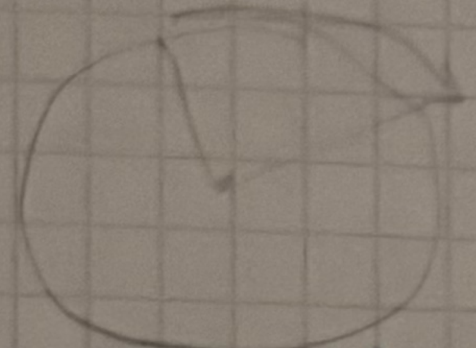
HSV : hue , saturation, value

Hue : color type measured in degrees around the colour wheel



Saturation : color intensity or purity

0 – 1 0 – 100 %

Value : brightness of the colour

0 – 1 0 – 100 %

$\boxed{\text{Aufgabe 3}}$

- to generate an image with raytracing , vector rays are needed

$\vec{r}(t) = \vec{p} + t\vec{d}$

- starting point $p = (-1, -1, 1)^T$ → you choose one particular ray
- direction vector $d = (1, 1, 0)^T$ → tells the ray in which direction to travel from p

1.1 Calculate the points of intersection of the view ray with a sphere with radius $r = 3$ and the center $M = (5, 5, 1)$

$\vec{R}(t) = \vec{p} + t\vec{d}$

Which of the two points of intersection lies in the direction of the view ray and is therefore visible to the viewer?

In ray tracing :

→ a view ray leaves the camera

→ travels in one direction

→ it does not bend unless it hits a surface and reflects / refracts direction vector

$\vec{r}(t) = -\vec{i} - \vec{j} + \vec{k} + t(\vec{i} + \vec{j})$

$\vec{r}(t) = (t-1)\vec{i} + (t-1)\vec{j} + \vec{k}$

- the ray moves only on Ox, Oy axes

- a point lies on the sphere if

$$\|r(t) - M\|^2 = R^2$$

the length of a vector

$M = (5, 5, 2)$

$\|x\| = \sqrt{1^2 + (-2)^2 + 3^2}$ $\quad x = (1, -2, 3)$

$\|(t-1)\vec{i} + (t-1)\vec{j} + \vec{k} - 5\vec{i} - 5\vec{j} - 2\vec{k}\|$

$= \|(t-6)\vec{i} + (t-6)\vec{j} - \vec{k}\|$

$= \sqrt{(t-6)^2 + (t-6)^2 + (-1)^2}$

$t^2 - 12t + 36 + t^2 - 12t + 36 + 1 = 9$

$2t^2 - 24t + 73 = 9$

$2t^2 - 24t + 64 = 0$

$t^2 - 12t + 32 = 0$ $\to t_1 = 4$, $\to t_2 = 8$

$(t-6)^2 - 4 = 0$

$(t-6)^2 = 4$

$t - 6 = -2$

$t - 6 = 2$

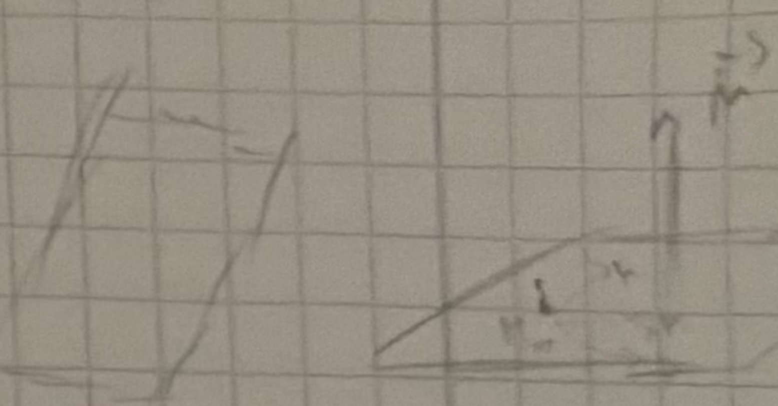
- both t values are positive $\to$ both intersections lie along the forward direction of the ray

$t = 4 \to r(4) = \boxed{(3, 3, 1)}$ - entry point

$t = 8 \to r(8) = (7, 7, 1)$ - exit point behind the sphere surface from the viewer's perspective

(1.2) plane $d=3$ from the origin
n normal with direction $(1,1,0)^T$ plane located in front of the camera

point of intersection with the view ray



$P: nx = d$ plane equation

$nx = (1,1,0)^T (x,y,z)^T = x+y$

$x+y=3$

$n(x-P_0)=0$

$nx = nP_0 \to d$

$\overrightarrow{M_0M}\,\overrightarrow{N}=0$

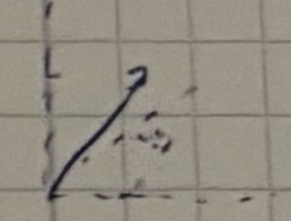
view ray $r(t) = (t-1)\vec{i} + (t-1)\vec{j} + \vec{k}$

$= (-1+t, -1+t, 1)^T$

$\vec{v}=(1,1,3)$

$-1+t+(-1)+t=3$

$-2+2t=3$ $t=\frac{5}{2}=2,5$



the point of intersection is $\boxed{(1,5, 1,5, 1)^T}$

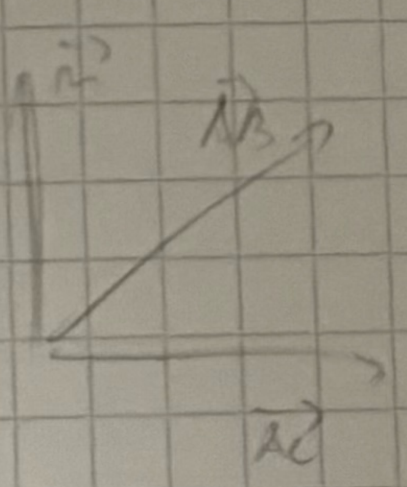
(1.3.) $r(t) = (-1+t, -1+t, 1)^T$

$A=(6,0,0)^T$ $B=(0,6,0)^T$ $C=(0,0,6)^T$

$n = (B-A)\times(C-A)$ - plane of the triangle

$B-A = (-6,6,0)$

$C-A = (-6,0,6)$



$\overrightarrow{AB} = (-6,6,0)^T$

$\overrightarrow{AC} = (-6,0,6)^T$

. the normal vector to a plane is perpendicular to every vector in a plane

$$\vec{n} = \overrightarrow{AB}\times\overrightarrow{AC} = \begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} = \begin{pmatrix} 36 \\ 36 \\ 36 \end{pmatrix} \to \vec{n} = (1,1,1)^T$$

$P_2: n\cdot(x-A) = 0$

$\Rightarrow P: x+y+z=6$

$r(t) = (-1+t, -1+t, 1)^T$

$-1+t-1+t+1=6$ $t>0$

$-1+2t=6$ $t=\frac{7}{2}$

. the point is inside the plane

$\boxed{x(2,5, 2,5, 1)}$

$2,5+2,5+1=6$

BRUNNEN