1 Introduction

The aim of this lab is to measure the spring constant of a helical spring and to compare it to the theoretical value. Afterwards, the period of vibration will also be measured, while also measuring the effect of spring mass on this value.

2 Theory

Hooke's law, as defined by the 17th century physicist Robert Hooke, states that the force exerted by a spring is directly proportional to the elongation or compression of the spring.

A helical spring is a part of wire coiled or wound in the shape of a helix. When a spring with the spring constant k and the unstretched length l_0 is stretched or compressed to the length l_e, the force exerted by the spring will be a relation between k and Δl. For small deflections Hooke's law is formulated in Equation 1.

$$F_e = -kx = -k(l_e - l_0) \tag{1}$$

Where:

- F_e is the force exerted by the spring
- k is the spring constant
- x is the displacement
- l_e is the stretched length of the spring
- l_0 is the unstretched length of the spring

Considering an ideal spring, we mount a system made of a helical spring and a mass at the end vertically, where it gets to an equilibrium position. The force of the spring in the equilibrium position is equal to the weight force of the object.

$$kx_e = mg \tag{2}$$

When displacing the mass from the equilibrium position, the system will start oscillating. Using Newton's second law, Equation 3 is determined.

$$m\frac{d^2x}{dt^2} = mg - kx = mg - k(x_e + x') \tag{3}$$

Since:

$$kx_e = Cst$$
$$\frac{d^2x}{dt^2} = \frac{d^2x'}{dt^2}(x_e \text{ is constant, } x = x_e + x')$$

The equation becomes:

$$\frac{d^2x'}{dt^2} = -\frac{k}{m}x' \tag{4}$$

Where x' is the position relative to the equilibrium position.

Solving the differential equation, Equation 4 an expression for the relative position is found:

$$x'(t) = Acos(\sqrt{\frac{k}{m}}t + \Phi) \tag{5}$$

Where:

- $x'(t)$ is the position relative to the equilibrium position
- A is the amplitude
- Φ is the phase angle (constant value that depends on initial conditions)

The time period is given by Equation 6:

$$T_{id} = 2\pi\sqrt{\frac{m}{k}} \tag{6}$$

However, since the impact of the spring's weight cannot be ignored, the formulas have to be modified to include the moment of inertia, which affects k. To account for this, the corrected formula is used:

$$T_{cor} = 2\pi\sqrt{\frac{m + \frac{m_S}{3}}{k}} \tag{7}$$

These formulas are used to validate Hooke's law and determine the spring constant. The corrected formula for period can also be used to compare measurements to theoretical values.

3 Method and Materials

3.1 Experimental setup

The setup for the experiment consists of a spring mounted to a vertical stand and a ruler next to the spring. Weight is added progressively, changing displacements to accurately calculate the spring constant. The mass is measured using a digital scale, displacement is measured with a ruler and the period is measured using a chronometer. Figure 1 shows the experimental setup.

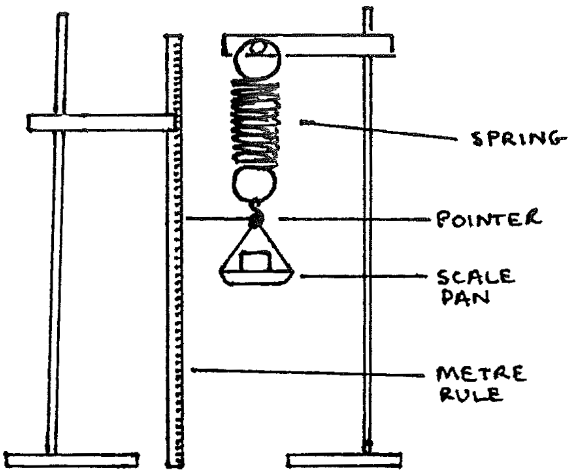

Figure 1: Drawing of the experiment setup(reft)

3.2 Measuring instruments

- Vertical meter ruler with 1mm increment (±0.5 mm uncertainty)
- Digital scale (±0.01 g uncertainty)
- Helical spring
- Smartphone used as chronometer (±0.01 s)
- Spring stand

3.3 Method

For the first experiment, the spring is attached to the vertical stand with a ruler by its side. Weights with known mass are added to the spring, measuring the elongation which are used to calculate the spring constant k.

For the second experiment, weights are attached to the spring, the weights are displaced and the time for 30 oscillations to happen is measured three times. By repeating the experiment with multiple masses, the theoretical values can be more accurately compared to the experimental results.

4 Results spring constant

4.1 Measurements

The measured values for the mass of the holder plus discs m, the unstretched length l_0, the stretched length l_1, the elongation x_e and the spring constant k for both the short and long spring are described in tables 1 and 2.

Table 1: Measurements for the short spring

mass of discs [g]	l_0 [cm]	l_1 [cm]	x_e [m]	mg [N]	k [Nm^{-1}]	Δk [Nm^{-1}]
31.1	4.7	13.1	0.08	0.31	3.63	0.16
52.1	4.7	18.5	0.14	0.51	3.70	0.10
70.8	4.7	24.4	0.20	0.69	3.53	0.07
91.9	4.7	29.95	0.25	0.90	3.57	0.05
110.46	4.7	35.4	0.31	1.08	3.53	0.05

Table 2: Measurements for the long spring

mass of the discs [g]	l_0 [cm]	l_1 [cm]	x_e [m]	mg [N]	k [Nm^{-1}]	Δk [Nm^{-1}]
31.14	17.35	18.7	0.014	0.31	22.63	5.93
52.14	17.35	19.7	0.024	0.51	21.77	3.27
70.8	17.35	20.65	0.033	0.69	21.05	2.26
91.9	17.35	21.6	0.043	0.90	21.21	1.77
110.46	17.35	22.7	0.054	1.08	20.25	1.34

4.2 Graphs

By plotting the values in Tables 1 and 2, the spring constant of each spring can be determined. A graph for each spring, with the weight as a function of elongation is plotted, is shown in Figures 2 and 3.

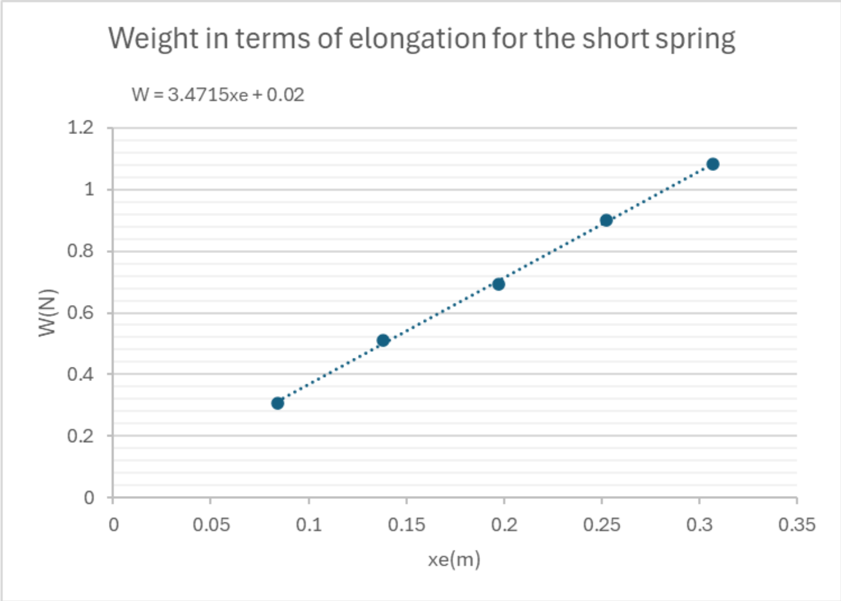

Figure 2: Weight as a function of elongation for the short spring

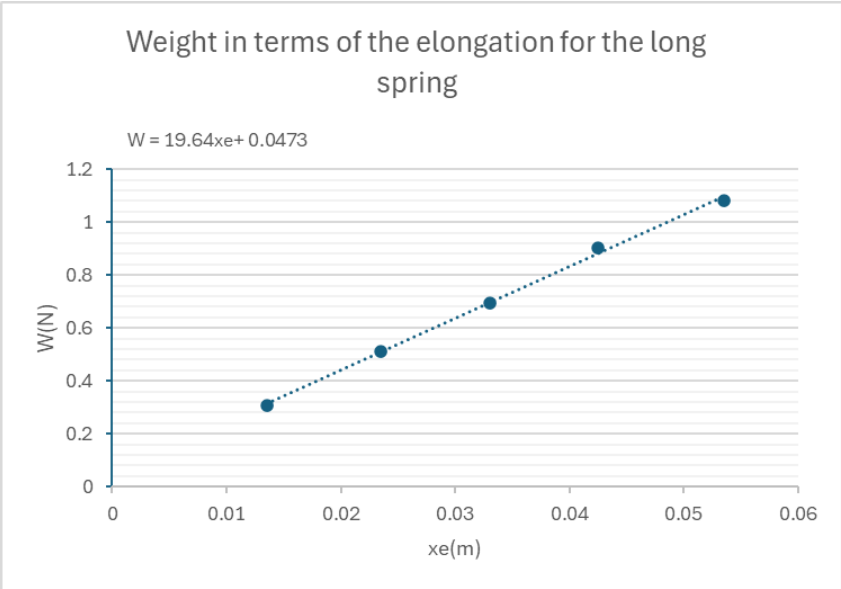

Figure 3: Weight as a function of elongation for the long spring

4.3 Calculations

Using the measured values, the spring constants can be determined. To determine x_e the following equation is used:

$$x_e = l_1 - l_0 \tag{8}$$

To calculate the force that is applied to the spring, the weight equation is used:

$$W = mg \tag{9}$$

By using equation 2 the spring constant can be therefore calculated.

4.3.1 Example calucaltion

By using the first data point of table 1, the following calculations are made to determine the spring constant.

Firstly, the weight of the discs and hook are calculated:

$$W = m \cdot g = 31.1 \text{ g} \cdot 0.001 \frac{\text{g}}{\text{kg}} \cdot 9.81 \frac{\text{kg}}{\text{N}} = 0.305091 \text{ N} \approx 0.31 \text{ N}$$

Afterwards, the elongation is calculated:

$$x_e = l_1 - l_0 = 13.1 \text{ cm} - 4.7 \text{ cm} = 8.4 \text{ cm} \approx 0.08 \text{ m}$$

Finally, the spring constant can be calculated using equation 2:

$$kx_e = W \Rightarrow k = \frac{W}{x_e} = \frac{0.305 \text{ N}}{0.084 \text{ m}} = 3.6309 \text{ Nm}^{-1} \approx 3.63 \text{ Nm}^{-1}$$

4.3.2 Error calculations

As the mass was calculated using a digital scale, the uncertainty is the sum of the smallest division(unc).

$$\Delta m = 0.01 \text{ g} = 0.01 \text{ g} \cdot 10^{-3} \text{ kg}$$

Since both l_0 and l_1 were measured using two markers on a ruler, the uncertainty in the measurement is equal to half of the smallest division(unc).

$$\Delta l_0 = \Delta l_1 = \frac{0.05 \text{ cm}}{2} = 0.025 \text{ cm} = 0.025 \cdot 10^{-2} \text{ m}$$