Multirate Kalman Filter for Sensor Data Fusion

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***Abstract*—This paper presents the adaptation of multirate Kalman filter to the multi sensor data fusion problem. Sensors operating at different resolutions and having different blurs observe the same phenomenon. The observations are modeled as the output of the analysis branches of a nonuniform filter bank with subband noise added to each branch. Multirate Kalman filter is used for obtaining minimum mean square error estimate from each sensor observation. The estimates weighted by the errror covariances in a specified way are then fused. Simultation results show an improvement in the SNR (Signal to Noise Ratio) performance with the additional sensor data. The additional sensor observations even at lower resolution, improves the over all estimate considerably.**

I. Introduction

Data fusion is a technique in which data from different sensors observing the same phenomenon is combined to obtain more insight and accuracy [4]. Target tracking is one of the main applications of multisensor data fusion [2], [5]. This is a statastical estimation problem. When all the sensors are operating at the same resolution, the estimation techniques designed are limited to the sensor resolution. In general different sensors are employed to capture different attributes i.e, individual sensors are used for observing position, velocity and acceleration. The resolution of all the sensors may not be the same. So to fuse data from these different sensors, a solution in the domain of multiresolution sensor data fusion is required. In image processing same scene is captured by different sensors [9], operating at different sampling rates and with different system fuctions. The operating spectrum of the sensor, the position of the sensor from the scence and practical constraints like portability, disturbances in the position, contribute to the heterogeneous system functions as well as resolutions. The solutions to the problems of target tracking by dynamic system state estimation and those regarding to image fusion have developed almost independently. In this paper we unify and exploit the strengths of both the approaches.

Carlson [10] presented a method to fuse data from decentralised processors/sensors. In his method the sampling rate of all the sensors was assumed to be the same. Another approach to the data fusion problem is based on wavelet methods. In the wavelet based decomposition of signals, the class of signals are defined over the dyadic tree and a data fusion method was developed by Chou et. al [1]. In their work while dealing with the finite lengths, signal reconstruction and boundary problems arise [6]. Here the sampling rate ratio between different sensors is limited to the powers of 2. Zhang et. al [5] also address the problem of multiresolution estimation. In their work the states of the dynamic system at different resolutions are related by a state projection equation. Also the ratio of sampling rates are restricted to the power 2. Yan et. al [6] extended the work in [5] to asynchronous sensors operating at any sampling rate ratio. These work are applicable to the case where multiple sensors observe different attributes at different resolutions. They do not address the case where same attribute is observed at different resolutions. Such a scenario better represents applications in the area of image fusion [12], computer vision etc., [4].

Therrien et. al [3], [11] gave the model for multirate observations of the same attribute and developed the multirate Weiner filter. The obsevations were modeled as the output of a linear filter followed by down sampling operation. Here the sampling rate ratios of different sensors is not restricted to powers of 2. The model is closely related to multispectral, multiresolution image fusion [12]. The design of statastically matched synthesis filter bank [13] is also close to this work. The nonuniform filter bank with subband noise [9] also resembles the observation model developed in this work.

In this paper we use the nonuniform filter bank model for the multi sensor observations. Multirate Kalman filter [2] is applied to each observation to estimate and interpolate the original signal from the lower resolution signals. The estimate from each sensor is weighted by the respective error variance as in [10], [6] and fused. Simulation results show a considerable the improvement in SNR performance with the additional data, even if the additional data is at lower resolution.

This paper is organised as follows. Section II presents modeling of sensor data observations as the output of nonuniform filter bank analysis branches with subband noise. Section III reviews state space representation of ARMA signals and Multirate Kalman filter. Section IV explains the recovery of signal from state vector and fusion of estimates based on the respective error variances. Section V presents the simulation results. Section VI contains the conclusions.

II. Model for Sensor Observation data

Consider a process observed by a sensor, which samples at greater than Nyquist rate. The observed samples constitute the sequence $y(n)$. Fig. 1 illustrates the discrete time model

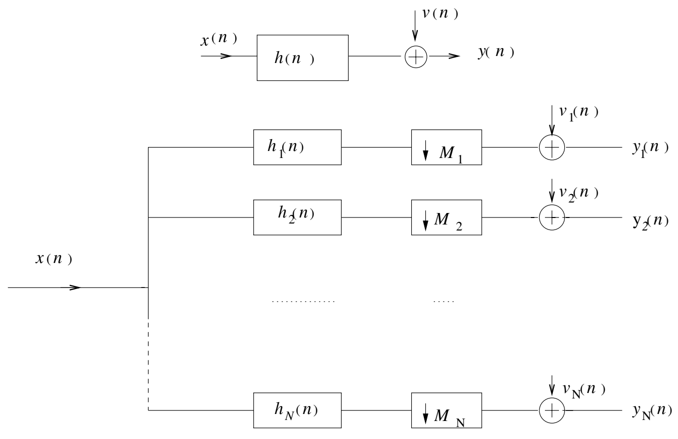

Fig. 1. Simple observation model

for the observation $y(n)$. The sensor effect is modeled by a Linear Time Invariant (LTI) system characterised by the impulse response $h(n)$. The measurement errors in the sensor, the environment noise and the quantization noise are modeled as additive noise $v(n)$.

This is a very simplistic representation of actual measurement process. A more correct representation requires us to take in to account individual blurs and sampling rates of each sensor. Each sensor can be modeled as an LTI system followed by a downsampler. The LTI system characterises the sensor blur and down sampling accounts the sensor resolution limitation. Each observation is further distorted by an additive noise as stated earlier in this section. The impulse response of the sensor blur is given by $h_i(n)$, M_i is the down sampling factor and $v_i(n)$ is the additive noise in the respective observation $y_i(n)$, for i =1, 2 $\ldots N$. Here N is the number of sensors. Fig. 1 also illustrates the model for the set of observations $y_i(n)$. It should be noted here that the model is the analysis stage of a nonuniform filter bank [9] with subband additive noise. Given the downsampling factor M_i's are not related, filter bank may or may not be maximally decimated.

III. State space model development

Multirate Kalman filter [2] is an extension of the standard Kalman filter, performing interpolation and estimation simultaneously. In this section we start with the state space model for the ARMA process, develop multirate state space model and explain the observation model.

A. State space model for ARMA(p, q) process

Kalman filter is a state estimation algorithm which minimises the mean square error. To apply Kalman filter to any problem, state space model for the process and the observation equation are required. Now we introduce the state space model for the ARMA process. If $x(n)$ is an ARMA(p, q) process then it will satisfy the following difference equation [8].

$$x(n) = \sum_{i=1}^{p} a_i x(n-i) + \sum_{j=0}^{q} b_j w(n-j) \qquad (1)$$

where $b_0 = 1$ without loss of generality.

Note that ARMA(p, q) process $x(n)$ can be viewed as the output of an LTI system driven by white noise $w(n)$, hence satisfies (1). The state equation is given as,

$$\mathbf{z}(n+1) = \mathbf{A}\mathbf{z}(n) + \mathbf{b}w(n+1) \qquad (2)$$

where,

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{0}_{1\times(\overline{p}+q)} \\ \mathbf{A}_2 \end{bmatrix}$$

$$\mathbf{z}(n) = \begin{bmatrix} w(n-(q-1)) \\ \cdot \\ \cdot \\ w(n) \\ x(n-(\overline{p}-1)) \\ \cdot \\ \cdot \\ x(n) \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} \mathbf{0}_{(q-1)\times 1} \\ 1 \\ \mathbf{0}_{(\overline{p}-1)\times 1} \\ 1 \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & 1 & . & . & . & . \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 0 & . & . & 0 & 1 & 0 & . & . & 0 \\ 0 & 0 & . & . & 0 & 0 & 1 & . & . & 0 \\ . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . \\ b_q & . & . & . & b_1 & a_{\overline{p}} & . & . & . & a_1 \end{bmatrix}$$

The augment variable $\overline{p}$ is chosen to be greater than p. $a_i = 0$ for $i = p, p+1, \ldots, \overline{p}$.

As shown in Fig. 1 the observation $y(n)$ is obtained by passing $x(n)$ through an LTI system with impulse response $h(n)$ and corrupted by an additive noise $v(n)$. This is a simple model for the sensor without considering any limitation in the sensor resolution. The observation $y(n)$ can be obtained from the state vector $\mathbf{z}(n)$ as follows,

$$y(n) = \mathbf{H}\mathbf{z}(n) + v(n) \qquad (3)$$

where, $\mathbf{H} = \begin{bmatrix} \mathbf{0}_{(1\times q)} & h_{\overline{p}-1} & . & . & . & h_0 \end{bmatrix}$. The filter length in general may not be equal to $\overline{p}$. We chose $\overline{p}$ to be larger than the length of the filter and the ARMA process order p for better estimation of $x(n)$. The filter length is made equal to $\overline{p}$ by padding zeros.

The white gaussian noise $w(n)$ and $v(n)$ are uncorrelated to each other.

$$\begin{aligned} E\{w(n)w(m)\} &= Q_w(n)\delta(n-m) \\ E\{v(n)v(m)\} &= Q_v(n)\delta(n-m) \\ E\{w(n)v(m)\} &= 0, \quad \forall n, m \in \mathbf{Z} \end{aligned}$$

The state space model developed is for the original signal which is sampled at a sufficient rate. This is no longer helpful when the sensors are operating at lower resolutions. So in the following section a Multirate state space model which helps in the modeling of the multi resolution sensor observation is developed.

B. Multirate state space Model

The observation $y(n)$ in (3) is decimated by a factor M. So $y(n)$ is available only at sample values $n = 0, M, 2M, \ldots$. Hence the state space model developed in the previous section is extended to the steps of M increment in n as follows,

$$\begin{aligned}\mathbf{z}(n+2) &= \mathbf{A}\mathbf{z}(n+1) + \mathbf{b}w(n+2) \\ &= \mathbf{A}\left(\mathbf{A}\mathbf{z}(n) + \mathbf{b}w(n+1) \right) + \mathbf{b}w(n+2) \\ &= \mathbf{A}^2\mathbf{z}(n) + \begin{bmatrix} \mathbf{A}\mathbf{b} & \mathbf{b} \end{bmatrix} \begin{bmatrix} w(n+1) \\ w(n+2) \end{bmatrix}\end{aligned}$$

Proceeding along the same lines ,

$$\mathbf{z}(n+M) = \mathbf{A}^M\mathbf{z}(n) + \mathbf{B}_M\mathbf{w}_M(n) \tag{4}$$

where $\mathbf{w}_M(n)$ is the block driving noise and $\mathbf{B}_M$ the parameter matrix are defined as follows.

$$\mathbf{w}_M(n) = \begin{bmatrix} w(n+1) \\ w(n+2) \\ . \\ . \\ w(n+M) \end{bmatrix},$$

$$\mathbf{B}_M = \begin{bmatrix} \mathbf{A}^{M-1}\mathbf{b} & \mathbf{A}^{M-2}\mathbf{b} & . & . & . & \mathbf{b} \end{bmatrix}$$

The covariance matrix of the block driving noise $\mathbf{w}_M(n)$ is,

$$E\{\mathbf{w}_M(n)\mathbf{w}_N^T(m)\} = Q_M(n)\delta(n-m),$$

where

$$Q_M(n) = \begin{bmatrix} Q_w(n+1) & & 0 \\ & . & \\ & & . \\ 0 & & Q_w(n+M) \end{bmatrix}$$

where $n, m = 0, M, 2M \ldots$

Since $v(n)$, the additive noise is uncorrelated with the driving noise $w(n)$, it is also uncorrelated with the block driving vector $\mathbf{w}_M(n)$.

IV. Multirate Kalman filter

This section explains the Multirate Kalman filtering algorithm to estimate the state vector $\mathbf{z}(n)$ from the observations $y_i(n)$, which are corrupted by noise as well as at lower resolution. The observations $y_i(n)$ are to be interpolated and the noise effect has to be reduced. Thus Multirate Kalman filter performs both estimation and interpolation simultaneously. The desired signal $x(n)$ is embedded in the state vector $\mathbf{z}(n)$. Each state vector has $\overline{p}$ samples of the original signal $x(n)$. It is to be noted that $\overline{p}$ is chosen sufficiently large for estimation and interpolation. In the following discussion, $\hat{\mathbf{z}}(n/j)$ is the estimate of $\mathbf{z}(n)$ given the observation sequence $\{y(0), y(1), \ldots, y(j)\}$. The respective error is given by $\mathbf{e}(n/j) = \mathbf{z}(n) - \hat{\mathbf{z}}(n/j)$ and the error covariance matrix is $\mathbf{P}(n/j) = E\{\mathbf{e}(n/j)\mathbf{e}^T(n/j)\}$.

The estimation step of Multirate kalman filter [2] is given as

$$\begin{aligned}\hat{\mathbf{z}}(n+M/n+M) &= (\mathbf{I} - \mathbf{k}(n+M)\mathbf{H})\mathbf{A}^M\hat{\mathbf{z}}(n/n) \\ &+\mathbf{k}(n+M)y(n+M), \qquad n = 0, M, 2M, \ldots.\end{aligned} \tag{5}$$

.

The Kalman gain $\mathbf{k}(n)$ and the error covariance matrices are recursively calculated using,

$$\begin{aligned}\mathbf{k}(n+M) = \mathbf{P}(n+M/n)\mathbf{H}^T(&\mathbf{H}\mathbf{P}(n+M/n)\mathbf{H}^T \\ &+Q_v(n+M))^{-1}\end{aligned} \tag{6}$$

$$\mathbf{P}(n+M/n) = \mathbf{A}^M\mathbf{P}(n/n)\mathbf{A}^{MT} + \mathbf{B}_M\mathbf{Q}_M(n)\mathbf{B}_M^T \tag{7}$$

$$\mathbf{P}(n+M/n+M) = (\mathbf{I} - \mathbf{k}(n+M)\mathbf{H})\mathbf{P}(n+M/n) \tag{8}$$

We can estimate the state vector $\mathbf{z}(n)$ by applying the above equations recursively.

V. Signal Reconstruction and Fusion

A. Reconstruction of the signal

From the state vector estimate $\hat{\mathbf{z}}(n/n)$ we reconstruct the observation signal $x(n)$ in blocks of M sample points. we had chosen $\overline{p}$ to be greater than M, which helps in the interpolation. The accuracy of the estimate depends on the number of the observation samples. The most recent samples of the signal $x(n)$ are extracted from the state vector estimate $\hat{\mathbf{z}}(n/n)$ as follows,

$$\begin{aligned}\hat{\mathbf{x}}_M(n) &= \begin{bmatrix} \widehat{x}(n-(M-1)) \\ . \\ . \\ . \\ \widehat{x}(n) \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{0}_{M\times\overline{p}+q-M}\mathbf{I}_{M\times M} \end{bmatrix} \widehat{\mathbf{z}}(n/n)\end{aligned} \tag{9}$$

where $n = 0, M, 2M \ldots$. In the above equation we are picking up recent M elements of $x(n)$ in the vector $\hat{\mathbf{z}}(n/n)$. The vector $\hat{\mathbf{x}}_M(n)$ is the estimate of $\mathbf{x}_M(n)$, i.e., $\widehat{x}(k)$ is the estimate of $x(n)$ at the sampling instant $n = k$. This implies that the error covariance matrix of the observed signal vector $\mathbf{x}_M(n)$ is obtained from the lower pricipal block diagnal element of size $M \times M$ in $\mathbf{P}(n/n)$. The last M diagnal elements in $\mathbf{P}(n/n)$ give the error variance in the respective samples of the estimated signal.

B. Data Fusion

The estimates from each sensor observations obtained from Multirate Kalman filtering has to be fused. Let $\widehat{x}(n, i)$ be the signal estimated by the i^{th} sensor and the respective estimation error variance be $P(n,i)$. Then the fused estimate is given as follows [10],

$$\widehat{x}(n) = \sum_{i=1}^{N} \alpha_{n,i}\widehat{x}(n,i) \quad (10)$$

where

$$\alpha_{n,i} = (\sum_{j=1}^{N} P^{-1}(n,j))^{-1} P^{-1}(n,i)$$

It should be noted here that the estimate with more error variance is given less weightage. The weights are time dependent and change along with the variations in the error variances. The sum of all the weights applied to the sensor estimates is unity, which implies that the fusion process is lossless.

VI. Simultaion Results

In this section simulation results are presented. The results demonstrate the effectiveness of the fusion algorithm. We chose the original signal as the ARMA(4,2) process. The coefficients chosen were a_1=2.760, a_2=-3.809, a_3=2.654, a_4=-0.924, $b1$=-0.20, b_2=0.04 [2]. The augment variable is chosen to be $\overline{p}$=5. The impluse response of the sensors were chosen as $h(n) = \delta(n)$. This choice aids simulation, while not affecting the validity of the theory for a more general blur. The Kalman filter is initialised with state vector $\mathbf{z}_0 = [\ 0\ .\ .\ .\ 0\ ]^T$ and the error covariance $\mathbf{P}_0 = \mathbf{I}$. The performance is measured in terms of SNR in dB. In Fig. 1, for a white additive noise $v(n)$, the effect of placing the down sampler before or after the adder is same. Hence we are defining the SNR of the low resolution signal as follows,

$$\text{SNR}{=}10\log_{10}\left(\frac{\sum_{k=iM} x^2(k)}{\sum_{k=iM} v^2(k)}\right)$$

In this simulation we first consider the two sensors case. The direct signal at a fixed SNR 0 dB is considered along with a sub sampled signal (M=2). The SNR of the sub sampled observations was varied from -2 dB to 10 dB . Fusion algorithm is applied and the estimated output signal SNR is calculated as

$$\text{SNR}{=}10\log_{10}\left(\frac{\sum_{k} x^2(k)}{\sum_{k}(x(k)-\widehat{x}(k))^2}\right)$$

Fig. 2 shows the plot of SNR of the high resolution reconstructed waveform versus the SNR of the low resolution waveform. The direct sequence is at 0 dB and the decimated sequence even though at -2 dB contributes significantly to the fused estimate. Even very small amount of redundancy is effectively exploited by the fusion algorithm. In Fig. 3 we show a comparision between the original waveform and the estimated waveform, when the sub sampled signal is at 4 dB. The plot shows the accuracy of the estimation technique, demonstrates why this technique is popular in the tracking applications.

To better visualise how the Multirate Kalman filter and the fusion algorithm improve the overall estimate we now

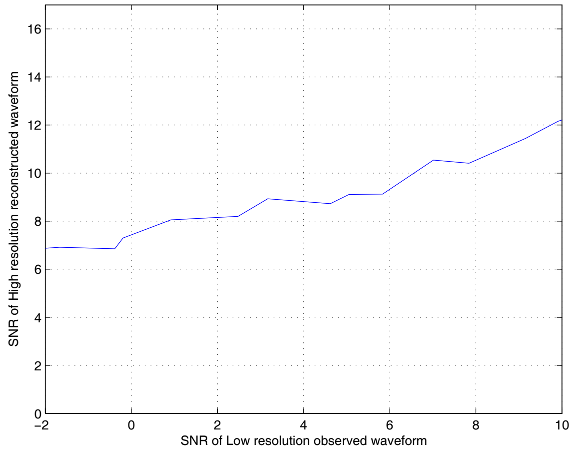

Fig. 2. Data fusion performance of two sensors

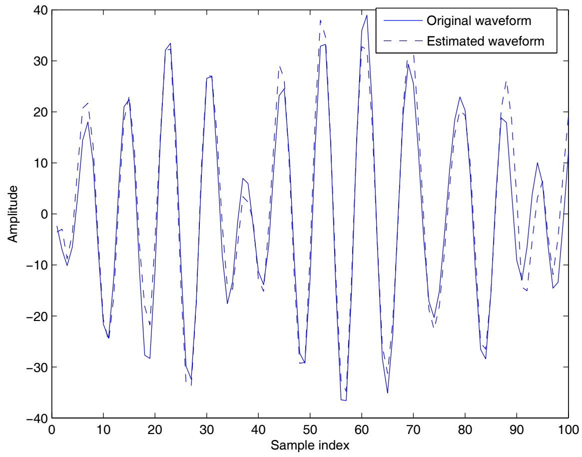

Fig. 3. Estimated waveform with original waveform

coinsider the case of three sensors. Two sensors whose observations are at a lower resolution $(M = 2,3)$ are taken along with a direct sequence. This example is chosen to show that there is no restriction on the ratio of sub sampling rates of different sensors. The SNR values of all the sensor observations are varied from -3 dB to 12 dB and the SNR of the estimates from individual sensors is plotted along with the fused estimate. Fig. 4 clearly demonstrates the improvement in the fused estimate.

VII. Conclusion

In this paper we have used Multirate Kalman filter for the data fusion problem where the observations of the sensors, operating at different resolutions, are modeled as the analysis stage of a nonuniform filter bank. The sensor resolutions can be of any integer ratio. The estimates of the original signal obtained from each of the sensor observations are combined by appropriately weighting them in accordance with their

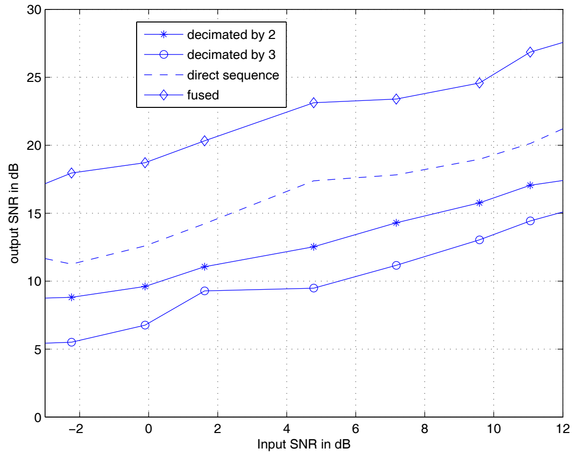

Fig. 4. Data fusion performance of three sensors

respective error variances. The simulation results demonstrate the application potential of the theory. This theory has the potential to be extended to the 2-D case and applied to multi spectral image fusion.

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