December 13, 2016

Hello Darya,

There appears to be an error in Algorithm 5 in Chapter 4 of your thesis. It starts from line 15 on page 69, where you wrote

$$\|\boldsymbol{x} - \boldsymbol{a}_i\| \ge \|\boldsymbol{x}_k - \boldsymbol{a}_i\| + \partial \|\boldsymbol{x}_k - \boldsymbol{a}_i\|^T (\boldsymbol{x} - \boldsymbol{a}_i)$$

which is incorrect and should be changed to

$$\| \boldsymbol{x} - \boldsymbol{a}_i \| \ge \| \boldsymbol{x}_k - \boldsymbol{a}_i \| + \partial \| \boldsymbol{x}_k - \boldsymbol{a}_i \|^T (\boldsymbol{x} - \boldsymbol{x}_k)$$

This change triggers several subsequent changes:

• 4 lines above Eq. (4.14a), change the expression to

$$- \| \boldsymbol{x}_{k} - \boldsymbol{a}_{i} \| - \partial \| \boldsymbol{x}_{k} - \boldsymbol{a}_{i} \|^{T} (\boldsymbol{x} - \boldsymbol{x}_{k}) \leq -(1 - \gamma) z_{i}$$

• change Eq. (4.14c) to

$$-\|\boldsymbol{x}_{k}-\boldsymbol{a}_{i}\|-\partial\|\boldsymbol{x}_{k}-\boldsymbol{a}_{i}\|^{T}(\boldsymbol{x}-\boldsymbol{x}_{k}) \leq -(1-\gamma)z_{i}, i=1,2,...,m$$

- In Algorithm 5 on page 70, Step 2, make the same change in the 4<sup>th</sup> line there.
- The MATLAB code for Algorithm 5 needs to be revised accordingly.
- The simulations in Sec. 4.2.2 need to be re-performed using the revised code. I would expect performance improvement when the revised code is used.

My suggestion is that you can make these changes and revisions in your thesis as soon as you have time (please send me a copy of the revised thesis), but the revised thesis cannot be submitted to FGS until the oral exam is complete. Doing the revision right now would save your time in January.

Based on the above correction, you can easily verify that Eq. (4.14c) can be written as

$$(\boldsymbol{x} - \boldsymbol{a}_i)^T \left( \frac{\boldsymbol{x}_k - \boldsymbol{a}_i}{\|\boldsymbol{x}_k - \boldsymbol{a}_i\|} \right) \ge (1 - \gamma) z_i$$

which turns out to be exactly the same as what I got in May 2016 when the nonconvex constraint  $\|x - a_i\| \ge z_i$  was relaxed from a different perspective. In other words, after all the method you developed in Sec. 4.2 coincides with my thoughts and I don't have anything further to improve it at the moment.

Best wishes,

**WSL**