Lab-4

1.a)

ROC, Causality, and Stability :

Here N(z)=1 and

H(z)=N(z)/D(z)

- N – number (a positive integer) of unique ROC possible for this $H(z)$
- ROC – Nx2 matrix (of non-negative real numbers) with each row [r1, r2] indicating an ROC of the form $r1 < |z| < r2$
- C – length N binary vector (1 if the corresponding system is causal, else 0)
- S – length N binary vector (1 if the corresponding system is stable, else 0)

Code for Function:

```
*function [N, ROC, C, S] = roc_cs(p2)
    N = length(p2)+1;
    ROC = zeros(N,2);
    C = zeros(N,1);
    S = zeros(N,1);
    C(N) = 1;
    for i = 1:N
        if i ~= N
            ROC(i,2) = abs(p2(i));
        end
        if i ~= 1
            ROC(i,1) = abs(p2(i-1));
        end
    end
    ROC(N,2) = Inf;
   v = 0;
   x=0;
for j = 1:length(p2)
    if abs(p2(j)) == 1
        v = 1;
```

```
    end
end

for i = 1:N-1
    if v ~= 1 && p2(i) < 1 && p2(i+1) > 1
        S(i+1) = 1;
        x = 1;
        break
    end
    if v~=1 && p2(i)>1
        S(i) = 1;
        x=1;
        break
    end
end
if v ~=1 && x ~= 1
        S(N) = 1;
end
end
```

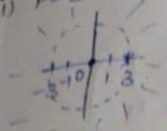
1.b)

Lab-4

Z-Transform

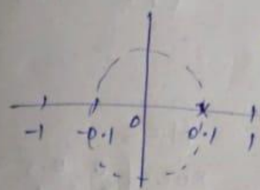
7.1) ROC, causality and Stability

b) (i) $P = 3$



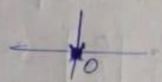
$\boxed{N = 2}$ $ROC = \begin{bmatrix} 0 & 3 \\ 3 & Inf \end{bmatrix}$ $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(ii) $P = 0.1$



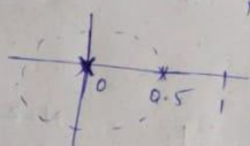
$N = 2$ $ROC = \begin{bmatrix} 0 & 0.1 \\ 0.1 & Inf \end{bmatrix}$ $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(iii) $P = 0$



$N = 1$ $ROC = [0 \;\; Inf]$ $C = [1]$ $S = [1]$

(iv) $P = [0, 0.5]$

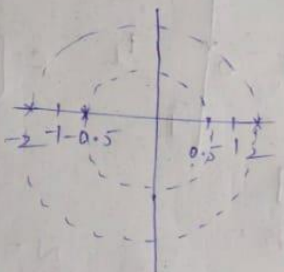


$N = 2$

$ROC = \begin{bmatrix} 0 & 0.5 \\ 0.5 & Inf \end{bmatrix}$

$C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(v) $P = [2, -0.5]$
$= [-0.5, 2]$

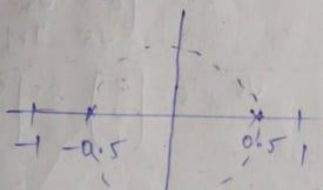


$N = 3$

$ROC = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 2 \\ 2 & Inf \end{bmatrix}$

$S = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(vi) $P = [-0.5, 0.5]$



$N = 2$

$ROC = \begin{bmatrix} 0 & 0.5 \\ 0.5 & Inf \end{bmatrix}$

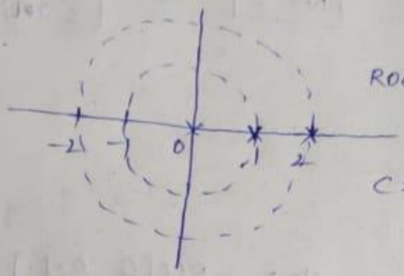
$C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(vii) $P = [2, 2, 2]$
$= [2]$

$N = 2$

$ROC = \begin{bmatrix} 0 & 2 \\ 2 & Inf \end{bmatrix}$ $C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

(viii) $P = [0, 1, 2]$



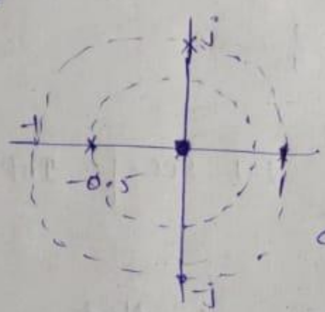
$N = 3$

$ROC = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 2 & Inf \end{bmatrix}$

$C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(ix) $P = [-0.5, j]$

$|j| = 1$



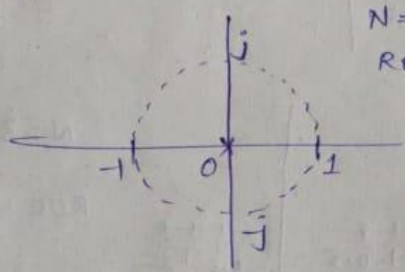
$N = 3$

$ROC = \begin{bmatrix} 0 & 0.5 \\ 0.5 & 1 \\ 1 & Inf \end{bmatrix}$

$C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(x) $P = [0, j, -j]$

$|j| = |-j| = 1$



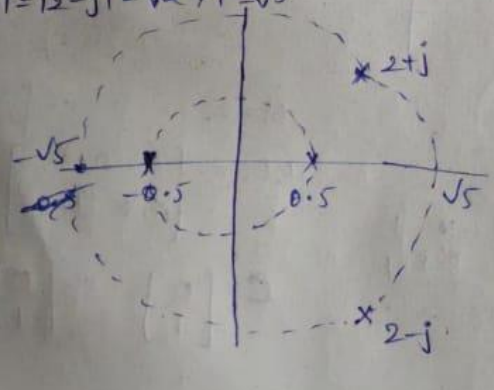
$N = 2$

$ROC = \begin{bmatrix} 0 & 1 \\ 1 & Inf \end{bmatrix}$

$C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(xi) $P = [0.5, -0.5, 2+j, 2-j]$

$|2+j| = |2-j| = \sqrt{2^2+1} = \sqrt{5}$



$N = 3$

$ROC = \begin{bmatrix} 0 & 0.5 \\ 0.5 & \sqrt{5} \\ \sqrt{5} & Inf \end{bmatrix}$

$C = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $S = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

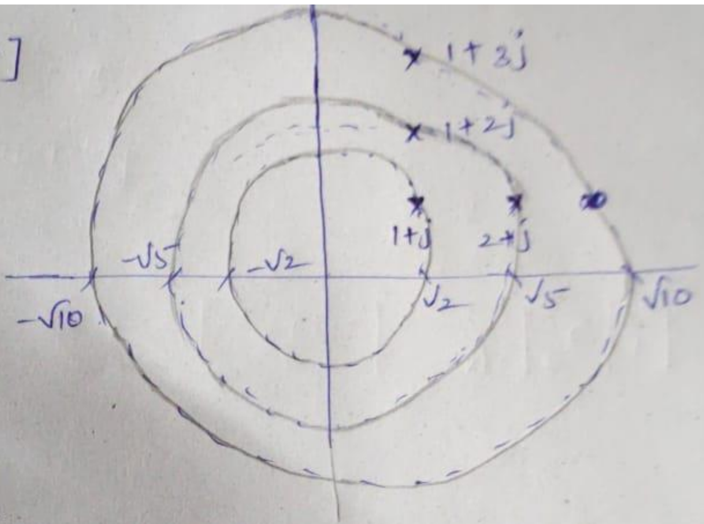
(xii) $p = [1+j, 1+2j, 1+3j, 2+j]$

$|1+j| = \sqrt{1^2+1^2} = \sqrt{2}$.

$|2+j| = |1+2j| = \sqrt{1^2+2^2} = \sqrt{5}$.

$|1+3j| = \sqrt{1+3^2}$

$= \sqrt{10}$.



$N = 4$

$$ROC = \begin{bmatrix} 0 & \sqrt{2} \\ \sqrt{2} & \sqrt{5} \\ \sqrt{5} & \sqrt{10} \\ \sqrt{10} & Inf \end{bmatrix} \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad S = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Verification with MATLAB:

When p=3

```
2

0     3
3   Inf

0
1

1
0
```

When p=0.1

```
  2

      0    0.1000
 0.1000       Inf

  0
  1

  0
  1
```

When p=0

```
  1

  0   Inf

  1

  1
```

When p=[0,0.5]

```
  2

      0    0.5000
 0.5000       Inf

  0
  1

  0
  1
```

When p=[2,-0.5]

```
  3

      0    0.5000
 0.5000    2.0000
 2.0000       Inf

  0
  0
  1

  0
  1
  0
```

When p=[-0.5,0.5]

```
  2

      0    0.5000
 0.5000       Inf

  0
  1

  0
  1
```

When p=[2,2,2]

```
  2

  0     2
  2   Inf

  0
  1

  1
  0
```

When p=[0,1,2]

```
  3

  0     1
  1     2
  2   Inf

  0
  0
  1

  0
  0
  0
```

When p=[-0.5,j]

```
  3

       0    0.5000
  0.5000    1.0000
  1.0000       Inf

  0
  0
  1

  0
  0
  0
```

When p=[0,-j,j]

```
  2

  0     1
  1   Inf

  0
  1

  0
  0
```

When p=[0.5,-0.5,2+j,2-j]

```
  3

       0    0.5000
  0.5000    2.2361
  2.2361       Inf

  0
  0
  1

  0
  1
  0
```

When p=[1+j,1+2j,1+3j,2+j]

```
    4

         0    1.4142
    1.4142    2.2361
    2.2361    3.1623
    3.1623       Inf

    0
    0
    0
    1

    1
    0
    0
    0
```

Code for script:

```
p = [1+1i,1+2i,1+3i,2+1i];
p1 = unique(abs(p));

p2 = p1;

for i = 1 : length(p1)
    if p1(i) == 0
        k = 1;
        p2 = p1(p1 ~= 0);
        break
    end
end
[N, ROC, C, S] = roc_cs(p2);
disp(N);
disp(ROC);
disp(C);
disp(S);
```

```
**Here we are getting the same output in the case of the
calculation part and in the case of MATLAB part.
```

2.

Given a LTI system

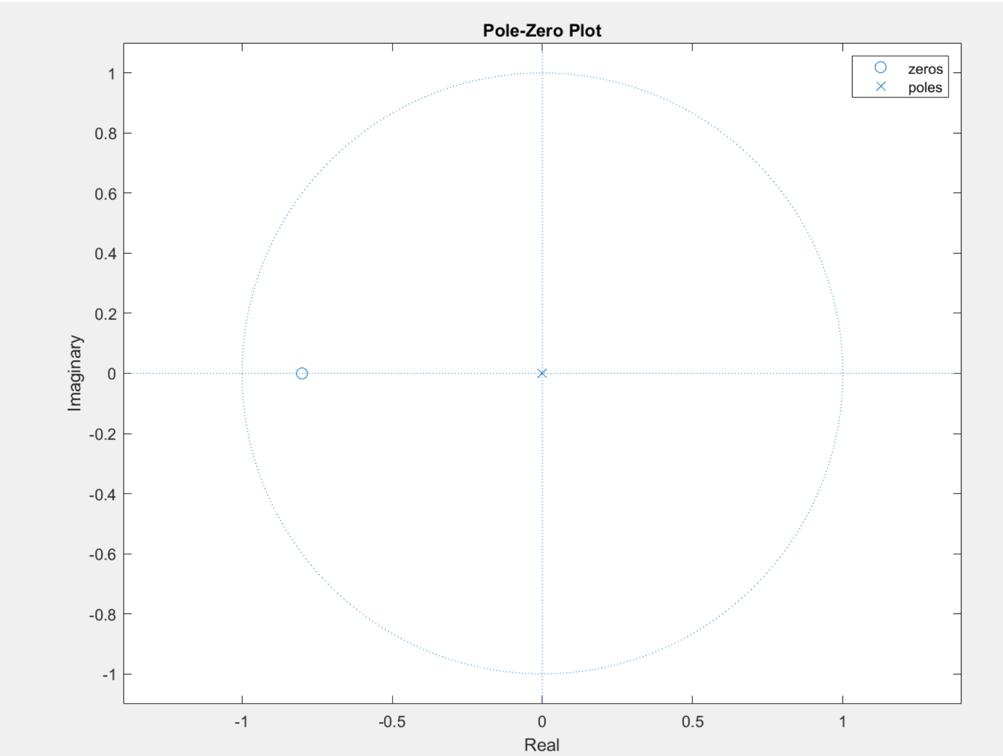
H(z)=z/(z+p)   where p belongs to (-1,1)

a)p=0.8

Code in MATLAB:

```
p=0.8;
d=[1,p];
figure;
zplane(d,1)
title('Pole-Zero Plot');
xlabel('Real');
ylabel('Imaginary');
legend('zeros','poles');
```

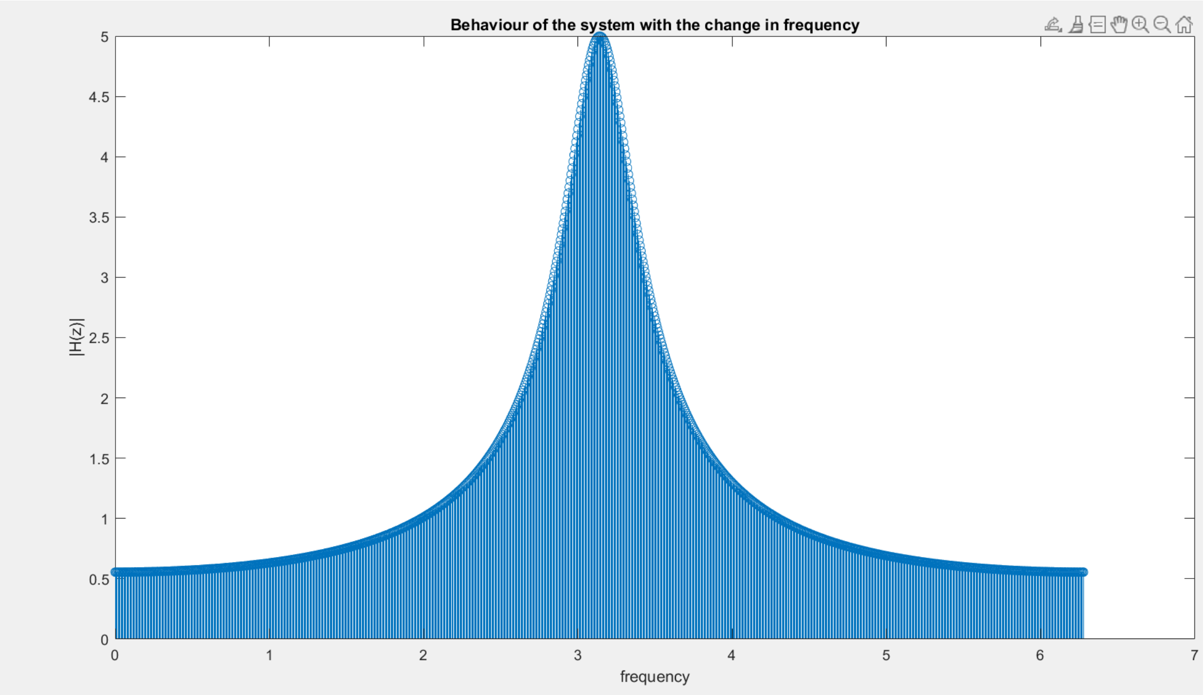
Plot:



b)

Code :

```
n=1001;
p=0.8;
d=[1,p];
[y,x]=freqz(1,d,n,"whole");
figure;
stem(x,abs(y));
title('Behaviour of the system with the change in frequency')
xlabel('frequency')
ylabel('|H(z)|')
```



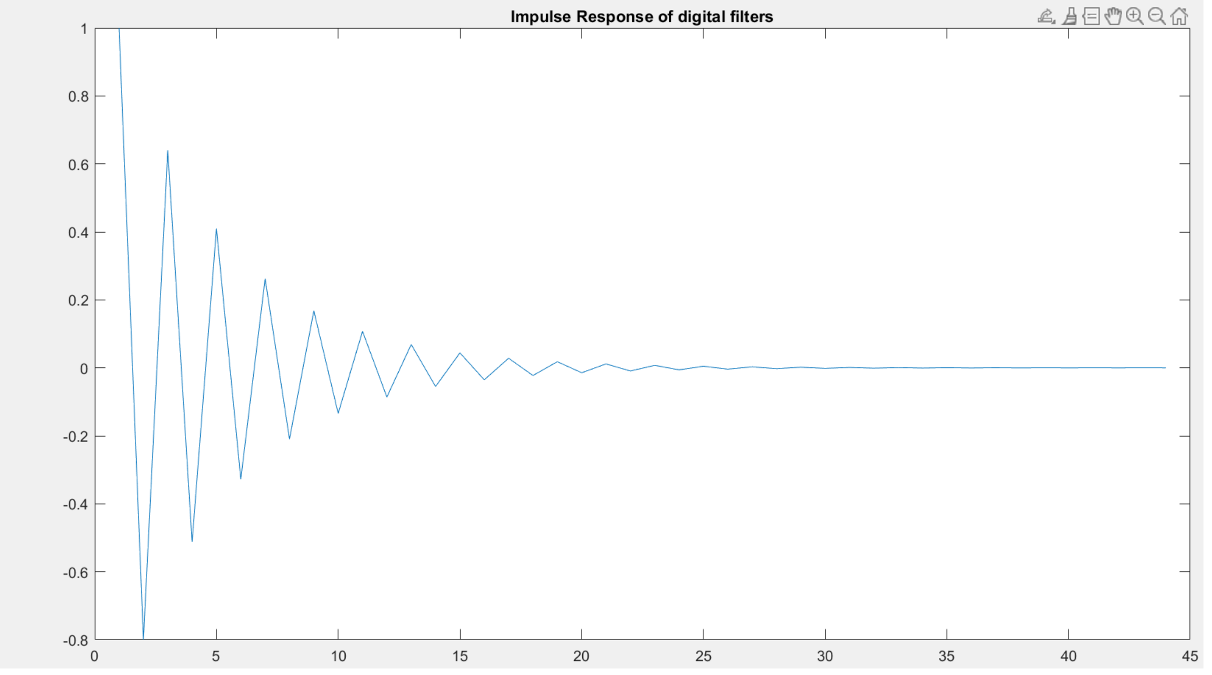
c)

Code :

```
n=1001;
p=0.8;
d=[1,p];
[y,x]=freqz(1,d,n,"whole");
figure;
stem(x,abs(y));
```

```
title('Behaviour of the system with the change in frequency')
xlabel('frequency')
ylabel('|H(z)|')
```

Plot:



From the Plot we can say that it is an IIR filter.

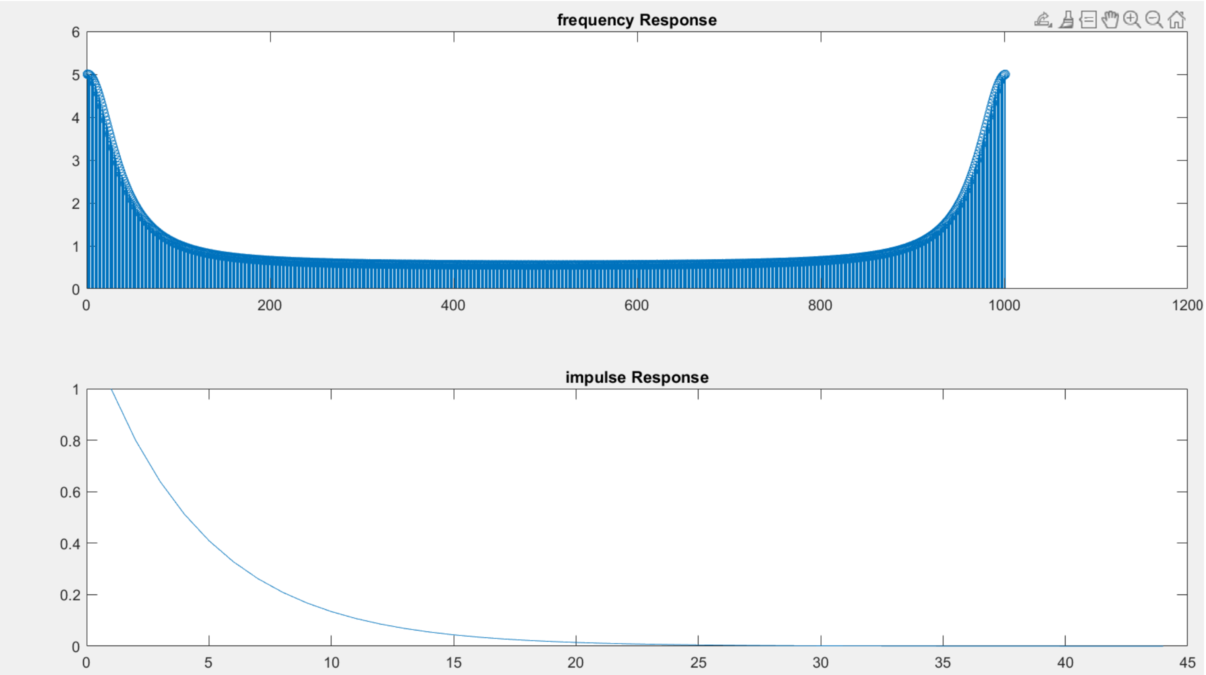
(from the definition of FIR and IIR filters

We know that the IIR is a type of filter that generates impulse response of infinite duration for a dynamic system. And the FIR filter provides an impulse response of a finite period.) here in the plot it is generating for the infinite duration .

In this case, the system is a first-order system, so there is only one possible impulse response

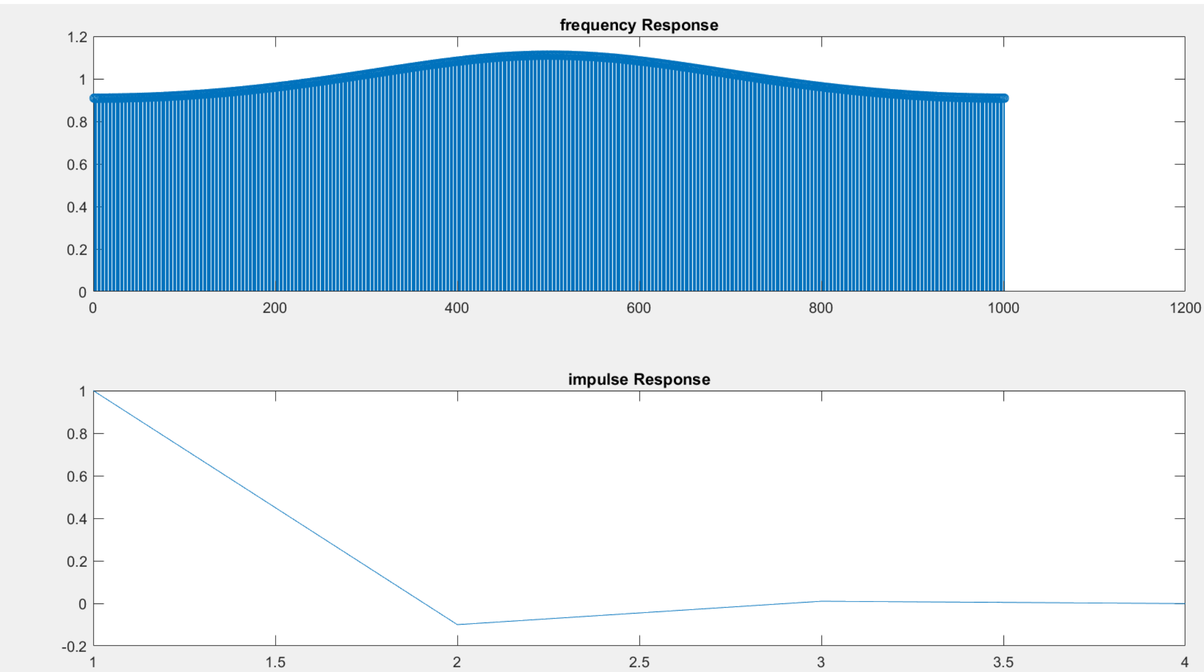
d)

p=-0.8



When you repeat the analysis for p = -0.8 in the magnitude response plot (Part d), you may see a response that resembles a high-pass or band-reject filter, depending on the exact value of 'p'. The response may not have a peak, but it should be stable.

P=0.1



For p = 0.1, the impulse response is likely to decay smoothly without significant oscillations, indicating a well-behaved, stable system.

Code:

```
n=1001;
p=0.1;
d=[1,p];
y1=freqz(1,d,n,"whole");
y2=impz(1,d);

figure
subplot(2,1,1)
stem(y1)
title('frequency Response')

subplot(2,1,2)
plot(y2)
title('impulse Response')
```

e)

Here the system function is

$H(z)=(z-p^{-1})/(z-p)$ and

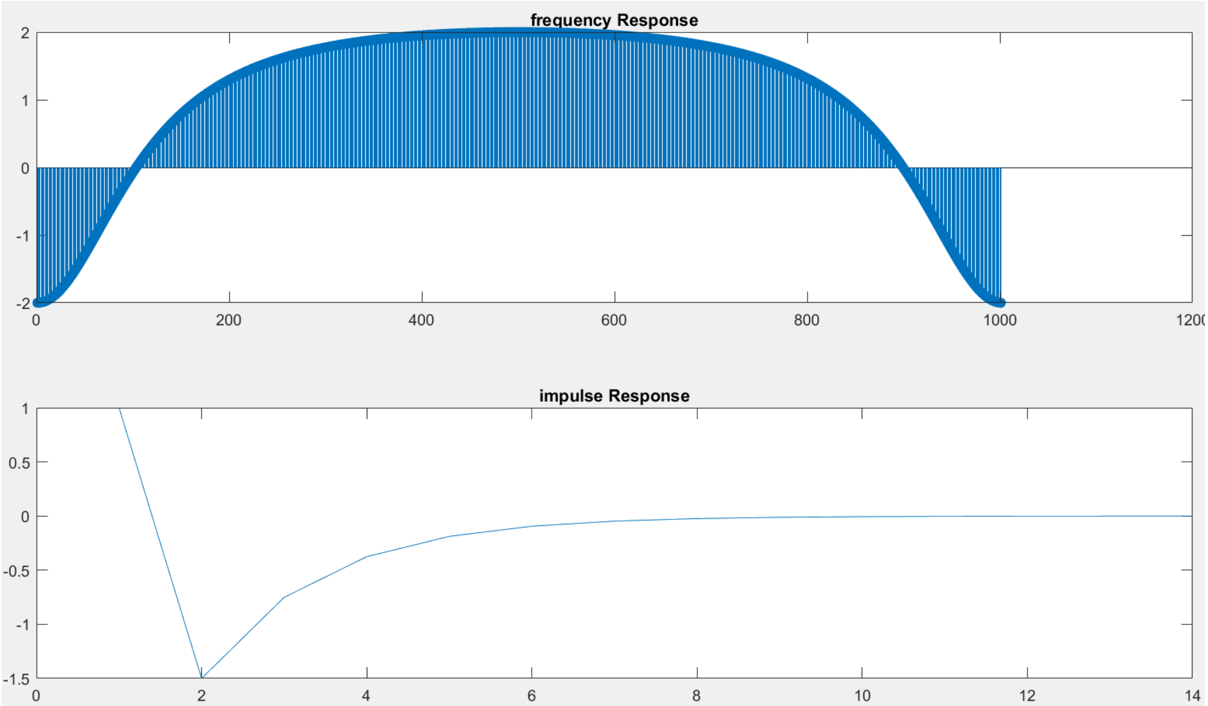
P=0.5

Code:

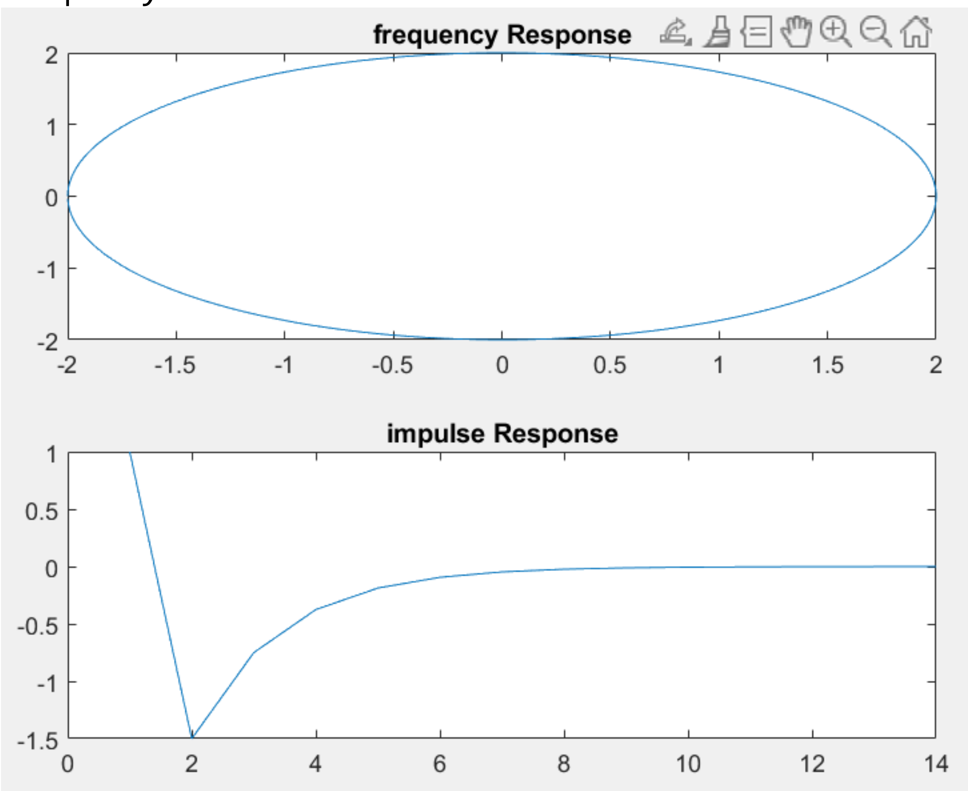
```
p=0.5;
N=1001;
n=[1,-1/p];
d=[1,-p];
y1=freqz(n,d,N,"whole");
y2=impz(n,d);
figure
subplot(2,1,1)
stem(y1,"filled")
title('frequency Response')

subplot(2,1,2)
plot(y2)
title('impulse Response')
```

Plot:



(Or)
The graphs are between magnitude of frequency response and frequency.



Here it is representing the IIR filter as per the plot.

3) LTI system with complex poles

Here the transfer function is
$H(z)=z^2-(2\cos\theta)z+1/ z^2-(2r\cos\theta)z+r^2$ where r belongs to (0,1) and θ belongs to [0,pi]
a)

Here
The Numerator is $z^2-(2\cos\theta)z+1$
and the denominator is $z^2-(2r\cos\theta)z+r^2$
to slove the equation for the poles we need to equate the denominator to the zero.and for finding the zeros we need to equate the numerator to zero.
By solving we can observe the zeros and poles as

For the numerator, we can use the quadratic formula:

$$z_{\text{zero}} = \frac{2\cos(\theta) \pm \sqrt{4\cos^2(\theta) - 4}}{2} = \cos(\theta) \pm j\sqrt{1 - \cos^2(\theta)}$$

For the denominator, we can also use the quadratic formula:

$$z_{\text{pole}} = \frac{2r\cos(\theta) \pm \sqrt{4r^2\cos^2(\theta) - 4r^2}}{2} = r\cos(\theta) \pm jr\sin(\theta)$$

Code:

```
r_values = [0.25,0.5, 0.75, 0.99];
t_values = [pi/4, pi/2,2*pi/3, 3*pi/4];

for i = 1:length(r_values)
    for j = 1:length(t_values)
        r = r_values(i);
        t = t_values(j);

        n = [1, -2*cos(t), 1];
        d = [1, -2*r*cos(t), r*r];

        figure;
        subplot(2, 1, 1);
```

```
        zplaneplot(n, d);
        title(['Pole-Zero Plot for r = ', num2str(r), ' and t = ', 
num2str(t)]);

        [y, x] = freqz(n, d,'whole');
        subplot(2, 1, 2);
        plot(x, abs(y));
        title(['Frequency Response for r = ', num2str(r), ' and t = 
', num2str(t)]);
        xlabel('Frequency');
        ylabel('|H(z)|');
    end
end
```

Plot:

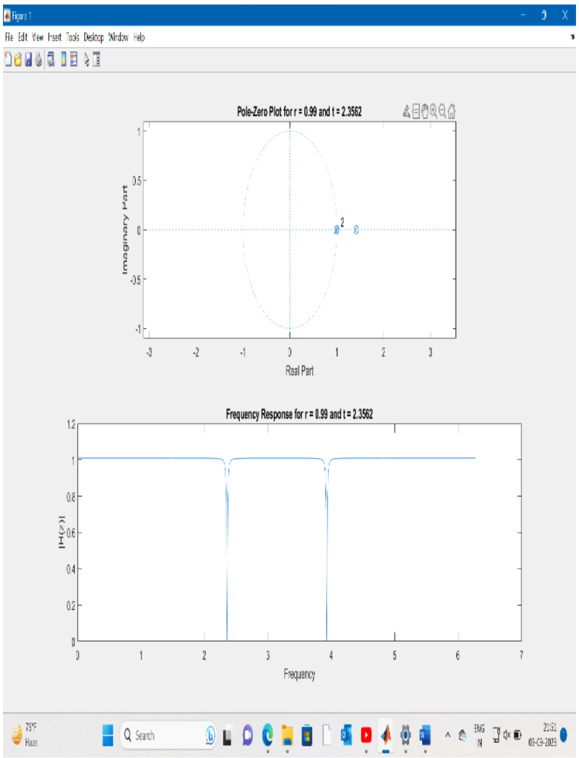


Figure 1
File Edit View Insert Tools Desktop Window Help
Pole-Zero Plot for r = 0.99 and t = 2.3562
Imaginary Part
Real Part
Frequency Response for r = 0.99 and t = 2.3562
|H(z)|
Frequency

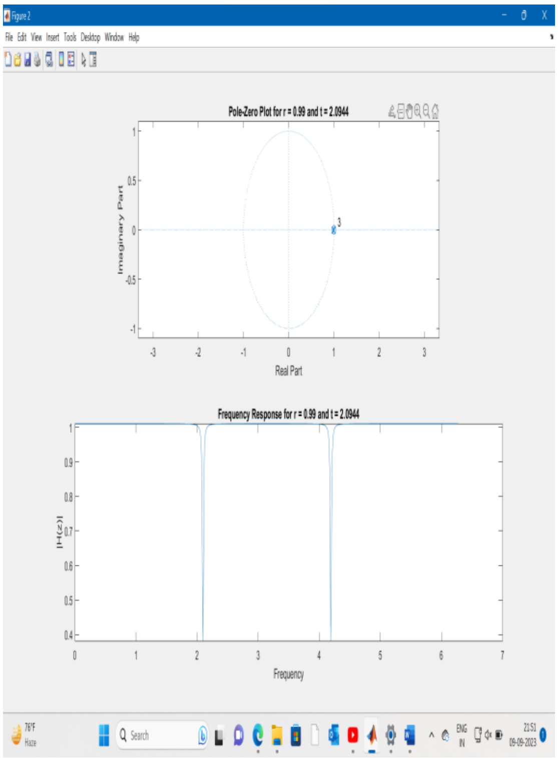


Figure 2
File Edit View Insert Tools Desktop Window Help
Pole-Zero Plot for r = 0.99 and t = 2.0944
Imaginary Part
Real Part
Frequency Response for r = 0.99 and t = 2.0944
|H(z)|
Frequency

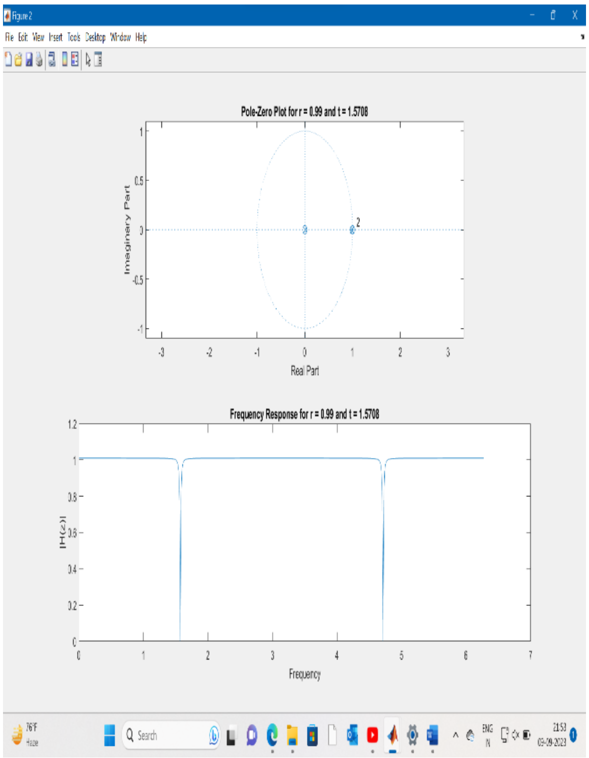
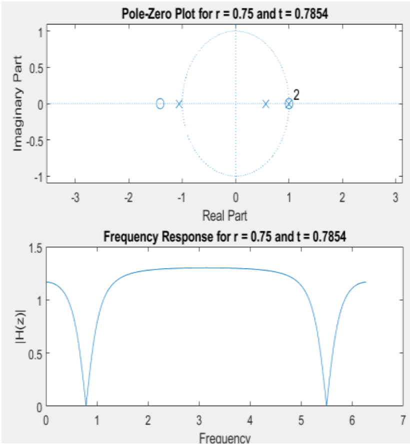
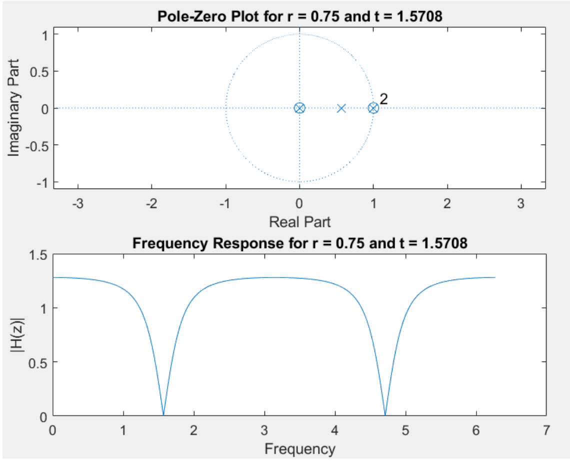
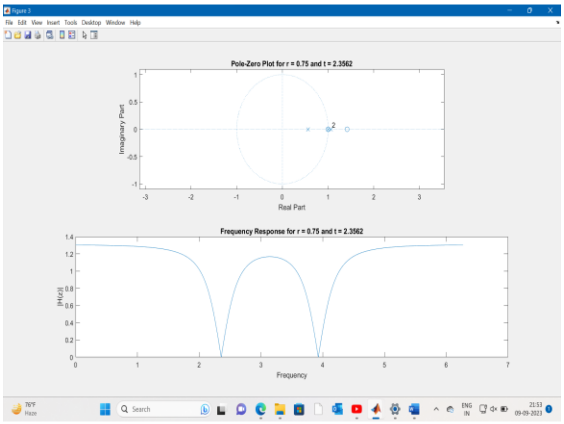
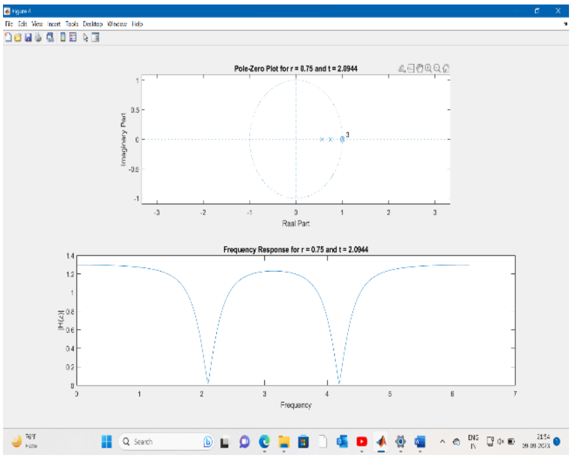


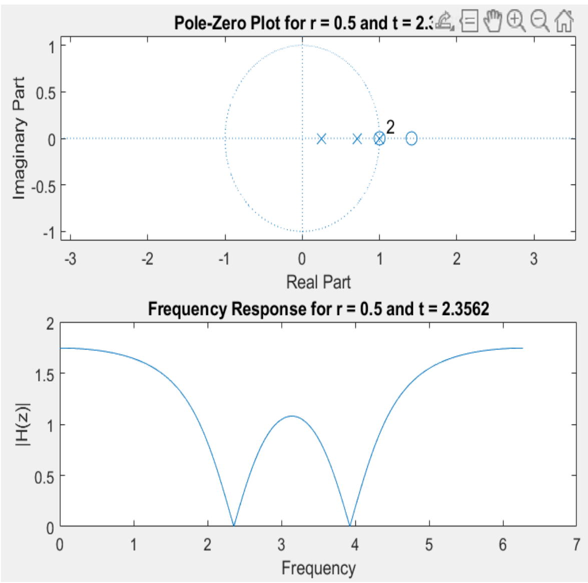
Figure 2
File Edit View Insert Tools Desktop Window Help
Pole-Zero Plot for r = 0.99 and t = 1.5708
Imaginary Part
Real Part
Frequency Response for r = 0.99 and t = 1.5708
|H(z)|
Frequency

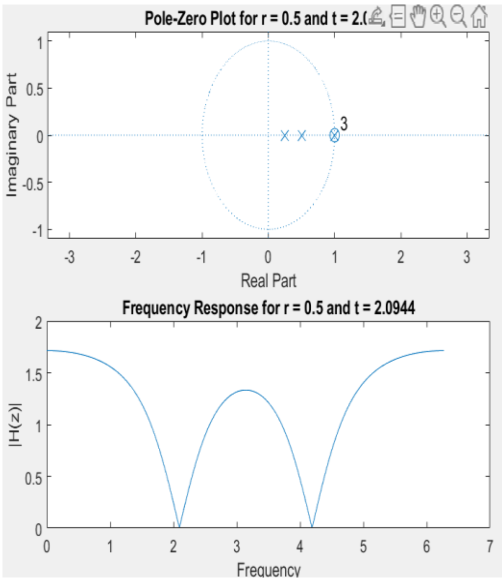

Pole-Zero Plot for r = 0.75 and t = 0.7854
Imaginary Part
Real Part
Frequency Response for r = 0.75 and t = 0.7854
|H(z)|
Frequency


Pole-Zero Plot for r = 0.75 and t = 1.5708
Imaginary Part
Real Part
Frequency Response for r = 0.75 and t = 1.5708
|H(z)|
Frequency


Pole-Zero Plot for r = 0.75 and t = 2.3562
Real Part
Frequency Response for r = 0.75 and t = 2.3562
Frequency


Pole-Zero Plot for r = 0.75 and t = 2.0944
Real Part
Frequency Response for r = 0.75 and t = 2.0944
Frequency


Imaginary Part
Real Part
Frequency Response for r = 0.5 and t = 2.3562
|H(z)|
Frequency


Imaginary Part
Real Part
Frequency Response for r = 0.5 and t = 2.0944
|H(z)|
Frequency

3.b)

The system can be both causal and stable if all poles of the transfer function are located inside the unit circle ($|z|<1$) In the complex plane. To find the ROC , we need to know the values of z for which the H(z) converges .

Here the denominator of H(z) is a quadratic polynomial , the ROC will be a ring-shaped region in the complex plane.

The ROC will include the unit circle ($|z|<1$) and exclude the area outside the unit circle.

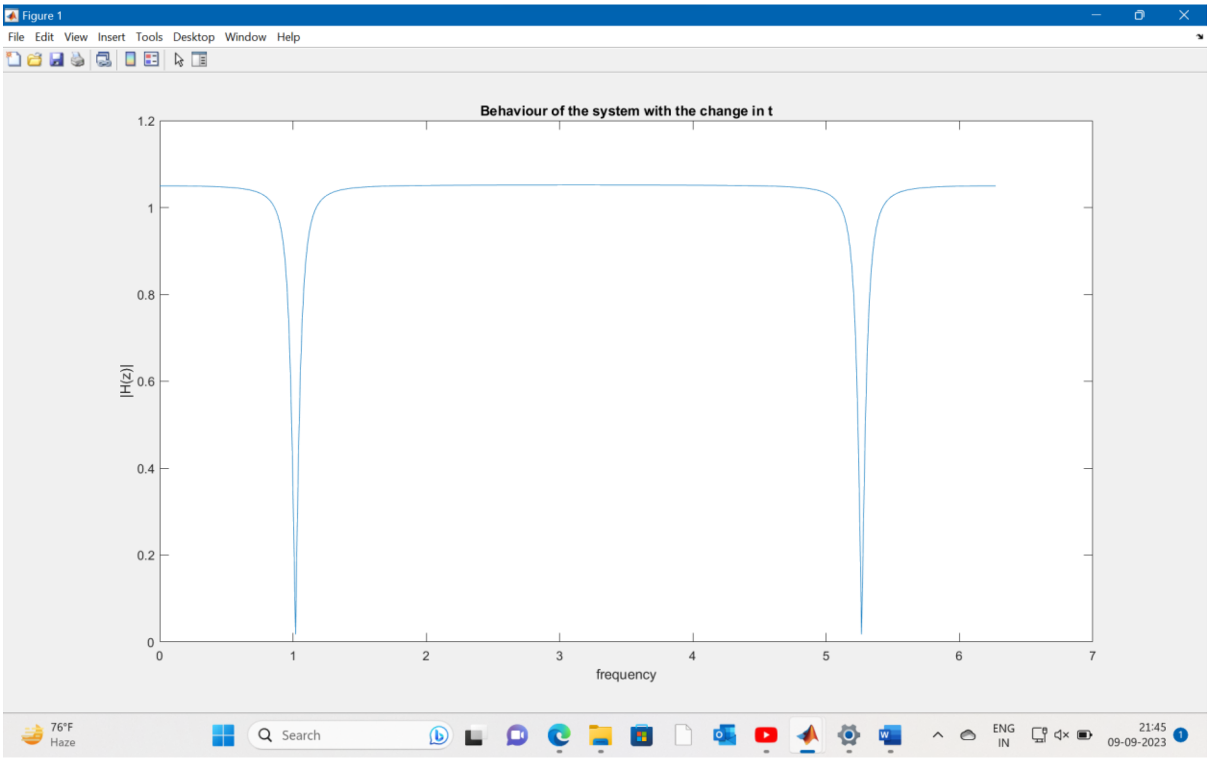
3.c)

Code:

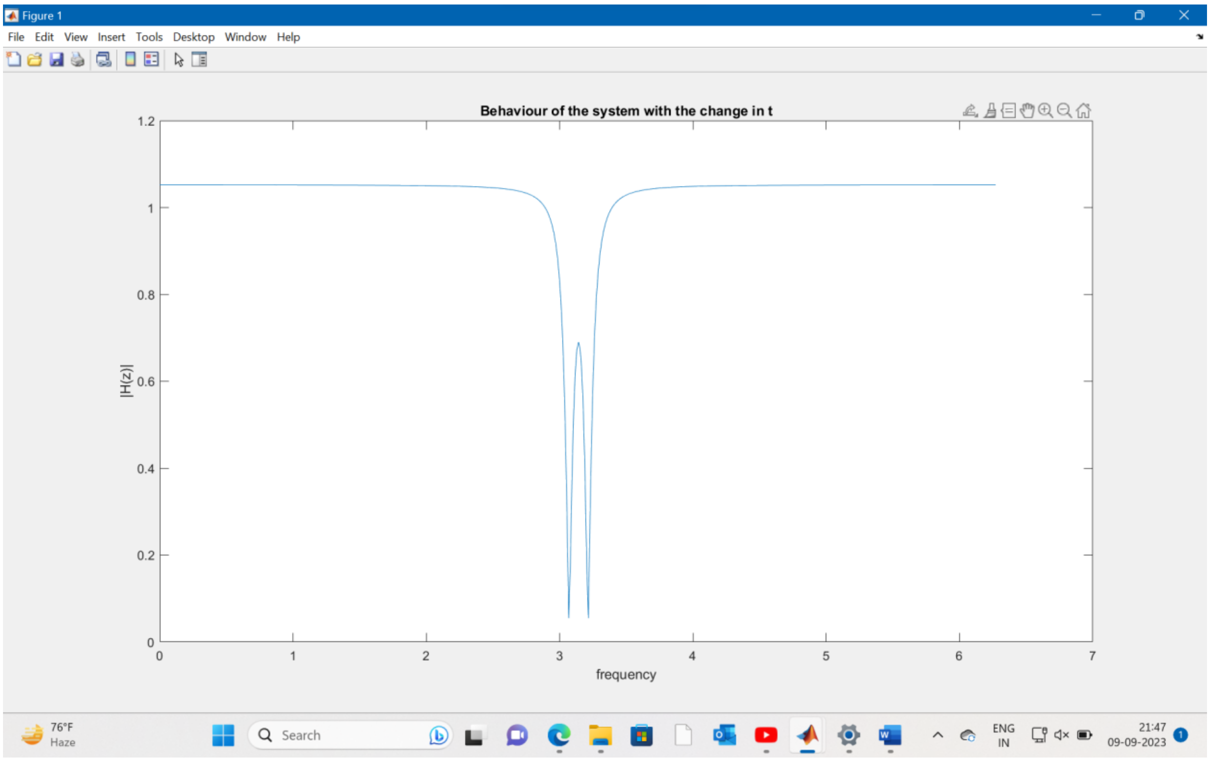
```
r=0.95;
t=60;
n=[1,-2*cos(t),1];
d=[1,-2*r*cos(t),r*r];
[y,x]=freqz(n,d,'whole');

figure;
plot(x,abs(y));
title('Behaviour of the system with the change in t')
xlabel('frequency')
ylabel('|H(z)|')
```

When θ=pi/4



When θ=3*pi/4



When θ=pi/2

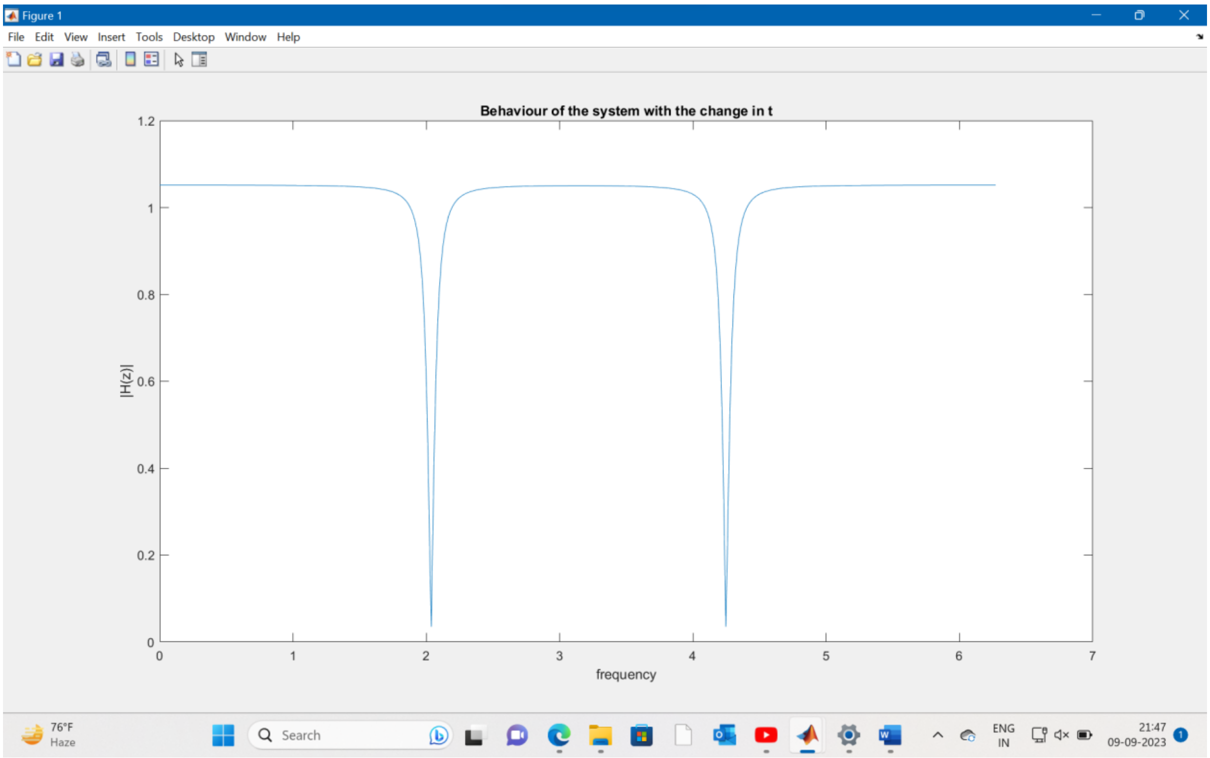


Figure 1
File Edit View Insert Tools Desktop Window Help
Behaviour of the system with the change in t
|H(z)|
frequency
1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7
76°F
Haze
Search
ENG
IN
21:47
09-09-2023

When θ=pi/3

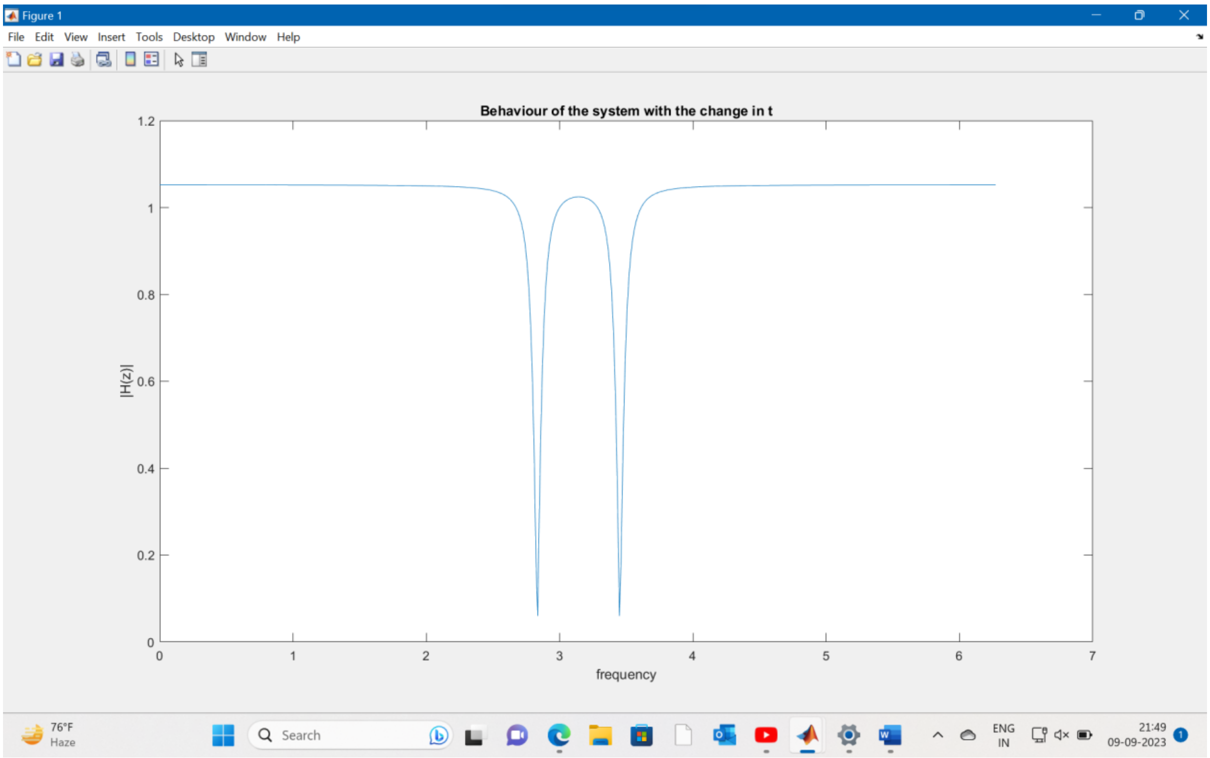
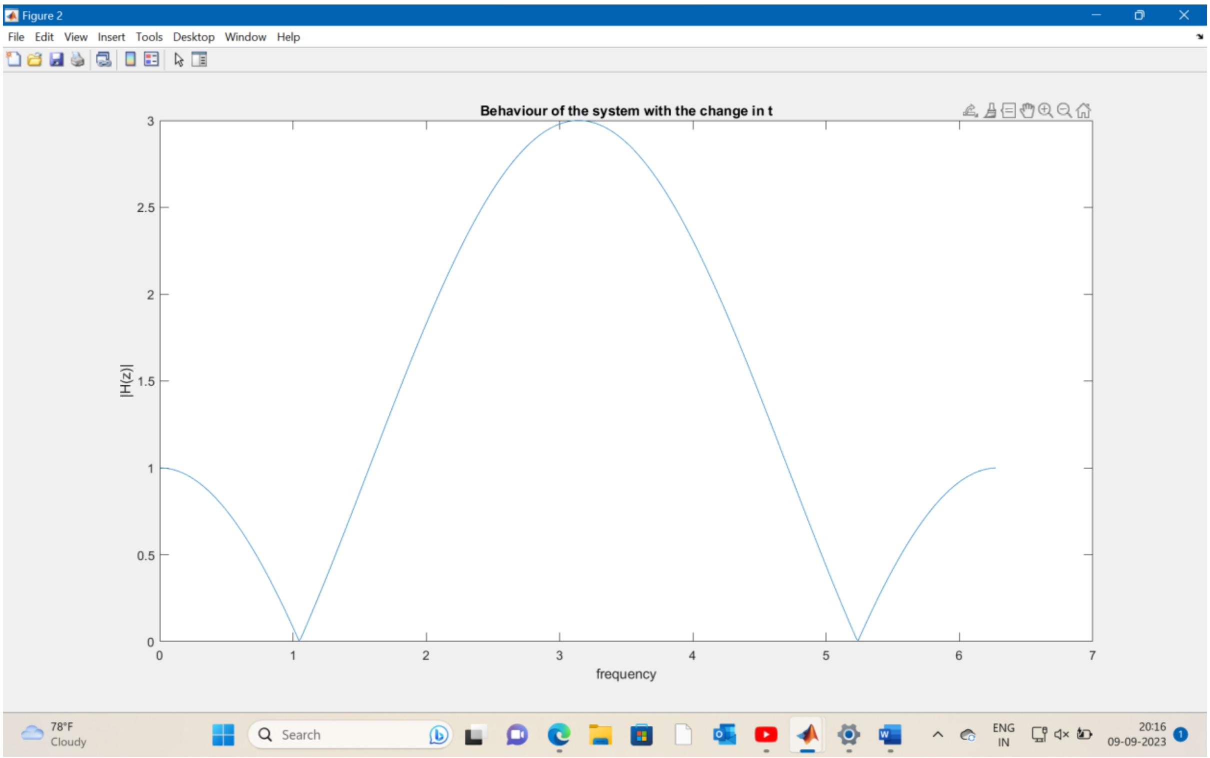


Figure 1
File Edit View Insert Tools Desktop Window Help
Behaviour of the system with the change in t
|H(z)|
frequency
1.2
1
0.8
0.6
0.4
0.2
0
0 1 2 3 4 5 6 7
76°F
Haze
Search
ENG
IN
21:49
09-09-2023

3.d)

here $\theta=60^0$

when r=0



When r=1

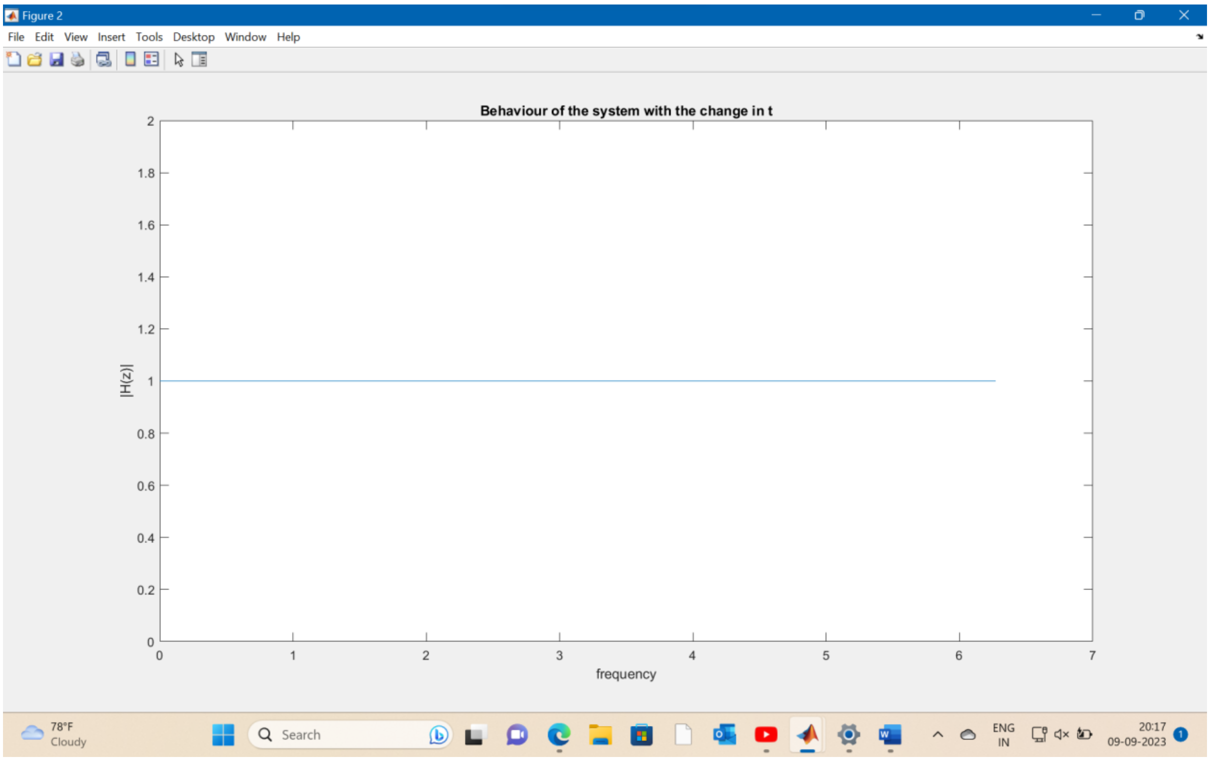


Figure 2
File Edit View Insert Tools Desktop Window Help
Behaviour of the system with the change in t
|H(z)|
frequency
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0 1 2 3 4 5 6 7
78°F
Cloudy
Search
ENG
IN
20:17
09-09-2023

When r=10

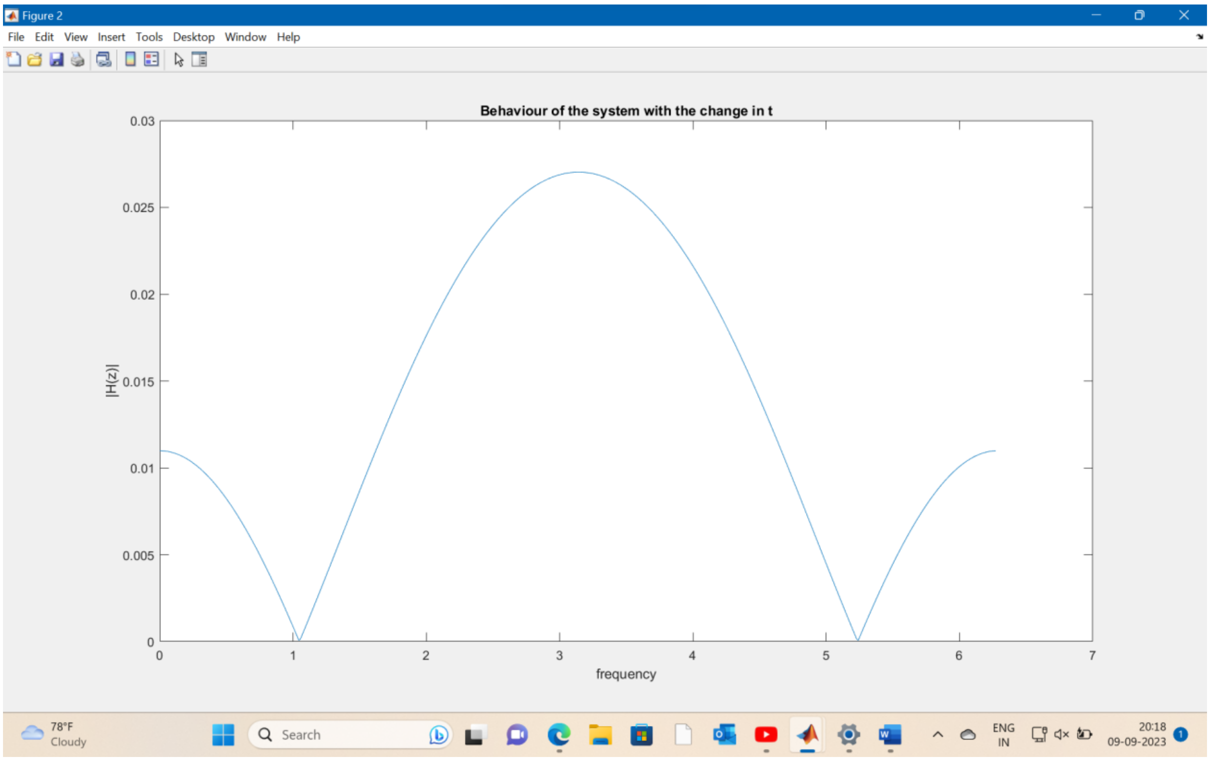
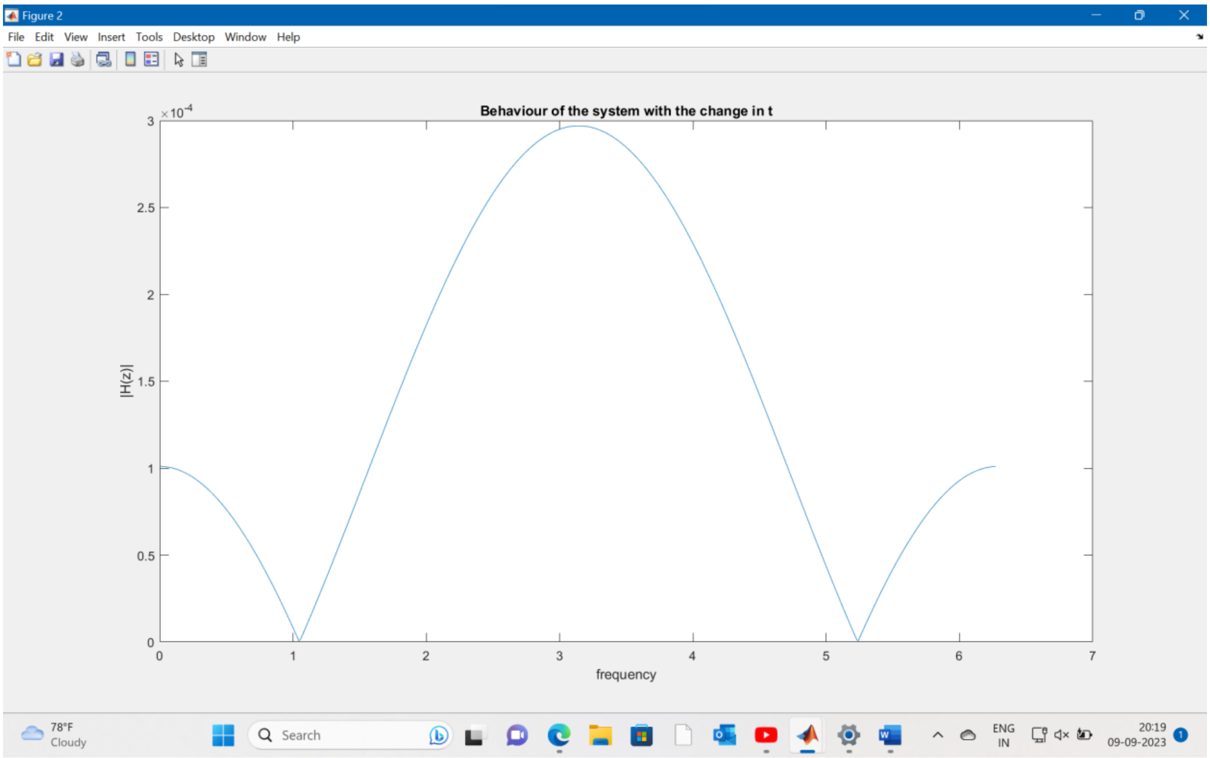


Figure 2
File Edit View Insert Tools Desktop Window Help
Behaviour of the system with the change in t
|H(z)|
frequency
0
0.005
0.01
0.015
0.02
0.025
0.03
0 1 2 3 4 5 6 7
78°F
Cloudy
Search
ENG
IN
20:18
09-09-2023

When r=100

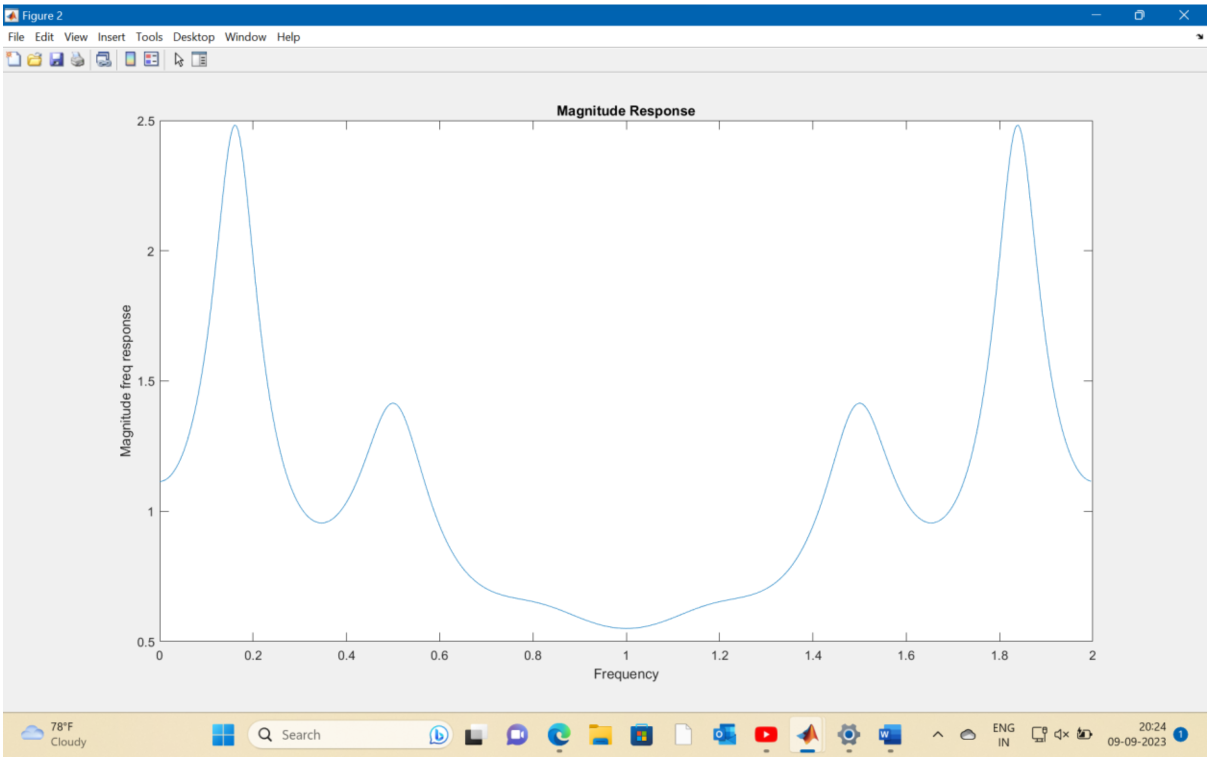


4. LTI system with multiple poles:

Here the transfer function is given as

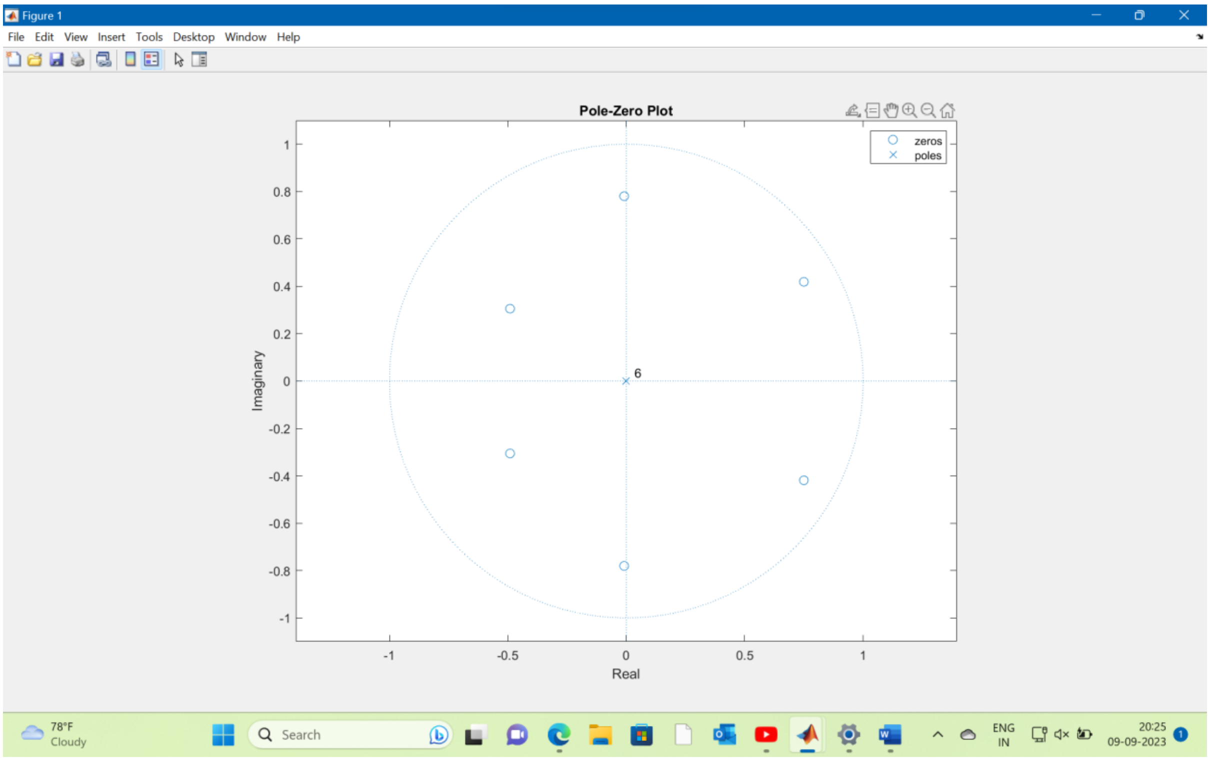
$H(z)=1/(1-0.5z^{-1}+0.2z^{-2}-0.1z^{-3}+0.007z^{-4}+0.14z^{-5}+0.15z^{-6})$

a)



Here from the observation of plot we can you say that the poles are located at the peaks of the curves . so here the peaks are symmetrical with respect to the frequency =1 .

4)b.



Here from the both plots we can say that the poles are conjugate pairs here there are 6 poles i.e 3 conjugate pairs of poles. Here the poles with highest peak are located near to the origin and the next second highest peak are next near to the origin.