

Physical Problem Solving: Assignment 1

Darix, SAMANI SIEWE

September 28, 2024

Exercise 1

I) This is not absolutely true. let,s consider two cases :

- without air resistance or force of fraction: in this case, heaviest object arrives at the same time as the lightest object
- with air resistance or force of friction: when an object is heavier it falls faster in the gravitational field because the gravitational force depends on the mass of the object and the heavier the object the more the gravitational force would tend to bring the object back to the ground faster.

II) A parachutist launches from the top of an airplane. What is the air resistance force acting on the parachutist knowing that the parachutist has a mass of 70kg and that he drops at a constant speed ?

When a parachutist takes off from an airplane, the forces acting on the parachutist are:

- force of gravitational of parachutist : $\vec{F} = m\vec{g}$,
- the force of air resistance: \vec{f}

We known that the parachutist drops at a constant speed so the acceleration of parachutist is zero:

According to the second law of Newton we have :

$$\begin{aligned}\sum \vec{F}_{\text{ext}} &= \vec{0} \\ \implies \vec{f} + \vec{F} &= \vec{0} \\ \implies \vec{f} + m\vec{g} &= \vec{0} \\ \implies \vec{f} &= -m\vec{g} \\ \implies \begin{pmatrix} 0 \\ f_y \end{pmatrix} &= -m \begin{pmatrix} 0 \\ g \end{pmatrix} \\ \implies \begin{cases} f_x = g_x = 0 \\ f_y = -m(g) = -75 * 9.8 \end{cases} \end{aligned}$$

so, $|f| = 735 \text{ N}$

III) $g = -10\lambda \text{ m/s}^2$ where $\lambda > 0$, $V_0 = 80 \text{ m/s}$

- 1 Find the time taken for the rock to return to the ground again for $\lambda = 4$.
there is only one force acting on the rock in this case: the gravitational force \vec{W}
According to the second law of Newton we have:

$$\begin{aligned}\sum \vec{F}_{\text{ext}} &= m\vec{a} \\ \vec{W} &= m\vec{a} \\ m\vec{g} &= m\vec{a} \\ \vec{g} &= \vec{a} \\ \begin{pmatrix} 0 \\ -g \end{pmatrix} &= \begin{pmatrix} a_x \\ a_y \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \Rightarrow \begin{cases} a_x = 0 \\ a_y = -g \end{cases} &\Rightarrow \begin{cases} v_x = v_{0x} = 0 \\ v_y = -gt + v_{0y} = -gt + v_0 \end{cases} \\ \Rightarrow \begin{cases} x = 0 \\ y = -\frac{1}{2}gt^2 + v_0t \end{cases} \end{aligned}$$

At $t = t_{rg}$ $y = 0 \Rightarrow 0 = -\frac{1}{2}gt^2 + v_0t$

$$\begin{aligned} 0 &= -\frac{1}{2}gt^2 + v_0t \\ 0 &= t(-\frac{1}{2}gt + v_0) \end{aligned}$$

$$\Rightarrow t = 0 \quad \text{or} \quad t = \frac{2v_0}{g}$$

we know at $t = 0$ it's a time when rock is launched, so the time taken for the rock to return to the ground again is $t = \frac{2v_0}{g}$

Now, let's calculate for $\lambda = 4$

$$\begin{aligned} t &= \frac{2v_0}{g} \\ &= \frac{2v_0}{10\lambda} \\ &= \frac{2 * 80}{10 * 4} \\ &= 4 \end{aligned}$$

so, $t = 4$ s

2 Find the maximum height reached by the rock.

To solve this question firstly we need to compute the time taken to achieve the maximum height, let's denote this time by t_{max}

we know at t equal to the time taken to arrive to the maximum height $v_y = 0$ and express below v_y

$$\begin{aligned} 0 &= -gt_{max} + v_0 \\ \Rightarrow t_{max} &= \frac{v_0}{g} \end{aligned}$$

$$\begin{aligned} H_{max} = y_{max} &= -\frac{1}{2}g(\frac{v_0}{g})^2 + v_0 * \frac{v_0}{g} \\ &= -\frac{1}{2}\frac{v_0^2}{g} + \frac{v_0^2}{g} \\ &= \frac{1}{2}\frac{v_0^2}{g} \end{aligned}$$

so, the maximum height reached by the rock is $H_{max} = \frac{1}{2}\frac{v_0^2}{g}$

3 Find the velocity with which it hits the ground.

we have compute below the time taken to return to the ground $t = \frac{2v_0}{g}$, just replace this time in the express of velocity that we have express below

$$\begin{aligned}
v_y &= -gt + v_0 \\
&= -g \frac{2v_0}{g} + v_0 \\
&= -2V_0 + v_0 \\
&= -v_0
\end{aligned}$$

so, the velocity when rock return to the ground is $v = \begin{pmatrix} 0 \\ -v_0 \end{pmatrix}$ and $|v| = v_0 = 80$ m/s

Exercise 2

A basketball player, $h = 210$ cm tall, throws a ball at an angle $\theta = 30^\circ$ to the horizontal. The basket is $H = 304$ cm tall and the player is positioned at the distance $D = 402$ cm from the basket. At what initial speed v_0 should the player throw the ball so that it passes through the center of the basket ?

In order to solve this problem we need to express the equation of parabola of the motion, and after if the ball player's passes through the center of the basket $x = D$ and $y = H - h$

According to the second law of Newton, we have

$$\begin{aligned}
\sum \vec{F}_{ext} &= m\vec{a} \\
\vec{W} &= m\vec{a} \\
m\vec{g} &= m\vec{a} \\
\vec{g} &= \vec{a} \\
\begin{pmatrix} 0 \\ -g \end{pmatrix} &= \begin{pmatrix} a_x \\ a_y \end{pmatrix} \\
\implies \begin{cases} a_x = 0 \\ a_y = -g \end{cases} &\implies \begin{cases} v_x = v_{0x} = v_0 \cos(\theta) \\ v_y = -gt + v_{0y} = -gt + v_0 \sin(\theta) \end{cases} \\
\implies \begin{cases} x = v_0 \cos(\theta)t & (1) \\ y = -\frac{1}{2}gt^2 + v_0 \sin(\theta)t & (2) \end{cases}
\end{aligned}$$

in (1) we have $t = \frac{x}{v_0 \cos(\theta)}$ and when we replace this equation of time in (2) we have

$$\begin{aligned}
y &= -\frac{1}{2}g \left(\frac{x}{v_0 \cos(\theta)} \right)^2 + v_0 \sin(\theta) \left(\frac{x}{v_0 \cos(\theta)} \right) \\
y &= -\frac{1}{2}g \frac{x^2}{v_0^2 \cos^2(\theta)} + x \tan(\theta) \\
\implies y - x \tan(\theta) &= -\frac{1}{2}g \frac{x^2}{v_0^2 \cos^2(\theta)} \\
\implies \frac{2(x \tan(\theta) - y)}{g} &= \frac{x^2}{v_0^2 \cos^2(\theta)} \\
\implies \frac{g}{2(x \tan(\theta) - y)} &= \frac{v_0^2 \cos^2(\theta)}{x^2} \\
\implies v_0^2 &= \frac{x^2 g}{2 \cos^2(\theta)(x \tan(\theta) - y)}
\end{aligned}$$

$$\implies v_0 = \sqrt{\frac{x^2 g}{2 \cos^2(\theta)(x \tan(\theta) - y)}} \text{ because } v_0 > 0$$

Now, let's replace $x = D$ and $y = H - h$, we have :

$$\begin{aligned}
 v_0 &= \sqrt{\frac{D^2 g}{2 \cos^2(\theta)(D \tan(\theta) - (H - h))}} \\
 &= \sqrt{\frac{402^2 * 9.8}{2 \cos^2(30)(402 \tan(30) - (304 - 210))}} \\
 v_0 &= 87.43
 \end{aligned}$$

Exercise 3

A) A block slides without friction down a fixed, inclined plane. The angle of the incline is $\alpha = 60^\circ$ from horizontal.

a) Draw the block on the inclined plane and name all the forces acting on the block.

without air resistance or the force of friction we have the following force acting to the block:

- the force of gravity: $\vec{W} = m\vec{g}$
- the normal force: \vec{F}_N

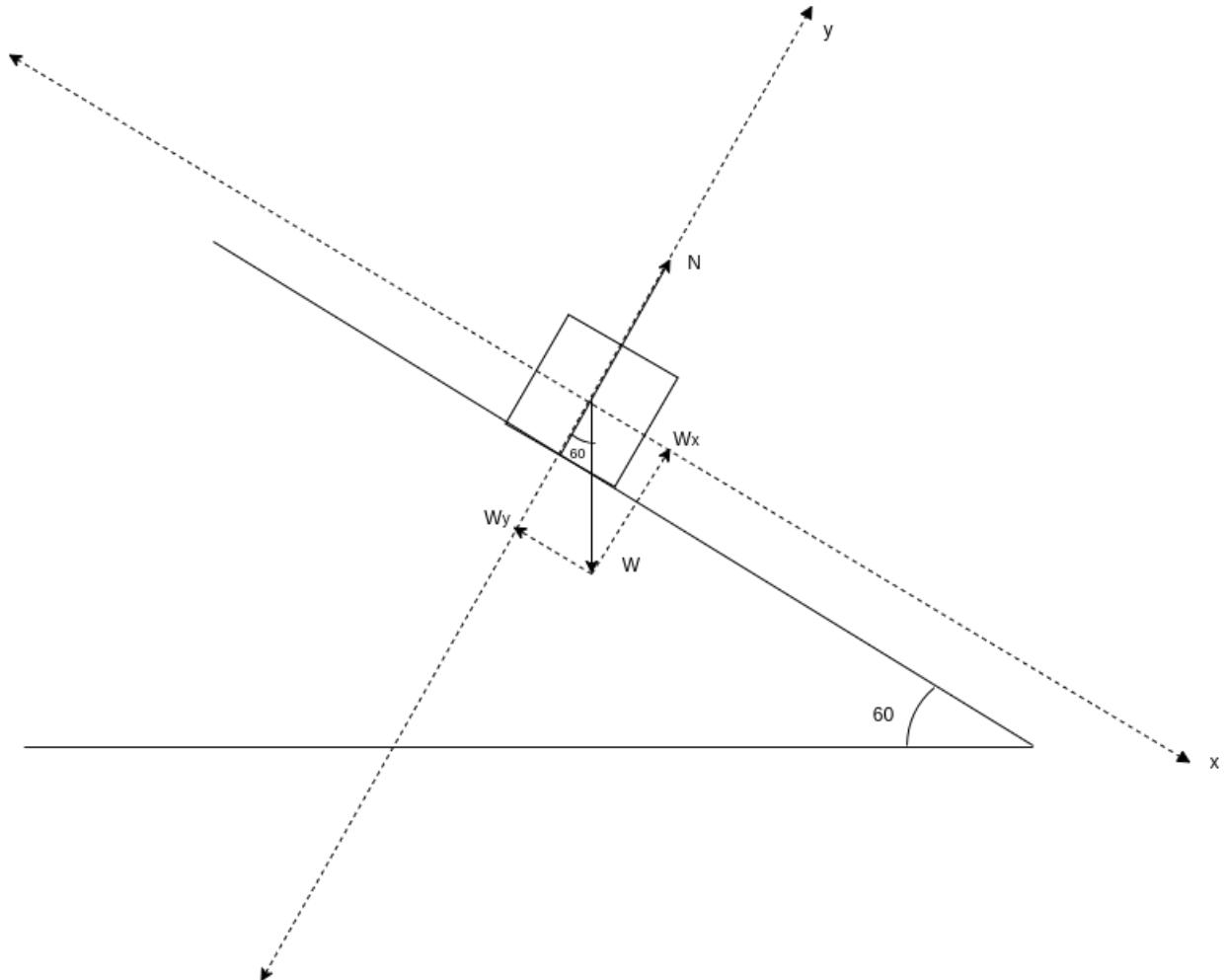


Figure 1: force acting to the Block

b) Using physical law to be stated, compute the acceleration of the block

let's state the second law of Newton:

The second law states that the acceleration of an object is dependent upon two variables - the net force acting upon the object and the mass of the object. The acceleration of an object

depends directly upon the net force acting upon the object, and inversely upon the mass of the object. As the force acting upon an object is increased, the acceleration of the object is increased. As the mass of an object is increased, the acceleration of the object is decreased.

$$\sum \vec{F}_{ext} = m\vec{a}$$

using this law we have :

$$\begin{aligned}\sum \vec{F}_{ext} &= m\vec{a} \\ \vec{W} + \vec{N} &= m\vec{a} \\ \begin{pmatrix} W \sin(\theta) \\ -W \cos(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ N \end{pmatrix} &= \begin{pmatrix} a_x \\ 0 \end{pmatrix}\end{aligned}$$

$$\implies a_x = mg \sin(\theta)$$

$$so, the acceleration is a = \begin{pmatrix} mg \sin(\theta) \\ 0 \end{pmatrix}$$

c Find velocity and position as a function of time

Using the expression of acceleration we compute below we have:

$$v = \begin{cases} v_x = mg \sin(\theta)t \\ v_y = 0 \end{cases} \quad \text{and}$$

$$O\vec{M} = \begin{cases} x = \frac{1}{2}mg \sin(\theta)t^2 \\ y = 0 \end{cases}$$

- B)** Do the same work as above by assuming now that the block is moving and that it is subject to sliding friction. Determine the acceleration of the block for the angle $\alpha = 60^\circ$ assuming the frictional force obeys $f = \mu F_N$ where $\mu = 0.3$ is the coefficient of kinetic friction.

when block have a fraction force the force acting on the block are :

- force of gravity: \vec{W}
- normal force: \vec{F}_N
- force of friction: \vec{f}

$$\begin{aligned}\sum \vec{F}_{ext} &= m\vec{a} \\ \vec{W} + \vec{F}_N + \vec{f} &= m\vec{a} \\ \begin{pmatrix} W \sin(\theta) \\ -W \cos(\theta) \end{pmatrix} + \begin{pmatrix} 0 \\ N \end{pmatrix} + \begin{pmatrix} -\mu F_N \\ 0 \end{pmatrix} &= m \begin{pmatrix} a_x \\ 0 \end{pmatrix}\end{aligned}$$

$$\implies a_x = mg \sin(\theta) - \mu F_N \quad \text{and} \quad F_N = mg \cos(\theta) = \frac{mg}{2}$$

$$\implies v = \begin{cases} v_x = \frac{1}{m} (mg \sin(\theta) - \mu F_N) t \\ v_y = 0 \end{cases}$$

$$\implies O\vec{M} = \begin{cases} x = \frac{1}{2m} (mg \sin(\theta) - \mu F_N) t^2 \\ y = 0 \end{cases}$$

$$v = \begin{cases} v_x = \frac{1}{m} (mg \sin(\theta) - \mu F_N) t = \frac{1}{m} (mg \sin(60) - 0.15mg) t = g (\sin(60) - 0.15) t = 0.71gt = 7.01t \\ v_y = 0 \end{cases} \quad \text{and}$$

$$O\vec{M} = \begin{cases} x = \frac{1}{2m} (mg \sin(\theta) - \mu F_N) t^2 = \frac{1}{2m} (mg \sin(60) - 0.15mg) t^2 = 0.35gt^2 = 3.5t^2 \\ y = 0 \end{cases}$$