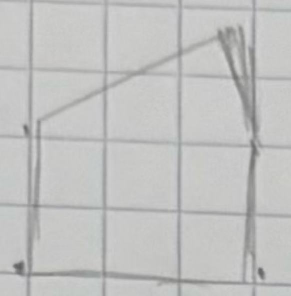
$\boxed{\text{Blatt 5}}$

Aufgabe 1

• a house in the form of lines between 5 points 2D

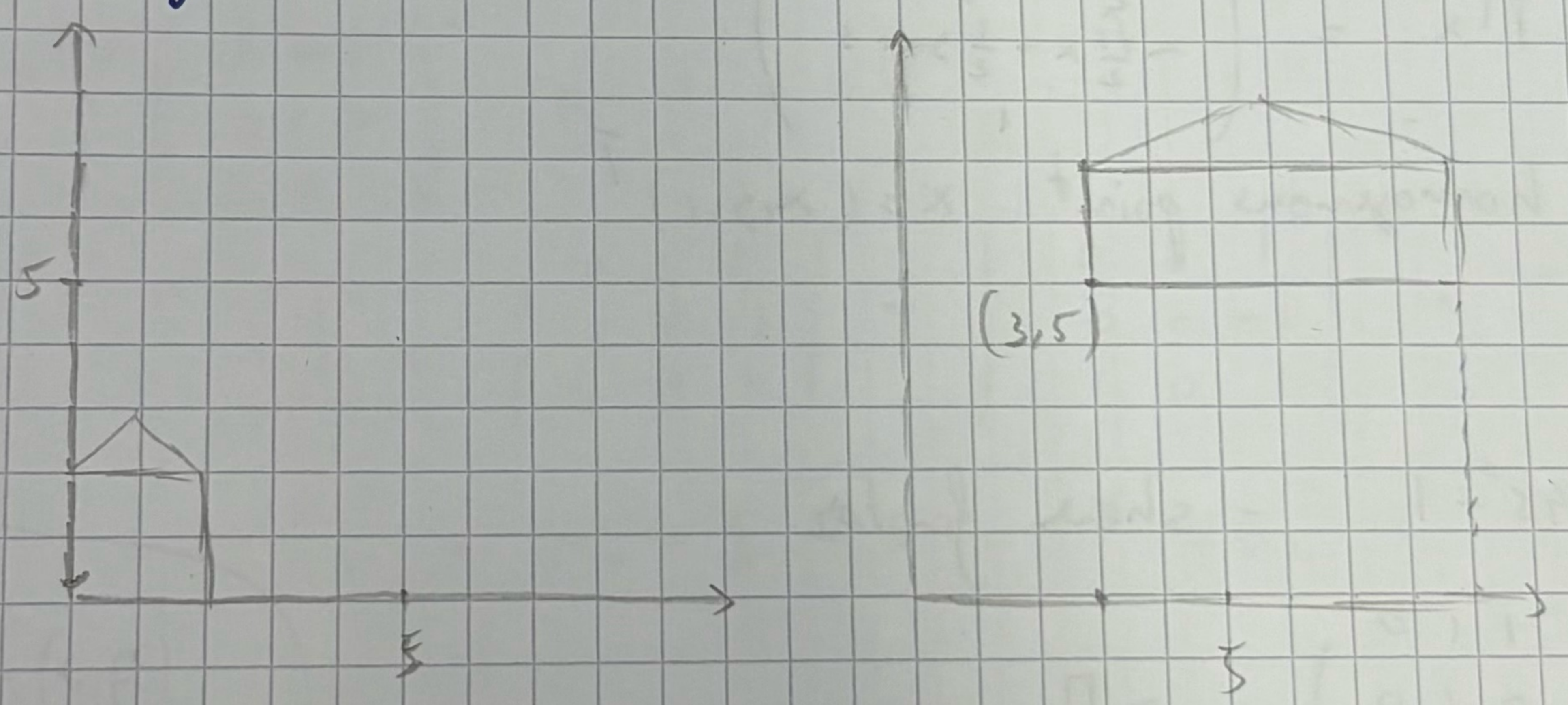


(0,0) lower left corner

• create matrices M that transform the points of the house x (in homogenous coordinates, i.e., $x \in \mathbb{R}^3$) by multiplication

$$\hat{x} = Mx$$

1. Scale and translate the house so that it is streched by a factor of 3 in horizontal direction and the lower left corner is moved to position (3,5)



$S = \mathrm{diag}(3,1,1)$ $\quad T = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad S = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

M

2D point – 3-vector $\begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$

$$M = TS = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix}$$

first scale then translate – matrix multiplication right to left

$$\hat{x} = Mx = \begin{pmatrix} 3 & 0 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} 3x+3 \\ y+5 \\ 1 \end{pmatrix}$$

2. Rotated $60°$ clockwise and then the lower left corner is located at $(3,3)$

$$R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$\theta = -60°$

$$R = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \qquad R = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M = TR = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 3 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\hat{x} = Mx = \begin{pmatrix} \frac{1}{2}x + \frac{\sqrt{3}}{2}y + 3 \\ -\frac{\sqrt{3}}{2}x + \frac{1}{2}y + 3 \\ 1 \end{pmatrix}$$

for any homogenous point $x = (x, y, 1)^T$

3. $\tan 45° = 1$ - shear factor

$$S = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = M$$

$$\hat{x} = Mx = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = \begin{pmatrix} x+y \\ y \\ 1 \end{pmatrix}$$

$\underline{\underline{(7,7) \to (0,0)}}$

$(7,7)$

to match around

4. $\theta = -120°$ - negative for clockwise

$\cos(-120°) = -\frac{1}{2}$ $\quad \sin(-120) = -\frac{\sqrt{3}}{2}$

$$R = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad T(7,7) = \begin{pmatrix} 1 & 0 & 7 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{pmatrix} \qquad T(-7,7) = \begin{pmatrix} 1 & 0 & -7 \\ 0 & 1 & -7 \\ 0 & 0 & 1 \end{pmatrix}$$

$$M = T(7,7)\, R\, T(-7,7) = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & \frac{21 - 7\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & \frac{21 + 7\sqrt{3}}{2} \\ 0 & 0 & 1 \end{pmatrix}$$

rotating about a point

Aufgabe 2

$$A = \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$$

$$B = \left\{ \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \right\}$$

$$T_{BA} = T_{BB_c} T_{B_cA} = -\frac{1}{11} B I_3 = -\frac{1}{11} B$$

$$T_{B_cA} = A = I_3$$

$$\frac{T_{BB_c}}{\det T_{BB_c}} = T_{B_cB}^{-1} \Rightarrow T_{BB_c} = \det T_{BB_c} \cdot T_{B_cB}^{-1}$$

$$\det(T_{B_cB}^{-1}) = \begin{vmatrix} 2 & 3 & 1 \\ 5 & 2 & -3 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 2 & 1 \\ 5 & 5 & -3 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 2 & 1 \\ 0 & 5 & -3 \\ 1 & 0 & 1 \end{vmatrix}$$

$$= -2 \begin{vmatrix} 0 & -3 \\ 0 & 1 \end{vmatrix} + \begin{vmatrix} 0 & 5 \\ 1 & 0 \end{vmatrix} = -2(+3) - 5 = -11$$

$$\det(T_{BB_c}) = \frac{1}{\det T_{B_cB}^{-1}} = -\frac{1}{11}$$

$$T_{BB_c} = -\frac{1}{11} T_{B_cB}^{-1} = -\frac{1}{11} B$$

$$P_A = O_B^A + T_{BA} P_B \qquad x_A = O + B x_B$$

$$n_{B \to A} = \begin{pmatrix} 2 & 3 & 1 & 1 \\ 5 & 2 & -3 & 2 \\ 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x_A = O + B \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 1 \\ 5 & 2 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ 6 \end{pmatrix}$$

BRUNNEN

(Aufgabe 3)

1. $(2,1)$ by $90°$

$$R_{90°} = \begin{pmatrix} \cos 90° & -\sin 90° \\ \sin 90° & \cos 90° \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\hat{x} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

2. $R\begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} \to \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}$ rotation of $90°$ around the z axis

$R(0,0,\mathcal{K})$
$\mathcal{K} = 90°$

$$R = \begin{pmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\hat{x} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}$$

3. skalierung $S(x) = \begin{pmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$(x,y,z) \longmapsto (kx, y, z)$$

mirroring mirror on yz-Axis → x-negat

$$M(x) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(x,y,z) \to (-x, y, z)$$

4. $R_a R_b \neq R_b R_a$ immediately matricies are commutative

$$R_x(90°)\, R_y(90°)\, R_z(90°)\, \vec{x} = (0,0,1)^T$$

rotate along the x axis $R_z(90°)\, R_y(90°)\, R_x(90°)\, \vec{x} = (0,0,-1)$