ASSIGNMENT - 2

SUBMITTED BY :-

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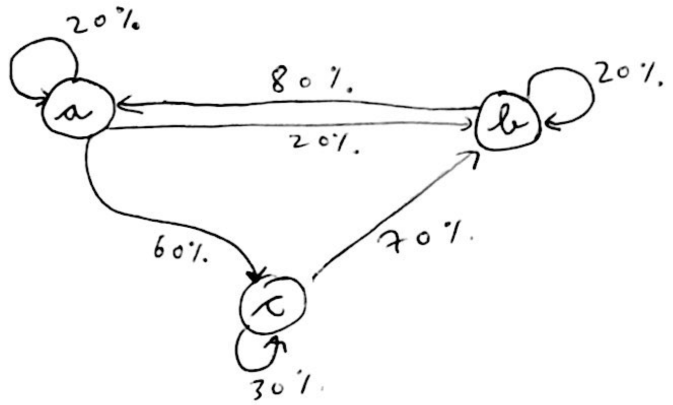
ANS 1



(a) State - Transition Probability Matrix (M) =

$$\begin{bmatrix} 0.2 & 0.2 & 0.6 \\ 0.8 & 0.2 & 0.0 \\ 0.0 & 0.7 & 0.3 \end{bmatrix}$$

(b) Given, $P(S_1 == a) = 1$

$P(S_3 == a) = ?$

Applying, Mini - Forward Algorithm given below :-

$P(x_1) =$ known

$$P(x_t) = \sum_{x_{t-1}} P(x_{t-1}, x_t)$$

$$= \sum_{x_{t-1}} P(x_t \mid x_{t-1}) \, P(x_{t-1}) \longrightarrow (i)$$

Using this,

$P(S_1 == a) = 1$

hence, $P(S_1 == b) = 0$ & $P(S_1 == c) = 0$

for S_2 :-

$$P(S_2 == a) = P(S_2 = a \mid S_1 = a) \times P(S_1 = a)$$
$$+ P(S_2 = a \mid S_1 = b) \times P(S_1 = b) +$$
$$P(S_2 = a \mid S_1 = c) \times P(S_1 = c)$$

$$= 0.2 \times 1 + 0.8 \times 0 + 0 \times 0 = 0.2$$

$$P(S_2 == b) = P(S_2 = b \mid S_1 = a) \times P(S_1 = a)$$
$$+ P(S_2 = b \mid S_1 = b) \times P(S_1 = b)$$
$$+ P(S_2 = b \mid S_1 = c) \times P(S_1 = c)$$

$$= 0.2 \times 1 + 0.2 \times 0 + 0.7 \times 0 = 0.2$$

$$P(S_2 == c) = P(S_2 = c \mid S_1 = a) \times P(S_1 = a)$$
$$+ P(S_2 = c \mid S_1 = b) \times P(S_1 = b)$$
$$+ P(S_2 = c \mid S_1 = c) \times P(S_1 = c)$$

$$= 0.6 \times 1 + 0 \times 0 + 0.3 \times 0 = 0.6$$

Hence, $P(S_2 == a) = 0.2$
$P(S_2 == b) = 0.2$
$P(S_2 == c) = 0.6$

for S_3 :-

$$P(S_3 == a) = \sum_{S_2} P(S_3 = a \mid S_2)\ P(S_2)$$

$$= P(S_3 = a \mid S_2 = a) \times P(S_2 = a)$$
$$+ P(S_3 = a \mid S_2 = b) \times P(S_2 = b)$$
$$+ P(S_3 = a \mid S_2 = c) \times P(S_2 = c)$$

$$= 0.2 \times 0.2 + 0.8 \times 0.2 + 0 \times 0.6 = \boxed{0.2}$$

(c) For equilibrium distrib^n :-

$$P_\infty(X) = P_{\infty+1}(X) = \sum_x P(X \mid x)\, P_\infty(x)$$

We can solve it using linear equations or iteratively for each S_1 till convergence.

Solving using linear eq^n :-

$0.2a + 0.8b = a$ —(1)

$0.6a + 0.3c = c$ —(2)

$0.2a + 0.2b + 0.7c = b$ —(3)

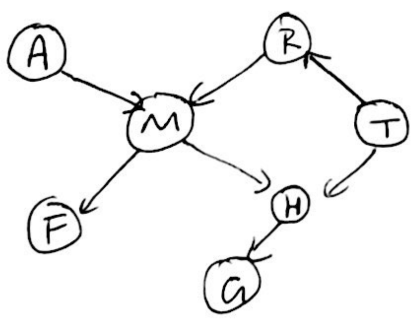
We know, $a + b + c = 1$ —(4)

After solving above equations we get :-

$a = 0.35,\ b = 0.35,\ c = 0.30$

Hence, equilibrium distrib^n is $\langle 0.35, 0.35, 0.3 \rangle$

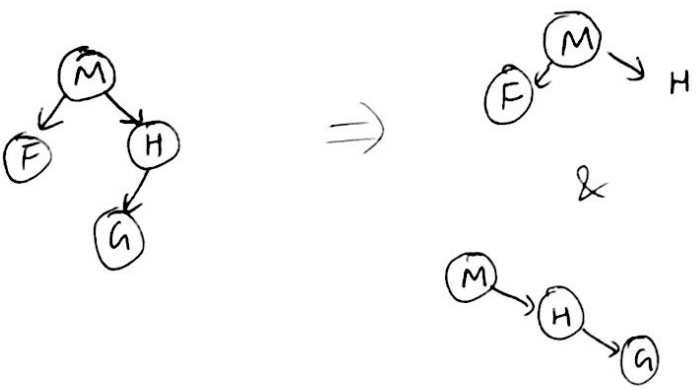
ANS 2



Part (a)

(i) $P(F, G) = P(F)\, P(G)$

We can see atleast 1 active path when we use d-seperation rules :-



active using common cause as M is not observable

active using causal chain as H is not observable

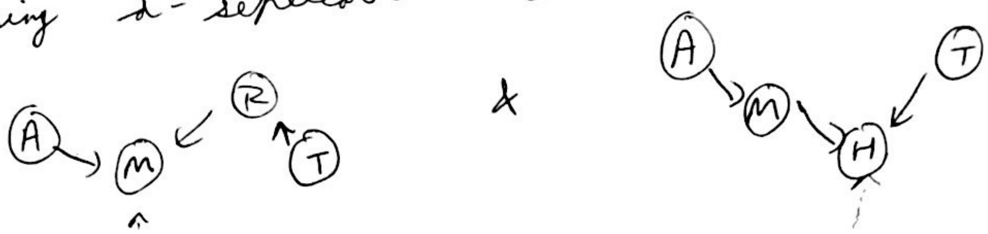
As there is an active path from F to G, we can't guarantee that $P(F, G) = P(F) \cdot P(G)$

Hence, statement is **False**

(ii) $P(A, T) = P(A) \cdot P(T)$

To verify this we check whether an active path exists.

Using d-seperation rules:-



Both inactive due to common effect

Not observable

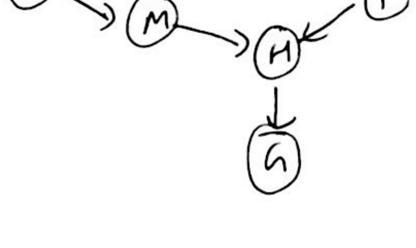
As there are no active paths from A to T. Hence, they are independent.

Hence, statement is **True**

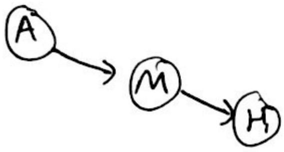
(iii) $P(A, T \mid R, G) = P(A \mid R, G) \; P(T \mid R, G)$

To verify this we check whether an active path exists.

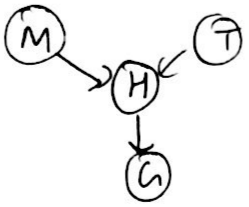
Using d-seperation rules:-



The path is active as.



active as M is not observable



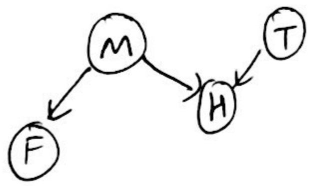
active as G is observable, hence, H will be observable using common effect.

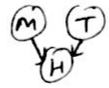
The whole path is active hence conditional indepence not guaranteed.

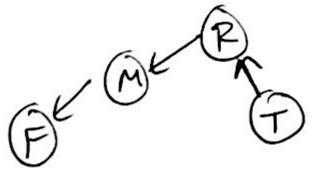
Hence, the statement is **False**

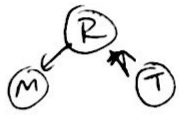
(iv) $P(F, T \mid R) = P(F \mid R)\, P(T \mid R)$

We look for active paths using d-seperation rules:-



This is inactive due to  common effect as H is not observable



This is inactive due to  clause which is inactive as R is observable

Hence, no active paths between F to T. conditional indepence gaurenteed.

Hence, statement is **true**

(v) $P(A, M \mid G) = P(A \mid M) \times P(M \mid G)$

L.H.S using conditional independence ~~rule~~ rule

$$P(A, M \mid G) = \frac{P(A, M, G)}{P(G)} \quad \text{—(1)}$$

$$P(A, M, G) = P(A) \times P(M \mid A) \times P(G \mid M, A)$$

Eq^n 1 becomes $\dfrac{P(A) \times P(M \mid A) \times P(G \mid M, A)}{P(G)}$

using bayes rule :-

$$\frac{P(A) \times \dfrac{P(A \mid M) \times P(M)}{P(A)} \times P(G \mid M, A)}{P(G)}$$

$$\Rightarrow \frac{P(A \mid M) \times P(M) \times P(G \mid M, A)}{P(G)} \quad \text{—(2)}$$

we know, $P(M \mid G) = \dfrac{P(G \mid M)}{P(G)} \times P(M)$

hence, $\dfrac{P(M)}{P(G)} = \dfrac{P(M \mid G)}{P(G \mid M)}$

Therefore, eq^n (2) becomes :-

$$\frac{P(A \mid M) \times P(M \mid G) \times P(G \mid M, A)}{P(G \mid M)}$$

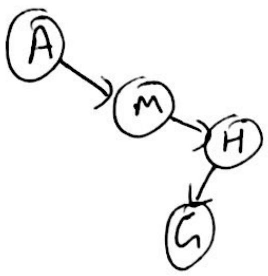
Hence for above eq^n to be equal to RHS we need to prove,

$$P(G \mid M, A) = P(G \mid M)$$

i.e. if $P(G \mid M, A) = P(G \mid M)$ is true, hence given statement will be true.

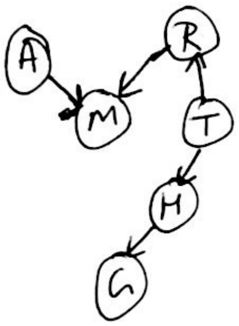
Using deseperation we get 2 paths :-

Path 1



This is inactive as M is observable in $A \to M \to H$

Path 2



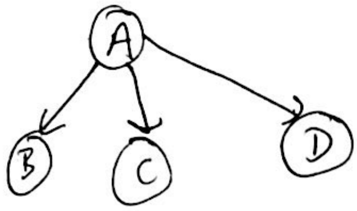
This is active as all the triplets are active.

Hence, $P(G \mid M, A) = P(G \mid M)$ is false

Hence, $P(A, M \mid G) = P(A \mid M) \times P(M \mid G)$ is also false

ANS - 3

(i)



For A we need 4 variables as 5^{th} can be calculated by summing all 4 variables & ~~&~~ subtracting them from 1.

Hence, for A, variables required = 4

For B, C, D each :

Number of parents = 1

value of $x = 5$

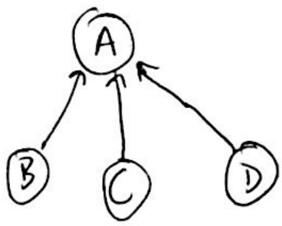
$\therefore \quad (x-1) \cdot x^n = (5-1) \times 4$

$= 20$

Hence, total minimum variabels required

$= 20 + 20 + 20 + 4 = 64$

(ii)



Here, B, C, D require 4 variables each as they have no parents.

Hence, ~~for~~ B, C, D, total variables

$= 4 + 4 + 4 = 12$

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For A, number of parents = 3

~~number of variable(x) = 5~~

value of x = 5

Therefore, number of variables $= (5)^3 \times 4$

$= 500$

Hence, minimum number of variables required to represent the bayesion network is

512