1 Introduction

DC power sources are an essential part of circuits. The aim of the following experiments is to study the proprieties of these sources. Electromotive force, ideal-source current, open-circuit voltage, short-circuit current, and internal resistance will be studied for voltage sources.

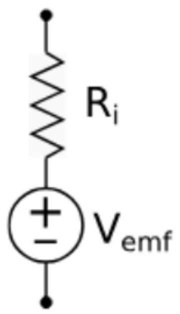
The behaviour of series and parallel source arrangements will also be studied.

2 Background/ Theory

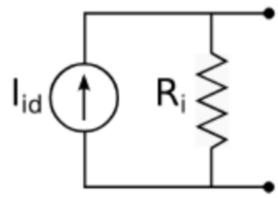
DC power sources refer to either voltage or current sources of which the output does not diminish with time. A source is called an ideal voltage source when the output voltage does not change in relation to the load resistance. For current sources, the current remains constant regardless of the load resistor.

In physical applications however, the current or voltage will depend on the load resistance. Therefore, power sources will be represented as a combination between an ideal source and an internal resistance. For voltage sources, the resistance will be connected in series with the ideal source, whereas for current sources, the resistor will be connected in parallel. (book)

Figures 1(a) and 1(b) display the aforementioned representation of power sources, both for current and voltage sources.(lab text)



(a) Schematic representation of voltage source



(b) Schematic representation of current source

Figure 1: Schematic representation of power sources

2.1 Source characteristics

Using source transformation, all power sources can be represented as voltage sources with an internal resistance(book). Therefore, going forward, sources will be displayed as shown in Figure 1(a). This representation is called the voltage-equivalent circuit. By adding a load resistance to the power source, essential characteristics of the source can be discovered.

Figure 2 displays the circuit which will be used to determine these characteristics(labtext).

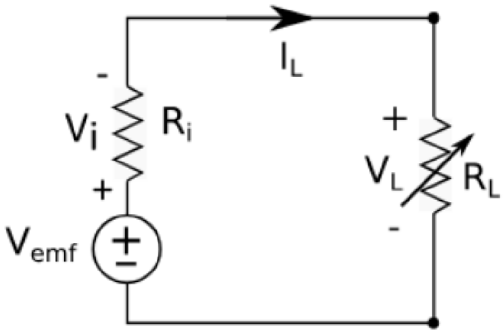


Figure 2: Simple load circuit of real voltage source

Firstly, the voltage of the ideal source is called the electromotive force, V_{emf}. By measuring the load current, this voltage can be related to the load voltage by observing that the same current passes through the internal resistance as well.

Equation 1 displays the relationship between current, load voltage, and electromotive force(labtext).

$$V_L = V_{emf} + I_L R_i \tag{1}$$

Since the internal resistance, R_i, is constant, the relationship between load voltage and current is a linear one.

Figure 3 displays this relationship for a real voltage source such as the one shown in Figure 1(a)(labtext).

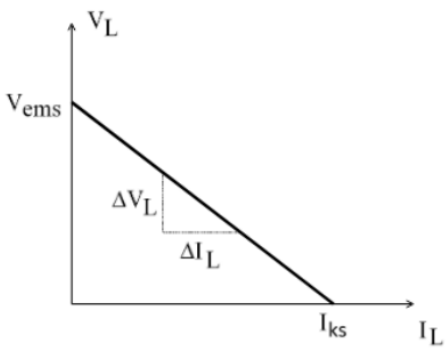


Figure 3: Typical V_L - I_L graph used to determine source characteristics

By determining the slope of the linear relationship between I_L and V_L, the internal resistance can be found(labtext):

$$R_i = -\frac{\Delta V_L}{\Delta I_L} \tag{2}$$

By looking at the x and y intercepts of the graph, both the electromotive force and the short circuit current can be found. Therefore, all of the sources characteristics can be found by analyzing the Voltage-Current graph.

A relationship between all of the characteristics is found equation 3(labtext):

$$I_{sc} = \frac{V_{emf}}{R_i} \tag{3}$$

2.2 Series and parallel sources

By connecting multiple voltage sources in series, higher V_{emf} values can be provided. The equivalent internal resistance will be equal to the sum of the individual sources internal resistances.

When connecting multiple sources in parallel, the equivalent internal resistance will be lowered. The lowering of the internal resistance will be governed by the parallel resistance equivalence(labtext):

$$R_i = \left(\sum_j \frac{1}{R_{i,\ j}} \right)^{-1} \tag{4}$$

Since sources should only be connected in parallel when they have the same electromotive force, the equivalent V_{emf} will be equal to that of one source(labtext).

2.3 Maximum power transfer

3 Method and materials

All circuits in the experiments were assembled using the connection boards. All measurements, including voltage and current, were determined using an AM-520 HVAC mutlimeter. 1.5 V batteries were used as voltage sources. Both a decade resistor, and two fixed resistors of 62 Ω and 150 Ω.

Throughout the experiments, both the load current, I_L, and load voltage, V_L, are measured at every itteration.

3.1 Experimental Set-up determination of internal resistance

The set-up for the first experiment is the same as the one described in Figure 2, where two different fixed resistors are used for R_i. The value of the resistors are 62 Ω and 150 Ω. A decade resistor is used to vary the load resistor in order to complete the necessary measurements.

To determine the source characteristics, the following experimental flow is established. Firstly the open circuit voltage is determined, thus establishing V_{emf}. Afterwards the decade resistor's value is adjusted such that a certain load current is established. The value of I_L varies between 0.5 mA and 5 mA in increments of 0.5 mA. After each adjustment, both the load voltage and the resistance of the decade resistor is noted down.

Figure 4 displays the physical experimental set-up used to gather the measurements for single sources.

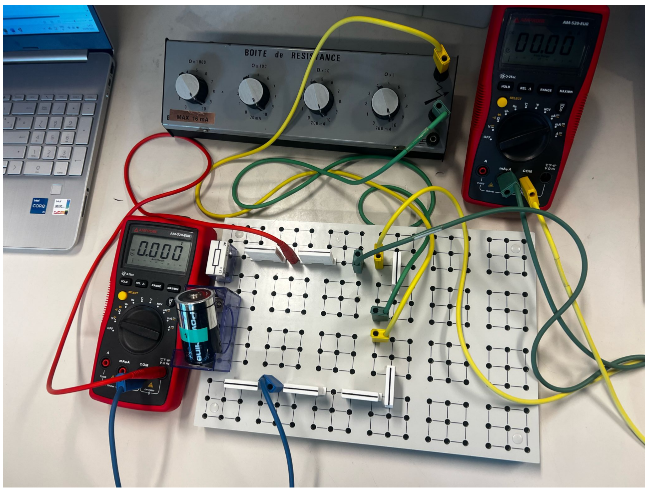
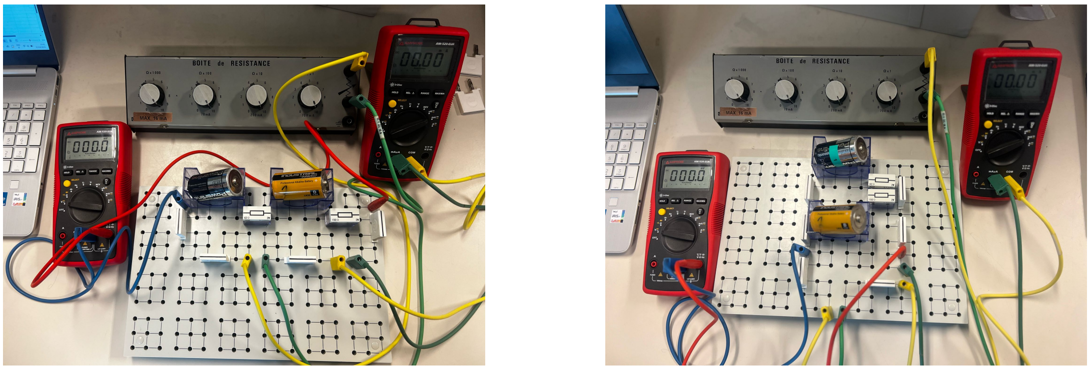


Figure 4: Experimental set-up for the first experiment

To measure the effect of combining multiple sources together, the same method of measurement is used, whilst having the two different types of sources connected, either in parallel, or in series.

Figures 5(a) and 5(b) show the physical set-up of the combined source circuits.



(a) Series sources circuit

(b) Parallel sources circuit

Figure 5: Experimental set-ups for the combined sources

3.2 Experimental set-up maximum power transfer

To determine the maximum power transfer, the same set-up as the last experiment is used, without the ammeter. For source 1, the value of the load resistor is varied between 30 Ω and 120 Ω in increments of 10, whereas source 2, the value varies between 70 Ω and 250 Ω. The voltage across the load resistor is measured each time. Additionally, the resistance of the decade resistor is measured after each change.

Figure 6 displays the physical set-up used to determine the maximum power transfer of each source.

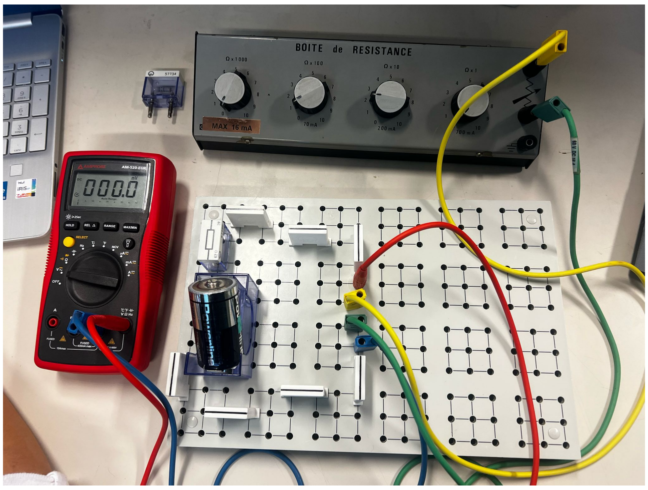


Figure 6: Experimental set-up of the power transfer experiment

4 Determination of internal resistance

4.1 Measurement results

After following the experimental flow as described in section 3.1, the determined voltages are noted down. Afterwards the fixed resistors used as internal resistances are measured using an ohmmeter. These are determined to be 61.5 Ω and 148.2 Ω respectively.

The measured load voltage of every source variation and the achieved load current are displayed in Table 1.

Table 1: Measurement results used to determine internal resistances

I_L mA	V_{L_1} V	V_{L_2} V	$V_{L_{ser}}$ V	$V_{L_{par}}$ V
0	1.49	1.49	3.004	1.496
0.5	1.459	1.415	2.897	1.475
1	1.429	1.342	2.793	1.453
1.5	1.398	1.268	2.687	1.431
2	1.367	1.193	2.584	1.409
2.5	1.336	1.121	2.48	1.388
3	1.305	1.047	2.375	1.366
3.5	1.275	0.974	2.271	1.344
4	1.244	0.899	2.168	1.322
4.5	1.213	0.827	2.063	1.3
5	1.183	0.752	1.959	1.279

4.2 Graphs

By plotting the load voltage in terms of I_L, all of a source's characteristics can be determined.

Figure 7 displays the relationship between load current and voltage for source 1, having an internal resistance of 61.5 Ω.

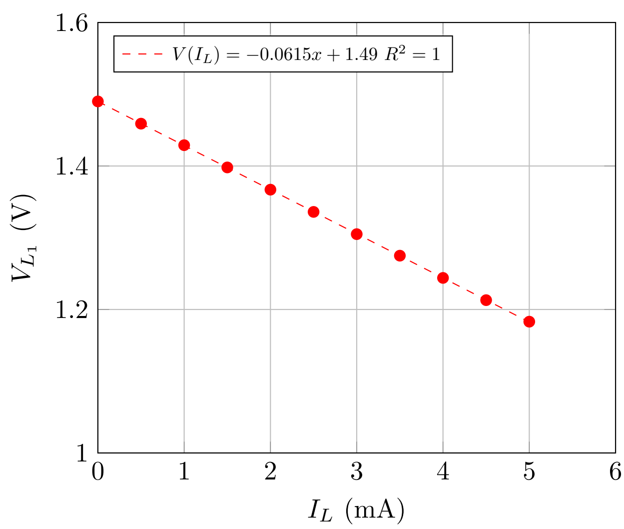


Figure 7: Load voltage of the first source in terms of load current

Figure 8 shows the relationship between I_L and V_{L_2}, source 2 having an internal resistance of 148.2 Ω

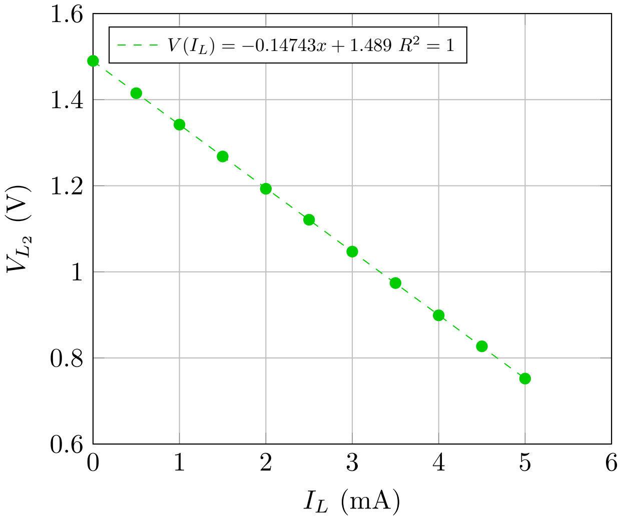


Figure 8: Load voltage of the second source in terms of load current

Figure 9 displays the voltage current dependency for the series source arrangement.

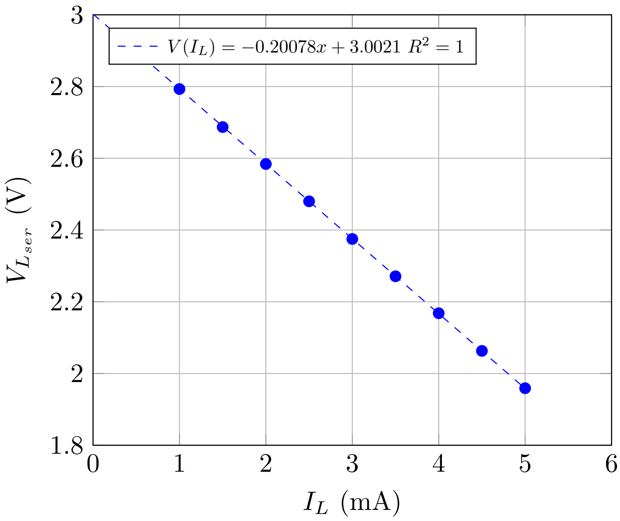


Figure 9: Load voltage of the series source arrangement in terms of load current

Figure 10 represents the linear relationship between load current and load voltage for the parallel source arrangements.

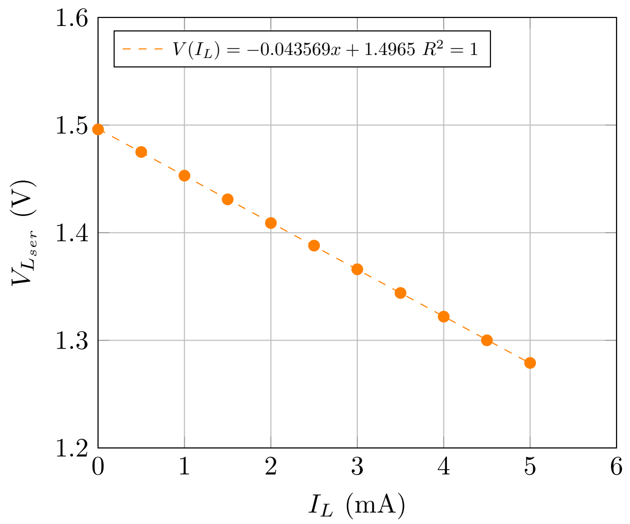


Figure 10: Load voltage of the parallel source arrangement in terms of load current

4.3 Calculations

By utilizing the equations described in section 2.1, the characteristics of both sources can be determined.

The result of these calculations is displayed in table 2.

	R_i Ω	V_{emf} V	I_{sc} mA
Source 1	61.5	1.49	24.22
Source 2	147.4	1.49	10.11
Series	208.8	3	14.37
Parallel	43.6	1.49	34.34

To determine the error of the internal resistance measurements, Microsoft Excel's LINEST function is used. This function returns information about linear regression of a set of samples. One of these values is the standard error for V_L. This value can be used to calculate the error of the internal resistance. This is done by using equation 5.This will also be used to determine the error of I_{sc}.

$$\Delta R_i = R_i \frac{\Delta V}{V} \tag{5}$$

Therefore, the following calculations are made:

$$\Delta R_i = 61.5 \frac{6.5 \cdot 10^{-5}}{1.183} \approx 0.1\ \Omega$$
$$\Delta I_{sc} = 21.22 \frac{6.5 \cdot 10^-5}{1.183} \approx 0.01\ \text{mA}$$

Therefore the values of R_i and I_{sc} for source one are:

$$R_i = (61.5 \pm 0.1)\ \Omega$$
$$I_{sc} = (21.22 \pm 0.01)\ \text{mA}$$

However, the error of V_{emf} will be determined using the error of the multimeter measurements(mm). This is done using equation 6.

$$\Delta V_{emf} = \frac{V}{100} + 3LSD \tag{6}$$

By introducing the measured value for source one, the following calculation is achieved:

$$\Delta V_{emf} = \frac{1.49}{100} + 0.003 \approx 0.02 \text{ V}$$

Therefore, the value for electromotive force is:

$$V_{emf} = (1.49 \pm 0.02) \text{ V}$$

The error of the internal resistance fixed resistors will also be determined by using the error of the multimeter. The formula used is shown in equation 7.

$$\Delta R = \frac{1.2 \cdot R}{100} + 2LSD \tag{7}$$

The sample calculation for the internal resistance of the first source, R_{i_1}, is:

$$\Delta R_{i_1} = \frac{1.2 \cdot 61.5}{100} + 0.002 \approx 0.8 \ \Omega$$

Therefore, the values of the measured internal resistances are:

$$R_{i_1} = (61.5 \pm 0.8) \ \Omega$$
$$R_{i_2} = (148.2 \pm 2) \ \Omega$$

4.4 Discussion