Physical Problem Solving: Assignment 2

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1. Two blocks, A and B, with masses $m_A = 10kg$ and $m_B = 3kg$ respectively, are connected by a massless string that runs over a massless, frictionless pulley. The mass A is placed on an incline that makes an angle of $\alpha = 40°$ with the horizontal. The coefficients of friction between A and the incline are $\mu_s = 0.56$ for the static friction and $\mu_k = 0.25$ for kinetic.

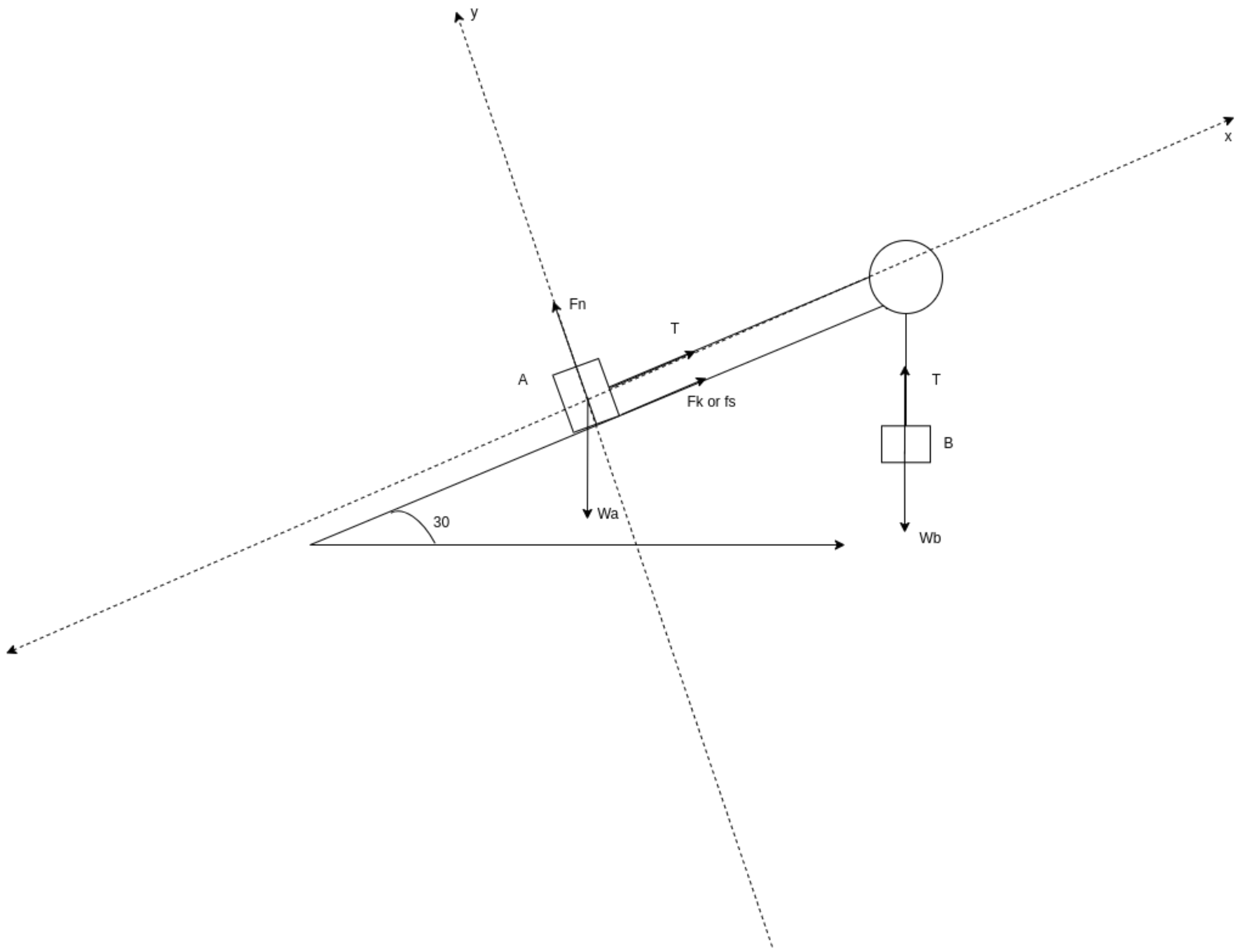

Figure 1: The forces acting of the system

(a) Find the components of the weight of A along and perpendicular to the incline.

$$W = \begin{cases} W_x = W_A \sin\alpha = m_A g \sin\alpha = 10 * 9.8 * \sin 40 = 62.99 = 63 \\ W_y = W_A \cos\alpha = m_A g \sin\alpha = 10 * 9.8 \cos 40 = 75.07 \end{cases}$$

(b) What is the maximal static friction F_s^{max} that can act between A and incline?

the static friction occur when the object is on the rest: let's considered the object A

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{F_N} + \vec{T} + \vec{F_s} = \vec{0}$$

$$\begin{pmatrix} m_A g \sin\alpha \\ -m_A g \sin\alpha \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} + \begin{pmatrix} -T \\ 0 \end{pmatrix} + \begin{pmatrix} -F_s \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

based on this system $F_N = m_a g \sin\alpha = 10 * 9.8 * \sin 40 = 63N$ and we know the $F_s^{max} = \mu_s F_N = 0.56 * 63 = 35.28N$

so, $F_s^{max} = \mu_s F_N = 0.56 * 63 = 35.28N$

(c) Find acceleration of the system if the friction were absent.
if the friction is absent, Forces Acting on the Blocks are :

i. the force acting on A:
   - Weight of A: $\vec{W_A}$
   - Normal force: $\vec{F_N}$
   - Tension force: $\vec{T}$

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{F_N} + \vec{T} + = m_A \vec{a}$$

$$\begin{pmatrix} m_A g \sin\alpha \\ -m_A g \cos\alpha \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} + \begin{pmatrix} -T \\ 0 \end{pmatrix} = m_A \begin{pmatrix} a \\ 0 \end{pmatrix}$$

and we have $m_a a = m_A g \sin\alpha - T$ (1)

ii. the force acting on B:
   - Weight of B: $\vec{W_B}$
   - Tension force: $\vec{T}$

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{T} + = m_A \vec{a}$$

$$\begin{pmatrix} 0 \\ -m_B g \end{pmatrix} + \begin{pmatrix} 0 \\ T \end{pmatrix} = m_B \begin{pmatrix} 0 \\ a \end{pmatrix}$$

and we have $T = m_B a + m_B g$ (2)

replace (2) in (1) we have :

$$\begin{aligned} m_a a &= m_A g \sin\alpha - (m_B a + m_B g) \\ &= m_A g \sin\alpha - m_B a - m_B g \\ \implies (m_A + m_b) a &= m_A g \sin\alpha - m_B g \\ \implies a &= \frac{g}{m_A + m_B}(m_A \sin\alpha - m_B) \\ a &= \frac{9.8}{10+3}(10 \sin 40 - 3) \\ &= 2.58 \end{aligned}$$

so, $|a| = 2.58 \quad m/s^2$

(d) With the friction active, if the system were starting from rest, would this system remain at rest? If not, would A start moving up or down the incline?

to answer this question we must proceed as in the previous question by taking into account the friction forces and consider a direction for the displacement if we find a null value for the acceleration it would mean that the system remains at rest, if we find a negative value it would mean that the system moves in the opposite direction chosen and finally a positive value would mean that the system moves in the same direction as the chosen system.

Now let's considered that the A moving down by using the figure 10 we have

if the friction is active, Forces Acting on the Blocks are :

i. the force acting on A:
   - Weight of A: $\vec{W_A}$
   - Normal force: $\vec{F_N}$
   - Tension force: $\vec{T}$
   - kinetic friction force: $\vec{F_k}$

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{F_N} + \vec{T} + \vec{F_k} = m_A\vec{a}$$
$$\begin{pmatrix} m_A g \sin\alpha \\ -m_A g \cos\alpha \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} + \begin{pmatrix} -T \\ 0 \end{pmatrix} \begin{pmatrix} -F_k \\ 0 \end{pmatrix} = m_A \begin{pmatrix} a \\ 0 \end{pmatrix}$$

and we have $m_A a = m_A g \sin\alpha - T - F_k = m_a a = m_A g \sin\alpha - T - \mu_k F_N$ and $F_N = m_A g \cos\alpha$

and $m_a a = m_A g \sin\alpha - T - \mu_k m_A g \cos\alpha$ (1)

ii. the force acting on B:
   - Weight of B: $\vec{W_B}$
   - Tension force: $\vec{T}$

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{T} + = m_A\vec{a}$$
$$\begin{pmatrix} 0 \\ -m_B g \end{pmatrix} + \begin{pmatrix} 0 \\ T \end{pmatrix} = m_B \begin{pmatrix} 0 \\ a \end{pmatrix}$$

and we have $T = m_B a + m_B g$ (2)

replace (2) in (1) we have :

$$\begin{aligned} m_A a &= m_A g \sin\alpha - (m_B a + m_B g) - \mu_k m_A g \cos\alpha \\ &= m_A g \sin\alpha - m_B a - m_B g - \mu_k m_A g \cos\alpha \\ \implies (m_A + m_b)a &= m_A g \sin\alpha - m_B g - \mu_k m_A g \cos\alpha \\ \implies a &= \frac{g}{m_A + m_B}(m_A \sin\alpha - m_B - \mu_k m_A \cos\alpha) \\ a &= \frac{9.8}{10+3}(10 \sin 40 - 3 - 0.25 * 10 * \cos 40) \\ &= 1.14 \end{aligned}$$

so, $|a| = 1.14 \quad m/s^2$

Because the acceleration is positive this denote that we have selected the good direction, so the object A moving down

(e) Find acceleration of the system, if it were started with A moving up the incline and the friction were active.

when the object A moving up, according to the figure 10 we have

if the friction is active, Forces Acting on the Blocks are :

i. the force acting on A:
   - Weight of A: $\vec{W_A}$
   - Normal force: $\vec{F_N}$
   - Tension force: $\vec{T}$
   - kinetic friction force: $\vec{F_k}$

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{F_N} + \vec{T} + \vec{F_k} = m_A \vec{a}$$
$$\begin{pmatrix} -m_A g \sin\alpha \\ -m_A g \cos\alpha \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} + \begin{pmatrix} T \\ 0 \end{pmatrix} \begin{pmatrix} -F_k \\ 0 \end{pmatrix} = m_A \begin{pmatrix} a \\ 0 \end{pmatrix}$$

   and we have $m_a a = -m_A g \sin\alpha + T - F_k = m_a a = -m_A g \sin\alpha + T - \mu_k F_N$ and $F_N = m_A g \cos\alpha$

   and $m_a a = -m_A g \sin\alpha + T - \mu_k m_A g \cos\alpha$ (1)

ii. the force acting on B:
   - Weight of B: $\vec{W_B}$
   - Tension force: $\vec{T}$

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{T} + = m_A \vec{a}$$
$$\begin{pmatrix} 0 \\ m_B g \end{pmatrix} + \begin{pmatrix} 0 \\ -T \end{pmatrix} = m_B \begin{pmatrix} 0 \\ a \end{pmatrix}$$

   and we have $T = -m_B a + m_B g$ (2)

replace (2) in (1) we have :

$$\begin{aligned} m_a a &= -m_A g \sin\alpha + (-m_B a + m_B g) - \mu_k m_A g \cos\alpha \\ &= -m_A g \sin\alpha - m_B a + m_B g - \mu_k m_A g \cos\alpha \\ \Longrightarrow \ (m_A + m_B) a &= -m_A g \sin\alpha + m_B g - \mu_k m_A g \cos\alpha \\ \Longrightarrow \ a &= \frac{g}{m_A + m_B}(-m_A \sin\alpha + m_B - \mu_k m_A \cos\alpha) \\ a &= \frac{9.8}{10+3}(-10 * \sin 40 + 3 - 0.25 * 10 * \cos 40) \\ &= -4.02 \end{aligned}$$

so, $|a| = 4.02 \quad m/s^2$ and the value of a is negative that show the object A can't moving up.

(f) Finally, find acceleration of the system, if it were started with A moving down the incline and the friction were active.

when the object A moving up, according to the figure 10 we have:

if the friction is active, Forces Acting on the Blocks are :

i. the force acting on A:
   - Weight of A: $\vec{W_A}$
   - Normal force: $\vec{F_N}$

- Tension force: $\vec{T}$
- kinetic friction force: $\vec{F_k}$

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{F_N} + \vec{T} + \vec{F_k} = m_A\vec{a}$$

$$\begin{pmatrix} m_A g\sin\alpha \\ -m_A g\cos\alpha \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} + \begin{pmatrix} -T \\ 0 \end{pmatrix} \begin{pmatrix} -F_k \\ 0 \end{pmatrix} = m_A \begin{pmatrix} a \\ 0 \end{pmatrix}$$

and we have $m_a a = m_A g\sin\alpha - T - F_k = m_a a = m_A g\sin\alpha - T - \mu_k F_N$ and $F_N = m_A g\cos\alpha$

and $m_a a = m_A g\sin\alpha - T - \mu_k m_A g\cos\alpha$ (1)

ii. the force acting on B:
- Weight of B: $\vec{W_B}$
- Tension force: $\vec{T}$

$$\sum \vec{F_{ext}} = \vec{0}$$

$$\vec{W_A} + \vec{T} + = m_A\vec{a}$$

$$\begin{pmatrix} 0 \\ -m_B g \end{pmatrix} + \begin{pmatrix} 0 \\ T \end{pmatrix} = m_B \begin{pmatrix} 0 \\ a \end{pmatrix}$$

and we have $T = m_B a + m_B g$ (2)

replace (2) in (1) we have :

$$\begin{aligned} m_a a &= m_A g\sin\alpha - (m_B a + m_B g) - \mu_k m_A g\cos\alpha \\ &= m_A g\sin\alpha - m_B a - m_B g - \mu_k m_A g\cos\alpha \\ \implies (m_A + m_b)a &= m_A g\sin\alpha - m_B g - \mu_k m_A g\cos\alpha \\ \implies a &= \frac{g}{m_A + m_B}(m_A\sin\alpha - m_B - \mu_k m_A\cos\alpha) \\ a &= \frac{9.8}{10+3}(10\sin 40 - 3 - 0.25 * 10 * \cos 40) \\ &= 1.14 \end{aligned}$$

so, $|a| = 1.14 \quad m/s^2$

2. A 10 kg block starts from rest at the top of a smooth 40-meter-high hill, which is inclined at an angle of 30° to the horizontal. After descending the incline, the block continues along a horizontal surface with a coefficient of kinetic friction $\mu_k = 0.3$ for a distance of 50 meters. After the horizontal surface, the block encounters a smooth ramp inclined at 20° and travels up until it comes to a complete stop due to gravity.

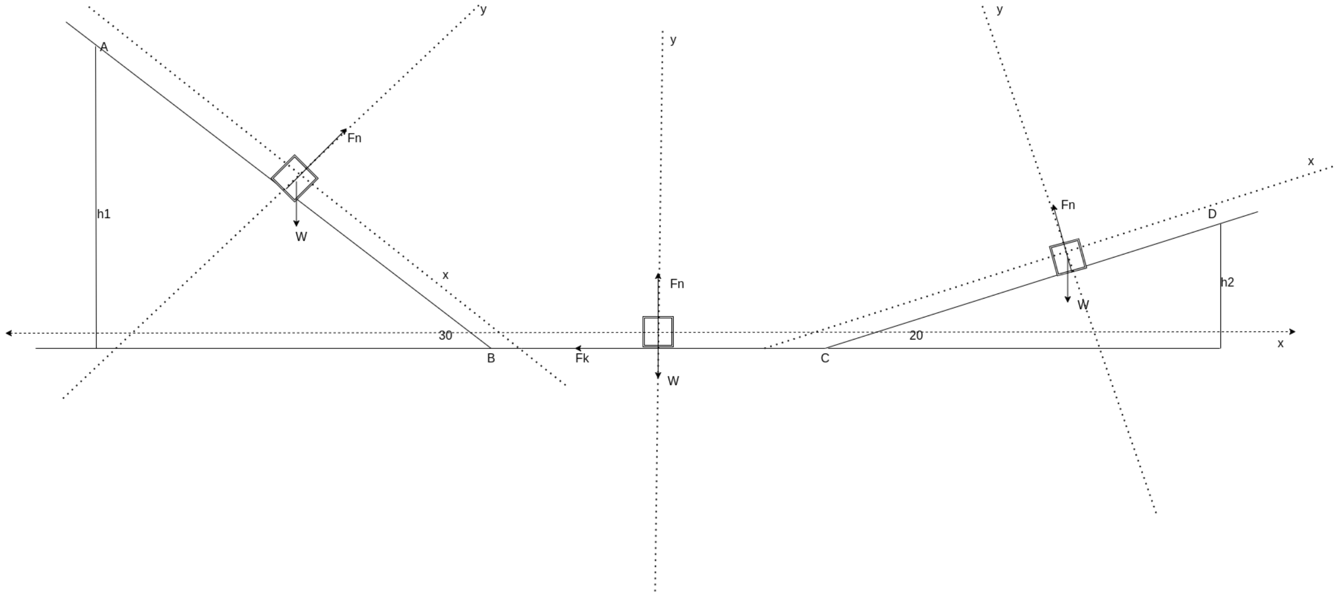

Figure 2: force acting to the object during the trajectory on the three parts

(a) Calculate the speed of the block at the bottom of the hill.
let's calculate V_B By apply the the principle of conservation of energy between A and B we have:

$$\begin{aligned} E_A &= E_B \\ mgh_1 &= \frac{1}{2}mV_b^2 \\ \implies V_b &= \sqrt{2gh_1} \qquad \text{or} \qquad h_1 = 40m \\ &= \sqrt{2*9.8*40} \\ &= 28 \end{aligned}$$

so, $V_B = 28$ m/s^2

(b) Determine the loss of mechanical energy due to friction over the 50-meter horizontal surface.

$$\begin{aligned} W_{B-C}(\vec{F_k}) &= \vec{F_k} * \vec{BC} = F_k * BC\cos(\vec{F_k}, \vec{BC}) \\ &= F_k * BC\cos 180) \\ &= -F_k * BC \\ &= -\mu_k mg * BC \\ &= -0.3 * 10 * 9.8 * 50 \\ &= -1470 \end{aligned}$$

so, $W_{B-C} = -1470$ J

(c) Find the speed of the block at the base of the second 20° incline after crossing the rough patch.
let's find V_C, we known on the 50-meter horizontal surface (BC), the loss mechanical energy is the work force of the force of friction on BC.

$$
\begin{aligned}
W_{B-C}(\vec{F_k}) &= E_B - E_C \\
&= \frac{1}{2}mV_B^2 - \frac{1}{2}mV_C^2 \\
\implies mV_c^2 &= -2W_{B-C}(\vec{F_k}) + mV_B^2 \\
\implies V_c &= \sqrt{\frac{-W_{B-C}(\vec{F_k}) + mV_B^2}{m}} \\
&= \sqrt{\frac{-2 * 1470 + 10 * 28^2}{10}} \\
&= 22.13
\end{aligned}
$$

so $V_C = 22.13$ m/s^2

(d) Calculate the distance the block travels up the 20^{co} ramp before coming to rest.

By apply the principle of conservation of energy we have:

$$
\begin{aligned}
E_c &= E_D \\
\frac{1}{2}mV_C^2 &= mgh_D = mgd \sin 20 \\
\implies d &= \frac{V_C^2}{2g \sin 20} \\
&= \frac{22.13^2}{2 * 9.8 \sin 20} \\
&= 73.05
\end{aligned}
$$

so $d = 73.05$ m

(e) Determine the total time taken for the entire journey, starting from the top of the 40-meter hill until the block comes to rest on the $20°$ ramp.

in order to calculate this time, we need to calculate the time on each part:

- on AB :

$$\sum \vec{F_{ext}} = m\vec{a}$$

$$
\begin{gathered}
\vec{W} + \vec{F_N} + = m\vec{a} \\
\begin{pmatrix} mg \sin \alpha \\ -mg \cos \alpha \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} = m_A \begin{pmatrix} a \\ 0 \end{pmatrix}
\end{gathered}
$$

$$\implies V = \begin{cases} V_x = gt \sin \alpha \\ V_y = 0 \end{cases}$$

when object completed AB $V_C = 28$ and

$$
\begin{aligned}
VC_x &= gt_{AB} \sin \alpha \\
\implies t_{AB} &= \frac{V_{C_x}}{g \sin \alpha} \\
&= \frac{28}{9.8 \sin 30} \\
&= 5.71s
\end{aligned}
$$

so $t_{AB} = 5.71$ s

- on BC:

$$\sum \vec{F_{ext}} = m\vec{a}$$

$$\vec{W} + \vec{F_N} + \vec{F_k} = m\vec{a}$$
$$\begin{pmatrix} 0 \\ -mg \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} + \begin{pmatrix} -\mu_k mg \\ 0 \end{pmatrix} = m\begin{pmatrix} a \\ 0 \end{pmatrix}$$

$$\implies V = \begin{cases} V_x = -\mu_k gt + V_B \\ V_y = 0 \end{cases}$$

when object completed BC $V_x = V_C$ and $V_{C_x} = -\frac{1}{2}\mu_k g t_{BC} + V_B$

$$\begin{aligned} V_{C_x} &= -\frac{1}{2}\mu_k g t_{BC} + V_B \\ \implies 2(V_B - V_{C_x}) &= \mu_k g t_{BC} \\ \implies t_{AB} &= \frac{2(V_B - V_{C_x})}{\mu_k g} \\ &= \frac{2(28 - 22.13)}{0.3 * 9.8} \\ &= 1.99 = 2 \end{aligned}$$

$t_{BC} = 2$ s

- on CD :

$$\sum \vec{F_{ext}} = m\vec{a}$$

$$\vec{W} + \vec{F_N} + = m\vec{a}$$
$$\begin{pmatrix} -mg\sin\alpha \\ -mg\cos\alpha \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} = m_A\begin{pmatrix} a \\ 0 \end{pmatrix}$$

$$\implies V = \begin{cases} V_x = -gt\sin\alpha + V_C \\ V_y = 0 \end{cases}$$

when object completed AB $V_D = 0$ and

$$\begin{aligned} V_D &= -g t_{CD}\sin\alpha + V_C \\ \implies t_{CD} &= \frac{V_C}{g\sin\alpha} \\ &= \frac{22.13}{9.8\sin 20} \\ &= 6.6 \end{aligned}$$

so, $t_{CD} = 6.6$ s

The total time is $ttotal = t_{AB} + tBC + t_{CD} = 5.71 + 2 + 6.6 = 14.31$ s

3. turbo-alternator group receives its energy from a waterfall with a height h = 40 m and a flow rate $D = 156$ m^3/s. The available electrical power at the output of the turbo-alternator group is Pu = 50 MW.

   (a) Determine the overall efficiency of the installation.
   $P_u = \tau P = \tau\rho g h D \implies \tau = \frac{P_u}{\rho g h d} = \frac{50*10^6}{10^3*9.8*156*40} = 0.81$

   so, the efficiency of the installation is : 81.76%

(b) Where in the system could the main energy losses occur?

during the fall there is loss of mechanical energy and there is also a loss of electrical energy from the turbo alternator due to the resistance

(c) We want to increase the useful power of this plant to $P_u = 60$ MW. To achieve this, a dam is used to increase the height of the waterfall. Assuming the overall efficiency of the installation remains the same as calculated above, calculate the new height h of the waterfall.

$h = \frac{P_u}{\tau \rho g D} = \frac{60*10^6}{0.8176*10^3*9.8*156} = 48$

so the new height h is 48 m

Problem:

Consider a particle of mass m = 1 kg moving on the horizontal x-axis with a potential:

$$U = \frac{1}{2} w_0^1 x^2$$

subject to a friction force $F_friction = -2\gamma x$ and a driving force $F_driving = \alpha \cos(t + \phi)$ The equation of motion is therefore given as:

$$\ddot{x} + 2\gamma\dot{x} + w_0^2 x = \alpha \cos(t + \phi) \tag{1}$$

This is a harmonic oscillator with damping (friction) and driving. To study the solutions of this equation and other relevant physical quantities, including potential and kinetic energy, the free parameters of the problem are considered. The initial position is x0, the initial velocity is x_0, the frequency is w_0, and the parameters are γ, α, and ϕ.

A- Simple Harmonic Oscillator

1. By assuming that $\gamma = 0$ and $\alpha = 0$, the equation (1) can be written as:

$$\ddot{x} + w_0 x = 0 \tag{2}$$

Solve numerically the equation (2) by using the Euler method with $\omega_0 = 1$, and starting at t = 0, $x_0 = 1$, and $x_0 = 0$. Plot the approximated and the exact solutions on the same graph.

let's calculate the exact solution of this differentials equations

$$r^2 = -w_0^2 = -1$$
$$\implies r = +i \quad \text{or} \quad r = -i$$

$$x(t) = Acost(t) + Bsin(t)$$

$$\begin{cases} x(0) = 1 \\ \dot{x}(0) = 0 \end{cases} \implies \begin{cases} A = 1 \\ B = 0 \end{cases}$$

so the exact solution is $x(t) = \cos(t)$

based on the figure 3 when h=0.1 the absolute error increase over the time

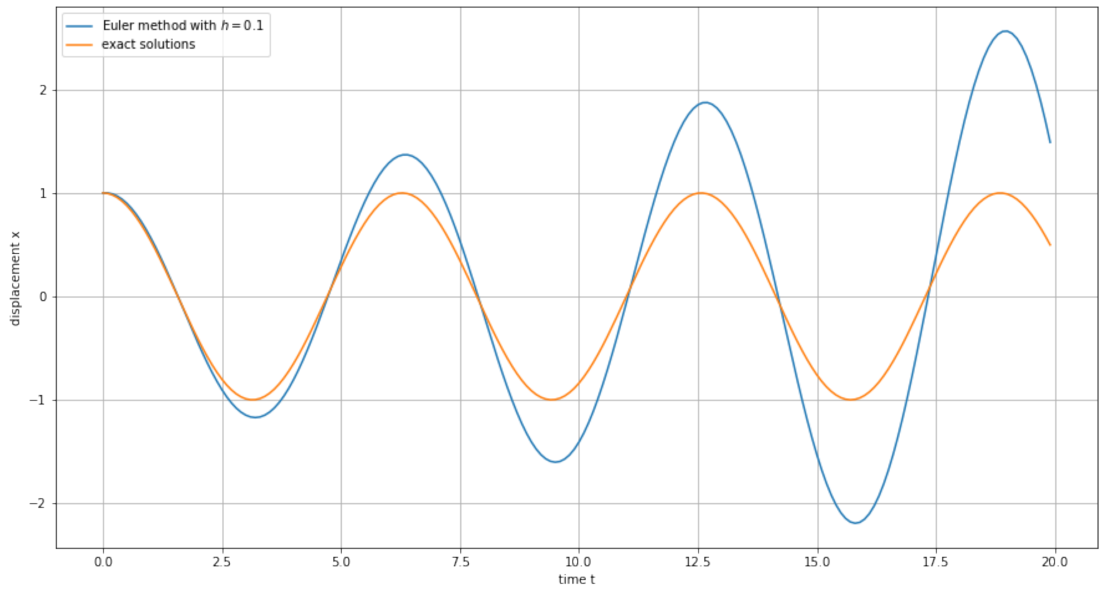

Figure 3: Euler with the step iteration h = 0.1 on [0, 20] and exact solution

based on figure 4 when h=0.01 the absolute error is very small and Euler's numerical solution is much closer to the exact solution but when h is small the numerical solution needs to do more calculation and it take many time to complete.

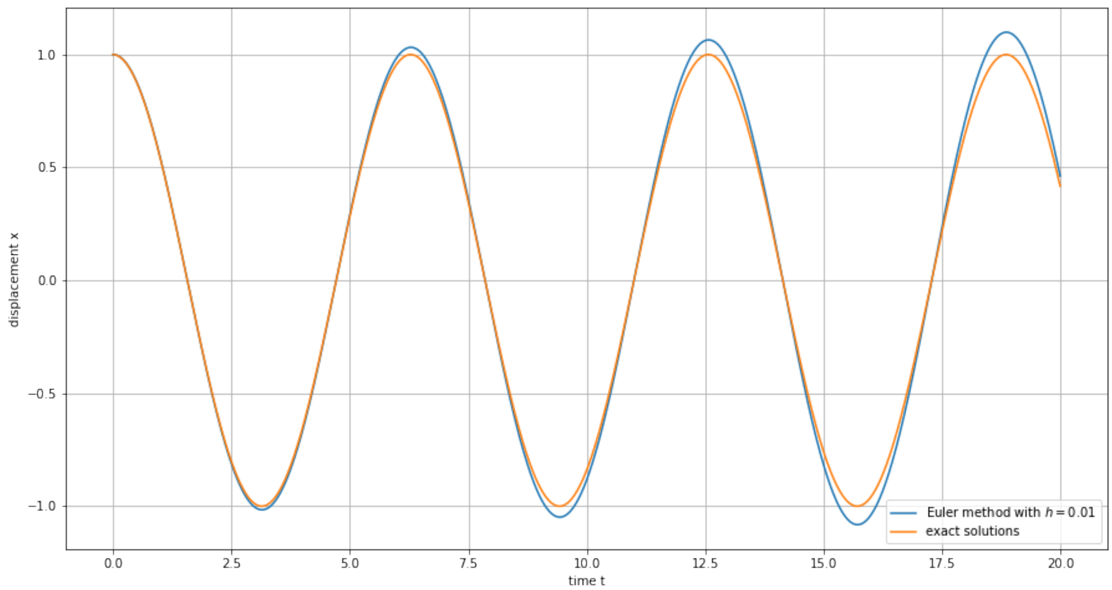

Figure 4: Euler with pas iteration h =0.01 on [0, 20] and exact solution

2. Using **odeint**, write a code that produces the following two graphs for the simple harmonic oscillator.

   (a) First graph: position x(t) and velocity $\dot{x}(t)$ which are both functions of time.

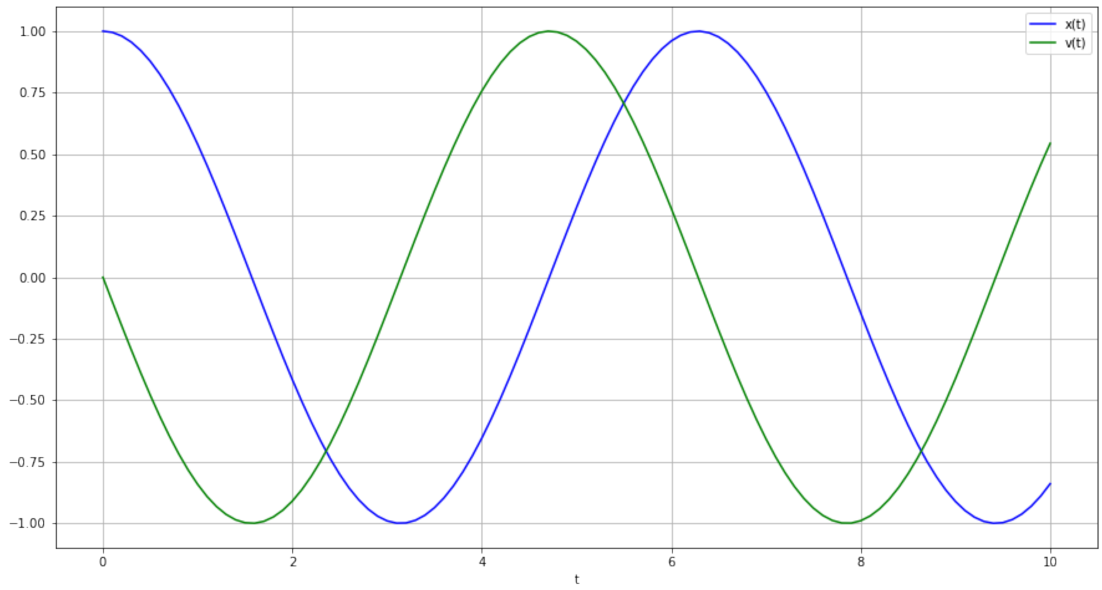

Figure 5: position x(t) and velocity v(t)

(b) Second graph: potential energy U(t) and kinetic energy K(t) which are both functions of time. Based on the figure 6 the system is Conservative because there are transfer energy between potential energy and kinetic energy. when kinetic energy is at its highest point potential energy is at its lowest point. so, in this case the mechanical energy in each time is constant because it is linear.

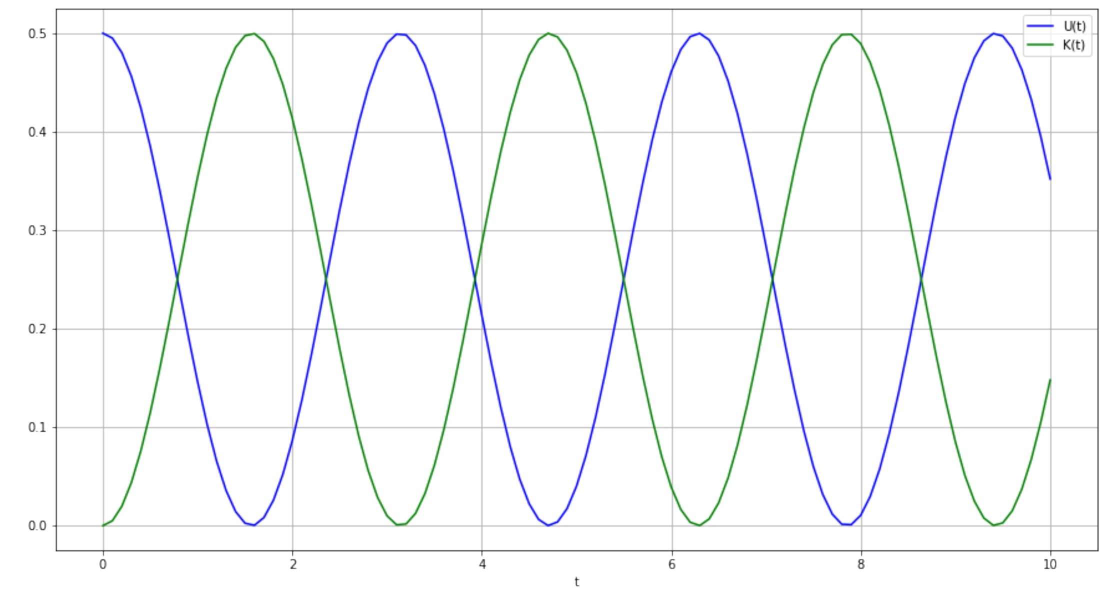

Figure 6: Potential Energy U(t) and kinetic Energy K(t)

B- Damped Oscillator

In this case, we take $\gamma = 0.1$ and $\alpha = 0$, which corresponds to the damped regime given as

$$\ddot{x} + 0.2\dot{x} + w_0^2 x = 0$$

The solution exhibits an oscillatory behavior with exponential damping. Therefore, repeat question (2) in part (A) using the odeint approach.

(a) First graph: position x(t) and velocity $\dot{x}(t)$ which are both functions of time.

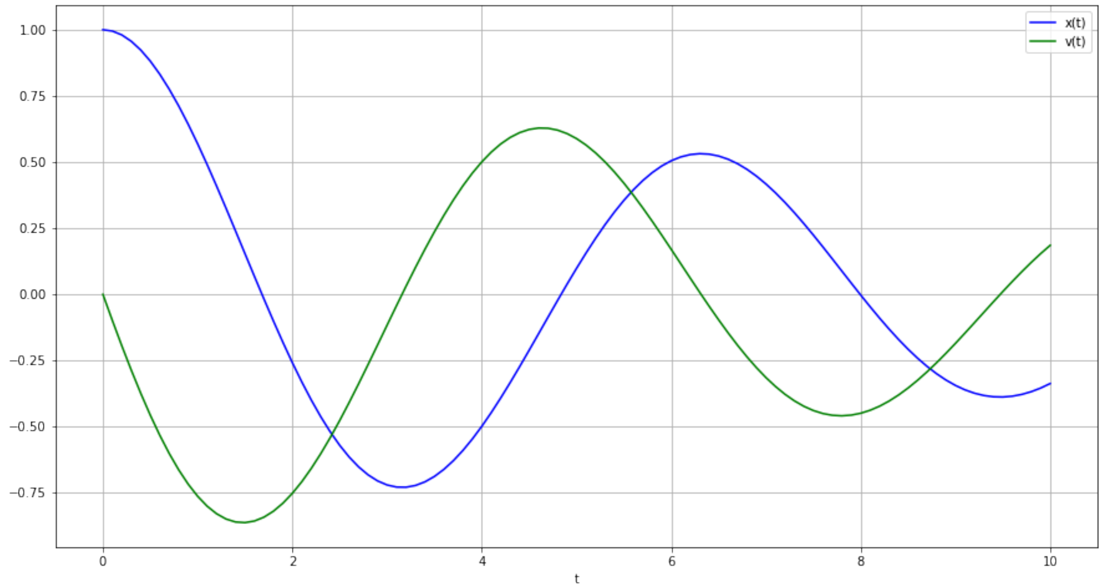

Figure 7: position x(t) and velocity v(t)

(b) Second graph: potential energy U(t) and kinetic energy K(t) which are both func- tions of time.

In this case the potential energy and kinetic energy decrease over the time based on the resultant of figure 8. so at one moment of the particle stop due of force of friction.

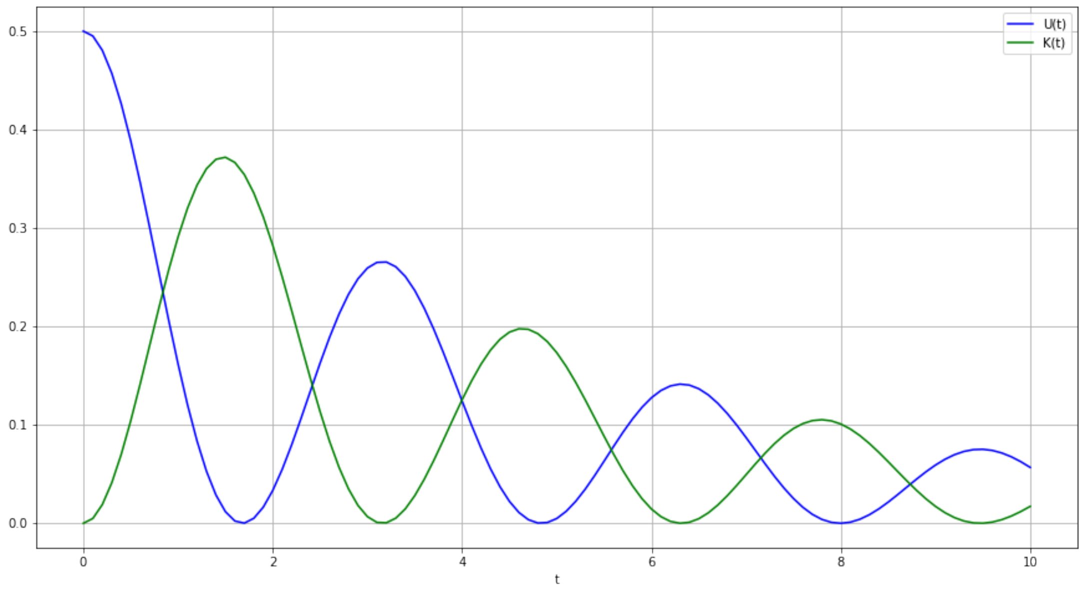

Figure 8: Potential Energy U(t) and kinetic Energy K(t)

C- Damped Oscillator with Driving

In this case, the motion is initially damped, as in the previous case ($\gamma = 0.1$), but after some time, the driving force takes over (with $\alpha = 1$) and becomes dominant. Thus, the motion is again oscillatory, with the oscillations dictated by the driving. Using the same approach as in the equation (1), produce the graphs as in the part (A) in question (2) using the odeint approach.

(a) First graph: position x(t) and velocity $\dot{x}(t)$ which are both functions of time.

because we don't give ϕ i take the value 0.

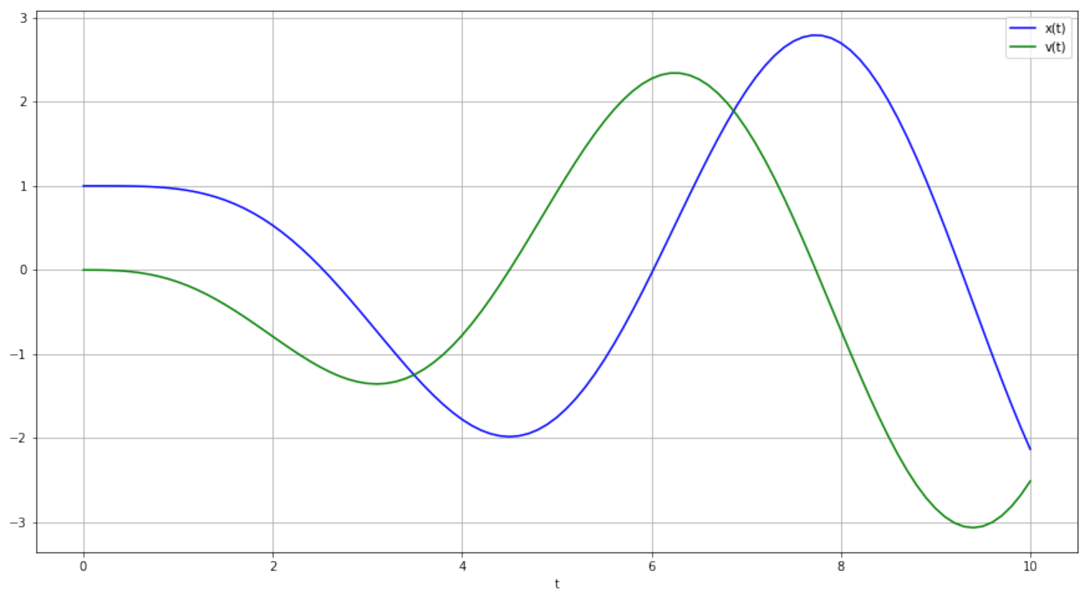

Figure 9: position x(t) and velocity v(t)

(b) Second graph: potential energy U(t) and kinetic energy K(t) which are both functions of time.

This case is opposite to the previous case because the potential energy and kinetic energy increase over the time due the driving force.

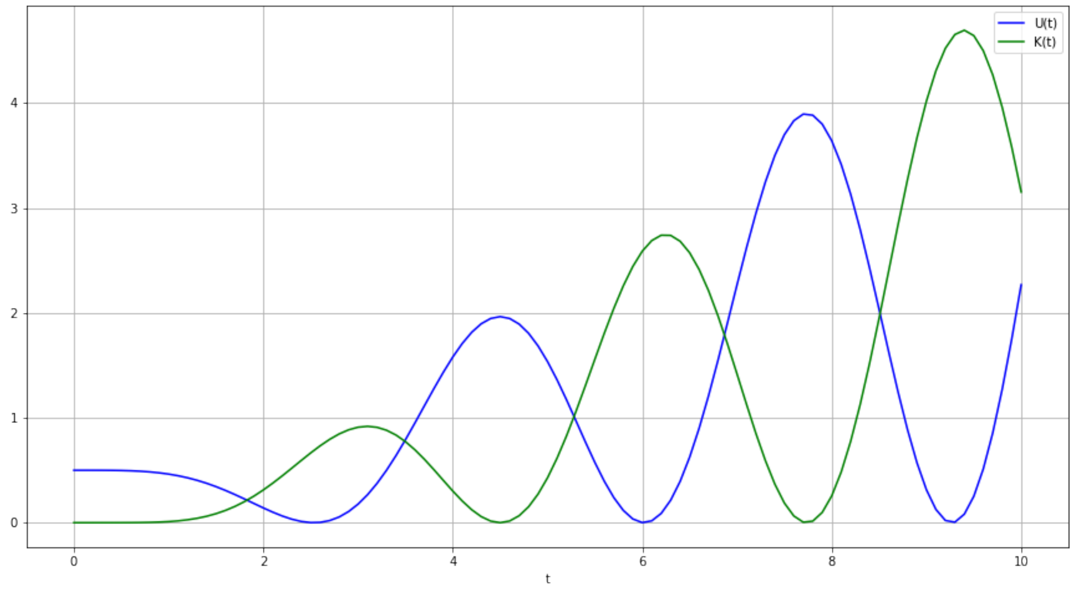

Figure 10: Potential Energy U(t) and kinetic Energy K(t)