HW #2 Solutions

1) a)

$$M = \begin{bmatrix} 0.2 & 0.2 & 0.6 \\ 0.8 & 0.2 & 0 \\ 0 & 0.7 & 0.3 \end{bmatrix}$$

b)

$$P(S_3 = a \mid S_1 = a) = \sum_{S_2 \in \{a,b,c\}} P(S_2, S_3 = a \mid S_1 = a)$$

$$= P(S_2 = a, S_3 = a \mid S_1 = a) + P(S_2 = b, S_3 = a \mid S_1 = a) + P(S_2 = c, S_3 = a \mid S_1 = a)$$

$$= 0.2 \times 0.2 + 0.2 \times 0.8 + 0.6 \times 0 = 0.04 + 0.16 = 0.2$$

c) $\pi M = \pi$

$$\Rightarrow [\pi_1 \; \pi_2 \; \pi_3] \begin{bmatrix} 0.2 & 0.2 & 0.6 \\ 0.8 & 0.2 & 0 \\ 0 & 0.7 & 0.3 \end{bmatrix} = [\pi_1 \; \pi_2 \; \pi_3]$$

$$\Rightarrow \begin{cases} 0.2\pi_1 + 0.8\pi_2 = \pi_1 \\ 0.2\pi_1 + 0.2\pi_2 + 0.7\pi_3 = \pi_2 \\ 0.6\pi_1 + 0.3\pi_3 = \pi_3 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases} \Rightarrow \begin{aligned} \pi_1 &= \frac{7}{20} \\ \pi_2 &= \frac{7}{20} \\ \pi_3 &= \frac{6}{20} \end{aligned}$$

2) a)

i) False : F and G are not d-separated, so they are not independent.
This statement is True if F and G and independent.

ii) True: A and T are d-separated, so they are independent.
This statement is True if A and T are independent.

iii) False: A and T are not d-separated if R and G are observed.
so they are not independent when R and G are observed.

iv) True: F and T are d-separated if R is observed.
Therefore they are independent when R is observed.

v) False: We first rewrite the statement:

$$P(A, M \mid G) = P(A \mid M)\, P(M \mid G)$$

$$\frac{P(A, M, G)}{\cancel{P(G)}} = P(A \mid M) \frac{P(M, G)}{\cancel{P(G)}} \Rightarrow P(A, M, G) = P(A \mid M)\, P(M, G)$$

$$\Rightarrow \div P(M) \Rightarrow P(A, G \mid M) \stackrel{?}{=} P(A \mid M)\, P(G \mid M)$$

A and G are not d-separated if M is observed. Therefore A and G are not independent if M is observed.

3) a)

$$P(A, B, C, D) = \underbrace{P(A)}_{5-1} \underbrace{P(B \mid A)}_{5(5-1)} \underbrace{P(C \mid A)}_{5(5-1)} \underbrace{P(D \mid A)}_{5(5-1)} \Rightarrow \text{Total \# of parameters} = 4 + 20 + 20 + 20 = \boxed{64}$$

b)

$$P(A, B, C, D) = \underbrace{P(A \mid B, C, D)}_{5 \times 5 \times 5 \times (5-1)} \underbrace{P(B)}_{5-1} \underbrace{P(C)}_{5-1} \underbrace{P(D)}_{5-1}$$

$$\Rightarrow \text{Total \# of parameters} = 500 + 4 + 4 + 4 = \boxed{512}$$