1) $\int \tan^{5} x \sec^{3} x \, dx$

1) POTENZA DISPARI

$\to \int \tan^{4} \sec^{2} \tan x \sec x$ | $\tan^{4} x \to (\tan^{2} x)^{2}$

$\to \int (\tan^{2} x)^{2} \sec^{2} \tan x \sec x$ | $\tan^{2} x = \sec^{2} x - 1$

$\to \int (\sec^{2} x - 1)^{2} \sec^{2} \tan x \sec x$

SOSTITUZIONE
$$\begin{cases} u = \sec x \\ du = \sec x \tan x \end{cases}$$

$\to d' \sec \to \frac{\sin x}{\cos^{2} x} \to \frac{\sin x}{\cos x}$

$\to \int (u^{2} - 1)^{2} u^{2} \cdot du$

$\to \int (u^{4} - 2u^{2} + 1) u^{2} = u^{6} - 2u^{4} + u^{2} \, du$

$\tan x = \frac{\sin x}{\cos x}$ $\sec x = \frac{1}{\cos x}$

$\to \tan^{2} = \frac{\sin^{2} x}{\cos^{2} x}$ $\sec^{2} x = \frac{1}{\cos^{2} x}$

$\tan^{2} x + \sec^{2} x = 1$

$\to \frac{\sin^{2} x}{\cos^{2} x} + \frac{1}{\cos^{2} x} = \frac{\sin^{2} x + \cos^{2} x}{\cos^{2} x}$

$\sin^{2} x + \cos^{2} x = 1$

$\to \frac{1}{\cos^{2} x}$

$$\rightarrow \int u^6 - 2u^4 + u^2 \, du \qquad u = \sec x$$

$$\hookrightarrow \int x^i \rightarrow \frac{x^{i+1}}{i+1} \rightarrow \frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3} \Rightarrow$$

$$\underline{\frac{\sec^7}{7} - \frac{2\sec^5}{5} + \frac{\sec^3}{3}}$$

2) $$\int \frac{\cos(2x)}{\sin x + \cos x} \, dx \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

$$\rightarrow \int \frac{\cos^2 x - \sin^2 x}{\sin x + \cos x} \rightarrow \# \, (x-y)^2 \rightarrow (x-y)(x+y)$$

$$\hookrightarrow (\cos x + \sin x)(\cos x - \sin x)$$

$$\rightarrow \int \frac{(\cos x - \sin x)\cancel{(\cos x + \sin x)}}{\cancel{\sin x + \cos x}} \rightarrow \int \cos x - \sin x \, dx$$

$$\rightarrow \underline{\sin x + \cos x + C}$$

3) $$\frac{x^2+1}{x^4 - x^2 + 1} \, dx \rightarrow \frac{\cancel{x^2}\left(1 + \frac{1}{x^2}\right)}{\cancel{x^2}\left(x^2 - 1 + \frac{1}{x^2}\right)} \Rightarrow \frac{1 + \frac{1}{x^2}}{x^2 - 1 + \frac{1}{x^2}} \, dx$$

Factor out

$\rightarrow x^2 - 1 + \frac{1}{x^2} \Rightarrow x^2 - 2 + \frac{1}{x^2} + 1 \quad (-1+1)$

$\rightarrow \left[x - \frac{1}{x}\right]^2 + 1$

ci servono per fare diventare un quadrato di binomio (il 2)

$\rightarrow \left(x - \frac{1}{x}\right)^2 = x^2 - 2 + \frac{1}{x^2}$

$\rightarrow \int \begin{cases} u = x - \frac{1}{x} \\ du = 1 + \frac{1}{x^2} \end{cases} \rightarrow \int \frac{du}{u^2+1} \rightarrow \frac{1}{u^2+1} \rightarrow \text{arc tan}$

$\tan^{-1}$

$\rightarrow \tan^{-1}(u) \rightarrow \tan^{-1}\left(x - \frac{1}{x}\right) + c$

4) $\int (x + e^x)^2 \, dx$

$\rightarrow \int x^2 + 2x e^x + e^{2x} \, dx$

$\frac{x^3}{3} \qquad \int 2x e^x + \frac{e^{2x+1}}{2x+1} ?$

INTEGRAZIONE PER PARTI

$\rightarrow \int 2x \, e^x$

$\int f(x) \cdot g(x) \, dx =$

$$F(x)\,g(x) - \int F(x)\cdot g'(x)$$

$$\rightarrow e^x 2x - \int e^x \cdot 2$$

$$\hookrightarrow e^x 2x - 2e^x$$

$$\rightarrow \frac{x^3}{3} + 2x e^x - 2e^x + \frac{1}{2} e^{2x}$$

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5) $\int \operatorname{cosech}^3 x \, \sec x \, dx$ → cosecante sec

$\hookrightarrow \frac{1}{\sin x}$ $\quad \hookrightarrow \frac{1}{\cos x}$ $\quad \geq \frac{1}{\sin} \quad \frac{1}{\cos}$

$$\rightarrow \int \frac{1}{\sin^3 x \cos x}$$

→ voglio ordini più così su cui lavorare → $1 = \sin^2 x + \cos^2 x$

$$\frac{\sin^2 x + \cos^2 x}{\sin^3 x \cos x} \rightarrow \frac{\cancel{\sin^2 x}}{\sin^{\cancel{3}\,1} x \cos x} + \frac{\cos^{\cancel{2}} x}{\sin^3 x \cancel{\cos x}}$$

$$\hookrightarrow \frac{1}{\sin x \cos x} + \frac{\cos x}{\sin^3 x} =$$

$1 \rightarrow \sin^2 x + \cos^2 x$

$$\rightarrow \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\sin x \cos x} \quad \left\{ \frac{f'(x)}{f(x)} \right.$$

$\hookrightarrow \frac{\sin x}{\cos x} \rightsquigarrow -\log|\cos x| + \log(\sin x)$

$\hookrightarrow \log|\cos x|$

$\rightarrow \int \frac{\cos x}{\sin^3 x}\,dx \rightarrow \frac{\cos x}{\sin x} \cdot \frac{1}{\sin^2 x}\,dx$

$\hookrightarrow$ cotangente $\cdot \frac{1}{\sin^2 x}$

$$\begin{cases} u = \cot \\ du = -\frac{\overbrace{\sin^2 x + \cos^2 x}^{1}}{\sin^2 x} \end{cases}$$

$\hookrightarrow -\frac{1}{\sin^2 x}$

$-du = \frac{1}{\sin^2 x}$

$\rightarrow \int \cot x \cdot \frac{1}{\sin^2 x}\,dx =$

$\int u \cdot -du \rightsquigarrow -\int u\,du \rightsquigarrow -\frac{u^2}{2} \rightarrow -\frac{\cot^2}{2}$

6) $\int \frac{\cos x}{\sin^2 x - 5\sin x - 6}\,dx$ $\begin{cases} u = \sin x \\ du = \cos x \end{cases}$

$\hookrightarrow \int \frac{du}{u^2 - 5u - 6}$ $\Rightarrow$ fattorizzazione $\rightsquigarrow (u-6)(u+1)$

$\hookrightarrow \int \frac{1}{(u-6)(u+1)} \rightarrow \frac{\frac{1}{7}}{u-6} + \frac{-\frac{1}{7}}{u+1}$ $\Rightarrow$ fr o e

mai voglio

$\rightarrow \frac{1}{7}\log|u-6| - \frac{1}{7}\log|u+1|$

$\rightsquigarrow \frac{1}{7} \log\left|\frac{\sin x - 8}{\sin x + 1}\right| + C$

7) $\int \frac{1}{\sqrt{e^x}} \Rightarrow \int \frac{1}{e^{\frac{x}{2}}}\,dx \Rightarrow \int e^{-\frac{x}{2}}\,dx$

$\rightsquigarrow -2e^{-\frac{x}{2}}\,dx$ # avem mentrat

$e^{2x} \to \frac{1}{2}e^{2x}$

$e^{-\frac{x}{2}} \Rightarrow -2e^{-\frac{x}{2}} \to \frac{-2}{\sqrt{e^x}} + C$

8) $\int \frac{e^x\sqrt{e^x - 1}}{e^x + 3}$ $\begin{cases} u = \sqrt{e^x - 1} \\ du = \frac{e^x}{2\sqrt{e^x-1}}\,dx \Rightarrow dx = \frac{2\sqrt{e^x-1}}{e^x} \end{cases}$ $\underbrace{2\sqrt{e^x-1}}_{u}$

$\rightsquigarrow 2\int \frac{\cancel{e^x}\, u}{u^2+1+3} \cdot \frac{u}{\cancel{e^x}}\,du$ $u = \sqrt{e^x - 1}$

$u^2 + 1 = e^x$

$\rightsquigarrow 2\int \frac{u^2 + 4 - 4}{u^2+4} \rightsquigarrow \frac{u^2+4}{u^2+4} - \frac{4}{u^2+4}$

$\downarrow$ 1 $\downarrow$ 4

$\int 1\,dx - 4\int \frac{1}{u^2+4}\,dx$

$\rightsquigarrow 2^2$

$\int \frac{1}{4+x^2}$

$$\hookrightarrow \int \frac{}{4\left(1+\frac{x^2}{4}\right)} = \frac{1}{4}\int \frac{1}{1+\frac{x^2}{4}} \quad \frac{1}{\left(\frac{x}{2}\right)^2}$$

$$\rightarrow \frac{1}{4}\, 2 \int \frac{\frac{1}{2}}{1+\left(\frac{x}{2}\right)^2}\, dy \rightarrow \frac{1}{1+u^2}$$

$$\begin{cases} u = \frac{x}{2} \\ du = \frac{1}{2} \end{cases}$$

$$\hookrightarrow \frac{1}{2}\operatorname{arctg}\left(\frac{x}{2}\right)+C$$

- $\int 1\, dx \sim u \int \frac{1}{u^2+4}\, dx$ $\quad u = \sqrt{e^x - 1}$

$\quad\quad\quad\quad\quad\quad 2^2$

$$\hookrightarrow 2\left(\int 1\, du - 2\operatorname{arctg}\left(\frac{u}{2}\right)\right.$$

$$2\sqrt{e^x-1} - 4\operatorname{arctg}\left(\frac{\sqrt{e^x-1}}{2}\right)$$

8) $\int \frac{1}{x+\sqrt{x}} \rightarrow \frac{1}{x+x^{\frac{1}{2}}} \rightarrow \frac{1}{x^{\frac{3}{2}}}$ no

$\hookrightarrow \sqrt{x}(\sqrt{x}+1) \Rightarrow \frac{1}{\sqrt{x}(\sqrt{x}+1)}$

$$\begin{cases} u = \sqrt{x}+1 \\ du = \frac{1}{2\sqrt{x}}\, dx \\ dx = 2\sqrt{x} \end{cases}$$

$\to \frac{1}{\sqrt{x}(\sqrt{x}+1)} \Rightarrow \frac{1}{\cancel{\sqrt{t}}(u)} \cdot 2\cancel{\sqrt{t}}\,dt \to 2\int \frac{1}{u}$

↳ $2\log(u)$

↳ $\sqrt{x}+1+c$

10) $\int_{-1}^{5} |x-3|\,dx$ GRAFICAMENTE

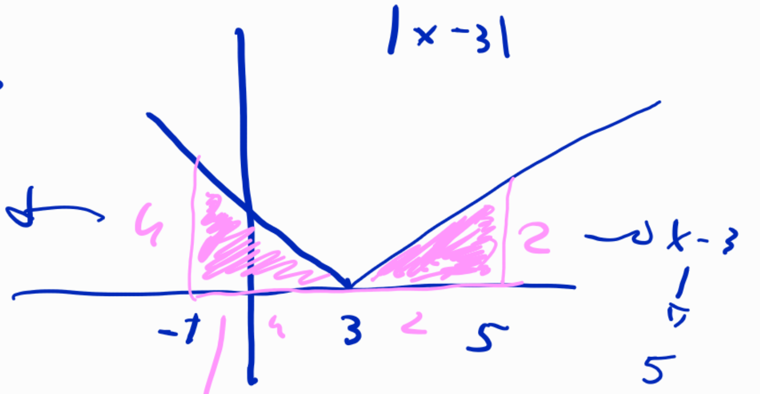
$x-3 \to$

↳

$x-3$

$\downarrow$

-1



↳ TRIANGOLI:

$\frac{\text{BASE} \cdot \text{ALTEZZA}}{2}$

$T_1 = \frac{4 \cdot 4}{2} = 8$

$T_2 = \frac{2 \cdot 2}{2} = 2$

$8+2=10$

$\frac{x^2}{2}\Big|_{-1}^{5} - 3x\Big|_{-1}^{5}$

↳ $\left(\frac{25}{2}-\frac{1}{2}\right)-(-1-5)$ ✗

↳ $\frac{25-1}{2} \Rightarrow 12+6=18$?

↳ $\frac{x^2}{2}-3x\Big|_{-1}^{5} \Rightarrow \left(\frac{25}{2}-5\right)-\left(\frac{1}{2}+1\right) \Rightarrow$ ✗

$\frac{25-10}{2} \Rightarrow \frac{15}{2}-\frac{3}{2}=\frac{15-3}{2}=\frac{12}{2}=6$

$\int^{5} 15$ ✗

$$\frac{x}{2}\Big|_{-1} - 3x\Big|_{-1} \Rightarrow \left(\frac{25}{2} - \frac{1}{2}\right) - (15 - 3) \Rightarrow$$

$$12 - 12 = 0\ ?$$

$$\frac{x^2}{2} - 3x\Big|_{-1}^{5} \Rightarrow \left(\frac{25}{2} - 15\right) - \left(\frac{1}{2} + 3\right)$$

$\times$

$$\hookrightarrow \frac{25 - 30}{2} \Rightarrow -\frac{5}{2} - \frac{7}{2}\ ?$$

11) $$\int \frac{\sin y}{\sec^{2019} y}\, dy \rightarrow \int \cos^{2019} x \sin x\, dy$$

$\hookrightarrow \frac{1}{\sec^{2019}} \rightarrow \cos^{2019}$?

$\underbrace{\qquad}_{\text{INVERSI}}$

$$\begin{cases} u = \cos x \\ du = -\sin x \end{cases}$$

$$\rightarrow \int -u^{2019} \cdot du \rightarrow -\int u^{2019}$$

$$\hookrightarrow -\frac{u^{2020}}{2020} \Rightarrow -\frac{\cos^{2020}}{2020}\ el$$

12) $$\int x \sin^{-1} x \quad dx$$

INTEGRAZIONE ... $F(x) \cdot g(x) - \int ...$

$$\to -\sin^{-1}x\,\sqrt{1-x^2}+\int 1\,dx$$

$$\left\{\begin{array}{ll}\sin^{-1}x & \dfrac{x}{\sqrt{1-x^2}} \\ & \downarrow \\ & \dfrac{1}{-1} \\ & -\sqrt{1-x^2}\end{array}\right.$$

$$\hookrightarrow \text{deriv.} = \boxed{\frac{1}{2\sqrt{1-x^2}}\cdot -2x}$$

$$\left(\int f(x)\cdot g'(x) = f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx\right.$$

$$-\sin^{-1}x\,\sqrt{1-x^2}+x+C$$

13) $\displaystyle\int \frac{2\sin x}{\sin(2x)}\,dx \to 2\sin x\cos x \Rightarrow \int \frac{\cancel{2\sin x}}{\cancel{2\sin x}\cos x}$

$$\int \frac{1}{\cos x}\,dx \Rightarrow \int \sec x$$

$$\to \int \frac{\sec x\,(\sec x+\tan x)}{(\sec x+\tan x)}$$

$$\left\{\begin{array}{l} u = \sec x + \tan x \\ du = \frac{1}{1}\,\sec x\tan x + \sec^2 x\,dx \end{array}\right.$$

$$dx = \frac{1}{\sec x\tan x+\sec^2 x}$$

$$\to \int \frac{\cancel{\sec x}\cdot u}{u}\cdot\frac{1}{\sec x\tan x+\sec^2 x}\,dx$$

$$\hookrightarrow \frac{1}{\cancel{\sec x}(\tan x+\sec x)} \Rightarrow \frac{u}{u^2} \to \frac{1}{u}$$

$\to \log|u| \to \log|\sec x + \tan x|$

14) $\int \cos^2(2x)\, dx$

$\hookrightarrow \cos(2x)\cos(2x)$

$\cos(2x) =$

$\cos^2 x - \sin^2 x$

$\int (\cos^2 x - \sin^2 x)(\cos^2 x - \sin^2 x)$

$\begin{cases} u = \cos^2 x - \sin^2 x \\ du = \end{cases}$

$\to \int \cos^2(2x)\,dx = \quad \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

$2\theta \to 2x \to 2 \cdot 2x$

$\to \frac{1}{2}\int 1 + \cos(4x)\,dx$

$x \qquad \frac{\sin(4x)}{4} \to \frac{1}{2}\left(x + \frac{1}{4}\sin(4x)\right) + c$

15) $\int \frac{1}{x^3+1}\,dx \Rightarrow (x+1)(x^2-x+1)$ Fattorizzazione STRANA

$\to \frac{1}{(\ldots)} = \frac{A}{\ldots} + \frac{Bx+C}{\ldots}$ $\quad A \to \frac{1}{3}, \quad B \to 0 \to \ldots, \quad C \to \frac{2}{3}$

$(x+1)(x^2-x+1)$ $\quad (x+1) \quad x^2-x+1$

$\rightarrow \frac{1}{3} + C = 1 \Rightarrow C = \frac{2}{3} \quad (x=0)$

$(x=1) \Rightarrow \frac{1}{2} = \frac{1}{6} + B + \frac{2}{3}$

CASO STRANO DI FATTORIZZAZIONE

1 2

$$\frac{1}{3}\int \frac{1}{x+1}\,dx - \frac{1}{3}\int \frac{x-2}{x^2-x+1}\,dx$$

$$\rightarrow \int \frac{x-2}{x^2-x+1} \qquad \frac{d}{dx}(x^2-x+1) = 2x-1 \neq x-2$$

$\hookrightarrow$ VOGLIO che $\frac{f'(x)}{f(x)} \rightarrow$ MOLTIPLICO e DIVIDO PER DUE

$$\frac{1}{2}\int \frac{2(x-2)}{x^2-x+1} = \frac{2x-4}{x^2-x+1} \rightarrow (2x-1)-3$$

$-4 \rightarrow$ diventa essere -1

$\hookrightarrow \frac{d}{dx}x^2-x+1$

2

$1 \quad \frac{1}{3}\int \frac{1}{x+1}\,dx - \frac{1}{6}\left[\int \frac{2x-1}{x^2-x+1}\,dx - \int \frac{3}{x^2-x+1}\right.$ 2 3

$\frac{1}{x^2+y^2}$

$3 \quad -\int \frac{3}{x^2-x+1} \to \left(x^2-x+\frac{1}{4}\right)+\frac{3}{4}$, $1 = \frac{1}{4}+\frac{3}{4}$

$\hookrightarrow \left(x-\frac{1}{2}\right)^2+\frac{3}{4} \to \left(\frac{\sqrt{3}}{2}\right)^2$

$1 \to \frac{1}{3}\log|x+1| - \frac{1}{6}\log(x^2-x+1)$

$3 \to \frac{1}{\cancel{2}}\,\frac{\cancel{2}}{\sqrt{3}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)$ (SOSTITUZIONE)

$\hookrightarrow \left(\frac{2}{\sqrt{3}}\left(x-\frac{1}{2}\right)\right)$

16) $\int x\,\underline{\sin^2 x}$

$\hookrightarrow$ RIDUZIONE POTENZA

$\sin^2 x = \frac{1}{2}(1-\cos(2x))$

$$\to \frac{1}{2}\int x\left(1-\cos(2x)\right) \Rightarrow \frac{1}{2}\int x - \int x\cos(2x)$$

$$\qquad\qquad\qquad\qquad\qquad 1 \qquad\quad 2$$

2 integrazione per parti $\Rightarrow \int f(x)g'(x) = f(x)g(x) - \int f'(x)g(x)$

$\cancel{\#}\cos(2x) = \cos^2 x - \sin^2 x$

derivo

$$-\int x\cos(2x)\,dx \qquad \begin{cases} -x \to -1 \\ \cos(2x) \to \text{integro} \to \dfrac{\sin(2x)}{2} \end{cases}$$

$$\to \frac{1}{2}\left(\frac{1}{2}x^2\right. \qquad \text{integral} \Rightarrow -\frac{x\sin(2x)}{2} - \int \frac{\sin(2x)}{2}$$

$$\to -\frac{1}{4}\cos(2x)$$

$$\to \frac{1}{2}\left(\frac{1}{2}x^2 - \frac{1}{2}x\sin(2x) - \frac{1}{4}\cos(2x)\right) \Rightarrow$$

$$\frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x) + c$$

17) $\int \left(x + \frac{1}{x}\right)^2 dx$

$\hookrightarrow x^2 + 2 + \frac{1}{x^2} dx \Rightarrow \frac{x^3}{3} + 2x + \frac{1}{x^2} \to x^{-2} \to \frac{x^{-1}}{-1} = -\frac{1}{x}$

18) $\int \frac{3}{x^2+4x+29} dx \to x^2+4x+29 \Rightarrow \frac{3}{(x+2)^2+25} \to 5^2$

$\int \frac{3}{(x+2)^2+5^2} dx \to 3\int \frac{1}{(x+2)^2+5^2}$

ou $3\int \frac{1}{(x+2)^2+5^2} dx \to \text{Tg}^{-1}$?

$\hookrightarrow \frac{3}{5} \text{Tg}^{-1}\left(\frac{x+2}{5}\right) + C$

19) $\int \cos^5 x \, dx$

$\hookrightarrow \frac{\cos x}{\sin x} \to \frac{\cos^5 x}{\sin^5 x}$

$\cos^2 x + \sin^2 x = 1$

$$\Rightarrow \frac{\cos^4 y}{\sin^5 y} \cos y \, dy \Rightarrow \frac{(1-\sin^2 x)^2}{\sin^5 x} \cos x \, dx$$

$$\begin{cases} u = \sin x \\ du = \cos x \end{cases} \Rightarrow \frac{(1-u^2)^2}{u^5} du$$

$$\int \frac{(1-u^2)^2}{u^5} dy \Rightarrow \frac{1-2u^2+u^4}{u^5}$$

$$\int \frac{1}{u^5} - 2\int \frac{u^2}{u^{5}} + \int \frac{\cancel{u^4}}{u^{5}}$$

$$\hookrightarrow \int u^{-5} - 2\int u^{-3} + \int u^{-1}$$

$$\frac{u^{-4}}{-4} - \cancel{2}\frac{u^{-2}}{\cancel{-2}} + \ln(u) \qquad u = \sin x$$

$$\frac{1}{\sin} \Rightarrow \text{cosecant}$$

20) $\int^{1} \left(\frac{\tan x}{x^4 - x^2 + 1} \right) = 0$ DISPARI

-1 ... positivo ... $\Rightarrow$ ... -1 ... $+1$

21) $\int \sin^3 x \cos^2 x \, dx$

no

$\Rightarrow \int (\sin x)(\sin^2 x)(1-\sin^2 x)\, dx$ $\quad \left\{ u = \sin x \right.$

$\Rightarrow \int (\sin x)(1-\cos^2 x)(\cos^2 x)$ $\quad \left\{ \begin{array}{l} u = \cos x \\ du = -\sin x \, dx \end{array} \right.$ $\quad dx = \dfrac{1}{-\sin x}$

$\int \cancel{\sin x}(1-u^2)\,u^2 \dfrac{1}{\cancel{-\sin x}}$

$-\int u^2 - u^4 \Rightarrow -\int u^2 + \int u^4$

$\Rightarrow$ $\quad u = \cos x$

$-\dfrac{u^3}{3} + \dfrac{u^5}{5} \Rightarrow -\dfrac{\cos^3 x}{3} + \dfrac{\cos^5 x}{5}$

22) $\int \dfrac{1}{x^2\sqrt{x^2+1}}\, dx$

$\hookrightarrow x^2\left(1+\frac{1}{x^2} \to x^{-2}\right)$

$\hookrightarrow \boxed{\sqrt{x^2+1} \to \sqrt{x^2}\sqrt{1+x^{-2}}}$

$$\to \int \frac{1}{x^2\sqrt{x^2}\sqrt{1+x^{-2}}} \to \int \frac{1}{x^3\sqrt{1+x^{-2}}} \Rightarrow \int \frac{x^{-3}}{\sqrt{1+x^{-2}}}$$

$$\begin{cases} u = 1+x^{-2} \\ du = -2x^{-3}\,dx \end{cases} \to \int \frac{\cancel{x^{-3}}}{\sqrt{u}} \cdot -\frac{1}{2\cancel{x^{-3}}}$$

$dx = -\frac{1}{2x^{-3}}\,du$

$$-\frac{1}{2}\int \frac{1}{\sqrt{u}} \Rightarrow \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \Rightarrow 2\sqrt{u}$$

$\to \frac{-1+2}{2} = \frac{1}{2}$

$$-\frac{1}{\cancel{2}}\,\cancel{2}\sqrt{u} \Rightarrow -\sqrt{u} \Rightarrow -\sqrt{1+x^{-2}}+C$$

23) $\int \sin x \sec x \tan x \, dx$

$\hookrightarrow \int \sin x \frac{\cancel{1}}{\cos x}\frac{\sin x}{} \, dx \quad N()$

$$\int \frac{\sin^2 x}{\cos x} \quad \begin{cases} u = \sin x \\ du = \cos x \, dx \\ dx = \frac{1}{\cos x} \end{cases} \rightarrow \int \frac{u^2}{du}$$

$$\rightarrow \frac{u^3}{3} \rightarrow \frac{\sin^3 x}{3}$$

$$\int \sin x \sec x \tan x \, dx \Rightarrow \frac{\sin x}{\cos x} \cdot \tan x = \tan^2 x \rightarrow (\sec^2 x - 1)$$

$\sec x = \frac{1}{\cos}$

integrale di $\sec^2 x \rightarrow$ tang oppure tangente $\sec^2 x$

$$-1 \rightarrow x \quad \rightarrow \tan x - x + C$$

24) $\int \sec^3 dy$

$\hookrightarrow \frac{1}{\cos^3} \rightarrow \int \sec^2 \cdot \sec x \quad \begin{cases} \sec^2 x \rightarrow \tan x \\ \sec x \rightarrow \sec^2 x \end{cases}$ integ

$$\frac{d}{dx} \frac{1}{\cos} = \frac{-\sin}{\cos^2 x} \rightarrow \sec x \tan x$$

$$\tan x \cdot \sec x - \int \tan x \cdot \sec x \tan x$$

$$\hookrightarrow \sec x \tan x - \int \sec^3 x \, dy + \int \sec x$$

$\tan^2 x \sec x$

$(\sec^2 x - 1) \sec x =$

3

$\sec^3 t - 2\sec t$

Recall

$$\int \sec^3 x \, dx = \sec x \tan x - \cancel{\int \sec^3 x \, dx} + \int \sec x$$

$+\sec^3 x$

→ aggiungo $\int \sec^3 x \to$ tutti e due

$+\sec^3 x$

$$\to 2\int \sec^3 x \, dx = \sec x \tan x + \log|\sec x + \tan x|$$

$$\to \int \sec^3 x \, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\log|\sec x + \tan x|$$

24) $\displaystyle \frac{1}{x\sqrt{9x^2-1}}\,dx \Rightarrow \int \frac{1}{x\sqrt{x^2\left(9-\frac{1}{x^2}\right)}} \Rightarrow \frac{1}{x\sqrt{x^2}\sqrt{9-\frac{1}{x^2}}}$

$$\int \frac{1}{x^2\sqrt{9-\frac{1}{x^2}}}$$

$\sqrt{9x^2-1}$

$\to (3x)^2$

$3x = \sec\theta$

$x = \frac{\sec\theta}{3}$

$dx = \frac{1}{3}\sec x \tan x$

$\to \sec^{-1}(3x) + c$

26) $\displaystyle \int \cos\sqrt{x}\,dx$

$u = \sqrt{x}$

$x = u^2$

$dx = 2u$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$du = 2\sqrt{x}$

?

$\to \int \cos u \cdot 2u \, du$ PER PARTI $\to$ $\sin u \cdot 2u - \int 2\sin u$

$$\left\{\begin{array}{cc} D & I \\ 2u & \cos u \\ \downarrow & \downarrow \\ 2 & \sin u \end{array}\right.$$

$\Downarrow$

$-2\int \sin u$

$\to +2\cos u$

$F(x) \cdot g(x) - \int F(x) \cdot g'(x)$

Un pad tecnic:

	D	I
PRIMO	$2u$	$\cos u$
SECONDO	-2	$\sin u$
	0	$-\cos u$

$\to 2u\sin u + 2\cos u$

27) $\int \operatorname{cosecante} x$

$\hookrightarrow \dfrac{(\operatorname{cosecante} x - \cot x)\operatorname{cosecante} x}{(\operatorname{cosecante} x - \cot x)}$

$\left\{\begin{array}{l} u = \operatorname{cosecante} x - \cot x \\ du = (-\operatorname{cosecante} x \cot x + \operatorname{cosecante}^2 x)\,dx \end{array}\right.$

RIFACCIO

$\int \csc x \, dx \Rightarrow \int \dfrac{\csc x \cdot (\csc x + \cot x)}{(\csc x + \cot x)} = $

$= \int \csc^2 x + \csc x \cot x$ ... $\left\{ u = \csc x + \cot x \right.$

$$\int \frac{\quad}{\csc x + \cot x}\,dx$$

$$\left\{ du = -\csc x \cot x - \csc^2 x\, dx \right.$$

BHO cl ckeso

$$dx = \frac{du}{-\csc x \cot x - \csc^2 x}$$

$$\to \int \frac{\cancel{\csc^2 + \csc x \cot x}}{u} \cdot \frac{du}{-(\cancel{\csc x \cot x + \csc^2 x})} \Rightarrow -\int \frac{1}{u}\,du$$

$$-\int \frac{1}{u}\,du \Rightarrow -\log|u| = -\log|\csc x + \cot x| + C$$

28) $\int \sqrt{x^2+4x+13}\,dx$

cerchiamo un bel quadrato di binomio $\to x^2+4x+4$

$$\sqrt{x^2+4x+4+9} \to \sqrt{(x+2)^2+3^2}$$

$x+2 = 3\tan\theta$

$dx = 3\sec^2\theta\, d\theta$

$$\to \int \sqrt{(3\tan\theta)^2+3^2}\,dx \Rightarrow \int \sqrt{(3\tan\theta)^2+3^2} \cdot 3\sec^2\theta\, d\theta$$

29) $\int e^{2x}\cos 3x\, dx$ per parti

	D		I
$+$	e^{2x}	$\searrow$	$\cos x$
$-$	$2e^{2x}$	$\searrow$	$\sin x$
$+$	$4e^{2x}$		$-\cos x$

$$\int e^{2x}\cos x\,dx \Rightarrow e^{2x}\sin x + \cos x\, 2e^{2x}$$

Case standard $\Rightarrow$

D	I
e^{2x}	$\cos x$
$2e^{2x}$	$\sin x$
$4e^{2x}$	$-\cos x$

Continue

$$\sin x\, e^{2x} - 2\int \sin x\, 2 e^{2x}$$

$$\rightarrow e^{2x}\sin x + \cos x\, 2e^{2x} - 4\int e^{2x}\cos x\,dx$$

Recap

$$\int e^{2x}\cos x\,dx = e^{2x}\sin x + \cos x\, 2e^{2x} - 4\int e^{2x}\cos x\,dx$$

$+4\int e^{2x}\cos x\,dx$ $\qquad$ $+4\int e^{2x}\cos x\,dx$

$$\rightarrow \int e^{2x}\cos x\,dx = \frac{1}{5}e^{2x}\sin x + \frac{2}{5}e^{2x}\cos x + C$$

20) $\displaystyle\int_{x=3}^{x=3} (x-3)^9\,dx$ $\qquad$ $\begin{cases} u = x-3 \\ du = dx \end{cases}$

$$\to \int_0^2 u^9\,du \to \frac{u^{10}}{10}\Big|_0^2 \to \frac{2^{10}}{10} \to \frac{1024}{10} \to \frac{512}{5}$$

$2^{10} \to 1024$

31) $\int \frac{1}{\sqrt{x-x^{\frac{3}{2}}}}\,dx \to x\left(1-\frac{x^{\frac{3}{2}}}{x^1} \to \frac{3}{2}-1=\frac{3-2}{2} \Rightarrow \frac{1}{2}\right)$

$$\to \int \frac{1}{\sqrt{x\left(1-x^{\frac{1}{2}}\right)}} \to \frac{1}{\sqrt{x}\sqrt{(1-\sqrt{x})}}\,dx$$

$$\begin{cases} u = 1-\sqrt{x} \\ du = -\frac{1}{2\sqrt{x}}\,dx \\ du \cdot -2\sqrt{x} \end{cases}$$

$$\to \int \frac{1}{\cancel{\sqrt{x}}\cdot\sqrt{u}} \cdot -2\cancel{\sqrt{x}} = -2\int \frac{1}{\sqrt{u}} \to \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \quad -\frac{1}{2}+1 = \frac{-1+2}{2} = \frac{1}{2}$$

$$\to \cancel{-2}\int \frac{u^{\frac{1}{2}}}{\cancel{\frac{1}{2}}} \to -\sqrt{u} \to -\sqrt{1-\sqrt{x}} + C$$

32) $\int \frac{1}{\sqrt{x-x^2}}\,dx \Rightarrow \frac{1}{\sqrt{x}\sqrt{1-x}} \to (\sqrt{x})^2$

$$\to \int \frac{1}{\sqrt{x}\sqrt{1-(\sqrt{x})^2}}\,dx$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

$dx = 2\sqrt{x}$

$$\to \int \frac{1}{\cancel{\sqrt{x}}\sqrt{1-u^2}}\,2\cancel{\sqrt{x}}\,du \to 2\int \frac{1}{\sqrt{1-u^2}}$$

$$\to \underline{2\sin^{-1}(u)}$$

33) $\int e^{2\ln x}\,dx$

$$\to \int \cancel{e}^{\cancel{\ln}(x^2)}\,dx \to \int x^2 \to \frac{x^2}{3} + C$$

inverse

34) $\int \frac{\ln x}{\sqrt{x}}\,dx$

D	I
$+\ \ln x$	1
$-\ \frac{1}{x}$	$\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$
	$2\sqrt{x}$

$$2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,dx \qquad \to \frac{1}{x}\cdot 2\sqrt{x} = \frac{1}{\sqrt{x}}$$

$$\to 2\sqrt{x}\ln x - 4\sqrt{x} + C$$

$\Rightarrow f'(x)\cdot g(y)$

33) $\int \frac{1}{e^x + e^{-x}}$ MOLTIPLICHIAMO PER $\frac{e^x}{e^x}$

$\Rightarrow \int \frac{1}{(e^x+e^{-x})} \frac{e^x}{e^x} \Rightarrow \int \frac{e^x}{(e^x)^2+1}$ $\begin{cases} u = e^x \\ du = e^x\,dx \end{cases}$

$\rightarrow \int \frac{du}{u^2+1} \rightarrow \tan^{-1}(\overset{e^x}{u}) + c$

36) $\int \log_{2} x \, dx \Rightarrow \int \frac{\log x}{\log 2} dx$

$\frac{1}{\log 2} \int \log x \, dx$ $\begin{cases} D & I \\ \log x & 1 \\ \frac{1}{x} & x \end{cases}$ $\Rightarrow \log x \cdot x - \int \frac{1}{\cancel{x}} \cdot \cancel{x}$

$\rightarrow \underline{\log x \cdot x - x}$

37) $\int x^3 \sin(2x) \, dx$

$\begin{cases} D & I \\ 1 \; x^3 & \sin(2x) \\ 2 \; 3x^2 & -\frac{\cos(2x)}{2} \\ 3 \; 6x & -\frac{\sin(2x)}{4} \\ 4 \; 6 & +\frac{1}{8}\cos(2x) \\ 0 & \frac{1}{16}\sin(2x) \end{cases}$

$$\to -\frac{1}{2}\frac{x^3\cos(2x)}{2} + \frac{3}{4}x^2\sin(2x) + \frac{3}{4}x\cos(2x) - \frac{3}{8}\sin(2x)$$

1 2 3 4

38) $\int x^2\sqrt[3]{1+x^3}\,dx$

$$\begin{cases} u = 1+x^3 \\ du = 3x^2\,dx \\ dx = \dfrac{1}{3x^2} \end{cases}$$

$$\frac{1}{3}+1=\frac{1+3}{3}$$

$$\to \int \cancel{x^2}\sqrt[3]{u}\,\frac{1}{3\cancel{x^2}} \to \frac{1}{3}\int u^{\frac{1}{3}} \to \frac{u^{\frac{1}{3}+1}}{\frac{1}{3}+1}$$

$$\frac{1}{3}\frac{u^{\frac{4}{3}}}{\frac{4}{3}} \to \frac{1}{4}\sqrt[3]{u^4} \to \frac{1}{4}\sqrt[3]{(1+x^3)^4}$$

$$\Rightarrow \frac{1}{4}(1+x^3)\sqrt[3]{(1+x^3)^3} + c$$

39) $\int \frac{1}{(x^2+4)^2}\,dx$

$$\begin{cases} x = 2\tan\theta \\ dx = 2\sec^2\theta\,d\theta \end{cases}$$

$$\{ \quad (2\tan\theta)^2 \Rightarrow 4\tan^2\theta \to 4(\sec^2\theta - 1)$$

$4\sec^2 x \cancel{-4} \cancel{+4} \to \int \frac{1}{(4\sec^2)^2} \cdot 2\sec^2\theta \, d\theta$

$\to \frac{1}{8 \cdot \cancel{16}\sec^{\cancel{4}\,2} x} \, \cancel{2}\sec^{\cancel{2}} x \to \int \frac{1}{8\sec^2 x} \Rightarrow \frac{1}{8}\int \frac{1}{\sec^2 x}$

$\frac{1}{C_1} = \cos^2 x$

POLBRI

$\frac{1}{8}\int \cos^2 x \to \cos^2 x = \frac{1}{2}(1 + \cos(2x))$

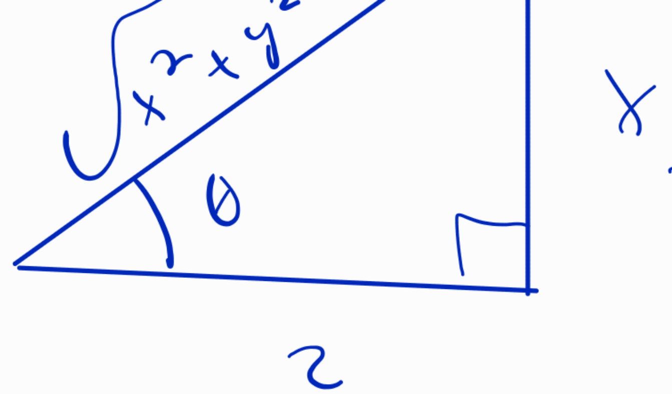
$\frac{1}{16}\int 1 + \cos(2x) \Rightarrow \frac{1}{16}\left(\theta + \frac{1}{\cancel{2}}\sin(2\theta)\right)$

$x \qquad \frac{\sin(2x)}{2}$

$\sin(2x) = \cancel{2}\sin x \cos x$

$\to \frac{1}{16}\theta + \frac{1}{16}\sin\theta\cos\theta$

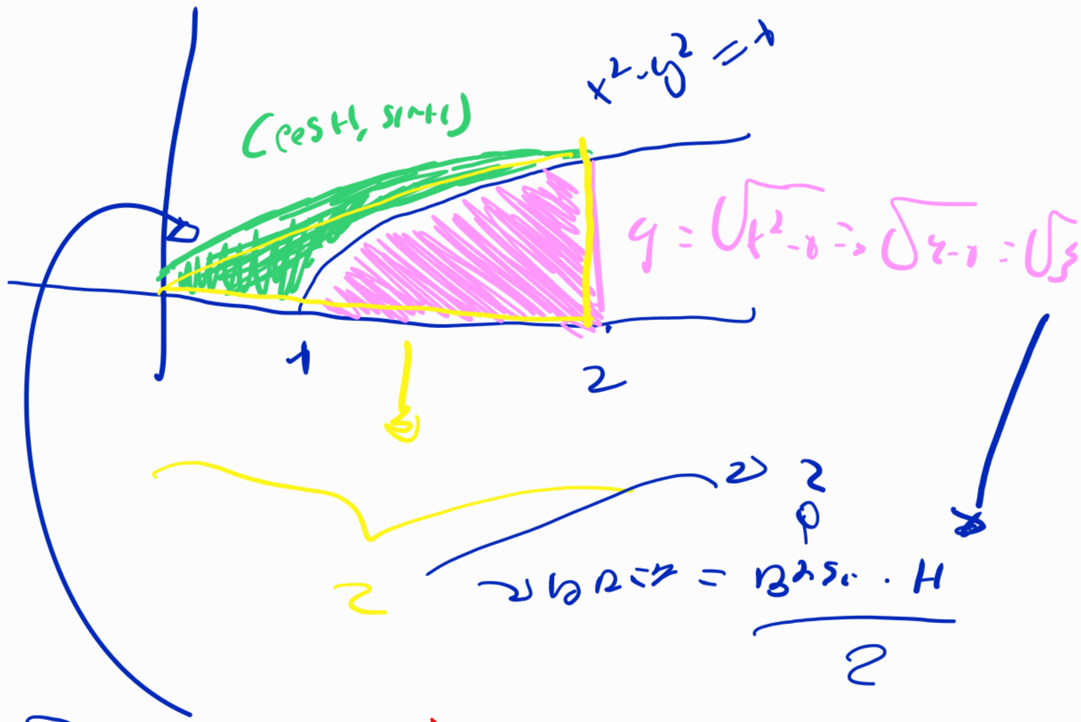
Ricorda che $\tan\theta = \frac{x}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{x}{2}\right)$



$$\to \frac{1}{16}\theta + \frac{1}{16}\sin\theta\cos\theta \Rightarrow \frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{16}\,\frac{x}{\sqrt{x^2+y^2}}\;\frac{2}{\sqrt{x^2+y^2}}$$

40) $\displaystyle\int_1^2 \sqrt{x^2-1}\,dy$

$\hookrightarrow$



$$\to \cancel{\frac{2}{2}}\sqrt{3} - \frac{T}{2}$$

$$\begin{cases} \cosh(t) = 2 \\ T = \cos^{-1}(2) \\ \sinh T = \sqrt{3} \end{cases}$$

$t \Rightarrow \operatorname{arctan}^{-1}(\sqrt{3})$

41) $\int \sinh x \, dx \Rightarrow$

$\left\{ \sinh x \Rightarrow \frac{e^x - e^{-x}}{2} \, dx \right.$

$\hookrightarrow \int \frac{e^x - e^{-x}}{2} \, dx$

$\hookrightarrow \frac{1}{2}(e^x + e^{-x}) \, dx \rightarrow \cosh x$

$\left\{ \cosh x = \frac{e^x + e^{-x}}{2} \right.$

42) $\int \sinh^2 x \, dx \rightarrow \frac{e^x - e^{-x}}{2}$

$\hookrightarrow \left(\frac{1}{2} e^x - e^{-x}\right)^2 = \frac{e^{2x} - 2 + e^{-2x}}{4}$ $\quad 2 \cdot e^x \cdot e^{-x} = 1$

$\cosh x$

$\# \frac{e^{2x} + e^{-2}}{2}$

$\frac{e^{2x} - 2 + e^{-2x}}{4} \Rightarrow \frac{1}{4}(e^{2x} - 2 + e^{-2x})$

$$\to \frac{1}{4}(-x) + \frac{1}{2}\cosh \to \frac{e^{2x}+e^{-2}}{2} \to -\frac{1}{2}+\frac{1}{2}\cosh x$$

$$\to \int -\frac{1}{2}+\frac{1}{2}\cosh x \to \int \frac{1}{2}(-1+\cosh x \cdot \frac{1}{2}\int -1+\cosh(2x)$$

$$\frac{1}{2}\left(-x+\frac{1}{2}\sinh(2x)\right)$$

$$-\frac{1}{2}x + \frac{1}{4}\sinh(2x)+c$$

43) $\int \sinh^3 x \, dx$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\to \int \sinh^2 x \cdot \sinh x \, dx$$

$$x^2+y^2=1$$

$$\cosh^2 t - \sinh^2 t = 1$$

$$\to \int (\cosh^2 x - 1)\cdot \sinh x \, dx$$

sostitution

$$\begin{cases} u = \cosh x \\ du = \sinh x \, dx \\ dx = \frac{1}{\sinh} \end{cases}$$

$$\to \int (u^2-1)\,\cancel{\sinh x}\cdot\frac{1}{\cancel{\sinh x}}$$

$$\rightarrow \int u^2 - 1 \rightarrow \frac{u^3}{3} - u \rightarrow \frac{\cos^3 x}{3} - \cos x$$

44) $\int \frac{1}{\sqrt{x^2+1}}\,dx$

$$\left[\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}\right.$$

$\downarrow$

deriv. di $\sinh^{-1} x$

$$\left[\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1+x^2}}\right.$$

$\hookrightarrow \sinh^{-1} x + c$

45) $\int \log\left(x + \sqrt{1+x^2}\right) dx$

$$\sinh^{-1} x = \log\left(x + \sqrt{1+x^2}\right)$$

D	I
$\sinh^{-1} x$	1
$\frac{1}{\sqrt{1+x^2}}$	x

$\rightarrow x \sinh^{-1} x - \sqrt{1+x^2} + c$

46) $\int \tan x \, dx \rightarrow \int \frac{\sin x}{\cos x}\,dx$

$$\begin{cases} u = \cos x \\ du = \sin x\, dx \end{cases}$$

$\downarrow$

$\frac{}{u} \to \log|u| \to \log|\cosh x| + C$

47) $\int \operatorname{sech} x \, dx \to \int \frac{1}{\cosh x} \, dx$

$\sinh^2 x + \cosh^2 x = 1$

$\int \frac{1}{\cosh x} \, dx \cdot \frac{\cosh x}{\cosh x} \to \int \frac{\cosh x}{\cosh x \cosh x} \, dx \to \int \frac{\cosh x}{\boxed{\cosh^2 x}} \, dx$

$\to \int \frac{\cosh x}{1 + \sinh^2 x} \, dx$

Sostituzione

$\begin{cases} u = \sinh x \\ du = \cosh x \end{cases}$

$\to \int \frac{\cancel{\cosh x}}{1 + u^2} \cdot \frac{1}{\cancel{\cosh x}} \to \int \frac{1}{1+u^2} \to \tan^{-1}(u)$

$\sinh x$

eseguendo variabile

48) $\int \sqrt{\tanh x} \, dx$

$\begin{cases} u = \sqrt{\tanh x} \\ x = \tanh^{-1}(u^2) \\ dx = \frac{2u}{1-(u^2)} \, du \end{cases}$

$\to \int u \cdot \frac{2u}{1-u^4} \to \int \frac{2u^2}{1-u^4} \, du$

$\to (1-u^2)(1+u^2)$

$\to \int \underbrace{u^2 + u^2}_{2u^2}$ aggiungo $-1 + 1$

$$\int \frac{\ldots}{(1-u^2)(1+u^2)} \longrightarrow \int \frac{-(1-u^2)}{(1-u)^3(1+u)^2} + \frac{1+u^2}{(1+u)^2(1+u^2)}$$

$$\downarrow$$

$$\tanh^{-1}(u) + \tan^{-1}(u)$$

49) $\int \tanh^{-1} x \, dx$

PER PARTI $\rightarrow \int f(x)\cdot g'(x) = f(x)\cdot g(x) - \int f'(x)\cdot g(x)$

D	I
$\tanh^{-1} x$	1
$\frac{1}{1-x^2}$	x

$$\tanh^{-1} x \cdot x - \int \frac{x}{1-x^2} \rightarrow f'(x)\cdot g(x)$$

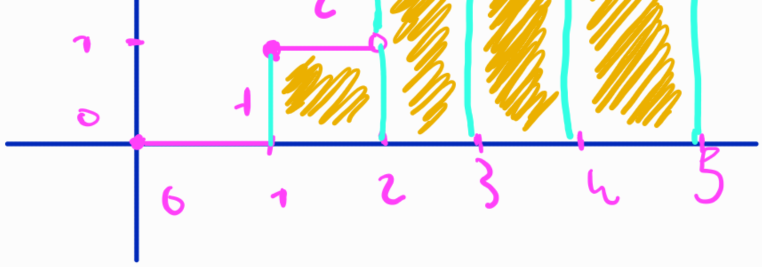
$$\tanh^{-1} x \cdot x - \frac{1}{2}\int \frac{2x}{1-x^2} \rightarrow \frac{1}{2}\cdot \log|1-x^2|$$

$$\rightarrow \tanh^{-1} x \cdot x - \frac{1}{2}\log|1-x^2|$$

50) $\int_0^5 \lfloor x \rfloor \, dx$

Li si fanno $\rightarrow$ 0-1, 1-2, 2-3, 3-4, 4-5

$= 1+2+3+4 = \boxed{10}$



$$\tan^2 x + 1$$

51) $\int \sec^6 x \, dx = $ $\quad \sec^4 x \to (\sec^2 x)^2 \to (\tan^2 x + 1)^2$

$$\to \int (\tan^2 x + 1)^2 \sec^2 x \, dx$$

$\begin{cases} u = \tan x \\ du = \sec^2 x \to \frac{1}{\cos^2 x} \end{cases}$

$dx \to \frac{1}{\sec^2 x}$

$$\to \int (u^2 + 1)^2 \cancel{\sec^2 x} \frac{du}{\cancel{\sec^2 x}} \to \int u^4 + 2u^2 + 1$$

$$\to \frac{u^5}{5} + 2\frac{u^3}{3} + u \to \frac{\tan^5 x}{5} + 2\frac{\tan^3 x}{3} + \tan x + C$$

52) $\int \frac{1}{(5x-2)^4} dx$

$\begin{cases} \text{PER SOSTITUZIONE VIENE UNA FIGATA} \\ u = 5x - 2 \end{cases}$

$du = 5 \, dx$

$dx = \frac{1}{5}$

$$\to \int \frac{1}{u^4} \cdot \frac{1}{5} \, du$$

$$\to \frac{1}{5} \cdot \frac{u^{-4+1}}{-4+1} \to -\frac{1}{15} \frac{1}{u^3}$$

53) $\int \log(1+x^2)\,dx$

$$\begin{cases} D & I \\ \log(1+x^2) & 1 \\ \frac{2x}{1+x^2} & x \end{cases}$$

$\log(1+x^2)\cdot x - \int \frac{2x^2}{1+x^2} \to -2\int \frac{x^2}{1+x^2}\,dx$

↳ provo a +1 per annullare

$x\log(1+x^2) - 2\int \frac{x^2+1-1}{1+x^2} \to \frac{\cancel{x^2+1}}{\cancel{1+x^2}} - \frac{1}{1+x^2}$

↳ $\frac{x^2+1-1}{1+x^2}$

$1 \to x$

↳ $\tan^{-1}$

$x\log(1+x^2) - 2x + 2\tan^{-1}x$

54) $\int \frac{1}{x^4+x} \to \frac{1}{x(x^3+1)}$ NO $\to$

$\to x^4(1+x^{-3}) \to \frac{1}{x^4(1+x^{-3})} \to \frac{x^{-4}}{1+x^{-3}}$ $\begin{cases} u = 1+x^{-3} \\ du = -3x^{-4}\,dx \end{cases}$

$\to \frac{\cancel{x^{-4}}}{u}\cdot\frac{1}{-3\cancel{x^{-4}}} \to -\frac{1}{3}\int \frac{1}{u} \to -\frac{1}{3}\log|1+x^{-3}| + c$

55) $\int \frac{1-\tan x}{1+\tan x}\,dx$

$1-\frac{\sin}{\cos x} \Rightarrow \frac{\cos x-\sin x}{\cos x}$

$1+\tan \to 1+\frac{\sin x}{\cos x}$

$\frac{\cos x+\sin x}{\cos x}$

$\frac{\;\frac{\cos x-\sin x}{\cancel{\cos x}}\;}{\;\frac{\cos x+\sin x}{\cancel{\cos x}}\;}$

$\Rightarrow \int \frac{\cos x-\sin x}{\cos x+\sin x}$

$\begin{cases} u = \cos x+\sin x \\ du = -\sin x+\cos x \end{cases}$

$\Rightarrow \int \frac{\cancel{\cos x-\sin x}}{u}\cdot\frac{du}{\cancel{\cos x-\sin x}} \Rightarrow \int \frac{1}{u} =$

$\Rightarrow \ln|\cos x+\sin x|+C$

56) $\int x\sec x\tan x\,dx$

$\sec x \to$ derivata: $\sec x\tan x$

integr

DUR ARAM

D	I
x	$\sec x\tan x$

$\to \sec x \to$ integr

$\ln|\sec x+\tan x|$

$1 \rightarrow \sec x$

$\rightarrow \log|\sec x + \tan x|$

$x \sec x - \int \sec x$

$x \sec x - \log|\sec x + \tan x| + c$

5f) $\int \sec^{-1} x \, dx$ per parti

D	I
$x \quad \sec^{-1}$	1
$-\frac{1}{x\sqrt{x^2-1}}$	x

$$x \sec^{-1} x - \int \frac{1}{x\sqrt{x^2-1}} \cdot \cancel{x}$$

$$g'(x) \cdot f(x)$$

$$-\int \frac{1}{\sqrt{x^2-1}}$$

cambio di variabile

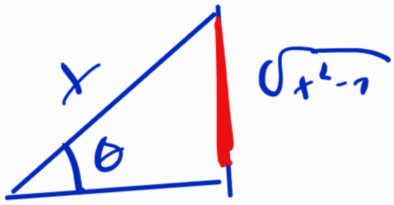
$$\begin{cases} x = \sec\theta \\ dx = \sec\theta \tan\theta \, d\theta \end{cases}$$

$$\int \frac{1}{\sqrt{\sec^2\theta - 1}} \sec\theta \tan\theta \, d\theta$$

$\hookrightarrow \tan^2\theta \rightarrow \frac{1}{\sqrt{\tan^2\theta}} \rightarrow \frac{1}{\cancel{\tan\theta}} \sec\theta \, \cancel{\tan\theta}$

$\hookrightarrow \log|\sec\theta + \tan\theta|$

$$x \sec^{-1} x - \log|x + \sqrt{x^2-1}| + c$$



58) $\int \frac{1-\cos x}{1+\cos x}$

$$\begin{cases} \cos^2\theta = \frac{1}{2}\left(1+\cos(2\theta)\right) \\ \sin^2\theta = \frac{1}{2}\left(1-\cos(2\theta)\right) \end{cases}$$

$\sin^2\theta = \frac{1}{2}\left(1-\cos(2\theta)\right)$

$\frac{1}{2}$ multiplicam per 2 →

$2\sin^2\theta = \cancel{2}\,\frac{1}{\cancel{2}}\left(1-\cos(2\theta)\right) \rightarrow 2\sin^2\theta = 1-\cos(2\theta) \rightarrow$

$2\sin^2 x = 1-\cos\left(\frac{x}{2}\right)$, 2 tem com la $1+\cos x$

$$\rightarrow \int \frac{\cancel{2}\sin^2\left(\frac{x}{2}\right)}{\underbrace{\cancel{2}\cos^2\left(\frac{x}{2}\right)}_{\text{tangent}^2 \rightarrow \sec^2 - 1}} \rightarrow \int \sec^2\left(\frac{x}{2}\right) - 1$$

$2\tan\left(\frac{x}{2}\right) - x + C$

59) $\int$

$$\int x^2\sqrt{x+4} \to x^2\sqrt{4\left(\frac{x}{4}+1\right)} \to x^2\cdot 2\cdot\sqrt{\frac{x}{4}+1}$$

AH

$$2\int x^2\sqrt{\frac{x}{4}+1} \to \sqrt{\frac{x+4}{4}}$$

$$\to \begin{cases} u = x+4 \to x = u-4 \\ du = \cancel{dx} \end{cases} \to \int x^2\sqrt{u} \to \int (u-4)^2\sqrt{u}\,du$$

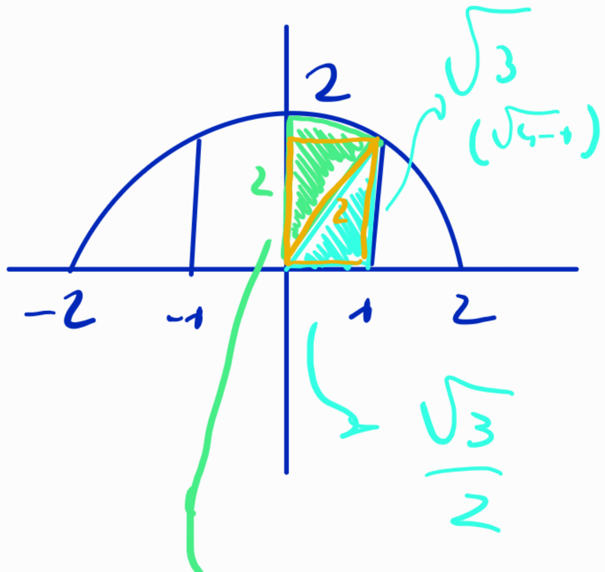
$x^2 \to (u-4)^2$

$$\to \int (u^2-8u+16)\,u^{\frac{1}{2}} \to u^{\frac{5}{2}} - 8u^{\frac{3}{2}} + 16u^{\frac{1}{2}}$$

$$\frac{2}{7}u^{\frac{7}{2}} - \frac{2\cdot 8}{5}u^{\frac{5}{2}} + \frac{2\cdot 16}{3}u^{\frac{3}{2}} + C$$

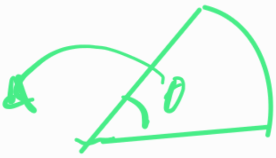
$u \to x+4$

10) $$\int_{-1}^{1}\sqrt{4-x^2}\,dx$$



$\int_{}^{1}$ $\int_{}^{1}$

$$\int_{-1}^{0} \to 2$$



$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\boxed{A = \frac{1}{2}R^2\theta}$

$$2\cdot\left(\frac{1}{2}\cdot 1\cdot\sqrt{3} + \frac{1}{2}(2^2)\cdot\frac{\pi}{6}\right)$$

$$\to \sqrt{3} + \frac{2}{3}\pi$$

61) $\int \sqrt{x^2+4x}\, dx$

Si completa il quadrato

$\to x^2+4x+4-4$

$$\int \sqrt{(x+2)^2-2^2}\, dx$$

CAMBIO VARIABILE

$$\begin{cases} x+2 = 2\sec\theta \\ dx = 2\sec\theta\tan\theta \end{cases}$$

$$\to \int \sqrt{(2\sec\theta)^2-2^2}\, dx \to 2\sec\theta\tan\theta$$

$\hookrightarrow \sqrt{4\sec^2\theta-4} \to \sqrt{4(\sec^2\theta-1)}$

$\hookrightarrow \sqrt{4\tan^2\theta} = 2\tan\theta$

$$\int 2\tan\theta \cdot 2\sec\theta\tan\theta \, d\theta$$

$$4\int \underbrace{\tan^2\theta}_{\sec^2\theta - 1}\sec\theta \, d\theta \rightarrow 4\int (\sec^2\theta - 1)\sec\theta$$

$$\rightarrow 4\int \sec^3\theta - \sec\theta \rightarrow$$

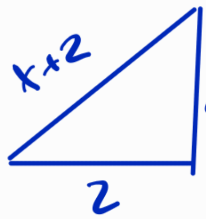
$$\left(\int \sec^3\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| \right.$$

$$-\frac{1}{2}\ln|\sec\theta + \tan\theta|$$

$$\left. \int \sec\theta = -\ln|\sec\theta + \tan\theta| \right) \cdot 4$$

$$\tan\theta = \frac{\sqrt{x^2+4x}}{2}$$

$\sec\theta = \dfrac{x+2}{2}$ → ipotenusa, 2 → base



$\sqrt{(x+2)^2 - 2^2}$

$\sqrt{x^2+4x}$

$$\rightarrow 2 \cdot \frac{x+2}{2} \cdot \frac{\sqrt{x^2+4x}}{2} - 2\ln\left|\frac{x+2}{2} + \frac{\sqrt{x^2+4x}}{2}\right| + C$$

62) $\left(x^2 \, (x^3) \right.$ → si può essere da destra

$$\to \frac{1}{3}\int 3x^2\, e^{x^3}\, dx \to \frac{1}{3} e^{x^3} + c$$

63) $\int x^3 e^{x^2}\, dx \to$

Per PARTI $\to \int f(x) g'(x) = f(x)\cdot g(x) - \int g(x)\cdot f'(x)$

D	I
x^2	$x\, e^{x^2}$
$2x$	$\frac{1}{2} e^{x^2}$

$$\to \underbrace{\frac{1}{2}}_{g(x)} e^{x^2} \overbrace{x^2}^{f(x)} - \int x\, e^{x^2}\, dx$$

$$\hookrightarrow \underbrace{\frac{1}{2} e^{x^2}}_{g(x)} \cdot \underbrace{2\cancel{x}}_{f'(x)}$$

$$\frac{1}{2}\int 2x\, e^{x^2} \to \frac{1}{2}\int 2x\, e^{x^2} \to \frac{1}{2} e^{x^2} \quad S)$$

$$\frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2} + C$$

64) $\int \tan x \underbrace{\ln(\cos x)}\, dx$

derivata → tangente

tangente

$$\begin{cases} u = \ln|\cos x| \\ du = \frac{1}{\cos x} \cdot -\sin x \to dx \end{cases}$$

$du = -\tan x$

$\rightarrow -\int u \, du \rightarrow -\frac{u^2}{2} \rightarrow -\frac{\ln^2(\cos x)}{2}$

63) $\int \frac{1}{x^3 - 4x^2} \, dx$

$\hookrightarrow x^2(x-4) \rightarrow \frac{1}{x^2(x-4)} = \frac{Ax+B}{x^2} + \frac{C}{x-4}$

$\frac{A}{x} + \frac{B}{x^2}$

$\rightarrow \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-4}$ $\quad x = 1 \rightarrow$ LASCIO PER CARICO

66) $\int \sin x \cos(2x) \, dx$

$\hookrightarrow 2\cos^2 x - 1$

cos(2x) = cos2 x − sin2 x = 1 − 2 sin2 x = 2 cos2 x − 1

$\rightarrow \int \sin x (2\cos^2 - 1) \Rightarrow u = \cos x$

$du = -\sin x$

$-\int 2u^2 - 1 \rightarrow -2 \cdot \frac{u^3}{3} + u$

67) $\int 2^{\cos x} \cdot \frac{\ln(\cos 2)}{?}$? No

$\hookrightarrow x^{\ln 2 + 1}$

$\rightarrow \left(e^{\ln 2}\right)^{\ln x}$?

08) $\int \sqrt{1+\cos(2x)}$?

$\hookrightarrow \cos 2x + \sin 2x = 1 - 2\sin 2x = 2\cos(2x) - 1$

$\rightarrow \cos^2 x = \dfrac{1+\cos(2x)}{2}$

$\sim \int \sqrt{\dfrac{1+\cos(2x)}{2} \cdot 2} \rightarrow \int \sqrt{2\cos^2 x}$

$\sqrt{2} \int \cos x \, dx \rightarrow \sqrt{2} \sin x + C$

09) $\int \dfrac{1}{1+\tan x} \, dx \rightarrow$ Rican b $\dfrac{1 - \tan x}{1+\tan x}$

$\hookrightarrow \int (1 - \tan x + \tan x$

$$\int \frac{\ldots}{1 + \tan x}$$

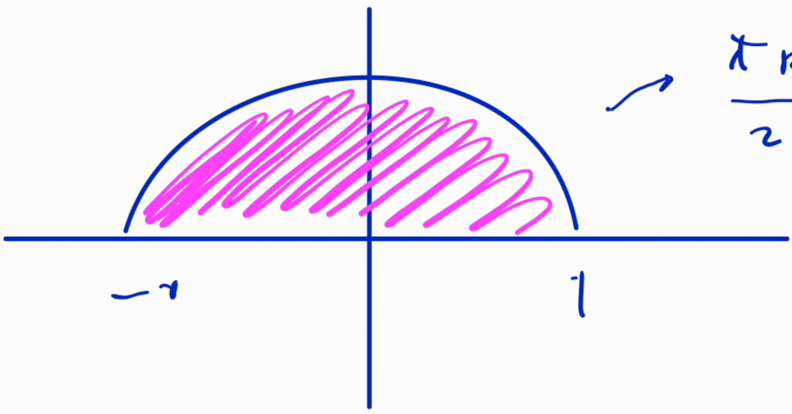
$$\to \frac{1}{2} \int \frac{1 + 1 - \tan x + \tan x}{1 + \tan x} \to \frac{1}{2}\left(\int \frac{1 - \tan x}{1 + \tan x} + \int \frac{1 \cancel{+ \tan x}}{1 \cancel{+ \tan x}} dx \right)$$

$$\frac{1}{2} \ln |\cos x + \sin x| + \frac{x}{2} + C$$

10) $$\int_{\frac{1}{e}}^{e} \frac{\sqrt{1 - \log x^2}}{x} \, dx \qquad \begin{cases} u = \log x \\ du = \frac{1}{x} \end{cases}$$

$$\frac{1}{e} \to u = \log \frac{1}{e} \to \begin{matrix} \log e = 1 \\ \log e^{-1} = -1 \end{matrix}$$

$$\to \int_{-1}^{1} \frac{\sqrt{1 - u^2}}{\cancel{x}} \cdot \cancel{x} \to \int_{-1}^{1} \sqrt{1 - u^2}$$



$$\frac{\pi r^2}{2} \to$$ metà circonferenza

$$\frac{\pi (1)^2}{2} \to \boxed{\frac{\pi}{2}}$$

7.1) $\int \frac{1}{\sqrt[3]{x}+1}$ $\begin{cases} u = \sqrt[3]{x}+1 \\ x = (u-1)^3 \end{cases}$

$\rightarrow \int \frac{1}{u} 3(u-1)^2 du \rightarrow 3\int \frac{u^2-2u+1}{u} du$

$\begin{bmatrix} u = \sqrt[3]{x}+1 \end{bmatrix}$

$3\int \frac{u^{\cancel{2}}}{\cancel{u}} - \frac{2\cancel{u}}{\cancel{u}} + \frac{1}{u}\, du \rightarrow 3\frac{u^2}{2} - 6u + 3\ln|u| + C$

7.2) $\int \frac{1}{\sqrt[3]{x+1}}$ $\begin{cases} u = x+1 \\ du = dx \end{cases}$ $\rightarrow \int \frac{1}{\sqrt[3]{u}} \rightarrow \frac{u^{-\frac{1}{3}+1}}{-\frac{1}{3}+1}$ $\frac{2}{3}$

$\frac{3}{2}\sqrt[3]{u^2} \rightarrow (x+1) \rightarrow \frac{3}{2}\sqrt[3]{(x+1)^2}\, dx + C$

7.3) $\int (\sin x + \cos x)^2 dx = \int \sin^2 x + 2\sin x \cos x + \cos^2 x\, dx$

$\rightarrow \sin^2 x + \cos^2 x = 1 \rightarrow \int 1 + 2\sin x \cos x$

$\downarrow$

x

$\int 2\sin x \cos x$ $\begin{cases} \rightarrow 2\sin x \cos x = \sin(2x) \\ \text{sostituzione} \end{cases}$

$\int \sin(2x) \to \dfrac{-\cos(2x)}{2}$ | $\begin{cases} u = \cos x \\ du = -2\sin x \end{cases}$

$\to -2\int u\,du$

$-\dfrac{2u^2}{2} \to -u^2 \to -\cos^2 x$?

(4) $\int 2x \log(x+1)$

Pick	Pencil
0	1
$\log(x+1)$	$2x$
$\frac{1}{x+1}$	x^2
	$\frac{x^3}{3}$

$\int f(x)\cdot g'(x) =$

$f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx$

$\to \log(x+1)\cdot x^2 - \int x^2 \cdot \frac{1}{x+1}\,dx$

$\log(x+1)\,x^2 - \int \frac{x^2}{x+1}$

$\int \frac{x^2}{}$

$\int \frac{}{1+x}$ { LO SUDDIVIDO $\sim$ $x+1 / x^2$

$x+1 \;/\; x^2$ con $x \to x(x+1) - x^2+x$

$-(x^2+x)$

$-x$

$-(-x-1)$ $-1 \to -(x+1) \to -x-1$

1

$\to \frac{x^2}{1+1} : \left(x-1+\frac{1}{x+1}\right) = \frac{1}{2}\left(\frac{x^2}{2} - x + \log(x+1)\right)$

$\Downarrow$

$\frac{(x-1)(x+1)+(x+1)}{x+1}$ $x^2 - \cancel{x} + \cancel{x} - \cancel{x} + x + \cancel{1}$

ym $= \frac{x(x+1) - \cancel{(x+1)} + \cancel{x+1}}{(x+1)}$ $\to x^2+x$

B+10

NON CO VOGLIAMO

75) $\int \frac{}{x(1+\sin^2(\ln x))} dx$ $\begin{cases} u = \ln x \\ du = \frac{1}{x} \end{cases}$

$$\int \frac{1}{1+\sin^2(x)} du \rightarrow$$ DIVIDO TUTTO PER $\cos^2 x$

$$\rightarrow \int \frac{\frac{1}{\cos^2 x}}{\frac{1}{\cos^2 x} + \frac{\sin^2(x)}{\cos^2 x}} \rightarrow \int \frac{\sec^2 x}{\sec^2 x + \tan^2 x} dx$$

$\sec^2 x = \tan^2 x + 1$

$$\int \frac{\sec^2 x}{2\tan^2 x + 1} du$$ $\begin{cases} v = \tan u \\ dv = \sec^2 x \, du \end{cases}$

$$\rightarrow \int \frac{1}{2v^2+1} dv = \int \frac{1}{(\sqrt{2}v)^2+1} \quad \tan^{-1}$$

$$\frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2}\,v) \rightarrow (\sqrt{2} \cdot \tan(\ln x)) + C$$

$\rightarrow \int$ VOGLIAMO TOGLIERE LA

76) $\int \sqrt{\frac{1-x}{1+x}}$ Rbalc

$\hookrightarrow \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} \to \frac{1-x}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{1-x^2}} - \int \frac{x}{\sqrt{1-x^2}}$

$\frac{1}{\sqrt{1-x^2}} \Rightarrow \sin^{-1}$

f'/f primz

$\sqrt{1-x^2}$

$\longrightarrow \sin^{-1} x + \sqrt{1-x^2}$

77) $\int x^{\frac{x}{\ln x}} dx = x^{\frac{x}{\ln x}} + c$

Yorktin su Eu $(e^{\ln x})^{\frac{x}{\ln x}} \to \cancel{\ln x} \frac{x}{\cancel{\ln x}}$

$\to \int e^x \longrightarrow e^x + c$

78) $\int \sin^{-1}(\sqrt{x})\, dy$ $\quad \int u = \sqrt{x}$

$$\begin{cases} x = u^2 \\ dx = 2u\,du \end{cases}$$

$$\int 2u \sin^{-1}(u)$$

D	I
$\sin^{-1} u$	$2u$
$-\frac{1}{\sqrt{1-u^2}}$	$\frac{\cancel{2}u^2}{\cancel{2}}$

$$\int f(x)g'(x) = f(x)g(x) - \int f'(x)\cdot g(x)$$

$$u^2 \sin^{-1}(u) + \int \frac{1}{\sqrt{1-u^2}} \cdot u^2 \, du$$

$g(x)$ $f(x)$

→ per prima

$$\begin{cases} u = \sin\theta \\ du = \cos\theta \, d\theta \end{cases}$$

$$\to \int \frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}} \to \frac{\sin^2\theta}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}\, d\theta$$

$\hookrightarrow \sqrt{\cos^2\theta}$

$$\int \sin^2\theta \, d\theta \qquad \sin^2\theta = \frac{1 - \cos(2x)}{2}$$

$$\hookrightarrow \frac{1}{2}\int 1 - \cos(2\theta)\, d\theta$$

$$\to -\frac{1}{2}\left(\theta - \frac{1}{\cancel{2}} \sin(2\theta)\right) \qquad \sin(2\theta) =$$

$2\sin x\cos x$

$$-\frac{1}{2}\theta+\frac{1}{2}\sin\theta\cos\theta$$

masino

79) $\int \tan^{-1} x \, dx$ integrazione per parti

D	I
	1
$\tan^{-1} x$	$+$
$\frac{1}{1+x^2}$	x

$$\int f(x)\,g'(x) = f(x)g(x) - \int f'(x)g(x)$$

$$x\tan^{-1}x - \int \frac{x}{1+x^2}$$

$$-\int \frac{x}{1+x^2}\cdot\frac{2}{2} \to -\frac{1}{2}\int\frac{2x}{1+x^2} \to -\frac{1}{2}\log|1+x^2|$$

80) $\int_0^5 f(x)\,dx$, $f(x)\begin{cases}10 & x\le 2\\ 3x^2-2 & x>2\end{cases}$ a tratti

$$\to \int^{2} 10\,dx + \int^{5} 3x^2-2\,dx$$

$$\int_0 \qquad \int_2$$

$$\frac{1}{0}$$

$$10x\Big|_0^2 \to 20-0=20$$

$$b \quad \frac{\cancel{3}x^3}{\cancel{3}}-2x\Big|_2^5 \Rightarrow (125-10)-(8-4)$$

$$115-4=$$

$$111$$

$$20+111= 131$$

81) $\int \frac{\sin\left(\frac{1}{x}\right)}{x^3}\,dy$

$$\sin\left(\frac{1}{x}\right)$$

D	I
$\frac{1}{x}$	$\frac{\sin\left(\frac{1}{x}\right)}{x^2}$
$-\frac{1}{x^2}$	$\cos\left(\frac{1}{x}\right)$

$$\frac{\cos\left(\frac{1}{x}\right)}{x}-\int -\frac{\cos\left(\frac{1}{x}\right)}{x^2}$$

$$\int f(x)g'(x)=f(x)g(x)-\int f'(x)g(x)$$

$$\Rightarrow \frac{\cos\left(\frac{1}{x}\right)}{x}-\sin\left(\frac{1}{x}\right)+C$$

82) $\int \frac{x-1}{x^4-1}\,dy \Rightarrow \frac{\cancel{x-1}}{(x^2-1)(x^2+1)}$

$$\left\{ x^4-\cancel{x^2+x^2}-1 \right. \quad \text{GL}$$

$$\swarrow \; \downarrow$$

$$(\cancel{x-1})(x+1)$$

$$\rightarrow \frac{1}{(t+1)(t^2+1)} \rightarrow \frac{A}{t+1} + \frac{Bx+C}{x^2+1} \quad \text{USE } x=0$$

83) $\int \sqrt{1+\left(x-\frac{1}{4x}\right)^2}\, dx$

→ svolge il quadrato

$$\rightarrow \int \sqrt{\underbrace{1}_{} + x^2 - \underbrace{\frac{1}{2}}_{} + \frac{1}{4x^2}}\, dx$$

$$\rightarrow \int \sqrt{\underbrace{x^2 + \frac{1}{2} + \frac{1}{16x^2}}}\, dx$$

→ è un quadrato → $\left(x+\frac{1}{4x}\right)^2$

$$\rightarrow \int \cancel{\sqrt{\left(x+\frac{1}{4x}\right)^2}} \rightarrow \int \left(x+\frac{1}{4}\right)\frac{1}{x}$$

$$\frac{x^2}{2} + \frac{1}{2}\ln|x|$$

84) $\int \frac{e^{\tan x}}{1-\sin^2 x}$ → segue la forma → $f'(x)\, e^{f(x)} \rightarrow e^{f(x)}$

$$\hookrightarrow \int \frac{e^{\tan x}}{\cos^2 x} \quad \frac{1}{\cos^2 x} = \sec^2 x \rightarrow \overset{f'(x)}{\sec^2 x}\, e^{\overset{f(x)}{\tan x}} = e^{\tan x}$$

85) $\int \frac{\tan^{-1} x}{x^2}\,dx$

PER PARTI $\to \int f(x)\,g'(x) = f(x)\,g(x) - \int f'(x)\,g(x)$

D I

$\tan^{-1} x$ $\quad \frac{1}{x^2}$

$\frac{1}{1+x^2}$ $\quad -\frac{1}{x}$

$f'(x)$ $\quad g(x)$

$- \int \frac{1}{1+x^2} \cdot \int \frac{1}{x(1+x^2)}$

$\to \int \frac{1}{\underbrace{x(1+x^2)}_{x^2(x^{-2}+1)}}\,dx = \int \frac{1}{x \cdot x^2 (x^{-2}+1)} = \int \frac{x^{-3}}{x^{-2}+1}$

$= \int \frac{x^{-3}}{x^{-2}+1}$ segue $\int \frac{f'(x)}{f(x)} = \log|f(x)|$

$-\frac{1}{2}\int \frac{-2x^{-3}}{x^{-2}+1} = -\frac{1}{2}\log(x^{-2}+1)$

$\to -\frac{\tan^{-1} x}{x} - \frac{1}{2}\log(x^{-2}+1) + c$

86) $\int \frac{\tan^{-1} x}{1+x^2}\,dx$

$\tan^{-1} x \to$ h derivata é

1

$$\frac{}{1+x^2}$$

$\Rightarrow$ Sashniwrn

$u = \tan^{-1}x$

$du = \frac{1}{1+x^2}$ $\rightarrow$ $\displaystyle\int \frac{u}{\cancel{1+x^2}} \cdot \cancel{1+x^2}\, du$

$$\frac{1}{2}\left(\tan^{-1}x\right)^2$$

8-f) $\displaystyle\int (\ln x)^2\, dx$

D	I
$(\ln x)^2$	1
$-\frac{2\ln x}{x}$	x

$\rightarrow (\ln x)^2 \cdot x - \displaystyle\int \frac{2\ln x}{\cancel{x}} \cdot \cancel{x}$

$(\ln x)^2 \cdot x - 2\displaystyle\int \ln x \rightarrow$

D	I
$\ln x$	1
$\frac{1}{x}$	x

$x\ln x - \displaystyle\int \frac{1}{\cancel{x}} \cdot \cancel{x} \rightarrow 2(x\ln x - x)$

$\Rightarrow x(\ln x)^2 - 2x\ln x + 2x + C$

88) $\displaystyle\int \frac{\sqrt{x^2+4}}{x^2}\,dx$

NO

$\Rightarrow \sqrt{x^2+4+2x-2x} \rightarrow \dfrac{\sqrt{(x+2)^2-2x}}{x^2}$

Cambio var

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$ $\rightarrow \displaystyle\int \frac{\sqrt{4\tan^2\theta+4}}{4\tan^2\theta}\, 2\sec^2\theta\, d\theta$

$\sqrt{4\tan^2\theta+4} = \sqrt{4(\tan^2\theta+1)} \rightarrow \cancel{\sqrt{4}}\sqrt{\boxed{\dfrac{\tan^2\theta+1}{\sec^2\theta}}}$

$\displaystyle\int \frac{\cancel{2}\sec\theta}{\cancel{4}\tan^2\theta}\cancel{2}\sec^2\theta\, d\theta = \int \frac{\sec^3\theta}{\tan^2\theta}\, d\theta$

$\Rightarrow \displaystyle\int \frac{1}{\cos^{\cancel{3}}\theta}\cdot\frac{\cancel{\cos^2\theta}}{\sin^2\theta}\, d\theta \rightarrow \int \frac{1}{\cos\theta\cdot\sin^2\theta}\, d\theta$

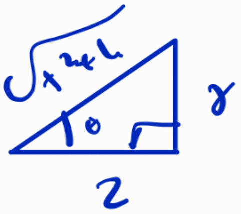
$1 \quad \cos^2\theta + \sin^2\theta \quad \dfrac{\cos^2\theta}{} \quad \cancel{\sin^2\theta}$

$$\rightarrow \frac{\quad}{\cos\theta \cdot \sin^2\theta} \rightarrow \frac{\quad}{\cos\theta \cdot \sin^2\theta} \quad \frac{\quad}{\cancel{\cos\theta \cdot \sin^2\theta}} \quad \frac{\quad}{\cancel{\cos\theta \cdot \sin^2\theta}}$$

$$\rightarrow \frac{\cos\theta}{\sin\theta} + \frac{1}{\cos\theta}$$

$$\rightarrow \ln|\sec\theta + \tan\theta| - \csc\theta$$

$\tan\theta = \frac{x}{2}$ →



$$\ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| - \frac{\sqrt{x^2+4}}{x}$$

8g) $\int \frac{\sqrt{x+4}}{x}\,dx$

$u = \sqrt{x+4}$

$x = u^2 - 4$

$dx = 2u$

$$\rightarrow \int \frac{u}{u^2-4} \cdot 2u \rightarrow \int \frac{2u^2}{u^2-4}$$

division → $u^2-4 \,/\, 2u^2$, quotient 2

$-(2u^2-8)$

8

$$\rightarrow \int 2 + \frac{8}{(u-2)(u+2)}$$

$$\rightarrow \int 2 + \frac{2}{u-2} + \frac{-2}{u+2}$$

$u = \sqrt{x+4}$

$2u \qquad \frac{1}{x}\cdot 2 \qquad \frac{x}{u-2} \Rightarrow +2\ln|u-2| - 2\ln|u+2|$

91) $\displaystyle\int \frac{x}{1+x^4}\,dx$ $\quad (x^2)^2 \quad \to \frac{1}{2}\int \frac{\overset{f'(x)}{2x}}{1+\underbrace{(x^2)^2}_{f(x)}} \to \tan^{-1} x \to \frac{1}{2}\tan^{-1}(x^2)$

92) $\displaystyle\int e^{\sqrt{x}}\,dx \Rightarrow$

$u = \sqrt{x}$

$x = u^2$

$dx = 2u$

$\to \displaystyle\int e^{u}\, 2u \to 2\int u\, e^{u}\, du$ ← parts prm

D	I
u	e^u
1	e^u

$2(u \cdot e^u - e^u) =$

$2u\,e^u - 2e^u \qquad \{ u = \sqrt{x} \Rightarrow 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}}$

93) $\displaystyle\int \frac{1}{\csc^3 x}\,dx$

$\quad \to \quad \sin^3 x$

$\displaystyle\int \sin^3 x\,dx \Rightarrow \int \sin^2 x \cdot \sin x\,dx \Rightarrow \int (1-\cos^2 x)\sin x$

$\int \sin^3 x\, dx \to \int (\sin^2 x) \sin x\, dx \to \int (1-\cos^2 x) \sin x\, dx$

$u = \cos x$

$du = -\sin x$

$\to \int 1-u^2\, du \to u - \frac{u^3}{3}$ ~ $\cos x$

94) $\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\, dx = \frac{1}{2}(\sin^{-1} x)^2 + C$

↳ $\frac{1}{\sqrt{1-x^2}}$ is the derivative of $\sin^{-1} x$

$u = \sin^{-1} x$

$du = \frac{1}{\sqrt{1-x^2}}$

$\to \frac{u}{\cancel{\sqrt{1-x^2}}} \cdot \cancel{\sqrt{1-x^2}} \to \int u \to \frac{u^2}{2}$

95) $\int \sqrt{1+\sin(2x)}\, dx \to \sqrt{(\sin x + \cos x)^2}$

↳ $\sin^2 x + \cos^2 x$ ↳ $2 \sin x \cos x$

$\int \sin x + \int \cos x\, dx \to -\cos x + \sin x + C$

96) $\int \sqrt[4]{x} \to \frac{x^{\frac{1}{4}+1}}{\frac{1}{4}+1}$ $\frac{1}{4}+1 = \frac{1+4}{4} = \frac{5}{4}$ $\frac{x^{\frac{5}{4}}}{\frac{5}{4}} = \frac{4}{5} x^{\frac{5}{4}}$

$4\sqrt[4]{x^5} \to x\sqrt[4]{x}$

5

97) $\int \frac{1}{1+e^x}dx \quad \frac{1+e^x-e^x}{1+e^x}dx = \int \frac{\cancel{1+e^x}}{\cancel{1+e^x}}dx \;\; x$

$-\int \frac{e^x}{1+e^x} \quad \frac{f'(x)}{f(x)} \to \ln|1+e^x|$

98) $\int \sqrt{1+e^x}\,dx$

sostituzione

$u = \sqrt{1+e^x}$

$du = \frac{e^x}{2\sqrt{1+e^x}}dx$

$\to 2\int \frac{u^2}{e^x \to u^2-1} \quad \int \frac{2u^2}{u^2-1} \to u^2-1 \,/\, 2u^2 \quad \frac{2}{}$

$\frac{-(2u^2-2)}{+2}$

$\int 2 + \frac{2}{(u-1)(u+1)} \to \frac{1}{u-1} + \frac{-1}{u+1}$

$2u \qquad \ln|u-1| - \ln|u+1|$

$\hookrightarrow \sqrt{1+}$

99) $\int \frac{\sqrt{\tan x}}{\sin(2x)} \to \frac{\sqrt{\frac{\sin x}{\cos x}}}{2\sin\cos}$ WO

$u = \sqrt{\tan u} \to \tan u = u^2$

$\int \sqrt{\tan}$

$$\to \int \frac{\sqrt{\tan x}}{2\sin x \cos x}$$

$du = \frac{\sec^2 x}{2\sqrt{\tan x}}\,dx$

$$\to \int \frac{u}{\cancel{2}\sin x\cos x}\cdot\frac{\cancel{2}u}{\sec^2 x} \Rightarrow \frac{u^2 \cos^{\cancel{2}} x}{\sin x\,\cancel{\cos x}}$$

$$\to \int \frac{u^2 \cos x}{\sin x}\,du \to \int u^2 \cot x\,du$$

$\cot x \to \frac{1}{\tan x}$

$$\int u^2\cdot\left(\frac{1}{u^2}\right) \qquad \frac{1}{\tan x} \to u^2 \to \int 1 \to u \to \sqrt{\tan x}$$

100) $$\int \frac{1}{1+\sin x}\,dx \to \frac{1}{1+\sin x}\frac{(1-\sin x)}{(1-\sin x)} \to \frac{1-\sin x}{1-\cancel{\sin x}+\cancel{\sin x}-\sin^2 x}$$

$\to \cos^2 x$

$$\to \int \frac{1-\sin x}{\cos^2 x}\,dx \to \frac{1}{\cos^2 x} = \sec^2 x$$

$$\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \to \left(\frac{\sin x}{\cos x}\cdot\frac{1}{\cos x}\right)$$

$$\to \sec^2 x - \tan x \sec x$$

$$\int \sec^2 y \, dy - \int \tan y \sec y \, dy$$

$\tan y$

$\sec y$