Mathematical Problem Solving

Group 18

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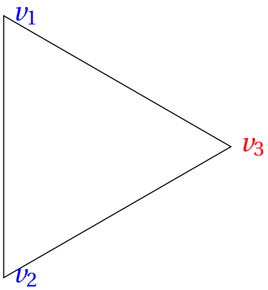
Outline

Problem Statement

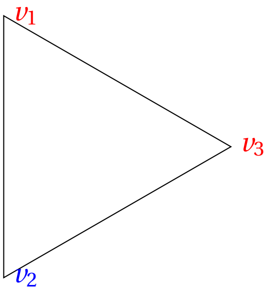
Problem

The Vertices of a regular 11-gons are colored blue or red. Prove that there are vertices of the same color, which form an isoscles triangle. What about a regular 12-gon ? For What regular n-gons is the assertion valid ?

Regular 3-gons



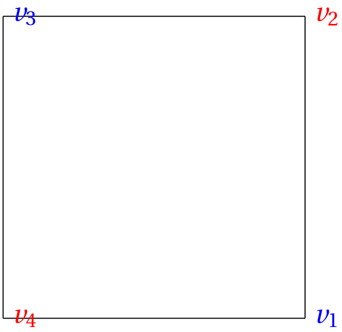
(a) 3-gons with one red and two blue



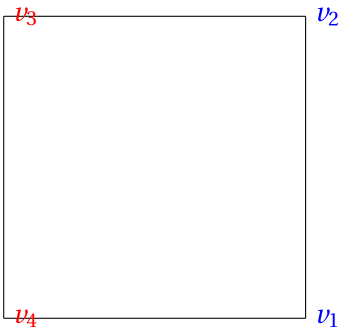
(b) 3-gons with 2 reds and one blue

Figure 1: 3-gons

Regular 4-gons



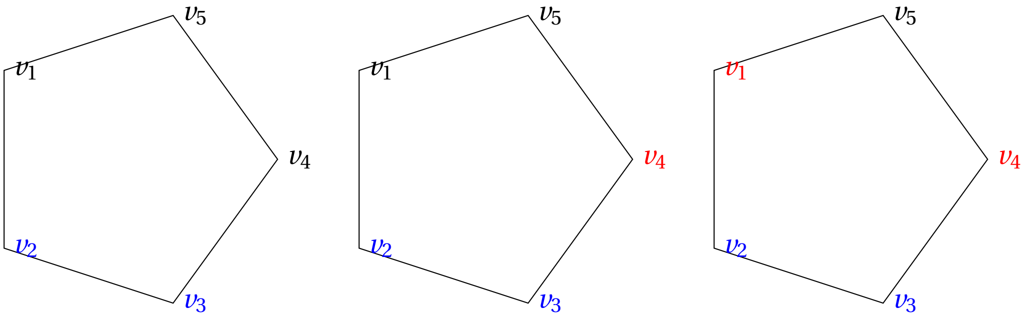
(a) 4-gons with two reds and two blues



(b) 4-gons with two reds and two blues

Figure 2: 4-gons

Regular 5-gons



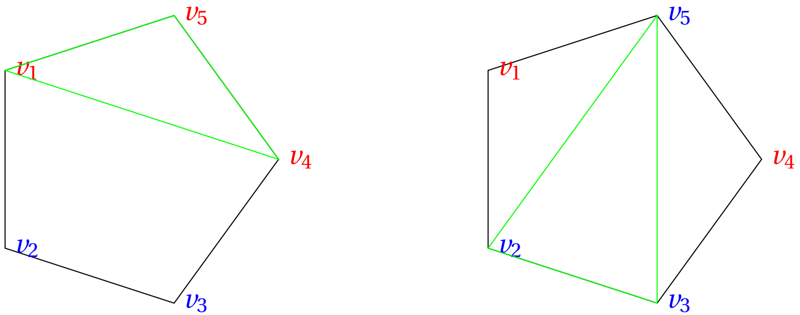
(a) 5-gons with 2 blues and zero red

(b) 5-gons with two blues and one red

(c) 5-gons with 2 reds and two blues

Figure 3: 5-gons

Regular 5-gons

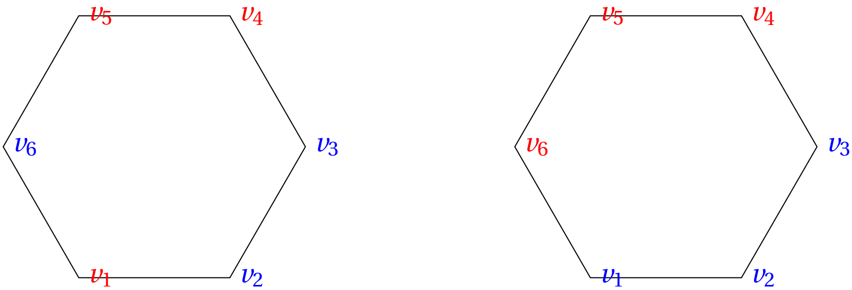


(a) 5-gons with 2 blues and three reds

(b) 5-gons with three blues and two reds

Figure 4: 5-gons

Regular 6-gons

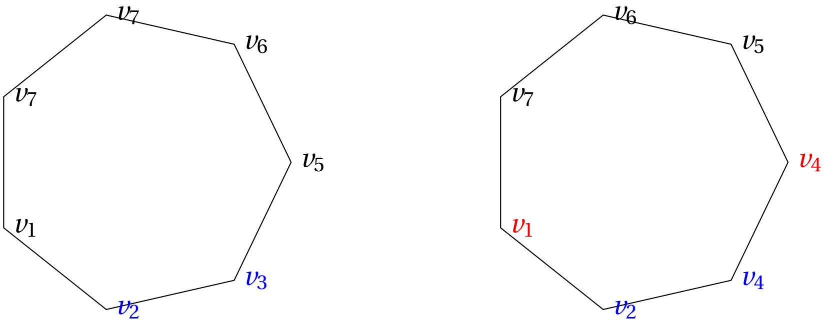


(a) incorrect case 6-gons with three reds and three blues

(b) correct case 6-gons with three reds and three blues that work

Figure 5: 6-gons

Regular 7-gons

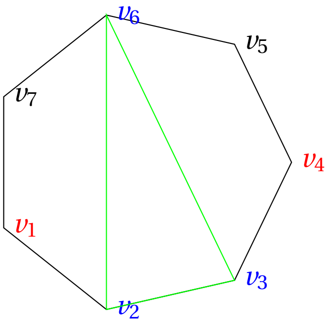


(a) 7-gons with 2 blues and zero red

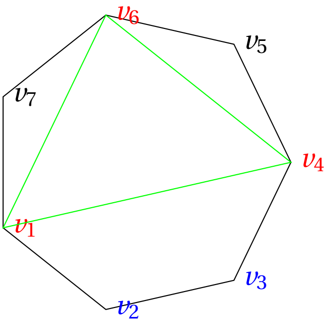
(b) 7-gons with 2 blues and two reds

Figure 6: 7-gons

Regular 7-gons



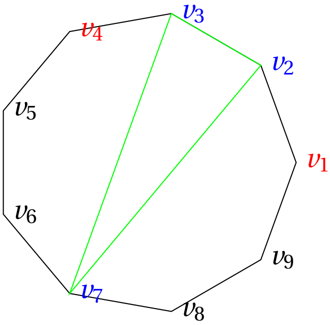
(a) 7-gons with three blues and two reds



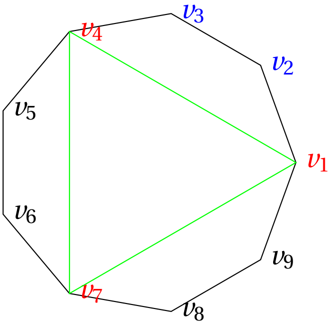
(b) 7-gons with three reds and two blues

Figure 7: 7-gons

Regular 9-gons



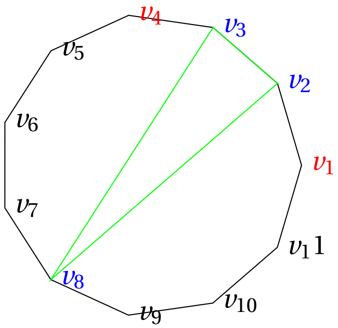
(a) 9-gons with three blues and two reds



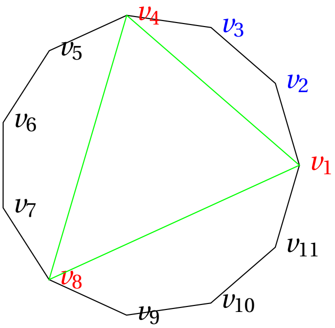
(b) 9-gons with two blues and three reds

Figure 8: 9-gons

Regular 11-gons



(a) 11-gons with three blues and two reds



(b) 11-gons with three reds and two blues

Figure 9: 11-gons

Generalization

- odd number : $\forall n \geq 5$, we can always get an isosceles triangles formed by tree vertices with same colors.
- Even number : $\forall n \geq 8$, we can always get an isosceles triangles formed by tree vertices with same colors.

Proof

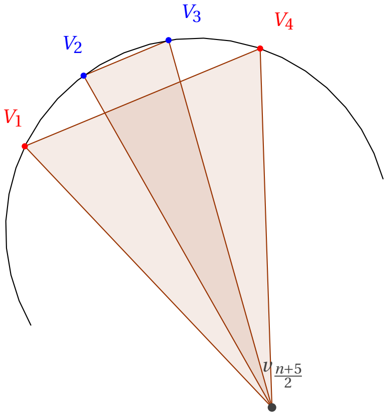


Figure 10: n-gons for n odd numbers

Proof for odd numbers greater or equal than 5

Let's denote by $D(i, j)$ the minimum number of segment(segment between two consecutive vertices) between vertices i and j,

$$D(i, j) = min(|i - j|, n - |i - j|)$$

Let's denote the vertices of regular n-gon by $v_1, v_2, \ldots, v_n, i = 1, \ldots, n$ and write $v_i = B$ or $v_i = R$, if vertices v_i is colored Blue, respectively Red. since n is odd, there are two neighboring vertices colored the same color; let the vertices be v_2 and v_3 and the color be B.

- now, if $v_1 = B$, we have a isosceles triangle $v_1 v_2 {}_{v3}$
- if $v_4 = B$, we have a isosceles triangle $v_2 v_3 {}_{v4}$

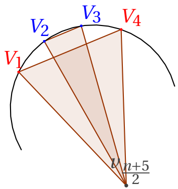


Figure 11: n-gons for n odd numbers

Proof for odd numbers greater or equal than (suite)

Thus, let's assume that $v_1 = v_4 = R$, But Then,

- if $v_{\frac{n+5}{2}} = B$, we have a isosceles triangle $v_2 v_{\frac{n+5}{2}} v_3$ because

$$D(v_2, v_{\frac{n+5}{2}}) = min\left(\frac{n+5}{2} - 2, n - \left(\frac{n+5}{2} - 2\right)\right) = min\left(\frac{n+1}{2}, \frac{n-1}{2}\right) = \frac{n-1}{2}$$

$$D(v_3, v_{\frac{n+5}{2}}) = min\left(\frac{n+5}{2} - 3, n - \left(\frac{n+5}{2} - 3\right)\right) = min\left(\frac{n-1}{2}, \frac{n+1}{2}\right) = \frac{n-1}{2}$$

so, $D(v_2, v_{\frac{n+5}{2}}) = D(v_3, v_{\frac{n+5}{2}}) = \frac{n-1}{2}$

Proof for odd numbers greater or equal than (suite and end)

- if $v_{\frac{n+5}{2}} = R$, we have a isosceles triangle $v_1 v_{\frac{n+5}{2}} v_4$ because

$$D(v_1, v_{\frac{n+5}{2}}) = min\left(\frac{n+5}{2} - 1, n - \left(\frac{n+5}{2} - 1\right)\right) = min\left(\frac{n+3}{2}, \frac{n-3}{2}\right) = \frac{n-3}{2}$$

$$D(v_4, v_{\frac{n+5}{2}}) = min\left(\frac{n+5}{2} - 4, n - \left(\frac{n+5}{2} - 4\right)\right) = min\left(\frac{n-3}{2}, \frac{n+3}{2}\right) = \frac{n-3}{2}$$

so, $D(v_1, v_{\frac{n+5}{2}}) = D(v_4, v_{\frac{n+5}{2}}) = \frac{n-3}{2}$

Conclusion

During this presentation we saw that to prove this problem it is essential to consider two cases: the odd case and the even case and that for each case it works from a certain rank

Thank you for listening !