Localization Algorithms in Passive Sensor Networks

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Outline

1. Motivation
2. Basic Localization Systems and Methods
3. Iterative Re-Weighting Least-Squares Methods for Source Localization
4. Penalty Convex-Concave Procedure for Range-based Localization
5. Conclusions and Future Work

Introduction

- Navigation: outdoor; indoor
- Surveillance
- Localization of emergency callers
- Emergency and rescue operations / first responders
- Self-organizing networks
- Asset monitoring and tracking
- Other commercial location-based services
- ...

Introduction

- Ranging methods
  - range measurements (Time Of Arrival)
  - range-difference measurements (Time-Difference of Arrival)
  - received signal strength
- Angle Of Arrival Techniques
- Survey-Based Systems (fingerprinting)
  - memoryless systems (SVM, NN)
  - memory systems (Bayesian inference, grid-based Markov)
  - channel impulse response fingerprinting non-RF features

Basic Localization Systems and Methods

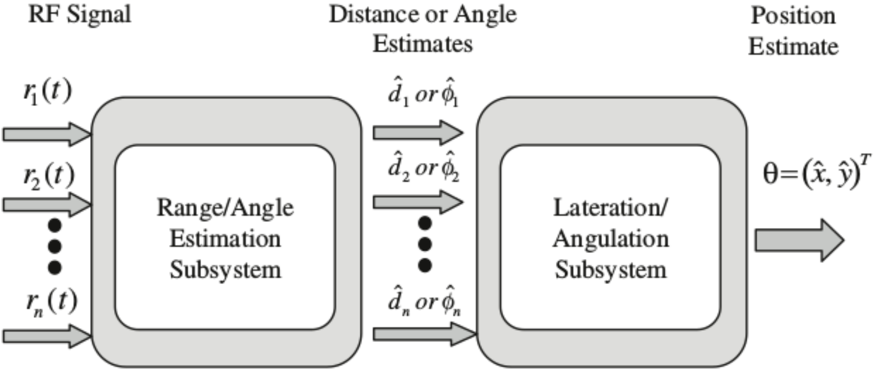

Figure: Classical geolocation system. Range or angle information is extracted from received RF signals. Location is then estimated by lateration/angulation techniques [GeoLoc].

Time Of Arrival Localization (TOA)

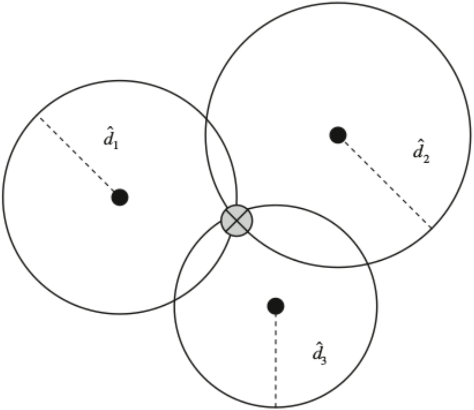

Figure: TOA-based trilateration. Range measurements to at least three BS make up a set of nonlinear equations that can be solved to estimate the position of a signal source [GeoLoc].

Time Of Arrival Localization (TOA)

The nonlinear least squares (NLLS) source location extimate $\hat{\boldsymbol{x}}$ is found by

$$\hat{\boldsymbol{x}} = \text{argmin}_{\boldsymbol{x}} \left\{ \sum_{i=1}^{m} \beta_i \left(d_n^i - \|\boldsymbol{x} - \boldsymbol{a}_i\| \right)^2 \right\}$$

where

$\boldsymbol{a}_i$- a vector of known coordinates of reference points (sensors)

d_n^i - a noisy range measurement associated with it

β_i - a weight used to emphasize the degree of confidence in the measurement

m - the number of sensors.

Time-Difference Of Arrival Localization (TDOA)

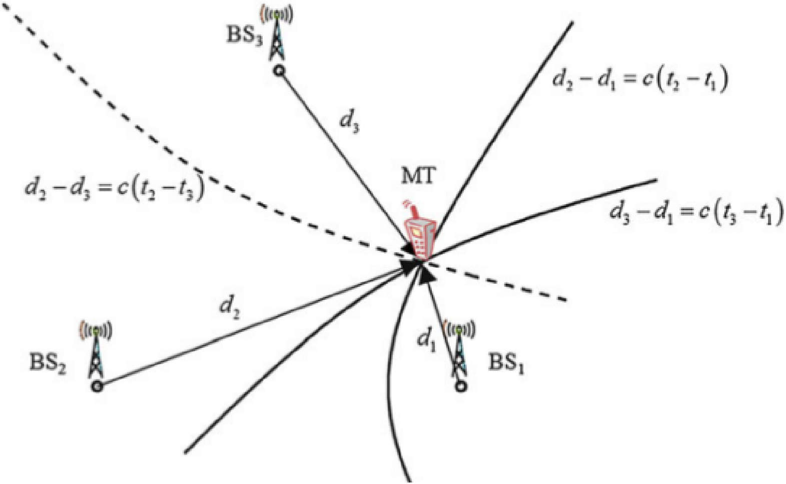

Figure: Example of observed time-difference of arrival (O-TDOA) method [GeoLoc].

Time-Difference Of Arrival Localization (TDOA)

Given the range-difference measurements

$$d_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x} - \boldsymbol{a}_0\| = \|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x}\|, \text{for } i = 1, 2, \ldots, m$$

The standard NLLS location estimate $\hat{\boldsymbol{x}}$ is found by

$$\hat{\boldsymbol{x}} = \underset{\boldsymbol{x}}{\text{minimize}} \sum_{i=1}^{m} \left(\|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x}\| - d_n^i \right)^2$$

with

$\boldsymbol{a}_i$- a vector of known coordinates of reference points (sensors)
d_n^i - a noisy range-difference measurement associated with it
m - the number of sensors.

Methods Based on Received Signal Strength (RSS-based)

The relationship between the RSS reading and the distance can be approximated by

$$P_x(d) = P_0(d_0) - 10 n_p \log_{10}\left(\frac{d_i}{d_0}\right) + X_\sigma$$

where
$P_0(d_0)$ - a reference power in dB milliwatts at a reference distance d_0 away from the transmitter
n_p - the pathloss exponent
X_σ - the log-normal shadow fading component with variance σ^2
d_i - the distance between the mobile devices and the ith base station
σ and n_p are environment dependent

Why Least Squares

- Least squares (LS) algorithms for range-based localization:
  - geometrically meaningful
  - provide low complexity solutions with competitive accuracy
- However:
  - the error measure is non-convex
  - excludes many local methods, that are iterative
- Solutions obtained using global localization techniques such as semidefinite programming (SDP) are not optimal in LS sense.

Iterative Re-Weighting Least-Squares Methods for Source Localization

- Methods developed by A. Beck, P. Stoica, J. Li [BSL2008] for *squared* range LS (SR-LS) and *squared* range difference LS (SDR-LS) problems allow to obtain exact and *global* solutions.
- The results produced are merely approximations of the original LS problems because SR-LS and SRD-LS are no longer an ML solutions.
- Proposed iterative procedure where the SR-LS (or SRD-LS) algorithm is applied to a *weighted* sum of squared terms and special weights construction allow to obtain a solution which is considerably closer to the original range-based (or range-difference-based) LS solution.

Source Localization From Range Measurements

Measurement Model

- Throughout it is assumed that *range measurements* obey the model

$$r_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| + \varepsilon_i, \quad i = 1, \ldots, m.$$

where $\{\boldsymbol{a}_1, \ldots, \boldsymbol{a}_m\}$ - given array of m sensors;
$\boldsymbol{a}_i \in R^n$ contains n coordinates of the ith sensor in space R^n;
r_i - received noisy distance reading from the ith sensor;
ε_i - unknown noise associated with measurement from the ith sensor.

- The problem can be stated as to estimate the exact source location $\boldsymbol{x} \in R^n$ from noisy range measurements $\boldsymbol{r} = [r_1 \ r_2 \ldots r_m]^T$.

Source Localization From Range Measurements

LS Formulations

- The range-based least squares (R-LS) estimate refers to the solution of the problem

$$\underset{\boldsymbol{x}}{\text{minimize}}\, f(\boldsymbol{x}) = \sum_{i=1}^{m} (r_i - \|\boldsymbol{x} - \boldsymbol{a}_i\|)^2 \tag{R}$$

- If $\varepsilon \sim N(0, \boldsymbol{\Sigma})$ and $\boldsymbol{\Sigma} \propto \boldsymbol{I}$, then the R-LS solution of problem (R) is identical to the ML location estimator.

- Unfortunately, the objective in (R) is highly non-convex, possessing many local minimizers even for small-scale systems.

Source Localization From Range Measurements

LS Formulations

- Alternatively, location estimate can be obtained by solving the *squared range based LS* (SR-LS) problem [BSL2008]

$$\underset{\boldsymbol{x}}{\text{minimize}} \sum_{i=1}^{m} (\|\boldsymbol{x} - \boldsymbol{a}_i\|^2 - r_i^2)^2 \tag{SR}$$

- The SR-LS estimate is no longer an ML solution, hence, only an approximation of the original R-LS problem.
- To reduce the gap between the two solutions we propose a weighted SR-LS (WSR-LS) problem:

$$\underset{\boldsymbol{x}}{\text{minimize}} \sum_{i=1}^{m} w_i \left(\|\boldsymbol{x} - \boldsymbol{a}_i\|^2 - r_i^2\right)^2 \tag{WSR}$$

Source Localization From Range Measurements

An Iterative Re-Weighting Strategy

- WSR-LS with properly chosen weights facilitates an excellent approximation of the R-LS estimate.

- The main idea is to use the weights $w_i, i = 1, \ldots, m$ to tune the objective in (WSR) toward the objective in (R).

$$\underbrace{w_i \left(\|\boldsymbol{x} - \boldsymbol{a}_i\|^2 - r_i^2\right)^2}_{\text{in (WSR)}} \leftrightarrow \underbrace{\left(\|\boldsymbol{x} - \boldsymbol{a}_i\| - r_i\right)^2}_{\text{in (R)}}$$

An Iterative Re-Weighting Strategy

- By writing the ith term in (WSR) as

$$w_i \left(\|\boldsymbol{x} - \boldsymbol{a}_i\|^2 - r_i^2 \right)^2 = w_i \left(\|\boldsymbol{x} - \boldsymbol{a}_i\| + r_i \right)^2 \underbrace{\left(\|\boldsymbol{x} - \boldsymbol{a}_i\| - r_i \right)^2}_{\text{same as in (R)}}$$

  we note that the objective in (WSR) would be the same as in (R) if the weight w_i was assigned to $1 / \left(\|\boldsymbol{x} - \boldsymbol{a}_i\| + r_i \right)^2$.

- Evidently, such weight assignments cannot be realized.

Source Localization From Range Measurements

An Iterative Re-Weighting Strategy

- In the proposed iterative procedure we solve a weighted SR-LS sub-problem,where at each iteration the weights are fixed:

$$\underset{x}{\text{minimize}} \sum_{i=1}^{m} w_i^{(k)} \left(\|\boldsymbol{x} - \boldsymbol{a}_i\|^2 - r_i^2 \right)^2 \qquad \text{(IRWSR)}$$

- for $k = 1$ all weights $\{w_i^{(1)}, i = 1, \dots, m\}$ are set to unity;
- for $k \geq 2$ the weights $\{w_i^{(k)}, i = 1, \dots, m\}$ are assigned using the previous iterate $\boldsymbol{x}_{k-1}$ as

$$w_i^{(k)} = \frac{1}{\left(\|\boldsymbol{x}_{k-1} - \boldsymbol{a}_i\| + r_i\right)^2}.$$

Source Localization From Range-Difference Measurements

Problem Statement

- It is assumed that the range-difference measurements obey the model:

$$d_i = \|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x} - \boldsymbol{a}_0\| = \|\boldsymbol{x} - \boldsymbol{a}_i\| - \|\boldsymbol{x}\|, \quad i = 1, \dots, m$$

  where $\boldsymbol{a}_0$ - reference sensor placed at the origin.

- The standard range-difference LS (RD-LS) problem is formulated as

$$\underset{\boldsymbol{x} \in R^n}{\text{minimize}}\, F(\boldsymbol{x}) = \sum_{i=1}^{m} (d_i + \|\boldsymbol{x}\| - \|\boldsymbol{x} - \boldsymbol{a}_i\|)^2 \qquad \text{(RD)}$$

Source Localization From Range-Difference Measurements

SRD-LS and WSRD-LS formulations

- An approximation of the RD-LS solution can be obtained by solving the *squared range difference based LS* (SRD-LS) problem.
- By re-writing the measurements model as $d_i + \|\boldsymbol{x}\| = \|\boldsymbol{x} - \boldsymbol{a}_i\|$ and squaring both sides, we obtain

$$-2d_i\|\boldsymbol{x}\| - 2\boldsymbol{a}_i^T\boldsymbol{x} = g_i, \quad i = 1, \ldots, m$$

where $g_i = d_i^2 - \|\boldsymbol{a}_i\|^2$. The SRD-LS solution can be obtained by minimizing following criterion:

$$\underset{\boldsymbol{x} \in R^n}{\text{minimize}} \sum_{i=1}^{m} \left(-2\boldsymbol{a}_i^T\boldsymbol{x} - 2d_i\|\boldsymbol{x}\| - g_i\right)^2$$

Source Localization From Range-Difference Measurements

Improved Solution Using Iterative Re-weighting

- We now present a method for improved solutions over SRD-LS solutions.

- We consider the weighted SRD-LS problem

$$\underset{\boldsymbol{x} \in R^n}{\text{minimize}} \sum_{i=1}^{m} w_i \left(-2\boldsymbol{a}_i^T \boldsymbol{x} - 2d_i \|\boldsymbol{x}\| - g_i \right)^2 \qquad \text{(WSRD)}$$

where weights w_i for $i = 1, \ldots, m$ are *fixed* nonnegative constants.

Source Localization From Range-Difference Measurements

Improved Solution Using Iterative Re-weighting

- The ith term of the objective function in (WSRD) can be written as:

$$
\begin{aligned}
& w_i \left(-2d_i \|\boldsymbol{x}\| - 2\boldsymbol{a}_i^T \boldsymbol{x} - g_i \right)^2 \\
= & w_i \left(d_i + \|\boldsymbol{x}\| + \|\boldsymbol{x} - \boldsymbol{a}_i\| \right) \underbrace{\left(d_i + \|\boldsymbol{x}\| - \|\boldsymbol{x} - \boldsymbol{a}_i\| \right)}_{\text{same as in RD}}
\end{aligned}
$$

- If weights w_i were set to $1/\left(d_i + \|\boldsymbol{x}\| + \|\boldsymbol{x} - \boldsymbol{a}_i\| \right)^2$ the objective in (WSRD) would be the same as in (RD).

Source Localization From Range-Difference Measurements

Improved Solution Using Iterative Re-weighting

- We employ an iterative procedure where the weights in the kth iteration are assigned to

$$w_i^{(k)} = \frac{1}{(d_i + \|\boldsymbol{x}_{k-1}\| + \|\boldsymbol{x}_{k-1} - \boldsymbol{a}_i\|)^2}, i = 1, \dots, m$$

with $\{w_i^{(1)} = 1, i = 1, \dots, m\}$.

- We will refer to the derived problem as the iterative re-weighted SRD-LS (WSRD-LS) problem and the solution obtained as IRWSRD-LS solution.

Performance Evaluation for SR-LS and IRWSR-LS

- We can see that IRWSR-LS solutions offer considerable improvement over SR-LS solutions.

Table: Averaged MSE for SR-LS and IRWSR-LS methods by noise level

σ	**SR - LS**	**IRWSR-LS**	**Improvement (%)**
1e-03	2.03251062e-06	1.19962894e-06	41
1e-02	1.83717590e-04	1.24797437e-04	32
1e-01	1.83611315e-02	1.22233840e-02	33

Performance Evaluation for SRD-LS and IRWSRD-LS

Table: Averaged MSE for SRD-LS and IRWSRD-LS methods by noise level

σ	**SRD - LS**	**IRWSRD-LS**	**Improvement (%)**
1e-04	1.38301598e-08	8.22705918e-09	40
1e-03	1.60398717e-06	1.03880406e-06	35
1e-02	1.11632818e-04	6.67785604e-05	40
1e-01	1.20947651e-02	7.20891487e-03	40
1e+0	1.57050323e+00	9.70756420e-01	40

Conclusions

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- These methods are developed by transforming the SR-LS and SRD-LS algorithms [BSL2008] into an iterative procedure so that a weighted SR-LS (SRD-LS) objective asymptotically approaches the original R-LS objective.

Conclusions

- New global methods for locating a radiating source based on noisy range or range difference measurements have been proposed.
- These methods are developed by transforming the SR-LS and SRD-LS algorithms [BSL2008] into an iterative procedure so that a weighted SR-LS (SRD-LS) objective asymptotically approaches the original R-LS objective.
- Proposed algorithms are found to outperform the existing methods.

Q & A

Appendix

Source Localization From Range Measurements

Weighted Squared Range Least Squares Formulation

- Following [BSL2008], we convert (WSR) into a GTRS as

$$\underset{\boldsymbol{y}\in R^{n+1}}{\text{minimize}} \, \|\boldsymbol{A}_w \boldsymbol{y} - \boldsymbol{b}_w\|^2 \tag{1a}$$

$$\text{subject to:} \quad \boldsymbol{y}^T \boldsymbol{D} \boldsymbol{y} + 2\boldsymbol{f}^T \boldsymbol{y} = 0 \tag{1b}$$

where $\boldsymbol{y} = [\boldsymbol{x}^T \ \alpha]^T$, $\alpha = \|\boldsymbol{x}\|$, $\boldsymbol{A}_w = \boldsymbol{\Gamma A}$ and $\boldsymbol{b}_w = \boldsymbol{\Gamma b}$ with fixed $\boldsymbol{\Gamma} = \text{diag}\left(\sqrt{w_1}, \ldots, \sqrt{w_m}\right)$, and

$$\boldsymbol{A} = \begin{pmatrix} -2\boldsymbol{a}_1^T & 1 \\ \vdots & \vdots \\ -2\boldsymbol{a}_m^T & 1 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} r_1^T - \|\boldsymbol{a}_1\|^T \\ \vdots \\ r_m^T - \|\boldsymbol{a}_m\|^T \end{pmatrix} \tag{2}$$

$$\boldsymbol{D} = \begin{pmatrix} \boldsymbol{I}_{n\times n} & \boldsymbol{0}_{n\times 1} \\ \boldsymbol{0}_{1\times n} & 0 \end{pmatrix}, \boldsymbol{f} = \begin{pmatrix} \boldsymbol{0} \\ -0.5 \end{pmatrix} \tag{3}$$

Source Localization From Range Measurements

The Algorithm

1. Input data: Sensor locations $\{\boldsymbol{a}_i, i = 1, \ldots, m\}$, range measurements $\{r_i, i = 1, \ldots, m\}$, maximum number of iterations k_{max} and convergence tolerance ζ.
2. Generate data set $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{D}, \boldsymbol{f}$ using (2) and (3). Set $k = 1, w_i^{(1)} = 1$ for $i = 1, \ldots, m$.
3. Set $\boldsymbol{\Gamma}_k = \text{diag}\left(\sqrt{w_1^{(k)}}, \ldots, \sqrt{w_m^{(k)}}\right)$, $\boldsymbol{A}_w = \boldsymbol{\Gamma}_k \boldsymbol{A}$ and $\boldsymbol{b}_w = \boldsymbol{\Gamma}_k \boldsymbol{b}$.
4. Solve the WSR-LS problem (IRWSR) via (1) to obtain its global solution $\boldsymbol{x}_k$.
5. If $k = k_{max}$ or $\|\boldsymbol{x}_k - \boldsymbol{x}_{k-1}\| < \zeta$, terminate and output $\boldsymbol{x}_k$ as the solution; otherwise, set $k = k + 1$, update weights $\{w_i^{(k)}, i = 1, \ldots, m\}$ and repeat from Step 3).

Source Localization From Range-Difference Measurements

Weighted Squared Range-Difference Least Squares Formulation

- By introducing new variable $\boldsymbol{y} = [\boldsymbol{x}^T \ \|\boldsymbol{x}\|]^T$ and noticing nonnegativity of the component y_{n+1} problem (WSRD) is converted to

$$\underset{\boldsymbol{y} \in R^{n+1}}{\text{minimize}} \|\boldsymbol{B}_w \boldsymbol{y} - \boldsymbol{g}_w\| \tag{4a}$$

$$\text{subject to: } \boldsymbol{y}^T \boldsymbol{C} \boldsymbol{y} = 0 \tag{4b}$$

$$y_{n+1} \geq 0 \tag{4c}$$

- where $\boldsymbol{B}_w = \boldsymbol{\Gamma} \boldsymbol{B}$, $\boldsymbol{g}_w = \boldsymbol{\Gamma} \boldsymbol{g}$, $\boldsymbol{\Gamma} = \text{diag}\{\sqrt{w_1}, \ldots, \sqrt{w_m}\}$, $\boldsymbol{g} = [g_1 \ldots g_m]^T$ and

$$\boldsymbol{B} = \begin{pmatrix} -2\boldsymbol{a}_1^T & -2d_1 \\ \vdots & \vdots \\ -2\boldsymbol{a}_m^T & -2d_m \end{pmatrix}, \boldsymbol{C} = \begin{pmatrix} \boldsymbol{I}_n & \boldsymbol{0}_{n\times 1} \\ \boldsymbol{0}_{1\times n} & -1 \end{pmatrix} \tag{5}$$

The Algorithm

1. Input data: Sensor locations $\{\boldsymbol{a}_i, i = 0, 1, \ldots, m\}$ with $\boldsymbol{a}_0 = \mathbf{0}$, range-difference measurements $\{d_i, i = 1, \ldots, m\}$, maximum number of iterations k_{max} and convergence tolerance ξ.
2. Generate data set $\{\boldsymbol{B}, \boldsymbol{g}, \boldsymbol{C}\}$ using (5). Set $k = 1$, $w_i^{(1)} = 1$ for $i = 1, \ldots, m$.
3. Set $\boldsymbol{\Gamma}_k = \text{diag}\left(\sqrt{w_1^{(k)}}, \ldots, \sqrt{w_m^{(k)}}\right)$, $\boldsymbol{B}_w = \boldsymbol{\Gamma}_k \boldsymbol{B}$ and $\boldsymbol{g}_w = \boldsymbol{\Gamma}_k \boldsymbol{g}$.
4. Solve WSRD-LS problem (4) to obtain its global solution $\boldsymbol{x}_k$.
5. If $k = k_{max}$ or $\|\boldsymbol{x}_k - \boldsymbol{x}_{k-1}\| < \xi$, terminate and output $\boldsymbol{x}_k$ as the solution; otherwise, set $k = k + 1$, update weights $\{w_i^{(k)}, i = 1, \ldots, m\}$ and repeat from Step 3).

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