Bài tập Ch7 Bài 2. Phương pháp Taylor bậc cao

1. Use Taylor's method of order two to approximate the solutions for each of the following initia problems.

a. $y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 0,$ with $h = 0.5$

b. $y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1,$ with $h = 0.5$

c. $y' = 1 + y/t, \quad 1 \le t \le 2, \quad y(1) = 2,$ with $h = 0.25$

d. $y' = \cos 2t + \sin 3t, \quad 0 \le t \le 1, \quad y(0) = 1,$ with $h = 0.25$

2. Use Taylor's method of order two to approximate the solutions for each of the following initia problems.

a. $y' = e^{t-y}, \quad 0 \le t \le 1, \quad y(0) = 1,$ with $h = 0.5$

b. $y' = \dfrac{1+t}{1+y}, \quad 1 \le t \le 2, \quad y(1) = 2,$ with $h = 0.5$

c. $y' = -y + ty^{1/2}, \quad 2 \le t \le 3, \quad y(2) = 2,$ with $h = 0.25$

d. $y' = t^{-2}(\sin 2t - 2ty), \quad 1 \le t \le 2, \quad y(1) = 2,$ with $h = 0.25$

3. Repeat Exercise 1 using Taylor's method of order four.

4. Repeat Exercise 2 using Taylor's method of order four.

5. Use Taylor's method of order two to approximate the solution for each of the following initia problems.

a. $y' = y/t - (y/t)^2, \quad 1 \le t \le 1.2, \quad y(1) = 1,$ with $h = 0.1$

b. $y' = \sin t + e^{-t}, \quad 0 \le t \le 1, \quad y(0) = 0,$ with $h = 0.5$

c. $y' = (y^2 + y)/t, \quad 1 \le t \le 3, \quad y(1) = -2,$ with $h = 0.5$

d. $y' = -ty + 4ty^{-1}, \quad 0 \le t \le 1, \quad y(0) = 1,$ with $h = 0.25$

6. Use Taylor's method of order two to approximate the solution for each of the following initia problems.

a. $y' = \dfrac{2-2ty}{t^2+1}, \quad 0 \le t \le 1, \quad y(0) = 1,$ with $h = 0.1$

b. $y' = \dfrac{y^2}{1+t}, \quad 1 \le t \le 2, \quad y(1) = -(\ln 2)^{-1},$ with $h = 0.1$

c. $y' = (y^2 + y)/t, \quad 1 \le t \le 3, \quad y(1) = -2$, with $h = 0.2$

d. $y' = -ty + 4t/y, \quad 0 \le t \le 1, \quad y(0) = 1$, with $h = 0.1$

7. Repeat Exercise 5 using Taylor's method of order four.

8. Repeat Exercise 6 using Taylor's method of order four.

9. Given the initial-value problem

$$y' = \frac{2}{t}y + t^2e^t, \quad 1 \le t \le 2, \quad y(1) = 0,$$

with exact solution $y(t) = t^2(e^t - e)$:

a. Use Taylor's method of order two with $h = 0.1$ to approximate the solution, and compare it wi the actual values of y.

b. Use the answers generated in part (a) and linear interpolation to approximate y at the followin values, and compare them to the actual values of y.

i. $y(1.04)$ **ii.** $y(1.55)$ **iii.** $y(1.97)$

c. Use Taylor's method of order four with $h = 0.1$ to approximate the solution, and compare with the actual values of y.

d. Use the answers generated in part (c) and piecewise cubic Hermite interpolation to approxima y at the following values, and compare them to the actual values of y.

i. $y(1.04)$ **ii.** $y(1.55)$ **iii.** $y(1.97)$

10. Given the initial-value problem

$$y' = \frac{1}{t^2} - \frac{y}{t} - y^2, \quad 1 \le t \le 2, \quad y(1) = -1,$$

with exact solution $y(t) = -1/t$:

a. Use Taylor's method of order two with $h = 0.05$ to approximate the solution, and com with the actual values of y.

b. Use the answers generated in part (a) and linear interpolation to approximate the following of y, and compare them to the actual values.

i. $y(1.052)$ **ii.** $y(1.555)$ **iii.** $y(1.978)$

c. Use Taylor's method of order four with $h = 0.05$ to approximate the solution, and com with the actual values of y.

d. Use the answers generated in part (c) and piecewise cubic Hermite interpolation to appro the following values of y, and compare them to the actual values.

i. $y(1.052)$ **ii.** $y(1.555)$ **iii.** $y(1.978)$

11. A projectile of mass $m = 0.11$ kg shot vertically upward with initial velocity $v(0) = 8$ m due to the force of gravity, $F_g = -mg$, and due to air resistance, $F_r = -kv|v|$, where g and $k = 0.002$ kg/m. The differential equation for the velocity v is given by

$$mv' = -mg - kv|v|.$$

a. Find the velocity after $0.1, 0.2, \ldots, 1.0$ s.

b. To the nearest tenth of a second, determine when the projectile reaches its maximun begins falling.

12. Use the Taylor method of order two with $h = 0.1$ to approximate the solution to

$$y' = 1 + t\sin(ty), \quad 0 \le t \le 2, \quad y(0) = 0.$$