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Dynamic Testing of Wholesale Power Market Designs: An Open-Source Agent-Based Framework

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Abstract In April 2003 the U.S. Federal Energy Regulatory Commission proposed a complicated market design—the *Wholesale Power Market Platform* (*WPMP*)—for common adoption by all US wholesale power markets. Versions of the WPMP have been implemented in New England, New York, the mid-Atlantic states, the Midwest, the Southwest, and California. Strong opposition to the WPMP persists among some industry stakeholders, however, due largely to a perceived lack of adequate performance testing. This study reports on the model development and open-source implementation (in Java) of a computational wholesale power market organized in accordance with core WPMP features and operating over a realistically rendered transmission grid. The traders within this market model are strategic profit-seeking agents whose learning behaviors are based on data from human-subject experiments. Our key experimental focus is the complex interplay among structural conditions, market protocols, and learning behaviors in relation to short-term and longer-term market performance. Findings for a dynamic 5-node transmission grid test case are presented for concrete illustration.

This article is an abridged version of Sun and Tesfatsion (2007a).

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1 Introduction

The meltdown in the restructured California wholesale power market in the summer of 2000 has shown what can happen when a poorly designed market mechanism is implemented without proper testing. The California crisis is believed to have resulted in part from strategic behaviors encouraged by inappropriate market design features (Borenstein 2002). Following the California crisis, many energy researchers have eloquently argued the need to combine structural understanding with economic analysis of incentives in order to develop wholesale power market designs with good real-world performance characteristics; see, for example, Amin (2004).

In April 2003 the U.S. Federal Energy Regulatory Commission proposed the *Wholesale Power Market Platform* (*WPMP*) as a template for all U.S. wholesale power markets (FERC 2003). As detailed in Wilson (2002), this design entails an integrated rather than unbundled market form; it recommends the operation of wholesale power markets by Independent System Operators (ISOs) or Regional Transmission Organizations (RTOs) using locational marginal pricing to price energy by the location of its injection into or withdrawal from the transmission grid. Versions of this design have been implemented in New England (ISO-NE), New York (NYISO), the mid-Atlantic states (PJM), the Midwest (MISO), the Southwest (SPP), and California (CAISO). Joskow (2006, p. 6) reports that ISO/RTO operated energy regions now include over 50% of the generating capacity in the US; see Fig. 1.

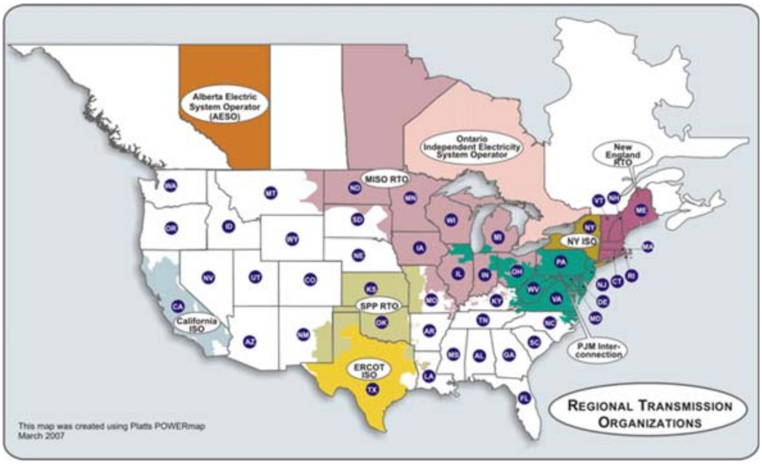

Fig. 1 Existing and proposed ISO/RTO-operated U.S. wholesale power markets (*Source:* FERC, http://www.ferc.gov/industries/electric/indus-act/rto/rto-map.asp)

The complexity of the WPMP market design has made it extremely difficult to undertake economic and physical reliability studies of the design using standard statistical and analytical tools. Strong opposition to the market design thus persists among some industry stakeholders due in part to a perceived lack of sufficient performance testing.

In recent years, however, powerful new agent-based computational tools have been developed to analyze this degree of complexity. A variety of commercial agent-based frameworks are now available for the study of restructured electricity markets; see, for example, the EMCAS framework developed by researchers at the Argonne National Laboratory (Conzelmann et al. 2004). In addition, researchers such as Bower and Bunn (2001), Nicolaisen et al. (2001), Veit et al. (2006) and Widergren et al. (2004) have used agent-based models to study important aspects of restructured electricity markets.[1]

In a preliminary study (Koesrindartoto et al. 2005), we examined the feasibility and potential fruitfulness of *Agent-based Computational Economics* (*ACE*) specifically for the study of the WPMP market design. ACE is the computational study of economic processes modeled as dynamic systems of interacting agents.[2]

Building on this prior work, the present study reports on the development and implementation of an ACE framework for testing the dynamic efficiency and reliability of the WPMP market design. This framework—referred to as *AMES* (*A*gent-based *M*odeling of *E*lectricity *S*ystems)—models strategic traders interacting over time in a wholesale power market that is organized in accordance with core WPMP features and that operates over a realistically rendered transmission grid. To our knowledge, AMES is the first non-commercial open-source framework permitting the computational study of the WPMP design.

To help ensure empirical input validity, the AMES framework has been developed by means of an iterative participatory modeling approach.[3] Specifically, we are engaging with industry participants and policy makers in an ongoing collaborative learning process involving four repeated stages of analysis: fieldwork and data collection; scenario discussion and role-playing games; agent-based model development; and intensive computational experiments. We are relying heavily on business practices from two adopters of the WPMP design (New England and the Midwest) for our implementation of market structure, market architecture, and dispatch and pricing solutions. We have also incorporated reinforcement learning representations for the electricity traders that are based on findings from human-subject multi-agent game experiments conducted by Roth and Erev (1995).[4]

We are currently using the AMES framework to investigate the intermediate-term performance of wholesale power markets operating under the WPMP market design.

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[1] See Tesfatsion (2006a) for extensive annotated pointers to agent-based electricity research.

[2] See Axelrod and Tesfatsion (2006), Tesfatsion (2006b), and Tesfatsion and Judd (2006) for extensive introductory materials on ACE.

[3] See Barreteau (2003) for a fuller discussion of iterative participatory modeling, also called *companion modeling*. For more general materials on empirical validation methods for agent-based computational models, see Tesfatsion (2006c) and Windrum et al. (2007).

[4] Real-world market traders are understandably reluctant to discuss with us the precise manner in which they determine their supply offers and demand bids, so indirect identification methods must be used.

In particular, we are exploring the extent to which this design is capable of supporting the efficient, profitable, and sustainable operation over time of existing generation and transmission facilities, despite possible attempts by some market participants to gain individual advantage through strategic pricing, capacity withholding, and induced transmission congestion.

To illustrate concretely the potential usefulness of the AMES framework for this purpose, experimental findings are reported below for a dynamic extension of a static five-node transmission grid test case used extensively for training purposes by the ISO-NE and PJM. In the static training case, the generators are assumed to report their true cost and production capacity attributes to the ISO; the possibility that generators might engage in strategic reporting behavior is not considered. In contrast, the AMES generators use reinforcement learning to decide the exact nature of the supply offers (marginal cost functions and production intervals) that they daily report to the AMES ISO for use in the WPMP day-ahead market. We show that all of the AMES generators learn over time to implicitly collude on the reporting of higher-than-true marginal costs, thus considerably raising total variable costs of operation at the ISO-determined "optimal" solutions.

Our longer-run goal is to develop AMES into a framework that facilitates intensive computational experiments for research and teaching purposes. Specifically targeted framework features include:

- Research/teaching-grade tool (small to medium-scale grid size);
- Operational validity (structure, architecture, and behavioral dispositions);
- Permits dynamic testing with learning traders;
- Permits intensive sensitivity experiments;
- Open source (full access to implementation);
- Easy modification (extensible/modular architecture).

We envision academic researchers and teachers using this framework to increase their qualitative understanding of the dynamic operation of restructured wholesale power markets. Industry participants should be able to use the framework to familiarize themselves with market rules and to test business strategies. And policy makers should find the framework useful for conducting intensive experiments to explore the performance of actual or proposed market designs from a social welfare viewpoint. In particular, does a design encourage the efficient and reliable operation of existing generation and transmission capacity in the short term, and does it provide appropriate incentives for investment in new generation and new transmission capacity in the longer term?

An overview of the AMES wholesale power market framework is presented in Sect. 2, and detailed configuration settings for the AMES transmission grid, energy traders, and ISO are presented in Sect. 3. Experimental findings for a dynamic five-node transmission grid test case are presented in Sect. 4 making use of the configuration settings from Sect. 3. Concluding remarks are given in Sect. 5.

2 Overview of the AMES Framework

The AMES wholesale power market framework is programmed in Java using RepastJ, a Java-based toolkit designed specifically for agent-based modeling in the social sciences.[5] The framework is modular, extensible, and open source in order to provide a useful foundation for further electricity research.[6]

The AMES framework currently incorporates in stylized form several core elements of the WPMP market design as implemented by the New England Independent System Operator (ISO-NE) and the Midwest Independent System Operator (MISO), respectively. By adhering closely to the architecture of these regional energy markets, we have been able to take advantage of the business practice manuals, training guides, and reports publicly released by the ISO-NE (2007) and the MISO (2007) for use by their market participants. These publications provide a wealth of specific implementation details missing from the more abstract WPMP template.

As depicted in Figs. 2–4, the core elements of the WPMP market design that have been incorporated into the AMES framework to date are as follows:

- The AMES wholesale power market operates over an AC transmission grid for DMax successive days, with each day D consisting of 24 successive hours $\mathrm{H} = 00, 01, \ldots, 23$.
- The AMES wholesale power market includes an Independent System Operator (ISO) and a collection of energy traders consisting of Load-Serving Entities (LSEs) and Generators distributed across the nodes of the transmission grid.[7]
- The AMES ISO undertakes the daily operation of the transmission grid within a two-settlement system consisting of a Real-Time Market and a Day-Ahead Market, each separately settled by means of *locational marginal pricing*.[8]
- During the afternoon of each day D the AMES ISO determines power commitments and *locational marginal prices* (*LMPs*)[9] for the Day-Ahead Market for day D + 1 based on Generator supply offers and LSE demand bids (forward financial contracting) submitted during Hours 00–11 of day D.

[5] See Tesfatsion (2006d) for resources related to the agent-based toolkit RepastJ. Agent-based researchers are increasingly making use of powerful object-oriented programming (OOP) languages such as Java, C++, or C# either directly or through some form of agent-based toolkit. Weisfeld (2003) provides an excellent introduction to OOP. For a general annotated listing of OOP software and toolkits suitable for agent-based modeling, see Tesfatsion (2006e).

[6] In particular, the goal of the larger NSF project encompassing the development of the AMES framework (McCalley et al. 2005) is to explore ways of achieving a more effectively integrated US energy transportation network encompassing electricity, gas, coal, and water subsectors. The longer-term plan is to incrementally extend the AMES framework to include consideration of these related energy subsectors.

[7] An *Independent System Operator* (*ISO*) is an organization charged with the primary responsibility of maintaining the security of a power system and often with system operation responsibilities as well. The ISO is independent to the extent that it does not have a conflict of interest in carrying out these responsibilities, such as an ownership stake in generation or transmission facilities within the power system. A *Load Serving Entity* (*LSE*) is an electric utility, transmitting utility, or Federal power marketing agency that has an obligation under Federal, State, or local law, or under long-term contracts, to provide electrical power to end-use (residential or commercial) consumers or to other LSEs with end-use consumers. An LSE aggregates individual end-use consumer demand into "load blocks" for bulk buying at the wholesale level. A *Generator* is a unit that produces and sells electrical power in bulk at the wholesale level. A *node* is a point on the transmission grid where power is injected or withdrawn.

[8] *Locational marginal pricing* is the pricing of electrical power according to the location of its withdrawal from, or injection into, a transmission grid.

[9] A *locational marginal price* (*LMP*) at any particular node is the least cost of meeting demand at that node for one additional unit of power, i.e. for one additional megawatt (MW).

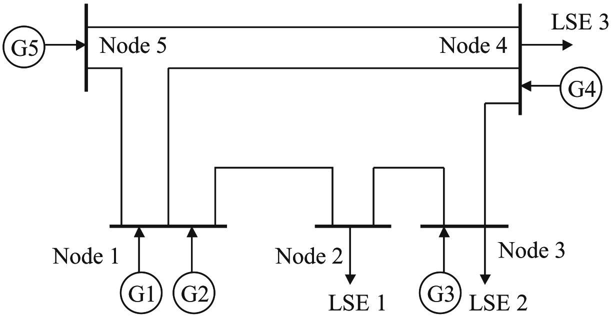

Fig. 2 A five-node transmission grid configuration

- **Traders**
  - Sellers and buyers
  - Follow market rules
  - Learning abilities
- **Independent System Operator**
  - System reliability assessments
  - Day-ahead bid-based unit commitment
  - Real-time dispatch
- **Two-Settlement Process**
  - Day-ahead market (double auction, financial contracts)
  - Real-time market (settlement of differences)
- **AC Transmission Grid**
  - Sellers and buyers located at various transmission nodes
  - Congestion managed by locational marginal pricing (LMP)

Fig. 3 AMES core features

- At the end of each day D the AMES ISO produces and posts a day D + 1 commitment schedule for Generators and LSEs and settles these financially binding contracts on the basis of day D + 1 LMPs.
- Any differences that arise during day D + 1 between real-time conditions and the day-ahead financial contracts settled at the end of day D must be settled in the Real-Time Market for day D + 1 at real-time LMPs for day D + 1.
- Transmission grid congestion in the Day-Ahead Market is managed via the inclusion of congestion components in LMPs.

Five additional elements that will subsequently be incorporated into AMES to reflect more fully the dynamic operational capabilities of the WPMP market design are: (a) *market power mitigation measures*; (b) *bilateral trading*, which permits longer-term contracting; (c) a market for *financial transmission rights*[10] to permit AMES

[10] A *financial transmission right* (*FTR*) purchased on a transmission line *from* node A *to* node B entitles the holder to a compensation if the LMP at node B exceeds the LMP at node A, and obligates the holder to make a payment if the LMP at node A exceeds the LMP at node B. See Sun (2006).

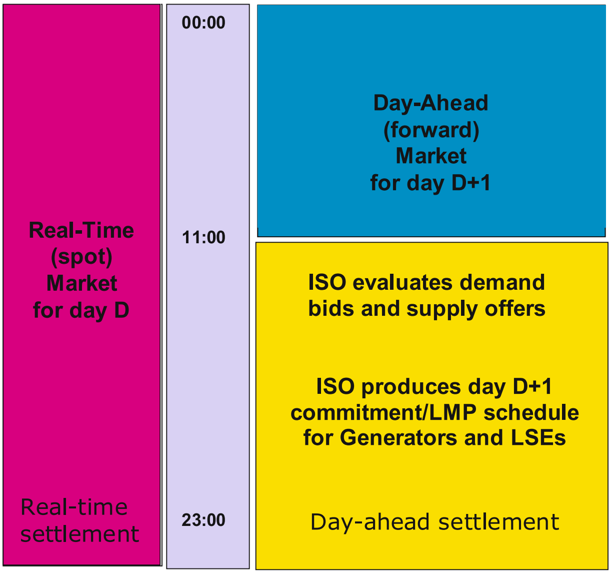

Fig. 4 Activities of the AMES ISO during a typical day D

traders to hedge against transmission congestion costs arising in the Day-Ahead Market; (d) *security constraints* incorporated into the DC-OPF problems solved by the AMES ISO for the Real-Time Market and Day-Ahead Market as a hedge against system disturbances; and (e) a (*Resource Offer*) *Re-Bid Period*[11] during each day D as part of a resource adequacy assessment undertaken by the AMES ISO to help ensure that forecasted loads and reserve requirements are always met. Figures 5 and 6 schematically depict the architecture and dynamic flow of this extended AMES framework.

As explained more carefully in Sect. 3.5 below, the AMES ISO determines hourly power commitments/dispatch levels and LMPs for the Day-Ahead Market and Real-Time Market by solving *DC Optimal Power Flow* (*OPF*) problems that approximate underlying AC-OPF problems. To handle these aspects, we have developed an accurate and efficient strictly convex quadratic programming (SCQP) solver module, *QuadProgJ*, wrapped in an outer DC-OPF data conversion shell, *DCOPFJ* (Sun and Tesfatsion 2006, 2007b). The AMES ISO solves its DC-OPF problems by invoking QuadProgJ through DCOPFJ.

As detailed in Sect. 3.6 below, trader learning is implemented in the AMES framework by a reinforcement learning module, *JReLM*, developed by Gieseler (2005). JReLM can implement a variety of different reinforcement learning methods, permitting

[11] Here we follow the MISO market architecture and terminology. The ISO-NE implements a similar design feature during each day D called the "(Real-Time Energy Market) Supply Re-Offer Period."

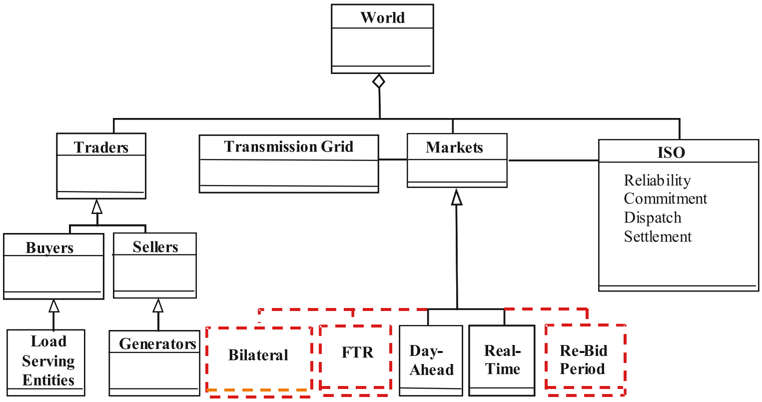

Fig. 5 AMES architecture (Agent hierarchy)

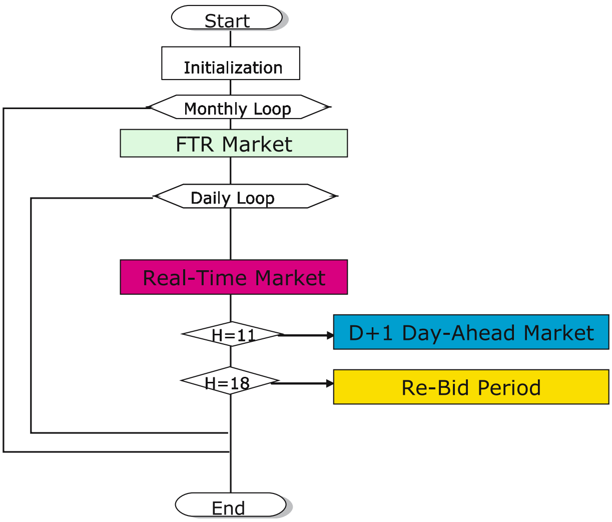

Fig. 6 AMES dynamic market activities: global view

flexible representation of trader learning within this family of methods. In later extensions of AMES, other possible trader learning methods (e.g. social mimicry and belief learning) will also be considered.

The QuadProgJ/DCOPFJ and JReLM modules for ISO grid operation and trader learning constitute the core components supporting the implementation of the AMES wholesale power market framework. This implementation is schematically depicted in Fig. 7.

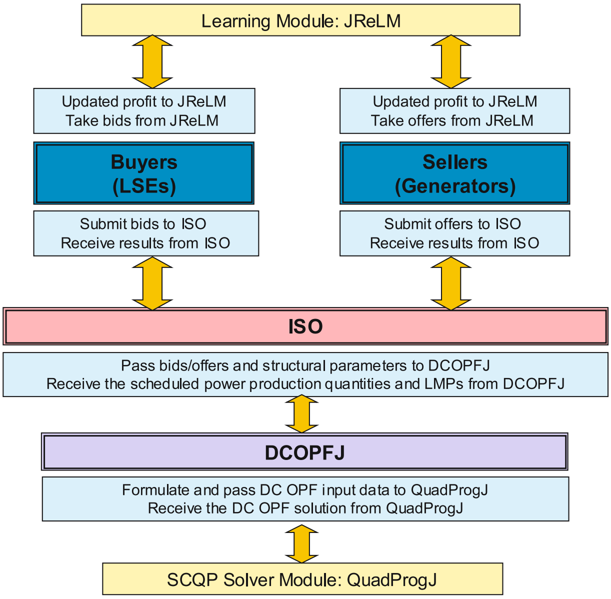

Fig. 7 Core module components of the AMES framework

3 Configuration of the AMES Framework

3.1 Overview

This section provides basic configuration information for the AMES wholesale power market framework as currently implemented. All subsequently reported experiments make use of these configurations.

For later ease of reference, the admissible exogenous variables for the AMES framework are depicted and defined in Table 1 and the endogenous variables are depicted and defined in Table 2.[12] These variable depictions and definitions will be used throughout the remainder of this study.

3.2 Structural Configuration of the AMES Transmission Grid

The structural specification of transmission grids is complicated due to the underlying physical relations governing power flows. Below we briefly summarize the AMES grid specification to indicate the care that has been taken to properly account for these underlying relations. The interested reader is referred to Sun and Tesfatsion (2006) and references therein for a more complete and rigorous discussion of this specification.

The AMES transmission grid is an alternating current (AC) grid modeled as a balanced three-phase network with $N \geq 1$ branches and $K \geq 2$ nodes. The

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[12] Only persistent variables appear in these tables. Locally scoped variables temporarily introduced to carry out method implementations are not included.

Table 1 Admissible exogenous variables for the AMES framework

Variable	Description	Admissibility restrictions
K	Total number of transmission grid nodes	$K > 0$
N	Total number of distinct network branches	$N > 0$
I	Total number of generators	$I > 0$
J	Total number of LSEs	$J > 0$
I_k	Set of generators located at node k	$\text{Card}(\cup_{k=1}^{K} I_k) = I$
J_k	Set of LSEs located at node k	$\text{Card}(\cup_{k=1}^{K} J_k) = J$
S_o	Base apparent power (three-phase MVAs)	$S_o \geq 1$
V_o	Base voltage (line-to-line kVs)	$V_o > 0$
V_k	Voltage magnitude (kVs) at node k	$V_k = V_o, k = 1, \ldots, K$
p_{Lj}	Real power load (MWs) withdrawn by LSE j	$p_{Lj} \geq 0, j = 1, \ldots, J$
km	Branch connecting nodes k and m (if one exists)	$k \neq m$
BR	Set of all distinct branches km, $k < m$	$BR \neq \emptyset$
X_{km}	Reactance (ohms) for branch km	$X_{km} = X_{mk} > 0, km \in BR$
B_{km}	$[1/X_{km}]$ for branch km	$B_{km} = B_{mk} > 0, km \in BR$
P_{km}^{U}	Thermal limit (MWs) for real power flow on km	$P_{km}^{U} > 0, km \in BR$
δ_1	Reference node 1 voltage angle (radians)	$\delta_1 = 0$
π	Soft penalty weight for voltage angle differences	$\pi > 0$
Money_i^{o}	Initial money holdings ($) for Gen i	$\text{Money}_i^{o} > 0, i = 1, \ldots, I$
Cap_i^{L}	True lower production limit (MWs) for Gen i	$\text{Cap}_i^{L} \geq 0, i = 1, \ldots, I$
Cap_i^{U}	True upper production limit (MWs) for Gen i	$\text{Cap}_i^{U} > \text{Cap}_i^{L}, i = 1, \ldots, I$
a_i, b_i	True cost coefficients ($/MWh, $/MW^{2}h) for Gen i	$b_i > 0, i = 1, \ldots, I$
$\text{MC}_i(p)$	$\text{MC}_i(p) = a_i + 2b_i p$ = Gen i's true MC function	$\text{MC}_i(\text{Cap}_i^{L}) > 0, i = 1, \ldots, I$
FCost_i	Fixed costs (hourly prorated) for Gen i	$\text{FCost}_i \geq 0, i = 1, \ldots, I$
M_i	Cardinality of the action domain AD_i for Gen i	$M_i \geq 1, i = 1, \ldots, I$
Mj_i	Integer-valued density-control parameter for AD_i	$\prod_{j=1}^{3} Mj_i = M_i, i = 1, \ldots, I$
RIMax_i^{L}	Range-index parameter for AD_i construction	$\text{RIMax}_i^{L} \in [0, 1), i = 1, \ldots, I$
RIMax_i^{U}	Range-index parameter for AD_i construction	$\text{RIMax}_i^{U} \in [0, 1), i = 1, \ldots, I$
RIMin_i^{C}	Range-index parameter for AD_i construction	$\text{RIMin}_i^{C} \in (0, 1], i = 1, \ldots, I$
SS_i	Slope-start control parameter for AD_i construction	$SS_i > 0, i = 1, \ldots, I$
$q_i(0)$	Initial propensity (learning)	Any real value, $i = 1, \ldots, I$
C_i	Cooling parameter (learning)	$C_i > 0, i = 1, \ldots, I$
r_i	Recency parameter (learning)	$0 \leq r_i \leq 1, i = 1, \ldots, I$
e_i	Experimentation parameter (learning)	$0 \leq e_i < 1, i = 1, \ldots, I$

Table 2 Endogenous variables for the AMES framework

Variable	Description
p_{Gi}	Real power injection (MWs) by Gen $i = 1, \ldots, I$
δ_k	Voltage angle (radians) at node $k = 2, \ldots, K$
LMP_k	Locational marginal price ($/MWh) at node $k = 1, \ldots, K$
P_{km}	Real power (MWs) flowing in branch $km \in \text{BR}$
PGen_k	Total real power injection (MWs) at node $k = 1, \ldots, K$
PLoad_k	Total real power withdrawal (MWs) at node $k = 1, \ldots, K$
PNetInject_k	Total net real power injection (MWs) at node $k = 1, \ldots, K$
Profit_i	Realized profit ($/h) for Gen $i = 1, \ldots, I$
Money_i	Cumulative money holdings ($) for Gen $i = 1, \ldots, I$
Cap_i^{RL}	Reported lower production limit (MWs) for Gen $i = 1, \ldots, I$
Cap_i^{RU}	Reported upper production limit (MWs) for Gen $i = 1, \ldots, I$
a_i^{R}, b_i^{R}	Reported cost coefficients ($/MWh, $/MW^{2}h) for Gen $i = 1, \ldots, I$

reactance on each branch is assumed to be a total branch reactance (rather than a per mile reactance), meaning that the branch length is already taken into account. All transformer phase angle shifts are assumed to be zero, all transformer tap ratios are assumed to be 1, all line-charging capacitances are assumed to be 0, and the temperature is assumed to remain constant over time.

The AMES transmission grid is assumed to be *connected* in the sense that it has no isolated components; each pair of nodes k and m is connected by a linked branch path consisting of one or more branches. If two nodes are in direct connection with each other, it is assumed to be through at most one branch, i.e., branch groups are not explicitly considered. However, complete connectivity is *not* assumed. That is, node pairs are *not* necessarily in *direct* connection with each other through a single branch.

For per unit normalization in DC-OPF implementations, it is conventional to specify base value settings for apparent power (in megavoltamperes MVA) and voltage (in kilovolts kV). For the AMES transmission grid, the base apparent power, denoted by S_o, is assumed to be measured in three-phase MVAs, and the base voltage, denoted by V_o, is assumed to be measured in line-to-line kVs.

It is also assumed that *Kirchoff's Current Law* (*KCL*) governing current flows in electrical networks holds for the AMES transmission grid for each hour of operation. As detailed in (Kirschen and Strbac, 2004, Sect. 6.2.2.1), KCL implies that real and reactive power must each be in balance at each node. Thus, real power must also be in balance across the entire grid, in the sense that aggregate real power withdrawal plus aggregate transmission losses must equal aggregate real power injection.

In wholesale power markets restructured in accordance with the WPMP market design, the transmission grid is overlaid with a commercial network consisting of "pricing locations" for the purchase and sale of electric power. A *pricing location* is a location at which market transactions are settled using publicly available LMPs. For simplicity, it is assumed that the set of pricing locations for AMES coincides with the set of transmission grid nodes.

3.3 Structural Configuration of the AMES LSEs

The AMES LSEs purchase bulk power in the AMES wholesale power market each day in order to service customer demand (load) in a downstream retail market. The user specifies the number J of LSEs as well as the location of these LSEs at various nodes of the transmission grid. LSEs do not engage in production or sale activities in the wholesale power market. Hence, LSEs purchase power only from Generators, not from each other.

For initial simplicity, the current study makes the usual empirically-based assumption that the downstream retail demands serviced by the AMES LSEs exhibit negligible price sensitivity and hence reduce to daily load profiles. In addition, the LSEs are modeled as passive entities who submit these daily load profiles into the Day-Ahead Market as their demand bids without strategic consideration. Specifically, at the beginning of each day D each LSE j submits a daily load profile into the day-ahead market for day D+1. This daily load profile indicates the real power demand p_{Lj}(H) (in MWs) that must be serviced by LSE j in its downstream retail market for each of 24 successive hours H.

3.4 Structural Configuration of the AMES Generators

The AMES Generators are electric power generating units. The user specifies the number I of Generators as well as the location of these Generators at various nodes of the transmission grid. Generators sell power only to LSEs, not to each other. Each AMES Generator i is user-configured with a production technology, learning capabilities, and an initial level Money_i^o of money holdings. Here we elaborate on Generator production technologies; learning capabilities are separately taken up in Subsect. 3.6 below.

With regard to production technology, it is assumed that each Generator has variable and fixed costs of production. However, Generators do not incur no-load, startup, or shutdown costs, and they do not face ramping constraints.[13]

More precisely, the technology attributes assumed for each Generator i take the following form. Generator i has lower and upper production limits (in MWs), denoted by Cap_i^L and Cap_i^U, that define the *feasible production interval* for its hourly real-power production level p_{Gi} (in MWs).[14] That is, for each i,

$$\text{Cap}_i^L \le p_{Gi} \le \text{Cap}_i^U \tag{1}$$

In addition, Generator i has a *total cost function* giving its total costs of production per hour for each p_{Gi}. This total cost function takes the form

$$\text{TC}_i(p_{Gi}) = a_i \cdot p_{Gi} + b_i \cdot p_{Gi}^2 + \text{FCost}_i \tag{2}$$

where a_i (\$/MWh), b_i (\$/MW^{2}h), and FCost_i (\$/h) are exogenously given constants. Note that $\text{TC}_i(p_{Gi})$ is measured in dollars per hour (\$/h). Generator i's *total variable cost function* and (*hourly prorated*) *fixed costs* for any p_{Gi} are then given by[15]

$$\text{TVC}_i(p_{Gi}) = \text{TC}_i(p_{Gi}) - \text{TC}_i(0) = a_i \cdot p_{Gi} + b_i \cdot p_{Gi}^2 \tag{3}$$

[13] As is standard in economics, *variable costs* are costs that vary with the level of production, and *fixed costs* are costs such as debt and equity obligations associated with plant investments that are not dependent on the level of production and that are incurred even if production ceases. As detailed by Kirschen and Strbac (2004, Sect. 4.3), the concept of *no-load costs* in power engineering refers to *quasi-fixed* costs that would be incurred by Generators if they could be kept running at zero output but that would vanish once shut-down occurs. *Startup costs* are costs specifically incurred when a Generator starts up, and *shutdown costs* are costs specifically incurred when a Generator shuts down. Finally, *ramping constraints* refer to physical restrictions on the rates at which Generators can increase or decrease their outputs.

[14] In the current AMES framework, the lower production limit Cap_i^L for each Generator i is a firm "must run" minimum real-power production level. That is, if Cap_i^L is positive, then shutting down Generator i is not an option for the AMES ISO.

[15] Quadratic functions as in Eq. 3 are commonly used to represent generator total variable costs (i.e. costs of operation) in power systems research; for example, see Shahidehpour et al. (2002). Variable costs in actual wholesale power markets primarily reflect fuel and labor costs, and additional study is needed to gauge the extent to which quadratic functions can adequately represent these costs.

and

$$\text{FCost}_i = \text{TC}_i(0) \tag{4}$$

respectively. Finally, the *marginal cost function* for Generator i takes the form

$$\text{MC}_i(p_{Gi}) = a_i + 2 \cdot b_i \cdot p_{Gi} \tag{5}$$

At the beginning of each day D, each Generator i reports a *supply offer* $s_i^R(\text{D})$ to the AMES ISO for use in each hour H of the Day-Ahead Market for day D+1. This supply offer consists of a *reported marginal cost function* (i.e., *supply schedule*)

$$\text{MC}_i^R(p_{Gi}) = a_i^R + 2 \cdot b_i^R \cdot p_{Gi} \tag{6}$$

defined over a *reported feasible production interval*[16]

$$\text{Cap}_i^{RL} \le p_{Gi} \le \text{Cap}_i^{RU} \tag{7}$$

This supply offer can be *strategic* in the sense that the reported cost coefficients a_i^R and b_i^R in Eq. 6 can deviate from Generator i's true cost coefficients a_i and b_i in Eq. 5 and the reported feasible production interval $[\text{Cap}_i^{RL}, \text{Cap}_i^{RU}]$ in Eq. 7 can deviate from Generator i's true feasible production interval $[\text{Cap}_i^{L}, \text{Cap}_i^{U}]$ in Eq. 1.

Suppose Generator i is located at node k, and suppose Generator i in some day D reports a supply offer $s_i^R(\text{D})$ to the AMES ISO for the day D+1 Day-Ahead Market (along with all other Generators). Let LMP_k denote the node-k locational marginal price (LMP) that is then subsequently determined by the AMES ISO in day D for some hour H of day D+1, and let p_{Gi}^* denote the real power that Generator i has been cleared to inject at node k in hour H of day D+1. Then the (possibly negative) profit accruing to Generator i in day D from the day-D settlement of this financially binding contract for hour H of day D+1 is

$$\text{Profit}_i^{\text{new}}(p_{Gi}^*) = \text{LMP}_k \cdot p_{Gi}^* - \text{TC}_i(p_{Gi}^*) \tag{8}$$

[16] As emphasized by Cain and Alvarado (2004), the implications of supply offer formats for the operation of wholesale power markets is an important topic in need of further study. Here we follow the basic form of the generator supply offers required by the MISO (2007) and ISO-NE (2007): namely, non-decreasing supply schedules accompanied by minimum and maximum real power production capacities. However, we assume *linear* supply schedules to ease the specification of the learning problem for the AMES Generators whereas the MISO and ISO-NE require step-function supply schedules. (Interestingly, in the ISO-NE the generators can check a "UseOfferSlope" box permitting the ISO to approximate their step-function supply schedules by smoother curves.) In addition, for initial simplicity, we follow the current practice of the ISO-NE in only permitting the AMES Generators to submit *one* supply offer to be used for *each* hour of the Day-Ahead Market, whereas the MISO permits generators to submit a separate supply offer for each hour of the Day-Ahead Market.

Moreover, as a result of this settlement, the updated cumulated money holdings for Generator i are given by

$$\text{Money}_i^{\text{new}} = \text{Money}_i^{\text{prev}} + \text{Profit}_i^{\text{new}}(p^*_{Gi}) \tag{9}$$

Since Generator i's profits (Eq. 8) can be negative, it is clear from Eq. 9 that Generator i faces a risk of *insolvency*, i.e., a risk that its money holdings will run out. Any Generator that becomes insolvent must immediately exit the market, which results in the loss of its production capacity to the market. Furthermore, no entry of new generation is permitted in the current implementation of the AMES framework.

3.5 Structural Configuration of the ISO

As in actual ISO-managed wholesale power markets operating under the WPMP market design, the AMES ISO during each day D is charged with determining a schedule of optimal power commitments and LMPs for each hour of the Day-Ahead Market in day D+1. This schedule is conditional on LSE-reported demand bids, Generator-reported supply offers, thermal limits on branch flows, and nodal balance constraints ensuring supply equals demand (load) at each transmission grid node.

As usual, "optimal" is interpreted to mean that total net surplus is maximized. The resulting optimization problem is known as a *bid-based AC optimal power flow* (*OPF*) problem. As typically done in actual markets, the AMES ISO approximates this difficult bid-based AC-OPF problem by means of a simpler bid-based DC-OPF problem in which real power constraints are linearized and reactive power constraints are ignored. A brief discussion of this bid-based DC-OPF problem will now be given.[17]

Recall from Sect. 3.3 that the AMES LSEs are currently modeled as non-strategic entities servicing price-insensitive loads whose reported demand bids in each day D take the form of their true daily load profiles. In this case the maximization of total net surplus reduces to the minimization of Generator-reported total variable cost. Using the variable definitions in Tables 1 and 2, the bid-based DC-OPF problem solved by the AMES ISO in day D for each hour of the Day-Ahead Market in day D+1 is then as follows:

Minimize Generator-reported total variable cost

$$\sum_{i=1}^{I} \left[a_i^R p_{Gi} + b_i^R p_{Gi}^2 \right] \tag{10}$$

with respect to real-power production levels and voltage angles

$$p_{Gi}, i = 1, \ldots, I; \quad \delta_k, k = 1, \ldots, K$$

[17] Sun and Tesfatsion (2006) motivate in detail the form of the objective function and constraints for this bid-based DC-OPF problem as well as explaining carefully how it is derived from an AC-OPF problem given certain standard simplifying assumptions.

subject to:

Real power balance constraint for each node $k = 1, \ldots, K$:

$$0 = \text{PLoad}_k - \text{PGen}_k + \text{PNetInject}_k \tag{11}$$

where

$$\text{PLoad}_k = \sum_{j \in J_k} p_{Lj} \tag{12}$$

$$\text{PGen}_k = \sum_{i \in I_k} p_{Gi} \tag{13}$$

$$\text{PNetInject}_k = \sum_{km \text{ or } mk \in BR} P_{km} \tag{14}$$

$$P_{km} = B_{km}[V_o]^2[\delta_k - \delta_m] \tag{15}$$

Real power thermal constraints for each branch $km \in BR$:

$$|P_{km}| \leq P^U_{km} \tag{16}$$

Reported real-power production constraints for each Generator $i = 1, \ldots, I$:

$$\text{Cap}_i^{RL} \leq p_{Gi} \leq \text{Cap}_i^{RU} \tag{17}$$

Voltage angle setting at reference node 1:

$$\delta_1 = 0 \tag{18}$$

As shown in Sun and Tesfatsion (2006), this DC-OPF problem can equivalently be represented in the numerically desirable form of a strictly convex quadratic programming (SCQP) problem if the balance constraints Eq. 11 are used to eliminate the voltage angles δ_k by substitution. However, this elimination prevents direct generation of solution values for LMPs since, by definition, the LMP for node k is the solution value for the multiplier (shadow price) for the kth nodal balance constraint.

For this reason, we replace the standard DC-OPF objective function (Eq. 10) with the following augmented form:

$$\sum_{i=1}^{I} \left[a_i^R p_{Gi} + b_i^R p_{Gi}^2\right] + \pi \left[\sum_{km \in BR} [\delta_k - \delta_m]^2\right], \tag{19}$$

where π is a positive soft penalty weight on the sum of squared voltage angle differences. As carefully demonstrated in Sun and Tesfatsion (2006), the augmented DC-OPF objective function (Eq. 19) provides a number of benefits based on both physical and mathematical considerations.

First, the resulting augmented DC-OPF problem now has a numerically desirable SCQP form permitting the direct generation of solution values for LMPs as well as for real power production levels, branch flows, and voltage angles. Second, the validity of the DC-OPF problem as an approximation for the underlying AC-OPF problem relies on an assumption of small voltage angle differences, and the augmented DC-OPF problem permits this assumption to be subjected to systematic sensitivity tests through variations in the penalty weight π. Third, solution differences between the non-augmented and augmented forms of the DC-OPF problem can be reduced to arbitrarily small levels by selecting an appropriately small value for π.

To solve this augmented DC-OPF problem, the AMES ISO invokes the SCQP solver QuadProgJ through an outer shell DCOPFJ. More precisely, as illustrated below in Sect. 4, the AMES ISO passes to DCOPFJ current DC-OPF input data in standard (SI) units together with base apparent power and voltage values S_o and V_o. DCOPFJ converts this SI input data into per unit (pu) form and performs all needed matrix and vector representations. DCOPFJ then invokes QuadProgJ to solve for LMPs, voltage angles, real power production levels, real power branch flows, and various other useful quantities with all internal calculation carried out in pu terms. QuadProgJ then passes these pu solution values back to DCOPFJ, which outputs them in SI units.

In future studies, the AMES ISO will also have to solve DC-OPF problems for the Real-Time Market to settle any differences that arise between day-ahead commitments and real-time conditions due to system disturbances (e.g., sudden line outages or changes in demand). However, in our initial experiments with the AMES framework we are not considering system disturbances that would cause such differences to arise. Consequently, all load obligations are fully met through Day-Ahead Market transactions and the Real-Time Market is inactive.

3.6 Learning Configuration for the AMES Generators

In general, multiple Generators at multiple nodes could be under the control of a single generation company ("GenCo"). This control aspect is critically important to recognize for the study of real-world strategic trading. This situation can be handled in the AMES framework by permitting coordinated learning across Generators controlled by a single GenCo.

For initial simplicity, however, the AMES Generators are currently modeled as autonomous energy traders with strategic learning capabilities; see Fig. 8. Each AMES Generator adaptively selects its supply offers on the basis of its own past profit outcomes using a version of a stochastic reinforcement learning algorithm developed by Roth–Erev (1995) based on human-subject experiments, hereafter referred to as the *VRE learning algorithm*. This section briefly outlines the implementation of the VRE learning algorithm for an arbitrary Generator i.

```
Public Access:
// Public Methods
    getMarketProtocols(posting, trade, settlement);
    getMarketProtocols(ISO market power mitigation);
    Methods for receiving data;
    Methods for retrieving stored Generator data.

Private Access:
// Private Methods
    Method for calculating my expected profits;
    Method for calculating my actual profit outcomes;
    Method for updating my supply offers (LEARNING).
// Private Data
    My capacity, grid location, cost fct., current wealth... ;
    Data recorded about external world (dispatch schedule...);
    Address book (communication links).
```

Fig. 8 A computational generator (Seller)

Suppose it is the beginning of the initial day D = 1, and Generator i must choose a supply offer from its *action domain* AD_i to report to the AMES ISO for the Day-Ahead Market in day D + 1. As will be seen below in Sect. 3.6, for learning purposes the only relevant attribute of AD_i is that it has finite cardinality $M_i \geq 1$.[18]

The *initial propensity* of Generator i to choose supply offer $m \in AD_i$ is given by $q_{im}(0)$. In general, these initial propensities can be any real numbers as specified by the AMES user. However, the default setting used in this study is that these initial propensities are equal. That is, we specify a fixed value $q_i(0)$ such that

$$q_{im}(0) = q_i(0) \quad \text{for all supply offers } m \in AD_i \tag{20}$$

Now consider the beginning of any day $\mathrm{D} \geq 1$, and suppose the current propensity of Generator i to choose supply offer $m \in AD_i$ is given by $q_{im}(\mathrm{D})$. The *choice probabilities* that Generator i uses to select a supply offer for day D are constructed from these propensities as follows:[19]

$$p_{im}(\mathrm{D}) = \frac{\exp(q_{im}(\mathrm{D})/C_i)}{\sum_{j=1}^{M_i} \exp(q_{ij}(\mathrm{D})/C_i)}, \quad m \in AD_i \tag{21}$$

In Eq. 21, C_i is a *cooling parameter* that affects the degree to which Generator i makes use of propensity values in determining its choice probabilities. As $C_i \to \infty$,

[18] The construction of each Generator i's action domain AD_i is outlined in Sect. 3.7, below, and carefully explained in Sun and Tesfatsion (2007a, Appendix). The key issue is how to construct the sets AD_i to give each Generator an economically meaningful and realistically flexible selection of supply offers without introducing hidden structural biases favoring some Generators over others.

[19] In the original algorithm developed by Erev and Roth (1998) and Roth and Erev (1995), the choice probabilities are defined in terms of relative propensity levels. Here, instead, use is made of a "simulated annealing" formulation in terms of exponentials. As will be seen below in Eq. 22, in the current context the propensity values can take on negative values if sufficiently large negative profit outcomes are experienced, and the use of exponentials ensures that the choice probabilities remain well defined even in this event.

then $p_{im}(\mathrm{D}) \to 1/M_i$, so that in the limit Generator i pays no attention to propensity values in forming its choice probabilities. On the other hand, as $C_i \to 0$, the choice probabilities (Eq. 21) become increasingly peaked over the particular supply offers m having the highest propensity values $q_{im}(\mathrm{D})$, thereby increasing the probability that these supply offers will be chosen.

At the end of day D, the current propensity $q_{im}(\mathrm{D})$ that Generator i associates with each supply offer $m \in AD_i$ is updated in accordance with the following rule. Let m' denote the supply offer that was *actually* selected and reported into the Day-Ahead Market by Generator i in day D, and let $\mathrm{Profit}_{im'}(\mathrm{D})$ denote the profits (positive or negative) attained by Generator i in the settlement of the Day-Ahead Market at the end of day D in response to its choice of supply offer m'. Then, for each supply offer $m \in AD_i$,[20]

$$q_{im}(\mathrm{D}+1) = [1 - r_i]q_{im}(\mathrm{D}) + \mathrm{Response}_{im}(\mathrm{D}), \tag{22}$$

where

$$\mathrm{Response}_{im}(\mathrm{D}) = \begin{cases} [1 - e_i] \cdot \mathrm{Profit}_{im'}(\mathrm{D}) & \text{if } m = m' \\ \\ e_i \cdot q_{im}(\mathrm{D})/[M_i - 1] & \text{if } m \neq m', \end{cases} \tag{23}$$

where $m \neq m'$ implies $M_i \geq 2$. The introduction of the *recency parameter* r_i in Eq. 22 acts as a damper on the growth of the propensities over time. The *experimentation parameter* e_i in Eq. 23 permits reinforcement to spill over to some extent from a chosen supply offer to other supply offers to encourage continued experimentation with various supply offers in the early stages of the learning process.

Generator i faces a trade-off in each day D between information exploitation and information exploration. The VRE learning algorithm outlined above resolves this trade-off by ensuring continual exploration but at a typically declining rate. More precisely, under the VRE learning algorithm, note that Generator i in day D does *not* necessarily choose a supply offer with the highest accumulated profits to date. Given a suitably small value for e_i, selected supply offers generating the highest accumulated profits tend to have a relatively higher *probability* of being chosen, but there is always a chance that other supply offers will be chosen instead. This ensures that Generator i continues to experiment with new supply offers to some degree, even if its choice probability distribution becomes peaked at a particular selected supply offer because of relatively good profit outcomes. This helps to reduce the risk of premature fixation

[20] The response function appearing in Eq. 22 modifies the response function appearing in the original algorithm developed by Erev and Roth (1998) and Roth and Erev (1995). The modification is introduced to ensure that learning (updating of choice probabilities) occurs even in response to zero-profit outcomes, which are particularly likely to arise in initial periods when Generator i is just beginning to experiment with different supply offers and the risk of overbidding to the point of non-dispatch is relatively high. See Koesrindartoto (2002) for a detailed discussion and experimental exploration of this zero-profit updating problem with the original Roth–Erev learning algorithm. See Nicolaisen et al. (2001) for a detailed motivation, presentation, and experimental application of the modified response function.

on suboptimal supply offers in the early stages of the decision process when relatively few supply offers have been tried.

In summary, the complete VRE learning algorithm applied to Generator i is fully characterized once user-specified values are provided for the number M_i of feasible supply offer selections in AD_i, the initial propensity value $q_i(0)$ in Eq. 20, the cooling parameter C_i in Eq. 21, the recency parameter r_i in Eq. 22, and the experimentation parameter e_i in Eq. 23. It is interesting to note, in particular, that the VRE learning algorithm is well-defined for any action domain AD consisting of finitely many elements M_i, regardless of the precise nature of these elements.

3.7 Construction of Generator Action Domains

The construction of action domains (supply offer choice sets) for the AMES Generators is a critical modeling issue. Empirical sensibility suggests these action domains should permit flexible choice from among a wide range of possible supply offers, and that the degree of flexibility should be roughly similar across the Generators. On the other hand, computational practicality suggests the number of supply offers included in each action domain should not be unduly large.

This subsection briefly summarizes how action domains have been constructed for the AMES Generators in accordance with these objectives. A detailed rigorous discussion can be found in Sun and Tesfatsion (2007a, Appendix).

As explained in Sect. 3.6, at the beginning of each day D each AMES Generator i uses VRE reinforcement learning to choose a supply offer s_i^R to report to the AMES ISO for each hour H of the day D+1 Day-Ahead Market. Each supply offer s_i^R takes the form of a reported marginal cost function

$$\mathrm{MC}_i^R(p) = a_i^R + 2b_i^R p \tag{24}$$

defined over a reported feasible production interval

$$\mathrm{Cap}_i^{RL} \leq p \leq \mathrm{Cap}_i^{RU} \tag{25}$$

Here a_i^R and b_i^R are Generator i's reported cost coefficients, p denotes Generator i's hourly real-power production level, and Cap_i^{RL} and Cap_i^{RU} are Generator i's reported lower and upper real-power production limits.

Each AMES Generator i chooses its supply offers s_i^R from an action domain AD_i with finite positive cardinality M_i. In keeping with the modeling goals of empirical sensibility and computational practicality, the action domain AD_i for each AMES Generator i is constructed under five simplifying assumptions. First, we assume Generator i only reports upward-sloping marginal cost functions (Eq. 24), i.e., $b_i^R > 0$. Second, we assume Generator i only reports non-trivial feasible production intervals (Eq. 25), i.e., $\mathrm{Cap}_i^{RL} < \mathrm{Cap}_i^{RU}$. Third, we assume Generator i only reports marginal cost curves that lie on or above its true marginal cost curve over the range of its accompanying reported production intervals. Fourth, we assume Generator i always reports its true lower production limit.[21] Fifth, we assume Generator i always reports an upper production limit that is less than or equal to its true upper production limit.

Let a supply offer s_i^R for Generator i be called *admissible* if the corresponding reported marginal cost function $\mathrm{MC}_i^R(p)$ and reported lower and upper production limits Cap_i^{RL} and Cap_i^{RU} are in compliance with the five simplifying assumptions. As shown in Sun and Tesfatsion (2007a, Appendix), given any positive value for a *slope-start* parameter SS_i for Generator i, any 4-dimensional vector s_i^A consisting of four components in percentage form can be uniquely mapped into an admissible supply offer s_i^R for Generator i.

Referring to Table 1 for precise variable definitions, one can then construct a matrix AD_i for Generator i characterized by three integer-valued density-control parameters $M1_i$, $M2_i$, and $M3_i$ (with $M1_i \times M2_i \times M3_i = M_i$) and three range-index parameters RIMax_i^L, RIMax_i^U, and RIMin_i^C in percentage form that has the following property: For any given $SS_i > 0$, the M_i rows of this matrix constitute M_i distinct vectors s_i^A in percentage form that can be transformed uniquely into M_i distinct admissible supply offers s_i^R for Generator i. Consequently, the matrix AD_i effectively constitutes an action domain for Generator i consisting of M_i admissible supply offers s_i^R.

Moreover, if the parameters $(M1_i, M2_i, M3_i, \mathrm{RIMax}_i^L, \mathrm{RIMax}_i^U, \mathrm{RIMin}_i^C, SS_i)$ are set identically across the Generators and the above construction is applied for each Generator $i = 1, \ldots, I$, the result is a collection $\{AD_i : i = 1, \ldots, I\}$ of Generator-specific action domains that have equal cardinalities and whose supply-offer elements s^R provide similar densities of coverage of the regions lying above the Generators' true marginal cost curves.

4 Dynamic Five-Node Test Case

4.1 Overview

Consider a situation in which five Generators and three LSEs are distributed across a 5-node transmission grid as depicted in Fig. 2. An interesting aspect of this transmission grid is that not all nodes are directly connected; for example, node 5 is not directly connected to either node 2 or node 3.

Originally due to John Lally (2002), this five-node transmission grid configuration is now used extensively in ISO-NE/PJM training manuals to solve for DC-OPF solutions at a given point in time conditional on variously specified marginal costs and production limits for the generators and variously specified price-insensitive loads for the LSEs. The implicit assumption in all of these static training exercises is that the true cost and

[21] As explained in footnote 14, the Generators' reported lower production limits are treated as firm by the AMES ISO. Since the current version of AMES lacks market power mitigation rules, the AMES Generators could ensure themselves arbitrarily high profits if they were permitted to *report* arbitrarily high lower production limits into the Day-Ahead Market. For this reason, it is assumed in the current study that the AMES Generators are closely monitored by the AMES ISO with regard to these lower production limits, ensuring that they always report their true lower limits. In the actual MISO and ISO-NE energy markets, generators are requested to report their true lower and upper production limits, but it is not clear from the MISO and ISO-NE business practices manuals just how closely generators are actually monitored to ensure compliance.

Table 3 Dynamic 5-node test case—DC-OPF structural input data (SI)

Base values									
S_o	V_o								
100.0	10.0								
K [a]	π [b]								
5	0.05								
Branch									
From	To	lineCap[c]	X [d]						
1	2	250.0	0.0281						
1	4	150.0	0.0304						
1	5	400.0	0.0064						
2	3	350.0	0.0108						
3	4	240.0	0.0297						
4	5	240.0	0.0297						
Gen ID	atNode	FCost	a	b	Cap^L	Cap^U	Init$		
1	1	1600.0	14.0	0.005	0.0	110.0	$1.0*M*		
2	1	1200.0	15.0	0.006	0.0	100.0	$1.0*M*		
3	3	8500.0	25.0	0.010	0.0	520.0	$1.0*M*		
4	4	1000.0	30.0	0.012	0.0	200.0	$1.0*M*		
5	5	5400.0	10.0	0.007	0.0	600.0	$1.0*M*		
LSE									
ID	atNode	L-00[e]	L-01	L-02	L-03	L-04	L-05	L-06	L-07
1	2	350.00	322.93	305.04	296.02	287.16	291.59	296.02	314.07
2	3	300.00	276.80	261.47	253.73	246.13	249.93	253.73	269.20
3	4	250.00	230.66	217.89	211.44	205.11	208.28	211.44	224.33
ID	atNode	L-08	L-09	L-10	L-11	L-12	L-13	L-14	L-15
1	2	358.86	394.80	403.82	408.25	403.82	394.80	390.37	390.37
2	3	307.60	338.40	346.13	349.93	346.13	338.40	334.60	334.60
3	4	256.33	282.00	288.44	291.61	288.44	282.00	278.83	278.83
ID	atNode	L-16	L-17	L-18	L-19	L-20	L-21	L-22	L-23
1	2	408.25	448.62	430.73	426.14	421.71	412.69	390.37	363.46
2	3	349.93	384.53	369.20	365.26	361.47	353.73	334.60	311.53
3	4	291.61	320.44	307.67	304.39	301.22	294.78	278.83	259.61

[a] Total number of nodes
[b] Soft penalty weight π for voltage angle differences
[c] Upper limit P^U_{km} (in MWs) on the magnitude of real power flow in branch km
[d] Reactance X_{km} (in ohms) for branch km
[e] L-H: Load L (in MWs) for hour H, where H = 00, 01, ..., 23

true production limits of the generators are known. Nowhere is any mention made of the possibility that generators in real-world ISO-managed wholesale power markets might learn to exercise market power over time through strategic reporting of their cost and production attributes.

In this section we illustrate how the AMES wholesale power market framework can be used to transform these static training exercises into a more realistic dynamic form with strategic learning. Detailed grid, production, and load input data for a specific dynamic five-node test case are provided in Table 3.[22] As seen in this table, and

[22] The transmission grid configuration, reactances, locations of the Generators and LSEs, and initial hour-0 load levels in Table 3 are taken from Lally (2002). The general shape of the LSE load profiles is adopted from a 3-node example presented in Shahidehpour et al. (2002, pp. 296–297).

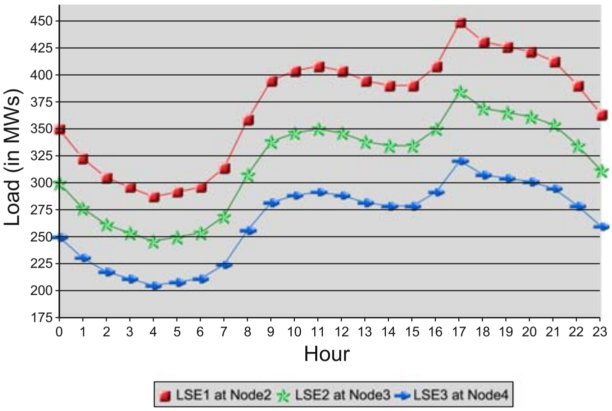

Fig. 9 24 Hour load distribution for the dynamic 5-node test case

depicted graphically in Fig. 9, the daily load profile for each LSE is price insensitive and peaks at hour 17. Note, also, that Generator 4 is a "peaker" unit with relatively high hourly marginal costs $\mathrm{MC(p)} = 30 + 0.024p$ for each p, where p denotes hourly real-power production in megawatts (MWs). Also, each Generator has a finite upper limit Cap^U on its hourly real power production.

We report below our findings for two experimental treatments. In the first benchmark "no learning" treatment, the Generators are assumed to report to the ISO their true marginal cost functions and true production limits. In the second "learning" treatment, the Generators can report strategic supply offers to the ISO. More precisely, the Generators still must report their true production limits to the ISO, but they can now learn over time what marginal cost attributes to report to the ISO in an attempt to increase their profit earnings.[23] All runs for both treatments were carried out on a laptop PC: namely, a Compaq Presario 2100 running under Windows XP SP2 (mobile AMD Athlon XP 2800+2.12 GHz, 496 MB of RAM). For the no-learning treatment (one run), the run time was approximately 4.3 s. For the learning treatment (20 runs), the average run time was 4.5 min.

Our findings for the no-learning treatment are detailed in Sect. 4.3. These findings reveal the complicated effects of daily load profiles, transmission congestion, and production limits on LMP determination over time, even in the absence of strategic supply-offer reporting by Generators.

Our findings for the learning treatment are detailed in Sect. 4.3. The existence of price-insensitive loads provides a potentially golden opportunity for the two largest Generators 3 and 5 to exercise market power. Note from Table 3 that the peak load in hour 17 is 1153.59, and that the combined capacity of the smallest three Generators 1, 2, and 4 is only 410 MWs. It follows that this peak load cannot be met unless Generator 3 (520 MWs) and Generator 5 (600 MWs) are both dispatched to some

[23] The Generators thus behave as if they were in a leader-follower game with the ISO. Since the Generators as currently implemented do not explicitly recognize the presence of rival Generators in their choice environments, there is no strategic interaction among the Generators per se.

Table 4 No-learning dynamic 5-node test case—solution value (SI) for real power branch flow P_{km}, with associated thermal limit P^U_{km}, for each distinct branch km

Hour	P^{a}_{12}	P_{14}	P_{15}	P_{23}	P_{34}	P_{45}
00	250.00	129.65	−255.77	−100.00	−67.47	−187.82
01	250.00	126.71	−253.27	−72.93	−80.32	−184.27
02	250.00	124.77	−251.61	−55.04	−88.81	−181.93
03	250.00	123.79	−250.77	−46.02	−93.09	−180.74
04	250.00	122.83	−249.95	−37.16	−97.30	−179.58
05	250.00	123.31	−250.36	−41.59	−95.19	−180.16
06	250.00	123.79	−250.77	−46.02	−93.09	−180.74
07	250.00	125.75	−252.45	−64.07	−84.52	−183.11
08	250.00	130.61	−256.60	−108.86	−63.26	−188.98
09	250.00	134.51	−259.92	−144.80	−46.20	−193.69
10	250.00	135.49	−260.76	−153.82	−41.92	−194.87
11	250.00	135.97	−261.17	−158.25	−39.81	−195.45
12	250.00	135.49	−260.76	−153.82	−41.92	−194.87
13	250.00	134.51	−259.92	−144.80	−46.20	−193.69
14	250.00	134.03	−259.51	−140.37	−48.30	−193.11
15	250.00	134.03	−259.51	−140.37	−48.30	−193.11
16	250.00	135.97	−261.17	−158.25	−39.81	−195.45
17	250.00	98.83	−346.76	−198.62	−63.15	−175.88
18	250.00	137.64	−274.17	−180.73	−29.93	−199.96
19	250.00	137.91	−262.83	−176.14	−31.32	−197.80
20	250.00	137.43	−262.42	−171.71	−33.42	−197.22
21	250.00	136.45	−261.58	−162.69	−37.71	−196.03
22	250.00	134.03	−259.51	−140.37	−48.30	−193.11
23	250.00	131.11	−257.02	−113.46	−61.08	−189.58
	P^U_{12}	P^U_{14}	P^U_{15}	P^U_{23}	P^U_{34}	P^U_{45}
	250.00	150.00	400.00	350.00	240.00	240.00

[a] In accordance with the usual convention, the real power P_{km} flowing along a branch km is positively valued if and only if real power is flowing from node k to node m

extent. Consequently, if these profit-seeking Generators had full structural information, their reported marginal costs should be as high as permitted by their action domains. The question is whether the simple VRE reinforcement learning algorithm permits these Generators to learn to exercise this potential market power.

As detailed below in Sect. 4.3, the answer is a resounding "yes." All five Generators learn to implicitly collude on higher-than-true reported marginal costs. Moreover, the marginal costs reported by Generators 3 and 5 typically are near or at the highest possible levels permitted by their action domains. Production and LMP solutions differ dramatically from the production and LMP solutions obtained for the no-learning treatment reported in Sect. 4.2. The result is a substantial increase in the total variable cost of operation at the ISO-determined "optimal" Day-Ahead Market DC-OPF solution for each hour of each day.

Table 5 No-learning dynamic 5-node test case—solution values (SI) for real power production levels and associated upper production limits, together with LMPs (nodal balance constraint multipliers) and minimum total variable cost

Hour	p_{G1}^*	p_{G2}^*	p_{G3}^*	p_{G4}^*	p_{G5}^*	LMP_1	LMP_2	LMP_3	LMP_4	LMP_5	minTVC
0	110.0	13.9	332.5	0.0	443.6	15.17	35.50	31.65	21.05	16.21	19587.1
1	110.0	13.4	269.4	0.0	437.5	15.16	33.95	30.39	20.60	16.13	17107.3
2	110.0	13.2	227.7	0.0	433.5	15.16	32.92	29.55	20.30	16.07	15556.8
3	110.0	13.0	206.7	0.0	431.5	15.16	32.40	29.13	20.15	16.04	14800.9
4	110.0	12.9	186.0	0.0	429.5	15.15	31.89	28.72	20.00	16.01	14076.1
5	110.0	13.0	196.3	0.0	430.5	15.16	32.15	28.93	20.07	16.03	14436.5
6	110.0	13.0	206.7	0.0	431.5	15.16	32.40	29.13	20.15	16.04	14800.9
7	110.0	13.3	248.8	0.0	435.6	15.16	33.44	29.97	20.45	16.10	16330.2
8	110.0	14.0	353.2	0.0	445.6	15.17	36.01	32.06	21.20	16.24	20433.9
9	110.0	14.6	437.0	0.0	453.6	15.18	38.08	33.74	21.81	16.35	24043.6
10	110.0	14.7	458.0	0.0	455.6	15.18	38.60	34.16	21.96	16.38	24993.9
11	110.0	14.8	468.4	0.0	456.6	15.18	38.85	34.37	22.03	16.39	25467.5
12	110.0	14.7	458.0	0.0	455.6	15.18	38.60	34.16	21.96	16.38	24993.9
13	110.0	14.6	437.0	0.0	453.6	15.18	38.08	33.74	21.81	16.35	24043.6
14	110.0	14.5	426.7	0.0	452.6	15.17	37.82	33.53	21.73	16.34	23583.1
15	110.0	14.5	426.7	0.0	452.6	15.17	37.82	33.53	21.73	16.34	23583.1
16	110.0	14.8	468.4	0.0	456.6	15.18	38.85	34.37	22.03	16.39	25467.5
17	2.1	0.0	520.0	108.9	522.6	14.02	78.24	66.07	32.61	17.32	31038.5
18	107.4	6.1	520.0	0.0	474.1	15.07	45.55	39.78	23.90	16.64	28006.9
19	110.0	15.1	510.1	0.0	460.6	15.18	39.88	35.20	22.33	16.45	27422.4
20	110.0	15.0	499.8	0.0	459.6	15.18	39.63	35.00	22.26	16.43	26931.9
21	110.0	14.9	478.7	0.0	457.6	15.18	39.11	34.57	22.11	16.41	25945.9
22	110.0	14.5	426.7	0.0	452.6	15.17	37.82	33.53	21.73	16.34	23583.1
23	110.0	14.1	363.9	0.0	446.6	15.17	36.28	32.28	21.28	16.25	20879.5
	Cap_1^U	Cap_2^U	Cap_3^U	Cap_4^U	Cap_5^U						
	110.0	100.0	520.0	200.0	600.0						

4.2 Treatment 1: Generators Report True Supply Data

Suppose each Generator submits its true marginal cost function and true production limits into the Day-Ahead Market. That is, suppose Generators do not report strategic supply offers. In this case, the augmented DC-OPF problem solved by the ISO for each hour H involves the minimization of true Generator total variable cost (subject to a small voltage angle difference penalty) conditional on LSE loads, nodal balance constraints, true Generator upper and lower production limits, and upper and lower thermal limits on each branch of the transmission grid; compare Sect. 3.5.

Tables 4 and 5 report outcomes in standard (SI) units obtained for this dynamic 5-node test case by means of QuadProgJ invoked through DCOPFJ. These outcomes include optimized solution values for real power branch flows, production levels, LMPs (nodal balance constraint multipliers), and minimum total variable cost for 24 successive hours in the Day-Ahead Market.

These outcomes reveal that branch congestion occurs between node 1 and node 2 (and only these nodes) in each of the 24 h. This can be verified by examining column P_{12} in Table 4, which shows that the real power flow P_{12} on branch $km = 12$ is at its upper thermal limit (250 MWs) for each hour. The direct consequence of this

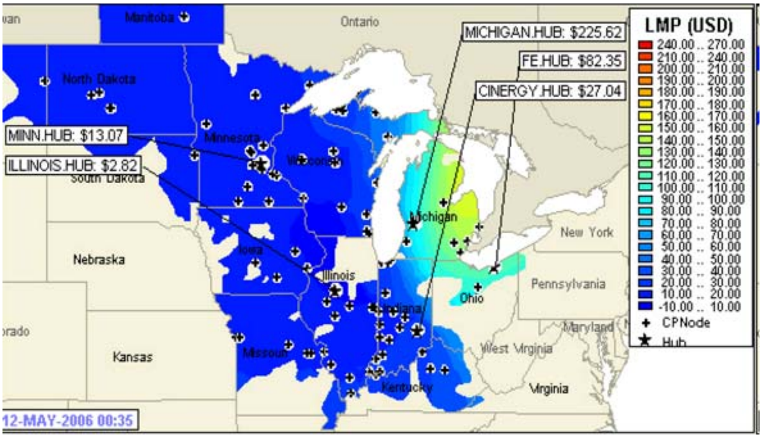

Fig. 10 LMP separation and spiking in the MISO real-time market (*Source:* Midwest ISO, http://www.midwestmarket.org/page/LMP%20Contour%20Map%20&%20Data)

branch congestion is the occurrence of widespread LMP separation, i.e., the LMP values differ across all nodes for each hour. This can be verified by examining output columns LMP_1–LMP_5 in Table 5.

Examining this LMP data more closely, it is seen that LMP_2 and LMP_3 (the LMPs for nodes 2 and 3) exhibit a sharp change in hour 17, increasing between hour 16 and hour 17 by about 100% and then dropping back to more normal levels in hour 18 and beyond. Interestingly, this type of sudden spiking in LMP values is also observed empirically in MISO's Dynamic LMP Contour Map for real-time market prices, which is updated every five minutes; see, for example, Fig. 10.

The rather dramatic LMP spiking in hour 17 can be traced to several factors. First, as seen in Fig. 9, the load profile for each LSE peaks at hour 17. Second, when solving the DC-OPF problem to meet the high load in hour 17, the ISO has to take into consideration the thermal limit constraining the flow of power on branch $km = 12$ as well as the upper limit Cap^U constraining the production of Generator 3. Both of these constraints turn out to be binding in hour 17. As seen in Table 4, the real power flow in branch $km = 12$ is at its upper limit (250 MWs) for all 24 h. As seen in Table 5, Generator 3 is dispatched in hour 17 at its upper production limit (520 MWs).

Given the configuration of the transmission grid, to meet the hour 17 peak load the ISO is forced to back down (relative to hour 16) the less expensive production of Generators 1 and 2 and to use instead the more expensive production of the "peaker" Generator 4. After the peak hour 17, the load returns to lower levels. The ISO is then able to schedule Generator 1 and Generator 2 at their more normal levels, with Generator 1 at its upper production limit, and to avoid scheduling any production from Generation 4; note from Table 3 that Generator 4's minimum production level (Cap^L) is 0. Furthermore, the LMPs drop back to their more normal levels after hour 17.

These illustrative 5-node test case outcomes for 24 successive hours in the Day-Ahead Market raise intriguing economic issues concerning the operation of ISO-managed wholesale power markets in the presence of inequality constraints on branch flows and production levels. The strong sensitivity of the optimized LMP and real

Table 6 Dynamic 5-node test case – action domain and learning input data

Action domain parameters

Gen ID	$M1$	$M2$	$M3$	RIMax^L	RIMax^U	RIMin^C	SS
1	10	10	1	0.75	0.75	1.00	0.001
2	10	10	1	0.75	0.75	1.00	0.001
3	10	10	1	0.75	0.75	1.00	0.001
4	10	10	1	0.75	0.75	1.00	0.001
5	10	10	1	0.75	0.75	1.00	0.001

Learning parameters

Gen ID	$q(0)$	C	r	e
1	6000.0	1000.0	0.04	0.97
2	6000.0	1000.0	0.04	0.97
3	6000.0	1000.0	0.04	0.97
4	6000.0	1000.0	0.04	0.97
5	6000.0	1000.0	0.04	0.97

Initial seed values for all 20 runs

RunID	InitialSeed	RunID	InitialSeed
01	695672061	11	−597305450
02	857398845	12	−494232424
03	507304343	13	−158932839
04	748974391	14	−934341230
05	494375928	15	−734837588
06	289658396	16	−219860821
07	158324732	17	−845925752
08	324702357	18	−367413463
09	903534301	19	−629523701
10	205753353	20	−257802760

power production values to changes in the set of binding (active) constraints is of particular interest.

Equally intriguing, however, is whether the Generators might learn to make use of the outcomes for any particular operating day D to change their reported supply offers for day D+1 and beyond. The next section considers this issue.

4.3 Treatment 2: Generators Report Strategic Supply Offers

Now suppose, in contrast to Treatment 1, that the Generators do not necessarily report their true marginal costs to the ISO for the Day-Ahead Market. Rather, using the VRE stochastic reinforcement learning algorithm detailed in Sect. 3.6, with parameter values as specified in Table 6, each profit-seeking Generator learns over time which marginal cost function to report to the ISO based on the profits it has earned from previously reported functions.

To control for random effects, outcomes for the learning treatment are reported below in the form of mean and standard deviation values obtained for 20 runs using the 20 different seed values reported in Table 6.[24] Across all 20 runs, 422 simulated

[24] Each Generator implements VRE learning by means of its own JReLM learning module, which must be initialized with a seed value for its pseudo-random number generator. Each initial seed value reported

Table 7 Learning dynamic 5-node test case—mean and standard deviation for solution value (SI) on day 422 for real power branch flow P_{km}, with associated thermal limit P^U_{km}, for each distinct branch km

Hour	$\overline{P_{12}}$	P^{SD}_{12}	$\overline{P_{14}}$	P^{SD}_{14}	$\overline{P_{15}}$	P^{SD}_{15}	$\overline{P_{23}}$	P^{SD}_{23}	$\overline{P_{34}}$	P^{SD}_{34}	$\overline{P_{45}}$	P^{SD}_{45}
00	249.3	3.3	77.0	9.2	−116.5	10.4	−100.7	3.3	−120.3	9.6	−103.9	11.6
01	248.0	6.4	74.8	11.7	−113.2	13.9	−74.9	6.4	−130.8	13.1	−100.9	14.7
02	247.1	9.1	73.7	13.3	−111.3	16.7	−58.0	9.1	−137.3	16.2	−99.4	16.9
03	246.4	10.5	73.2	14.2	−110.3	18.3	−49.6	10.5	−140.1	17.8	−98.7	18.1
04	245.5	12.0	72.6	15.2	−108.9	20.1	−41.6	12.0	−142.8	19.5	−97.8	19.4
05	246.0	11.2	72.9	14.7	−109.6	19.2	−45.6	11.2	−141.5	18.6	−98.2	18.7
06	246.4	10.5	73.2	14.2	−110.3	18.3	−49.6	10.5	−140.1	17.8	−98.7	18.1
07	247.5	7.8	74.1	12.5	−112.1	15.3	−66.5	7.8	−134.1	14.7	−100.0	15.8
08	249.4	2.9	77.8	8.5	−117.3	9.5	−109.5	2.9	−116.5	8.7	−104.8	10.6
09	249.8	1.1	80.7	5.6	−120.6	5.9	−145.0	1.1	−101.0	5.7	−108.6	7.0
10	249.9	0.6	81.4	5.1	−121.4	5.2	−154.0	0.6	−97.1	5.1	−109.5	6.3
11	249.9	0.4	81.8	4.8	−121.9	4.9	−158.3	0.4	−95.1	4.9	−110.0	6.0
12	249.9	0.6	81.4	5.1	−121.4	5.2	−154.0	0.6	−97.1	5.1	−109.5	6.3
13	249.8	1.1	80.7	5.6	−120.6	5.9	−145.0	1.1	−101.0	5.7	−108.6	7.0
14	249.7	1.3	80.3	5.9	−120.2	6.3	−140.7	1.3	−102.9	6.1	−108.1	7.4
15	249.7	1.3	80.3	5.9	−120.2	6.3	−140.7	1.3	−102.9	6.1	−108.1	7.4
16	249.9	0.4	81.8	4.8	−121.9	4.9	−158.3	0.4	−95.1	4.9	−110.0	6.0
17	250.0	0.0	85.3	3.2	−125.5	3.2	−198.6	0.0	−77.0	3.3	−114.4	3.9
18	250.0	0.0	83.6	4.1	−123.8	4.1	−180.7	0.0	−85.2	4.2	−112.3	5.1
19	250.0	0.0	83.2	4.2	−123.4	4.2	−176.1	0.0	−87.3	4.3	−111.7	5.1
20	250.0	0.0	82.9	4.3	−123.0	4.3	−171.7	0.0	−89.3	4.4	−111.3	5.3
21	250.0	0.2	82.1	4.6	−122.3	4.6	−162.7	0.2	−93.2	4.7	−110.4	5.7
22	249.7	1.3	80.3	5.9	−120.2	6.3	−140.7	1.3	−102.9	6.1	−108.1	7.4
23	249.4	2.7	78.1	8.1	−117.7	9.0	−114.0	2.7	−114.5	8.3	−105.3	10.1
	P^U_{12}		P^U_{14}		P^U_{15}		P^U_{23}		P^U_{34}		P^U_{45}	
	250.0		150.0		400.0		350.0		240.0		240.0	

trading days was the maximum time it took for all five Generators to "converge" to a sharply peaked choice probability distribution in which a probability of 0.999 was assigned to a single supply offer.[25] Consequently, all learning outcomes reported below are for day 422.

For simplicity, each Generator i selects supply offers from its action domain using VRE reinforcement learning with commonly specified values for the four learning parameters $\{q(0), C, r, e\}$; cf. Sect. 3.6. In addition, to ensure equal cardinalities and similar densities, each Generator i's action domain AD_i is constructed using commonly specified values for the six action-domain parameters $\{M1, M2, M3, \text{RIMax}^L, \text{RIMax}^U, SS\}$; cf. the discussion in Sect. 3.7. These parameter value specifications are listed in Table 6.

Table 7 provides detailed numerical solution values (means and standard deviations) for branch flows on day 422. Recalling that the thermal limit on branch $km = 12$ is

Footnote 24 continued
in Table 6 is used to generate five pseudo-random numbers, one for each Generator. Each of these numbers is then used in turn as the initial seed value for the corresponding Generator's JReLM learning module.

[25] The *mean* convergence time across the 20 runs was actually only 62 simulated trading days with an actual computing time of about 4.5 min.

Table 8 Learning dynamic 5-node test case—means and standard deviations for solution values (SI) on day 422 for real power production levels

Hour	$\overline{p^*_{G1}}$	p^{*SD}_{G1}	$\overline{p^*_{G2}}$	p^{*SD}_{G2}	$\overline{p^*_{G3}}$	p^{*SD}_{G3}	$\overline{p^*_{G4}}$	p^{*SD}_{G4}	$\overline{p^*_{G5}}$	p^{*SD}_{G5}
00	110.00	0.00	99.80	0.88	280.40	10.92	189.37	29.60	220.42	21.84
01	109.92	0.36	99.64	1.59	220.92	17.07	185.74	37.25	214.17	28.21
02	109.85	0.67	99.53	2.10	182.18	22.66	182.11	42.50	210.73	32.93
03	109.81	0.83	99.47	2.35	163.20	25.51	179.72	45.31	208.98	35.57
04	109.78	0.98	99.42	2.60	144.96	28.69	177.50	48.31	206.74	38.51
05	109.80	0.91	99.45	2.48	154.08	27.03	178.61	46.79	207.86	37.00
06	109.81	0.83	99.47	2.35	163.20	25.51	179.72	45.31	208.98	35.57
07	109.88	0.52	99.59	1.84	201.60	19.83	184.36	39.92	212.17	30.52
08	110.00	0.00	99.81	0.86	300.60	9.85	190.23	27.16	222.16	19.91
09	110.00	0.00	99.82	0.80	382.48	5.95	193.70	18.22	229.20	12.83
10	110.00	0.00	99.82	0.79	403.03	5.22	194.57	16.43	230.97	11.41
11	110.00	0.00	99.83	0.78	413.12	4.92	195.00	15.65	231.84	10.81
12	110.00	0.00	99.82	0.79	403.03	5.22	194.57	16.43	230.97	11.41
13	110.00	0.00	99.82	0.80	382.48	5.95	193.70	18.22	229.20	12.83
14	110.00	0.00	99.82	0.81	372.38	6.36	193.27	19.19	228.33	13.60
15	110.00	0.00	99.82	0.81	372.38	6.36	193.27	19.19	228.33	13.60
16	110.00	0.00	99.83	0.78	413.12	4.92	195.00	15.65	231.84	10.81
17	110.00	0.00	99.84	0.71	506.19	3.25	197.68	10.36	239.88	7.11
18	110.00	0.00	99.83	0.74	464.70	4.18	197.02	13.32	236.04	9.13
19	110.00	0.00	99.83	0.75	454.09	4.26	196.73	13.57	235.14	9.30
20	110.00	0.00	99.83	0.76	443.90	4.37	196.30	13.91	234.36	9.53
21	110.00	0.00	99.83	0.77	423.24	4.69	195.43	14.97	232.71	10.29
22	110.00	0.00	99.82	0.81	372.38	6.36	193.27	19.19	228.33	13.60
23	110.00	0.00	99.81	0.86	311.06	9.30	190.67	25.92	223.06	18.93
	Cap^U_1		Cap^U_2		Cap^U_3		Cap^U_4		Cap^U_5	
	110.0		100.0		520.0		200.0		600.0	

250 MWs, note that congestion occurs on branch $km = 12$ in the peak hour 17 (and for several hours thereafter) in all 20 runs. Moreover, although the *mean* flow on branch $km = 12$ is slightly below the thermal limit in other hours, in fact this branch is congested during all 24 h of day 422 in all but three of the 20 runs. Moreover, no other branch is ever congested. These findings are similar to the no-learning treatment, in which branch $km = 12$ (and only this branch) was found to be persistently congested.

Tables 8 and 9 provide detailed numerical solution values (means and standard deviations) for real power production levels and LMPs, respectively, on day 422. Table 10 gives the ordinate coefficient a^R and slope coefficient b^R for the (linear) marginal cost function reported to the ISO on Day 422 by each of the five Generators in each of the 20 runs. In the following discussion we highlight various aspects of these outcomes that differ significantly from the corresponding outcomes presented for the no-learning treatment in Sect. 4.2.

Figure 11 displays the (mean) solution values obtained for production for each of the 24 h on day 422, along with the corresponding solution values obtained for day

Table 9 Learning dynamic 5-node test case—means and standard deviations for solution values (SI) on day 422 for LMPs (nodal balance constraint multipliers)

Hour	$\overline{\mathrm{LMP}_1}$	LMP_1^{SD}	$\overline{\mathrm{LMP}_2}$	LMP_2^{SD}	$\overline{\mathrm{LMP}_3}$	LMP_3^{SD}	$\overline{\mathrm{LMP}_4}$	LMP_4^{SD}	$\overline{\mathrm{LMP}_5}$	LMP_5^{SD}
00	52.74	12.33	110.30	58.16	99.39	48.02	69.40	21.56	55.70	13.06
01	52.70	12.26	100.56	49.61	91.49	41.16	66.56	19.44	55.16	12.82
02	52.68	12.23	94.18	44.34	86.32	36.92	64.69	18.12	54.81	12.67
03	52.66	12.22	91.02	41.79	83.75	34.86	63.77	17.46	54.63	12.60
04	52.63	12.23	87.96	39.38	81.27	32.90	62.86	16.84	54.45	12.54
05	52.65	12.23	89.49	40.57	82.51	33.86	63.32	17.15	54.54	12.57
06	52.66	12.22	91.02	41.79	83.75	34.86	63.77	17.46	54.63	12.60
07	52.69	12.24	97.38	46.96	88.91	39.03	65.63	18.78	54.98	12.75
08	52.75	12.37	113.52	61.04	102.01	50.33	70.35	22.28	55.87	13.15
09	52.79	12.56	126.59	73.16	112.61	60.05	74.15	25.31	56.58	13.52
10	52.80	12.62	129.87	76.28	115.27	62.55	75.11	26.09	56.75	13.61
11	52.80	12.65	131.48	77.83	116.57	63.79	75.58	26.48	56.84	13.66
12	52.80	12.62	129.87	76.28	115.27	62.55	75.11	26.09	56.75	13.61
13	52.79	12.56	126.59	73.16	112.61	60.05	74.15	25.31	56.58	13.52
14	52.78	12.53	124.98	71.64	111.30	58.83	73.68	24.93	56.49	13.47
15	52.78	12.53	124.98	71.64	111.30	58.83	73.68	24.93	56.49	13.47
16	52.80	12.65	131.48	77.83	116.57	63.79	75.58	26.48	56.84	13.66
17	52.73	12.81	147.26	92.89	129.34	75.90	80.10	30.38	57.58	14.07
18	52.80	12.81	139.68	85.72	123.22	70.13	77.95	28.50	57.26	13.93
19	52.80	12.78	138.00	84.10	121.86	68.83	77.46	28.08	57.17	13.87
20	52.80	12.75	136.38	82.54	120.55	67.58	77.00	27.68	57.09	13.82
21	52.81	12.68	133.09	79.38	117.88	65.04	76.05	26.87	56.93	13.71
22	52.78	12.53	124.98	71.64	111.30	58.83	73.68	24.93	56.49	13.47
23	52.76	12.39	115.19	62.56	103.36	51.54	70.83	22.66	55.96	13.19

422 in the absence of Generator learning.[26] In the no-learning treatment, note that the "peaker" (high cost) Generator 4 is only dispatched to produce energy at the peak load hour 17. In the learning treatment, however, Generator 4 is able to use strategic supply offers to ensure it is dispatched at approximately its upper production limit (200 MWs) throughout each hour of the day. Also, in the no-learning treatment the "cheap" Generator 5 is regularly dispatched at a high production level during each hour of the day, but in the learning treatment it is backed way down because its strategic supply offers make it appear to be a relatively more expensive Generator.

This heavier reliance on costlier generation in the learning treatment substantially increases the total variable cost of operation. Indeed, as seen in Fig. 12, the minimum total variable cost of operation under the learning treatment is roughly three times higher than under the no-learning treatment.

Figure 13 graphically depicts the 24-h (mean) LMP solution values for the learning treatment along with the 24-h LMP solution values for the no-learning treatment. Interestingly, although the LMPs for the learning treatment are considerably higher than the LMPs for the no-learning treatment, they are also less volatile around the peak load hour 17. Consequently, the ISO is not able to use the appearance of price spikes

[26] Given the stationarity of the daily load profiles and the Generators' cost functions and production limits, and the absence of system disturbances, in the no-learning treatment the 24-h outcomes obtained for any one day are the same as for any other day.

Table 10 Learning dynamic 5-node test case—ordinate coefficients (a^r) and slope coefficients (b^r) for the linear marginal cost functions reported to the ISO by the five generators on day 422 in each of the 20 runs, with summary statistics

Run	a_1^R	b_1^R	a_2^R	b_2^R	a_3^R	b_3^R	a_4^R	b_4^R	a_5^R	b_5^R
1	21.0	0.031824	18.0	0.000005	75.0	0.036059	36.0	0.090005	40.0	0.033335
2	24.0	0.036370	25.7	0.011694	100.0	0.288465	32.7	0.067325	40.0	0.100003
3	24.0	0.036370	18.0	0.126012	75.0	0.100964	30.0	0.273000	40.0	0.066669
4	42.0	0.017360	15.0	0.063857	100.0	0.019232	72.0	0.000003	40.0	0.023811
5	18.7	0.060614	16.4	0.027279	37.5	0.072118	45.0	0.000002	30.0	0.025002
6	33.6	0.013889	25.7	0.011694	75.0	0.036059	32.7	0.048682	40.0	0.006668
7	15.3	0.013890	45.0	0.020460	25.0	0.112115	36.0	0.030003	40.0	0.033335
8	24.0	0.054552	16.4	0.027279	42.9	0.082420	60.0	0.000002	30.0	0.075003
9	14.0	0.054026	22.5	0.010233	100.0	0.096156	32.7	0.048682	30.0	0.050002
10	15.3	0.069431	16.4	0.081828	75.0	0.024040	40.0	0.020003	40.0	0.100003
11	16.8	0.054553	22.5	0.056257	37.5	0.072118	36.0	0.030003	40.0	0.000001
12	42.0	0.095461	30.0	0.000005	60.0	0.028848	51.4	0.025717	40.0	0.066669
13	16.8	0.038189	16.4	0.114557	75.0	0.024040	32.7	0.005182	30.0	0.035002
14	14.0	0.054026	20.0	0.000005	75.0	0.144234	60.0	0.000002	30.0	0.075003
15	24.0	0.009922	16.4	0.016370	30.0	0.039231	36.0	0.018003	40.0	0.006668
16	28.0	0.090917	16.4	0.114557	42.9	0.041211	36.0	0.000002	30.0	0.050002
17	21.0	0.095464	15.0	0.168000	37.5	0.018030	40.0	0.009094	40.0	0.016668
18	21.0	0.047734	16.4	0.016370	37.5	0.018030	40.0	0.020003	30.0	0.050002
19	14.0	0.011240	16.4	0.027279	75.0	0.024040	30.0	0.029400	40.0	0.033335
20	24.0	0.109100	15.0	0.006000	100.0	0.000001	45.0	0.037503	30.0	0.050002
Mean	22.7	0.049747	20.2	0.044987	63.8	0.063871	41.2	0.037631	36.0	0.044859
SD	8.4	0.030342	7.2	0.049954	25.6	0.065315	11.4	0.060501	5.0	0.029031
Min	14.0	0.009922	15.0	0.000005	25.0	0.000001	30.0	0.000002	30.0	0.000001
Max	42.0	0.109100	45.0	0.168000	100.0	0.288465	72.0	0.273000	40.0	0.100003

in peak load hours to detect the considerable exercise of market power by the learning Generators. Rather, some form of direct auditing of the Generators' cost attributes would seem to be required.

Figure 14 displays the (mean) marginal cost functions that the five Generators report to the ISO on day 422, along with their true marginal cost functions. Despite the absence of any explicit collusion, all five Generators have learned to report higher-than-true marginal cost functions with respect to both ordinate and slope. In the case of Generators 3 and 5, the two largest generating units, the increase is substantial; these two Generators quickly learn to report a marginal cost function that is near or at the highest level permitted in their action domains.[27] Clearly the core aspects of the WPMP market design currently captured in the AMES framework do not provide sufficient mechanisms to prevent Generators from exercising substantial market power through strategic reporting of supply offers.

These findings can be compared with the findings of Wolfram (1999), who determined empirically that the (pre-NETA) uniform-price auction design in effect for the UK wholesale power market at the time of her study provided incentives for generators

[27] More precisely, the lower and upper range-index values implied by these Generators' reported marginal cost curves typically converge with rapidity to values that are near or at their highest permitted range-index levels $\text{RIMax}^L = 0.75$ and $\text{RIMax}^U = 0.75$; cf. Table 6.

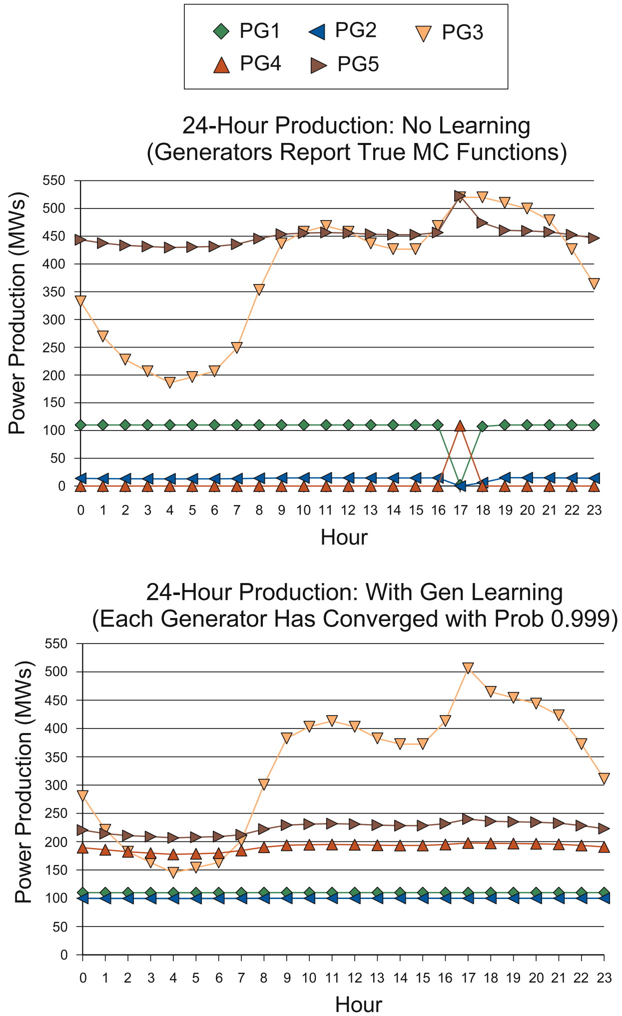

Fig. 11 Dynamic 5-node test case solution values for 24-h real power production levels (day 422)—generator learning compared with no learning

to raise prices above costs. In addition, Mount (2000) uses a simple analytical framework to show how generators facing normally-distributed demand in a wholesale power market operated as a uniform-price auction have an incentive to submit supply offers that far exceed their true costs. The AMES Day-Ahead Market collapses to a uniform-price auction only in the absence of transmission congestion; LMP separation occurs when any branch is congested. However, using simple reinforcement learning with no explicit collusion, the AMES Generators quickly co-learn how to submit supply offers that result in substantial market power whether or not LMP separation occurs.

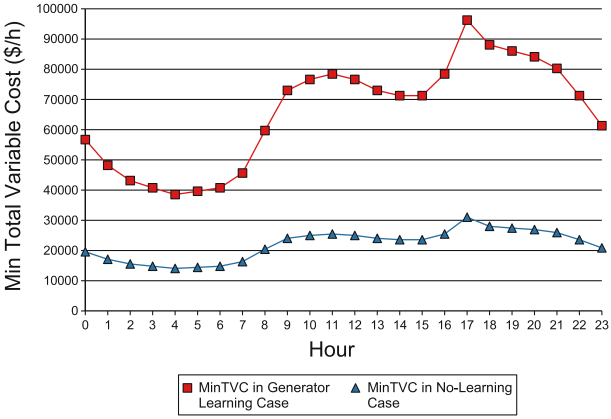

Fig. 12 Dynamic 5-node test case solution values for 24-h minimum total variable cost (day 422)—generator learning compared with no learning

5 Concluding Remarks

The North American power transmission grid has been called "the largest and most complex machine in the world" (Amin 2004, p. 31). An extraordinary experiment is under way to see whether the physical operation of this complex machine can be successfully married with a restructured commercial architecture encouraging increased reliance on demand and supply forces. Smart electrical devices permitting more distributed physical control of the grid are being introduced along with market designs permitting more decentralized pricing and allocation mechanisms, a trend one commentator has called "electricity's third great revolution" (Mazza 2003).

Stakeholders, policy makers, and researchers all clearly recognize the critical need for this experiment to succeed (FERC 2007). Nevertheless, the issues raised by this experiment are extremely challenging. How to analyze the potential dynamic performance of a system comprising multiple distributed entities, some physical and some human, all with finite information and computational capabilities? How to properly take into account the stability limits of physical components as well as the strategic behaviors of human participants responding to the incentives deliberately or inadvertently presented by system design features?

Agent-based modeling tools have been specifically developed to handle these types of complexities, hence it is not surprising to find agent-based researchers actively involved in this electricity restructuring movement. As detailed by Davidson and McArthur (2005) and by Widergren et al. (2006), multi-agent systems are attracting significant research interest for power system applications. Indeed, the *IEEE Working Group on Multi-Agent Systems in Power Engineering* is charged with exploring the benefits, applications, and advanced functionality that can be provided for power systems through agent technology. The members of this IEEE MAS Working Group include economists as well as engineers, and academics as well as industry stakeholders.

In this study we explore the potential usefulness of agent-based tools for investigating the efficiency and reliability of the Wholesale Power Market Platform (WPMP), a

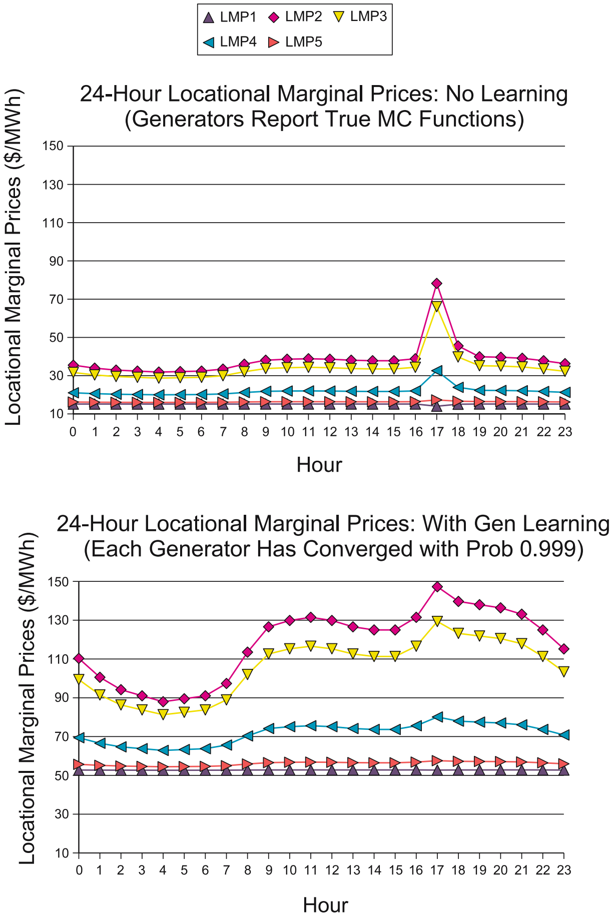

Fig. 13 Dynamic 5-node test case solution values for 24-h LMPs (day 422)—generator learning compared with no learning

market design proposed by the U.S. Federal Energy Regulatory Commission for common adoption by all U.S. wholesale power markets (FERC 2003). We first describe a newly developed agent-based computational laboratory—the AMES framework—that models a wholesale power market operating in accordance with core WPMP features over a realistically rendered transmission grid subject to congestion effects. Using a dynamic 5-node test case for concrete illustration, we then explore the extent to which these core WPMP features permit and even encourage the exercise of market power by Generators through strategic reporting of supply offers.

More precisely, in the dynamic 5-node test case the AMES ISO does not know the AMES Generators' true cost attributes. Rather, in each operating day D, the AMES ISO must formulate its DC-OPF problem for each hour of the Day-Ahead Market for day D + 1 based on the cost attributes reported to it by the Generators. The

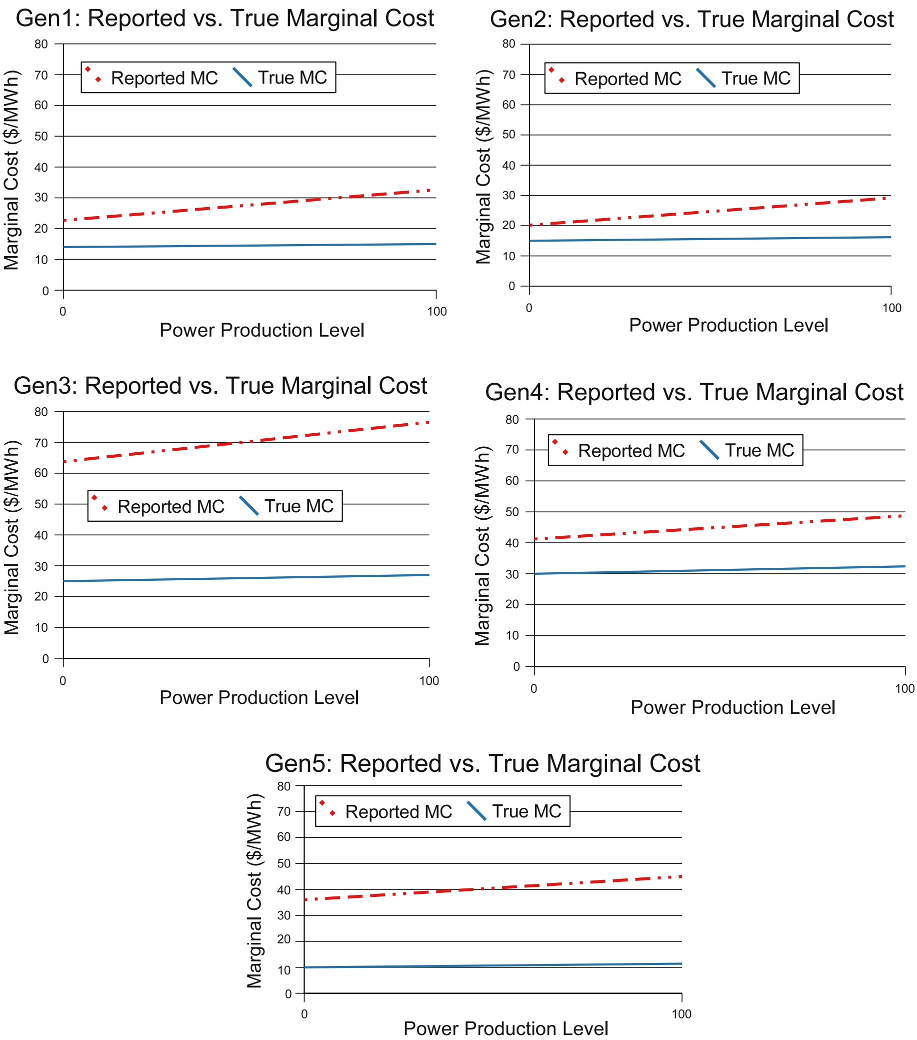

Fig. 14 Dynamic 5-node test case—mean reported marginal cost function versus true marginal cost function for each generator (Day 422)

profit-seeking Generators learn over time what cost attributes to report to the ISO using a simple reinforcement learning algorithm based on past profit outcomes.

As seen in Sect. 4, in a typical run the Generators converge within 62 simulated trading days to supply offer selections for which their reported marginal cost functions are uniformly higher than their true marginal cost functions, in some cases substantially higher, despite the absence of any explicit collusion. The resulting "optimal" DC-OPF solutions determined by the AMES ISO appear to have desirable properties, e.g., low LMP volatility during peak load hours and congestion on only one branch. In fact, however, total variable costs of operation are roughly three times higher than they would have been had the Generators reported their true cost attributes. As captured in the current AMES framework, the core WPMP design features do not prevent the considerable exercise of market power by Generators.

As detailed in Sect. 2, the AMES framework needs to be further extended to incorporate additional key aspects of the WPMP design that could significantly impact the efficiency and reliability of market operations. Moreover, initial conditions and parameter specifications need to be more carefully calibrated to match real-world conditions. Nevertheless, we believe the preliminary findings reported in this study suggest the great potential of agent-based computational models to help ensure a successful restructuring of the electric power industry through intensive sensitivity experiments.

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