Cultural Bootstrapping of Quantity Terminologies Based on Approximate Number Sense

Content areas: Agent-Based Simulation and Emergence, Cognitive Modeling, Natural Language Semantics

Abstract

We present an agent-based model and agent-based simulations where a population engages in communicative interactions in a shared environment to bootstrap a language. We focus on the part of language that expresses quantities (so-called quantity terminologies) and on the properties such expression share across (almost) all natural languages (so-called linguistic universals). We look at two universals in the semantic domain of quantities: convexity and monotonicity. We test whether they can be explained in terms of human number perception as described by the theory of approximate number sense (ANS). Using a state-of-the-art language coordination model, with the perceptual layer substituted by the ANS, we evolve communicatively usable quantity terminologies. We compare the degrees convexity and monotonicity of languages evolving with and without ANS. The results suggest that ANS supports the development of monotonicity and, to a lesser extent, convexity.

1 Introduction

Even though natural languages seem quite different from each other, linguists observed many striking similarities. It is widely believed that languages share some non-trivial properties, known as *linguistic universals*. They have been hypothesised at many levels of linguistic analysis, including phonology [Hyman, 2008], syntax [Chomsky, 1965; Croft, 1990], and semantics [Barwise and Cooper, 1981].

Once a linguistic universal is attested, the question arises as to the source of its ubiquity. Common answers relate universality to human cognition [Christiansen and Chater, 2016], communication [Kemp and Regier, 2012], complexity [Rolando *et al.*, 2018], and learnability [Tiede, 1999; Gierasimczuk, 2009; Carcassi *et al.*, 2019; Steinert-Threlkeld and Szymanik, 2019; Steinert-Threlkeld and Szymanik, 2020]. In this paper we look at how the shared subsystem of the human cognition responsible for the perception of numerosities can shape the fragment of language for denoting quantities (henceforth, *quantity terminologies*), supporting the development of these universals.

Human cognition is hypothesized to be equipped with an evolutionarily old mechanism of number cognition called the *approximate number sense* (ANS, for short) [Dehaene, 1997]. ANS allows for instant perception of quantities at the cost of accuracy, with an error proportional to the intensity (cardinality) of the perceived input. An intricate issue is whether the perceptual constraints of the ANS support the emergence of regularities such as convexity and monotonicity. To answer this question we employ a methodology of computer simulations where a community of artificial agents engages in communicative interactions in a shared environment to bootstrap a language. Such models, based on similar principles, has been used extensively in AI and language evolution studies (see, e.g., [Steels, 2012]). We use one of the state-of-the-art solutions to the language coordination problem, originally applied to color terms [Steels and Belpaeme, 2005] and transfer it to the domain of quantities by substituting its perceptual layer by the ANS. We show that the emergent quantity terminologies are communicatively usable. This proves that the model constitutes a hypothetical mechanistic theory of the transition from the stage without quantity terminology to the stage in which such a terminology exists. Finally, we contrast the degrees of convexity and monotonicity of terminologies resulting from the cognitively-plausible ANS-based model with the respective degrees obtained from a cognitively-neutral model, where the perception error is small and independent of the perceived input.

The model is described in Section 2. Section 3 defines notions of *meaning* and *pragmatic meaning* in the model and introduces two semantic universals that can be attributed to quantity terms: *convexity* and *monotonicity*. Section 4 constitutes the experimental part. Section 5 concludes the paper.

2 Language coordination model

Stimuli

For simplicity, in our model we deal directly with quantities (henceforth, *stimuli*), which represent quantitative properties of the observed situations. That is consistent with the semantic literature, where a quantity term is formalised as a quantifier defined to be a (closed on isomorphism) class of situations (models) to which it can be truthfully applied [Peters and Westerståhl, 2006]. For example, *Some boys are rude* is true if, in given situation, the set of boys intersected with the set of rude children is nonempty. Formally, the quantifier denoted by *some* is the class of models (A, B) such that $A \cap B \neq \emptyset$.

A **numeric-based stimulus** is a positive integer. It can be interpreted as the number of objects presented to an agent. A **quotient-based stimulus** is a reduced fraction $\frac{n}{k}$. It can be interpreted as the ratio of n objects of a given kind among all k objects presented to an agent. In experiments, a numeric-based stimulus is sampled from the uniform discrete probability distribution $\mathcal{U}\{1, 20\}$; a quotient-based stimulus is obtained by first sampling a denominator $k \sim \mathcal{U}\{1, 20\}$ and then a non-greater numerator $n \sim \mathcal{U}\{1, k\}$.

Interaction between agents is grounded in a shared **context**. A context is a set of two stimuli, either numeric-based or quotient-based, depending on the chosen simulation.

Agents' perception is founded on simple internal representations of stimuli called **reactive units**. Reactive units are then combined to form higher-level representations called **categories**. Reactive units and categories are recruited in a **discrimination game** in order to differentiate between stimuli within the context; in case of discriminative success or failure, categories are suitably adapted.

Reactive units

Reactive units code the perception of numerosity called approximate number sense [Dehaene, 1997]. The ANS representation of n is

$$R_n \sim \mathcal{N}(n, \gamma n), \tag{1}$$

where $\mathcal{N}(n, \gamma n)$ is the normal distribution with mean n and standard deviation γn, with $\gamma = 0.1$.

Note that due to approximate nature of representation, there is always some uncertainty regarding the exact value causing a given sensation. Moreover, numbers further from the exact value are less likely to cause the sensation. Finally, the greater the number, the wider the scope of plausible numerocities causing the sensation. The latter property is a realization of the Weber law [Fechner, 1966].[1] Examples of numeric-based reactive units are presented in Figure 1.

Reactive unit representing a numeric-based stimulus n is simply R_n. In practice, R_n is approximated—the actual probability density function associated with R_n is not supported on $\mathbb{R}$ but on a sufficiently long segment encompassing almost 100% of the true probability mass. The ANS representation of a quotient-based stimulus $\frac{n}{k}$ is the ratio distribution:

$$R_{n/k} \sim R_n / R_k. \tag{2}$$

Since R_n, R_k are gaussian distributions, we could, in principle, compute the probability density function of $R_{n/k}$ based on known analytical expressions (see, e.g., [Hinkley, 1969; Pham-Gia *et al.*, 2006]). In practice, we resort to an empirical approximation of $R_{n/k}$. We first draw two equally-sized large samples $(n_i)_{i=1}^{s}$ and $(k_i)_{i=1}^{s}$ from R_n and R_k, respectively. Then we derive a density histogram based on ratios $(\frac{n_i}{k_i})_{i=1}^{s}$. Finally, we obtain a desired approximation by interpolating the resulting histogram, see Figure 1 for reactive units of quotient-based stimuli. If the exact form of a reactive unit is irrelevant, we denote it by z, possibly with subscripts. When we write $z(x)$, we mean $p_z(x)$, i.e., the value of the probability density function associated with z for $x \in \mathbb{R}$.

While simple numerical expressions have been empirically studied with respect to their ANS representations, quotients have not. We decided to follow here a possible mathematically justified interpretation of quotients. We leave the cognitive validation of this proposal to further work.

Categories

Each category consists of a finite number of reactive units $z_1, z_2, \ldots, z_k$ and corresponding weights $w_1, w_2, \ldots, w_k$, $w_i \geq 0$. The activation pattern of a category c is given by:

$$c(x) = \Sigma_{i=1}^{k} w_i z_i(x). \tag{3}$$

A response of c to a stimulus q is defined by:

$$< c|R_q >= \int_{-\infty}^{+\infty} c(x) R_q(x) dx. \tag{4}$$

The rationale behind (4) is twofold. First, the stimulus is not perceived directly but through reactive unit so we cannot assume that the response of c to q is simply $c(q)$. Second, intuitively, the response of c to q should be proportional to the joint contributions of c and R_q over the same regions. For example, if the activation of c and z for a given region is high, the response should be higher, accordingly. Similarly, if the activation of z is low for a given region, the response should also be low. These intuitive properties are realized by the scalar product of c and z, as shown in equation (4).[2]

Discrimination Game

A discrimination game is an individual cognitive mechanism in which, on an input of a stimulus, the agent outputs a category which distinguishes that stimuli (the topic) from other stimuli in the context [Steels, 1997]. An agent is presented with a context q, q' with $q_T \in \{q, q'\}$ being the topic. Let C be the set of categories of the agent. If $C = \emptyset$, the game fails. If $\langle c|R_q\rangle = 0$ or $\langle c|R_{q'}\rangle = 0$ for all $c \in C$, the game fails. Let $c = \underset{c \in C}{\text{argmax}} \langle c|R_q\rangle$ and $c' = \underset{c \in C}{\text{argmax}} \langle c|R_{q'}\rangle$. If $c \neq c'$, the game is successful and the winning category (c if $q = q_T$, c' otherwise) is returned. If $c = c'$, the discrimination game fails.

Discriminative success (DS) describes how well an agent copes with the problem of distinguishing topic from the rest of the context in a series of games. Let $ds_j^a = 1$ if the j-th discrimination game of an agent a is successful. Otherwise, let $ds_j^a = 0$. The cumulative discriminative success of an agent a at game j for the last n games (we use $n = 50$) is:

$$DS(n)_j^a = \frac{1}{n} \Sigma_{i=max(0,j-n)}^{j} ds_i^a. \tag{5}$$

If a, j are known/irrelevant, we write DS to mean $DS(50)_j^a$.

[1] In Steels and Belpaeme [2005], the standard deviation of reactive units is constant. This is justifiable because the CIELAB space is antropocentric: equal distances between colors in that space correspond to approximately equal perception of color distance by humans.

[2] In Steels and Belpaeme [2005], a response of c to a color stimulus $\overline{x}$ is $c(\overline{x})$. This is justified because $\overline{x}$ stands for a complex activation pattern reduced to a single point in the CIELAB space.

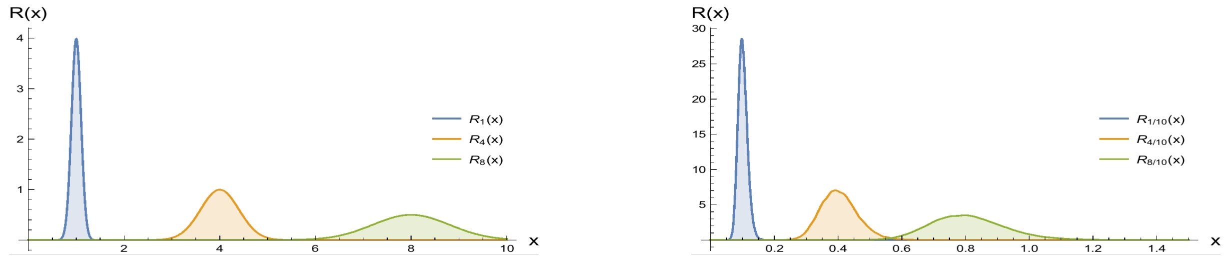

Figure 1: Examples of reactive units for numeric-based stimuli $1, 4$ and 8 (left) and quotient-based stimuli $1/10, 4/10$ and $8/10$ (right). The reactive unit $R_{8/10}$ is visibly different from a normal distribution because of its longer right tail.

Success When a discrimination game is successful, the winning category is reinforced to enhance its chances of winning similar games in the future. Let c be the winning category. Let w_i, z_i, $i = 1, 2, \ldots, k$, be the weights and reactive units of c, respectively. Reinforcement is computed according to the following expression:

$$w_i = w_i + \beta \cdot \langle z_i | R_{q_0} \rangle, \tag{6}$$

i.e., the weights of c are increased proportionally to the corresponding reactive units' responses to the topic. β is called the learning rate. In our simulations we assume $\beta = 0.2$.

Failure An agent repairs the system of categories to avoid a similar malfunction in the future.

1. Suppose $C = \emptyset$. We set $C = \{c\}$, where c is a new category with one reactive unit R_{q_T} and weight 0.5.
2. Suppose $\langle c|R_q\rangle = 0$ or $\langle c|R_{q'}\rangle = 0$ for all $c \in C$. A new category is added in the same way as in the first point above (for q, if q does not yield any positive response, otherwise for q').
3. Suppose $c = c'$. If $DS < \delta$, we set $C := C \cup \{c\}$, where c is a new category with one reactive unit R_{q_T} and weight 0.5. If $DS \geq \delta$, the category c (and thus c') is adapted by appending R_{q_T} with initial weight 0.5. δ is called the discriminative threshold and set to 95%.

After the completion of the game and the associated repairs, the weights of all reactive units of all categories of the agent are decreased by a small factor $0 < \alpha < 1$ ($\alpha = 0.2$), $w_i = \alpha w_i$, which results in slow forgetting of categories. If all weights of a category are smaller than 0.01, the category is deleted.

Language and Guessing Game

A dictionary is a collection of strings (obtained using the `gibberish` package [Haskins, 2016]). A language is a weighted map $L : F \times C \to [0, \infty)$ between agent's dictionary F and her set of categories C. $L(f, c)$ is the strength of the connection between f and c. We write $F(L)$ and $C(L)$ to denote the dictionary and the set of categories of L.

A guessing game involves the speaker S and the hearer H facing a shared context [Steels, 1997]. The speaker wants to turn hearer's attention to one stimulus in the context—the topic. Only the speaker is aware which stimulus is the topic. The goal of the speaker is to utter a word so that the hearer, upon hearing it, will be able to guess the topic.

Let F_R, C_R, L_R be the dictionary, the categories and the language of the agent $R \in \{S, H\}$.

1. S and H face a common context: q_1, q_2.
2. S plays the discrimination game for the context q_1, q_2, with the topic $q_S \in \{q_1, q_2\}$. If the winning category c_S is found the game continues. Otherwise the game fails.
3. S searches for $f \in F_S$ such that $L_S(f, c_S) > 0$.
   If $L_S(f, c_s) = 0$ for all $f \in F_S$ or $L_S = \emptyset$, we set $F_S := F_S \cup \{f\}$ and $L_S(f, c_S) = 0.5$. The game fails.
   Suppose $L_S(f, c^s) = 0$ for some $f \in F_S$. Let $f_1, \ldots, f_n$ be all such words. S chooses $f \in \{f_1, \ldots, f_n\}$ such that $L_S(f, c^S) \geq L_S(f_i, c^S)$, for $i = 1, 2, \ldots, n$, and conveys f to the hearer.
4. H checks whether $f \in F_H$.
   If $f \notin F_H$, the game fails and q_S is revealed to H. We set $F_H := F_H \cup \{f\}$). H plays the discrimination game for q_1, q_2 with the topic q_S. If the winning category c is found, we set $L_H(f, c) = 0.5$.
   Suppose $f \in D_H$. Let $c_1, c_2, \ldots, c_k$ be all categories $c \in C_H$ such that $L_H(f, c) > 0$. H chooses c_H from $c_1, \ldots, c_k$ such that $L(f, c_H) \geq L(f, c_i)$ for $i = 1, 2, \ldots, k$. Let $q_H = \underset{q \in \{q_1, q_2\}}{\operatorname{argmax}}\, resp(c_H, q)$).
5. If $q_S = q_H$, the game is successful. Otherwise, it fails.

Success S increases the strength between c_S and f by δ_{inc} and decreases associations between c_S and other words by δ_{inh}. H increases the strength between c_H and f by δ_{inc} and decreases the strength of competing words with c_H by δ_{inh}.

Failure S decreases the strength between c_S and f by δ_{dec}. H decreases the strength between c_H and f by δ_{dec}.

We assume $\delta_{inc} = \delta_{inh} = \delta_{dec} = 0.2$.

As we mentioned in the introduction, the above-described model is largely based on the work on color terminologies in [Steels and Belpaeme, 2005]. We adapt their architecture to the domain of quantities by changing the layer of stimuli, reactive units, categories and, in part, the structure of the discrimination game. Using a similar agent architecture and communication model across different parts of language paints a uniform picture of language evolution.

3 Convexity and Monotonicity

We begin with the notion of meaning and pragmatic meaning for which convexity and monotonicity can be defined. Let Q be the set of stimuli. The **meaning** of a word f in a language

L is defined by:

$$[f]^L =_{df} \{q \in Q : \Sigma_{c \in C(L)} L(f,c) \langle c|R_q\rangle > 0\}. \quad (7)$$

In words, a stimulus q contributes to the meaning of f if some category c associated with f gives a positive response to q.

A justification for (7) comes from the guessing game. Let q, q' be a context with the topic q. The speaker utters f. The hearer searches for the category with the highest association to f and finds c. To choose q and win the game, it suffices that $\langle c|R_q\rangle > \langle c|R_{q'}\rangle$. In principle, any stimulus q generating a positive response to c can outperform (in terms of the response magnitude) some other stimulus q'. Hence, the interpretative process on the part of the hearer suggests that q indeed contributes to the meaning of f.

Now we turn to the notion of **pragmatic meaning** which depends on the context and interactions with other words:

$$[f]^L_p = \{q : \exists\, c\, [f \in \underset{\substack{f' \in F(L)\\ L(f',c)>0}}{\operatorname{argmax}} L(f',c), c \in \underset{\substack{c \in C(L)\\ \langle c|R_q\rangle>0}}{\operatorname{argmax}} \langle c|R_q\rangle]\}. \quad (8)$$

Less formally, q is an element of the pragmatic meaning of f if f is maximally (and non-negatively) associated with c that gives a maximal (and non-negative) response to q. The former condition corresponds to to between-word interaction while the latter one— to context-dependence.

The notion of **convexity** has paved its way into cognitive research thanks to the theory of conceptual spaces [Gärdenfors, 2000; Gärdenfors, 2014]—meanings can always be represented geometrically, and natural categories must be convex regions in the associated conceptual space. For evidence supporting this theory for logical words like quantifiers, see, e.g., [Jäger, 2010; Chemla *et al.*, 2019].

Convexity of a set in a conceptual space is usually defined using an in-between relation that should satisfy several constraints [Gärdenfors, 2000]. However, as observed in [Chemla *et al.*, 2019], one might depart from a strictly ordered set for which a natural in-between relation (defined by: b is between a and c if $a < b < c$ or $c < b < a$) satisfies the required axioms of in-betweenness. In our case, a strictly ordered set is the set of stimuli $(Q, <)$ with the *less than* relation. A definition of convexity by Chemla et el. [2019] states that $S \subseteq Q$ is **convex** if for any $a, b, c \in S$: if b is between a and c then $b \in S$. Hence, the pragmatic meaning of f in L is convex if it is a convex set in the space of stimuli Q. The above definition of convexity of meaning is sufficient for our purposes and can be derived from a more subtle analysis of convexity for quantifiers presented in [Chemla *et al.*, 2019].

The notion of **monotonicity** plays a prominent role in the traditional model-theoretic semantics [Peters and Westerståhl, 2006]. It is classified among quantifier universals, i.e., certain well-behaved properties which are believed to hold for the denotations of all naturally occurring simple determiners (see, e.g., [Keenan and Stavi, 1986; Barwise and Cooper, 1981]). Consider the following sentences:

Many good chess players know advanced tactics. (9)

Many good chess players know tactics. (10)

(10) follows from (9). The only difference in the above sentences is that *advanced tactics* is replaced by a more inclusive expression *tactics*. The inference would remain valid if we replaced any expression A with a more general expression B. This shows that the validity of the above inference is entirely based on the meaning of the quantifier *many*. The property of meaning that guarantees that is called (upward) monotonicity. In other words, a quantifier is upward monotone if whenever it can be truly applied to a set of objects it can be truly applied to any of its supersets.

Another variant of monotonicity is called downward monotonicity. Consider the following sentences:

Few beginning chess players know tactics. (11)

Few beginning chess players know advanced tactics. (12)

Again, (12) follows from (11) and the only difference is that *tactics* is substituted by a more specific term *advanced tactics*. The inference would be valid if we replaced any expression B with a more specific expression A. A quantifier is monotone if it is either upward or downward monotone.

We attribute monotonicity to meanings of words in our model. The justification of this is that a pragmatic meaning represents only the fragment of the overall meaning—the one that is most contextually and linguistically salient. For example, even though *most* can be truly used in situations where all objects posses a given property, it then might make more sense to use *all* instead.

To pinpoint the notion of monotonicity in our model model, let f be a word in language L. We say that $[f]^L$ is **monotone** if it is upward closed with respect to $\leq$ (i.e., if $q \in [f]^L$ and $q \leq q'$ then $q' \in [f]^L$) or downward closed with respect to $\leq$ (if $q' \in [f]^L$ and $q \leq q'$ then $q \in [f]^L$).

4 Experiments

The code for experiments and data visualization is written in Python and Mathematica. Experiments are based on parameters the values of which are mentioned earlier in the text.A simulation consists of 30 trials. Within a trial, 10 agents evolve quantity terminologies across 3000 steps. At each step agents are paired randomly to play a guessing game. There are separate simulations for numeric-based and quotient-based stimuli. We have varied whether agents are equipped with the ANS or not. This gives us 4 conditions.

First, we show that the model defined in Section 2 is valid, i.e., that it provides a solution to the language coordination problem in the domain of quantities. Next we check the extent to which convexity and monotonicity are represented in languages emerging with and without ANS.

Model validity

We use three types of metrics to evaluate validity: discriminative success, communicative success, and number of active terms in the lexicon. Similar measures have been used in previous studies [Steels and Belpaeme, 2005; Pauw and Hilferty, 2012]. Details about metrics are presented below, followed by the presentation of results and discussion.

The cumulative discriminative success of a population A for the last n games is defined as:

$$CDS(n)_j = \frac{1}{|A|} \Sigma_{a \in A} DS(n)^a_j, \quad (13)$$

where $DS(n)^a_j$ is defined in (5). We use $n = 50$. The measure of communicative success CS reflects the percentage of successful linguistic interactions in a population. The minimal requirement is that at some point CS will grow above 50% and stay above that threshold later on.[3]

Formally, let $cs^a_j = 1$ if the j-th guessing game of an agent a is successful. Otherwise, let $cs^a_j = 0$. The definitions of cumulative communicative success $CS(n)^a_j$ of an agent a, and of cumulative communicative success $CCS(n)^A_j$ of a population A, at game j for the last n guessing games are defined in the same way as $DS(n)^a_j$ and $CDS(n)^A_j$, respectively (replace DS with CS and ds with cs).

Finally, we also track the number of active words in the lexicon. We call a word $f \in L$ active if $[f]^L_p \neq \emptyset$. The active lexicon of an agent is the set of her active words. It represents the actually used fragment of the entire language of an agent.

Figure 2 shows the results. The discriminative success attains levels mostly above 80%. This shows that agents successfully discriminate the topic from the rest of the context in the vast majority of discrimination games. The communicative success lags behind the discriminative success, varying between 60% and 90%, with an average between 70% and 80% which is strictly above the minimal usability success rate of 50%. This shows that the evolving languages are communicatively useful, allowing the interlocutors to successfully communicate about the topic in the majority of guessing games. The languages evolving for numeric-based stimuli have lexicons with 6 active terms on average. The active lexicons for quotient-based stimuli have mean size 4 (see also Figures 4 and 6).[4]

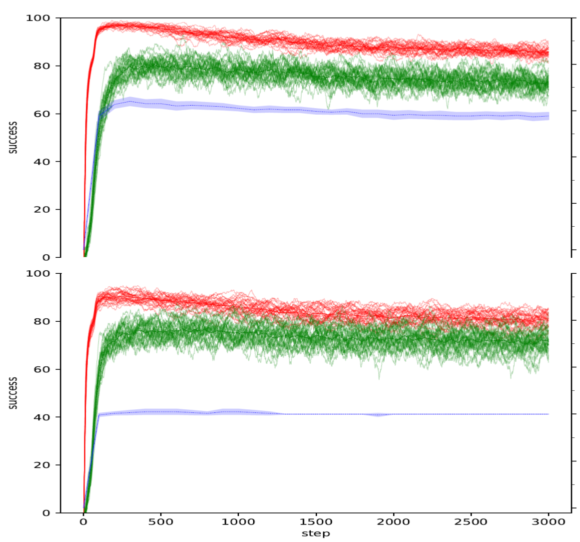

Figure 2: Cumulative discriminative success (red), communicative success (green), and mean size of active lexicon (blue, scale on the right) for numeric-based (top) and quotient-based stimuli (bottom). Each red/green line corresponds to a single trial.

The results seem to justify the conclusion that the model defined in Section 2 solves the language coordination problem for the domain of quantities. It then may serve as a hypothetical mechanistic theory explaining the transition from the stage of having no quantity terminology to the stage in which such a terminology exists. This allows us to turn our attention to assess the influence of ANS on convexity and monotonicity.

ANS, Convexity and Monotonicity

We compare the levels of convexity and monotonicity for languages evolving with and without ANS. The condition without ANS is obtained by assuming $R_n \sim \mathcal{N}(n, \sigma^2)$, $\sigma = 1/3$, which makes number perception more precise with an error independent from n.

The convexity of a language for a single agent is calculated as the number of active words that have convex pragmatic meanings, divided by the size of the active lexicon. The population-level convexity is the average convexity of languages of all agents in the population.

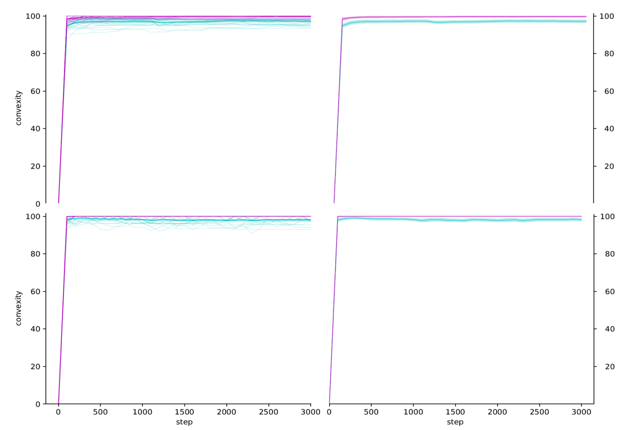

Figure 3: Population-level convexity for each trial (left) and averaged across trials (right) with ANS (magenta) and without ANS (cyan) for numeric-based (top) and quotient-based stimuli (bottom).

Figure 3 shows the results. We observe that convexity appears naturally with high frequency in all conditions but also that ANS representation facilitates convexity to a larger extent. The convexity of pragmatic meanings is clearly visible in Figure 4 where each vertical section represents the active lexicon at a given step. All emerging terminologies are convex—at step 3000 there are no words that contain 'holes' in the depicted quantity intervals.

The monotonicity of a language for a single agent is calculated as the number of monotone meanings corresponding to words from the active lexicon, divided by the size of the active lexicon. The population-level monotonicity is given by the average monotonicity of languages of all agents in the population. The plots in Figure 5 represent population-level monotonicity for all 30 trials.

[3]Notice that CS will never be greater than DS since successful communication entails successful discrimination.

[4]As a side note, we mention that both success metrics attain higher levels for the model without ANS. This should not be surprising—turning ANS off leads to more precise number perception and the evolution of more fine-grained terminologies.

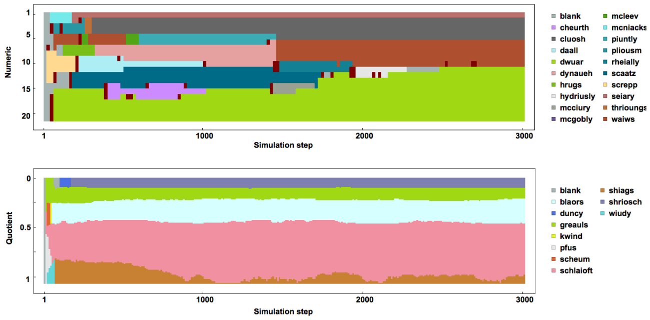

Figure 4: Active lexicon for numeric-based (top) and quotient-based stimuli in the ANS condition for agent 4 (compare with the data for the same agent in Figure 6).

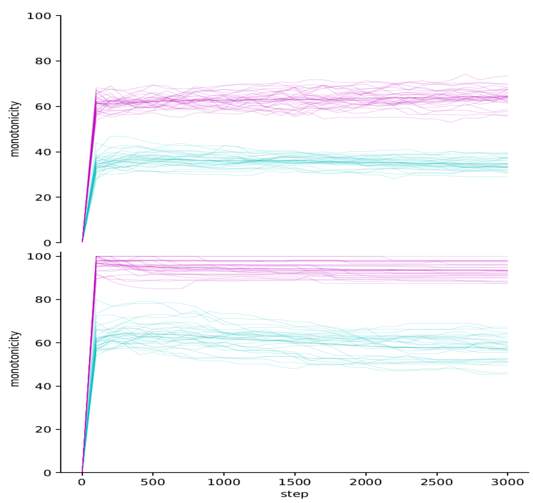

Figure 5: Plots for the numeric-based and quotient-based stimuli are to the left and to the right, respectively. Condition without ANS is given in cyan, while the ANS condition in magenta.

In this case, there is no doubt that ANS contributes significantly to the emergence of monotonicity in language. The difference in monotonicity levels between numeric-based and quotient-based condition is most likely due to the fact that ANS allows for more fine-grained categorization of the number line $1, 2, \ldots, 20$ than the quotient line between 0 and 1.

5 Conclusions

We presented an agent-based model evolving quantity expressions (quantifiers). Experimenting with quantity expressions in agent-based simulation is a recent topic (for different kind of coordination experiment with quantifiers see [Pauw and Hilferty, 2012] and [Kalociński *et al.*, 2018]). Our model in the present paper is based on the existing approach to simulate the emergence of color expressions [Steels and Belpaeme, 2005]). The perceptual layer of our model is inspired by the cognitive model of approximate number sense (ANS). Agent-based simulations based on agent interaction in so-called guessing games result in communicatively usable quantity expressions. This proves that the model solves the language coordination problem for the domain of quantities and corroborates the robustness of the original model.

To get a better understanding of the resulting quantity terminologies let us look at Figures 4 and 6. Each color represents a different word, whose semantics spans over a range of stimuli. We see words appearing and disappearing with time. We can also observe that after the initial variation the terminologies start to regularize structurally, following the ANS-based perception—there are several words with restrictive fine-grained meaning, for small quantities, for larger ones the words have broader meanings. Figure 6 illustrates the fact that even though the languages of individual agents differ among each other, the satisfactory level of communication is still possible to achieve. The common cognitive apparatus and the pressure to cooperate in communication probably contribute to that effect.

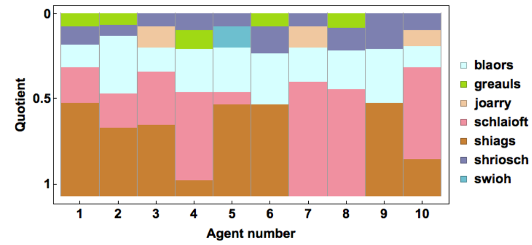

Figure 6: Active lexicons of all agents from a selected trial at the last step of the simulation for quotient-based stimuli.

Finally, we have looked at two widely-discussed semantic universals: convexity and monotonicity. ANS turns out to facilitate both properties, but the positive effect is especially evident in the case of monotonicity. We can speculate about the causes of that effect. ANS certainly extends the actual scope of a category, especially for larger inputs, because the long-tailed categories can provide positive responses even for distant quantities. Moreover, the more relaxed, vague semantics of quantity expressions under ANS likely allows an easier upward or downward merge with other categories.

We can conclude that ANS does not contribute significantly to the emergence of convexity. The source of convexity, for the most part, stems from other layers of the model. This is an intriguing observation as the model does not rule out non-convexity (unlike the convexity operator in [Pauw and Hilferty, 2012]), and it even makes room for it because categories can be formed from distant reactive units or the same word can be linked to distant categories. However, convexity is not immune to the nature of perception. We have observed that ANS facilitates convexity to the larger extent than precise number perception, leading, for the most part, to fully convex terminologies. We can hypothesize that if one is attached to the intuition that cognitive concepts should be convex in the appropriate conceptual space, it is more plausible to assume vague meanings of words.

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