Short-Term Electricity Market Auction Game Analysis: Uniform and Pay-as-Bid Pricing

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***Abstract*—This paper analyzes the competing pricing mechanisms of uniform and pay-as-bid pricing in an electricity market. Game theory and auction theory are adopted to analyze the strategic behavior of a big player and a small player in a short-term auction game. Contrary to what would be expected from the conclusion of the "revenue equivalence theorem," we prove that for a two-player static game the Nash Equilibrium (NE) under pay-as-bid pricing will yield less total revenue in expectation than under uniform pricing when demand is inelastic. To confirm this theoretical result we simulated the model using a mixed-strategy NE solver. We extended the model to an elastic demand case and showed that pay-as-bid pricing also led to a larger expected total demand being served when demand is elastic.**

***Index Terms*—Auction theory, electricity market, game theory, market design, market power.**

I. Introduction

GAME THEORY and auction theory have proven useful to understand markets based on auctions, including electricity markets, and especially in modeling auction participants, auction types, and potential market performance [1]–[3]. In a single unit auction, Vickrey proved that "English" and "Dutch" type auctions will yield the same expected revenue under the assumptions of risk neutral participants and privately known value drawn from a common distribution [4]. Vickrey's result is embodied in the "Revenue Equivalence Theorem" (RET) [3].

The RET also applies to multi-unit auctions when no buyer wants more than one of the k available objects [3]. The RET does not, however, apply to general multi-unit auctions, which is a more appropriate model for an electricity market. Opinions have differed on whether or not RET-like results apply to electricity markets.

For general multi-unit auctions, the effect of auction type on outcome has been a research focus [5], [6]. Wilson showed that there are "seemingly collusive" Nash Equilibria (NE) in a uniform pricing auction for shares [5]. Back and Zender compared NE under uniform pricing and discriminatory pricing in the U.S. Treasury auction [6]. They also argued that uniform pricing could yield collusive equilibria and that the seller's expected revenue would be smaller than in the equilibria under discriminatory pricing [6].

References [3]–[6] describe "buyers' auctions" where there are several strategic bidders offering to buy. In contrast, electricity markets are typically organized as "sellers' auctions" with strategic sellers. In the rest of this paper, we will focus on "sellers' auctions" as models of electricity markets.

In electricity markets, a huge volume of commodities are being traded or will be traded through multi-unit auctions. The economic effects of trading in electricity markets have been very serious, as have been observed in the California crisis and England and Wales reform [7], [8].

The choice between uniform and pay-as-bid pricing for electricity auctions has been one of most important issues in newly deregulated electricity markets [9]–[13]. Federico and Rahman studied the perfect competition and monopoly case under both uniform and discriminatory (pay-as-bid) pricing in an electricity auction [9]. They noted the tradeoff between efficiency and consumer surplus. Fabra set up a two-player auction game and considers it in a static and a dynamic setting [10]. She noted that in the static setting the profit level of suppliers will be greater under uniform pricing and that in the dynamic setting uniform pricing can facilitate collusion. V́zquez, Rivier, and Pérez-Arriaga noted that an entry barrier for small players could exist under a pay-as-bid auction and that it can be harmful in the long run [12].

Wolfram illustrated a simple electricity auction example to explain the differences between discriminatory and uniform pricing auctions [11]. She criticized the England and Wales reform, which involved a switch from uniform pricing to discriminatory pricing, illustrating with an example where there is equal revenue between the uniform pricing and discriminatory pricing cases. The blue ribbon panel report on a proposed change from uniform pricing to pay-as-bid pricing in California concluded that the expectation of a price drop due to the proposed change was dubious and that a change to discriminatory pricing could discourage increased competition in the long run [13].

In this paper, we deal with the issue of uniform pricing and pay-as-bid pricing in electricity markets from the viewpoint of the effect of strategic bidding and market power in the short term. We ignore long-run effects in this study but recognize that long-run implications are also important.

The auction model in this paper follows the basic framework of the sealed-bid multiple-unit auction in [14]. In order to address alternative auction pricing mechanisms in electricity markets, we use a specific two-player auction game model having

Manuscript received April 13, 2004. This work was supported in part by the National Science Foundation under Grant ECS-0082506 and by the Lower Colorado River Authority. Paper no. TPWRS-00181-2003.

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Digital Object Identifier 10.1109/TPWRS.2004.836192

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a "big" player with market power due to owning a large fraction of the installed capacity and a strategic "small" player. The example system that we use has a similar configuration to that examined in [11] but it contains a potential structural problem of market power. We show that the potential structural problem can yield a different conclusion to that in [11]. That is, we show that the equilibrium revenue under uniform pricing is different from that under pay-as-bid pricing. In particular, we show that revenue equivalence does not hold in a simple multi-unit auction model of an electricity market consisting of two players and no transmission constraints. Revenue equivalence is also unlikely to hold for more realistic cases with more than two players and with transmission constraints.

We first analyze the two-player auction game to show the core reasoning of each player under each auction pricing mechanism. We show that the profit maximizing bidding strategy can be dependent on the auction pricing mechanism. Then we analyze the gaming in terms of bid price itself and compare the expected revenue under each pricing mechanism. "Revenue inequivalence" under the two price mechanisms is explained and proved based on the concept of the static mixed-strategy NE.

To confirm the theoretical result, we simulated the model using the Lemke and Howson algorithm [15], [16] and obtained the mixed-strategy NE for the example. We compared the expected total revenues from the given NEs under both pricing mechanisms. We also extended our model to an elastic demand case and compared the total expected market cleared demands under both pricing mechanisms using the Lemke and Howson algorithm.

We have not extended the theory nor the simulation to more than two players nor to the case of transmission constraints. The computational effort for multi-player games tends to rise rapidly with the number of players. A discussion of approaches to solving multi-player electricity market models appears in [17]. Nevertheless, the main conclusion that revenue equivalence does not hold is likely to hold in multi-player games with transmission constraints.

II. Auction Model and Assumptions

We follow the basic framework of the sealed-bid multi-unit auction model for the electricity market [14]. We assume a uniform tradable energy unit size (for example, 50 MW) for the auction. We consider a sellers' auction model given fixed demands as in usual electricity pools. As mentioned in the introduction, typical auction theories model buyers' auctions [3]–[6]. However most of the analysis still holds for a sellers' auction under similar assumptions. A seller owning energy unit G_i having production cost c_i sets a bid price p_i for that unit. Bid winners will be determined by the ordered price bid stack given the demand level. The general multi-unit auction in the electricity market given N units of demand is illustrated in Fig. 1. The bid prices are shown as solid bars, potentially set higher than the costs of the corresponding units of production.

We define two pricing mechanisms for multi-unit auctions in electricity markets: uniform and pay-as-bid pricing.

Definition 1: [Uniform pricing]: Under the uniform pricing structure, the marginal bid block, as illustrated in Fig. 1, sets the

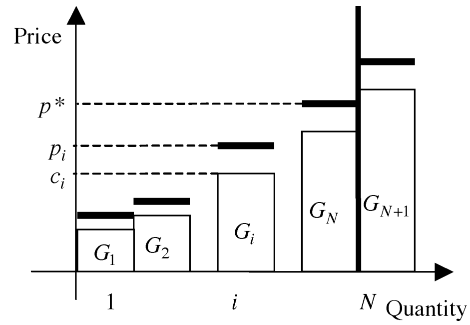

Fig. 1. Multi-unit auction in the electricity market.

uniform market clearing price p^*. An individual seller's profit π is defined as $\pi = Kp^* - \sum_{j=1}^{K} c_{i(j)}$ where the seller wins K blocks of bid, namely blocks $i(1), \ldots, i(K)$. The total revenue under uniform pricing is defined by $TR_{\text{uni}} = Np^*$.

Definition 2: [Pay-as-bid pricing]: Under the pay-as-bid pricing structure, every winning block gets its bid price as its income. An individual seller's profit is defined as $\pi = \sum_{j=1}^{K}(p_{i(j)} - c_{i(j)})$ when the seller wins K blocks of bid, namely blocks $i(1), \ldots, i(K)$. The total revenue under pay-as-bid pricing is defined by $TR_{\text{pay}} = \sum_{j=1}^{N} p_{i(j)}$.

If we assume that each seller bids only one energy block then the RET holds between both pricing mechanisms [3]. To illustrate the case where each seller has only one energy block, let us consider a perfect information case and suppose that all costs and prices are a multiple of a minimum bid increment or decrement size ϵ. Assume that the blocks are ordered so that $c_1 < c_2 < \ldots < c_{N+1}$. Under uniform pricing the marginal cost seller will set the market clearing price as $p^* = c_{N+1} - \epsilon$. The rest of the sellers who own G_i, $i < N$ will behave as price takers. Under the pay-as-bid pricing, every seller who owns G_i, $i < N$ will maximize its profit by bidding at the price of $p_i = c_{N+1} - \epsilon$. Under both auction mechanisms the total revenue $TR_{\text{uni}} = TR_{\text{pay}} = N(c_{N+1} - \epsilon)$.

Similar reasoning applies to a multi-unit auction example without market power described by Wolfram [11]. However, the reasoning above does not seem to hold when there are sellers with market power [9], [10]. Since the exercise of market power was the impetus for considering a change from uniform to pay-as-bid in both California and England and Wales, it is important to represent market power. Different pricing mechanisms can potentially result in different equilibrium results in the presence of market power.

To consider the market power issue, we consider an oligopolistic auction game. In order to analyze the market power problem under uniform and pay-as-bid pricing electricity auctions, we set up a simple two-player game model. We assume a big player with market power and a small player. Both players will behave strategically to maximize their profits. Demand level and cost functions of each player are assumed to be available publicly. In electricity markets these assumptions can be justified by widespread knowledge of information about the heat rate of each unit, fuel costs, and accurate forecasts of demand [7].

For a clearer analytical understanding of the situation, we take the simplest configuration as illustrated in Fig. 2. In Fig. 2,

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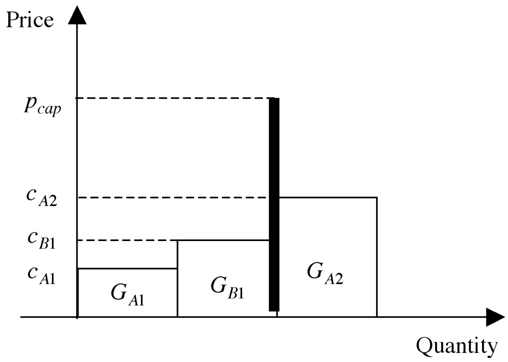

Fig. 2. A two-player game configuration.

TABLE I
PLAYER A'S CATEGORIZED BID PRICE SET

Strategy (X_A)	Range of bid price p_A
W (Withholding)	$p_W = p_{cap}$
U (Undercutting)	$1/2(p_{cap} + c_{A2}) \le p_U < p_{cap}$
T (Timid)	$c_{A2} \le p_T < 1/2(p_{cap} + c_{A2})$

player A owns two energy units G_{A1}, G_{A2} with cost c_{A1}, c_{A2}, respectively. Player B owns only one unit G_{B1} costing c_{B1}, with $c_{A1} < c_{B1} < c_{A2}$. We assume two units of demand with a price cap in the market of p_{cap}. Each player will simultaneously bid prices p_{A1}, p_{B1}, p_{A2} with all prices no larger than p_{cap}. As a tie-breaking rule, we assume that player A is declared the winner if both A and B declare equal bids that are both marginal. (Other assumptions for tie-breaking lead to essentially the same results although the analysis differs in details.)

Under economic dispatch based on actual marginal costs, the marginal cost of supplying additional demand is c_{A2}. This is the competitive price for demand. (Most pool markets, however, clear based on the last accepted block, rather than the next block. That would yield a clearing price of c_{B1} given bids equal to marginal costs.)

III. Equilibrium Under Uniform and Pay-as-Bid Pricing

A. Strategic Bid Price Classification

In order to describe the strategic behaviors of player A and B in the two-player model, we classify the bid strategies of player A and B. We divide player A's price bid actions into three categorized bid price set in Table I.

"W (Withholding)" represents player A attempting to exercise extreme market power. By withholding G_{A2} and raising the price of G_{A1} to the extreme p_{cap}, player A tries to maximize his profit.

"U (Undercutting)" represents another strategic action of player A based on the belief that undercutting player B's bid price is more profitable. The price range of p_U is determined through the profit comparison with "Withholding"

$$\pi_{\text{Undercutting}} > \pi_{\text{Withholding}}. \tag{1}$$

TABLE II
PLAYER B'S CATEGORIZED BID PRICE SET

Strategy (X_B)	Range of bid price p_B
S (Safe)	$p_S < 1/2(p_{cap} + c_{A2})$
R (Risky)	$1/2(p_{cap} + c_{A2}) \le p_R \le p_{cap} - \epsilon$

TABLE III
PLAYER A'S PROFITS UNDER UNIFORM PRICING

Case	Value
π_A^{TS}	$u(p_T > p_S)(p_T - c_{A1}) + u(p_T \le p_S)(2p_T - c_{A1} - c_{A2})$
π_A^{TR}	$2p_T - c_{A1} - c_{A2}$
π_A^{US}	$p_U - c_{A1}$
π_A^{UR}	$u(p_U > p_R)(p_U - c_{A1}) + u(p_U \le p_R)(2p_U - c_{A1} - c_{A2})$
π_A^{WS}	$p_W - c_{A1}$
π_A^{WR}	$p_W - c_{A1}$

TABLE IV
PLAYER B'S PROFITS UNDER UNIFORM PRICING

Case	Value
π_B^{TS}	$u(p_T > p_S)(p_T - c_{B1})$
π_B^{TR}	0
π_B^{US}	$p_U - c_{B1}$
π_B^{UR}	$u(p_U > p_R)(p_U - c_{B1})$
π_B^{WS}	$p_W - c_{B1}$
π_B^{WR}	$p_W - c_{B1}$

This condition implies

$$2p_U - c_{A2} - c_{A1} \ge p_{\text{cap}} - c_{A1}. \tag{2}$$

From (2), we have

$$p_U \ge \frac{1}{2}(p_{\text{cap}} + c_{A2}). \tag{3}$$

"T (Timid)" represent the remaining range of prices that A can bid.

Player B has two options. He can take the conservative bid by bidding $p_B < 1/2(p_{\text{cap}} + c_{A2})$ (and to maximize profits under this condition would choose the largest bid that is less than $1/2(p_{\text{cap}} + c_{A2})$.) We call this "S (Safe)." A second option is for player B to take the risk of being undercut. We call this "R (Risky)." The categorized bid price set of player B are summarized into Table II.

B. Auction Game Under Uniform Pricing

We analyze the auction game under uniform pricing based on the classification in Section III-A. Under uniform pricing the resulting profits of the two players are summarized in Table III and Table IV, respectively. We use the notation $u(.)$ for a Boolean function such that $u(\text{True}) = 1$ and $u(\text{False}) = 0$. Each entry $\pi_k^{X_A X_B}$ in Tables III and IV represents the player k's profit when players A and B choose strategies $X_A \in \{W, U, T\}$ and $X_B \in \{S, R\}$, respectively.

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TABLE V
PLAYER A'S BEST RESPONSE UNDER UNIFORM PRICING

X_B	Player A's best response
S	W
R	$U_{p_U<p_R}$ $=\{p_U \| p_U < p_R\} \subset U$

TABLE VI
PLAYER B'S BEST RESPONSE UNDER UNIFORM PRICING

X_A	Player B's best response
T	$S_{p_S<p_T} = \{p_S \| p_S < p_T\} \subset S$
U	S and $R_{p_R<p_U} = \{p_R \| p_R < p_U\} \subset R$
W	S and R

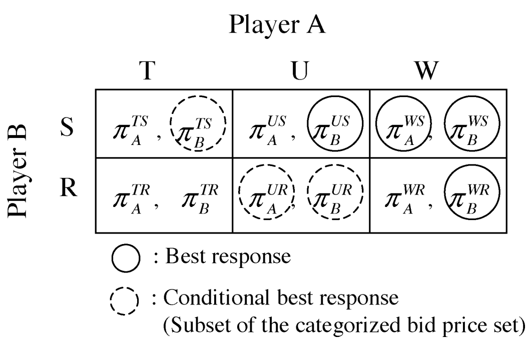

Fig. 3. Best responses in the two-player auction game under the uniform pricing mechanism.

TABLE VII
PLAYER A'S PROFITS UNDER PAY-AS-BID PRICING

Case	Value
π_A^{TS}	$u(p_T > p_S)(p_T - c_{A1}) + u(p_T \le p_S)(2p_T - c_{A1} - c_{A2})$
π_A^{TR}	$2p_T - c_{A1} - c_{A2}$
π_A^{US}	$p_U - c_{A1}$
π_A^{UR}	$u(p_U > p_R)(p_U - c_{A1}) + u(p_U \le p_R)(2p_U - c_{A1} - c_{A2})$
π_A^{WS}	$p_W - c_{A1}$
π_A^{WR}	$p_W - c_{A1}$

From Table III and IV we can analyze each player's best response to the other player's strategic choice. Table V shows player A's best response given player B's strategic choice X_B under uniform pricing. Table VI shows player B's best response given player A's strategic choice X_A under uniform pricing.

By aggregating the results of best response analysis in Tables V and VI, we can get the information about the NE for the auction game. When a player's best response is conditionally limited (for example, when $X_B =$ R, player A's best response is $X_A =$ U only if the price p_U is less then p_R), we call this a "conditional" best response hereafter. Using the best responses in Tables V and VI, we illustrate the normal form game result in Fig. 3.

A pure strategy NE is determined by the intersection of best responses. The choice set ($X_A =$ W, $X_B =$ S), is a NE. We can represent the NE by the range of player A's and player B's bid prices

$$\left\{ (p_A^*, p_B^*) \,|p_A^* = p_{\text{cap}}, \quad p_B^* < \frac{1}{2}(p_{\text{cap}} + c_{A2}) \right\}. \quad (4)$$

The intersection of "conditional" best responses yields no pure strategy NE because each player will change their bid price in turn repeatedly to respond to the other's strategic choice.

TABLE VIII
PLAYER B'S PROFITS UNDER PAY-AS-BID PRICING

Case	Value
π_B^{TS}	$u(p_T > p_S)(p_S - c_{B1})$
π_B^{TR}	0
π_B^{US}	$p_S - c_{B1}$
π_B^{UR}	$u(p_U > p_R)(p_R - c_{B1})$
π_B^{WS}	$p_S - c_{B1}$
π_B^{WR}	$p_R - c_{B1}$

TABLE IX
PLAYER A'S BEST RESPONSE UNDER PAY-AS-BID PRICING

X_B	Player A's best response
S	W
R	$U_{p_U<p_R}$ $=\{p_U \| p_U < p_R\} \subset U$

TABLE X
PLAYER B'S BEST RESPONSE UNDER UNIFORM PRICING

X_A	Player B's best response
T	$S_{p_S<p_T} = \{p_S \| p_S < p_T\} \subset S$
U	$R_{p_R<p_U} = \{p_R \| p_R < p_U\} \subset R$
W	R

C. Auction Game Under Pay-as-Bid Pricing

We analyze the auction game under pay-as-bid pricing by similar steps to that for unform pricing. Table VII and VIII show, respectively, player A's and player B's profits in the possible cases.

Player A's profit for each case under pay-as-bid pricing is the same as under uniform pricing. Player B's profit for each case under pay-as-bid pricing is changed since he is not paid the uniform market clearing price anymore.

From Table VII and VIII we can analyze each player's best response to the other player's strategic choice. Table IX shows player A's best response given player B's strategic choice X_B under pay-as-bid pricing. Table X shows player B's best response given player A's strategic choice X_A under pay-as-bid pricing.

By aggregating the results of the best response analysis, we can get the information about the NE for the auction game. We illustrate the results in Fig. 4.

In this case, we have no pure strategy NE. However, there will still exist a mixed-strategy NE. By eliminating the player A's dominated strategy "Timid" and player B's dominated strategy

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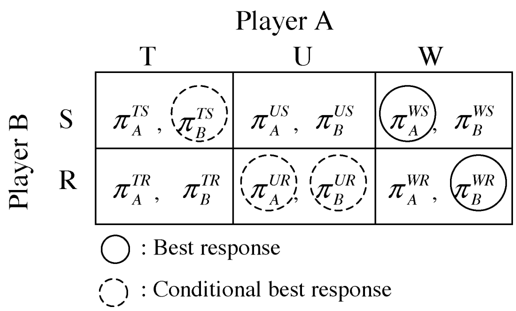

Fig. 4. Best responses in the two-player auction game under the pay-as-bid pricing mechanism.

TABLE XI
PROFITS IN THE INELASTIC DEMAND CASE

Region	Uniform		Pay-as-bid	
	π_A (\$)	π_B (\$)	π_A (\$)	π_B (\$)
(1)	$2p_A - 30$	0	$2p_A - 30$	0
(2)	$p_A - 10$	$p_A - 15$	$p_A - 10$	$p_B - 15$

"Safe", we get the set of "rationalizable" price strategies P_A^R and P_B^R for each player in (5) [2]

$$P_A^R = \left\{ p_A | \frac{1}{2}(p_{\text{cap}} + c_{A2}) \le p_A \le p_{\text{cap}} \right\}$$
$$P_B^R = \left\{ p_B | \frac{1}{2}(p_{\text{cap}} + c_{A2}) \le p_B \le p_{\text{cap}} - \epsilon \right\}. \quad (5)$$

That is, player A cannot improve its profit by bidding a price outside the set P_A^R and player B cannot improve its profit by bidding a price outside the set P_B^R.

Consider probability {mass} functions σ_A, σ_B over the set of rationalizable strategies P_A^R, P_B^R. The probability mass functions σ_A and σ_B satisfy

$$\sum_{p_A \in P_A^R} \sigma_A(p_A) = 1; \quad \forall p_A \in P_A^R, \quad \sigma_A(p_A) \ge 0,$$
$$\sum_{p_B \in P_B^R} \sigma_B(p_B) = 1; \quad \forall p_B \in P_B^R, \quad \sigma_B(p_B) \ge 0. \quad (6)$$

The expected profits for player A and B are defined by

$$\pi_A(\sigma_A, \sigma_B) = \sum_{p_A \in P_A^R} \sigma_A(p_A) \times \left\{ \sum_{p_B \in P_B^R} \sigma_B(p_B) \pi_A(p_A, p_B) \right\}$$
$$\pi_B(\sigma_A, \sigma_B) = \sum_{p_B \in P_B^R} \sigma_B(p_B) \times \left\{ \sum_{p_A \in P_A^R} \sigma_A(p_A) \pi_B(p_A, p_B) \right\}. \quad (7)$$

For σ_A and σ_B satisfying (6), the conditions for probability mass functions σ_A^* and σ_B^* to be a mixed-strategy NE under pay-as-bid pricing are

$$\forall \sigma_A, \pi_A(\sigma_A^*, \sigma_B^*) \ge \pi_A(\sigma_A, \sigma_B^*)$$
$$\forall \sigma_B, \pi_B(\sigma_A^*, \sigma_B^*) \ge \pi_B(\sigma_A^*, \sigma_B). \quad (8)$$

D. Revenue Comparison Between Uniform and Pay-as-Bid Pricing

We compare the total revenues under uniform and pay-as-bid pricing. We consider the expected total revenue when market players set NE strategy as their strategic choices. By checking the NE under each pricing mechanism, we have

Proposition 1: For the two-player multi-auction game with market power described in Section II, the expected total revenue under the uniform pricing auction is larger than under the pay-as-bid pricing auction.

Proof: From (4), we know the marginal bid is always p_{cap} at any NE under the uniform pricing auction. From definition 1 the expected total revenue at NE under uniform pricing auction is

$$TR_{\text{uni}}^* = TR_{\text{uni}}(p_A^*, p_B^*) = 2p_{\text{cap}}. \quad (9)$$

From (8) and definition 2, the expected total revenue under pay-as-bid pricing auction is represented by

$$TR_{\text{pay}}^* = \sum_{P_A^R} \sigma_A^*(p_A) \left\{ \sum_{P_B^R} \sigma_B^*(p_B) TR_{\text{pay}}(p_A, p_B) \right\}$$
$$= \sum_{P_B^R} \sigma_B^*(p_B) \left\{ \sum_{P_A^R} \sigma_A^*(p_A) TR_{\text{pay}}(p_A, p_B) \right\}. \quad (10)$$

Because there is no pure strategy NE, $\sigma_A^*(p_{\text{cap}}) < 1$, so since $TR_{\text{pay}}(p_A, p_B) < TR_{\text{pay}}(p_{\text{cap}}, p_B)$ for $p_A \ne p_{\text{cap}}$, we have that

$$\sum_{P_A^R} \sigma_A^*(p_A) TR_{\text{pay}}(p_A, p_B) < TR_{\text{pay}}(p_{\text{cap}}, p_B). \quad (11)$$

Similarly $\sigma_B^*(p_{\text{cap}} - \epsilon) < 1$ and so since, $\forall p_B \in P_B^R$, $TR_{\text{pay}}(p_{\text{cap}}, p_B) < TR_{\text{pay}}(p_{\text{cap}}, p_{\text{cap}} - \epsilon)$

$$\sum_{P_B^R} \sigma_B^*(p_B) TR_{\text{pay}}(p_{\text{cap}}, p_B) < TR_{\text{pay}}(p_{\text{cap}}, p_{\text{cap}} - \epsilon). \quad (12)$$

From (11) and (12), we have

$$TR_{\text{pay}}^* < TR(p_{\text{cap}}, p_{\text{cap}} - \epsilon)$$
$$= 2p_{\text{cap}} - \epsilon < 2p_{\text{cap}} = TR_{\text{uni}}^*. \quad (13)$$

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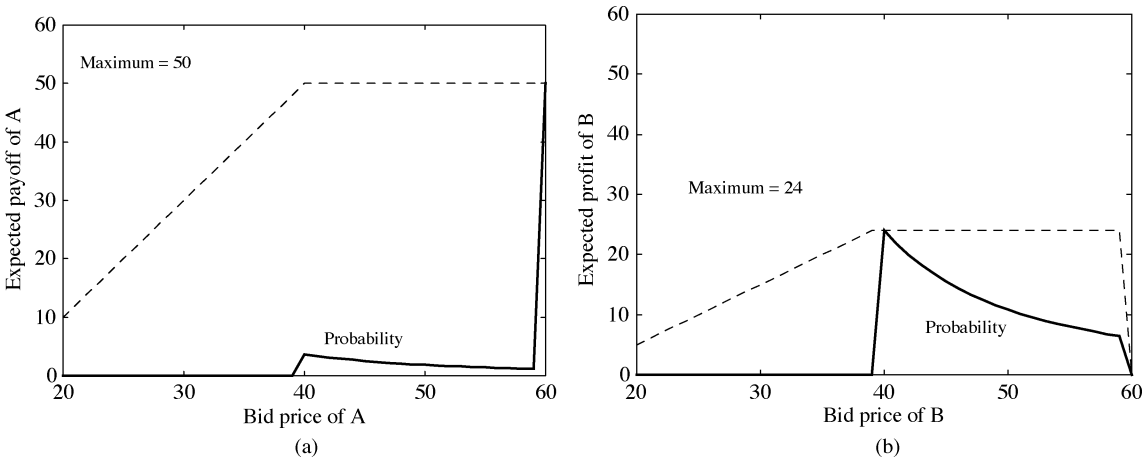

Fig. 5. Probability distribution on mixed-strategy NE in the inelastic demand case for pay-as-bid pricing for (a) Player A and (b) Player B. The solid curves represent the scaled probability distribution $\sigma_A^*(p_A)$ and $\sigma_B^*(p_B)$, while the dashed curves represent the expected profits $\sum_{p_B \in P_B^R} \sigma_B(p_B)\pi_A(p_A, p_B)$ and $\sum_{p_A \in P_A^R} \sigma_A(p_A)\pi_B(p_A, p_B)$ for the corresponding bid, given the distribution of the other player's bid.

TABLE XII
EXPECTED TOTAL REVENUES WITH DIFFERENT BID INCREMENT SIZES

ϵ	TR_{uni}^* ($)	TR_{pay}^* ($)
0.1	120	100.98
0.5	120	100.62
1	120	100.17
2	120	100.95

We will see in Section IV that TR_{pay}^* can be significantly smaller than TR_{uni}^*.

IV. SIMULATION

In order to confirm our analytical results, we simulate the short-term electricity market auction game with inelastic and elastic demand cases. For a quantitative result, we use bid prices directly instead of categorized bid price sets as in Section III. We apply the Lemke and Howson Algorithm [15], [16] to deal with the mixed-strategy NE. (The pure strategy NE is a special case of the mixed-strategy NE). The simulation steps follow.

1) Construct simulation bid price vectors $\mathbf{p}_A$ and $\mathbf{p}_B$ with the increment size ϵ.
2) Define vectors $\mathbf{x}$ and $\mathbf{y}$ for the probability masses σ_A and σ_B in (6), respectively.
3) Generate the profit matrix Π_A and Π_B given the game environment.
4) represent the mixed-strategy NE in (8) by

$$\mathbf{x}^{*T}\Pi_A\mathbf{y}^* \geq \mathbf{x}^T\Pi_A\mathbf{y}^*, \forall \mathbf{x} \in R^m : \sum_{i=1}^{m} x_i = 1, \mathbf{x} \geq 0$$
$$\mathbf{x}^{*T}\Pi_B\mathbf{y}^* \geq \mathbf{x}^{*T}\Pi_B\mathbf{y}, \forall \mathbf{y} \in R^n : \sum_{j=1}^{n} y_j = 1, \mathbf{y} \geq 0. \quad (14)$$

5) Convert conditions in (14) into a linear complementarity problem (LCP) and solve by a complementary pivot Algorithm [15].

A. Inelastic Demand Case

We simulated the two-player auction game analyzed in Section III. We set c_{A1}, c_{B1}, and c_{A2} as $10, $15, and $20, respectively. The price cap was set as $p_{\text{cap}} = \$60$. The simulation bid price range was $\$20 \leq p_A \leq \$60$ for player A and $\$15 \leq p_B \leq \$60$ for player B, respectively. The bid increment or decrement size was set as $\epsilon = \$1$. The elements of Π_A and Π_B are determined by Table XI.

The NE solution obtained by the algorithm under uniform pricing was a pure strategy NE of $p_A = 60$ and $p_B = 15$ consistent with the analysis in Section III. The expected total revenue is $TR_{\text{uni}}^* = 120$. The price is considerably above the competitive price.

The NE solution obtained under pay-as-bid pricing was a mixed-strategy NE of $\mathbf{x}^*$ and $\mathbf{y}^*$, which is represented by Fig. 5. The figure shows that the support of the bid price range is consistent with the range of "rationalizable" strategies $P_A^R = \{\$40 \leq p_A \leq \$60\}$ and $P_B^R = \{\$40 \leq p_B \leq \$59\}$ specified in (5). The expected total revenue was $TR_{\text{pay}}^* = \mathbf{x}^{*T}\mathbf{TR}_{\text{pay}}\mathbf{y}^* = 100.17$. Consistent with our theoretical analysis in Section III, TR_{pay}^* is less than $TR_{\text{uni}}^* = 120$. The expected price is still considerably above the competitive level, but closer to the system marginal cost of $5 than under uniform pricing.

We tried several different values of ϵ to see if there is any serious sensitivity to ϵ. The resulting expected total revenues are illustrated in Table XII. The NE and expected total revenue were not very sensitive to ϵ and confirm that TR_{pay}^* is significantly less than TR_{uni}^* for this example.

B. Elastic Demand Case

The theoretical development in Section III assumes inelastic demand, unit size generation blocks, and an enforced price cap. Potentially, proposition 1 depends critically on these assumptions. To test the extensibility of the proposition 1 to an elastic demand case, we extended the model by incorporating demand willingness-to-pay and different sized generation blocks.

We assume three demand blocks W_1, W_2, and W_3 that have different willingness-to-pay and size. The demand blocks are

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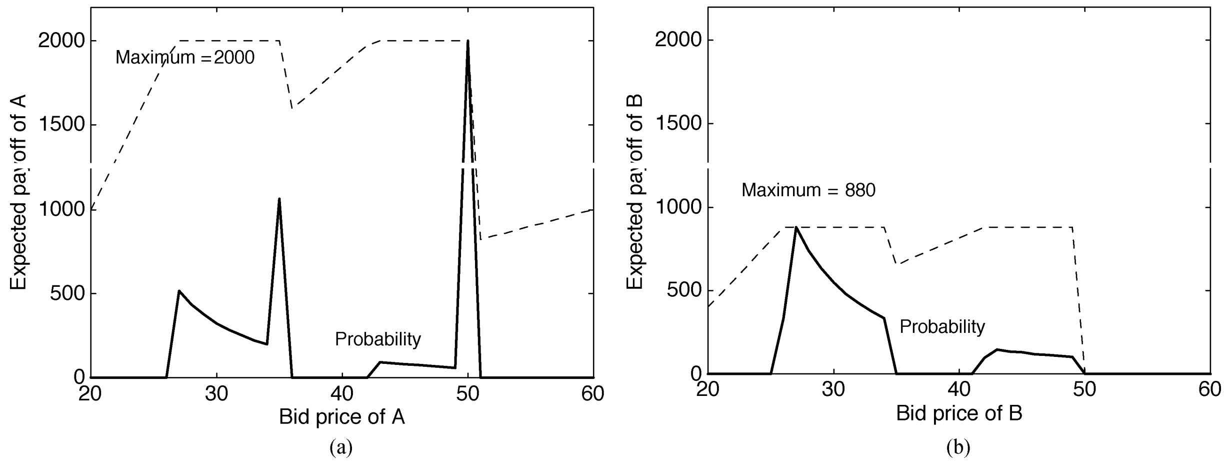

Fig. 6. Probability distribution on mixed-strategy NE in the elastic demand case for pay-as-bid pricing for (a) Player A and (b) Player B. The solid curves represent the scaled probability distribution $\sigma_A^*(p_A)$ and $\sigma_B^*(p_B)$, while the dashed curves represent the profits $\sum_{p_B \in P_B^R} \sigma_B(p_B)\pi_A(p_A, p_B)$ and $\sum_{p_A \in P_A^R} \sigma_A(p_A)\pi_B(p_A, p_B)$ for the corresponding bid, given the distribution of the other player's bid.

TABLE XIII
DEMAND AND SUPPLY FOR THE ELASTIC DEMAND CASE

Willingness-to-pay			Cost		
demand	price ($/MW)	size (MW)	supply	cost ($/MW)	size (MW)
W_1	60	100	G_{A1}	10	100
W_2	50	30	G_{B1}	15	80
W_3	35	20	G_{A2}	20	80

TABLE XIV
PROFITS IN THE ELASTIC DEMAND CASE UNDER UNIFORM PRICING

Region	π_A ($)	π_B ($)
(1)	$150p_A - 2000$	0
(2)	$130p_A - 1600$	0
(3)	$100p_A - 1000$	0
(4)	$70p_A - 700$	$80p_A - 1200$
(5)	$50p_A - 500$	$80p_A - 1200$
(6)	$20p_A - 200$	$80p_A - 1200$

TABLE XV
PROFITS IN THE ELASTIC DEMAND CASE UNDER PAY-AS-BID PRICING

Region	π_A ($)	π_B ($)
(1)	$150p_A - 2000$	0
(2)	$130p_A - 1600$	0
(3)	$100p_A - 1000$	0
(4)	$70p_A - 700$	$80p_B - 1200$
(5)	$50p_A - 500$	$80p_B - 1200$
(6)	$20p_A - 200$	$80p_B - 1200$

assumed to be bid in at their willingness-to-pay. As previously, we assume two strategic suppliers, players A and B. Player A owns two blocks, G_{A1} and G_{A2}, while player B owns one block, G_{B1}. The configuration for demand and supply shown in Table XIII. Under economic dispatch based on actual marginal costs, the marginal cost of supplying additional demand would be $15/MWh.

However, the supply is bid strategically. Supply that is cleared in the auction is paid either the market clearing price or its bid depending on the auction design.

The stacked supply bid and demand blocks determine the market clearing price and quantity. The price cap was set as $p_{\text{cap}} = \$60$. The simulation bid price range was $\$20 \le p_A \le \$60$ for player A and $\$15 \le p_B \le \$60$ for player B, respectively. The bid increment or decrement size was set as $\epsilon = \$1$. In Table XIV and Table XV, we illustrate the profit of player A and B in each region under the uniform and pay-as-bid pricing, respectively.

Under uniform pricing, a pure strategy NE of ($p_A = 50$, $p_B = 15$) was obtained by the algorithm resulting in a uniform clearing price of $50. Although there are other pure strategy NEs and a mixed-strategy NE, they all result in the same market clearing price and demand as ($p_A = 50$, $p_B = 15$). The expected market cleared demand TD_{uni}^* was 130 MW out of the total bid demand 150 MW. The uniform market clearing price was $50 yielding an expected total revenue of $TR_{\text{uni}} = \$6500$.

Under pay-as-bid pricing, a mixed-strategy NE is obtained. Fig. 6 represents the NE probability distribution over each player's strategies. Under pay-as-bid pricing, the expected market cleared demand is $TD_{\text{pay}}^* = 141.85$ MW out of the total bid demand 150 MW and the total expected revenue of $TR_{\text{pay}}^* = \$4726.2$. The expected price is much closer to the system marginal cost of 15 $/MW than under uniform pricing.

Under pay-as-bid pricing, the expected market cleared demand is more and the expected total revenue is less than under the uniform pricing. Table XVI shows the results with different bid increment sizes ϵ. Again, the results are not heavily dependent on the choice of ϵ, TR_{pay}^* is significantly lower than TR_{uni}^*, and more demand is cleared under pay-as-bid than uniform pricing.

V. DISCUSSION

The theoretical and computational analysis in this paper shows that the conclusion of the RET does not hold for a two-player model of an electricity market without transmission

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TABLE XVI
EXPECTED TOTAL REVENUES AND MARKET CLEARED DEMAND WITH DIFFERENT BID INCREMENT SIZES

ϵ	Uniform		Pay-as-bid	
	TR^*_{uni} (\$)	TD^*_{uni} (MW)	TR^*_{pay} (\$)	TD^*_{pay} (MW)
0.1	6500	130	4769	141.5
0.5	6500	130	4760.4	141.48
1	6500	130	4726.2	141.85
2	6500	130	4716.9	141.2

constraints. That is, unlike the conclusion of the RET, the market pricing mechanism *does* affect the market clearing price and total revenue.

The counterexample shows that the conclusion of the RET cannot hold in general in electricity markets. However, since the proof and simulation result in this paper are limited to a specific model with two asymmetric players, we can only conjecture about more general cases. In the following sections, we discuss conjectures for generalization of our results to multiple players and transmission constraints.

A. Conjecture for Games With Multiple Players

We conjecture that the observations made in this paper for a two-player game can also generalize to multiple player games. In particular, under uniform pricing, since most of market players have intra-marginal blocks, they can enjoy profit maximized revenues without any risk by letting players with market power keep the prices high. This would result in extremely high market price in the short term when supply is tight, as experienced in California.

In contrast, under pay-as-bid pricing most of market players cannot enjoy the profit maximized revenues without the risk of being "undercut." This would result in reducing total suppliers' revenue in the short term.

In future work, we plan to develop the methodology described in [17] to treat the case of multiple players. An extension of the theoretical analysis will also be attempted.

B. Conjecture for Transmission Constrained Systems

Transmission constraints can facilitate market power problems by isolating local markets from a central market [18]. When a local market is isolated and has players with market power, we conjecture that the same behavior pattern described in this paper will arise in the local market. However, exercise of market power also interacts with congestion management mechanisms, as described in [19]. This issue has not been considered in this paper.

VI. CONCLUSIONS

We analyzed a two-player static auction game that has a big player with market power and small player. We proved that the total expected revenue for the NEs under uniform pricing and pay-as-bid pricing were not equivalent. The total payment of consumers would be smaller under pay-as-bid pricing for the two-player game; however, under pay-as-bid the equilibrium is a mixed-strategy equilibrium, which is presumably undesirable from an operational point of view. We illustrated the simulation results of the model. We also extended the model to an elastic demand case and showed that pay-as-bid pricing also led to a larger expected total demand being served.

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