Challenge Problem: Lissajous Curves SOLUTIONS

Let's clear the global computing environment:

```
rm( list = ls() )
```

Problem 7: Lissajous Curves

A "Lissajous curve" is another beautiful type of curve that is defined by parametric equation for x and y, just as we saw for the Lemniscate of Bernoulli.

If you search for "Lissajous curve" in your favorite search engine you'll find many Internet references.

The Wikipedia entry is very thorough, and writes out the parametric equations right at the beginning of the article.

There are many different Lissajous curves, which depend on three parameters δ, a, and b.

The parameteric equation for the x coordinate is:

$$x(t) \;=\; \sin(at + \delta), \qquad 0 \leq t \leq 2\pi$$

The parameteric equation for the x coordinate is:

$$x(t) \;=\; \sin(bt) \qquad 0 \leq t \leq 2\pi$$

The Wikipedia article has a number of figures of Lissajous curves for various different choices of parameter values, and if you search a little you'll find diagrams of all the curves that we draw in this problem.

Part (a): Simple Lissajous curve

Let's graph a Lissajous curve!

We'll start with a very simple one, where $\delta = 0$, $a = 1$ and $b = 2$.

That means that the parametric equation for the x-coordinate is:

$$x(t) \;=\; \sin(t) \qquad 0 \leq t \leq 2\pi$$

The parametric equation for the y-coordinate is:

$$yx(t) \;=\; \sin(2t) \qquad 0 \leq t \leq 2\pi$$

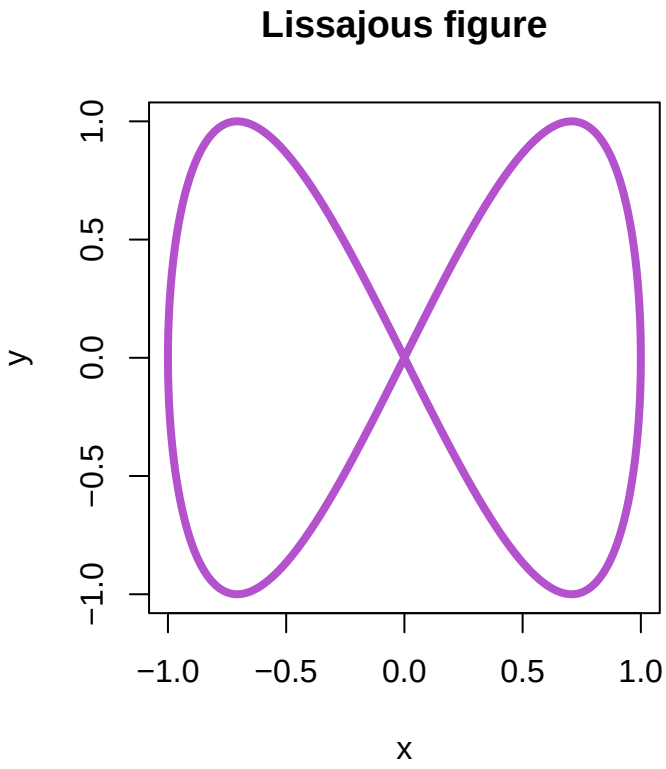
First, create a vector of parameter values from 0 to 2π using the `seq()` function.

Next, create an empty plotting region with no data, and make sure your graph is square.

Then plot this simple Lissajous figure.

Solution

```r
par( pty = "s" )

plot(
  x = NULL,
  xlim = c(-1, 1),
  ylim = c(-1, 1),
  main = "Lissajous figure",
  xlab = "x",
  ylab = "y"
)

parameter.vector <-
  seq(
    from = 0,
    to = 2 * pi,
    length.out = 1000
  )

x.vector <-
  sin( parameter.vector )

y.vector <-
  sin( 2 * parameter.vector )

lines(
  x = x.vector,
  y = y.vector,
  lty= "solid",
  lwd = 4,
  col = "mediumorchid3"
)
```



Part (b): Another Lissajous curve

Now let's graph another Lissajous curve.

Now we'll use the parameters $\delta = \pi/2$, $a = 3$, and $b = 4$.

That means that the parametric equation for the x-coordinate is:

$$x(t) \;=\; \sin\left(3t + \frac{\pi}{2}\right) \qquad 0 \le t \le 2\pi$$

The parametric equation for the y-coordinate is:

$$y(t) \;=\; \sin(4t) \qquad 0 \le t \le 2\pi$$

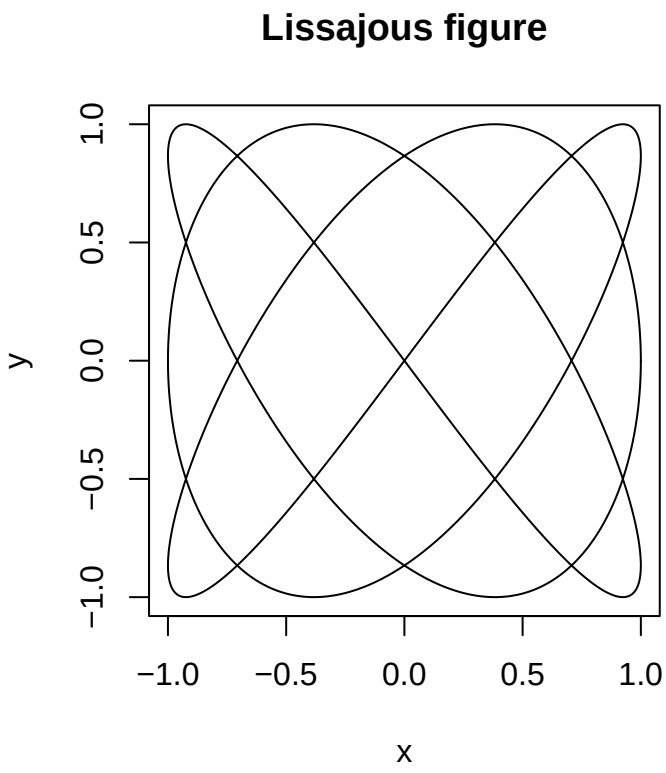
To draw this graph:

- Create an empty plot with no data, and use a square plot.
- Construct a vector of parameter values from 0 to 2π.
- Graph the Lissajous curve using the `lines()` function.

You are welcome and encouraged to copy the code that you wrote in part (a) and modify it.

Solution

```r
par( pty = "s" )

plot(
  x = NULL,
  xlim = c(-1, 1),
  ylim = c(-1, 1),
  main = "Lissajous figure",
  xlab = "x",
  ylab = "y"
)

parameter.vector <-
  seq(
    from = 0,
    to = 2 * pi,
    length.out = 1000
  )

x.vector <-
  sin( 3 * parameter.vector + pi/2 )

y.vector <-
  sin( 4 * parameter.vector )

lines(x.vector, y.vector)
```



Part (c): One more Lissajous curve

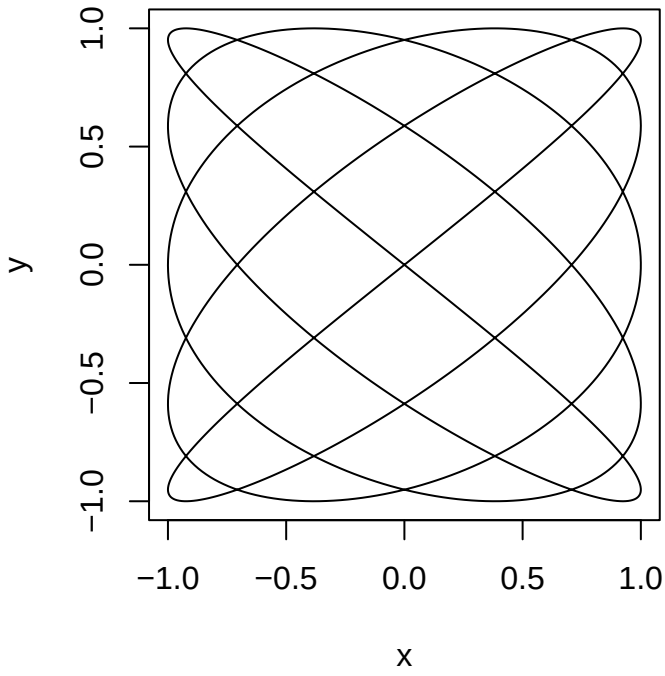
Now draw the curve with parameters $\delta = \pi/4$, $a = 5$, and $b = 4$.

Construct the graphing display just like before.

You're on your own for the parametric equations!

Solution

```
par( pty = "s" )

plot(
  x = NULL,
  xlim = c(-1, 1),
  ylim = c(-1, 1),
  main = "Lissajous figure",
  xlab = "x",
  ylab = "y"
)

parameter.vector <-
  seq(
    from = 0,
    to = 2 * pi,
    length.out = 1000
  )

x.vector <-
  sin( 5 * parameter.vector + pi/4 )

y.vector <-
  sin( 4 * parameter.vector )

lines(x.vector, y.vector)
```


Lissajous figure
y
x
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0