Convex Optimization (MA401)
B.S. VII Sem (2024-2025)
Practical Sheet 3

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1. Determine if the function satisfies the First Order Necessary Conditions (FONC) for optimality at the specified point:

   a) For $f(x) = x^2$ at $x = 1$ and $x = 2$ with the domain $\Omega = [1, \infty)$.

   b) For $f(x) = x^2$ at $x = 1$ and $x = 2$, with the domain $\Omega = [-1, \infty)$.

2. For a given matrix M, check whether it has full rank or not. Also, find the linearly independent columns of the matrix.

   a) $A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

   b) $B = \begin{bmatrix} 1 & 8 & 5 \\ 5 & 0 & 1 \\ 1 & 0 & 90 \end{bmatrix}$

3. Find the basis of the solution space of the following problem.

   a)
   $$\begin{aligned} x + y &= 2 \\ 2x + 2y &= 4 \end{aligned}$$

   b)
   $$\begin{aligned} x + y - z &= 3 \\ x - y - z &= 4 \\ 2x + 3y + 4z &= 0 \end{aligned}$$

   c)
   $$\begin{aligned} x + y + z + w &= 4 \\ x + 2y + 2z + w &= 8 \\ 2x + 3y + 3z + 2w &= 10 \end{aligned}$$