Mathematics

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Preface

Mathematics possesses not only truth, but supreme beauty,a beauty cold and austere, like that of a sculpture, and capable of stern perfection, such as only great art can show.
–Bertrand Russell

Contents

1 Introduction

Steps for developing a usable algorithm

- Model the Problem
- Find an algorithm to solve it.
- Fast Enough? Fits in memory?
- If not, figure out why.
- Find a way to address the problem.
- Iterate until satisfied

The scientific method

Mathematical analysis

2

Union-Find

2.1 Dynamic Connectivity

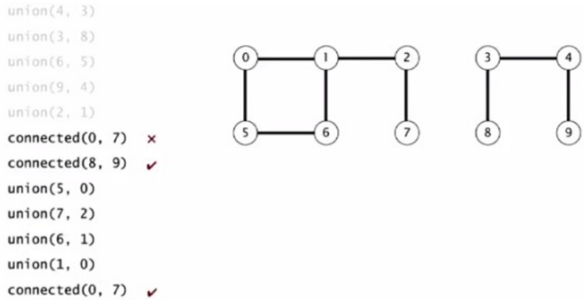
2.1.1 Applications involve manipulating objects of all types

- Pixels in a digital photo
- Computers in a network
- Friends in a social network.
- Transistors in a computer chip
- Variable name in Fortran program
- Metallic sites in a composite system

Given a set of N objects

Union command: connect two objects

Find/connected query: is there a path connecting the two objects?



```
union(4, 3)
union(3, 8)
union(6, 5)
union(9, 4)
union(2, 1)
connected(0, 7)   ×
connected(8, 9)   ✓
union(5, 0)
union(7, 2)
union(6, 1)
union(1, 0)
connected(0, 7)   ✓
```

2.1.2 Implementing the operations

Find Query Check if two objects are in the same component.

Union Command Replace components containing two objects with their union.

For example if you have [0][1 4 5][2 3 6 7] where each [X...X] represents the connectec components if you use the operation **union(2,5)** you will have [0][1 2 3 4 5 6 7]

2.1.3 Union-find data type (API)

Goal Design efficient data structure for union-find.

- Number of objects N can be huge.
- Number of operations M can be huge.
- Find queries and union commands may be intermixed.

Public Class UF	
UF(int N)	initialize union-find data structure with N objects(0 to N-1)
void union(int p, int q)	add connection between p and q
boolean connected(int p, int q)	are p and q in the same component ?
int find(int p)	component identifier for p(0 to N-1)
int count()	number of components

2.2 Quick Find

Data Structure

- Integer array id[] of size N.
- Interpretation: p and q are connected if they have the same id.