NOTE: Pointed Taylor Bubble Revisited P. Daripa
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The Taylor bubble problem consists of two fluids; a gas of negligible density in the interior of the bubble and an incompressible nonviscous fluid in the exterior of the bubble. The bubble is symmetric and infinitely long which rises under gravity at a speed U through a tube of width h. The dimensionless speed of the bubble is known as Froude number (F) and $F = U/\sqrt{gh}$. Here g is the gravitational acceleration. This bubble models the late stages of pure Rayleigh-Taylor instability.

The flow exterior of the bubble interface in the incompressible fluid is a potential flow which in theory would allow the free streamlines at the stagnation point (i.e., tip of the bubble) to separate at any arbitrary angle, θ_t. In the absence of surface tension, conservation of energy of fluid particles on the bubble interface (i.e., Bernoulli's equation) allows the bubbles, if they exist, to be either round, cusped, or pointed with $\theta_t = 120°$ at the tip (see [4]). There is a general consensus that these bubbles exist as solutions of this problem.

The numerical solutions of Vanden-Broeck [9] show a pointed bubble rising at a speed $F = 0.3577$. This is consistent with a conjecture of Garabedian [5].

There are still some open questions about the pointed bubble. The equations of this problem contain F as a free parameter. Vanden-Broeck [9] uses a Fourier collocation method and determines F numerically by treating F as a free parameter. He finds $F = 0.3577$. He resolves this problem but with the value of F prescribed a priori. He attempts to find pointed bubbles for values of F other than 0.3577. The results lead him to suggest that there are no other pointed bubbles.

In Section 3 of the paper, we present a higher order constraint at the tip which contains the selection mechanism of the tip angle 120° of the pointed bubbles. This constraint is referred to as "tip selection criterion" or TSC in short. The TSC allows computation of tip angle from local higher order derivatives at the tip. The computed tip angle should be equal to 120° for the correct pointed bubble.

We use the tip angle or equivalently a local higher order derivative at the tip as a continuation parameter in our numerical procedure to provide numerical evidence of the unique pointed bubble. Our convergence studies show that this unique pointed bubble rises at a speed $S \sim 0.35784$, accurate up to four decimal places. This is consistent with the result of Vanden-Broeck [9] except that our estimate of the speed differs from Vanden-Broeck's in the fourth decimal place. Using a desingularizaiton method [2], we obtain the same estimate of speed. We also investigate the nature of singularity at the tip of the bubble by studying the asymptotic behavior of the Fourier spectrum of this bubble.

The rest of the paper is as laid out as follows. The Section 2 contains the basic formulation of the problem. In Section 3 we describe the higher order constraint at the tip and describe the determination of tip angle from local higher order derivatives at the tip. In Section 4 we describe the numerical method. The numerical results are presented in Section 5 and are discussed in Section 6. Finally we conclude in Section 7.

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