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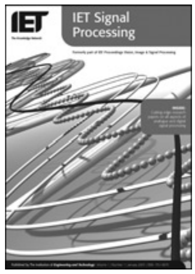

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Multidimensional scaling-based passive emitter localisation from range-difference measurements

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Abstract: The problem of passive emitter localisation from range difference measurements has received considerable attentions. Unlike most of ordinary hyperbolic location methods focusing on minimising a loss function with respect to the range-difference vector, a simple estimator is proposed by exploiting a new multidimensional scaling analysis. On the basis of the optimisation of a loss function related to a scalar product matrix, it is robust for large measurement noise. The bias and variance of the proposed estimator is also derived. It is shown that the new method attains the Cramér–Rao lower bound for Gaussian range-difference noise at moderate noise level. Moreover, the threshold effect occurs latter than the conventional methods as the measurement noise increases. Additionally, they have comparable computational complexity.

1 Introduction

Source localisation using measurements from an array of passive sensors has been an important problem in passive radar, surveillance and navigation system, global positioning system [1], mobile communications [2–5], video camera steering system [6, 7] and wireless sensor networks [8, 9]. One commonly used location parameter is the time difference of arrival (TDOA), that is proportional to the range difference between the emitter and a sensor pair. For three-dimensional (3D) positioning, each noise-free range difference defines a hyperboloid in which the emitter must lie. By using $M \geq 4$ sensors, the emitter location can be uniquely determined by the intersection of hyperboloids.

In practice, the range difference measurements are noisy which implies multiple intersection points and thus they are usually converted into a set of hyperboloidal equations, from which the emitter position is estimated with the knowledge of the signal propagation speed and sensor array geometry. Under such circumstances maximum likelihood (ML) estimation is a powerful method of estimating the source location. On the basis of the assumption of Gaussian distributed measurement noise, the ML location estimate is the globally optimal solution of the quadratic optimisation problem [10]. The ML estimator is an asymptotically unbiased, and it normally provides optimum performance and approaches the Cramér–Rao lower bound (CRLB) asymptotically (in the case of a large number of sensors or small measurement errors) [11]. A straightforward approach of the ML estimator for determining the source location is to solve the nonlinear optimisation problem directly, but computationally intensive. Apart from the direct methodology, another common technique that avoids solving the nonlinear problem is to linearise it, and then, the solution is found iteratively [10, 12]. However, this approach requires an initial estimate and cannot guarantee convergence to the correct solution unless the initial guess is close to it. On the other hand, an alternative approach which allows real-time implementation as well as ensures global convergence is to reorganise the nonlinear equations into a set of linear equations by introducing an extra variable that is a function of the emitter position. The linear equations could then be solved straightforwardly by using least-squares (LS) such as the spherical-interpolation (SI) method [13]. Furthermore, the relation between the extra parameter and the emitter location could also be exploited to improve the accuracy of the emitter location estimate, by using two-step weighted LS (WLS) method [14], or by employing the technique of Lagrange multipliers, which is refered to linear-correction LS (LCLS) estimator [7]. And the two-step WLS method has comparable performance with the LCLS when the covariance matrix of noise distribution is available or well estimated. Stoica and Li [15] clarify and streamline these efficient methods and put them into two categories basically as the unconstrained LS estimate [13] (also see the references therein) and the constrained LS method [7, 14] in an unified view. Instead of focusing on minimising a loss function with respect to a vector, multidimensional scaling (MDS) has been utilised for mobile location [3–5] from range measurements, and it is deduced from the optimisation of a loss function related to a scalar product matrix. Inspired by these works, in this paper, we extend the MDS technique from range-based (or TOA-based) systems to range-difference-based (or TDOA-based) system, and then derive a MDS-based emitter location approach, which also provides a computationally efficient and globally convergence solution.

MDS [9] is an attractive technique for analysing experimental data in psychology, geography and molecular biology, and it has been developed for mobile location from range measurements [3–5]. These methods provide a computationally simple, non-iterative solution and what is more, it is robust for large measurement noise than the conventional algorithms, which implies that the MDS method has a larger operation range. This is due to the fact that the dimension knowledge and eigen-structure information of the scalar product matrix is exploited in the MDS framework. With the MDS technique, the mobile location is usually estimated through two steps [3–5]. First, the scalar product matrix is constructed to represent the similarity by pairwise distances. Second, a set of linear equations related to the signal subspace or noise subspace of the scalar product matrix is solved to estimate the unknown mobile location in the LS sense. By the similar principle, it is expected that the MDS technique can be extended to the passive emitter location from the range difference measurements. Unfortunately, it does not work directly because the range between the emitter and receiver can not be obtained. In this paper, our major findings include: (i) by adding an extra pure imaginary dimension related to the range-differences to the position space, the MDS analysis is extended to passive emitter localisation successfully; (ii) we propose a new MDS analysis, in which a new formula about the scalar product matrix on the basis of the orthogonal property between the signal and noise subspaces is derived. Accordingly, a weighting matrix is designed to whiten the residuals about these linear equations. The accuracy of this estimator is shown to reach the CRLB at moderate noise level before the threshold effect occurs, and it has a larger operation range.

The rest of this paper is organised as follows. In Section 2, the measurement model and its CRLB are presented. In Section 3, we first extend the MDS framework to the hyperbolic localisation, and a new formula about the scalar product matrix is then derived with the new MDS analysis. The MDS-based estimator is proposed and further discussions are given in Section 4. Simulations are included to evaluate the estimator performance by comparison with SI method, one and two-step WLS methods as well as the CRLB in Section 5. Finally, some conclusions are derived in Section 6.

2 Measurement model and CRLB

We consider a 3D scenario where an array of M sensors is used to determine the position $\boldsymbol{x}_0 = [x, y, z]^{\mathrm{T}}$ of an emitter using range differences, where T denotes the transpose operator. The sensors position $\boldsymbol{x}_m = [x_m, y_m, z_m]^{\mathrm{T}}$, $m = 1, 2, \ldots, M$ are assumed known. The location problem requires at least $M = 4$ receivers to produce three range-differences. This paper focuses on the overdetermined scenario where the number of receivers is larger than 4. We assume that all the receivers are neither lying on a plane nor on a straight line, which guarantees that the location solution is unique. Without loss of generality, the first receiver is chosen as the reference receiver; therefore the range-differences from the receiver pair m and 1 is

$$d_{m1} = d_m - d_1 \quad m = 2, 3, \ldots, M \tag{1}$$

where $d_m = \sqrt{(\boldsymbol{x}_m - \boldsymbol{x}_0)^{\mathrm{T}}(\boldsymbol{x}_m - \boldsymbol{x}_0)}$, $m = 1, 2, \ldots, M$. In particular, we have $d_{11} = 0$ and $d_{01} = -d_1$.

In practice, the range difference measurement r_{m1} is corrupted by additive noise as

$$r_{m1} = d_{m1} + q_{m1} \quad m = 2, 3, \ldots, M \tag{2}$$

where q_{m1} is the measurement noise. The range difference is proportional to the TDOA in a constant-velocity propagation medium. The noisy measurement model is related to many unbiased TDOA estimators [16] and does not impose any distribution on the measurement errors vector $\boldsymbol{q} = [q_{21}, q_{31}, \ldots, q_{M1}]^{\mathrm{T}}$. Once the covariance matrix $\boldsymbol{\Sigma} = \mathrm{E}[\boldsymbol{q}\boldsymbol{q}^{\mathrm{T}}]$ has been obtained [7, 14, 16], where $E[\cdot]$ is the expectation operator, we can use them for the passive emitter localisation.

The CRLB is a fundamental lower bound on the variance of any unbiased estimator [11]. Therefore the CRLB serves as a benchmark for the performance of actual estimators. It is given by [11] for the emitter location from the Gaussian measurement vector $\boldsymbol{r} = [r_{21}, r_{31}, \ldots, r_{M1}]^{\mathrm{T}}$ of (2)

$$\mathrm{CRLB}(\boldsymbol{x}_0) = \left\{ \left(\frac{\partial \boldsymbol{d}}{\partial \boldsymbol{x}_0} \right)^{\mathrm{T}} \boldsymbol{\Sigma}^{-1} \left(\frac{\partial \boldsymbol{d}}{\partial \boldsymbol{x}_0} \right) \right\}^{-1} \tag{3}$$

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where $\boldsymbol{d} = [d_{21}, d_{31}, \ldots, d_{M1}]^{\mathrm{T}}$ and

$$\frac{\partial \boldsymbol{d}}{\partial \boldsymbol{x}_0} = \begin{bmatrix} \dfrac{(\boldsymbol{x}_2 - \boldsymbol{x}_0)^{\mathrm{T}}}{d_2} - \dfrac{(\boldsymbol{x}_1 - \boldsymbol{x}_0)^{\mathrm{T}}}{d_1} \\ \dfrac{(\boldsymbol{x}_3 - \boldsymbol{x}_0)^{\mathrm{T}}}{d_3} - \dfrac{(\boldsymbol{x}_1 - \boldsymbol{x}_0)^{\mathrm{T}}}{d_1} \\ \vdots \\ \dfrac{(\boldsymbol{x}_M - \boldsymbol{x}_0)^{\mathrm{T}}}{d_M} - \dfrac{(\boldsymbol{x}_1 - \boldsymbol{x}_0)^{\mathrm{T}}}{d_1} \end{bmatrix}_{(M-1)\times 3} \tag{4}$$

3 MDS analysis for range-difference localisation

3.1 MDS framework for hyperbolic Localisation

From the MDS procedure for source localisation, it is required to get the ranges between the source and the receivers. Since the distance between the receiver and the emitter cannot be obtained from range-differences, the MDS does not work directly for the passive emitter localisation. By introducing an extra pure imaginary dimension about the range-differences into the 3D space, the MDS framework can be extended to the passive emitter localisation in this special 4D complex space as follows.

In this special complex space, the coordinates related the mth receiver which can be expressed as $\boldsymbol{z}_m = [x_m, y_m, z_m, \mathrm{i}d_{m1}]^{\mathrm{T}}$, $m = 1, 2, \ldots, M$, where $\mathrm{i}^2 = -1$. The unknown coordinates become $\boldsymbol{z}_0 = [x, y, z, \mathrm{i}d_{01}]^{\mathrm{T}}$. We first construct a matrix of squared distances from these coordinates with dimension $(M+1) \times (M+1)$, which is denoted by $\boldsymbol{D}$ with (m, n) entry as

$$\begin{aligned} [\boldsymbol{D}]_{mn} &= [(\boldsymbol{z}_m - \boldsymbol{z}_n)]^{\mathrm{T}}[(\boldsymbol{z}_m - \boldsymbol{z}_n)] \\ &= (\boldsymbol{x}_m - \boldsymbol{x}_n)^{\mathrm{T}}(\boldsymbol{x}_m - \boldsymbol{x}_n) - (d_{m1} - d_{n1})^2, \\ m, n &= 0, 1, \ldots, M \end{aligned} \tag{5}$$

Note that the entries in the first column and the first row of $\boldsymbol{D}$ are all zeros because of $d_{01} = -d_1 = -\sqrt{(\boldsymbol{x}_1 - \boldsymbol{x}_0)^{\mathrm{T}}(\boldsymbol{x}_1 - \boldsymbol{x}_0)}$. Hence, this squared distance matrix is completely determined with the knowledge of known sensor array geometry and the range differences.

Defined a $(M+1) \times 4$ matrix $\boldsymbol{Z}_{\mathrm{c}}$ of the form

$$\boldsymbol{Z}_{\mathrm{c}} = [\boldsymbol{z}_0 - \boldsymbol{z}_{\mathrm{c}}, \boldsymbol{z}_1 - \boldsymbol{z}_{\mathrm{c}}, \ldots, \boldsymbol{z}_M - \boldsymbol{z}_{\mathrm{c}}]^{\mathrm{T}} \tag{6}$$

which is the centred coordinates matrix and parameterised by $\boldsymbol{z}_0$. Here $\boldsymbol{z}_{\mathrm{c}}$ is the centroid of all points, that is $\boldsymbol{z}_{\mathrm{c}} = 1/(M+1)\sum_{m=0}^{M} \boldsymbol{z}_m$. Following [3–5, 9], we define the scalar product matrix, namely, $\boldsymbol{B} = \boldsymbol{Z}_{\mathrm{c}}\boldsymbol{Z}_{\mathrm{c}}^{\mathrm{T}}$, which is rank-4 symmetric and positive semi-definite matrix with dimension $(M+1) \times (M+1)$. It can be obtained [3, 9]

$$\boldsymbol{B} = -\frac{1}{2}\boldsymbol{J}_{M+1}\boldsymbol{D}\boldsymbol{J}_{M+1} \tag{7}$$

where

$$\boldsymbol{J}_{M+1} = \boldsymbol{I}_{M+1} - \frac{1}{M+1}\mathbf{1}_{M+1}\mathbf{1}_{M+1}^{\mathrm{T}} \tag{8}$$

is the centring matrix, with $\boldsymbol{I}_{M+1}$ and $\mathbf{1}_{M+1}$ denoting the $(M+1) \times (M+1)$ identity matrix and $(M+1) \times 1$ column vector of all ones, respectively.

Then the passive emitter location problem related to the unknown parameter vector $\boldsymbol{z}_0$ including emitter position $\boldsymbol{x}_0$ and distance d_1 can be stated as follows

$$\begin{aligned} \hat{\boldsymbol{z}}_0 &= [\hat{x}, \hat{y}, \hat{z}, -\mathrm{i}\hat{d}_1]^{\mathrm{T}} \\ &= \arg\min_{\boldsymbol{z}_0} \|\hat{\boldsymbol{B}} - \boldsymbol{Z}_{\mathrm{c}}\boldsymbol{Z}_{\mathrm{c}}^{\mathrm{T}}\|_{\mathrm{F}}^2 \end{aligned} \tag{9}$$

where $\|\cdot\|_{\mathrm{F}}$ represents the Frobenius norm. $\hat{\boldsymbol{B}}$ is the noisy matrix of $\boldsymbol{B}$ and it can be obtained from (7) and the noisy squared matrix $\hat{\boldsymbol{D}}$ in the presence of range-difference measurement noises. This optimisation problem (9) has been studied in the classical MDS framework for several decades.

3.2 Classical MDS analysis

The optimisation problem (9) can be solved by spectral decomposition in the MDS framework. Decomposing the symmetric $\boldsymbol{B}$ by eigenvalue factorisation yields $\boldsymbol{B} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{\mathrm{T}}$, where $\boldsymbol{\Lambda} = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_{M+1}\}$ is the diagonal matrix of eigenvalues of $\boldsymbol{B}$ with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{M+1} \geq 0$, and $\boldsymbol{U} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_{M+1}]$ is an orthogonal matrix whose columns are the corresponding eigenvectors. Since $\mathrm{rank}(\boldsymbol{B}) = 4$, we have $\lambda_5 = \lambda_6 = \cdots = \lambda_{M+1} = 0$, and as a result, we obtain

$$\boldsymbol{B} = \boldsymbol{U}_{\mathrm{s}}\boldsymbol{\Lambda}_{\mathrm{s}}\boldsymbol{U}_{\mathrm{s}}^{\mathrm{T}} \tag{10}$$

and

$$\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{U}_{\mathrm{n}} = \boldsymbol{O}_{(M-3)\times(M-3)} \tag{11}$$

where $\boldsymbol{\Lambda}_{\mathrm{s}} = \mathrm{diag}\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$, and $\boldsymbol{U}_{\mathrm{s}} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4]$ and $\boldsymbol{U}_{\mathrm{n}} = [\boldsymbol{u}_5, \boldsymbol{u}_6, \ldots, \boldsymbol{u}_{M+1}]$ denote the signal and noise subspaces of the scalar product matrix $\boldsymbol{B}$, respectively. Here $\boldsymbol{O}_{(M-3)\times(M-3)}$ represents an $(M-3) \times (M-3)$ matrix consisting entirely of zeros. From the definition of $\boldsymbol{B}$, (11) implies that

$$\boldsymbol{Z}_{\mathrm{c}}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}} = \boldsymbol{O}_{4\times(M-3)} \tag{12}$$

Substituting (6) into (12), reorganising them and eliminating the imaginary unit i, we obtain

$$\boldsymbol{P}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}} = \boldsymbol{z}\boldsymbol{m}_{M+1}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}} \tag{13}$$

where $\boldsymbol{z} = [x, y, z, -d_1]^{\mathrm{T}}$, $\boldsymbol{m} = [1/(M+1) - 1, 1/(M+1), \ldots, 1/(M+1)]^{\mathrm{T}}$ with $(M+1) \times 1$ dimension, and

$$\boldsymbol{P} = \begin{bmatrix} -\dfrac{1}{M+1}\sum_{m=1}^{M}\boldsymbol{x}_m^{\mathrm{T}} & -\dfrac{1}{M+1}\sum_{m=1}^{M}d_{m1} \\ \boldsymbol{x}_1^{\mathrm{T}} - \dfrac{1}{M+1}\sum_{m=1}^{M}\boldsymbol{x}_m^{\mathrm{T}} & d_{11} - \dfrac{1}{M+1}\sum_{m=1}^{M}d_{m1} \\ \vdots & \vdots \\ \boldsymbol{x}_M^{\mathrm{T}} - \dfrac{1}{M+1}\sum_{m=1}^{M}\boldsymbol{x}_m^{\mathrm{T}} & d_{M1} - \dfrac{1}{M+1}\sum_{m=1}^{M}d_{m1} \end{bmatrix}_{(M+1)\times 4} \tag{14}$$

which is an $(M+1) \times 4$ matrix constructed from the sensor positions and the range differences.

Equation (13) is the subspace equation with respect to the unknown vector $\boldsymbol{z} = [x, y, z, -d_1]^{\mathrm{T}}$. It can be regarded as the extension of subspace approach for mobile location [5] in the MDS framework. However, the MDS-based subspace approach does not achieve the CRLB, even though the measurement noise is small. This has been evidenced in [3–5] for mobile location, because the residuals of (13) are not independently nor identically distributed any more in the presence of measurement noise. As a result, the solution in LS sense is not optimum from Gauss–Markov theorem [17]. Moreover, it is generally hard to whiten the residuals analytically because of the eigenvalue factorisation of the scalar product matrix.

3.3 New MDS analysis

To overcome the problem of classical MDS analysis, we derive a new set of linear equations, in which the residuals can be whiten. With the orthogonality between the signal and noise subspaces of the scalar product matrix, we have the following result.

Proposition: The relationship between the signal subspace and the coordinates of sensors and emitter can be expressed as

$$\boldsymbol{U}_{\mathrm{s}}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{m}^{\mathrm{T}} \\ \boldsymbol{P}^{\mathrm{T}}\end{bmatrix}^{\dagger}\begin{bmatrix}1 \\ \boldsymbol{z}\end{bmatrix} = \mathbf{0}_4 \tag{15}$$

where $\mathbf{0}_4$ denotes a 4×1 column vector of zero entries, and $(\cdot)^{\dagger}$ represents the Moore–Penrose inverse [17].

Proof: We note that (13) can be rewritten as

$$\begin{bmatrix}\boldsymbol{m}^{\mathrm{T}} \\ \boldsymbol{P}^{\mathrm{T}}\end{bmatrix}\boldsymbol{U}_{\mathrm{n}} = \begin{bmatrix}1 \\ \boldsymbol{z}\end{bmatrix}\boldsymbol{m}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}} \tag{16}$$

Multiplying both sides of (16) on the right by the vector $\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{m}$, and then dividing both sides by the non-zero factor $\boldsymbol{m}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}}\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{m}$ produce

$$\begin{bmatrix}\boldsymbol{m}^{\mathrm{T}} \\ \boldsymbol{P}^{\mathrm{T}}\end{bmatrix}\frac{\boldsymbol{U}_{\mathrm{n}}\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{m}}{\boldsymbol{m}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}}\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{m}} = \begin{bmatrix}1 \\ \boldsymbol{z}\end{bmatrix} \tag{17}$$

Since $\begin{bmatrix}\boldsymbol{m}^{\mathrm{T}} \\ \boldsymbol{P}^{\mathrm{T}}\end{bmatrix} \in \mathbb{R}^{5\times(M+1)}$, there are different noise subspace bases that satisfy (17) from the eigendecomposition theory. However, the matrix $\boldsymbol{U}_{\mathrm{n}}\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{m}/\boldsymbol{m}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}}\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{m}$ is unique in minimum norm sense, and is expressed as

$$\frac{\boldsymbol{U}_{\mathrm{n}}\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{m}}{\boldsymbol{m}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}}\boldsymbol{U}_{\mathrm{n}}^{\mathrm{T}}\boldsymbol{m}} = \begin{bmatrix}\boldsymbol{m}^{\mathrm{T}} \\ \boldsymbol{P}^{\mathrm{T}}\end{bmatrix}^{\dagger}\begin{bmatrix}1 \\ \boldsymbol{z}\end{bmatrix} \tag{18}$$

With the orthogonal property between signal subspace $\boldsymbol{U}_{\mathrm{s}}$ and noise subspace $\boldsymbol{U}_{\mathrm{n}}$, that is, $\boldsymbol{U}_{\mathrm{s}}^{\mathrm{T}}\boldsymbol{U}_{\mathrm{n}} = \boldsymbol{O}_{4\times(M-3)}$, we can obtain (15). □

Multiplying both sides of (15) on the left by the matrix $\boldsymbol{U}_{\mathrm{s}}\boldsymbol{\Lambda}_{\mathrm{s}}$, and applying the spectral decomposition in (10), (15) becomes

$$\boldsymbol{B}\begin{bmatrix}\boldsymbol{m}^{\mathrm{T}} \\ \boldsymbol{P}^{\mathrm{T}}\end{bmatrix}^{\dagger}\begin{bmatrix}1 \\ \boldsymbol{z}\end{bmatrix} = \mathbf{0}_{M+1} \tag{19}$$

which is a linear equation with respect to the unknown vector $\boldsymbol{z}$. It is noteworthy that in the presence of the measurement noise, we can obviate the analytical difficulty associated with the evaluation of the statistical prosperities of the residuals $\{\epsilon_m\}$ of (13), and determine the ones of (19) directly. Consequently, as expected, a better estimation performance may be obtained from this new set of linear equations by whitening the residuals $\{\epsilon_m\}$ about (19).

4 Proposed estimator based on MDS

4.1 Proposed estimator

For convenience, we define two matrices $\boldsymbol{R}$ and $\boldsymbol{A}$

$$\boldsymbol{R} = \begin{bmatrix}\boldsymbol{m}^{\mathrm{T}} \\ \boldsymbol{P}^{\mathrm{T}}\end{bmatrix}, \quad \boldsymbol{A} = \begin{bmatrix}\boldsymbol{m}^{\mathrm{T}} \\ \boldsymbol{P}^{\mathrm{T}}\end{bmatrix}^{\dagger} \tag{20}$$

The relationship between $\boldsymbol{R}$ and $\boldsymbol{A}$ is given as [17]

$$\boldsymbol{A} = \boldsymbol{R}^{\dagger} = \boldsymbol{R}^{\mathrm{T}}(\boldsymbol{R}\boldsymbol{R}^{\mathrm{T}})^{-1}, \quad \boldsymbol{R}\boldsymbol{A} = \boldsymbol{I}_5 \tag{21}$$

Replacing the true range difference d_{m1} by their noisy value r_{m1} yields the noisy matrices $\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{A}}$, and as a result, the $(M+1)\times 1$ dimensional residuals vector $\boldsymbol{\epsilon}$ of (19) is

$$\boldsymbol{\epsilon} = \hat{\boldsymbol{B}}\hat{\boldsymbol{A}}\begin{bmatrix}1 \\ \boldsymbol{z}\end{bmatrix} \tag{22}$$

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From (19) and (22), we have the approximate residuals vector with respect to the noise vector $\boldsymbol{q}$ for small measurement errors (Appendix)

$$\boldsymbol{\epsilon} = \boldsymbol{G}\boldsymbol{q} \tag{23}$$

where $\boldsymbol{G}$ is a $(M+1) \times (M-1)$ matrix defined in (40).

Reorganising (22), we have the vector equation with respect to the unknown vector $\boldsymbol{z}$ as

$$\boldsymbol{\epsilon} = \hat{\boldsymbol{B}}\hat{\boldsymbol{A}}_2\boldsymbol{z} + \hat{\boldsymbol{B}}\hat{\boldsymbol{A}}_1 \tag{24}$$

where $\hat{\boldsymbol{A}}_1$ is the first column of noisy matrix $\hat{\boldsymbol{A}}$, and other columns construct the matrix $\hat{\boldsymbol{A}}_2$ from the partition $\hat{\boldsymbol{A}} = [\hat{\boldsymbol{A}}_1 \vdots \hat{\boldsymbol{A}}_2]$.

A weighted least squares solution to the approximation matrix (24) is given to eliminate the correlation between the noisy scalar product matrix and the residuals vector $\boldsymbol{\epsilon}$, by [11, 14]

$$\hat{\boldsymbol{z}} = -\left(\hat{\boldsymbol{A}}_2^{\mathrm{T}}\hat{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{W}\hat{\boldsymbol{B}}\hat{\boldsymbol{A}}_2\right)^{-1}\hat{\boldsymbol{A}}_2^{\mathrm{T}}\hat{\boldsymbol{B}}^{\mathrm{T}}\boldsymbol{W}\hat{\boldsymbol{B}}\hat{\boldsymbol{A}}_1 \tag{25}$$

where $\boldsymbol{W}$ is the weighting matrix to whiten the residuals vector (23), and is chosen as [11]

$$\boldsymbol{W} = E\left[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^{\mathrm{T}}\right]^{-1} \tag{26}$$

The residuals vector in (23) is a Gaussian random vector with covariance matrix from the statistical propriety of $\boldsymbol{\Sigma}$

$$E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^{\mathrm{T}}] = \boldsymbol{G}E[\boldsymbol{q}\boldsymbol{q}^{\mathrm{T}}]\boldsymbol{G}^{\mathrm{T}} = \boldsymbol{G}\Sigma\boldsymbol{G}^{\mathrm{T}} \tag{27}$$

Notice that matrix $\boldsymbol{G}\Sigma\boldsymbol{G}^{\mathrm{T}}$ in (27) is singular for $\mathrm{rank}(\boldsymbol{G}) < M+1$. The weighting matrix $\boldsymbol{W}$ in (26) can be obtained from (27) via using regularisation techniques [18]

$$\boldsymbol{W} = \left(\boldsymbol{G}\Sigma\boldsymbol{G}^{\mathrm{T}} + \delta\boldsymbol{I}_{M+1}\right)^{-1} \tag{28}$$

where δ is the regularisation factor.

Let $\hat{\boldsymbol{z}} = \boldsymbol{z} + \Delta\boldsymbol{z}$, $\Delta\boldsymbol{z}$ can be obtained by using the perturbation approach when the measurement errors are small, as developed in [14]

$$\Delta\boldsymbol{z} \simeq -\left(\boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{B}\boldsymbol{A}_2\right)^{-1}\boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{G}\boldsymbol{q} \tag{29}$$

Taking the expected value on both sides of (29), and applying the fact that $E[\boldsymbol{q}] = \boldsymbol{0}_{M-1}$ yield

$$E[\Delta\boldsymbol{z}] \simeq \boldsymbol{0}_4 \tag{30}$$

which illustrates the unbiasedness of the proposed method for low measurement noise level. The covariance matrix is given from (28) and (29)

$$\mathrm{cov}(\hat{\boldsymbol{z}}) = E\left[\Delta\boldsymbol{z}\Delta\boldsymbol{z}^{\mathrm{T}}\right] \simeq \left(\boldsymbol{A}_2^{\mathrm{T}}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{B}\boldsymbol{A}_2\right)^{-1} \tag{31}$$

4.2 Implementation and discussion

With the new MDS analysis, a weighting matrix is designed to whiten the residuals of the set of the new linear equations in the proposed estimator. The weighting matrix (28) depends on the emitter position $\boldsymbol{x}_0$. At the beginning, we can use the LS solution of (24) from which to generate a better $\boldsymbol{W}$ to yield a more accurate solution.

The following summarises the proposed algorithm when using weighting matrix in (28):

1. Form $\hat{\boldsymbol{A}}$ from (20) and (14), and $\hat{\boldsymbol{B}}$ from (7) and (5), using range-differences r_{m1}, $m = 2, 3, \ldots, M$;

2. Find $\hat{\boldsymbol{z}} = [\hat{x}, \hat{y}, \hat{z}, -\hat{d}_1]$ from (25), using $\boldsymbol{W} = \boldsymbol{I}_{M+1}$;

3. Repeat the following one or two times:

(a) Update $\hat{\boldsymbol{A}}$ and $\hat{\boldsymbol{B}}$ by the emitter location estimate from $\hat{\boldsymbol{z}}$, and use (40) and (28) to form $\boldsymbol{W}$;

(b) Find $\hat{\boldsymbol{z}}$ from (25);

4. Select the first three entries of $\hat{\boldsymbol{z}}$ to generate the emitter position $\hat{\boldsymbol{x}}_0$.

Note that the regularisation factor δ can be determined by small positive constant such as the minimum singular value of the matrix $\boldsymbol{G}^{\mathrm{T}}\boldsymbol{G}$ in the implementation.

Remark 1: Although the extra variable is also introduced in the proposed estimator based on MDS technique, the additional process exploiting the relation between the extra variable and the emitter location from the estimate is not required. This is because that this relationship has been used in the formation of the squared distance matrix (5) in the special complex space.

Remark 2: Unlike most of ordinary hyperbolic location methods [7, 12–15], the proposed estimator is based on a new set of linear equations (19) which is deduced from the MDS analysis based on the optimisation of a loss function with respect to the scalar product matrix. With the new MDS analysis, a weighting matrix can be designed to whiten the residuals to obtain a better performance, although it is generally hard to perform analytically for the classical MDS-based equations (13) because of the eigenvalue factorisation of the scalar product matrix.

Remark 3: In the following, the computational complexity of the ordinary efficient hyperbolic location methods [7, 13–15] and the proposed estimator (25) is compared. Apart from the LS computation, the two-step WLS method [14] requires the few times iteration to form two weighting matrices in the two procedures, and the LCLS method [7] (or constrained LS method [15]) includes the computational process to obtain the Lagrange multiplier by iterative method, while the proposed MDS-based estimator requires one or two repeats to form a better weighting matrix (28). Hence, they have comparable computation complexity, and their complexity is a little more than that of the unconstrained LS methods [13, 15]. However, these differences will not be significant when the sensors number is small.

5 Simulations

This section contains simulation results to evaluate the performance of the proposed estimator by comparing with the SI method [13], the one-step WLS method [14, (14)](or named weighted SI method), two-step WLS method [14] (i.e. one of the constrained LS methods), the ML estimator as well as the CRLB (3). Notice that the two-step WLS method has comparable performance with the LCLS method [7], and they have better performance than the SI and one-step WLS methods [14]. The ML estimator is implemented using the exhaustive search with the step size 0.1 m, which is computationally intensive. We consider a five-receiver geometry, as shown in Fig. 1, with coordinates [300, 100, 150]m, [400, 150, 100]m, [300, 500, 200]m, [350, 200, 100]m and $[-100, -100, -100]$m. The estimation accuracy in terms of the norm of estimation bias defined as $\sqrt{(E[\hat{\boldsymbol{x}}_0] - \boldsymbol{x}_0)^{\mathrm{T}}(E[\hat{\boldsymbol{x}}_0] - \boldsymbol{x}_0)}$ and root mean squares error (RMSE) defined as $\sqrt{E[(\hat{\boldsymbol{x}}_0 - \boldsymbol{x}_0)^{\mathrm{T}}(\hat{\boldsymbol{x}}_0 - \boldsymbol{x}_0)]}$ are investigated for near- and far-field source as the measurement noise increases. The range-difference measurement errors vector $\boldsymbol{q} = [q_{21}, q_{31}, \ldots, q_{M1}]^{\mathrm{T}}$ are assumed zero-mean Gaussian process [7, 14]. The covariance matrix $\boldsymbol{\Sigma}$ is given $\sigma_d^2\boldsymbol{\Theta}$, where $\boldsymbol{\Theta}$ was set to 1 in the diagonal

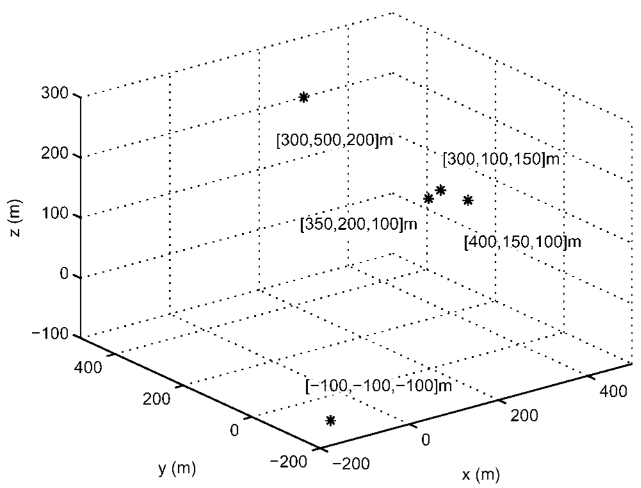

Figure 1 *Three-dimensional sensor array for passive localisation*

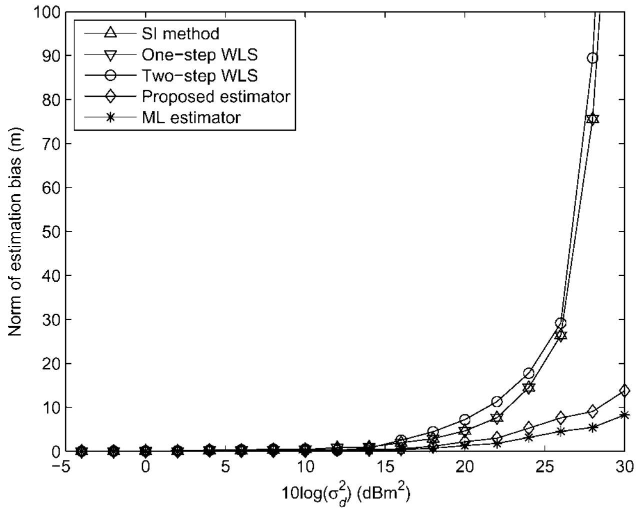

Figure 2 *Estimation bias against range difference error for near-field emitter located at (280, 325, 275)m*

elements and 0.5 otherwise, and σ_d^2 is the variance of the measurement noise q_{m1}. The results were averages of 5000 independent runs.

The first simulation is concerned with the estimation bias and RMSE as a function of range-difference noise power for near-field emitter located at $\boldsymbol{x}_0 = [280, 325, 275]$m. Two repetitions are applied for the one- and two-step WLS methods as well as the proposed estimator. As clearly shown in Fig. 2, the estimation bias grows as the noise measurement errors increase because of nonlinear nature of this problem. The ML estimator has the smallest bias for the five-receiver scenario. The two-step WLS method exhibits the worst bias among all the algorithms simulated. While the bias of the proposed estimator is not as small as that of the ML estimator, it is nonetheless significantly smaller than the SI method, the one and the two-step WLS methods. In Fig. 3, the RMSE results clearly demonstrate

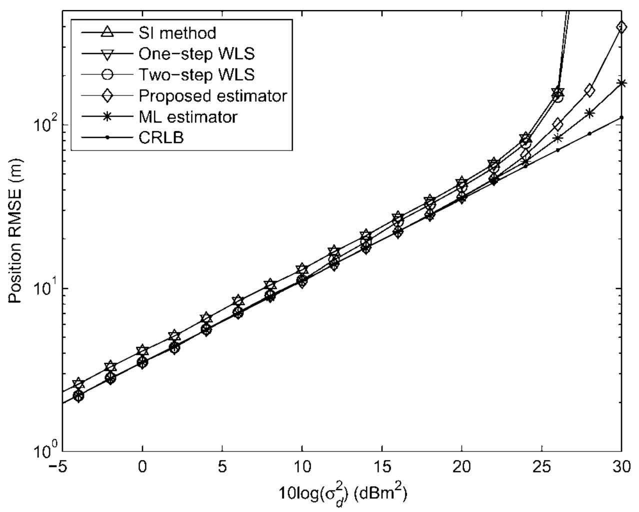

Figure 3 *RMSE against range difference error for near-field emitter located at (280, 325, 275)m*

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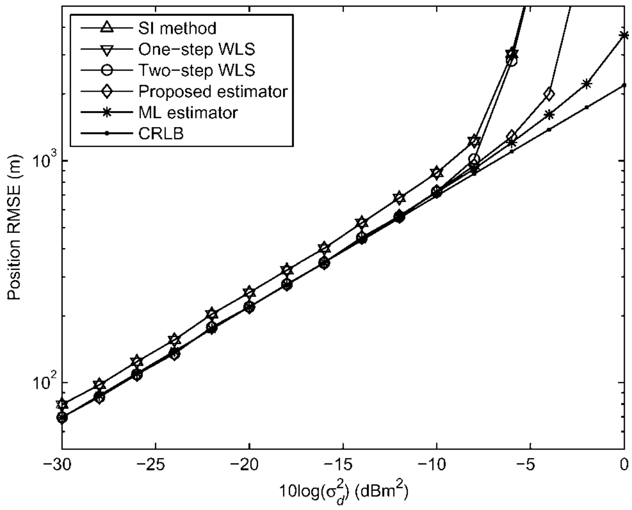

Figure 4 *Estimation bias against range difference error for far-field emitter located at (2800, 3250, 2750)m*

the superior performance of the proposed estimator compared with the SI method, the one and two-step WLS approaches. The proposed estimator is very close to the ML estimator in terms of the RMSE performance. Moreover, the two-step WLS method deviates from the CRLB at a noise power ~9 dB, whereas the proposed method gives inaccurate estimate at the noise power ~19 dB. The threshold effect, resulting from the nonlinear nature, of the proposed method occurs at a noise power that is ~10 dB later than that of the two-step WLS method as the noise power increases. This indicates that it has a larger operation range. The reason for this superiority is that the dimensional knowledge and eigen-structure information of the scalar product matrix are exploited and a weighting matrix (28) is designed to restrain the measurement noises in the matrix. Figs. 4 and Fig. 5 are the second simulation results for far-field emitter located at $\boldsymbol{x}_0 = [2800, 3250, 2750]$m. One repetition is applied. The proposed estimator has

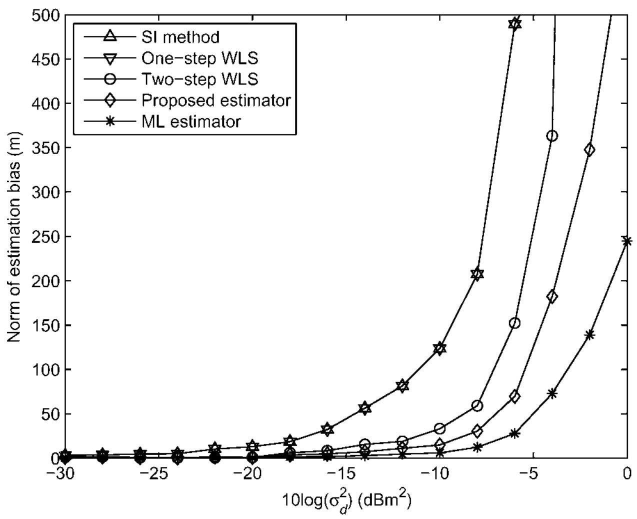

Figure 5 *RMSE against range difference error for far-field emitter located at (2800, 3250, 2750)m*

comparable bias and RMSE with the two-step WLS method, and the ML estimator appears to outperform the other four approaches. From Fig. 2 to Fig. 5, the SI method and one-step WLS method have the same performance, and both of them are inferior to the proposed estimator apparently. In addition, simulations show that in most cases, repeating solution computation one or two times is sufficient, and further repetition is not necessary.

6 Conclusions

We have addressed an accurate and computationally simple estimator for passive emitter localisation from range-difference measurements. The proposed estimator exploits the MDS technique, which is robust to large measurement noise. The accuracy of the estimate achieves the CRLB at low and moderate noise levels. Furthermore, the threshold effect of the new method occurs later than that of the conventional methods as the range difference measurement noise increases. It implies that the proposed estimator has a larger operation range. This new technique is attractive under the circumstances of large measurement errors.

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8 Appendix: Derivation of (23)

The residuals vector (23) with respect to the measurement noise vector $\boldsymbol{q}$ is derived as follows. In the presence of measurement noise, the noisy matrices $\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{A}}$ can be represented as

$$\hat{\boldsymbol{B}} = \boldsymbol{B} + \Delta\boldsymbol{B}, \quad \hat{\boldsymbol{A}} = \boldsymbol{A} + \Delta\boldsymbol{A} \tag{32}$$

where the symbol $\hat{(\cdot)}$ deontes the noisy value of the true one, $\Delta\boldsymbol{B}$ is a $(M+1)\times(M+1)$ dimensional error matrix of $\hat{\boldsymbol{B}}$, and $\Delta\boldsymbol{A}$ is a $(M+1)\times 5$ dimensional error matrix of $\hat{\boldsymbol{A}}$. Substituting (32) into (22) and ignoring the second-order error terms, from (19), the residuals vector $\boldsymbol{\epsilon}$ becomes

$$\boldsymbol{\epsilon} \simeq \Delta\boldsymbol{B}\boldsymbol{A}\begin{bmatrix}1\\ \boldsymbol{z}\end{bmatrix} + \boldsymbol{B}\Delta\boldsymbol{A}\begin{bmatrix}1\\ \boldsymbol{z}\end{bmatrix} \tag{33}$$

From (7), $\Delta\boldsymbol{B}$ can be obtained

$$\Delta\boldsymbol{B} = -\frac{1}{2}\boldsymbol{J}_{M+1}\Delta\boldsymbol{D}\boldsymbol{J}_{M+1} \tag{34}$$

where $\Delta\boldsymbol{D}$ is a $(M+1)\times(M+1)$ dimensional error matrix of $\hat{\boldsymbol{D}} = \boldsymbol{D} + \Delta\boldsymbol{D}$. By ignoring the second-order error terms of the measurement noise, the (m, n) entry of $\Delta\boldsymbol{D}$, except the first column and first row (these entries are all zeros in $\boldsymbol{D}$), can be represented as from (2) and (5)

$$[\Delta\boldsymbol{D}]_{mn} \simeq -2(d_{m1} - d_{n1})(q_{m1} - q_{n1}),\quad m, n = 1, 2, \ldots, M \tag{35}$$

in which q_{11} is defined as $q_{11} = 0$ for representation simplicity. Similarly, the error matrix $\Delta\boldsymbol{R}$ of $\hat{\boldsymbol{R}} = \boldsymbol{R} + \Delta\boldsymbol{R}$ can be expressed from (20) and (14), as

$$\Delta\boldsymbol{R} = \begin{bmatrix} \boldsymbol{0}_4 & \boldsymbol{0}_4 & & \cdots & \boldsymbol{0}_4 \\ -\frac{1}{M+1}\sum_{m=2}^{M} q_{m1} & -\frac{1}{M+1}\sum_{m=2}^{M} q_{m1} & \boldsymbol{0}_4 \times q_{21} - \frac{1}{M+1}\sum_{m=2}^{M} q_{m1} & \cdots & q_{M1} - \frac{1}{M+1}\sum_{m=2}^{M} q_{m1} \end{bmatrix}_{5\times(M+1)} \tag{36}$$

With the relationship between $\boldsymbol{R}$ and $\boldsymbol{A}$ in (21), the $(M+1)\times 5$ dimensional error matrix $\Delta\boldsymbol{A}$ can be obtained

$$\Delta\boldsymbol{A} = -\boldsymbol{R}^{\dagger}\Delta\boldsymbol{R}\boldsymbol{A} = -\boldsymbol{A}\Delta\boldsymbol{R}\boldsymbol{A} \tag{37}$$

by retaining the linear perturbation terms in $\hat{\boldsymbol{R}}\hat{\boldsymbol{A}} = \boldsymbol{I}_5$.

As a result, the residuals vector (33) is from (34) and (36)

$$\boldsymbol{\epsilon} \simeq -\frac{1}{2}\boldsymbol{J}_{M+1}\Delta\boldsymbol{D}\boldsymbol{J}_{M+1}\boldsymbol{A}\begin{bmatrix}1\\ \boldsymbol{z}\end{bmatrix} - \boldsymbol{B}\boldsymbol{A}\Delta\boldsymbol{R}\boldsymbol{A}\begin{bmatrix}1\\ \boldsymbol{z}\end{bmatrix} = \left(-\frac{1}{2}\boldsymbol{J}_{M+1}\boldsymbol{T}_1 - \boldsymbol{B}\boldsymbol{A}\boldsymbol{T}_2\right)\boldsymbol{q} \tag{38}$$

where

$$\boldsymbol{a} = \boldsymbol{A}\begin{bmatrix}1\\ \boldsymbol{z}\end{bmatrix}, \quad \bar{a} = \frac{1}{M+1}\boldsymbol{a}^{\mathrm{T}}\boldsymbol{1}_{M+1},$$

$$\boldsymbol{T}_2 = \begin{bmatrix} \boldsymbol{0}_4 & \boldsymbol{0}_4 & \cdots & \boldsymbol{0}_4 \\ a_2 - \bar{a} & a_3 - \bar{a} & \cdots & a_M - \bar{a} \end{bmatrix}_{5\times(M+1)}$$

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and

$$
\boldsymbol{T}_1 = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ c_{12} & c_{13} & \cdots & c_{1M} \\ -\sum_{m=1}^{M} c_{2m} & c_{23} & \cdots & c_{2M} \\ c_{32} & -\sum_{m=1}^{M} c_{3m} & \cdots & c_{3M} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M2} & c_{M3} & \cdots & -\sum_{m=1}^{M} c_{Mm} \end{bmatrix}_{(M+1)\times(M-1)} \tag{39}
$$

in which $c_{mn} = 2(a_n - \bar{a})(d_{m1} - d_{n1})$, $m = 1, 2, \ldots, M$, $n = 2, 3, \ldots, M$.

Hence, (38) can be rewritten as (23) by defining a $(M+1) \times (M-1)$ dimensional matrix

$$
\boldsymbol{G} = -\frac{1}{2}\boldsymbol{J}_{M+1}\boldsymbol{T}_1 - \boldsymbol{BAT}_2 \tag{40}
$$