Kannan, Mao, & Vucetic, 2006) for a more detailed description of the design of a SA algorithm for distance-based localization problems.

A properly designed SA has the advantage that it is robust against being trapped into a false local minimum. However SA is also well-known to be very computationally demanding.

The DV-Hop and DV-Distance Localization Algorithms

The *DV(distance vector)-hop* algorithm (Niculescu & Nath, 2001) utilizes the connectivity measurements to estimate locations of non-anchor nodes. The algorithm starts with all anchors broadcasting their locations to other nodes in the network. The messages are propagated hop-by-hop and there is a hop-count in the message. Each node maintains an anchor information table and counts the least number of hops that it is away from an anchor. When an anchor receives a message from another anchor, it estimates the average distance of one hop using the locations of both anchors and the hop-count, and sends it back to the network as a correction factor. When receiving the correction factor, a non-anchor node is able to estimate its distance to anchors and performs trilateration to estimate its location if its distances to at least three anchors are available.

The *DV-distance* algorithm is similar to the DV-hop algorithm except that it includes measured distances into the localization process. The main idea in the DV-distance algorithm is the propagation of measured distance among neighbouring nodes instead of hop count.

Since the proposal of the DV-hop and DV-distance algorithms, many other algorithms based on essentially the same principle were proposed which aims to improve the performance of the basic DV-hop and DV-distance algorithms under various conditions, e.g., in irregular networks or when there are additional information such as node distribution available. We refer interested readers to (Chris Savarese & Rabaey, 2002; Shang, Ruml, Zhang, & Fromherz, 2004) for more detailed discussion.

Statistical Location Estimation Techniques

In the early part of this chapter, we have mentioned in a number of places the use of the ML estimator for localization under various types of measurements. Denote the coordinator vectors of non-anchor nodes by $\boldsymbol{X}$ and the vector of all inter-sensor measurements by $\boldsymbol{Z}$. Denote by $f(\boldsymbol{Z})$ the distribution of $\boldsymbol{Z}$ so that $f\left(\boldsymbol{Z} \mid \boldsymbol{X}\right)$ is the conditional probability of $\boldsymbol{Z}$ when the non-anchor nodes are at $\boldsymbol{X}$. The ML estimator is given by

$$\hat{\boldsymbol{X}} = \arg\max_{\hat{\boldsymbol{X}}} f\left(\boldsymbol{Z} \mid \hat{\boldsymbol{X}}\right) \tag{13}$$

When the inter-sensor measurements can be modelled by the sum of their respective true values and additive Gaussian noises with zero mean and the same variance, the ML estimator is equivalent to an LS estimator. When the variances of additive Gaussian noises are different, the ML estimator is equivalent to a weighted LS estimator. All three estimators, i.e., the ML estimator, the LS estimator and the weighted LS estimator, have been widely used in both centralized and distributed localization algorithms.

Occasionally we may have prior knowledge on the possible locations of non-anchor nodes. In that case, the *maximum a posteriori (MAP) estimator* can be used, which utilizes the prior knowledge on non-anchor nodes' locations to obtain a more accurate estimate. Denote the *a priori* known distribution of the non-anchor nodes by $g(X)$. The MAP estimator is given in the following:

$$\hat{\boldsymbol{X}} = \arg\max_{\hat{\boldsymbol{X}}} f\left(\boldsymbol{Z} \mid \hat{\boldsymbol{X}}\right) g\left(\hat{\boldsymbol{X}}\right) \tag{14}$$

Note that the MAP estimator of $\boldsymbol{X}$ coincides with the ML estimator when the non-anchor nodes have equal probability to be distributed anywhere in the sensor network area, i.e., $g(\boldsymbol{X})$ is a constant function.

The above estimators have often been used to obtain a point estimate of the non-anchors' locations. In some applications, we are interested in knowing in which region a non-anchor node is located. Such knowledge is often useful in asset management for example. Both the ML estimator and the MAP estimator can be altered to generate such location information. Assume that the entire network area is divided into *M* regions and each region is labelled by $L_k, 1 \le k \le M$. Denote by $g(L_k)$ the *a priori* known probability that a non-anchor node is located in L_k. Denote by $f\left(\boldsymbol{Z} \mid L_k\right)$ the conditional probability of $\boldsymbol{Z}$ when the non-anchors node is in L_k. The region in which the non-anchor node is located given the measurements $\boldsymbol{Z}$ can be estimated using the MAP estimator as:

$$L_k = \arg \max_{L_i, 1 \le i \le M} f\left(\boldsymbol{Z} \mid L_i\right) g\left(L_i\right) \tag{15}$$

An ML estimate of the region in which the non-anchor is located can be obtained analogously. ***Chapter IX - Statistical Location Detection*** provides more detailed discussions on the topic and presents a localization algorithm in indoor WLAN environment based on the same principle as that in Equation (15).

A recent statistical approach in distributed sensor network localization is the use of *Bayesian filter-based localization* techniques (Kwok, Fox, & Meila, 2004). Different from other localization techniques whose outputs are deterministic estimates of non-anchors' locations, Bayesian filters probabilistically estimate sensors' locations from noisy measurements. The outputs of Bayesian filters are probability distributions of the estimated locations conditioned on all available sensor data. Such probability distribution is known as *belief* representing uncertainty in estimated locations. Bayesian filter-based localization techniques are often implemented as iterative algorithms which iteratively update and improve such beliefs as localization process proceeds and more accurate knowledge about the neighbouring sensors become available. This process is known as *belief propagation*. In (Ihler et al., 2005), based on the Bayesian filters, the sensor network localization problem is formulated as an inference problem on a graphical model and a variant of *belief propagation (BP*) techniques, the so-called *nonparametric belief propagation (NBP)* algorithm, is applied to obtain an approximate solution to the sensor locations. The NBP idea is implemented as an iterative local message exchange algorithm, in each step of which each sensor node quantifies its "belief" about its location estimate, sends this belief information to its neighbours, receives relevant messages from them, and then iteratively updates its belief using Bayes' formula. The iteration process is terminated only when some convergence criterion is met about the beliefs and location estimates of the sensors in the network. Because of the difficulty both in obtaining an analytical expression of the belief function and in updating the belief function analytically, particle filters (Kwok et al., 2004) are often used to represent beliefs numerically by sets of samples, or particles. The main advantages of the NBP algorithm and the use of particle filters are its easy implementation in a distributed fashion and sufficiency of a small number of iterations to converge. Furthermore it is capable of providing information about location estimation uncertainties and accommodating non-Gaussian measurement errors. These advantages make the approach particularly attractive in non-linear systems with non-Gaussian measurement errors.

RSS-Based Localization Techniques

Chapters IX-XI of this book give a thorough discussion on various aspects involved in the design and implementation of RSS-based localization systems. The number of chapters in this book, the number of research papers in the area and the number of deployed systems on RSS-based localization techniques properly reflects the huge interest in the research community and industry on the techniques. As mentioned previously in this chapter, RSS-based localization techniques can only provide a coarse-grained estimate of sensor locations. However almost every wireless device has the capability of performing

RSS measurements and RSS-based localization techniques meet the exact demand from industry on localization solutions with minimal hardware investment. It is this feature of RSS-based localization techniques that drives the tremendous interest in their research and developments.

As mentioned above, ***Chapter IX*** presents an RSS-based localization system for indoor WLAN environments. The entire network area is divided into several regions and the algorithm identifies the region in which the non-anchor node resides. The localization problem is formulated as a multi-hypothesis testing problem and the authors provide an asymptotic performance guarantee of the system. The authors further investigate the optimal placement of anchor nodes in the system. The optimal placement problem is formulated as a mixed integer *linear programming* problem and a fast algorithm is presented for solving the problem. Finally the proposed techniques are validated using testbed implementations involving MICAz motes manufactured by Crossbow.

Chapter X - Theory and Practice of Signal Strength-Based Localization in Indoor Environments starts with a brief overview of *indoor localization techniques* and then focuses on RSS-based techniques for indoor wireless deployments using 802.11 technology. The authors present an analytical framework that aims to ascertain the attainable accuracy of RSS-based localization techniques. It provides answers to questions like "Is there any theoretical limit to the localization accuracy using techniques based on signal strength?". The approach is based on the analysis of α-*region*s in location space: If the probability that the observed signal strength at the receiver is due to a transmitter located inside a certain region is α, then this certain region is called an α-*region*. The definition of α-region leads to an analytical approach for characterizing uncertainties in RSS-based localization. Several properties of the uncertainties are established, including that uncertainty is proportional to the variance in signal strength. This observation has resulted in several algorithms which aim at improving localization performance by reducing the variance. The authors also summarize issues that may affect the design and deployment of RSS-based localization systems, including deployment ease, management simplicity, adaptability and cost of ownership and maintenance. With this insight, the authors present the "LEASE" architecture for localization that allows easy adaptability of localization models. The chapter concludes with a discussion of some open issues in the area.

Chapter XI - On a Class of Localization Algorithms Using Received Signal Strength surveys and compares several RSS-based localization techniques from two broad categories: *point-based* and *area-based*. In point-based localization, the goal is to return a single point estimate of the non-anchor node's location while in area-based localization the goal is to return the possible locations of the non-anchor node as an area or a volume. The authors find that individual RSS-based localization techniques have similar limited performance in localization error (i.e., the distance between the estimated location and the true location) and reveal the empirical law that using 802.11 technology, with dense sampling and a good algorithm, one can expect a median localization error of about 3 m; with relatively sparse sampling, every 6 m, one can still get a median localization error of 4.5 m. Therefore it can be concluded that there are fundamental limitations in indoor localization performance that cannot be transcended without using qualitatively more complex models of the indoor environment, e.g., models considering every wall, desk or shelf, or by adding extra hardware in the sensor node above that required for communication, e.g., very high frequency clocks to measure the TOA. The authors also briefly describe a sample core localization system called *GRAIL (General purpose Real-time Adaptable Localization)*, which can be integrated seamlessly into any application that utilizes radio positioning via simple Application Program Interfaces (APIs). The system has been used to simultaneously localize multiple devices running 802.11 (WiFi), 802.15.4 (ZigBee) and special customized RollCallTM radios.

Localization Techniques Based on Machine Learning and Information Theory

In the earlier part of this section, we have mentioned some widely used WSN localization approaches and introduced the relevant chapters of this book. There exist other less conventional approaches in the literature as well, which complement the above widely used approaches, especially by providing alternative localization solutions suitable for various specific application domains and settings. ***Chapters XII and XIII of this book*** present two such approaches.

Chapter XII - Machine Learning Based Localization presents a machine learning approach to localization. *Machine learning* is an information science field, studying algorithms that improve automatically through experience. It is concerned with the design and development of algorithms and techniques that allow computers or computing systems to "learn" rules and patterns out of massive data sets automatically, using certain computational and statistical tools of regression, detection, classification, pattern recognition, and data cleaning as well as convex optimization techniques. Two key concepts used in machine learning are *kernels,* which can be considered as systems that describe similarities between objects, and *support vector machines*, supervised learning methods used for regression and classification. Machine learning has been used in a number of areas including syntactic pattern recognition, search engines, medical diagnosis, bioinformatics, object recognition in computer vision, game playing and robot locomotion.

Chapter XII discusses the application of machine learning methods to WSN localization based on formulation of the localization problem (i) as a classification problem and (ii) as a regression problem. Both problem definitions are RSS-based, and RSS measurements from anchors at various sample points distributed inside the sensor network area are used as training data for the support vector machines. In the classification problem based approach, the sensor network area is partitioned into (overlapping or non-overlapping) geographical regions, and a set of classes are defined to represent membership to these regions. Using RSS measurements received from anchors at the non-anchor node and rules established from the training data, the classes attached to the non-anchor node location estimate, which represent the regions where the non-anchor node is estimated to lie, are found. If the found classes are more than one then the localization algorithm returns the centroid of the intersection of the regions corresponding to these classes as the location estimate of the non-anchor node. If only a single class is found, then the location estimate is determined as the centroid of the corresponding region. The regression problem based approach exploits the correlation between the RSS measurements from anchors at the non-anchor node and the RSS measurements from anchors at sampling points. The non-anchor node is estimated to be at the centroid of the sampling points whose RSS measurements have the highest correlation with those of the non-anchor node.

Chapter XIII - Robust Localization Using Identifying Codes presents a different paradigm for robust WSN localization based on *identifying codes*, a concept borrowed from the *information theory* literature with links to covering and superimposed codes. The approach involves choosing a set of discrete sampling points and transmitters in a given region such that each discrete sampling point is covered by a distinct set of transmitters. The location of a non-anchor node is estimated to be at the location of the discrete sampling point, which is covered by the same set of transmitters as the non-anchor node. The major challenges involved in using this approach are choosing the set of transmitters and finding good and robust identifying codes. The chapter presents the basics of robust identifying codes, use of these codes in WSN localization, design and analysis of an identifying code based algorithm, and implementation of the proposed algorithm on a test bed at Boston University involving a 33mx76m indoor region (fourth floor of the Photonics building) and four transmitters (anchors). The identifying codes-based approach has the simplifying advantage that a non-anchor node only needs to know the set of transmitters it can detect in order to infer its location. This feature makes the approach robust to spurious connections or sensor failures and suitable for implementation in harsh environments, at the expense of reduced localization accuracy.

Evaluation of Localization Algorithms

It is often the case that a number of solutions exist for solving the same localization problem. A question naturally arises is how to evaluate and compare the performance of various localization solutions.

Evaluating the performance of localization algorithms is important for both researchers and practitioners, either when validating a new algorithm against the previous state of the art, or when choosing existing algorithms which best fit the requirements of a given WSN application. However, there is currently no agreement in the research and engineering community on the criteria and performance metrics that should be used for the evaluation and comparison of localization algorithms. Neither there exists a standard methodology which takes an algorithm through modelling, simulation and emulation stages, and into real deployment. Part of the problem lies in the large number of factors that may affect the performance of a localization algorithm, including but not limited to: the type of measurements being used and measurement errors, the distributions of anchor and non-anchor nodes, the density of network nodes which is usually measured by the average node degree, the geometric shape of the network area, whether or not there is any prior knowledge of the network, the wireless environment in which the localization technique is being deployed, the presence of NLOS conditions. Quite often a localization algorithm performing well in one scenario, e.g., in regular networks, does not deliver a good performance in another scenario, e.g., in irregular networks. A localization algorithm delivering an excellent performance in simulation environment may also not perform satisfactorily in real deployment. All these phenomena highlight the importance of building a scientific methodology for the evaluation of localization algorithms.

Chapter XIV - Evaluation of Localization Algorithms addresses the above challenges by introducing a methodological approach to the evaluation of localization algorithms. The chapter contains a discussion of evaluation criteria and performance metrics, which is followed by statistical/empirical simulation models and parameters that affect the performance of the algorithms and hence their assessment. Two contrasting localization studies are presented and compared with reference to the evaluation criteria discussed throughout the chapter. The chapter concludes with a localization algorithm development cycle overview: from simulation to real deployment. The authors argue that algorithms should be simulated, emulated (on test beds or with empirical data sets) and subsequently implemented in hardware, in a realistic WSN deployment environment, as a complete test of their performance. It is hypothesised that establishing a common development and evaluation cycle for localization algorithms among researchers will lead to more realistic results and viable comparisons.

Chapter XV - Accuracy Bounds for Wireless Localization Methods looks at evaluation methods for localization systems from a different perspective and takes an analytical approach to performance evaluation. The authors argue that evaluation methods for localization systems serve two purposes. First, they allow a network designer to determine the achievable performance of a localization system from a given network configuration and available measurements prior to the deployment of the system. Second, these tools can be used to evaluate the performance of an existing localization system to see if the potential location accuracy is being achieved or if further improvements are possible.

The authors present several methods for calculating performance bounds for node localization in WSNs. The authors point out that the widely used *Cramer-Rao bound* relies on several assumptions: (i) The environment is an LOS radio propagation environment; (ii) The location estimator is unbiased; (iii) No prior information on node's location is available. Obviously, not all these assumptions are valid in real applications. Indeed, most distance-based, AOA-based and TDOA-based location estimators are biased which makes the second assumption invalid. The authors advocate the use of the *Weinstein-Weiss* and *extended Ziv-Zakai lower bounds* to address the above problems. These bounds remain valid under NLOS conditions and can also use all available information for bound calculations. It is demonstrated that these bounds are tight to actual estimator performance and may be used to determine the available accuracy of location estimation from survey data collected in the network area.

EXPERIMENTAL STUDIES AND APPLICATIONS OF WSN LOCALIZATION

The earlier sections of this chapter and correspondingly ***Chapters II-XV*** have largely focused on measurement techniques, theoretical backgrounds, and algorithm design for WSN localization. Nevertheless, there exist various other issues to consider in order to guarantee that an actual real-time WSN localization system works properly and performs well. The amount and the type of these issues in general differ for different application domains and tasks. ***Chapters XVI-XVIII*** of this book present three different WSN localization application studies exemplifying such further issues.

Chapter XVI - Experiences in Data Processing and Bayesian Filtering Applied to Localization discusses algorithms and solutions for signal processing and filtering for localization and *location tracking* applications. Here, the term *location tracking* is used for estimation of the trajectory of an object based on sequential measurements. As opposed to localization in static networks in which sensor locations do not change with time, location tracking techniques are developed to meet the demand (in a large number of application domains) for knowledge of the time-varying location of a moving object, which can be a vehicle, a robot, a mobile sensor unit, a human operator, etc.

Chapter XVI explains some practical issues for engineers interested in implementing location tracking solutions and their experiences gained from implementation and deployment of several such systems. In particular, the chapter introduces the data processing solutions found appropriate for commonly used sensor types, and discusses the use of Bayesian filtering for solving position tracking problem. The use of particle filters is recommended as a flexible solution appropriate for tracking in non-linear systems with non-Gaussian measurement errors. Finally the authors also give a detailed discussion on the design of some of the indoor and outdoor position tracking systems they have implemented, highlighting major design decisions and experiences gained from test deployments. Note that, the basics of Bayesian filters and particle filters and their use in location estimation in static networks have been introduced in the subsection *Stochastic Optimization Based Localization Techniques* above, and Chapter ***XVI*** features a more detailed introduction to Bayesian and particle filters as well as *Kalman filters*, focusing more on their application in location tracking.

Chapter XVII - A Wireless Mesh Network Platform for Vehicle Positioning and Location Tracking presents an experimental study on the integration of Wi-Fi based wireless mesh networks and Bluetooth technologies for detecting and tracking travelling cars and measuring their speeds. The authors propose a wireless platform for these purposes and deploy a small-scale network of four access points to validate the proposal. The platform employs RSS measurements and is shown to be able to track cars travelling at speeds of 0 to 70 km/h. The platform is found to be cost-effective and is envisaged to be a significant contribution to intelligent transportation systems for road traffic monitoring.

The availability of physical locations enables a myriad of applications, as exemplified extensively throughout this book. A particular application domain that benefits from the availability of location information is sensor network routing. Specifically the prospects brought by recent developments in WSN localization have sparked interest on a category of routing algorithms, known as geographical routing (D. Chen & Varshney, 2007). Geographic routing utilizes the location information of sensors to make routing decisions. It does not require the establishment or maintenance of routes from sources to destinations. Sensor nodes do not need to store routing tables. These features make geographic routing an attractive option for routing in large scale sensor networks.

Chapter XVIII - Beyond Localization: Communicating Using Virtual Coordinator discusses an interesting aspect of the *geographic routing* problem and question: for the purpose of improved *geographic routing*, whether it would be more efficient to label sensors by information other than their physical locations. Specifically the chapter advocates labelling sensors by their *virtual coordinates*, which are not related to their physical coordinates, and let the geographic routing algorithm use these virtual coordinates for routing. The concept of virtual coordinates is based on the notion of *greedy embedding*. A greedy embedding of a geometric graph (*G,p*) is the geometric graph (*G,p'*) that has the same underlying graph *G*, i.e., the same edges interconnecting the same set of vertices, but having the vertices placed at different coordinates (*p'*) such that greedy routing always functions when sending a message between arbitrarily chosen nodes. *Greedy routing* refers to a simple geographic routing scheme in which a node always forwards a packet to the neighbour that has the shortest distance to the destination. The use of virtual coordinates greatly facilitates geographic routing and removes void areas which have been a major hurdle in the implementation of geographic routing algorithms. The authors then present an algorithm that assigns virtual coordinates to sensors and the algorithm has been validated by both simulations and experiment.

Chapter XVIII reveals some insight that may be of interest for some applications currently using the physical location information of sensors. Physical locations of sensors can, to a large extent, be considered as a means to label sensors. It is possibly the most intuitive and useful way of labelling the sensors so that people know where the sensors are located and where the measured information by sensors comes from. Location information cannot be replaced by other information in many applications. However, in some applications which do not necessarily need to know the physical location of sensors but rely on some sort of sensor labels for identification of sensors or supporting the correct functioning of the application, there may be more efficient ways to label sensors that facilitate the application. It is in this sense that ***Chapter XVIII*** motivates us to think beyond the horizon of localization.

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