Precalculus

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Preface

The goal of this notes, is to have a formal reference of the important subjects necessary to understand calculus and more advanced subjects.

Mathematics possesses not only truth, but supreme beauty, a beauty cold and austere, like that of a sculpture, and capable of stern perfection, such as only great art can show.
–Bertrand Russell

Contents

1 Foundations

1.1 Greek Alphabet

Letter	*Lower*	*Upper*
alpha	α	A
beta	β	B
gamma	γ	Γ
delta	δ	Δ
epsilon	ϵ	E
zeta	ζ	Z
eta	η	H
theta	θ	Θ
iota	ι	I
kappa	κ	K
lambda	λ	Λ
mu	μ	Mu

Letter	*Lower*	*Upper*
nu	ν	N
xi	ξ	Ξ
omicron	o	O
pi	π	Π
rho	ρ	P
sigma	σ	Σ
tau	τ	T
upsilon	υ	Υ
phi	ϕ	Φ
chi	χ	X
psi	ψ	Ψ
omega	ω	Ω

1.2 Language of mathematics

The language of mathematics is a system to describe concrete ideas.

1.3 Sets

A set is a collection of distinct objects.

1.3.1 Special Sets

There are some sets that hold a great mathematical importance and are used regularly everywhere so they have acquire their own names and their conventions.
The empty set is one example, is usually denoted by ∅ or . Different families of numbers have their own names as well like:

- Prime Numbers - ℙ or **P** = {2, 3, 5, 7, 11, 13, 17, 19, 23, ...}
- Natural Numbers - ℕ or **N** = {1, 2, 3, 4, ...} sometimes 0 is considered as well
- Integers - ℤ or **Z** = {..., -2, -1, 0, 1, 2, ...}
- Real - ℝ or **R** = Every number that can be found on the number line
- Complex Numbers - ℂ or **C** = Every number that can be expressed in the form $a + bi$
- Irrational Numbers - 𝕀 or **I** = any real number that cannot be expressed as a/b where a,b are integers
- Rational Numbers - ℚ or **Q** = any number that can be expressed as a/b where a,b are integers

1.3.2 Operations

There are several operations for construction new sets.

Unions

The union of $\mathbf{A}$ and $\mathbf{B}$ is denoted by $\mathbf{A} \cup \mathbf{B}$, can be also seen as the set of elements that belong to $\mathbf{A}$ or $\mathbf{B}$.

Intersections

The intersection of $\mathbf{A}$ and $\mathbf{B}$ is denoted by $\mathbf{A} \cap \mathbf{B}$, can be also seen as the set of elements that belong to $\mathbf{A}$ and $\mathbf{B}$. If $\mathbf{A}$ and $\mathbf{B}$ don't have any elements in common their intersection is the $\emptyset$ and they are said to be disjoint.

Complements

A set complement is everything else that does not belong in it. $\mathbf{A} \cap \mathbf{A} = \Omega$

2 Functions

A function f is a rule that assigns to each element x in a set A exactly one element, called $f(x)$, in a set B **MACHINEDIAGRAMHERE**

A function can be represented in four different ways :

- verbally
- algebraically
- visually
- numerically

2.1 Linear

A linear equation in one variable is an equation equivalent to one of the form

$$ax + b = 0$$

where a and b are real numbers and x is the variable.

2.2 Quadratic

A quadratic equation is and equation of the form

$$ax^2 + bx + c = 0$$

where a,b, and c are real numbers with a $\neq$ 0

zero-product property

$AB = 0 \iff A = 0$ or $B = 0$

factoring

Completing the square

Quadratic Formula

The roots of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PROOF:

$$\begin{aligned}
0 &= ax^2 + bx + c && \text{starting from the base polynomial} \\
\frac{-c}{a} &= x^2 + \frac{b}{a}x && \text{divide by a} \\
x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 &= -\frac{c}{a} + (\frac{b}{2a})^2 && \text{completing the square with } (\frac{b}{2a})^2 \\
(x + \frac{b}{2a})^2 &= \frac{-4ac + b^2}{4a^2} && \text{Perfect Square} \\
x + \frac{b}{2a} &= \pm\frac{\sqrt{b^2 - 4ac}}{2a} && \text{Take square root} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} && \text{subtract } \frac{b}{2a}
\end{aligned}$$

Descriminant: the discriminant of the general quadratic $ax^2 + bx + c = 0 (a \neq 0)$ is $D = b^2 - 4ac$ where

- if $D > 0$ then the equation has two distinct real solutions.
- if $D = 0$ then the equation has exactly one real solution.
- if $D < 0$ then the equation has no real solution.

2.3 Rational and Polynomial

2.4 Exponential and Logarithmic

2.4.1 Binomial Theorem

2.5 Trigonometric

2.6 Hyperbolics

3
Inequalities

4
Lines

The slope of a line or the steepness of a line is how quicly it rises or falls as we move from right to left. We define run to be the distance in the x axis and rise to be the corresponding distance in the y axis. The slope of a line can be then be expressed by

$$slope = rate of change = m = \frac{rise}{run} = \frac{y}{x} = \frac{y_2 - y_1}{x_2 - x_1}$$

General Equation of a line

The graph of every **linear equation** $Ax + By + C = 0$ if (A,B not both zero) is a line.

point-slope form of the equation of a line

$$y - y_1 = m(x - x_1)$$

slope-intercept form of the equation of a line

$$y = mx + b$$

parallel lines

two nonvertical lines are parallel $\iff$ they have the same slope

Perpendicular lines

two lines with slopes m_1 and m_2 are perpendicular $\iff$ $m_1 m_2 = -1$, that is their slopes are negative reciprocals:

$$m_2 = -\frac{1}{m_1}$$

also, a horizontal line $m = 0$ is perpendicular to a vertical line.

5
Geometry

6
Sequences and Series

7
Conics

8

Trigonometry