Textile Waste

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"All this I see, and I see that the fashion wears out more apparel than the man"

(William Shakespeare - *Much Ado About Nothing*, Act 3, Scene 3, Page 6)

Trends in fashion cause people to stop wearing clothes before they have even worn out. This leads to unnecessary accumulation of clothing and waste of money.

This is known as Fast Fashion

Companies create products that **intentionally break down** after few uses, shortening the consumer's time between purchases, increasing revenue for the company.

*This is known as **Planned Obsolescence***

The aim of our project is to predict where the textile waste generated by this type of business model ends up.

Bayesian Data Analysis

- Source
- First Look
- Final Form

Data
Source

The data that we used was gathered to generate the EPA's annual report[1] on advancing sustainable materials management

We only used the subset of data that was related to textiles

[1] Environmental Protection Agency. (2019). *Advancing Sustainable Materials Management: Facts and Figures Report.* Retrieved from: https://www.epa.gov/facts-and-figures-about-materials-waste-and-recycling/advancing-sustainable-materials-management

Data

Final Form

In the final dataframe, the years that the data was ordered by were all shifted so that 1950 became year 0

This was done to obtain a more reasonable intercept and slope

Table 1: Final dataframe

Materials generated	Materials recycled	Material combusted	Materials landfilled	year	original year
1760000	50000	0	1710000	10	1960
2040000	60000	10000	1970000	20	1970
2530000	160000	50000	2320000	30	1980
5810000	660000	880000	4270000	40	1990
9480000	1320000	1880000	6280000	50	2000
11510000	1830000	2110000	7570000	55	2005
13220000	2050000	2270000	8900000	60	2010
13690000	2070000	2540000	9080000	61	2011
14480000	2200000	2800000	9480000	62	2012
14840000	2220000	2960000	9660000	63	2013
15240000	2260000	3020000	9960000	64	2014
16060000	2460000	3060000	10540000	65	2015
16880000	2510000	3240000	11130000	66	2016
16890000	2570000	3170000	11150000	67	2017
17030000	2510000	3220000	11300000	68	2018

Bayesian Data Analysis

- Models
  - Separate Model
  - Hierarchical Model
  - Gaussian Linear Model
- Priors

Models

Bayesian
Models

Our data is split into three categories (materials recycled, materials combusted, and materials landfilled), and their differences are an aspect of interest

So, a model is needed which can differentiate between them

We chose to implement and compare the separate and the hierarchical models for this purpose

Models

Separate Model

The separate model assumes that each category is independent from other categories, and essentially gives each category its own Bayesian model

Hence, it considers the categories to have no relation between each other

Models

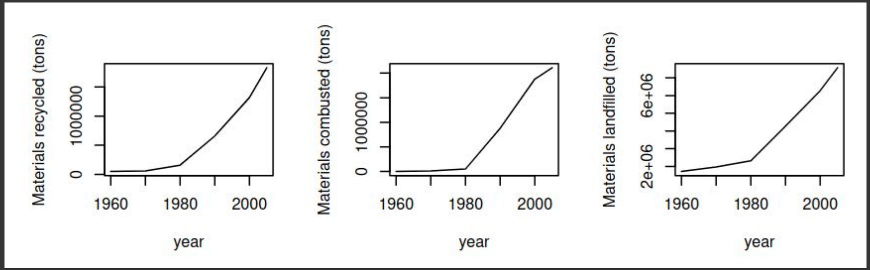
Hierarchical Model

The hierarchical model considers each category to be distinct but related enough that the data from one category can influence the predictions for another

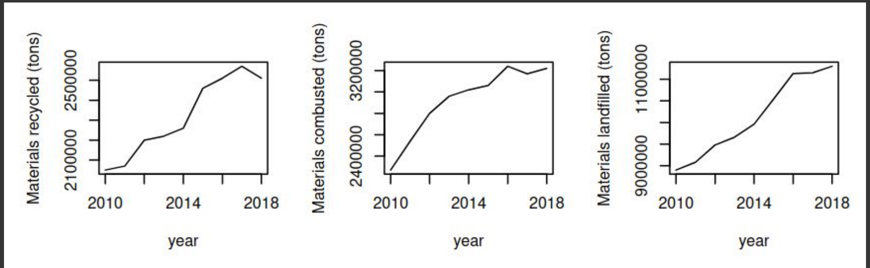
It provides separate models for each category but they share the same variance, and the means of the models share the same prior distribution constructed with hyperparameters, to keep them linked

Models

Gaussian Linear Model



1960-2005



2010-2018

As can be seen above, the data is approximately linear, with the year as the explanatory variable. Hence why we implemented both our models as Linear Gaussian Models.

This means that our parameters of interest are intercepts (α) and slopes (β) of the lines; and that the likelihood for this model is $y \sim N(\alpha + \beta * x, \sigma)$

Priors

Bayesian

Process of Prior Generation

Fitting a linear model to 1960-2005 data to get intercepts and slopes:

$\boldsymbol{\alpha} = (\alpha_{recycled'}\ \alpha_{combusted'}\ \alpha_{landfilled'}\ )$

$\boldsymbol{\beta} = (\beta_{recycled'}\ \beta_{combusted'}\ \beta_{landfilled'}\ )$

Calculating the standard deviations of data from 1960-2005:

$\boldsymbol{\sigma} = (\sigma_{recycled'}\ \sigma_{combusted'}\ \sigma_{landfilled'}\ )$

Bayesian

Process of Prior Generation

Separate Model

Alpha prior for category x:

$N(\alpha_x, 1000)$

α_x from $\boldsymbol{\alpha}$

A weak standard deviation (large number in comparison to intercept values)

Beta prior for category x:

$N(\beta_x, 10000)$

β_x from $\boldsymbol{\beta}$

A weak standard deviation (large number in comparison to slope values)

Sigma prior for category x:

$N(\sigma_x, 1000)$

σ_x from $\boldsymbol{\sigma}$

A weak standard deviation (large number in comparison to slope values)

Bayesian

Process of Prior Generation

Separate Model

$\alpha_i \sim N(\alpha_x, 1000)$
$\beta_i \sim N(\beta_x, 10000)$
$\sigma_i \sim N(\sigma_x, 1000)$
$y_i \sim N(\alpha_i + \beta_i * x, \sigma_i)$

	Alpha	Beta	Sigma
Materials recycled	N(0,1000)	N(39551,10000)	N(256043,1000)
Materials combusted	N(0,1000)	N(51622,10000)	N(333117,1000)
Materials landfilled	N(0,1000)	N(133578,10000)	N(843285,1000)

Separate model final priors (values rounded to 0 digits)

Bayesian

Process of Prior Generation

Hierarchical Model

Alpha mean prior:

N(mean(**α**), 100)

A weak standard deviation

Alpha standard deviation prior:

N(1000, 200)

Separate model α-prior standard deviation

A weak standard deviation

Beta mean prior:

N(mean(**β**), 1000)

A weak standard deviation

Beta standard deviation prior:

N(10000, 2000)

Separate model β-prior standard deviation

A weak standard deviation

Shared sigma prior:

N(mean(**σ**), 100000)

A weak standard deviation - Started smaller but had to be increased to improve posterior predictive checks

Bayesian

Process of Prior Generation

Hierarchical Model

α-mean ~ $N(\text{mean}(\boldsymbol{\alpha}), 100)$
α-sd ~ $N(1000, 200)$
β-mean ~ $N(\text{mean}(\boldsymbol{\beta}), 1000)$
α-sd ~ $N(10000, 2000)$
α_i ~ $N(\alpha\text{-mean}, \alpha\text{-sd})$
β_i ~ $N(\beta\text{-mean}, \beta\text{-sd})$
σ ~ $N(\text{mean}(\boldsymbol{\sigma}), 100000)$
y_i ~ $N(\alpha_i + \beta_i {}^* x, \sigma)$

	Alpha-mean	Alpha-sd	Beta-mean	Beta-sd	Sigma
Prior	N(0,100)	N(1000,200)	N(74917,1000)	N(10000,2000)	N(477481,1e+05)

Final hyperpriors (and sigma prior) of Hierarchical model (values rounded to 0 digits)

Bayesian Data Analysis

- Convergence Diagnostics
- Posterior Predictive Checks
- Predictive Performance Assessment
- Sensitivity Analysis
- Model Comparison

Convergence Diagnostics

Convergence Diagnostics

$\hat{R}$ and ESS

The hierarchical model had good R (<1.05) and ESS(>100) from the start. For the final priors the R and ESS are still very good.

For the separate model's initial priors, all the R and most ESS values indicated that the chains did not converge. This was first fixed by removing the sigma prior entirely. The sigma prior was later added back with different parameters.

Convergence Diagnostics

R and ESS

For Hierarchical Model

	R	bulk-ESS	tail-ESS
alpha[1]	1.00	2619	1914
beta[1]	0.99	3817	2364
alpha[2]	1.00	2130	1419
beta[2]	0.99	3863	2666
alpha[3]	1.00	2450	1599
beta[3]	1.00	3706	2090

For Separate Model

	R	bulk-ESS	tail-ESS
alpha[1]	1.00	2128	1278
beta[1]	0.99	3246	2515
alpha[2]	1.00	2269	1426
beta[2]	0.99	3295	2372
alpha[3]	1.00	2128	1215
beta[3]	1.00	3803	2778

Convergence Diagnostics

Hamiltonian Diagnostics

For our hierarchical model, there were some divergences in the initial priors. This was because:

a) the standard deviations used in the alpha_sigma and beta_sigma priors were small compared to the data and
b) we had summed the standard deviations instead of averaging them.

After fixing this there were no more divergences for the final priors.

For our separate model, there were no divergences in any iteration of the priors.

Posterior Predictive Checks

Analysis

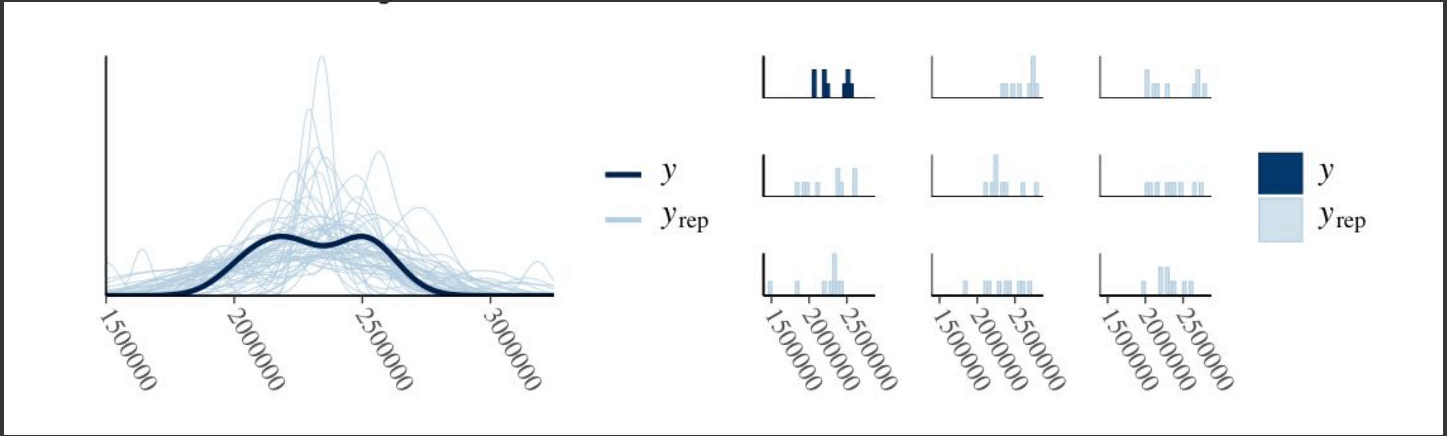
Posterior predictive checks

We predict data based on the models' posteriors and compare this to the observed data, looking for systematic differences between real and simulated data.

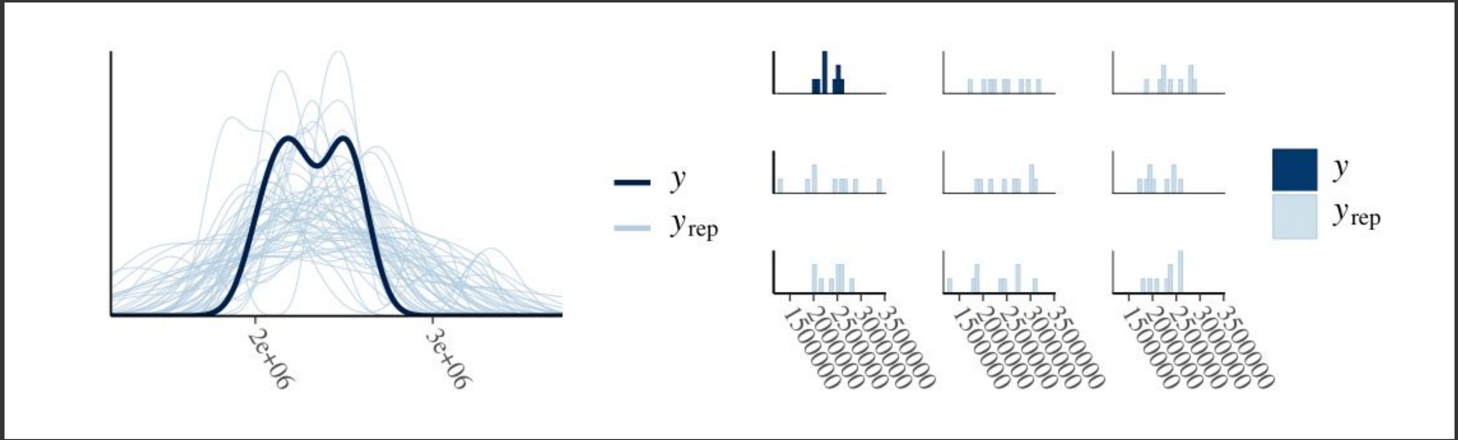
Posterior Predictive Checks

Materials Recycled

Separate Model



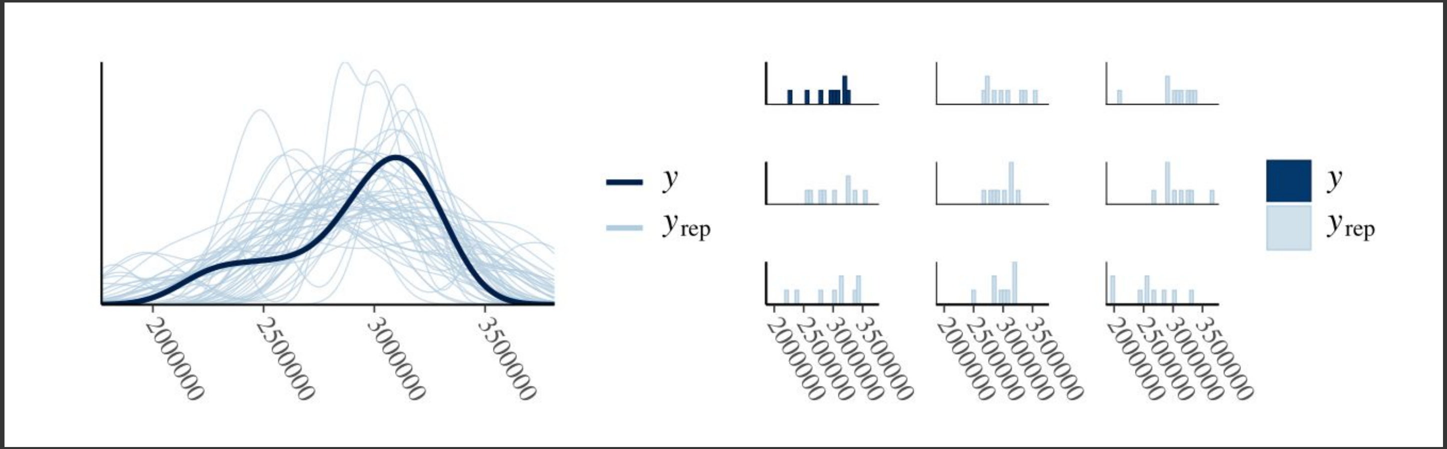
Hierarchical Model



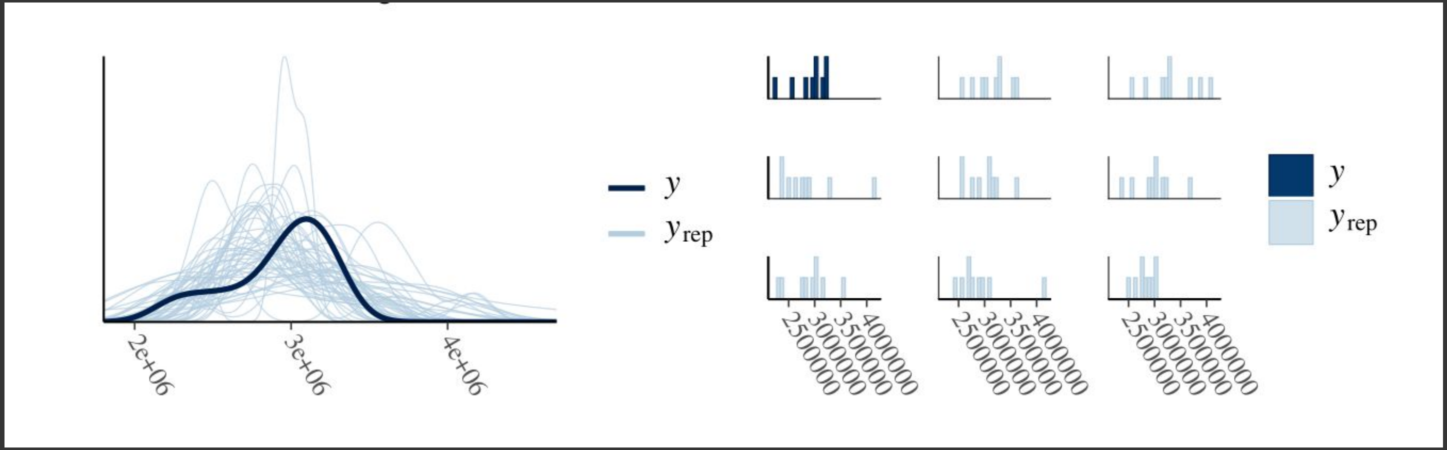
Posterior Predictive Checks

Materials Combusted

Separate Model



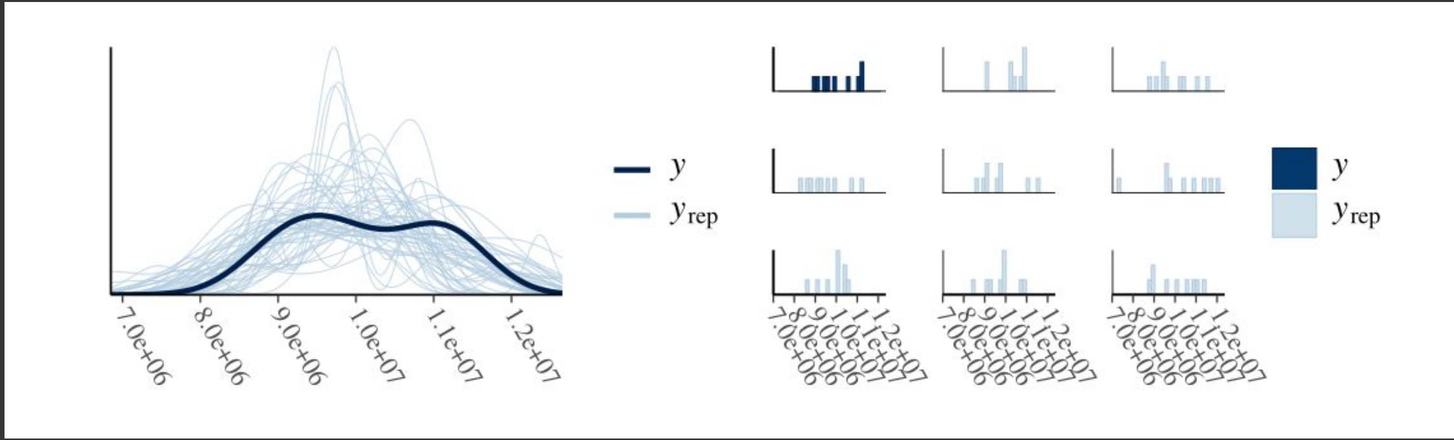
Hierarchical Model



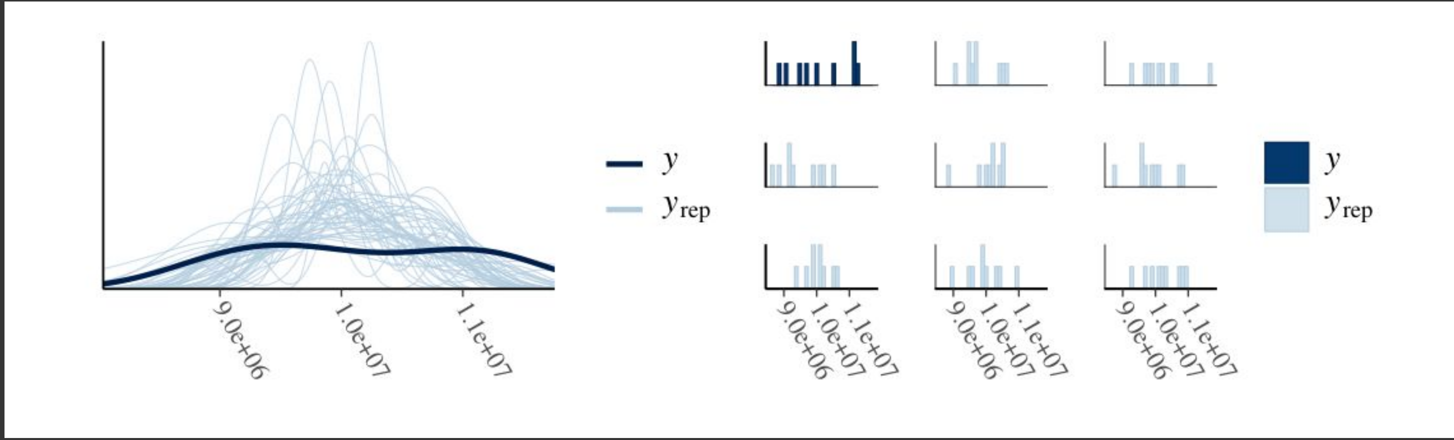
Posterior Predictive Checks

Materials Landfilled

Separate Model



Hierarchical Model



Predictive Performance Assessment

Analysis

Predictive Performance Assessment

Separate Model

	Materials recycled	Materials combusted	Materials landfilled
Real	2510000.0	3220000.0	11300000.0
Predicted mean	2457762.6	3108645.0	10429711.4
Predicted sd	269116.4	349642.1	900025.7

Hierarchical Model

	Materials recycled	Materials combusted	Materials landfilled
Real	2510000.0	3220000.0	11300000.0
Predicted mean	2508494.0	3130620.8	10634169.8
Predicted sd	407598.2	414242.3	410517.7

We fit the models excluding the year 2018, instead adding that year as the predicted year. We then compared the true values for 2018 to the predicted values.

Sensitivity Analysis

Analysis

Sensitivity Analysis

Two sets of priors were tested for the sensitivity analysis in addition to the final priors described above.

These priors are the “uniform prior” (used for every parameter), and the combination of N(10000,100) for mean parameters and N(0,50000) for standard deviation parameters which was called the “second prior” in this analysis.

“second prior” for separate model

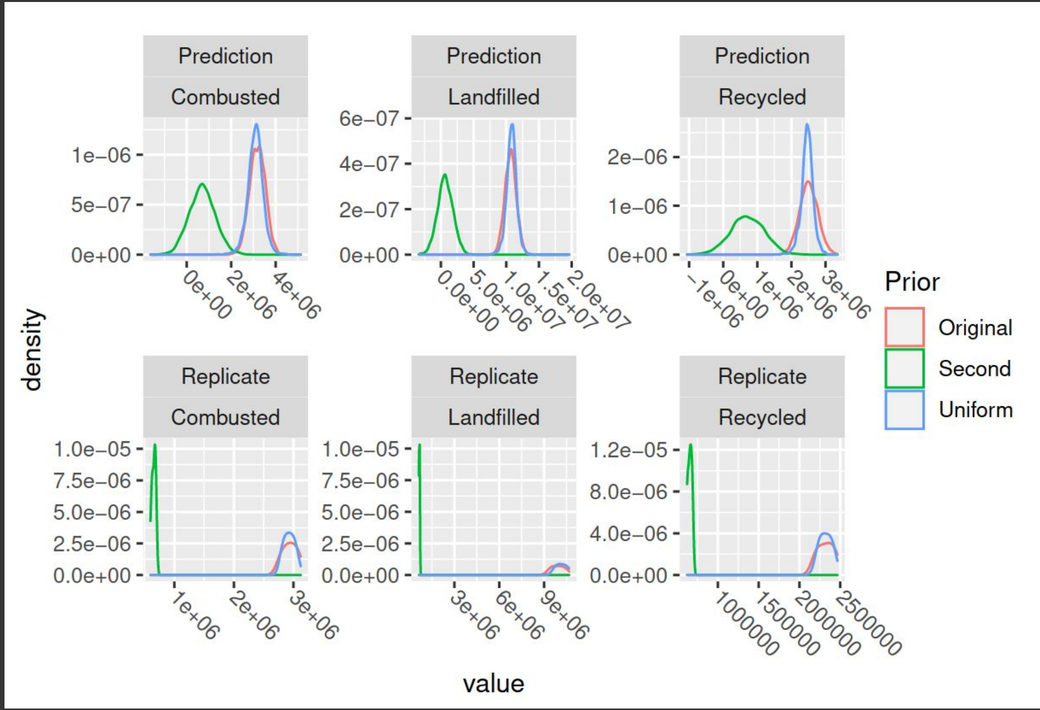
```
##    for (j in 1:J){
##      alpha[j] ~ normal(10000,100);
##      beta[j] ~ normal(10000,100);
##      sigma[j] ~ normal(0,50000);
##    }
```

“second prior” for hierarchical model

```
##     mu_alpha ~ normal(10000,100);
##     mu_beta ~ normal(10000,100);
##     sigma_alpha ~ normal(0,50000);
##     sigma_beta ~ normal(0,50000);
##     alpha ~ normal(mu_alpha, sigma_alpha);
##     beta ~ normal(mu_beta, sigma_beta);
##     sigma ~ normal(0,50000);
```

Analysis

Sensitivity Analysis - Separate

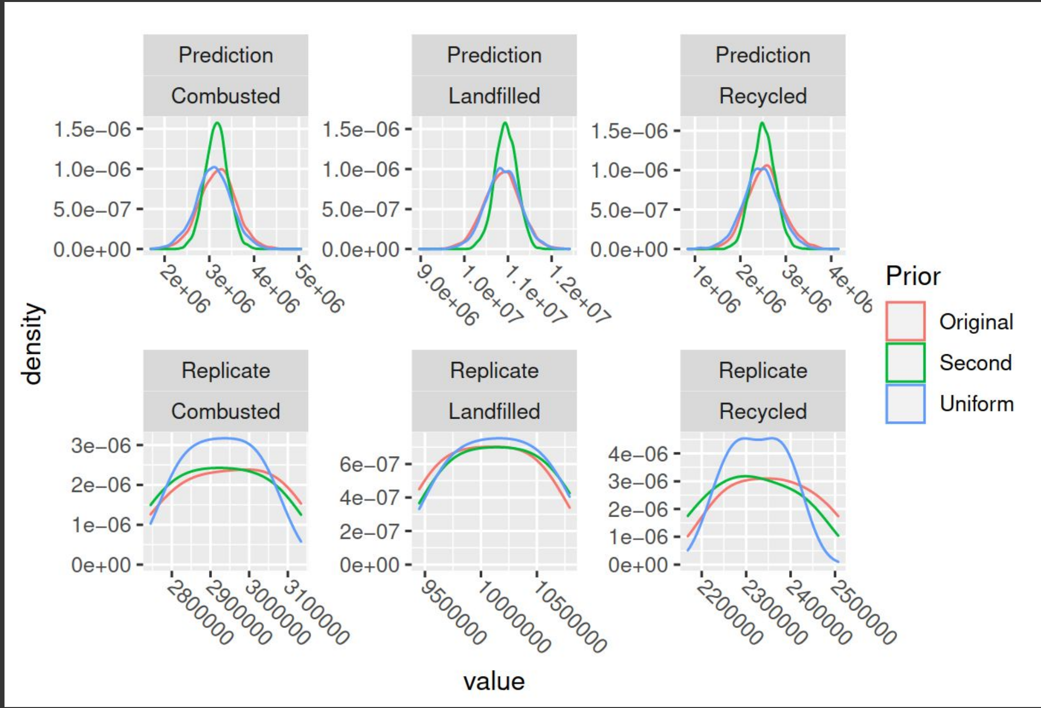


We see that the separate model is quite sensitive to prior choice

There is a small difference between the effect of the **"final priors"** and the **"uniform prior"**, but a large difference when the **"second prior"** is used

Analysis

Sensitivity Analysis - Hierarchical



We can see that the hierarchical model is much less sensitive to prior choice than the separate model.

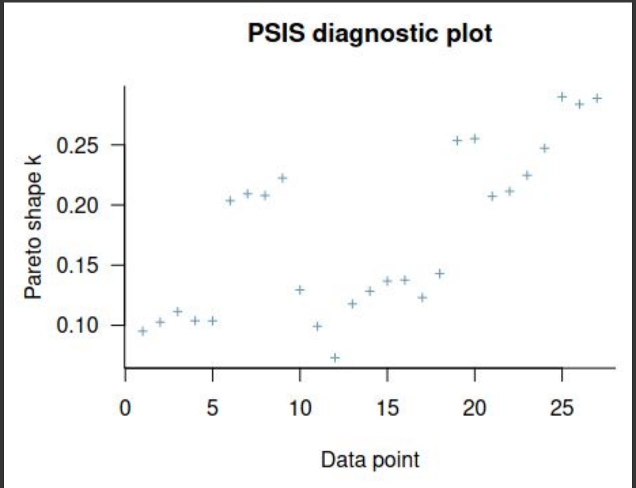
For the predicted quantities the prior choice doesn't have much effect, although the "second prior" results in a slightly narrower distribution

Model Comparison

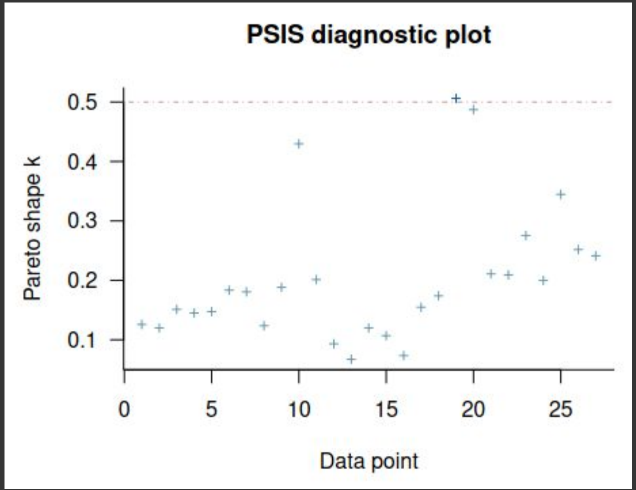
*Pareto Smoothed Importance Sampling

Analysis

Model Comparison (PSIS*)



PSIS plot for separate model



PSIS plot for hierarchical model

All observed k-hat values were <0.7

=> Importance sampling estimates are stable

*Leave-one-out cross-validation

Analysis

Model Comparison (LOO-CV*)

	Hierarchical	Separate
elpd	-383.98	-380.27
p_eff	2.97	0.97

LOO-CV results for both models

elpd - expected log predictive density
p_eff - effective number of parameters

Since the separate model has a slightly larger elpd value, it would be selected in accordance with LOO-CV

Conclusion

Bayesian Data Analysis

Conclusion

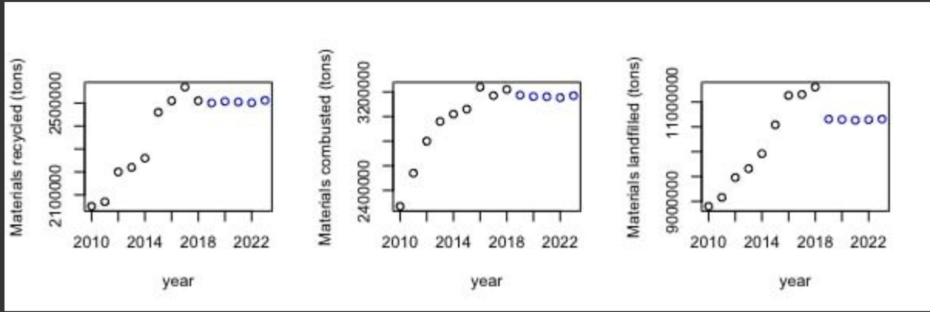
In accordance with both PSIS and LOO-CV, we can conclude that the separate model is superior to the hierarchical model in their current configurations, for this dataset

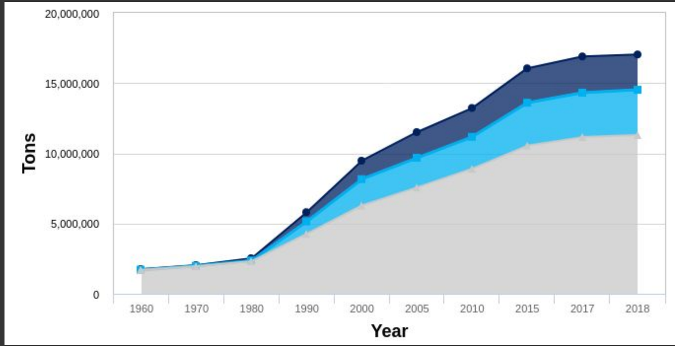
We have observed that the smaller the dataset, the more important the priors are in building a good model

This highlights the importance of domain expertise, and can be applied in fields where data is hard to gather e.g. measurements of particle accelerators or the treatment outcomes of patients with rare cancers

Bayesian Data Analysis
Conclusion

Grey: Landfilled
Light Blue: Combusted
Dark Blue: Recycled





According to our model, growth from 2018 onwards stagnates...

...which seems to be in line with the trends shown by the data[3]

[3] Environmental Protection Agency. (2019). *Advancing Sustainable Materials Management: Facts and Figures Report.* Retrieved from: https://www.epa.gov/facts-and-figures-about-materials-waste-and-recycling/textiles-material-specific-data

Report

Script