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# The Application of Bayesian Model Averaging in Compatibility of Stand Basal Area for Even-Aged Plantations in Southern China

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Stand growth-and-yield models include whole-stand models, individual-tree models, and diameter distribution models. Based on the growth data of Chinese fir (*Cunninghamia lanceolata* [Lamb.] Hook.) in Fenyi County, Jiangxi Province, in southern China, Bayesian model averaging (BMA) was used to forecast stand basal areas by combining these three types of models into a single predictive model. BMA is a statistical method that infers consensus predictions by weighting individual predictions based on their posterior probabilities, with the better performing predictions getting higher weights than the poorer performing ones. Furthermore, BMA accounts for model uncertainty as reflected by the variance. The variance of BMA can be decomposed into a between-model variance that reflects the model's consistency and a within-model variance that reflects the data variability. Results showed that the between-model variance was much greater than the within-model variance for all the stand basal area predictions. The resulting model produced accurate and reliable predictions, and the 95% confidence interval of BMA predictions encompassed the observations very well. The BMA method provided a consistent prediction of stand basal area from three types of models, thus improving compatibility among these models.

Keywords: compatibility, Bayesian model averaging, uncertainty, stand basal area

Forest growth-and-yield models are vitally important for forest management planning. Predicting the growth and yield of stands is a prerequisite for planning the management of forests at any level. Forest growth-and-yield models can be divided into three broad categories: whole-stand models, individual-tree models, and diameter distribution models (Munro 1974). Whole-stand models are developed on the information at stand level such as stand basal area and volume (Curtis et al. 1981, Vanclay 1994), whereas individual-tree models consider each tree as a unit (Zhang et al. 1997, Cao 2000, Cao et al. 2002). Diameter distribution models, in contrast, use statistical probability functions, such as the Weibull (Bailey and Dell 1973, Liu et al. 2004, Newton et al. 2005), beta (Gorgoso-Várela et al. 2008), or $S_B$ function (Wang and Rennolls 2005), to characterize stand structure. The Weibull function has been commonly used to model diameter distributions, and three main methods have been proposed to estimate the parameters of Weibull distribution: the maximum likelihood estimation, the percentile estimation, and moment estimation (MOM). Cao (2004)

proposed the cumulative distribution function (CDF) regression method and found that the method produced better results than those from the MOM for loblolly pine plantations because the CDF regression aimed to fit the CDF of diameter distribution. Lei (2008) reported that the MOM was superior for estimating the parameters of Weibull distribution for tree diameter distribution for Chinese pine. Liu et al. (2009) found that the MOM method and the CDF method performed almost equally in predicting parameters of the Weibull function in unthinned *Picea glauca* plantations.

There are advantages and disadvantages of each type of model. Whole-stand models can predict stand variables directly and provide rather general information about the future stand, but lack detailed tree-level information. On the other hand, whereas individual-tree models provide much more detailed information and diameter distribution models offer diameter structure, stand-level outputs from these two types of models often have an accumulation of errors that leads to poor accuracy and precision (Qin and Cao 2006, Zhang et al. 2010).

**Table 1. Summary statistics of stand-level and tree-level variables for Chinese fir, by data set.**

| Variable | Fit data ($n = 75$) | | | | Validation data ($n = 45$) | | | |
|---|---|---|---|---|---|---|---|---|
| | Minimum | Maximum | Mean | SD | Minimum | Maximum | Mean | SD |
| Dominant height (m) | 8.8 | 16.9 | 12.5 | 1.88 | 12.8 | 18.4 | 15.6 | 1.43 |
| Stand survival (trees $\cdot$ ha$^{-1}$) | 1,633 | 9,883 | 4,830 | 2,356.44 | 1,450 | 5,650 | 3,194 | 1,274.85 |
| Stand basal area (m$^2 \cdot$ ha$^{-1}$) | 20.07 | 65.94 | 44.44 | 9.70 | 21.00 | 60.59 | 48.66 | 9.40 |
| Arithmetic mean diameter (cm) | 7.08 | 18.14 | 11.47 | 2.90 | 10.79 | 19.84 | 14.16 | 2.57 |
| Dbh (cm) | 1.1 | 28.2 | 10.24 | 3.50 | 3.6 | 32.4 | 13.33 | 4.04 |

From a common sense point of view, we would expect stand-level outputs from different types of models to be similar. However, these models may still provide numerically inconsistent predictions due to model errors (Meng 1996). Daniels and Burkhart (1988) proposed a framework for an integrated system of forest models, in which constraints were applied so that stand outputs from models of different levels of resolution are similar. Zhang et al. (1993) and Qin and Cao (2006) applied the disaggregation method to maintain compatibility by linking stand-level and tree-level models. However, this method assumes that predictions from the stand-level model are superior, and it attempts to adjust outputs from the tree-level model to match those from the stand-level model. Yue et al. (2008) applied the forecast combination method to solve the compatibility problem between tree-level and stand-level models. Zhang et al. (2010) extended this method to improve the compatibility among tree-level, stand-level, and diameter distribution models.

An alternative for combining the three types of models is the Bayesian model averaging (BMA) method. BMA (Leamer 1978, Kass and Raftery 1995) was originally developed as an approach to combine inferences and forecasts from multiple competing models (Hoeting et al. 1999) and has been widely applied in several fields, such as social and political sciences (Bartels 1997, Montgomery and Nyhan 2010), ecology (Wintle et al. 2003), hydrology (Duan et al. 2007, Zhang et al. 2009), climate forecast (Raftery et al. 2005, Berliner and Kim 2008, Smith et al. 2009), and forest biomass (Li et al. 2008, Picard et al. 2012). BMA can clearly show the information update process and can also combine predictive distributions from different sources. In BMA, the predictive probability density function (PDF) of any quantity of interest is a weighted average of predictive PDFs based on the individual predictions, where the weights assigned to the different predictions arise naturally as posterior model probabilities. The contribution of each individual model to the averaging model is weighted by its posterior weight of evidence (Ellison 2004). In addition, BMA takes into account the model uncertainty, and it is easy to construct prediction intervals accounting for model uncertainty as well as weight uncertainty (Eklund and Karlsson 2005).

The objective of this study was to demonstrate how the BMA method, which accounts for model variances, is used to link whole-stand, individual-tree, and diameter distribution models so that the resulting stand basal area is consistent with the three types of models.

## Materials and Methods
### Data

The Chinese fir (*Cunninghamia lanceolata* [Lamb.] Hook.) stands located in Fenyi County, Jiangxi Province, in southern China were established in 1981. The plots were planted in a random block arrangement with the following tree spacings: 2 × 3 m (1,667

trees/ha), 2 × 1.5 m (3,333 trees/ha), 2 × 1 m (5,000 trees/ha), 1 × 1.5 m (6,667 trees/ha), and 1 × 1 m (10,000 trees/ha). Each spacing level was replicated three times. Each plot comprised an area of 20 × 30 m, and a buffer zone (two lines) consisting of similarly treated trees surrounded each plot. The diameter of each tree was measured after its height reached 1.3 m. Measurements were performed in each winter from 1983 to 1988 and then every other year until 2007. In the winter of 1998, there was a snow storm in Jiangxi Province, resulting in many dead trees for 1999 and 2001 in high-density plots. We therefore excluded the data between 1999 and 2001. In this study, the data consisted of 120 measurements, with a 2-year remeasurement interval, obtained from 1989 to 2007. Seventy-five plots were used for model development and another 45 plots for model validation. The fit data included all measurements from stands aged 10–18 years, and the validation data included all measurements from stands of aged 22–26 years, in 2-year increments. Summary statistics for both data sets are shown in Table 1.

### Stand-Level and Tree-Level Models

The stand-level models included equations for stand density, arithmetic mean diameter, stand basal area, diameter SD, and minimum diameter. The tree-level model consisted of diameter growth and tree survival equations. The nonlinear equations that formed the stand- and tree-level models are as follows (Qin and Cao 2006, Qin et al. 2007)

*Stand-level*:

$$B_2 = \exp\{(A_1/A_2)\ln(B_1) + (1 - A_1/A_2)[\phi_1 + \phi_2 H_1/\ln(N_1)]\} \quad (1a)$$

$$Dm_2 = \exp\{(A_1/A_2)\ln(Dm_1) + (1 - A_1/A_2)[\delta_1 + \delta_2 A_1/\ln(N_1) + \delta_3 Rs_1]\} \quad (1b)$$

$$Dsd_2 = \exp\{(A_1/A_2)\ln(Dsd_1) + (1 - A_1/A_2)[\gamma_1 + \gamma_2 A_1/\ln(N_1)]\} \quad (1c)$$

$$D\min_2 = \exp\{(A_1/A_2)\ln(D\min_1) + (1 - A_1/A_2)[\kappa_1 + \kappa_2 A_1/\ln(D\min_1)]\} \quad (1d)$$

$$N_2 = \exp\{(A_2/A_1)\ln(N_1) + (1 - A_1/A_2)[\beta_1 + \beta_2 A_1 + \beta_3 \ln(N_1)]\} \quad (1e)$$

*Tree-level*:

$$D_{i,2} = D_{i,1} + \exp\{\lambda_1 + \lambda_2 B_1 + \lambda_3/Rs_1 + \lambda_4/\ln(D_{i,1})\} \quad (1f)$$

$$P_{i,2} = \{1 + \exp[\mu_1 + \mu_2 A_1 + \mu_3 D_1/\ln(Dm_1) + \mu_4 \ln(N_1)]\}^{-1} \quad (1g)$$

where $Rs_j = (10,000/N_j)^{0.5}/H_j$ is relative spacing at age $A_j$, $j = 1, 2$, $N_j$ is number of trees per ha at age $A_j$, $H_j$ is dominant height (m) at age $A_j$, $Dm_j$ is arithmetic mean diameter (cm) at age $A_j$, $B_j$ is stand basal area (m$^2$/ha) at age $A_j$, $Dsd_j$ is diameter SD (cm) at age $A_j$, $Dmin_j$ is minimum diameter (cm) at age $A_j$, $D_{i,j}$ is diameter of tree $i$ at age $A_j$, $P_{i,2}$ is the probability that tree $i$ is survived the period from age $A_1$ to $A_2$, and $\beta_1, \beta_2, \ldots, \mu_4$ is parameters to be estimated.

For the tree-level model, estimates of stand basal area at age $A_2$ were obtained by summing up the basal area per ha represented by each tree, computed from the individual-tree diameter growth model (Equation 1f) and survival model (Equation 1g).

The seemingly unrelated regression method (SUR) suggested by Borders (1989) was used to simultaneously estimate parameters of the stand-level system (Equations 1a–1e). The fitting procedure involved the use of option SUR of the procedure MODEL in SAS (SAS Institute, Inc. 2008). Parameters of the tree diameter growth and survival equations (Equation 1f–1g) were estimated by use of nonlinear regression and maximum likelihood techniques.

### Diameter Distribution Model

The Weibull function has been widely used to quantify tree diameter distribution because of its flexibility in describing a wide range of unimodal distributions, the relative simplicity of parameter estimation, and its closed cumulative density functional form (Bailey and Dell 1973, Rennolls et al. 1985, Mabvurira et al. 2002, Lei 2008). The Weibull PDF is expressed as follows

$$f(x; a, b, c) = \left(\frac{c}{b}\right)\left(\frac{x-a}{b}\right)^{c-1} \exp\left[-\left(\frac{x-a}{b}\right)^c\right], x \geq a \quad (2)$$

where $x$ is dbh, $a$ is the location parameter, $b$ is the scale parameter, and $c$ is the shape parameter.

MOM is one of the most extensively applied parameter recovery methods for estimating Weibull parameters (Liu et al. 2004, Lei 2008). The Weibull location parameter ($a$) was computed from the predicted minimum diameter in the stand. Frazier (1981) found that $a = 0.5\hat{D}min_2$ produced minimum errors in terms of goodness of fit. A straightforward method to recover $b$ and $c$ involves predicted values of arithmetic and quadratic mean diameters ($\hat{D}m_2$ and $\hat{D}g_2$, respectively). A possible problem is that $\hat{D}g_2$ might be too close to or too far from $\hat{D}m_2$ and can even be smaller than $\hat{D}m_2$ if not properly constrained. Zhang et al. (2010) found that the Weibull parameters are sensitive to the difference between $\hat{D}m_2$ and $\hat{D}g_2$ and therefore might result in unstable estimations of $b$ and $c$. An alternative method is to use the predicted arithmetic mean diameter ($\hat{D}m_2$) and diameter variance ($\hat{D}var_2$) instead of $\hat{D}m_2$ and $\hat{D}g_2$ (Qin et al. 2007, Zhang et al. 2010). The Weibull parameters $b$ and $c$ are solved from the following set of equations

$$b = (\hat{D}m_2 - a)/\Gamma_1 \quad (3)$$

$$\hat{D}var_2 - b^2(\Gamma_2 - \Gamma_1^2) = 0 \quad (4)$$

where $\Gamma_1 = \Gamma(1 + 1/c)$, $\Gamma_2 = \Gamma(1 + 2/c)$, and $\hat{D}var_2 = (\hat{D}sd_2)^2$.

We first predicted the quadratic mean diameter from the parameters of the diameter distribution model and then computed an estimate of stand basal area

$$\hat{D}g_2^2 = b^2\Gamma_2 + 2ab\Gamma_1 + a^2 \quad (5)$$

$$\hat{B}_2 = (\pi/40000)(\hat{N}_2)(\hat{D}g_2^2) \quad (6)$$

### BMA

BMA involves averaging over all models and accounts for uncertainty about model form and assumptions (Raftery et al. 2005). Each model weight is equal to its posterior probability. We know that the stand basal areas from the stand-level model, tree-level model, and diameter distribution model are different, and the models perform differently. This is a source of uncertainty in estimating stand basal area. The typical approach, conditioning on a single model deemed to be "best," ignores this source of uncertainty and thus underestimates total uncertainty. BMA overcomes the problem by conditioning, not on a single best model, but on the entire ensemble of statistical models first considered (Raftery et al. 2005, Picard et al. 2012). Let $M = (M_1, \ldots, M_K)$ be the set of models considered. In this study, $K = 3$. The law of total probability given some data $Z$ tells us that the forecast PDF is given by

$$P(y|Z) = \sum_{k=1}^{K} P(y|M_k) P(M_k|Z) \quad (7)$$

where $P(y|M_k)$ is the forecast PDF under model $M_k$ alone, and $P(M_k|Z)$ is the posterior model probability of model $M_k$ given the data. The posterior model probabilities reflect how well $M_k$ fits the given data. They add up to 1, so that $\Sigma_{k=1}^{K}P(M_k|Z) = 1$, and they can be regarded as weights. The BMA PDF thus is a weighted average of its distribution under each model, weighted by their posterior model probabilities.

Let $y$ be the stand basal area to be forecasted. Each model prediction $f_k$ is then associated with a component PDF, $g_k(y|f_k)$. The BMA predictive PDF is then a mixture of the component PDFs, namely

$$P(y|f_1, \ldots, f_K) = \sum_{k=1}^{K} w_k g_k(y|f_k) \quad (8)$$

where $w_k$ is the posterior probability of model prediction $f_k$, which reflects the relative performance in the averaging model. The $w_k$ values are probabilities, and therefore they are positive and sum up to 1. In this study, $\Sigma_{k=1}^{3} w_k = 1$.

The posterior mean and variance of the BMA prediction can be expressed as (Raftery et al. 2005, Duan et al. 2007)

$$E[y|f_1, \ldots, f_K] = \sum_{k=1}^{K} w_k E[g_k(y|f_k)] = \sum_{k=1}^{K} w_k f_k \quad (9)$$

$$Var[y|f_1, \ldots, f_K] = \sum_{k=1}^{K} w_k (f_k - \sum_{i=1}^{K} w_i f_i)^2 + \sum_{k=1}^{K} w_k \sigma_k^2 \quad (10)$$

where $\sigma_k^2$ is the variance of specific model prediction $f_k$. In Equation 9, the expected BMA prediction is the average of individual predictions weighted by the likelihood. We should also note some properties of the BMA prediction. First, the BMA prediction gets higher weights from better performing models because the likelihood of a model is a measure of the agreement between the model predictions and observations. Second, the BMA variance is an uncertainty measure of the BMA predictions. It contains two terms as shown on the right-hand side of Equation 10: the first term denotes the between-model variance or the between-model spread; the second term represents the within-model variance or the expected uncertainty

conditional on one of the models being best (Duan et al. 2007, Picard et al. 2012).

We used the expectation-maximization (EM) algorithm for estimating the BMA weights and variance, as described by Raftery et al. (2005). The advantage of the EM algorithm is that it is relatively easy to implement and computationally efficient. Before we present the BMA algorithm, it is assumed that the conditional probability distribution $g_k(y|f_k)$ is Gaussian. Box-Cox transformation was used before the weights were estimated, so that the transformed variables will be close to the Gaussian distribution (Raftery et al. 2005, Picard et al. 2012). The log-likelihood function for Model 8 is given by

$$\ell(w_1, \ldots, w_K, \sigma^2) = \sum_{i=1}^{n} \left( \sum_{k=1}^{K} ln(w_k g_k(y|f_k)) \right) \quad (11)$$

where $n$ is the total number of observations. The EM algorithm casts the maximum likelihood problem as a "missing data" problem. The missing data may not be actual data. The missing data $z_{ki}$ is the posterior probability that stand basal area model $k$ is the best model for observation $i$. If the $k$th model ensemble is the best prediction, $z_{ki} = 1$; otherwise $z_{ki} = 0$. The EM algorithm is iterative and alternates between the E (or expectation) step and the M (or maximization) step. It starts with an initial guess, $\theta^{(0)}$, for parameter vector $\theta$. Details of this algorithm were discussed by Duan et al. (2007) and Picard et al. (2012).

## Evaluation of Methods

The three types of models were evaluated against the BMA method, based on the following criteria computed from the validation data.

Mean deviation (MD)

$$MD = \sum_{i=1}^{n} (y_i - \hat{y}_i)/n, \quad (12)$$

Root mean square error (RMSE)

$$RMSE = \sqrt{\sum_{i=1}^{n} (y_i - \hat{y}_1)^2/n} \quad (13)$$

Fit index (FI)

$$FI = 1 - \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 / \sum_{i=1}^{n} (y_i - \bar{y}_i)^2, \quad (14)$$

where $y_i$ are observed values of stand basal area at age $A_2$ for the $i$th observation, $\hat{y}_i$ are the predicted values of $y_i$, $\bar{y}_i$ are the mean values of $y_i$, and $n$ is the number of observations.

## Results

All of the parameters were significant at the 0.05 level (Table 2). $R^2$ values of the stand basal area model, arithmetic mean diameter model, diameter SD model, minimum diameter model, stand survival model, diameter growth, model and quadratic mean diameter from diameter distribution ranged from 0.7398 to 0.9949, and RMSE of tree survival was 0.1604. In terms of $R^2$ value, these models all performed well. The fit for the individual-tree diameter

**Table 2. Parameter estimates and model evaluation statistics for different equations for Chinese fir.**

| Equations | Attribute | Parameter | Estimate | SE | $R^2$ |
|---|---|---|---|---|---|
| 1a | Stand basal area (m²/ha) | $\phi_1$ | 5.3110 | 0.0903 | 0.9797 |
|  |  | $\phi_2$ | −0.6225 | 0.0632 |  |
| 1b | Arithmetic mean diameter (cm) | $\delta_1$ | 2.1173 | 0.0661 | 0.9944 |
|  |  | $\delta_w$ | 0.1200 | 0.0284 |  |
|  |  | $\delta_3$ | 3.9059 | 0.2397 |  |
| 1c | Diameter SD (cm) | $\gamma_1$ | 1.1367 | 0.1069 | 0.8947 |
|  |  | $\gamma_2$ | 0.1726 | 0.0637 |  |
| 1d | Minimum diameter (cm) | $\kappa_1$ | 2.0321 | 0.2532 | 0.7398 |
|  |  | $\kappa_2$ | 0.0335 | 0.0156 |  |
| 1e | Stand survival (trees/ha) | $\beta_1$ | 3.7979 | 0.4711 | 0.9949 |
|  |  | $\beta_2$ | −0.0388 | 0.0068 |  |
|  |  | $\beta_3$ | 0.5861 | 0.0504 |  |
| 1f | Diameter (cm) | $\lambda_1$ | 0.7973 | 0.0551 | 0.9356 |
|  |  | $\lambda_2$ | −0.0402 | 0.0030 |  |
|  |  | $\lambda_3$ | −0.0352 | 0.0153 |  |
|  |  | $\lambda_4$ | 0.9157 | 0.0186 |  |
| 1g | Tree survival | $\mu_1$ | −16.8858 | 4.1004 | 0.1604 |
|  |  | $\mu_2$ | 0.2793 | 0.0412 | (RMSE) |
|  |  | $\mu_3$ | −2.1696 | 0.1472 |  |
|  |  | $\mu_3$ | 1.8729 | 0.4369 |  |
|  | Quadratic mean diameter (cm) |  |  |  | 0.9905 |

Quadratic mean diameter was calculated from the diameter distribution model. The equation for $R^2$ is the same as that for the FI (Equation 14).

**Table 3. Posterior weights of the BMA model of stand basal area predictions, derived from three level types of models.**

| Model | $w$ |
|---|---|
| Tree-level model | 0.3274 |
| Stand-level model | 0.3391 |
| Diameter distribution model | 0.3335 |

growth model was quite good, with an $R^2$ value of 0.9356 (Table 2). This might be due to the short growth periods (2 years) in the data. In contrast, Cao (2000) reported fit index values between 0.19 and 0.21 for tree diameter growth prediction, based on growth intervals between 4 and 7 years.

The BMA weight estimated by the EM algorithm for the stand-level model was 0.3391, larger than 0.3335 for the diameter distribution model, and 0.3274 for the tree-level model (Table 3). Although the weight of the stand-level model was the largest, the differences among the weights of the three types of models were very small because the models performed similarly for predicting stand basal area (Figure 1).

The BMA predictive variance was decomposed into two parts: the between-model variance and the within-model variance. The between-model variance of stand basal area predictions for the validation data ranges from 0.07 to 7.05, whereas most of the within-model variance was smaller than 1. The between-model variance was much greater than the within-model variance for all the stand basal area predictions (Figure 2). The BMA predictions for stand basal area of validation data are shown in Figure 2, along with the 95% confidence interval. The expected predictions were very close to the observed values. The 95% confidence interval also encompassed the observations very well.

## Discussion

The three types of models were combined into a single model using BMA for improving compatibility of stand basal area predictions. BMA provides a solution that combines three types of models
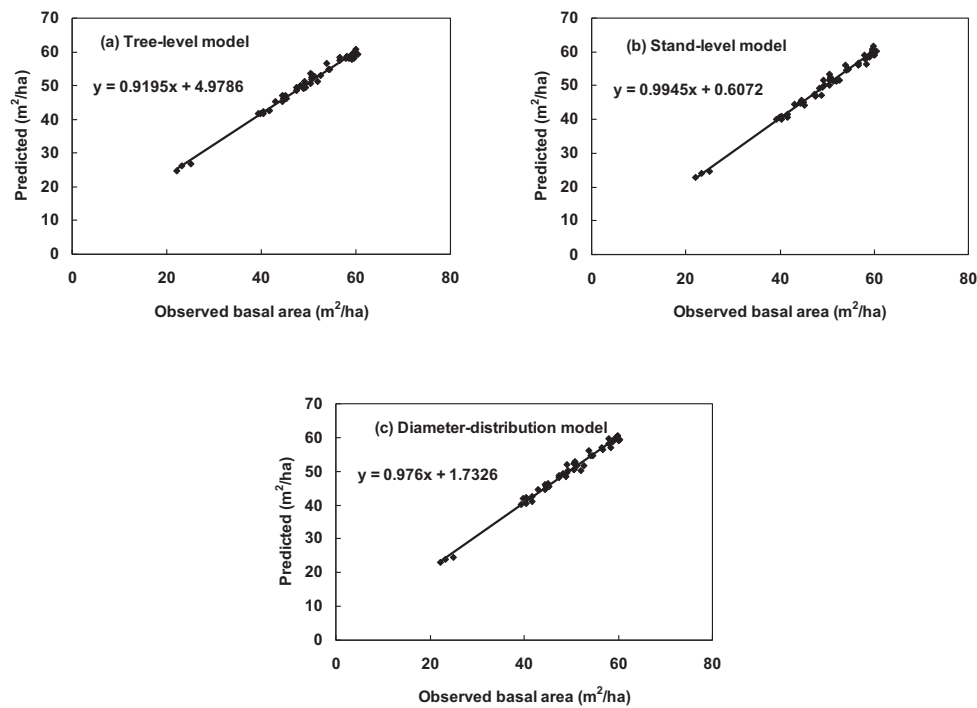
Figure 1. Relationship between predicted stand basal areas and observed stand basal areas of Chinese fir for the three types of models.
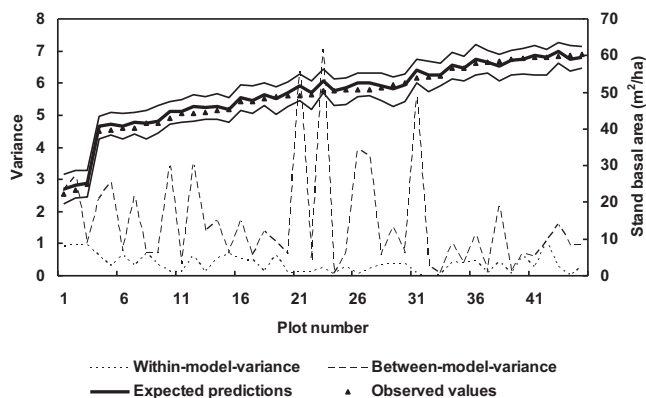


Figure 2. BMA predictive variance (dotted lines) and 95% confidence intervals (solid lines) of predicted stand basal areas for the BMA estimator for validation data against plot number. (The plots were numbered according to increasing basal area.)

Table 4. Evaluation criteria of stand basal area predictions from the BMA and from three types of models, computed from the validation data.

| Criteria | Tree-level | Stand-level | Diameter distribution | BMA |
|---|---|---|---|---|
| MD | −1.0506 | −0.3405 | −0.6480 | −0.5619 |
| RMSE | 1.5591 | 1.1085 | 1.1764 | 1.1445 |
| FI | 0.9713 | 0.9855 | 0.9836 | 0.9846 |

and takes advantages of the existing three models. It overcomes the problem of arbitrarily selecting a single best model and puts weight on individual models by their posterior model probabilities. Each weight reflects the relative model performance because it is the likelihood measure of a model being correct, given the data (Duan et al. 2007). Comparing the evaluation statistics from BMA and the three level models, we found that they are quite close, with the stand-level model performing slightly better than other models (Table 4). Without exception, the stand-level model produced better evaluation statistics than did the tree-level and diameter distribution models. Qin and Cao (2006) also found that the stand-level model predicted stand attributes better than did the tree-level model. The performance of the prediction after the BMA scheme is not expected to be better than that of the original prediction by individual models, especially when the individual models performances are similar (Figure 1), because BMA prediction is the weighted average of in-

dividual predictions with the sum of weights to be 1. In addition, in this article, we do not intend to argue whether the BMA technique is superior to a single model. We simply propose an alternative approach to solve the compatibility of stand basal area predictions from the three models. Zhang et al. (2010) proposed the forecast combination method to combine the three types of models to improve the compatibility of stand basal area. The method, however, does not explicitly account for all sources of forecast uncertainty and gives a point estimate but does not give an estimate on its variance just like the BMA method. Some form of postprocessing is necessary to provide predictive ensemble PDFs that are meaningful and can be used to provide accurate forecasts (Georgekakos et al. 2004, Wöhling and Vrugt 2008).

When the BMA predictive variance was decomposed, the between-model variance indicated the disagreement between the three types of models. These three models provide numerically different predictions due to model errors and constructions (Meng 1996), reflecting the between-model variance. The further apart the models' predictions are, the larger the between-model variance is. The BMA assembled the three models into a single model that took into account both between-model and within-model variances. This measure is a better description of predictive uncertainty than that in a non-BMA scheme, which estimates uncertainty based only on the ensemble spread (i.e., only the between-model variance is considered) or the single model (i.e., only the within-model variance is considered) and consequently leads to under-dispersive predictions

(Raftery et al. 2003, Duan et al. 2007). Kangas (1999) found that the Monte Carlo simulation method underestimated the variance of errors compared with computing the error variance directly from measured and predicted values from observed plots. The BMA method, based on empirical data, should provide a better estimate for uncertainty.

BMA takes account of the model uncertainty, and it is easy to construct prediction intervals accounting for model uncertainty as well as weight uncertainty (Eklund and Karlsson 2005). The three models are assumed to have different structures and uncertainty in parameter estimates. The BMA method produces more accurate and reliable predictions in resolving different forecasting problems than other available multimodel techniques (Ajami et al. 2007, Diks and Vrugt 2010).

It should also be noted that the data used in the study were from remeasured plots. A common method to model longitudinal data is the use of mixed-effects modeling techniques (Hall and Bailey 2001, Li et al. 2012). If these techniques had been used to model tree-level model and stand-level models, the performance of the models and subsequently of BMA would have been improved. Another issue is that the observations used for fitting equations came from the same plots as did the observations from the validation data. The validation procedure used in this study was therefore not rigorous, because of the lack of independence between the fit data and the validation data.

Each of the three broad types of forest growth-and-yield models (whole-stand, individual-tree, and diameter distribution) has success in predicting stand attributes such as basal area per unit area but produces inconsistent values. It makes sense therefore to improve the compatibility of stand basal area. The BMA estimator provided a consistent prediction of stand basal area from the three types of models, thus improving compatibility among these models.

## Conclusions

BMA accounts for the uncertainty of each level and yields accurate and reliable predictions. Although BMA performed slightly worse than the stand-level model, BMA was an alternative method to solve the compatibility of stand basal area predictions from the three models. By dealing with inconsistent stand basal area estimates from models of different levels of resolutions, BMA provides a solution that combines three types of models and takes advantages of the existing three models. It overcomes the problem of arbitrarily selecting a single "best model" and also provides a feasible integrated system of stand basal area growth models. This method could also be used for other stand variables, such as volume per unit area.

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