# Bài tập Phương pháp chia đôi

1. Use the Bisection method to find p_3 for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$.

2. Let $f(x) = 3(x + 1)(x - \frac{1}{2})(x - 1)$. Use the Bisection method on the following intervals to find p_3.
   a. $[-2, 1.5]$ **b.** $[-1.25, 2.5]$

3. Use the Bisection method to find solutions accurate to within 10^{-2} for $x^3 - 7x^2 + 14x - 6 = 0$ on each interval.
   a. $[0, 1]$ **b.** $[1, 3.2]$ **c.** $[3.2, 4]$

4. Use the Bisection method to find solutions accurate to within 10^{-2} for $x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ on each interval.
   a. $[-2, -1]$ **b.** $[0, 2]$ **c.** $[2, 3]$ **d.** $[-1, 0]$

5. Use the Bisection method to find solutions accurate to within 10^{-5} for the following problems.
   a. $x - 2^{-x} = 0$ for $0 \leq x \leq 1$
   b. $e^x - x^2 + 3x - 2 = 0$ for $0 \leq x \leq 1$
   c. $2x \cos(2x) - (x + 1)^2 = 0$ for $-3 \leq x \leq -2$ and $-1 \leq x \leq 0$
   d. $x \cos x - 2x^2 + 3x - 1 = 0$ for $0.2 \leq x \leq 0.3$ and $1.2 \leq x \leq 1.3$

6. Use the Bisection method to find solutions, accurate to within 10^{-5} for the following problems.
   a. $3x - e^x = 0$ for $1 \leq x \leq 2$
   b. $2x + 3 \cos x - e^x = 0$ for $0 \leq x \leq 1$
   c. $x^2 - 4x + 4 - \ln x = 0$ for $1 \leq x \leq 2$ and $2 \leq x \leq 4$
   d. $x + 1 - 2 \sin \pi x = 0$ for $0 \leq x \leq 0.5$ and $0.5 \leq x \leq 1$

7. **a.** Sketch the graphs of $y = x$ and $y = 2 \sin x$.
   b. Use the Bisection method to find an approximation to within 10^{-5} to the first positive value of x with $x = 2 \sin x$.

8. **a.** Sketch the graphs of $y = x$ and $y = \tan x$.
   b. Use the Bisection method to find an approximation to within 10^{-5} to the first positive value of x with $x = \tan x$.

9. **a.** Sketch the graphs of $y = e^x - 2$ and $y = \cos(e^x - 2)$.
   b. Use the Bisection method to find an approximation to within 10^{-5} to a value in $[0.5, 1.5]$ with $e^x - 2 = \cos(e^x - 2)$.

10. Let $f(x) = (x+2)(x+1)^2 x(x-1)^3(x-2)$. To which zero of f does the Bisection method converge when applied on the following intervals?

   a. $[-1.5, 2.5]$ b. $[-0.5, 2.4]$ c. $[-0.5, 3]$ d. $[-3, -0.5]$

11. Let $f(x) = (x+2)(x+1)x(x-1)^3(x-2)$. To which zero of f does the Bisection method converge when applied on the following intervals?

   a. $[-3, 2.5]$ b. $[-2.5, 3]$ c. $[-1.75, 1.5]$ d. $[-1.5, 1.75]$

12. Find an approximation to $\sqrt{3}$ correct to within 10^{-4} using the Bisection Algorithm. [*Hint:* Consider $f(x) = x^2 - 3$.]

13. Find an approximation to $\sqrt[3]{25}$ correct to within 10^{-4} using the Bisection Algorithm.

14. Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-3} to the solution of $x^3 + x - 4 = 0$ lying in the interval $[1, 4]$. Find an approximation to the root with this degree of accuracy.

15. Use Theorem 2.1 to find a bound for the number of iterations needed to achieve an approximation with accuracy 10^{-4} to the solution of $x^3 - x - 1 = 0$ lying in the interval $[1, 2]$. Find an approximation to the root with this degree of accuracy.

16. Let $f(x) = (x-1)^{10}$, $p = 1$, and $p_n = 1 + 1/n$. Show that $|f(p_n)| < 10^{-3}$ whenever $n > 1$ but that $|p - p_n| < 10^{-3}$ requires that $n > 1000$.

17. Let $\{p_n\}$ be the sequence defined by $p_n = \sum_{k=1}^{n} \frac{1}{k}$. Show that $\{p_n\}$ diverges even though $\lim_{n \to \infty}(p_n - p_{n-1}) = 0$.

18. The function defined by $f(x) = \sin \pi x$ has zeros at every integer. Show that when $-1 < a < 0$ and $2 < b < 3$, the Bisection method converges to

   a. 0, if $a+b < 2$ b. 2, if $a+b > 2$ c. 1, if $a+b = 2$

19. A trough of length L has a cross section in the shape of a semicircle with radius r. (See the accompanying figure.) When filled with water to within a distance h of the top, the volume V of water is

$$V = L\left[0.5\pi r^2 - r^2 \arcsin(h/r) - h(r^2 - h^2)^{1/2}\right].$$
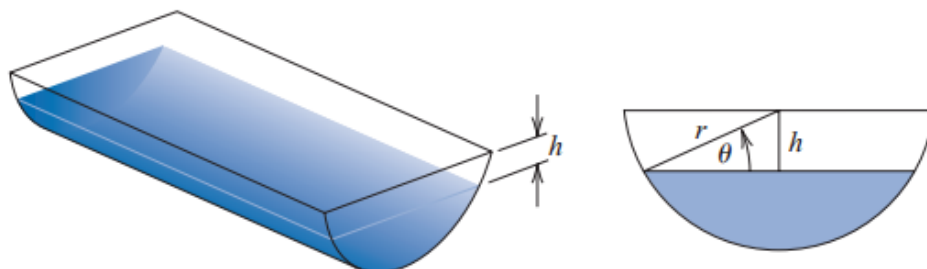
Suppose $L = 10$ ft, $r = 1$ ft, and $V = 12.4$ ft^3. Find the depth of water in the trough to within 0.01 ft.

20. A particle starts at rest on a smooth inclined plane whose angle θ is changing at a constant rate

$$\frac{d\theta}{dt} = \omega < 0.$$

At the end of t seconds, the position of the object is given by

$$x(t) = -\frac{g}{2\omega^2}\left(\frac{e^{\omega t} - e^{-\omega t}}{2} - \sin \omega t\right).$$

Suppose the particle has moved 1.7 ft in 1 s. Find, to within 10^{-5}, the rate ω at which θ changes. Assume that $g = 32.17$ ft/s^2.