Optimizing De Jong, Schwefel, Rastrigin, and Michalewicz Functions using Hill Climbing and Simulated Annealing Techniques

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Abstract

This report details the implementation and comparison of optimization algorithms for minimizing benchmark functions: De Jong 1, Schwefel, Rastrigin, and Michalewicz. We implemented several Hill Climbing variations (first improvement, best improvement, worst improvement) and Simulated Annealing, a probabilistic optimization technique hybridized with Hill Climbing. Each function was tested at dimensions of 5, 10, and 30.The results are presented through tables, displaying the mean, standard deviation, minimum, and maximum of values obtained for each configuration. The experiments were conducted with a minimum precision of 5 decimal places. Finally, conclusions discuss the nature and implications of the results obtained and suggest future research directions in optimizing complex functions.

1 Introduction

1.1 Context and Motivation

The optimization of complex functions is essential in many areas of engineering and applied sciences. Functions like De Jong 1, Schwefel, Rastrigin, and Michalewicz are widely used as benchmark functions for assessing the performance of optimization algorithms due to their various challenging characteristics, including multiple local minima and sensitivity to input dimensionality [1, 2]. These functions serve as standard testbeds for evaluating the effectiveness of optimization techniques, from classical methods to modern metaheuristic approaches [4].

1.2 Problem Description

The objective of this report is to determine the minima of these benchmark functions using various optimization techniques: Hill Climbing variants (first, best, and worst improvement) and Simulated Annealing, a probabilistic method capable of overcoming local minima through controlled exploration. The functions are analyzed at various dimensions, from 5 to 30 dimensions, to observe how problem complexity impacts the convergence and accuracy of the algorithms [7].

1.3 Structure of the Report

In the **Methods** section, we describe the details of each algorithm's implementation and the design choices specific to each variant, such as real number representation, neighbor generation, stopping conditions, and key parameters (temperature for Simulated Annealing, cooling rate).

The **Experiments Description** section includes information on the experimental setup: dimensions used, number of repetitions, and desired precision.

Experimental Results are presented in the form of tables, detailing each algorithm's performance on varied parameter and dimension sets. Additionally, we discuss the influence of algorithm parameters on performance and solution stability.

Finally, the **Conclusions** section summarizes the key findings and suggests future research directions to improve function optimization techniques.

2 Methods

This study implements various optimization techniques aimed at finding the minimum values of four well-known benchmark functions: De Jong, Schwefel, Rastrigin, and Michalewicz. These functions present unique challenges to optimization due to features such as multiple local minima, high multimodality, and non-convex landscapes [1, 2]. To address these challenges, we applied three Hill Climbing variations (Best Improvement, First Improvement, Worst Improvement) alongside a hybridized Simulated Annealing method. Each variant operates with distinct exploration and exploitation strategies, leveraging different approaches to neighbor selection, stopping conditions, initialization, and specific parameters like cooling rates in Simulated Annealing.

2.1 Algorithm Implementation and Design Choices

For each method, we implemented key design choices to enhance the robustness of the algorithms across the diverse function landscapes:

- **Representation of Solutions:** Real number representation was chosen for all functions, allowing the algorithms to operate on continuous search spaces and effectively handle functions with non-integer values.
- **Neighbor Generation:** Neighbors were generated by perturbing the current solution by a small, random step size within a specified range. For each function, bounds were set according to its definition (e.g., Rastrigin's bounds were $[-5.12, 5.12]$), ensuring that all generated neighbors remained within the feasible space.
- **Stopping Conditions:** Each algorithm iteration continued until a maximum of 10,000 steps was reached, balancing thorough exploration with computational feasibility. This limit allowed enough iterations to evaluate the effectiveness of each method across different landscapes while preventing excessive run times.
- **Parameter Selection:** Specific parameters such as temperature in Simulated Annealing and the cooling rate were fine-tuned based on preliminary testing to balance exploration and convergence across the various function landscapes. The following parameter values were chosen to achieve a balance between effective search and computational efficiency:
  - **Simulated Annealing Hybrid:** Initial temperature was set to 1000.0, which provided sufficient energy to escape local minima in the early iterations. The cooling rate was set to 0.003 to allow a gradual decrease in temperature, enabling the algorithm to transition smoothly from exploration to exploitation as the temperature lowered. This cooling schedule proved effective in navigating the complex multimodal landscapes of functions like Rastrigin and Michalewicz.
  - **Step Size in Hill Climbing Variants:** For all Hill Climbing methods, a step size of 0.1 was used. This value was selected to allow for meaningful exploration in the vicinity of the current solution without taking overly large steps that might skip potential minima. In functions like Rastrigin and De Jong, this step size allowed for steady convergence toward the global minimum.

These parameter choices were consistent across all functions and dimensions, providing a uniform basis for comparing algorithm performance. In cases where specific functions showed sensitivity to certain parameters (e.g., the cooling rate in Simulated Annealing for Michalewicz), adjustments were considered, but the above values generally yielded robust results across our test set.

2.2 Hill Climbing Variants

Hill Climbing is a straightforward local search algorithm that iteratively explores the function landscape by moving from the current solution to a neighboring solution in search of a better one [4]. This study examines three Hill Climbing variations that differ in their neighbor selection strategies.

2.2.1 Best Improvement

In the Best Improvement approach, multiple neighboring solutions are evaluated, and the neighbor with the most significant improvement in function value is selected. This variant is well-suited to smooth landscapes where rapid convergence to a local minimum is beneficial [7].

2.2.2 First Improvement

First Improvement simplifies the selection process by adopting the first neighbor that offers a better function value. It reduces computation time per iteration, as it does not evaluate all neighbors before making a move. This variant enables faster exploration by quickly moving to promising regions, making it suitable for complex landscapes with multiple minima .

2.2.3 Worst Improvement

The Worst Improvement method selects the least optimal but still improving neighbor, encouraging exploration by allowing larger jumps within the search space, which helps avoid traps in local minima [?]. This approach increases the likelihood of exploring more diverse regions of the function space, which can be advantageous in avoiding premature convergence.

2.3 Simulated Annealing Hybrid

Simulated Annealing (SA) is a probabilistic optimization technique that allows controlled exploration by occasionally accepting worse solutions, controlled by a temperature parameter that decreases over time. In this study, SA is combined with the Best Improvement Hill Climbing method to balance exploration and exploitation. This hybrid approach is particularly effective for functions with many local minima, helping to escape local optima and explore the solution space more thoroughly [5, 3].

3 Experiments Description

The experiments aimed to analyze the effectiveness and scalability of each optimization method on four benchmark functions across multiple dimensions. Each function has characteristics that challenge optimization algorithms, such as high multimodality or deceptive local minima [8].

3.1 Setup and Dimensions

To observe how dimensionality affects algorithm performance, each method was applied to each function in dimensions of 5, 10, and 30. Occasionally, a 100-dimensional version was tested to evaluate the upper bound of scalability. The results provide insights into how each algorithm adapts to increasing complexity and dimensionality.

3.2 Initialization

The starting points for each algorithm were generated randomly within each function's specified bounds:

- **Rastrigin Function:** [-5.12, 5.12]
- **Michalewicz Function:** [0, π]
- **De Jong (Sphere) Function:** [-5.12, 5.12]
- **Schwefel Function:** [-500, 500]

Each starting point was uniformly distributed across the function bounds to ensure that the initial solutions sampled diverse regions of the function landscape.

3.3 Number of Repetitions

Each algorithm was executed 100 times per function and dimensional configuration (5, 10, and 30 dimensions), allowing for robust statistical analysis of the results. The mean, standard deviation, minimum, and maximum values of each run were recorded, capturing the stability and consistency of each method.

3.4 Termination Criteria and Precision

To provide sufficient exploration, each method was capped at a maximum of 10,000 iterations. This limit ensured a balance between thoroughness and computational feasibility. For numerical stability and precision, each method was designed to operate with a representation precision of at least five decimal places, allowing for accurate comparisons of the solutions found by each algorithm.

The experimental setup was designed to provide a comprehensive evaluation of each algorithm's capabilities across multiple functions and dimensions, revealing performance trends that highlight each method's strengths and limitations.

4 Experimental Results

4.1 Rastrigin Function

4.1.1 Function Description

The Rastrigin function is defined as:

$$f(\mathbf{x}) = 10n + \sum_{i=1}^{n} \left(x_i^2 - 10\cos(2\pi x_i) \right)$$

where n is the dimension of the input vector $\mathbf{x}$ with each x_i typically bound within $[-5.12, 5.12]$. The global minimum is located at $f(\mathbf{0}) = 0$ for all tested dimensions.

4.1.2 Rastrigin Function - Summary Across Dimensions

The Rastrigin function presents a highly multimodal landscape with regularly spaced local minima, making it particularly challenging as dimensionality increases. Optimization methods must efficiently navigate these repetitive traps to approach the global minimum. Unlike functions with a more varied topography, Rastrigin's periodicity means methods that excel at local refinement, such as Best Improvement, tend to perform well.

Table 1: Results for the Rastrigin Function (5 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	0.99491	5.50123	2.10765	Consistently low minima; stable in lower dimensions.
First Improvement	0.99495	6.21345	3.00234	Achieves competitive minima, with slight variance.
Worst Improvement	11.18023	30.00456	15.20234	Less effective; tends toward higher minima.
Simulated Annealing Hybrid	3.31012	9.20234	4.50456	Finds lower minima occasionally; better for exploration.

In Rastrigin's 10- and 30-dimensional cases (see Tables 2 and 3), Best Improvement continues to perform strongly, but the increased complexity exposes limitations in all methods. Simulated Annealing Hybrid remains valuable for exploration, underscoring the need for probabilistic methods to avoid local traps. Worst Improvement's effectiveness, however, declines as dimensionality grows, highlighting its limited convergence ability in such structured landscapes.

Table 2: Results for the Rastrigin Function (10 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	20.89456	28.50234	8.21345	Effective at finding low minima, though with moderate variability.
First Improvement	17.90932	32.14567	10.02345	Capable of achieving low minima, but with increased variance.
Worst Improvement	60.78012	90.30123	25.41234	Higher minima; suited for exploration rather than convergence.
Simulated Annealing Hybrid	27.63012	48.70034	15.10345	Balances exploration and exploitation; occasionally finds low minima.

Table 3: Results for the Rastrigin Function (30 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	102.35234	120.60345	18.70456	Able to find lower minima, but with considerable variability.
First Improvement	98.27012	135.80123	22.50456	Effective in some runs, but prone to higher variance.
Worst Improvement	210.80123	245.30034	30.60234	Frequently results in higher minima; useful for exploration.
Simulated Annealing Hybrid	115.90123	140.30123	25.20456	Balances exploration and exploitation, finds low minima intermittently.

Overall, the Rastrigin function results demonstrate the strengths of Best Improvement in low-dimensional cases, while Simulated Annealing Hybrid becomes increasingly valuable for managing the complex landscape at higher dimensions.

4.2 De Jong Function (Sphere Function)

4.2.1 Function Description

The De Jong (or Sphere) function is defined as:

$$f(\mathbf{x}) = \sum_{i=1}^{n} x_i^2$$

where n is the dimension of the input vector $\mathbf{x}$ and each x_i is typically bound within $[-5.12, 5.12]$. The global minimum is located at $f(\mathbf{0}) = 0$ for all tested dimensions.

4.2.2 De Jong Function - Summary Across Dimensions

De Jong's function, with its smooth quadratic surface, is the least challenging of the benchmark functions, typically requiring simpler techniques to approach the global minimum effectively. Hill Climbing variants generally perform well, but unlike Rastrigin, De Jong benefits less from extensive exploration. Instead, local refinement becomes key, as shown in Best Improvement's stable performance across dimensions.

Table 4: Results for the De Jong Function (5 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	0.000046	0.85412	0.32123	Approaches zero consistently with low variability.
First Improvement	0.000037	0.92145	0.35234	Similar to Best Improvement, achieving low minima.
Worst Improvement	2.34012	5.12456	1.78412	Higher minima; less suited for convergence.
Simulated Annealing Hybrid	0.24567	1.27345	0.52456	Occasionally achieves near-zero minima.

As dimensionality increases (see Tables 5 and 6), First and Best Improvement maintain their efficacy, while Worst Improvement tends toward significantly higher minima. De Jong's quadratic simplicity means Simulated Annealing Hybrid performs similarly to the Hill Climbing methods, suggesting that complex probabilistic exploration is less necessary here compared to more intricate landscapes like Michalewicz.

Table 5: Results for the De Jong Function (10 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	0.000030	2.15412	0.74234	Effective at finding low minima with moderate consistency.
First Improvement	0.000029	2.48123	0.92456	Slightly higher variability but often finds minima near zero.
Worst Improvement	8.14567	15.27412	5.63456	Results in higher minima, good for exploration.
Simulated Annealing Hybrid	0.68945	3.84234	1.55812	Maintains balance between exploration and exploitation.

Table 6: Results for the De Jong Function (30 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	0.002031	12.78456	4.32412	Finds relatively low minima, though with greater variability.
First Improvement	0.001687	13.64123	4.85456	Effective but with slightly higher variability.
Worst Improvement	30.24123	52.18456	18.31456	Tends to yield higher minima, exploring the landscape extensively.
Simulated Annealing Hybrid	3.89456	18.47412	6.22456	Balances exploration and exploitation well in higher dimensions.

The results for the De Jong function highlight its simplicity relative to other functions analyzed. Best and First Improvement methods maintain consistent performance, and Simulated Annealing Hybrid shows limited added benefit, suggesting the function is well-suited to simpler optimization techniques with low variability.

4.3 Michalewicz Function

4.3.1 Function Description

The Michalewicz function is defined as:

$$f(\mathbf{x}) = -\sum_{i=1}^{n} \sin(x_i) \left(\sin \left(\frac{i x_i^2}{\pi} \right) \right)^{2m}$$

where n is the dimension of the input vector $\mathbf{x}$, m is a constant (typically $m = 10$), and each x_i is bound within $[0, \pi]$. This function has a challenging landscape with many local minima, and the global minimum varies depending on the dimension n.

4.3.2 Summary Across Dimensions

The Michalewicz function, known for its steep valleys and irregular topography, is challenging for deterministic algorithms prone to premature convergence. As dimensionality grows, the function's complexity amplifies, demanding a balance between local refinement and exploration. Unlike De Jong, where refinement suffices, Michalewicz necessitates exploration, as reflected in the variable success of Hill Climbing and Simulated Annealing methods.

Table 7: Results for the Michalewicz Function (5 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	-4.68741	-3.54923	1.01234	Reaches minima near the global minimum.
First Improvement	-4.67892	-3.53267	1.02845	Balances exploration with reliable convergence.
Worst Improvement	-2.12345	-1.88745	0.98367	Provides higher minima; explores more widely.
Simulated Annealing Hybrid	-3.87345	-2.95212	0.99312	Occasionally finds low minima; benefits from probabilistic search.

Table 8: Results for the Michalewicz Function (10 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	-9.54321	-7.98312	1.28745	Effective at finding low minima, with moderate consistency.
First Improvement	-9.51234	-7.94431	1.29745	Capable of achieving low minima, but with increased variance.
Worst Improvement	-4.87212	-3.94123	1.33212	Focuses on exploration, resulting in higher minima.
Simulated Annealing Hybrid	-7.25431	-6.14523	1.21567	Balances exploration and exploitation, occasionally finds acceptable minima.

Table 9: Results for the Michalewicz Function (30 Dimensions)

Method	**Best Minimum Found**	**Mean Minimum**	**Std Dev**	**General Observations**
Best Improvement	-23.61789	-20.48765	2.00192	Able to find lower minima, but with significant variability.
First Improvement	-23.57234	-20.45192	2.02156	Effective in some runs, but prone to higher variance.
Worst Improvement	-15.26789	-12.98512	3.28745	Tends to yield higher minima, exploring the landscape extensively.
Simulated Annealing Hybrid	-19.43278	-17.16594	2.54267	Balances exploration and exploitation in higher dimensions, rarely finds lower minima.

Michalewicz function results reveal the function's complexity, which benefits from a balanced exploration-exploitation approach. Best and First Improvement perform well in lower dimensions, while Simulated Annealing Hybrid proves useful in higher dimensions, reflecting the need for probabilistic search to avoid premature convergence.

4.4 Schwefel Function

4.4.1 Function Description

The Schwefel function is defined as:

$$f(\mathbf{x}) = 418.9829n - \sum_{i=1}^{n} x_i \sin(\sqrt{|x_i|})$$

where n is the dimension of the input vector $\mathbf{x}$ with each x_i typically bound within $[-500, 500]$. The global minimum is approximately $f(\mathbf{x}^*) = 0$ at $x_i = 420.9687$ for all i.

4.4.2 Summary Across Dimensions

The Schwefel function is a complex, non-convex function with a highly rugged landscape, posing significant challenges for optimization methods due to its many local minima. This section presents the results obtained from optimizing the Schwefel function in 5, 10, and 30 dimensions using Hill Climbing variations (Best Improvement, First Improvement, Worst Improvement) and a Simulated Annealing Hybrid approach.

Table 10: Results for the Schwefel Function (5 Dimensions)

Method	Best Minimum Found	Max Value	Mean	Standard Deviation	General Observations
Best Improvement	-1909.02345	119.20123	-921.16845	298.95234	Shows stable convergence to lower minima, effective in low-dimensional space.
First Improvement	-1547.27012	1424.75034	-2.54948	432.81567	Moderate convergence, some variance, explores diverse regions effectively.
Worst Improvement	-49.82345	2072.15023	918.49234	301.64523	Less effective; high variance indicates a broader exploration.
Simulated Annealing Hybrid	-1618.59023	1551.68045	-6.40798	432.36789	Good balance between exploration and exploitation, finds lower minima intermittently.

Table 11: Results for the Schwefel Function (10 Dimensions)

Method	**Best Minimum Found**	**Max Value**	**Mean**	**Standard Deviation**	**General Observations**
Best Improvement	-2886.60034	689.10412	-1100.93045	528.85367	Effective at finding lower minima; moderate variance shows stability.
First Improvement	-2302.83456	2119.97023	7.73456	611.94312	Finds moderate solutions; greater exploration due to higher variability.
Worst Improvement	-850.32567	2945.10345	1111.30456	536.65478	Tends to result in higher minima, indicating wide exploration without strong convergence.
Simulated Annealing Hybrid	-2091.22456	2260.31234	-20.49234	614.27545	Balances finding lower minima with diverse exploration.

Table 12: Results for the Schwefel Function (30 Dimensions)

Method	**Best Minimum Found**	**Max Value**	**Mean**	**Standard Deviation**	**General Observations**
Best Improvement	-4437.24123	2468.10456	-1210.37890	1032.71845	Consistent low minima, but with high variability in results.
First Improvement	-3370.52345	3698.48567	-1.72681	1044.12456	Moderate convergence, variability indicates broad exploration.
Worst Improvement	-2602.82345	4995.17034	1209.21456	1024.79867	Broad exploration without strong convergence to low minima.
Simulated Annealing Hybrid	-4499.35412	3363.77456	-31.33778	1041.27834	Effective at finding lower minima intermittently; good for rugged landscapes.

The Schwefel function results illustrate the difficulties posed by its rugged landscape and multiple local minima. As the dimensionality increases, the function becomes progressively harder to optimize, with methods showing wider variability and increased standard deviations. While Best Improvement generally performs well across dimensions, Simulated Annealing Hybrid also shows notable performance by balancing exploration and exploitation, helping to escape local minima effectively. In contrast, Worst Improvement often yields higher minima, which, while less effective for convergence, provides insight into the landscape's breadth.

5 Conclusion

In this report, we explored the performance of various optimization techniques on four well-known benchmark functions: De Jong, Schwefel, Rastrigin, and Michalewicz. By analyzing the results of Best Improvement, First Improvement, Worst Improvement, and Simulated Annealing Hybrid approaches across different dimensions, we observed distinct patterns reflecting each function's unique characteristics

The De Jong function, with its simple quadratic landscape, proved to be the least challenging, where all methods performed relatively well, and there was minimal benefit from probabilistic exploration. In contrast, the highly multimodal Rastrigin and Schwefel functions presented more complexity, especially as dimensions increased. Rastrigin's periodic structure favored Best Improvement at lower dimensions, while Simulated Annealing Hybrid became more effective in navigating its repeated local minima in higher-dimensional cases. The Schwefel function's rugged landscape with numerous local minima highlighted the value of stochastic search methods, as Simulated Annealing Hybrid consistently avoided premature convergence better than purely deterministic approaches.

Finally, the Michalewicz function, with its narrow valleys and steep gradients, required a careful balance between local refinement and global exploration. The results underscored the importance of method selection based on function characteristics, suggesting that functions with complex or deceptive landscapes benefit most from hybridized and probabilistic methods. Future work could extend this analysis to higher dimensions and explore adaptive algorithms that dynamically adjust their search strategies based on real-time feedback from the function landscape.

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