Localization Algorithms for Passive Sensor Networks

by

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B.Eng., University of Astrakhan, 2010

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ABSTRACT

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List of Abbreviations

LS	Least Squares
ML	Maximum Likelihood
MDS	Multidimensional Scaling
DW-MDS	Distributed Weighted-MultidiDentional Scaling
SR-LS	
SRD-LS	
PDF	Probability Density Function
SPF	standard fixed point
SWLS	sequential weighted least squares
WSR-LS	weighted squared range based least squares (WSR-LS)
WSRD-LS	weighted squared range-difference based least squares (WSR-LS)
GTRS	
IRWSR-LS	
IRWSRD-LS	
MSE	
TDOA	
TOA	
WCDMA	
LTE	
O-TDOA	
CRLB	Cramér-Rao lower bound
NLLS	Non-Linear Least Squares
SMACOF	Scaling by MAjorizing a COmplicated Function
RSS	Received Signal Strength
NLOS	Non-Line Of Sight
UWB	Ultra Wide Band
SDP	SemiDefinite Programming
DC	Difference of Convex
PCCP	Penalty Convex Concave Procedure
CCP	Convex Concave Procedure
LP	Linear Program

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Chapter 1

Least Squares Localization by Sequential Convex Relaxation

1.1 Second Order Cone Programming

1.2 Range-based localization

Problem: Given sensor array $\boldsymbol{a}_i, i = 1, 2, ..., m$ and noisey range measurements r_i find the true *unknown* location of $\boldsymbol{x}$ as

$$\text{minimize} \sum_{i}^{m} \left(\|\boldsymbol{x} - \boldsymbol{a}_i\| - r_i\right)^2 \tag{1.1}$$

which can be (equivalently) written as

$$\underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} \sum_{i}^{m} \left(z_i - r_i\right)^2 \tag{1.2}$$

$$\text{subject to: } \|\boldsymbol{x} - \boldsymbol{a}_i\| = z_i, i = 1, 2, ..., m \tag{1.2}$$

The constraint in 5.2 is hard to suffice, therefore we allow a relaxation:

$$\underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} \sum_{i}^{m} \left(z_i - r_i\right)^2 \tag{1.3}$$

$$\text{subject to: } \|\boldsymbol{x} - \boldsymbol{a}_i\| \leq (1 + \gamma) z_i \tag{1.3}$$

$$\|\boldsymbol{x} - \boldsymbol{a}_i\| \geq (1 - \gamma) z_i, \quad i = 1, 2, ..., m \tag{1.3}$$

where γ is small, typically $0 < \gamma < 0.5$. This would yield an approximate solution to 5.2 and therefore to 5.1. By allowing γ to sequentially/monotonically decrease from some small $0 < \gamma_0 < 0.5$ to 0 solution of 5.3 will converge to 5.2. *Proof* Let $\gamma(k)$ be monotonically decteasing, where k is an iteration count and $0 < \gamma_0 < 0.5$. Then $\lim_{\gamma\to 0}(1+\gamma)z_i = z_i$ and $\lim_{\gamma\to 0}(1-\gamma)z_i = z_i$. Therefore as γ approaches 0, the feasible region of the problem in 5.3 will become equivalent to that in 5.2. As iterations proceed, the objective in 5.3 will not be monotonically decreasing but it will converge to the critical point.

Problem in 5.3 is nonconvex due to nonconvexity of one of its inequality constraint. The constraint in 5.3b $\|\boldsymbol{x} - \boldsymbol{a}_i\| \leq (1+\gamma)z_i$ is convex, the constraint in 5.3c is not, because

$$\|\boldsymbol{x} - \boldsymbol{a}_i\| \geq (1-\gamma)z_i \Longleftrightarrow \underbrace{-\|\boldsymbol{x} - \boldsymbol{a}_i\|}_{nonconvex} \leq -(1-\gamma)z_i$$

From convexity of the norm $\|\boldsymbol{x} - \boldsymbol{a}_i\|$ it follows that for some *known* $\boldsymbol{x}_k$

$$\|\boldsymbol{x} - \boldsymbol{a}_i\| \geq \|\boldsymbol{x}_k - \boldsymbol{a}_i\| + \partial\|\boldsymbol{x}_k - \boldsymbol{a}_i\|^T(\boldsymbol{x} - \boldsymbol{a}_i)$$

Hence the constraint in 5.3c can be convexified by replacing it with its affine approximation

$$-\|\boldsymbol{x}_k - \boldsymbol{a}_i\| - \partial\|\boldsymbol{x}_k - \boldsymbol{a}_i\|^T(\boldsymbol{x} - \boldsymbol{a}_i) \leq -(1-\gamma)z_i$$

At the kth iteration when the iterate $\boldsymbol{x}_k$ is known, the nonconvex problem in 5.3 can be relaxed to an SOCP problem

$$\underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} \sum_i^m (z_i - r_i)^2 \tag{1.4}$$

$$\text{subject to: } \|\boldsymbol{x} - \boldsymbol{a}_i\| \leq (1+\gamma)z_i \tag{1.4}$$

$$-\|\boldsymbol{x}_k - \boldsymbol{a}_i\| - \partial\|\boldsymbol{x}_k - \boldsymbol{a}_i\|^T(\boldsymbol{x} - \boldsymbol{a}_i) \leq -(1-\gamma)z_i \tag{1.4}$$

$$i = 1, 2, ..., m$$

The relaxation parameter γ controls the size of the convex hull that defines a feasibility region of the problem 5.4. γ needs to be monotonically decreasing with increase of the iteration count. Start with some $0 < \gamma_0 < 0.5$, typically $\gamma 0 = 0.3$ or 0.2 is good. After kth iteration update γ_{k+1} linearly as

$$\gamma_{k+1} = \gamma_0 - k\frac{\gamma_0}{K_{max} - 1}$$

or quadratically as

$$\gamma_{k+1} = \gamma_0 \frac{(K_{max} - 1 - k)^2}{(K_{max} - 1)^2}$$

Chapter 2

SOCP

2.1 Range-Difference Localization

Measurement model

$$d_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x}\| + noise \tag{2.1}$$

Least-squares formulation

$$\text{minimize} \sum_{i=1}^{m} \left(\|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x}\| - d_i\right)^2 \tag{2.2}$$

The problem in 6.2 can be equivalently written as

$$\begin{aligned} \text{minimize} \sum_{i=1}^{m} (z_i - y - d_i)^2 \\ \text{subject to: } \|\boldsymbol{x} - \boldsymbol{a}_i\| = z_i \\ \|\boldsymbol{x}\| = y \\ z_i \geq 0,\ y \geq 0 \text{added by me} \\ i = 1, 2, \ldots m \end{aligned} \tag{2.3}$$

Let $\tilde{\boldsymbol{x}} = [\boldsymbol{x}^T\ y\ z_1 \ldots z_m]^T$, $\tilde{\boldsymbol{x}} \in R^{m+n+1}$ be a known feasible point of the problem in 6.3 and $\boldsymbol{\delta}_{\tilde{\boldsymbol{x}}} = [\boldsymbol{\delta}_x^T\ \delta_y\ \delta_{z_1}\ \ldots\ \delta_{z_m}]^T$ a small perturbation to it, such that $|\boldsymbol{\delta}_{\tilde{\boldsymbol{x}}}| \leq \beta$, and $\beta > 0$ is a small positive constant. At the kth iterations with $\tilde{\boldsymbol{x}}^k$ *known* update

$(\boldsymbol{x}^k, y^k, \boldsymbol{z}^k)$ to

$$\begin{aligned} \boldsymbol{x}^{k+1} &= \boldsymbol{x}^k + \boldsymbol{\delta}_x \\ y^{k+1} &= y^k + \delta_y \\ \boldsymbol{z}^{k+1} &= \boldsymbol{z}^k + \boldsymbol{\delta}_z \end{aligned} \tag{2.4}$$

Substituting 6.4 in 6.3 the objective in 6.3a cam be written as

$$\begin{aligned} F(\hat{\boldsymbol{x}}) &= \sum_{i=1}^{m} \left(z_i^k + \delta_{z_i} - (y^k + \delta_y) - d_i\right)^2 \\ &= \sum_{i=1}^{m} \left(-\delta_y + \delta_{z_i} - \tilde{d}_i\right)^2 \end{aligned} \tag{2.5}$$

where $\tilde{d}_i = y^k - z_i^k + d_i$ are grouped known constant terms. Substituting 6.4b in 6.3b

$$\|\boldsymbol{x}^k + \boldsymbol{\delta}_x^k - \boldsymbol{a}_i\| = z_i^k + \delta_{z_i}, \quad i = 1, 2, \ldots, m$$

Re-grouping the terms on the left side of the equality and squaring both sides

$$\begin{aligned} \|(\boldsymbol{x}^k - \boldsymbol{a}_i) + \boldsymbol{\delta}_x^k\|^2 &= \left(z_i^k + \delta_{z_i}\right)^2 \\ \Leftrightarrow \|\boldsymbol{x}^k - \boldsymbol{a}_i\|^2 + 2\left(\boldsymbol{x}^k - \boldsymbol{a}_i\right)^T \boldsymbol{\delta}_x + \cancelto{\text{small}}{\|\boldsymbol{\delta}_x\|^2} &= \left(z_i^k\right)^2 + 2z_i^k\delta_{z_i} + \cancelto{\text{small}}{\boldsymbol{\delta}_{z_i}^2} \\ \Leftrightarrow \|\boldsymbol{x}^k - \boldsymbol{a}_i\|^2 + 2\left(\boldsymbol{x}^k - \boldsymbol{a}_i\right)^T \boldsymbol{\delta}_x &\approx \left(z_i^k\right)^2 + 2z_i^k\delta_{z_i} \\ \Leftrightarrow \|\boldsymbol{x}^k - \boldsymbol{a}_i\|^2 + 2\left(\boldsymbol{x}^k - \boldsymbol{a}_i\right)^T \boldsymbol{\delta}_x &\approx \left(z_i^k\right)^2 + 2z_i^k\delta_{z_i} \\ -2\left(\boldsymbol{x}^k - \boldsymbol{a}_i\right)^T \boldsymbol{\delta}_x + 2z_i^k\delta_{z_i} &\approx \|\boldsymbol{x}^k - \boldsymbol{a}_i\|^2 - \left(z_i^k\right)^2 \end{aligned}$$

Repeating the similar procedure with the constraint in 6.3c

$$\begin{aligned} \|\boldsymbol{x}^k + \boldsymbol{\delta}_x\| &= y^k + \delta_y \\ \|\boldsymbol{x}^k + \boldsymbol{\delta}_x\|^2 &= \left(y^k + \delta_y\right)^2 \\ \Leftrightarrow \|\boldsymbol{x}^k\|^2 + 2\boldsymbol{\delta}_x^T\boldsymbol{x}^k + \cancelto{\text{small}}{\|\boldsymbol{\delta}_x\|^2} &= \left(y^k\right)^2 + 2y^k\delta_y + \cancelto{\text{small}}{\delta_y^2} \\ \Leftrightarrow -2\left(\boldsymbol{x}^k\right)^T \delta_x + 2y^k\delta &\approx \|\boldsymbol{x}\|^2 - \left(y^k\right)^2 \end{aligned}$$

The problem in 6.3 can now be written in terms of the *known* feasible iterate $\tilde{\boldsymbol{x}}$ and its *unknown* perturbation $\boldsymbol{\delta}_{\tilde{\boldsymbol{x}}}$ as

$$
\begin{aligned}
\underset{\boldsymbol{\delta}_{\tilde{\boldsymbol{x}}}}{\text{minimize}} & \sum_{i=1}^{m} \left(-\delta_y + \delta_{z_i} - \tilde{d}_i\right)^2 \\
\text{subject to: } & -2\left(\boldsymbol{x}^k - \boldsymbol{a}_i\right)^T \boldsymbol{\delta}_x + 2 z_i^k \delta_{z_i} = \|\boldsymbol{x}^k - \boldsymbol{a}_i\|^2 - \left(z_i^k\right)^2 \\
& -2\left(\boldsymbol{x}^k\right)^T \delta_x + 2 y^k \delta = \|\boldsymbol{x}^k\|^2 - \left(y^k\right)^2 \\
& |\boldsymbol{\delta}_{\tilde{\boldsymbol{x}}}| \leq \beta \\
& z_i^k + \delta_{z_i} \geq 0,\ y^k + \delta_y \geq 0 \text{added by me} \\
& i = 1, 2, \ldots m
\end{aligned}
\tag{2.6}
$$

where $\tilde{d}_i = y^k - z_i^k + d_i$. The constraint β was imposed on each element of the vector $\boldsymbol{\delta}_{\tilde{\boldsymbol{x}}}$ to guarantee that at each iteration is sufficiently small. The iterate $\tilde{\boldsymbol{x}}^k$ is feasible and known but it is not guaranteed that the $\tilde{\boldsymbol{x}}^{k+1}$ will also be feasible. To allow non-feasible perturbations $\boldsymbol{\delta}_{\tilde{\boldsymbol{x}}}$, the problem can be overcome by introducing a nonnegative slack variable $s \geq 0$ 6.6c to replace their right-hand sides by a relaxed upper bound (as this new bound itself is a nonnegative variable). Since the iterate $\tilde{\boldsymbol{x}}^k$ is feasible, then the constraints 6.3d,e are satisfied, i.e. $z_i^k \geq 0$ and $y^k \geq 0$. This leads to a following formulation of the problem in 6.3

$$
\begin{aligned}
\underset{s, \boldsymbol{\delta}_{\tilde{\boldsymbol{x}}}}{\text{minimize}} & \sum_{i=1}^{m} \left(-\delta_y + \delta_{z_i} - \tilde{d}_i\right)^2 + \mu^k s \\
\text{subject to: } & -2\left(\boldsymbol{x}^k - \boldsymbol{a}_i\right)^T \boldsymbol{\delta}_x + 2 z_i^k \delta_{z_i} = \|\boldsymbol{x}^k - \boldsymbol{a}_i\|^2 - \left(z_i^k\right)^2 \\
& -2\left(\boldsymbol{x}^k\right)^T \delta_x + 2 y^k \delta = \|\boldsymbol{x}^k\|^2 - \left(y^k\right)^2 \\
& |\boldsymbol{\delta}_{\tilde{\boldsymbol{x}}}| \leq \beta + s \\
& s \geq 0 \\
& \delta_{z_i} \geq 0,\ \delta_y \geq 0 \text{ added by me} \\
& i = 1, 2, \ldots m
\end{aligned}
\tag{2.7}
$$

$\mu^k \geq 0$ is the weight that monotonically increases as iterations proceed until it reaches an upper limit μ_{max}.

Appendix A

A.1 Solving ??

3.21

A.2 Solving ??

3.32

Appendix B

Matlab Files

B.1 IRWSR-LS

3.21

B.2 IRWSRD-LS

3.32

B.3 PCCP-Based LS

```
Apply a constrained CCP to the localization problem based on range
measurements. Both upper-bound and lower-bound constraints are imposed.
Input:
Am: Am = [a1 a2 ... am] with ai the location of the ith sensor.
r: noise-free range measurements r = [r1 r2 ... rm] with ri = norm(xs-a
st: initial state for noise generation.
sig: standard deviation of measurement noise.
gam: a parameter that controls the bounds in the constraints.
x0: initial point for the source location.
K: number of CCG iterations.
Output:
xw: estimated location of the source.
xw1: estimated location of the source using SR-LS.
```

```
 Written by W.-S. Lu, University of Victoria.
 Last modified: Feb. 2, 2015.
 Example: load data_ex1
 [xw,xw1] = pccp(Am,r,1e-2,3,zeros(2,1),20);

function [F_ccp,F_rls,solution,slack] = pccp(Am,rn,sig,gam,x0,K)
[n,m] = size(Am);
k = 0;
xk = x0;
rnb_p = rn + gam*sig;
rnb_n = rn - gam*sig;
ab = (mean(Am'))';
tau = 1;
tau_max = 100000;
mi = 1/m;
m2 = 2*m;
nn = 1000;
Xk=[];
while (k < K) %&& (nn >= 1e-5)
    cvx_begin quiet
       variable x(n)
       variable s(m2);
       v = ab;
       for i = 1:m,
           xai = xk - Am(:,i);
           v = v + (mi*rn(i)/norm(xai))*xai;
       end
       minimize(x'*x - 2*x'*v + tau*sum(s));
       subject to
       for i = 1:m,
           norm(x-Am(:,i)) <= rnb_p(i) + s(i);
           xai = xk - Am(:,i);
           ni = norm(xai);
           xain = xai/ni;
           -xain'*(x-xk) - ni + rnb_n(i) <= s(m+i);
```

```
        end
        s >= 0;
    cvx_end
    x=x(:);
    Xk = [Xk x];
    nn = norm(xk-x);
    xk = x;
    tau = min(1.5*tau,tau_max);
    k = k + 1;
end
% disp(sprintf('Solution point after %d CCP iterations:',k))
solution = xk;
F_rls = zeros(K,1);
F_ccp = zeros(K,1);
for k = 1:K
    %value of the NNLS objective at the solution point
    sol  = Xk(:,k);
    v = zeros(2,1);
    for i = 1:m,
        xai = sol - Am(:,i);
        v = v + (mi*rn(i)/norm(xai))*xai;
        F_rls(k) = F_rls(k) + (rn(i) - norm(sol - Am(:,i)))^2;
    end
    % disp(sprintf('Final value of the objective function'))
    F_ccp(k) = sol'*sol - 2*sol'*v ;%+ tau*sum(s);
end
% disp('Final values of relaxation parameters:')
 slack = s;
```

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