BDA Project

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Contents

1 Introduction

In light of the current world situation, with global warming and sustainability of natural resource consumption being the primary concerns of all generations, young and old, as a group we thought that we should create a project that contributed towards the global effort towards building a better future for everyone. We decided to tackle a subset of the larger issues of “Fast Fashion” and “Planned Obsolescence”, our goal being to “Predict the amount of textile waste in landfills.” and compare different types of waste management, generating predictions on landfilling, recycling and combustion of waste.

We have chosen data on US landfills and consumption habits, because it was the most reliably available. To achieve our goal, we design weakly informative priors, fit two models, compare them with LOO-CV, and perform posterior predictive checks and sensitivity analyses to verify our results.

2 Data

The data used in this project comes from the “Advancing Sustainable Materials Management: Facts and Figures Report” of the United States Environmental Protection Agency (EPA (2022)). The data is available for download through this link: https://edg.epa.gov/data/PUBLIC/OLEM/Materials_Municipal_Waste_Stream_1960_2018.xlsx.

The data is in Excel format and contains waste management data from the US for the years 1960-2018. This document contains a large amount of sheets, but for this project, only the “Materials generated”, “Materials recycled”, “Material combusted”, and “Materials landfilled” sheets were used, as they are the only ones with clear data for textiles.

The data from 1960 to 2000 is given every ten years, then there is data for 2005, and the data from 2010-2018 is given yearly. Due to this inconsistency in time intervals of data, only part of the years are used to fit the models. The years 2010-2018 were chosen, as they have the most regular data and they are the most recent. The rest of the data is used to help in estimating prior parameters.

Also, the sheet “Materials generated” is not used in the models as it isn’t interesting to the goal of the project, but it is used as an aid in prior parameter determination.

2.1 The final data

As the data is scattered in multiple sheets of an Excel file, the specific textile-related data had to be extracted from each sheet, combined, and the row and column names cleaned up a bit in order to acquire a final dataset. This results in a dataframe with categories “Materials generated”, “Materials recycled”, “Material combusted”, and “Materials landfilled”.

The year is also adjusted such that the year 1950 becomes the new year 0 (1950 is subtracted from each year). This is done so that the significantly long period of time before data collection where values for each category might have been 0 doesn’t affect the final models. For example, the first landfill in the US was established in 1937 (Zylberberg (2019)), so there would have been 1937 years of 0 values in the “Materials landfilled” category, which would have a significant impact on the (slope of the linear) models. Hence, a new year 0 was chosen in which all categories of waste management had already been implemented/invented.

The final dataframe can be seen in Table 1.

The final data is visualised in Figure 1 for years 1960-2005, and in Figure 2 for years 2010-2018.

Table 1: Final dataframe

Materials generated	Materials recycled	Material combusted	Materials landfilled	year	original year
1760000	50000	0	1710000	10	1960
2040000	60000	10000	1970000	20	1970
2530000	160000	50000	2320000	30	1980
5810000	660000	880000	4270000	40	1990
9480000	1320000	1880000	6280000	50	2000
11510000	1830000	2110000	7570000	55	2005
13220000	2050000	2270000	8900000	60	2010
13690000	2070000	2540000	9080000	61	2011
14480000	2200000	2800000	9480000	62	2012
14840000	2220000	2960000	9660000	63	2013
15240000	2260000	3020000	9960000	64	2014
16060000	2460000	3060000	10540000	65	2015
16880000	2510000	3240000	11130000	66	2016
16890000	2570000	3170000	11150000	67	2017
17030000	2510000	3220000	11300000	68	2018

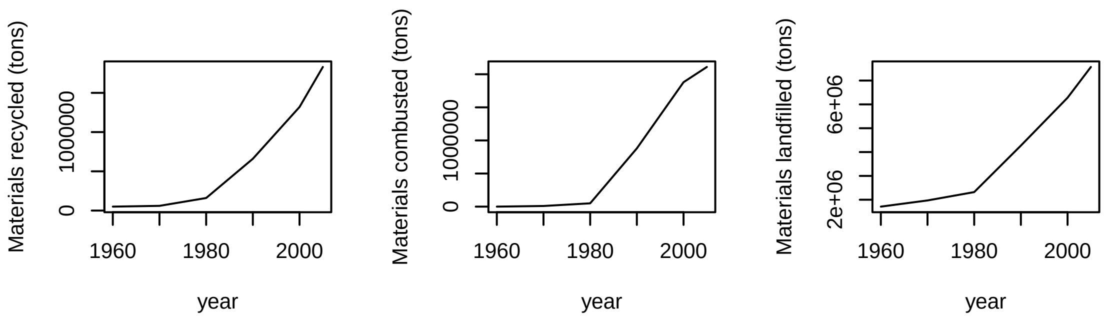

Figure 1: Data for years 1960-2005.

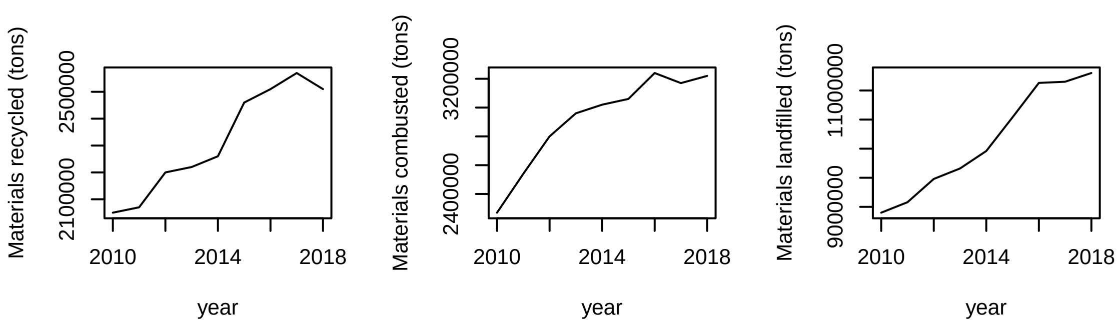

Figure 2: Data for years 2010-2018.

As can be seen from the figures, the data could likely be approximated decently well with a linear model.

3 The models

Since the data used for modeling in this project is split into three categories (materials recycled, materials combusted, and materials landfilled), and their differences are an aspect of interest, a model is needed which differentiates between them. Two good options for this are the separate model and the hierarchical model. Hence, in this project both of these models are implemented and compared.

3.1 Separate model

The separate model assumes that each category is independent from other categories, and essentially gives each category its own Bayesian model. Hence, it considers the categories to have no relation between each other.

3.2 Hierarchical model

The hierarchical model considers each category as being distinct from each other, but still similar enough that the measurements from one category can influence the predictions for another. To accommodate for this, the hierarchical model provides separate models for each category, however the models share the same variance and the means of the models share the same prior distribution constructed with hyperparameters. Hence, the hierarchical model considers the categories to be distinct, but still aknowledges some relation between them.

3.3 Linear Gaussian model

As was shown in the "Data" section, the data used to fit the models is approximately linear, with the explanatory variable being the year. Hence, both the separate and hierarchical models are implemented as linear gaussian models in this project. This means the parameters of interest are the intercepts (α) and slopes (β) of the lines describing the data. The likelihood for this model is as follows:

$$y \sim N(\alpha + \beta * x, \sigma).$$

4 Priors

4.1 Initial priors

Initial priors refer to the first priors which were tested.

4.1.1 Separate model

The priors for α, β, and σ are normal distributions, because those are convenient considering the likelihood is also a normal distribution. Since the separate model is in question, separate priors are needed for each category ("Materials recycled", "Material combusted", and "Materials landfilled").

To get the mean parameter of the α and β priors (μ in $N(\mu, \sigma)$), a linear model was fit for the 1960-2005 data of each category. Then, the means (μ) of the α priors for each category were set to be the maximum of 0 and the intercept extracted from the corresponding linear model (the intercept shouldn't be negative as waste management can't take on a negative value). Then, the means (μ) of the β priors for each category were set to be the slope extracted from the corresponding model.

The standard deviation parameters of the α and β priors (σ in $N(\mu, \sigma)$), were set to be 100 and 1000, respectively, for each category. This was chosen quite arbitrarily to represent the prior being weakly informative (the sd for β is bigger than that of α because the slopes had much larger values than intercepts).

The means for the σ priors were set to be the standard deviations of the data for the corresponding category from years 1960-2005. The standard deviations for the σ priors were set to be 1000 to represent weak informativeness.

Table 2: Separate model intial priors (values rounded to 0 digits).

	Alpha	Beta	Sigma
Materials recycled	N(0,100)	N(39551,1000)	N(746217,1000)
Materials combusted	N(0,100)	N(51622,1000)	N(970843,1000)
Materials landfilled	N(0,100)	N(133578,1000)	N(2457690,1000)

Table 3: Initial hyperpriors (and sigma prior) for the hierarchical model (values rounded to 0 digits).

	Alpha-mean	Alpha-sd	Beta-mean	Beta-sd	Sigma
Prior	N(0,10)	N(300,100)	N(74917,100)	N(3000,1000)	N(1391583,1000)

This resulted in the priors shown in Table 2.

4.1.2 Hierarchical model

For the hierarchical model, priors were needed for the means (μ in $N(\mu, \sigma)$) and standard deviations (σ in $N(\mu, \sigma)$) for both α and β, and then for the common standard deviation σ (5 priors in total: α-mean, α-sd, β-mean, β-sd, σ).

The prior for the α-mean was set to have the average of the intercept estimates from the three categories as its mean, and 10 as it's standard deviation because we were quite confident in the mean value of the α-mean (10 is a fairly small number so it made the prior informative).

The prior for the α-sd was set to have the sum of the standard deviations of the α priors from the separate model as its mean, and 100 as it's standard deviation to make the prior weakly informative.

The prior for the β-mean was set to have the average of the slope estimates from the three categories as its mean, and 100 as it's standard deviation because we were again quite confident in the mean value of the β-mean (100 is quite a small value compared to the average of the slopes, so this made the prior informative).

The prior for the β-sd was set to have the sum of the standard deviations of the β priors from the separate model as its mean, and 1000 as it's standard deviation to make the prior weakly informative (again, the standard deviation for β-sd prior is larger than for α-sd prior because the values for the slopes are much larger).

The prior for σ was set to have the mean of the standard deviations of the 1960-2005 data of each category as its mean, and 1000 as its standard deviation to make it weakly informative.

The resulting hyperpriors (and sigma prior) can be seen in Table 3, and their usage can be seen in the following formula:

$$\begin{aligned}
\alpha_i &\sim N(\alpha\text{-}mean, \alpha\text{-}sd) \\
\beta_i &\sim N(\beta\text{-}mean, \beta\text{-}sd) \\
y_i &\sim N(\alpha_i + \beta_i * x, \sigma)
\end{aligned}$$

4.2 Final priors

Final priors refer to the priors obtained after all necessary modifications from analysis (the changes are also described throughout the analysis).

4.2.1 Separate model

For the separate model, it was realised during analysis that the standard deviations of the α and β priors were too small compared to the values in the data, and the priors weren't actually weakly informative. Hence,

Table 4: Separate model final priors (values rounded to 0 digits).

	Alpha	Beta	Sigma
Materials recycled	N(0,1000)	N(39551,10000)	N(256043,1000)
Materials combusted	N(0,1000)	N(51622,10000)	N(333117,1000)
Materials landfilled	N(0,1000)	N(133578,10000)	N(843285,1000)

Table 5: Final hyperpriors (and sigma prior) for the hierarchical model (values rounded to 0 digits).

	Alpha-mean	Alpha-sd	Beta-mean	Beta-sd	Sigma
Prior	N(0,100)	N(1000,200)	N(74917,1000)	N(10000,2000)	N(477481,1e+05)

the standard deviation for the α prior was increased to 1000, and the standard deviation for the β prior was increased to 10000.

In addition, the mean of the σ prior was modified. The standard deviations of years 1960-2005 and years 2010-2018 were compared for the "Materials generated" data (which wasn't included in the model). The standard deviation is almost three times larger for 1969-2005 than for 2010-2018. Hence, the standard deviations of the σ priors were divided by this coefficient (sd-previous-years/sd-recent-years). As "Materials generated" is the sum of the other categories, it is sensible to assume that the trends in the former would be similar to those in the latter, so dividing by the coefficient is sensible as well.

These changes resulted in the priors show in Table 4.

4.2.2 Hierarchical model

For the hierarchical model, the standard deviations of the α-mean and β-mean priors were increased, because after some thought it was concluded that the standard deviations were too small compared to the scale of the data. The standard deviations were increased to 100 and 1000, respectively.

The parameters for the α-sd and β-sd priors were also recalculated according to the new separate model priors. Also, instead of summing the standard deviations of the separate model α and β priors, the average was taken to acquire the prior mean. This made more sense as the α-mean and β-mean prior means are also calculated as an average. The standard deviations of the α-sd and β-sd priors were changed accordingly to 200 and 2000, respectively, to represent weak informativity.

Similarly to the separate model, the mean of the σ prior was divided by the factor (sd-previous-years/sd-recent-years). In addition, the standard deviation of the σ prior had to be increased quite drastically, to 100000, for the posterior predictive checks to look good. This value still made sense with the scale of the data as the and the differences of standard deviations between the categories were larger than this value.

The resulting hyperpriors (and sigma prior) can be seen in Table 5.

5 Stan code

5.1 Separate

Stan code with final priors:

```
## data {
##   int<lower=0> N;  // number of observations
##   int<lower=0> J;  // number of categories
##   vector[N] x; // observation year
##   vector<lower=0>[J] y[N]; // observations
##   real xpred; // prediction year
```

```
##   vector[J] alpha_mean; //Means for alpha priors
##   vector[J] beta_mean; //Means for beta priors
##   vector[J] sds; // Standard deviations of each group (1960-2005)
## }
## parameters {
##   vector<lower=0>[J] alpha; // category intercepts
##   vector<lower=0>[J] beta; // category slopes
##   vector<lower=0>[J] sigma; // category stds
## }
## model {
##   // priors
##   for (j in 1:J){
##     alpha[j] ~ normal(alpha_mean[j], 1000);
##     beta[j] ~ normal(beta_mean[j], 10000);
##     sigma[j] ~ normal(sds[j], 1000);
##   }
##
##   // likelihood
##   for (j in 1:J)
##     y[,j] ~ normal(alpha[j]+beta[j]*x, sigma[j]);
## }
## generated quantities {
##   // 2019 prediction for combusted materials
##   real ypred_r = normal_rng(alpha[1] + beta[1]*xpred, sigma[1]);
##   // 2019 prediction for recycled materials
##   real ypred_c = normal_rng(alpha[2] + beta[2]*xpred, sigma[2]);
##   // 2019 prediction for landfilled materials
##   real ypred_l = normal_rng(alpha[3] + beta[3]*xpred, sigma[3]);
##   // Replicated data sets
##   real yrep_r[N] = normal_rng(alpha[1] + beta[1]*x, sigma[1]);
##   real yrep_c[N] = normal_rng(alpha[2] + beta[2]*x, sigma[2]);
##   real yrep_l[N] = normal_rng(alpha[3] + beta[3]*x, sigma[3]);
##   // For LOO-CV
##   vector[J*N] log_lik;
##
##   for (j in 1:J) {for (n in 1:N)
##     log_lik[n + (j-1)*N] = normal_lpdf(y[n,j] | alpha[j]+beta[j]*x[n], sigma[j]);
##     // "n + (j-1)*N" calculates the column index of the S-by-N
##     // matrix, from the row & column
##     // index of the observation.
##   }
## }
```

5.2 Hierarchical

Stan code with final priors:

```
## data {
##   int<lower=0> N;  // number of observations
##   int<lower=0> J;  // number of categories
##   vector[N] x; // observation year
##   vector<lower=0>[J] y[N]; // observations
##   real xpred; // prediction year
##   real sd_mean; // Standard deviations of each group (1960-2005)
##   real slope_mean;
```

```
##   real intercept_mean;
## }
## parameters {
##   real mu_alpha; // Alpha prior mean
##   real mu_beta; // Beta prior mean
##   real<lower=0> sigma_alpha; // Alpha prior std
##   real<lower=0> sigma_beta; // Beta prior std
##   vector<lower=0>[J] alpha; // category intercepts
##   vector<lower=0>[J] beta; // category slopes
##   real<lower=0> sigma; // Common positive std
## }
## model {
##   // priors
##   mu_alpha ~ normal(intercept_mean,100);
##   mu_beta ~ normal(slope_mean,1000);
##   sigma_alpha ~ normal(1000,200);
##   sigma_beta ~ normal(10000,2000);
##   alpha ~ normal(mu_alpha, sigma_alpha);
##   beta ~ normal(mu_beta, sigma_beta);
##   sigma ~ normal(sd_mean, 100000);
##
##   // likelihood
##   for (j in 1:J)
##     y[,j] ~ normal(alpha[j]+beta[j]*x, sigma);
## }
## generated quantities {
##   // Prediction for combusted materials
##   real ypred_r = normal_rng(alpha[1] + beta[1]*xpred, sigma);
##   // Prediction for recycled materials
##   real ypred_c = normal_rng(alpha[2] + beta[2]*xpred, sigma);
##   // Prediction for landfilled materials
##   real ypred_l = normal_rng(alpha[3] + beta[3]*xpred, sigma);
##   // Replicated data sets
##   real yrep_r[N] = normal_rng(alpha[1] + beta[1]*x, sigma);
##   real yrep_c[N] = normal_rng(alpha[2] + beta[2]*x, sigma);
##   real yrep_l[N] = normal_rng(alpha[3] + beta[3]*x, sigma);
##   vector[J*N] log_lik;
##
##   for (j in 1:J) {for (n in 1:N)
##     log_lik[n + (j-1)*N] = normal_lpdf(y[n,j] | alpha[j]+beta[j]*x[n], sigma);
##     // "n + (j-1)*N" calculates the column index of the S-by-N
##     // matrix, from the row & column
##     // index of the observation.
##   }
## }
```

6 Fitting the models

Both models were fit with the default parameters for stan(), meaning 4 chains with 2000 iterations and 1000 warmup, and random initial points. This can be seen in the following code snippet:

```
fit_hierarchical <- stan(file = "Hierarchical_project.stan", data = data_hierarchical)
fit_separate <- stan(file = "Separate_project.stan", data = data_separate)
```

Table 6: Hierarchical model Rhat and ESS.

	Rhat	bulk-ESS	tail-ESS
alpha[1]	1.0012301	2618.591	1913.814
beta[1]	0.9998127	3817.007	2364.252
alpha[2]	1.0012212	2130.101	1419.284
beta[2]	0.9999907	3862.829	2665.815
alpha[3]	1.0010881	2450.128	1599.429
beta[3]	1.0011899	3706.309	2089.699

Table 7: Separate model Rhat and ESS.

	Rhat	bulk-ESS	tail-ESS
alpha[1]	1.0032263	2128.048	1277.651
beta[1]	0.9999724	3245.738	2514.649
alpha[2]	1.0000218	2269.028	1425.593
beta[2]	0.9997853	3295.154	2371.927
alpha[3]	1.0002557	2127.864	1214.602
beta[3]	1.0009735	3802.737	2777.526

7 Convergence diagnostics

7.1 R-hat and ESS

7.1.1 Hierarchical

The hierarchical model had good Rhat and ESS from the start. For the final priors the Rhat and ESS are still very good. This means that all Rhat values are below 1.05 and all ESS values are larger than 100 which indicate convergence. Table 6 shows the Rhat and ESS values for the hierarchical model with final priors.

7.1.2 Separate

For the initial priors, all the separate model's Rhat values were greater than 1.05 and most ESS values were smaller than 100, meaning the chains did not converge. Initially this was fixed through simplifying the model by removing the sigma prior entirely. The sigma prior was later added back with different parameters (final sigma prior in the section on priors), and the Rhat and ESS values for the final model can be seen in Table 7. As can be seen from the table, all Rhat values are smaller than 1.05, and all ESS values are larger than 100, meaning the final separate model has converged.

7.2 Hamiltonian Monte Carlo (HMC) diagnostics

7.2.1 Hierarchical

HMC diagnostics for the final hierarchical model:

```
check_hmc_diagnostics(fit_hierarchical)
```

```
##
## Divergences:

## 0 of 4000 iterations ended with a divergence.

##
## Tree depth:

## 0 of 4000 iterations saturated the maximum tree depth of 10.
```

```
##
## Energy:

## E-BFMI indicated no pathological behavior.
```

The hierarchical model had a few divergences in the initial model. Changing adapt_delta didn't seem to do much, so instead priors were modified. At this point, a mistake in the alpha_sigma and beta_sigma priors was noticed. In the separate model, the standard deviation parameters for alpha and beta were too small compared to the values in the data, meaning the priors weren't really weakly informative. The parameters of the alpha_sigma and beta_sigma priors in the hierarchical model are calculated based on these standard deviations in the separate model. This calculation was also nonsensical, summing all the standard deviations instead of averaging them. Both of these issues were addressed, after which there were no more divergences in the hierarchical model, as can be seen above.

Note: More details about the prior modification can be found in the Priors section.

7.2.2 Separate

HMC diagnostics for the final separate model:

```
check_hmc_diagnostics(fit_separate)
```

```
##
## Divergences:

## 0 of 4000 iterations ended with a divergence.

##
## Tree depth:

## 0 of 4000 iterations saturated the maximum tree depth of 10.

##
## Energy:

## E-BFMI indicated no pathological behavior.
```

The separate model had no divergences or other issues with HMC for either the initial priors or the final priors.

8 Posterior predictive checks

8.1 Separate

Figures 3, 4, and 5 show posterior predictive checks for the final separate model. The density plots suggest that the model is an okay fit as the replicate draws seem somewhat similar to the true data, but the histograms don't look quite as good. However, considering the dataset is very small, this extent of fit is satisfactory for this project.

For the initial model without a sigma prior, the fit was worse. The distributions were clearly too wide, so a sigma prior was added (the final sigma prior is discussed in the Priors section). This made the fit much better.

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

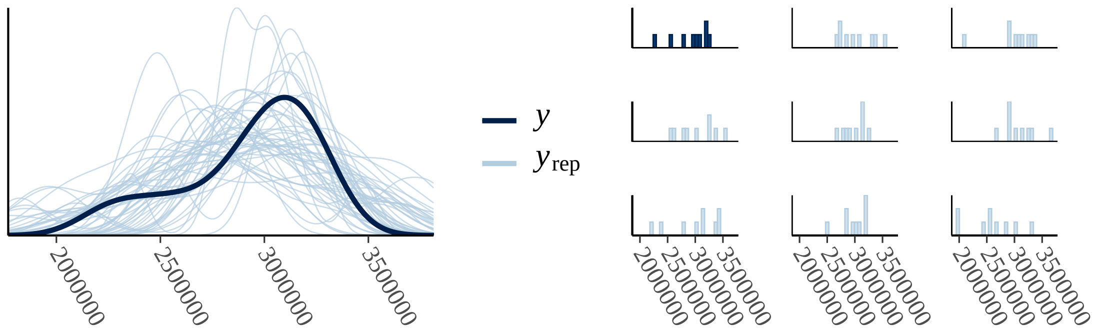

Figure 3: Posterior predictive check for materials combusted with the separate model.

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

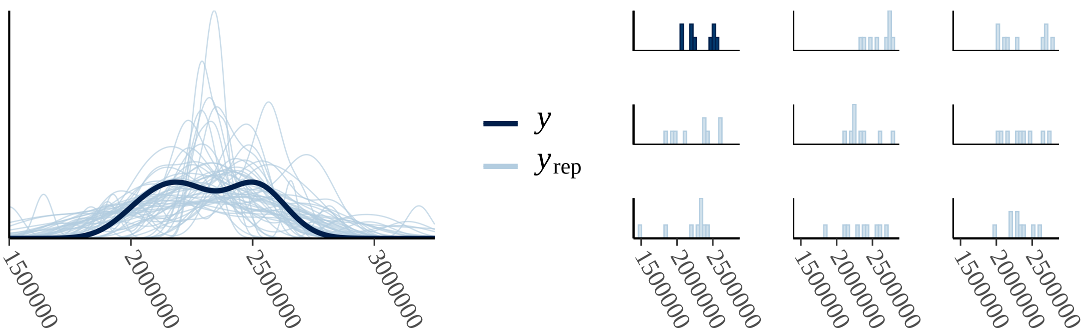

Figure 4: Posterior predictive check for materials recycled with the separate model.

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

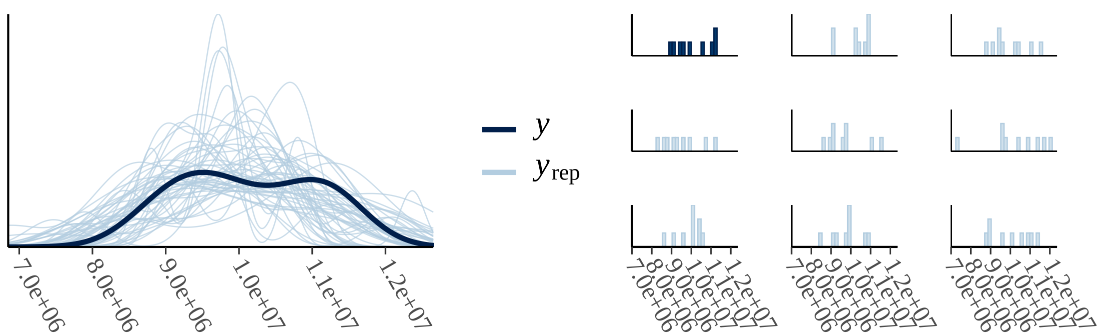

Figure 5: Posterior predictive check for materials landfilled with the separate model.

8.2 Hierarchical

Figures 6, 7, and 8 show posterior predictive checks for the final hierarchical model. The density plots suggest that the model is a somewhat decent (but worse than the separate model) fit as the replicate draws seem somewhat similar to the true data, but the histograms don't look quite as good. However, considering the dataset is very small again, this extent of fit is satisfactory for this project.

For the initial model without a sigma prior, the fit was much worse. The distributions were clearly too wide, so a sigma prior was added (the final sigma prior is discussed in the Priors section). This made the fit much better.

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

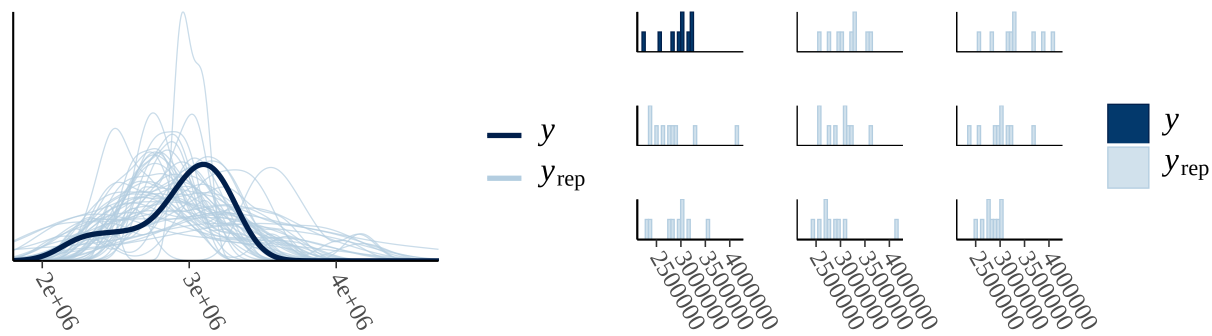

Figure 6: Posterior predictive check for materials combusted with the hierarchical model.

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

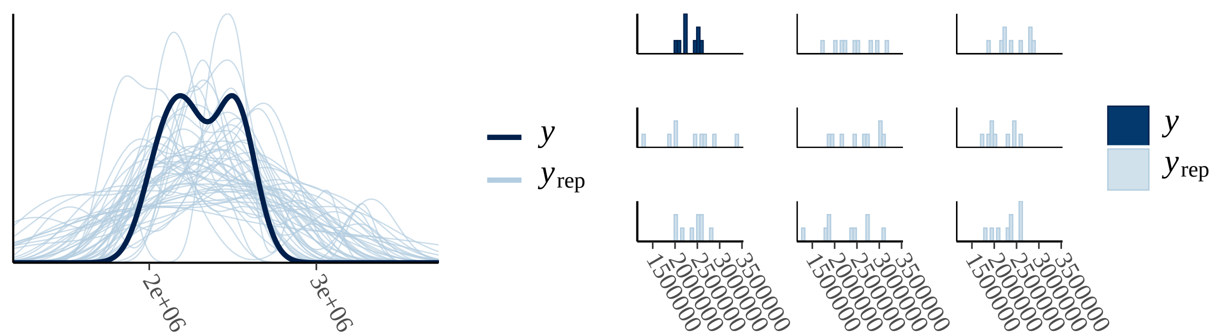

Figure 7: Posterior predictive check for materials recycled with the hierarchical model.

```
## `stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```

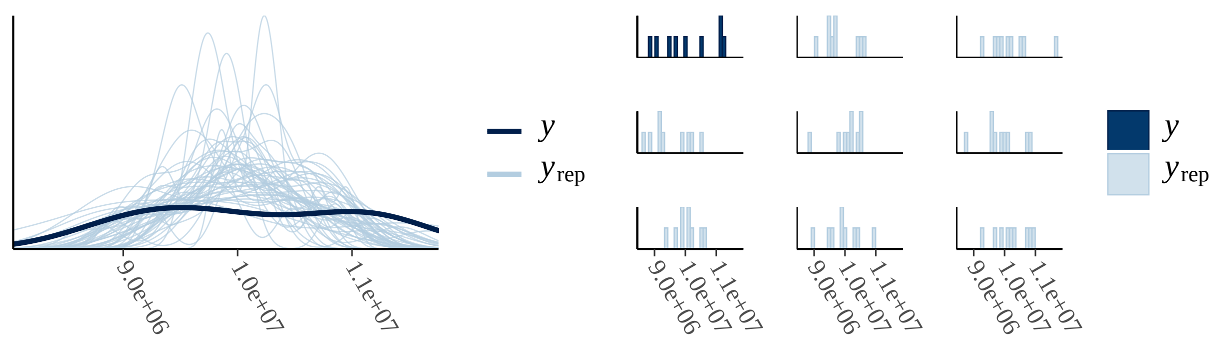

Figure 8: Posterior predictive check for materials landfilled with the hierarchical model.

Table 8: Hierarchical model predictive performance assessment.

	Materials recycled	Materials combusted	Materials landfilled
Real	2510000.0	3220000.0	11300000.0
Predicted mean	2508494.0	3130620.8	10634169.8
Predicted sd	407598.2	414242.3	410517.7

Table 9: Separate model predictive performance assessment.

	Materials recycled	Materials combusted	Materials landfilled
Real	2510000.0	3220000.0	11300000.0
Predicted mean	2457762.6	3108645.0	10429711.4
Predicted sd	269116.4	349642.1	900025.7

9 Predictive performance assessment

To assess performance, we can fit the models excluding the year 2018, instead adding that year as the predicted year. We can then compare the true values for 2018 to the predicted values.

The results for this analysis can be seen in Table 8 for the hierarchical model, and Table (**ref?**)(tab:perf-asses-s) for the separate model.

Both models seem to perform somewhat well and quite similarly. The means of the predictions are relatively close to the real values, though standard deviations are quite large. Although the values aren't incredibly accurate, they could be used as a rough measure for guidance in waste management planning for upcoming years. They could also be used as a rough measure of how much waste production needs to be reduced to stay at the same or a lower level as the previous year.

10 Sensitivity analysis

Two priors were tested for the sensitivity analysis in addition to the final priors described in the Priors section. These priors are the uniform prior (used for every parameter), and the combination of N(10000,100) for mean parameters and N(0,50000) for standard deviation parameters which was called the "Second" prior in this analysis.

Clarification of the Second prior for separate model:

```
##   for (j in 1:J){
##     alpha[j] ~ normal(10000,100);
##     beta[j] ~ normal(10000,100);
##     sigma[j] ~ normal(0,50000);
##   }
```

Clarification of the Second prior for hierarchical model:

```
##   mu_alpha ~ normal(10000,100);
##   mu_beta ~ normal(10000,100);
##   sigma_alpha ~ normal(0,50000);
##   sigma_beta ~ normal(0,50000);
##   alpha ~ normal(mu_alpha, sigma_alpha);
##   beta ~ normal(mu_beta, sigma_beta);
##   sigma ~ normal(0,50000);
```

10.1 Separate

Sensitivity analysis for the separate model can be seen in Figure 9.

```
ggplot(data=sep_sens_df, aes(x=value, color = Prior)) +
  geom_density() +
  facet_wrap(y_type ~ col_val, scales='free') +
  theme(axis.text.x = element_text(angle = -45, vjust = 0.5, hjust=0))
```

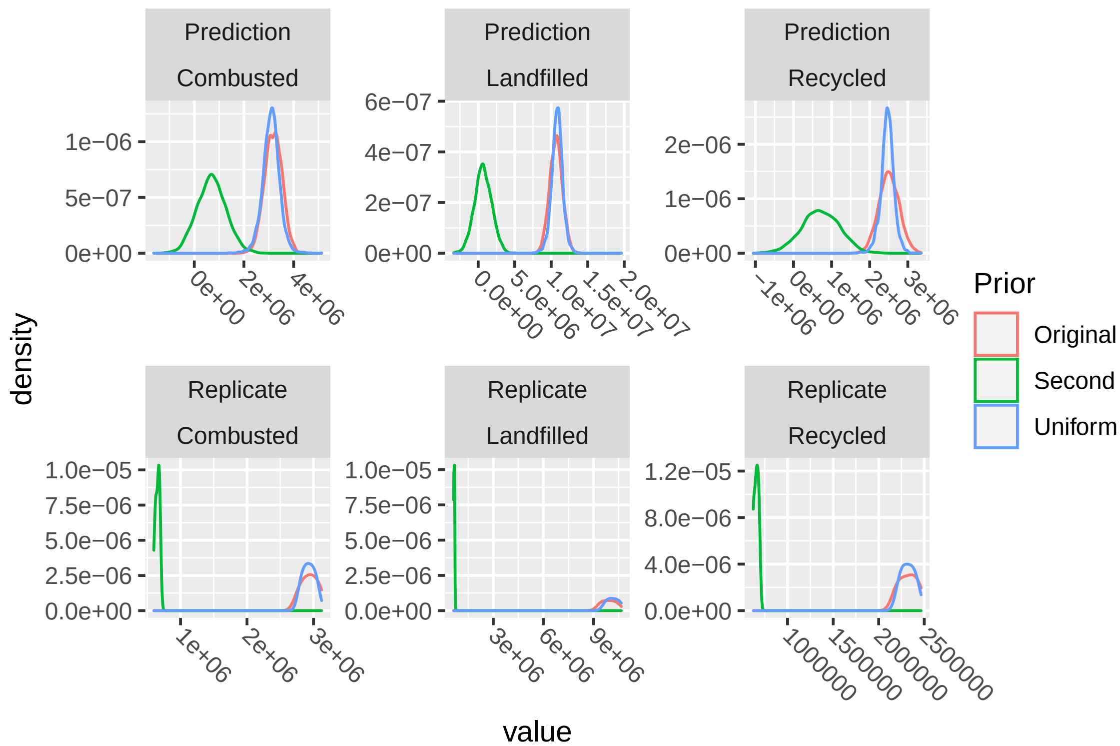

Figure 9: Sensitivity analysis for separate model. Prediction refers to 2019 prediction, and Replicate refers to the original dataset replicated with the model.

From the figure, we can see that the separate model is quite sensitive to prior choice. Although there isn't much difference between the effect of the final priors and the uniform prior, the "Second" prior makes quite a large difference.

10.2 Hierarchical

Sensitivity analysis for the hierarchical model can be seen in Figure 10.

```
ggplot(data=hier_sens_df, aes(x=value, color = Prior)) +
  geom_density() +
  facet_wrap(y_type ~ col_val, scales='free') +
  theme(axis.text.x = element_text(angle = -45, vjust = 0.5, hjust=0))
```

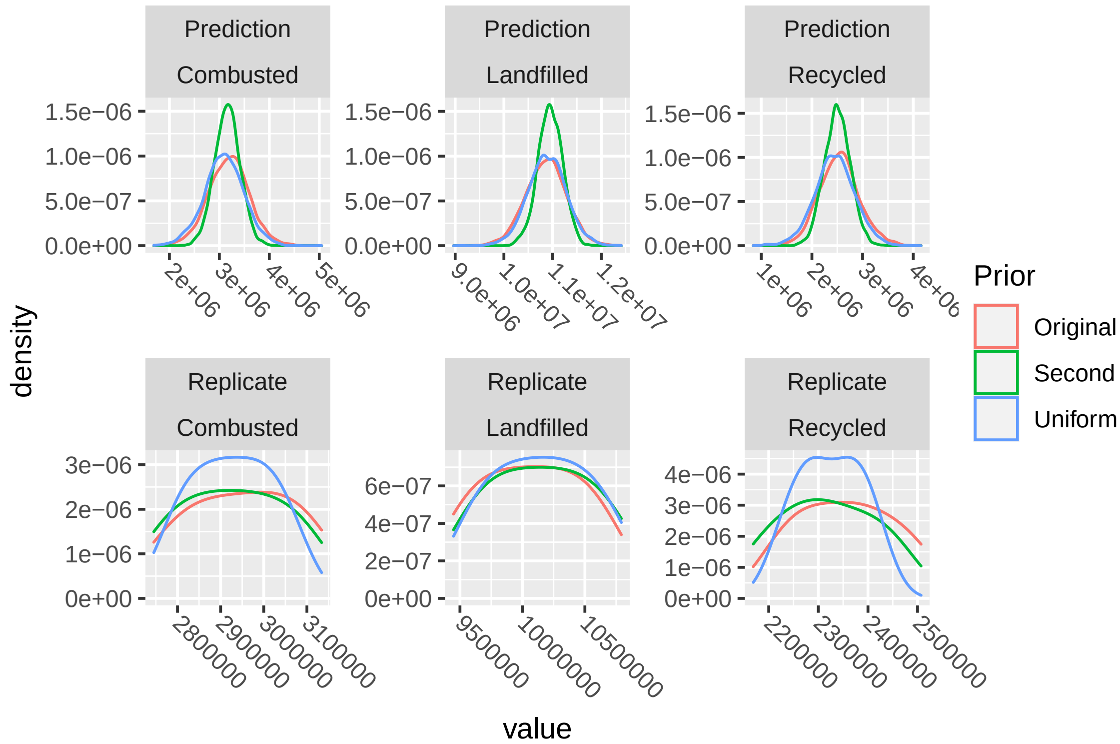

Figure 10: Sensitivity analysis for hierarchical model. Prediction refers to 2019 prediction, and Replicate refers to the original dataset replicated with the model.

From the figure, we can see that the hierarchical model is much less sensitive to prior choice than the separate model. For the predicted quantities the prior choice doesn't seem to have a significant effect, although the "Second" prior clearly results in a slightly narrower distribution than the others. For the replicated quantities, the prior choice doesn't have much effect on the "Landfilled" block, but the model seems more sensitive in the other blocks where the uniform prior is clearly causing some differences.

11 Model comparison (LOO-CV)

The k-hat values are visualised in Figure 11 for the separate model, and in Figure 12 for the hierarchical model. All k-values for both models are below 0.7, meaning the PSIS-LOO values for each model are reliable.

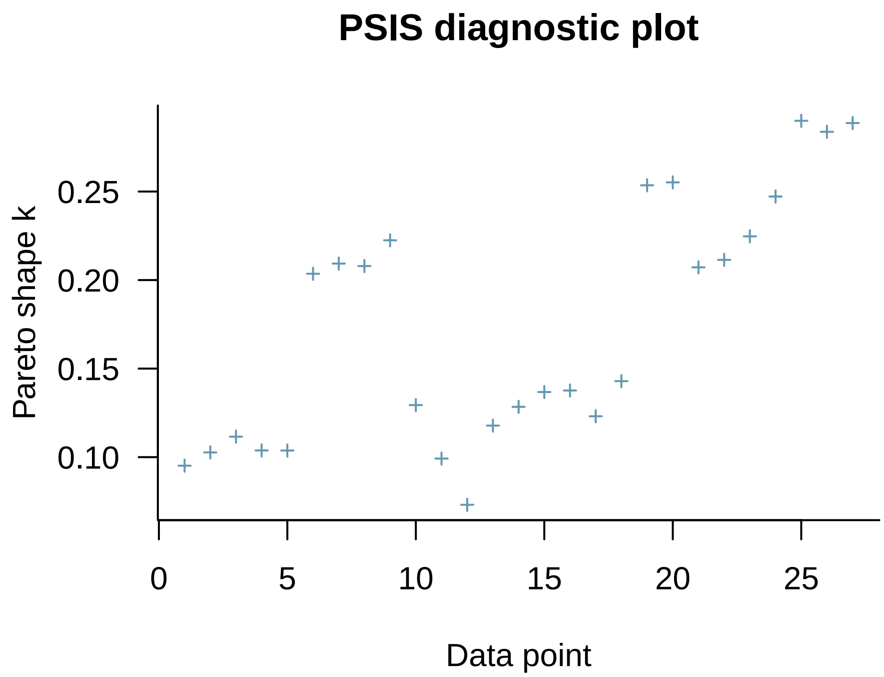

Figure 11: k-hat values for the separate model.

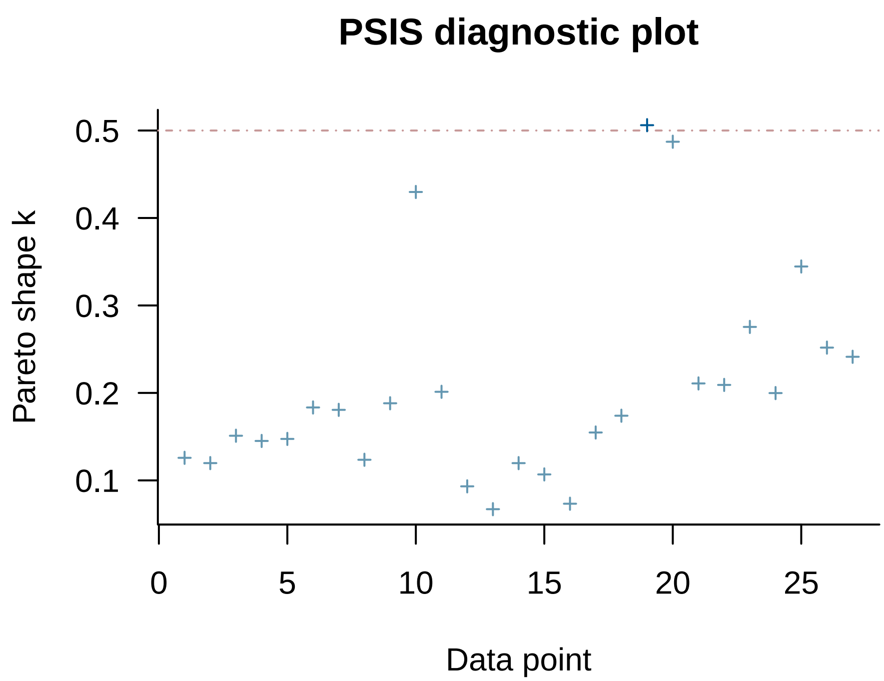

Figure 12: k-hat values for the hierarchical model.

Table 10: LOO-CV results

	Hierarchical	Separate
elpd	-383.98024	-380.2781614
p_eff	2.97216	0.9752602

The elpd values and effective number of parameters (p_eff) can be seen in Table 10 for both the hierarchical and separate models.

We can see that the separate model has a slightly larger elpd-value, hence the separate model would be selected according to PSIS-LOO.

12 Discussion

One area where this project could be improved is plotting. There were some issues with the formatting of the report and the scaling of the plots that could not be remedied in time.

Another aspect to be improved on would be the selection and cleaning of the dataset. Our chosen dataset has irregular time periods of data collection, and we had to account for this when designing our models. It would be preferable to have more data, that is grouped annually.

The fit of the model is made worse because of the small dataset, but we used our priors to try and mitigate that factor when ftting our models.

13 Conclusion

Through the course of this project, we have gone through the process of fitting our dataset to two Stan models, a hierarchical model and a separate model, both of which fit linear Gaussian models to the features. We compare the R-hat and ESS diagnostics of both models to see if they converge or not (they do.) We then use the HMC diagnostics to verify if random samples drawn from the posterior distribution converge.

We used these metrics in the early stages of the project to troubleshoot model design and changed parts of our model to optimize for them. A predictive performance assessment was conducted to test if the model produced good predictions, where we predicted the data of year 2018, and compared it with the real values.

Our final finding is that the separate model fit the data better.

We gained some insights in working with smaller datasets, and have observed that the smaller the dataset, the more important the priors are in building a good model. This highlights the importance of domain expertise, and has applications in fields with small datasets like the measurements of particle accelerators or the treatment outcomes of patients with rare cancers.

In a future project, we could spend more time researching the topic and building up domain expertise to generate more informative priors, which have a very large impact on the performances of our models. We could also fit more models to our data and perform more in-depth cross validation of the models.

14 Self reflection

We learned how to combine everything learned in the assignments into the evaluation of chosen models with a real-world dataset. We also learned how to adapt various model evaluation methods to a model with several categories. There were some challenges like unclean data and coordinating between group members, but we learned a lot along the way, and are satisfied with the results of our labours.

References

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