Algorithms

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Preface

Mathematics possesses not only truth, but supreme beauty, a beauty cold and austere, like that of a sculpture, and capable of stern perfection, such as only great art can show.

-Bertrand Russell

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Introduction

Steps for developing a usable algorithm

- Model the Problem
- Find an algorithm to solve it.
- Fast Enough? Fits in memory?
- If not, figure out why.
- Find a way to address the problem.
- Iterate until satisfied

The scientific method

Mathematical analysis



2.1 Dynamic Connectivity

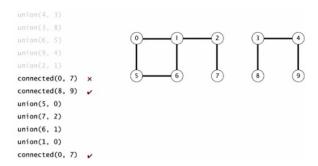
2.1.1 Applications involve manipulating objects of all types

- Pixels in a digital photo
- Computers in a network
- Friends in a social network.
- Transistors in a computer chip
- Variable name in Fortran program
- Metallic sites in a composite system

Given a set of N objects

Union command: connect two objects

Find/connected query: is there a path connecting the two objects?



2.1.2 Implementing the operations

Find Query Check if two objects are in the same component.

Union Command Replace components containing two objects with their union.

For example if you have $[0][1 \ 4 \ 5][2 \ 3 \ 6 \ 7]$ where each [X...X] represents the connected components if you use the operation **union(2,5)** you will have $[0][1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7]$

2.1.3 Union-find data type (API)

Goal Design efficient data structure for union-find.

- Number of objects N can be huge.
- Number of operations M can be huge.
- Find queries and union commands may be intermixed.

Public Class UF

UF(int N)	initialize union-find data structure with N objects(o to N-1)
void union(int p, int q)	add connection between p and q
boolean connected(int p, int q)	are p and q in the same component?
int find(int p)	component identifier for p(o to N-1)
int count()	number of components

2.2 Quick Find

Data Structure

- Integer array id[] of size N.
- Interpretation: p and q are connected if they have the same id.