ETH zürich
Departement Informatik

Theoretical Computer Science

Prof. Dr. Juraj Hromkovič
Dr. Hans-Joachim Böckenhauer

Final Exam

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Exercise 1

(a) Construct a deterministic finite automaton (in graphical representation) that accepts the language

$$L = \{w \in \{0,1\}^* \mid w \text{ contains the subword 001 exactly once}\}.$$

(b) For each state q of your automaton as constructed in exercise part (a), give the class $\mathrm{Kl}[q]$.

5+5 points

Exercise 2

(a) Prove that the language

$$L = \{0^m 1^n \mid m, n \geq 1 \text{ and } m \text{ divides } n\}$$

is not regular using one of the three methods presented in the lecture (Lemma 3.12 (called Lemma 3.3 in the German version of the book), pumping lemma or Kolmogorov complexity).

(b) Prove that every deterministic finite automaton accepting the language

$$L = \{w \in \{a,b\}^* \mid (|w|_a + 2|w|_b) \bmod 3 = 1 \text{ or } w \text{ ends with } b\}$$

has at least 5 states.

5+5 points

(please turn the page)

Exercise 3

(a) Formulate the pumping lemma for context-free languages.

(b) Use the pumping lemma for context-free languages to prove that the language

$$L = \{a^n b^m c^{n \cdot m} \mid m, n \in \mathbb{N}, m, n \geq 1\}$$

over the alphabet $\{a, b, c\}$ is not context-free.

3+7 points

Exercise 4

We consider the language

$$L_{\text{len2}} = \{\text{Kod}(M) \mid M \text{ is a TM over the input alphabet } \{0,1\} \text{ and } M \text{ accepts at least one word of length 2}\}.$$

Which of the following statements is true?

(i) $L_{\text{len2}} \in \mathcal{L}_{\text{R}}$,

(ii) $L_{\text{len2}} \in \mathcal{L}_{\text{RE}} - \mathcal{L}_{\text{R}}$,

(iii) $L_{\text{len2}} \notin \mathcal{L}_{\text{RE}}$.

Prove the statement you have recognized to be true. **10 points**

Exercise 5

(a) Describe how to enumerate all positive rational numbers, i. e., prove that $\mathbb{Q}^+$ is countable.

(b) Use the diagonalization method to construct a positive real number that is not contained in $\mathbb{Q}^+$.

(c) Argue why there exist real numbers that do not have a finite decimal representation.

4+3+3 points

Exercise 6

We consider the languages

$$\text{LARGE-CLIQUE} = \{(G,k) \mid G = (V,E) \text{ is an undirected graph containing a clique of size } k \geq |V|/3\}$$

and

$$\text{VERY-LARGE-CLIQUE} = \{(G,k) \mid G = (V,E) \text{ is an undirected graph containing a clique of size } k \geq |V| - 3\}.$$

Prove, for each of the two languages, either that it is NP-complete or that it is contained in P. **10 points**