Localization Algorithms for Passive Sensor Networks

by

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B.Eng., University of Astrakhan, 2010

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ABSTRACT

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List of Abbreviations

LS	Least Squares
ML	Maximum Likelihood
MDS	Multidimensional Scaling
DW-MDS	Distributed Weighted-Multi Dementional Scaling
SR-LS	Squared-Range Least Squares
SRD-LS	Squared-Range-Difference Least Squares
PDF	Probability Density Function
SPF	Standard Fixed Point
SWLS	Sequential Weighted Least Squares
WSR-LS	Weighted Squared Range-based Least Squares
WSRD-LS	weighted Squared Range-Difference-based Least Squares
GTRS	Generalized Trust Region Sub-problem
IRWSR-LS	Iterative Re-Weighting Squared Range-based Least Squares
IRWSRD-LS	Iterative Re-Weighting Squared Range-Difference-based Least Squares
MSE	Mean Squared Error

TDOA	Time Difference Of Arrival
TOA	Time Of Arrival
WCDMA	Wide Band Code Division Multiple Access
LTE	Long Term Evolution
O-TDOA	Observed Time Difference Of Arrival
CRLB	Cramér-Rao lower bound
NLLS	Non-Linear Least Squares
SMACOF	Scaling by MAjorizing a COmplicated Function
RSS	Received Signal Strength
NLOS	Non-Line Of Sight
UWB	Ultra Wide Band
SDP	SemiDefinite Programming
SOCP	Second-Order Cone Programming
QP	Quadratic Programming
DC	Difference of Convex
PCCP	Penalty Convex Concave Procedure
CCP	Convex Concave Procedure
LP	Linear Program

SOCP	Second Order Cone Programming

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Chapter 1

Least Squares Localization by Sequential Convex Relaxation

1.1 Range-based localization

The source localization problem discussed in this section involves a given array of m sensors placed in the $n = 2$ or 3 dimentional space with coordinates specified by $\{\boldsymbol{a}_1, \ldots, \boldsymbol{a}_m, \boldsymbol{a}_i \in R^n\}$. Each sensor measures its distance to a radiating source $\boldsymbol{x} \in R^n$. Throughout it is assumed that only noisy copies of the distance data are available, hence the *range measurements* obey the model

$$r_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| + \varepsilon_i, \quad i = 1, \ldots, m. \tag{1.0}$$

where ε_i denotes the unknown noise that has occurred when the ith sensor measures its distance to source $\boldsymbol{x}$. Let $\boldsymbol{r} = [r_1\ r_2 \ldots r_m]^T$ and $\boldsymbol{\varepsilon} = [\varepsilon_1\ \varepsilon_2 \ldots \varepsilon_m]^T$. The source localization problem can be stated as to estimate the true *unknown* location of $\boldsymbol{x}$ from noisy range measurements $\boldsymbol{r}$ as

$$\underset{\boldsymbol{x}}{\text{minimize}} \sum_i^m \left(\|\boldsymbol{x} - \boldsymbol{a}_i\| - r_i\right)^2 \tag{1.1}$$

The problem in (1.1) can be (equivalently) written as

$$\begin{aligned} \underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} \quad & \sum_{i}^{m} (z_i - r_i)^2 && (1.2a) \\ \text{subject to:} \quad & \|\boldsymbol{x} - \boldsymbol{a}_i\| = z_i, \quad i = 1, 2, ..., m && (1.2b) \\ & \boldsymbol{z} \geq \mathbf{0} && (1.2c) \end{aligned}$$

The constraint in 1.2 is hard to suffice, therefore we allow a relaxation:

$$\begin{aligned} \underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} \quad & \sum_{i}^{m} (z_i - r_i)^2 && (1.3a) \\ \text{subject to:} \quad & \|\boldsymbol{x} - \boldsymbol{a}_i\| \leq (1+\gamma) z_i && (1.3b) \\ & \|\boldsymbol{x} - \boldsymbol{a}_i\| \geq (1-\gamma) z_i, \quad i = 1, 2, ..., m && (1.3c) \end{aligned}$$

where γ is small, typically $0 < \gamma < 0.5$. This would yield an approximate solution to 1.2 and therefore to 1.1. By allowing γ to sequentially/monotonically decrease from some small $0 < \gamma_0 < 0.5$ to 0 solution of 1.3 will converge to 1.2. *Proof* Let $\gamma(k)$ be monotonically decteasing, where k is an iteration count and $0 < \gamma_0 < 0.5$. Then $\lim_{\gamma \to 0}(1+\gamma) z_i = z_i$ and $\lim_{\gamma \to 0}(1-\gamma) z_i = z_i$. Therefore as γ approaches 0, the feasible region of the problem in 1.3 will become equivalent to that in 1.2. As iterations proceed, the objective in 1.3 will not be monotonically decreasing but it will converge to the critical point.

Problem in 1.3 is nonconvex due to nonconvexity of one of its inequality constraint. The constraint in 1.3b $\|\boldsymbol{x} - \boldsymbol{a}_i\| \leq (1+\gamma) z_i$ is convex, the constraint in 1.3c is not

$$\|\boldsymbol{x} - \boldsymbol{a}_i\| \geq (1-\gamma) z_i \iff \underbrace{-\|\boldsymbol{x} - \boldsymbol{a}_i\|}_{nonconvex} \leq -(1-\gamma) z_i$$

From convexity of the norm $\|\boldsymbol{x} - \boldsymbol{a}_i\|$ it follows that for some *known* $\boldsymbol{x}_k$

$$\|\boldsymbol{x} - \boldsymbol{a}_i\| \geq \|\boldsymbol{x}_k - \boldsymbol{a}_i\| + \partial \|\boldsymbol{x}_k - \boldsymbol{a}_i\|^T (\boldsymbol{x} - \boldsymbol{a}_i)$$

Hence the constraint in 1.3c can be replaced with its affine approximation

$$-\|\boldsymbol{x}_k - \boldsymbol{a}_i\| - \partial \|\boldsymbol{x}_k - \boldsymbol{a}_i\|^T (\boldsymbol{x} - \boldsymbol{a}_i) \leq -(1-\gamma) z_i$$

At the kth iteration when the iterate $\boldsymbol{x}_k$ is known, the nonconvex problem in 1.3 can be converted to an SOCP problem

$$\underset{\boldsymbol{x},\boldsymbol{z}}{\text{minimize}} \quad \sum_{i}^{m} (z_i - r_i)^2 \tag{1.4a}$$

$$\text{subject to:} \quad \|\boldsymbol{x} - \boldsymbol{a}_i\| \leq (1+\gamma) z_i \tag{1.4b}$$

$$-\|\boldsymbol{x}_k - \boldsymbol{a}_i\| - \partial \|\boldsymbol{x}_k - \boldsymbol{a}_i\|^T (\boldsymbol{x} - \boldsymbol{a}_i) \leq -(1-\gamma) z_i, \quad i = 1, 2, ..., m \tag{1.4c}$$

The relaxation parameter γ controls the size of the convex hull that defines a feasibility region of the problem 1.4. γ needs to be monotonically decreasing with increase of the iteration count. Start with some $0 < \gamma_0 < 0.5$, typically $\gamma_0 = 0.3$ or 0.2 is good. After kth iteration update γ_{k+1} linearly as

$$\gamma_{k+1} = \gamma_0 - k \frac{\gamma_0}{K_{max} - 1}$$

or quadratically as

$$\gamma_{k+1} = \gamma_0 \frac{(K_{max} - 1 - k)^2}{(K_{max} - 1)^2}$$

1.2 Range-Difference Localization

1.2.1 Problem Formulation

In this section we focus on the problem of range-difference based localization given the time-difference of arrival information. TDOA localization, also known as multilateration, or hyperbolic positioning, is a method where the position of the mobile unit (signal source) can be determined using the differences in the TOAs from different base stations. By using this method the clock biases between the mobile units and base stations are automatically removed, since only the pairwise differences between the TOAs from base stations are considered [45]. A hyperbola is the basis for solving multilateration problems. In particular, the set of possible positions of a mobile unit that has a range difference of d_i from two given base stations BS_i and BS_0, placed at $\boldsymbol{a}_i$ and $\boldsymbol{a}_0$ respectively, is a hyperbola with vertex separation of d_i and focii located at $\boldsymbol{a}_i$ and $\boldsymbol{a}_0$. BS_0 is placed at the origin of the coordinate system, i.e. $\boldsymbol{a}_0 = \boldsymbol{0}_n$, and used as a reference station. Consider now a third base station BS_j at a third location. This would provide one extra independent measurement between BS_j and BS_0 and the source is located on the curve determined by the two intersecting hyperboloids.

Figure 1.1 illustrates an example of the range-difference localization based on TDOA measurements.

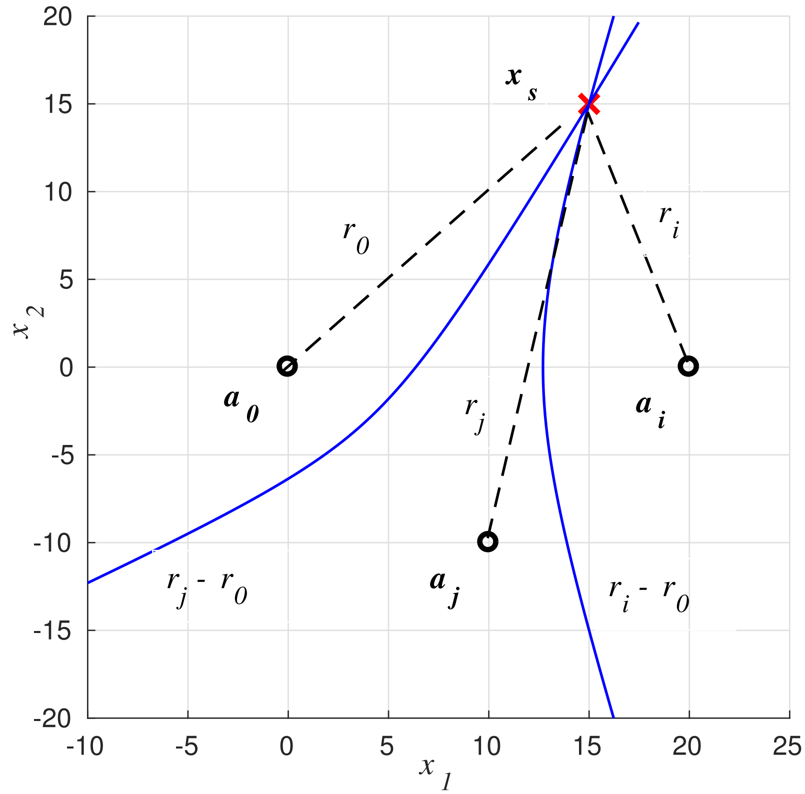

Figure 1.1: Range-difference localization. At least three base stations are required for the planar localization. The red cross indicates the location of the signal source. Sensors are placed at $\boldsymbol{a}_j = (10, -10)^T$, and $\boldsymbol{a}_0$ is the reference sensor. The time (range) differences $r_j - r_0$ and $r_i - r_0$ form two hyperboloids with focii located at $\boldsymbol{a}_i, \boldsymbol{a}_j$ and $\boldsymbol{a}_0$. Note that the hyperboloids are actually double sheeted, but for visual clarity only the halves which are part of the solution are shown. The intersection of these hyperboloids is the estimated position. The figure depicts the locus of possible source locations as one half of a two-sheeted hyperboloids.

The localization problem discussed in this section involves a given array of $m+1$ sensors placed in the $n = 2$ or 3 dimentional space with coordinates specified by $\{\boldsymbol{a}_1, \boldsymbol{a}_2, ..., \boldsymbol{a}_m, a_i \in R^n\}$ and $\boldsymbol{a}_0 = \boldsymbol{0}_n$ placed at the origin and used as a refference sensor. The localization problem here is to estimate the location of a radiating source $\boldsymbol{x}$ given the locations of the $m+1$ sensors and noise-contaminated range-difference measurements $\{d_i, i = 1, 2, \ldots, m\}$ where

$$d_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x}\| + \varepsilon_i, \text{ for } i = 1, 2, \ldots, m \tag{1.5}$$

Therefore, the standard range-difference LS (RD-LS) problem is formulated as

$$\text{minimize} \sum_{i=1}^{m} \left(\|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x}\| - d_i\right)^2 \tag{1.6}$$

As described in Sec.2.2 of the thesis, finding the solution to (1.6) is a non trivial problem and many approaches have been developed to address this problem. In the following we propose a new iterative procedure to tackle the RD-LS problem (1.6), with the goal of achieving a more accurate and robust solution. It operates by solving a QP problem at each iteration to find an increment vector that sequentially moves the initial estimate of the solution towards the minimum.

1.2.2 Sequential Convex Relaxation

We begin by re-writing the unconstrained problem in (1.6) as a constrained problem with second-order cone constraints

$$\begin{aligned} \underset{\boldsymbol{x}, y, \boldsymbol{z}}{\text{minimize}} \quad & \sum_{i=1}^{m} (z_i - y - d_i)^2 && (1.7a) \\ \text{subject to:} \quad & \|\boldsymbol{x} - \boldsymbol{a}_i\| = z_i && (1.7b) \\ & \|\boldsymbol{x}\| = y, \quad i = 1, 2, \ldots m && (1.7c) \end{aligned}$$

Assume we are in the kth iteration and we are to update the kth iterate $\{\boldsymbol{x}_k, y_k, \boldsymbol{z}_k\}$. Let the next iterate be

$$\begin{aligned} \boldsymbol{x}^{k+1} &= \boldsymbol{x}^k + \boldsymbol{\delta}_x && (1.8a) \\ y^{k+1} &= y^k + \delta_y && (1.8b) \\ \boldsymbol{z}^{k+1} &= \boldsymbol{z}^k + \boldsymbol{\delta}_z && (1.8c) \end{aligned}$$

where $\{\boldsymbol{\delta}_x, \delta_y, \boldsymbol{\delta}_z\}$ are such that the constraints in (1.7b) and (1.7c) are better approximated at $\{\boldsymbol{x}_{k+1}, y_{k+1}, \boldsymbol{z}_{k+1}\}$ in the sense that

$$
\begin{aligned}
\|\boldsymbol{x}_{k+1} - \boldsymbol{a}_i\| &\approx z_i^{k+1}, \quad i = 1, 2, \ldots, m \\
\|\boldsymbol{x}_{k+1}\| &\approx y_{k+1}
\end{aligned}
$$

namely,

$$
\begin{aligned}
\|\boldsymbol{x}_k + \boldsymbol{\delta}_x - \boldsymbol{a}_i\| &\approx z_i^k + \boldsymbol{\delta}_{z_i}, \quad i = 1, 2, \ldots, m \\
\|\boldsymbol{x}_k + \boldsymbol{\delta}_x\| &\approx y_k + \boldsymbol{\delta}_y
\end{aligned}
$$

By replacing the left-hand sides of the above equations with their first-order Taylor approximations, we obtain

$$
\begin{aligned}
\|\boldsymbol{x}_k - \boldsymbol{a}_i\| + \partial_x^T \|\boldsymbol{x}_k - \boldsymbol{a}_i\| \boldsymbol{\delta}_x &\approx z_i^k + \boldsymbol{\delta}_{z_i}, \quad i = 1, 2, \ldots, m \\
\|\boldsymbol{x}_k\| + \partial_x^T \|\boldsymbol{x}_k\| \boldsymbol{\delta}_x &\approx y_k + \boldsymbol{\delta}_y
\end{aligned}
$$

where ∂_x is the subdifferential operator with respect to variable $\boldsymbol{x}$. Assuming $\boldsymbol{x}_k \neq \boldsymbol{a}_i$ and $\boldsymbol{x}_k$ is nonzero, then

$$
\partial_x \|\boldsymbol{x}_k - \boldsymbol{a}_i\| = \frac{\boldsymbol{e}}{\|\boldsymbol{x}_k - \boldsymbol{a}_i\|} \text{ and } \partial_x \|\boldsymbol{x}_k\| = \frac{\boldsymbol{e}}{\|\boldsymbol{x}_k\|}
$$

where $\boldsymbol{e}$ is the all-one vector. Hence

$$
\|\boldsymbol{x}_k - \boldsymbol{a}_i\| + \frac{\mathbf{e}^T \boldsymbol{\delta}_x}{\|\boldsymbol{x}_k\|} \approx z_i^{(k)} + \delta_{z_j}, \quad i = 1, 2, ..., m \tag{1.9a}
$$

$$
\|\boldsymbol{x}_k\| + \frac{\mathbf{e}^T \boldsymbol{\delta}_x}{\|\boldsymbol{x}_k\|} \approx y_k + \delta_y \tag{1.9b}
$$

The objective in 1.7 can be written as

$$
\begin{aligned}
F(\boldsymbol{x}_{k+1}) &= \sum_{i=1}^{m} \left(z_i^{(k)} + \delta_{z_i} - (y_k + \delta_y) - d_i \right)^2 \\
&= \sum_{i=1}^{m} \left(-\delta_y + \delta_{z_i} - \tilde{d}_i^{(k)} \right)^2
\end{aligned}
$$

where

$$\tilde{d}_i^{(k)} = d_i - y_k - z_i^{(k)}$$

are grouped known constant terms. Based on the above, the problem to be solved in the kth iteration is formulated as

$$\underset{\boldsymbol{\delta}}{\text{minimize}} \quad f(\tilde{\boldsymbol{\delta}}) = \sum_{i=1}^{m} \left(-\delta_y + \delta_{z_j} - \tilde{d}_i^{(k)}\right)^2 \tag{1.10a}$$

$$\text{subject to:} \quad \|\boldsymbol{x}_k - \boldsymbol{a}_i\| + \frac{\boldsymbol{e}^T \boldsymbol{\delta}_x}{\|\boldsymbol{x}_k - \boldsymbol{a}_i\|} = z_i^{(k)} + \delta_{z_j}, i = 1, 2, ..., m \tag{1.10b}$$

$$\|\boldsymbol{x}_k\| + \frac{\boldsymbol{e}^T \boldsymbol{\delta}_x}{\|\boldsymbol{x}_k\|} = y_k + \delta_y \tag{1.10c}$$

$$\begin{bmatrix} -\beta \mathbf{1}_2 \\ -\min\{\beta, y_k\} \\ -\min\{\beta, \boldsymbol{z}_k\} \end{bmatrix} \leq \begin{bmatrix} \boldsymbol{\delta}_x \\ \delta_y \\ \boldsymbol{\delta}_z \end{bmatrix} \leq \begin{bmatrix} \beta \mathbf{1}_2 \\ \beta \\ \beta \mathbf{1}_m \end{bmatrix} \tag{1.10d}$$

The constraints in (1.10d) not only assure that the magnitude of each component in $\{\boldsymbol{\delta}_x, \delta_y, \boldsymbol{\delta}_z\}$ is no greater than β, but also they assure that all components of $\{y_{k+1}, \boldsymbol{z}_{k+1}\}$ are nonnegative as long as $\{y_k, \boldsymbol{z}_k\}$ are nonnegative, which are natural to impose as can be seen from (1.7b) and (1.7c) because they are vector norms. Obviously, the problem in (1.10) is a convex QP problem. One technical difficulty that may occur in solving problem (1.10) is that the feasible region defined by (1.10b), (1.10c), and (1.10d) may be empty. In such a case the constraints in problem (1.10) much be adequately relaxed in order for the problem to be solvable. To this end we rewrite (1.10) as

$$\underset{\boldsymbol{\delta}}{\text{minimize}} \quad f\left(\tilde{\boldsymbol{\delta}}\right) \tag{1.11a}$$

$$\text{subject to} \quad \boldsymbol{A}\tilde{\boldsymbol{\delta}} = \boldsymbol{b} \tag{1.11b}$$

$$\boldsymbol{C}\tilde{\boldsymbol{\delta}} \leq \boldsymbol{d} \tag{1.11c}$$

where

2) Form $\tilde{\boldsymbol{d}}_k, \boldsymbol{p}_k$ and $\boldsymbol{C}_k$ as

$$\tilde{\boldsymbol{d}}_k = \begin{bmatrix} d_1 + y^k - z_1^k \\ d_2 + y^k - z_2^k \\ \vdots \\ d_m + y^k - z_m^k \end{bmatrix}, \quad \boldsymbol{p}_k = \begin{bmatrix} \|\boldsymbol{x}^k\|^2 - \left(y^k\right)^2 \\ \|\boldsymbol{x}^k - \boldsymbol{a}_1\|^2 - \left(z_1^k\right)^2 \\ \vdots \\ \|\boldsymbol{x}^k - \boldsymbol{a}_m\|^2 - \left(z_m^k\right)^2 \end{bmatrix}$$

$$\boldsymbol{C}_k = \begin{bmatrix} -2\left(\boldsymbol{x}^k\right)^T & 2y^k & 0 & \dots & 0 \\ -2\left(\boldsymbol{x}^k - \boldsymbol{a}_1\right)^T & 0 & 2z_1^k & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -2\left(\boldsymbol{x}^k - \boldsymbol{a}_m\right)^T & 0 & 0 & \dots & 2z_m^k \end{bmatrix},$$

$\tilde{\boldsymbol{\delta}}^k = (\boldsymbol{\delta}_x^*, \delta_y^*, \boldsymbol{\delta}_z^*)$.

By introducing nonnegative slack variables $\boldsymbol{u}, \boldsymbol{v}$ and $\boldsymbol{w}$, we relax the problem in (1.11) to

$$\underset{\boldsymbol{\delta}}{\text{minimize}} \quad f\left(\tilde{\boldsymbol{\delta}}\right) + \tau \sum_{i=1}^{m+1} (u_i + v_i) + \tau \sum_{j=1}^{2(m+3)} w_j \tag{1.12a}$$

$$\text{subject to} \quad \boldsymbol{A}\tilde{\boldsymbol{\delta}} - \boldsymbol{b} = \boldsymbol{u} - \boldsymbol{v} \tag{1.12b}$$

$$\boldsymbol{C}\tilde{\boldsymbol{\delta}} - \boldsymbol{d} \leq \boldsymbol{w} \tag{1.12c}$$

$$\boldsymbol{u} \geq \boldsymbol{0}, \boldsymbol{v} \geq \boldsymbol{0}, \boldsymbol{w} \geq \boldsymbol{0} \tag{1.12d}$$

where $\tau > 0$ is a sufficiently large scalar. It is easy to verify that the feasible region defined by (1.12b) - (1.12d) is always nonempty. For example, if we fix $\tilde{\boldsymbol{\delta}} = \tilde{\boldsymbol{\delta}}_0$ arbitrarily, then obviously the point $\{\tilde{\boldsymbol{\delta}}_0, \boldsymbol{u}_0, \boldsymbol{v}_0, \boldsymbol{w}_0\}$ with

$$\boldsymbol{u}_0 = \max\{\boldsymbol{0}, \boldsymbol{A}\tilde{\boldsymbol{\delta}}_0 - \boldsymbol{b}\}, \quad \boldsymbol{v}_0 = \max\{\boldsymbol{0}, -(\boldsymbol{A}\tilde{\boldsymbol{\delta}}_0 - \boldsymbol{b})\}, \text{and } \boldsymbol{w}_0 = \max\{\boldsymbol{0}, \boldsymbol{C}\tilde{\boldsymbol{\delta}}_0 - \boldsymbol{d}\}$$

is a feasible point for problem (1.12). The penalty term tries to reduce the magnitudes of the slack variables while minimizing the original objective function. If the solution slack variables turn out to be all zero, then the solution of (1.12) also solves problem (1.11). Otherwise, we conclude that problem (1.11) in not solvable and the solution obtained by solving (1.12) is a reasonable candidate for the kth iteration to update $\{\boldsymbol{x}_k, y_k, \boldsymbol{z}_k\}$.

1.2.3 Numerical Results

For illustration purposes, the proposed algorithm was applied to a network with five sensors, and its performance was evaluated and compared with existing state-of-the-art methods by Monte Carlo simulations with a set-up similar to that of [15]. SR-LS solutions were used as performance benchmarks for the PCCP-based LS Algorithm. The system consisted of 5 sensors $\{\boldsymbol{a}_i, i = 1, 2, \ldots, 5\}$ randomly placed in the planar region in $[-15; 15] \times [-15; 15]$, and a radiating source $\boldsymbol{x}_s$, located randomly in the region $\{\boldsymbol{x} = [x_1; x_2], -10 \leq x_1, x_2 \leq 10\}$. The coordinates of the source and sensors were generated for each dimension following a uniform distribution. Measurement noise $\{\varepsilon_i, i = 1, \ldots, m\}$ was modelled as independent and identically distributed (i.i.d) random variables with zero mean and variance σ^2, with σ being one of four possible levels $\{10^{-3}, 10^{-2}, 10^{-1}, 1\}$. The range-difference measurements $\{d_i, i = 1, 2, \ldots, 5\}$ were calculated using (1.5). Accuracy of source location estimation was evaluated in terms of average of the squared position error error in the form $\|\boldsymbol{x}^* - \boldsymbol{x}_s\|^2$, where $\boldsymbol{x}_s$ denotes the exact source location and $\boldsymbol{x}^*$ is its estimation obtained by SR-LS and proposed methods, respectively. In our simulations parameter β was set to () and the number of iterations was set to (). The proposed method was implemented by using CVX [46] and implementation of SRD-LS followed [15]. The proposed algorithm was initialized with ...

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