Bài tập Xấp xỉ BPTT bằng các hàm liên tục và trực giao

1. Find the linear least squares polynomial approximation to $f(x)$ on the indicated i

a. $f(x) = x^2 + 3x + 2, \quad [0, 1];$

b. $f(x) = x^3, \quad [0, 2];$

c. $f(x) = \frac{1}{x}, \quad [1, 3];$

d. $f(x) = e^x, \quad [0, 2];$

e. $f(x) = \frac{1}{2}\cos x + \frac{1}{3}\sin 2x, \quad [0, 1];$

f. $f(x) = x \ln x, \quad [1, 3].$

2. Find the linear least squares polynomial approximation on the interval $[-1, 1]$ for the foll functions.

a. $f(x) = x^2 - 2x + 3$

b. $f(x) = x^3$

c. $f(x) = \frac{1}{x+2}$

d. $f(x) = e^x$

e. $f(x) = \frac{1}{2}\cos x + \frac{1}{3}\sin 2x$

f. $f(x) = \ln(x + 2)$

3. Find the least squares polynomial approximation of degree two to the functions and inter Exercise 1.

4. Find the least squares polynomial approximation of degree 2 on the interval $[-1, 1]$ for the fun in Exercise 3.

5. Compute the error E for the approximations in Exercise 3.

6. Compute the error E for the approximations in Exercise 4.

7. Use the Gram-Schmidt process to construct $\phi_0(x)$, $\phi_1(x)$, $\phi_2(x)$, and $\phi_3(x)$ for the following int

a. $[0, 1]$

b. $[0, 2]$

c. $[1, 3]$

8. Repeat Exercise 1 using the results of Exercise 7.

9. Obtain the least squares approximation polynomial of degree 3 for the functions in Exercise 1 the results of Exercise 7.

the results of Exercise 7.

10. Repeat Exercise 3 using the results of Exercise 7.

11. Use the Gram-Schmidt procedure to calculate L_1, L_2, and L_3, where $\{L_0(x), L_1(x), L_2(x), L_3(x)\}$ is an orthogonal set of polynomials on $(0, \infty)$ with respect to the weight functions $w(x) = e^{-x}$ and $L_0(x) \equiv 1$. The polynomials obtained from this procedure are called the **Laguerre polynomials**.

12. Use the Laguerre polynomials calculated in Exercise 11 to compute the least squares polynomials of degree one, two, and three on the interval $(0, \infty)$ with respect to the weight function $w(x) = e^{-x}$ for the following functions:

    a. $f(x) = x^2$ **b.** $f(x) = e^{-x}$ **c.** $f(x) = x^3$ **d.** $f(x) = e^{-2x}$

13. Suppose $\{\phi_0, \phi_1, \ldots, \phi_n\}$ is any linearly independent set in $\prod_n$. Show that for any element $Q \in \prod_n$, there exist unique constants $c_0, c_1, \ldots, c_n$, such that

$$Q(x) = \sum_{k=0}^{n} c_k \phi_k(x).$$

14. Show that if $\{\phi_0, \phi_1, \ldots, \phi_n\}$ is an orthogonal set of functions on $[a, b]$ with respect to the weight function w, then $\{\phi_0, \phi_1, \ldots, \phi_n\}$ is a linearly independent set.

15. Show that the normal equations (8.6) have a unique solution. [*Hint:* Show that the only solution for the function $f(x) \equiv 0$ is $a_j = 0, j = 0, 1, \ldots, n$. Multiply Eq. (8.6) by a_j, and sum over all j. Interchange the integral sign and the summation sign to obtain $\int_a^b [P(x)]^2 dx = 0$. Thus, $P(x) \equiv 0$, so $a_j = 0$, for $j = 0, \ldots, n$. Hence, the coefficient matrix is nonsingular, and there is a unique solution to Eq. (8.6).]