ETH zürich
Departement Informatik

Theoretical Computer Science

Prof. Dr. Juraj Hromkovič
Dr. Hans-Joachim Böckenhauer
http://www.ita.inf.ethz.ch/theoInf18

Final Exam

Zurich, February 7, 2019

Exercise 1

(a) Construct a deterministic finite automaton (in graphical representation) that accepts the language

$$L = \{awb \mid w \in \{a,b\}^* \text{ and } |w|_a \bmod 3 = |w|_b \bmod 3\}\,.$$

(b) For each state q of your automaton as constructed in exercise part (a), give the class $\mathrm{Kl}[q]$.

5+5 points

Exercise 2

(a) Prove, using the method of Kolmogorov complexity, that the language

$$L_1 = \{1^{n^3}0^n \mid n \in \mathbb{N}\}$$

is not regular.

(b) Prove that the language $L_2 = L(G)$ that is generated by the context-free grammar $G = (\{S\}, \{[,]\}, P, S\}$, where

$$P = \{S \to SS, S \to [S], S \to \lambda\}\,,$$

is not regular.

(c) Prove that every deterministic finite automaton accepting the language

$$L_3 = \{w \in \{a,b,c\}^* \mid (|w|_a + |w|_b) \bmod 3 = |w|_c \bmod 3 \text{ and } w \text{ ends with } c\}$$

has at least 4 states.

5+5+5 points

(please turn the page)

Exercise 3

(a) Formulate the pumping lemma for context-free languages.

(b) Use the pumping lemma for context-free languages to prove that the language

$$L = \{a^n b^{2n} c^{3n} \mid n \in \mathbb{N}\}$$

over the alphabet $\{a, b, c\}$ is not context-free.

(c) Construct a general grammar over the terminal alphabet $\{a, b, c\}$, that generates the language L from exercise part (b) and informally describe the idea behind your construction.

2+4+4 points

Exercise 4

(a) Show that $(L_{\text{diag}})^{\complement} \leq_{\text{EE}} L_{\text{U}}$ by giving a concrete reduction and proving that your reduction is correct.

(b) For any two words $w_1, w_2 \in \{0,1\}^*$ with $w_1 \neq w_2$ let L_{w_1,w_2} be the language defined as

$$L_{w_1,w_2} = \{\text{Kod}(M) \mid M \text{ is a TM and } w_1 \in L(M) \text{ and } w_2 \notin L(M)\}.$$

Show that, for all words $w_1, w_2 \in \{0,1\}^*$ with $w_1 \neq w_2$, we have $L_{\text{U}} \leq_{\text{EE}} L_{w_1,w_2}$ by giving a concrete reduction and proving that your reduction is correct.

(c) We consider the language

$$L_{\text{not-all-length-2}} = \{\text{Kod}(M) \mid M \text{ is a TM and } \Sigma^2 \not\subseteq L(M)\}.$$

Prove that $L_{\text{not-all-length-2}} \notin \mathcal{L}_{\text{RE}}$. To this end, you may use all results known from the lecture.

(d) Can there exist two languages $L_1 \notin \mathcal{L}_{\text{RE}}$ and $L_2 \in \mathcal{L}_{\text{RE}}$ such that $L_1 \leq_{\text{R}} L_2$? Justify your answer.

3+5+5+2 points

Exercise 5

The *subset sum problem* (SUBSET-SUM, for short) is the following decision problem: Given a finite set $S = \{s_1, \ldots, s_m\}$ of natural numbers and a natural number t, decide whether there exists a subset $U \subseteq S$ such that $\sum_{x \in U} x = t$.

The *set partition problem* (PARTITION, for short) is the following decision problem: Given is a set $S = \{s_1, \ldots, s_m\}$ of natural numbers. The question is whether S can be partitioned into two sets U_1 and U_2 such that $U_1 \cup U_2 = S$, $U_1 \cap U_2 = \emptyset$, and $\sum_{x \in U_1} x = \sum_{y \in U_2} y$.

(a) Prove that PARTITION $\in$ NP.

(b) Prove that SUBSET-SUM $\leq_{\text{p}}$ PARTITION.

2+8 points