Part III
Energy Auctions and Markets

Agent-based Modeling and Simulation of Competitive Wholesale Electricity Markets

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Abstract This paper sheds light on a promising and very active research area for electricity market modeling, that is, agent-based computational economics. The intriguing perspective of such research methodology is to succeed in tackling the complexity of the electricity market structure, thus the fast-growing literature appeared in the last decade on this field. This paper aims to present the state-of-the-art in this field by studying the evolution and by characterizing the heterogeneity of the research issues, of the modeling assumptions and of the computational techniques adopted by the several research publications reviewed.

Keywords Agent-based computational economics · Agent-based modeling and simulation · Electricity markets · Power system economics

1 Introduction

In the last decade, several countries in the world have been obliged to intensively regulate the electric sector, either for starting and supporting the liberalization process (e.g., Directives 96/92/EC and 2003/54/EC of the European Commission recommended all European countries to switch to market-based prices) or for amending and improving proposed regulations because of shortcomings in the market design or market failures (e.g., the 2001 California crisis has motivated the US Federal Energy Regulatory Commission to propose a common wholesale power market for adoption by all United States). In all cases, common restructuring proposals are to adopt complex market structures, where several interrelated market places are envisaged and where the integration of the national markets towards an inter-regional/continental perspective is encouraged (e.g., European Union and USA).

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A generic wholesale power market is composed by energy markets such as spot and forward markets for trading electricity, ancillary service markets for guaranteeing security in the provision, and even market couplings with foreign markets. Furthermore, wholesale electricity markets are vertically integrated with other commodity markets, mainly fuels markets, such as natural gas, oil, and coal markets, and moreover climate policy measures are imposing market-based mechanisms for climate change regulation to foster a carbon-free electricity production. The rationale for this complex structure relies upon the nature of the traded good and of the technologies of the production. Electricity is essentially not storable in large quantities, thus a real-time balancing in the production and consumption of electrical power must be carefully guaranteed. Furthermore, the electric sector is a highly capital-intensive industry due to economies of scale occurring mainly in the production sector. This favors market concentration in the supply side and increases the opportunity for the exertion of market power by a reduced number of market players. Furthermore, the producers' long-term investment plans must be supported by a clear, adequate, and also stable regulation. Thus regulatory interventions must be accurately devised and implemented.

Such fast-changing economical sector together with its complexity from both an economical and a technological viewpoint have fostered practitioners and power system/economics researchers to acquire new competencies and to adopt innovative research approaches to study realistic market scenarios and propose appropriate regulations for this original market environment. The several high-quality scientific contributions gathered in this book witness the broadness and multidisciplinarity of the research domain and the great interest around this topic. Several approaches have been proposed in the electricity markets literature. Generally speaking, classical analytical power system models are no longer valid because they are a poor fit to the new market-based context. Now all operation decisions are decentralized and strategically taken by each market operator so as to maximize individual trading profit. Similarly, standard analytical economic approaches, based on game-theoretic models, are usually limited to stylized market situations among few actors neglecting technological and economical constraints such as transmission networks. Furthermore, modeling aspects such as repeated interactions among market operators, transmission network constraints, incomplete information, multi-settlement market mechanisms, or market coupling are seldom, if never, addressed by the theoretical literature. On the other side, human-subject experiments have been proposed to complement theoretical approaches (Staropoli and Jullien 2006); however, realistically replicating the bidding behavior of a power generator in laboratory experiments requires strong expertise by the participants. Moreover, the relevant literature mainly focuses on market design issues concerning single stylized marketplaces, such as the day-ahead market session, thus neglecting the structural complexity of wholesale electricity markets. Conversely, computational methodologies offer an appealing approach to cope with the complexity of power market mechanisms by enabling researchers to realistically simulate such market environments.

This paper focuses attention to the computational literature and in particular with the promising approach of agent-based modeling and simulation (ABMS), which has now became a widespread methodology in the study of power systems and electricity markets. Current strands of research in power systems ABMS are distributed electricity generation models, retail and wholesale electricity market models. Space reasons as well as the fact that last topic has been the major strand of research in electricity market ACE have motivated the choice to focus on the literature about wholesale electricity market models.

It is worth mentioning that two literature reviews have recently appeared,[1] that is, Ventosa et al. (2005) and Weidlich and Veit (2008b). The first paper presents electricity market modeling trends enclosing both contributions from the theoretical and the computational literature. In particular, the authors identify three major trends: optimization models, equilibrium models, and simulation models. The second one is a critical review about agent-based models for the wholesale electricity market. Their focus is on the behavioral modeling aspect, with a discussion about methodological issues to raise awareness about critical aspects such as appropriate agent architecture and validation procedures.

This review paper offers an updated outlook on the vast and fast-growing literature, thus enhancing previous surveys (49 papers are reviewed in details). Moreover, this paper differs from previous reviews both for the chronological perspective and for the agent-based modeler perspective adopted to organize the contents of the writing. As far as concerns the former aspect, this paper reports three distinct summary tables where all papers are listed and compared in chronological order. This enables to highlight modeling trends with respect to different viewpoints, that is, research issues, behavioral adaptive models, and market models. As far as concerns the latter aspect, this paper discusses in separate sections the modeling solutions for the major agents populating an electricity market models, that is, market operators and market institutions, thus allowing the proper identification of the solutions proposed by each reviewed paper. As a final remark, this paper provides an overview of the challenging ABMS research methodology in the domain of electricity markets to facilitate newcomers and practitioners in the attempt to better disentangle in the vast literature and to present the state-of-the-art of the several computational modeling approaches.

The paper is structured as follows: Sect. 2 introduces notions about the agent-based modeling and simulation approach, from the perspective of agent-based computational economics. A specific focus is then devoted to the electricity market literature. Section 3 focuses on the behavioral modeling aspect by reviewing several papers and the variety of approaches adopted. On the other side, Sect. 4 lists and discusses the market models proposed. Large-scale models as decision support systems are also presented. For each of the three previous sections, a table is reported

[1] A recent working paper by Sensfuß et al. (2007) proposes a further literature review mainly focusing on the research activity led by some research groups active in the field. The discussion is grouped around three major themes, that is, the analysis of market power and design, the modeling agent decisions, and the coupling of long-term and short-term decisions. The authors also review papers dealing with electricity retail markets.

to summarize and highlight the major modeling features of each reviewed paper. Finally, Sect. 5 draws conclusions.

2 Agent-based Modeling and Simulation

ABMS refers to a very active research paradigm, which has gained more and more popularity among a wide community of scientist, ranging from natural science like biologist up to human and social science, that is, economists and sociologist, and engineers. The rationale is twofold: a scientific reason referring to the notion of complexity science and a technological one, that is, the increased availability of cheap and powerful computing facilities.

The development of ABMS in social sciences, and in particular in Economics, is closely linked with the pioneering work conducted at the Santa Fe Institute (SFI). SFI grouped 20 worldwide renowned experts from different disciplines, that is, economists, physicists, biologists, and computer scientists, to study the economy as an evolving complex system. Their authoritative work greatly influenced researchers by adopting agent-based models in the study of social systems.[2] Their philosophical approach can be summarily expressed by "If you didn't grow it, you didn't explain it," supporting the view of ABMS approach as a "generative social science" (Epstein 1999). The basic units of an agent-based model (ABM) are agents, which are considered entities characterized by autonomy, that is, there is no central or "top down" control over their behavior, and by adaption, that is, they are reactive to the environment. The notion of agent is better understood within the framework of complex adaptive systems (CAS) (Blume and Easley 1992). A CAS is a complex system, that is, a system of interacting units, that includes goal-directed units, that is, units that are reactive and that direct at least some of their reactions towards the achievement of built-in or evolved goals (Tesfatsion and Judd 2006). This definition enables agents to be entities ranging from active data-gathering decision-makers with sophisticated learning capabilities to passive world features with no cognitive functioning.

2.1 ACE: A New Paradigm for Economics

A modern market-based economy is an example of a CAS, consisting of a decentralized collection of autonomous agents interacting in various market contexts. Agent-based computational economics (ACE) (Tesfatsion and Judd 2006; Richiardi 2009) is the computational study of economic processes modeled as dynamic

[2] At SFI, Swarm (2009), the ancestor of agent-based software programming tools, was developed, and the Santa Fe Artificial Stock Market, one of the first agent-based financial market platforms, was realized (Ehrentreich 2002).

systems of interacting agents.[3] ACE researchers and practitioners are intrigued by the appealing perspective of taking into account in their economic models of several aspects of the procurement processes, the "dark side" of the economic modeling, that is, all economic events occurring in reality among customers and suppliers during negotiation and trading processes, which are constrained by economic and social institutions. These aspects are greatly simplified or even neglected by standard analytical economic theories; indeed, substituting equilibrium assumptions for procurement processes is one way to achieve a drastically simplified representation of an economic system.[4] The drawback but also the opportunity of this challenging methodological approach is to consider aspects such as asymmetric information, strategic interaction, expectation formation, transaction costs, externalities, multiple equilibria, and out-of equilibrium dynamics.

However, this fairly young methodological approach in spite of an apparently ease-of-use in modeling and simulations requires great expertise in implementing the models and in obtaining significant simulation results. To succeed with the modeling goals, ACE models must be carefully devised in their attempt to increase realism. A correct identification of irrelevant modeling aspects is still required, and in any case theoretical models are effective starting-points for the modeling activity. Moreover, ACE models must be adequately implemented and an object-oriented programming approach is strongly recommended for scalability and modularity. In particular, agent-oriented programming offers a suitable implementing framework for building artificial worlds populated by interacting agents (Jennings 2000). This is a strong point of the methodology that both the modeler and the software programmer perspectives usefully coincide. Several agent-based software tools[5] have appeared in the last 10 years and are fostering the adoption of such methodology.

2.2 ACE in Electricity Markets: Research Issues

The great versatility of ACE methodology has motivated researchers to adopt it for the study of several electricity market issues. Some researchers have applied agent-based models for examining electricity consumer behavior at the retail level, for example, Hämäläinen et al. (2000); Roop and Fathelrahman (2003); Yu et al. (2004); and Müller et al. (2007) or for studying distributed generation models, for example, Newman et al. (2001); Rumley et al. (2008); and Kok et al. (2008). This paper focuses on a third strand of research, that is, the study of competitive

[3] The "mother" of the expression agent-based computational economics (ACE) is Professor Leigh Tesfation at Iowa State University, who has been one of the leader of agent-based modeling and simulation applied to economics and also to electricity markets (Nicolaisen et al. 2000).

[4] For a detailed discussion on procurement process and neoclassical approaches to economic modeling refers to Tesfatsion and Judd (2006).

[5] A description of software tools for agent-oriented programming is available at Tesfatsion (2007).

wholesale electricity markets, which are mainly characterized by centralized market mechanisms such as double-auction markets.

Table 1 reports a broad selection of papers written in the last decade about ACE applied to the study of wholesale electricity markets.[6] These 49 works are the ones that are considered and reviewed in the rest of this paper. Table 1 lists the papers in a chronological order[7] to show how the researchers' focus of attention in the field has evolved throughout the years. Eight major research issues are highlighted from column 2 to column 9 from the most present to the less present in the considered literature. These aspects, which are not mutually exclusive, have been selected because they are the ones of major concern in the literature. In particular, these correspond to (1) market performances and efficiency, (2) market mechanisms comparison, (3) market power, (4) tacit collusion, (5) multi-settlement markets, (6) technological aspects affecting market performances, (7) diversification or specialization, and (8) divestiture or merging of generation assets. Finally, the last column of Table 1 reports a concise explanation of the objectives for every paper, and the last row reports the sum of the occurrences of the specific research issue in the literature.

The chronological listing points out the progressive increase in the number of contributions in this field throughout the last decade. The tendency pinpoints the still rising interest in the topic and in the computational approach. Table 1 further shows that there has not been a clear evolution in the study of the different research themes. The first three reported issues, that is, columns numbered one to three, have been and are the most addressed research areas since the early computational studies. Indeed, market performances and efficiency, market mechanisms comparison, and market power have been and are the major research issues also of the theoretical and experimental literature. The great emphasis placed on these issues denote the confidence that ACE researchers have on this approach to provide guidance for market design issues. The remaining columns, and in particular columns numbered 6–8, highlight further research areas, which have been currently investigated only by few research groups, which certainly deserve more research efforts. As a final remark, it is important to note that the majority of these papers are purely computational studies, that is, empirical validation is seldom addressed. This is a critical aspect that needs to be addressed by researcher to assess the effectiveness of their modeling assumptions. Furthermore, it is often difficult to compare simulation results of papers dealing with the same issue because of the great heterogeneity

[6] In this paper, all major ACE publications dealing with wholesale electricity markets known to the authors have been considered. The selection has been based on the surveys of previous review papers, most recent papers published by the major groups active in this field of research and their relevant cites. Moreover, the long-standing authors' knowledge in the field helped selecting further original contributions. It is worth remarking that due to the fast-growing and broad literature in this research area, authors may have probably, but involuntarily, missed citing some contributions that appeared in the literature. However, the authors believe that the paper provides a vast and representative outlook on the ACE literature on wholesale electricity markets.

[7] The papers are ordered in descending order with respect to the year of publication and if two or more papers have appeared in the same year, then an alphabetical descending order has been considered.

Table 1 Description of the research issues for each paper of the considered literature

Paper	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*Explanation*
Curzon Price (1997)	✓								Co-evolutionary programming technique to study Bertrand and Cournot competition, a vertical chain of monopolies and a simple model of electricity pool
Richter and Sheblé (1998)	✓								Study of bidding strategies with GA, with sellers endowed with price forecasting techniques
Visudhiphan and Ilic (1999)	✓		✓						Comparison of hour- and day-ahead spot market performances with different demand elasticities
Bagnall and Smith (2000)	✓	✓		✓					Study on UK wholesale electricity market with a complex agent architecture based on learning classifier systems
Bower and Bunn (2000)		✓							ACE investigation of bilateral and pool market system under UA and DA clearing price mechanisms
Lane et al. (2000)	✓		✓						Study of market power exertion by sellers in different oligopolistic scenarios with intelligent buyers
Nicolaisen et al. (2000)	✓	✓	✓						Market power analysis under different concentration and capacity scenarios
Bower and Bunn (2001b)		✓							ACE investigation of bilateral and pool market system under UA and DA clearing price mechanisms. The paper is similar to Bower and Bunn (2000)
Bower et al. (2001)								✓	Evaluation of mergers operations in the German electricity market
Bunn and Oliveira (2001)	✓				✓				Study about market performances for the NETA of UK electricity market
Day and Bunn (2001)		✓	✓					✓	Impact of divestiture proposals in the price market
Nicolaisen et al. (2001)	✓	✓	✓						Market power and efficiency analysis in the discriminatory double-auction
Visudhiphan and Ilic (2001)	✓	✓	✓						Investigation of market power abuse with generators strategically withholding capacity
Cau and Anderson (2002)	✓			✓					Study of bidding behavior of a simple duopolistic model of the Australian wholesale electricity market

(*Continued*)

Table 1 (Continued)

Paper	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*Explanation*
Koesrindartoto (2002)	✓	✓	✓						Study on DA efficiency with respect to Roth and Erev learning model specification
Bunn and Oliveira (2003)	✓		✓		✓				Study about the opportunity of exertion of market power by two companies in the UK electricity market
Cau (2003)	✓			✓					Investigation of tacit collusion behavior in an oligopolistic market based on the Australian wholesale power market
Visudhiphan (2003)	✓	✓	✓						Comparing several learning algorithms to study how simulated price dynamic is affected
Atkins et al. (2004)		✓			✓				Large scale model for studying electricity market
Bin et al. (2004)		✓	✓						Pricing mechanisms comparison for pool-based electricity market (UA, DA, EVE)
Minoia et al. (2004)	✓					✓			Proposal of an original model of pricing settelment rule for wholesale electricity markets providing incentives for transmission investments
Xiong et al. (2004)	✓	✓							Comparison of discriminatory and uniform auction markets in different demand scenarios
Bagnall and Smith (2005)	✓	✓		✓					Study on UK wholesale electricity market with a complex agent architecture based on learning classifier systems
Bunn and Martoccia (2005)			✓	✓					Market power and tacit collusion analysis of the electricity pool of England and Wales
Bakirtzis and Tellidou (2006)		✓	✓						Analysis of the effect of the market power under uniform and discriminatory settlement rule
Botterud et al. (2006)		✓	✓			✓			Analysis of the effect of congestion management on market price
Chen et al. (2006)				✓					Analysis of Cournot models of electricity pool

Krause et al. (2006)	✓					Study of market dynamics for network-constrained pool markets
Krause and Andersson (2006)		✓	✓			Market power analysis under different congestion management schemes
Ma et al. (2006)	✓			✓		Study of the bidding behavior of suppliers in network-constrained electricity market
Naghibi-Sistani et al. (2006)	✓					Study of simulation convergence properties with Nash equilibrium
Veit et al. (2006)	✓			✓		Impact of forward market on the spot price volatility
Weidlich and Veit (2006)	✓	✓		✓		Analysis of day ahead and real time market under discriminatory and uniform auctions
Banal-Estanol and Rupérez-Micola (2007)					✓	Study on how the diversification of generation portfolio influences market prices
Bunn and Oliveira (2007)		✓			✓	Investigation about impact of market design on technological diversification among portfolio generators
Guerci et al. (2007)	✓	✓				Detailed analysis of learning bidding behavior in UA and DA market mechanisms
Hailu and Thoyer (2007)	✓	✓				Study the performance of uniform, discriminatory, and Vickrey auctions
Ikeda and Tokinaga (2007)	✓					Analysis of electricity market performances with a genetic programming approach
Nanduri and Das (2007)		✓	✓			Market power analysis under different auction mechanisms
Sueyoshi and Tadiparthi (2007)	✓					Assessment of adaptive agent-based model as good estimator of electricity price dynamics

(*Continued*)

Table 1 (Continued)

Paper	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*Explanation*
Sun and Tesfatsion (2007)	✓								Presentation of the AMES simulator with a test bed case
Tellidou and Bakirtzis (2007)	✓		✓	✓					Analysis of market power and tacit collusion among producers
Guerci et al. (2008a)	✓	✓		✓					Study of tacit collusive behavior with two RL algorithm in UA and DA market mechanisms
Guerci et al. (2008b)	✓		✓						Study of market power in network-constrained electricity market with different learning algorithms
Li et al. (2008)	✓	✓							Analysis of LMP mechanisms in different price-cap and price-sensitive demand scenarios
Somani and Tesfatsion (2008)	✓		✓						Comparisons of market efficiency indexes for the detection of market power in wholesale power markets
Sueyoshi and Tadiparthi (2008a)		✓				✓			Study on how transmission limits affect the market price and its volatility in different market mechanism
Weidlich and Veit (2008a)	✓				✓				Analysis of German electricity market performances by an ACE model considering day-ahead, real-time balancing and CO2 emission allowances markets
Bunn and Day (2009)	✓	✓		✓					Comparison between real UK market performances and a detailed computational model of price formation
Total occurrences	34	25	18	9	7	3	2	2	

From column 2 to column 9, the following research issues are considered: (1) market performances and efficiency, (2) market mechanisms comparison, (3) market power, (4) tacit collusion, (5) multi-settlement markets, (6) technological aspects affecting market performances, (7) diversification or specialization, and (8) divestiture or merging of generation assets performances. Last row reports the sum of the occurrences of the specific research issue (column) in the behavioral modeling assumptions. Next section presents the heterogeneity of the behavioral modeling assumptions adopted for modeling market participants in wholesale electricity markets.

3 Behavioral Modeling

Key activity of the agent-based modeler concerns the ability to choose appropriate behavioral models for each type of agent. Important sources of heterogeneity, among agents populating an artificial economy, are not only economical or technological endowments such as marginal costs or revenues, generating capacity, but also behavioral aspects such as available information, backward- or forward-looking horizon, risk-aversion, and generically reasoning capabilities. This heterogeneity is exogenously set by the modeler and should be adequately justified by correct modeling assumptions.

In general, the ABSM literature has adopted several approaches for modeling agent's behavior ranging from zero-intelligence to complex black-box data-driven machine learning approaches such as neural networks, from social learning such as genetic algorithms to individual learning approaches such as reinforcement learning algorithms.[8] This great variability is present also in the ACE literature about electricity markets, even if the model choice is seldom adequately justified.

The summary Table 2 is reported to facilitate the comparison of the reviewed papers, for easily identifying key behavioral modeling aspects for each paper. This table lists in a chronological order papers to pinpoint the modeling trends in the last decade. Three major aspects are examined, that is, the type of adaptive behavioral model (columns 2–6), some features of the generation companies model (columns 7 and 8), and of the demand model (columns 9 and 10). In particular, columns 2–6 refer to the adaptive behavioral model chosen for suppliers, buyers, or even transmission operators. We have categorized the papers under five distinct classes in an attempt to gather all computational approaches in nonoverlapping groups. However, it is always difficult to exactly categorize each element, and so in some cases approximate solutions may have been proposed. The five classes are genetic algorithms (GA), co-evolutionary genetic algorithms (CGA), learning classifier systems (LCS), algorithm models tailored on specific market considerations (Ad-hoc), and Reinforcement Learning algorithms (RL). In the rest of the section, all such techniques will be adequately illustrated. As far as concerns columns 7 and 8, wholesale electricity market models usually focus on suppliers behavior, and so the table reports the bidding format of the spot market (Actions) and the suppliers characteristic of being generating units portfolio (Portf.). On the contrary, the demand-side is often

[8] A major distinction in ACE models regards the so-called social/population learning and individual learning models (Vriend 2000). The former refers to a form of learning where learned experiences are shared among the players, whereas the latter involves learning exclusively on the basis of his own experience.

Table 2 Categorization of agents' adaptive models and supply- and demand-side models for each paper

Paper	Adaptive model					Supply		Demand	
	GA	*CGA*	*LCS*	*Ad-hoc*	*RL*	*Action*	*Portf.*	*Elas.*	*Adapt.*
Curzon Price (1997)		✓				P			
Richter and Sheblé (1998)	✓					P,Q			
Visudhiphan and Ilic (1999)				✓		SF		✓	
Bagnall and Smith (2000)			✓			P			
Bower and Bunn (2000)					✓	P	✓	✓	
Lane et al. (2000)	✓					P		✓	✓
Nicolaisen et al. (2000)	✓					P		✓	✓
Bower et al. (2001)					✓	P	✓	✓	
Bower and Bunn (2001b)					✓	P	✓	✓	
Bunn and Oliveira (2001)				✓		P	✓	✓	✓
Day and Bunn (2001)				✓		SF	✓	✓	
Nicolaisen et al. (2001)					✓	P		✓	✓
Visudhiphan and Ilic (2001)				✓		P,Q			
Cau and Anderson (2002)		✓				SF			
Koesrindartoto (2002)					✓	P		✓	✓
Bunn and Oliveira (2003)				✓		P	✓	✓	✓
Cau (2003)		✓				SF	✓		
Visudhiphan (2003)				✓	✓	P,Q		✓	✓
Atkins et al. (2004)				✓		P,Q	✓	✓	✓
Bin et al. (2004)					✓	P	✓		
Minoia et al. (2004)				✓		P			
Xiong et al. (2004)					✓	P		✓	
Bagnall and Smith (2005)			✓			P			
Bunn and Martoccia (2005)				✓		P	✓	✓	
Bakirtzis and Tellidou (2006)					✓	P	✓		

Botterud et al. (2006)				✓		P	✓		
Chen et al. (2006)		✓				Q		✓	
Krause et al. (2006)					✓	SF			
Krause and Andersson (2006)					✓	SF		✓	✓
Ma et al. (2006)					✓	SF		✓	
Naghibi-Sistani et al. (2006)					✓	SF			
Veit et al. (2006)					✓	SF	✓	✓	
Weidlich and Veit (2006)					✓	P,Q			
Banal-Estanol and Rupérez-Micola (2007)					✓	P	✓		
Bunn and Oliveira (2007)				✓		Q		✓	
Guerci et al. (2007)					✓	P,Q			
Hailu and Thoyer (2007)					✓	SF			
Ikeda and Tokinaga (2007)		✓				P,Q	✓		
Nanduri and Das (2007)					✓	SF			
Sueyoshi and Tadiparthi (2007)					✓	P,Q		✓	✓
Sun and Tesfatsion (2007)					✓	SF			
Tellidou and Bakirtzis (2007)					✓	P,Q			
Guerci et al. (2008a)					✓	P,Q			
Guerci et al. (2008b)					✓	SF	✓		
Li et al. (2008)					✓	SF		✓	
Somani and Tesfatsion (2008)					✓	SF		✓	
Sueyoshi and Tadiparthi (2008a)					✓	P,Q		✓	✓
Weidlich and Veit (2008a)					✓	P,Q	✓		
Bunn and Day (2009)				✓		SF	✓	✓	
Total occurrences	3	5	2	12	28		18	24	11

The five classes for the adaptive models are genetic algorithms (GA), co-evolutionary genetic algorithms (CGA), learning classifier systems (LCS), algorithm models tailored on specific market considerations (ad-hoc), and reinforcement learning algorithms (RL). For the suppliers model, the bidding strategy (Actions) and the suppliers characteristic of being generating units portfolio (Portf.) are highlighted. Finally, the demand side model is described by the properties of being price-elastic (Elas.) or adaptive (Adapt.). A ✓notation is adopted for paper presenting the column feature; otherwise, an empty field means that the negation is valid.

represented by an aggregate price-inelastic or -elastic demand mainly without adaptive behavior, thus these two aspects are highlighted in columns 9 and 10 of the table (Elas. and Adapt., respectively).

It happens that some papers describe the great flexibility of a computational model by illustrating an extended software framework, but reports simulation studies in a simplified case. In this case, table's values refer to the simulation test case proposed.

3.1 Evolutionary Computation

Evolutionary computation (EC) is the use of computer programs that not only self-replicate but also evolve over time in an attempt to increase their fitness for some purpose. Evolutionary algorithms have been already widely adopted in economics and engineering for studying socio-economic systems. One major and most famous algorithm for EC is the genetic algorithm (GA).[9] GA are commonly adopted as adaptive heuristic search algorithm, but they have also been applied as a social learning (mimicry) rule across agents (Vriend 2000). They are premised on genetic and natural selection principles and on the evolutionary ideas of survival of the fittest. GAs are conceived on four analogies with evolutionary biology. First, they consider a genetic code (chromosomes/genotypes). Second, they express a solution (agents/phenotypes) from the code. Third, they consider a solution selection process, the survival of the fittest; the selection process extracts the genotypes that deserve to be reproduced. Finally, they implement the so-called genetic operators, such as elitism, recombination, and mutation operations, which are used to introduce some variations in the genotypes.

Key element for the implementation of the genetic algorithm is the definition of a utility function, or fitness function, which assigns a value to each candidate solution (potential phenotype). This evolving function is thus used to evaluate the interest of a phenotype with regard to a given problem. The survival of the corresponding solution or equivalently its number of offspring in the next generation depends on this evaluation.

A summarily description of the algorithm flow is: Initialization. Random generation of an initial population P_1 of N agents

for iteration t $= 1, \dots,$ T:

1. Evaluation of the fitness $F_t(n)$ for each agent $n = 1, \dots, N$ of the current population P_t.
2. Selection of the best-ranking phenotypes in P_t according to selection probabilities $\sigma(n)$. $\sigma(n)$ are proportional to $F_t(n)$.

[9] First pioneered by John Holland in the 1960s, genetic algorithms (GA) has been widely adopted by agent-based modelers. Introduction to GA learning can be found in Holland (1992) and Mitchell (1998).

3. Adoption of genetic operators to generate offspring. The offspring of an individual are built from copies of its genotype to which genetic operators are applied.
4. Evaluation of the individual fitnesses $F_t^*(n)$ of the offspring.
5. Generation of P_{t+1} by probabilistically selecting individuals from P_t according to $F_t(n)$ and $F_t^*(n)$.

end

This approach has been adopted by ACE researchers to mimic social learning rule. However, modifications to the standard single-population GA implementation have been proposed. Multi-population GAs have been introduced as coevolutionary computational techniques to extend conventional evolutionary algorithms. A multi-population GA models an ecosystem consisting of two or more species. Multiple species in the ecosystem coevolve, that is, they interact in the environment with each other without exchanging genetic material and thus without the possibility of hybridization, and they independently evolve by implementing a GA. This latter approach is commonly adopted by ACE researchers for implementing individual learning models, where each species is a specific agent endowed with a population of actions, for example, bidding-strategies, evolving according to GA rules.

3.1.1 Social Learning: Single-population Genetic Algorithms (GA)

In the early literature, the adoption of standard GA methods for evolving the economic systems concerned mainly single-population GA. These early attempts (Richter and Sheblé 1998; Lane et al. 2000; Nicolaisen et al. 2000) consider simple electricity market implementing double-auctions with a discriminatory price mechanism based on mid-point pricing. Lane et al. (2000) and Nicolaisen et al. (2000) implement in their market model also grid transmission constraints. The authors consider generation companies (GenCos) with single production-unit.

Richter and Sheblé (1998) is one of the first attempt to adopt GA for studying the bidding behavior of electricity suppliers. The GenCos forecast equilibrium prices according to four distinct techniques, that is, moving average, weighted moving average, exponentially weighted moving average, and linear regression. Each GenCo/chromosome is composed of three parts or genes for determining the offer price and the offer quantity and for choosing the price forecast method. Simulation results suggest that the followed GA approach furnishes reasonable findings.

The papers of Nicolaisen et al. (2000) and Lane et al. (2000) are closely related in their aims and approaches. They both adopt the same EPRI market simulator and they address the same research issue, that is, to study the exertion of market power in different market conditions. They both consider adaptive sellers and buyers. Price is the unique decision variable of market operators, being always quantity in their maximum production or purchase capacity, respectively. Different market scenarios are considered to test for market power. In particular, Lane et al. (2000) consider three market scenarios where sellers differ with respect to market concentrations and marginal costs and where the three buyers are identical. Nicolaisen et al. (2000) consider nine scenarios where both buyers and sellers differ in number and purchase or production capacities, being marginal revenues and costs always equal. In both papers, results are contradictory also with economic theory. The authors suggests that this approach is not suitable, in particular when few agents are considered. They envisage the adoption of individual learning techniques.

3.1.2 Individual Learning: Multipopulation or Coevolutionary Genetic Algorithms (CGA)

Curzon Price (1997) studies Bertrand and Cournot competition, a vertical chain of monopolies, and a simple model of an electricity pool identical to the one considered by von der Fehr and Harbord (1993), by means of co-evolutionary programming. This paper is the first attempt in the electricity market ACE literature to implement a multi-population GA (CGA). In particular, the author considers a simplified market scenario where only two sellers/producers, that is, two-population GA, are competing on prices or quantities and each producer autonomously adapts to the changing environment by implementing a GA where the solutions are his own strategies. Results show that this evolutionary approach is suitable to converge to Nash equilibria in pure strategies in this rather simple economic setting. The author suggests to extend this research by adopting richer evolutionary approaches such as learning classifier systems.

Cau and Anderson (2002) and Cau (2003) propose a wholesale electricity market model similar to the Australian National Electricity Market. Cau (2003) extend the model of Cau and Anderson (2002) by considering both step-wise and piece-wise linear functions as bidding strategies and by studying several market settings with different number of sellers (two and three), with asymmetric and symmetric cost structures and production capacities, and with producers with a portfolio of generating units. An important aspect is the fact that they consider condition-action rules, that is, they consider a mapping from a state (defined by the previous spot market price, the past market demand, and the forecast market demand) to a common set of possible actions (step-wise and piece-wise linear functions). The demand is assumed price-inelastic and uniformly randomly varying between a high and a low level. The aim of the authors is to study the agents ability to achieve tacit collusion in several market settings. Summarily, the author finds that tacit collusive bidding strategies can be learned by the suppliers in the repeated-game framework. This is true for both market bidding formats even with randomness in the environment. In particular, on the demand side, high overall demand, high uncertainty, and low price elasticity facilitate tacit collusion. On the supply side, situations where tacit collusion is easier to achieve are characterized by symmetry in cost and capacity and small hedging contract quantity. Furthermore, the author experimentally confirms that an increasing number of agents makes more difficult, but still possible, tacit collusion to occur in a sustainable way. Finally, the author states that the results are quite analogous to classical supergame theory.

Chen et al. (2006) address the issue of the emergence of tacit collusion in the repeated-game framework by means of a co-evolutionary computation approach. In particular, they consider a simplified model of an electricity market based on the classical Cournot model with three generating units. The authors propose to test their CGA models for convergence towards game-theoretical solutions, that are, the Cournot–Nash equilibrium and the solutions set Pareto-dominating the Cournot–Nash equilibrium to consider also equilibria in infinitely repeated games. In particular, they adopt a standard GA to study the bidding behavior with respect to Cournot–Nash equilibrium, whereas for studying tacit collusion they introduce a more complicated GA technique used for multiobjective optimization problems. Furthermore, the authors adopt two type of demand functions, the inverse linear and the constant elasticity demand functions. Under this respect, the authors aim to demonstrate the ability of their coevolutionary approach for finding Cournot–Nash equilibrium and for solving nonlinear Cournot models (the case with constant elasticity demand function).

Ikeda and Tokinaga (2007) investigate the properties of multi-unit multi-period double-auction mechanisms by a genetic programming (GP) approach, which is an extension of the coevolutionary GA. Each solution/agent in the population is a function (or computer program) composed of arithmetic operations, standard mathematical expressions, if–then type condition, and variables. Standard mutation, crossover, recombination operations are then applied to improve the fitness of the population. Producers are backward-looking adaptive agents selling electricity if they think that the cost to generate electricity is costly than buying electricity. The sellers are endowed with a Cobb–Douglas type production function and a multi-objective fitness function evaluating both profits and capacity utilization rate of their units. Two different demand market settings are then considered, a constant price-inelastic and a randomly time-varying price-inelastic demand scenarios. The authors compare their computational results in both market settings with a perfect competition scenario and furthermore they discuss about the price fluctuations of the simulation results, that is, spikes and volatility, with respect to real price performances. Finally, the authors propose two simplified market cases as control schemes to stabilize the price fluctuations. In particular, in the second case they introduce an exogenous force to control the auction price, and they assume that buyers additionally pay (get) some amount of money from sellers if market prices falls higher (lower) than a reference level, that is, the theoretical equilibrium value. The additional payment schemes work as factors to remove the instability, but simultaneously they produce unavoidable costs for untraded electricity.

3.2 *Individual Learning "Ad-hoc" Models*

3.2.1 Microsimulation Model with No Adaption

Bunn and Martoccia (2005) present a static microsimulation analysis based on an agent-based model of the electricity pool of England and Wales for studying unilateral and collusive market power. They do not consider a multiagent coevolutionary market setting, and the authors realize static scenario simulations by exogenously imposing bidding strategies to each market agent. This paper is not a proper ACE model in the sense that agents do not present autonomy in the decision-making; however, this paper illustrates how agent-based modeling framework can be adopted. The simulations consider a demand profile based on the NGC typical winter day and own estimates of real marginal cost functions. Starting from an empirical observation on generators supply functions that the mark-up is an increasing function of their units' marginal costs, they develop a behavioral model where generating portfolio companies consider to bid-up some of their plants beyond the competitive levels of marginal costs by a particular percentage mark-up value. The simulation results report the change in profits due to different mark-up levels. Then, the authors attempt to estimate the degree of tacit collusion present during the two divesture programs required by the UK Office for Electricity Regulation (OFFER). Results support the thesis that National Power in the early years (owning 50% of total capacity production) exercised unilateral market power and that in the later years, as the market concentration declined, a regime of tacit collusion occurred. The authors further conclude that market concentration measures, such as HHI, do not give a reliable diagnostic aid to the potential exercise of market power in the wholesale electricity sector.

3.2.2 Simple Behavioral Rules

Botterud et al. (2006) study the effect of transmission congestion management on the exertion of market power in electricity markets, in particular focusing on the ability of portfolio generating companies to raise prices beyond competitive levels. They adopt EMCAS software framework, which is a powerful and flexible agent-based simulation platform for electricity market (see Sect. 4.1). However, they do not exploit the complex agent architecture, but they consider simple adaptive behavioral rules to replicate the bidding strategy of generation companies. Only three strategies are considered. The first is the production cost bidding strategy, so that generation companies act as pure price-taker bidding the marginal production cost of their plants. The second regards physical withholding strategies where generation companies attempt to raise the market price by withholding units during hours when the expected system capacity is low. Random outages are also considered in the simulations. Finally, the third strategy implements an economic withholding strategy, that is, generation companies try to probe their influence on market prices by increasing (decreasing) the bid price with a specific percentage for the same unit in the same hour for the following day if the bid for a unit in a certain hour was accepted (not accepted). The authors test two pricing mechanisms a locational marginal pricing and a system marginal pricing (uniform price mechanisms) for two sequential markets comprising day-ahead (DA) and real-time (RT) sessions. A case study for an 11-node test power system is presented. An aggregate price-inelastic demand and eight generation companies are considered. Each GenCo owns three identical thermal plants, that is, one base load coal plant (CO), one combined cycle plant (CC) to cover intermediary load, and one gas turbine (GT) peaking unit. Three scenarios are then simulated: a base case, a physical withholding case, and an economic withholding case. Results show that the dispatch of the system does not depend on the market pricing mechanism. Furthermore, the results show that unilateral market power is exercised under both pricing mechanisms, even if locational marginal pricing scheme exhibits the largest changes in consumer costs and generation company profits.

3.2.3 Comparisons Among Standard Multiagent Learning and Ad-hoc Learning Models

A group of two researchers at the Massachusetts Institute of Technology repeatedly investigated (Visudhiphan and Ilic 1999, 2001, 2002; Visudhiphan 2003) the opportunity to adopt different agent-based models for studying market efficiency and market power issues in wholesale electricity markets. In their research activity, these researchers have compared own learning models based on electricity market considerations with learning models inspired by the multiagent learning literature.

In their first paper, Visudhiphan and Ilic (1999) consider a repeated day-ahead and hour-ahead spot markets adopting a uniform double-auction. Three generators submit bids as linear supply or single step supply functions. Pursuing their profit maximization strategy, they implement a derivative follower strategy (Greenwald et al. 1999) to learn how to adjust to the market conditions in near-real time. The agents exploit the available information about the past market clearing prices and their own marginal-cost functions to adjust the chosen strategy. On the demand side, time varying price-inelastic and -elastic loads are analyzed. The authors conclude that the market clearing price in the hour-ahead market model is lower than the day-ahead model and confirm that generators exert more market power facing price-inelastic demand, that is, prices are significantly higher. In Visudhiphan and Ilic (2001), they propose an own learning model for studying market power where agents can strategically withhold capacity. The algorithm reproduces a two-step decision process for setting bid quantity and price, respectively. The agents strategically withhold capacity if their expected profit increases. The bidding price strategy is richer and is decided on the basis of past market performances. The agent stores information about past market prices with respect to predefined load ranges and consider if market outcomes are a result of strategic or competitive behavior. Six possible price strategies for setting the marginal-unit bidding price, conditioned on such information, are available. Each strategy corresponds to a different statistical estimation performed on historic prices, for example, maximum, minimum, or mean price, or the sum of weighted prices. The authors conclude that the strategic outcomes exhibit prices higher than competitive ones for different level of demand and for both market settings of available capacity considered. Finally, Visudhiphan (2003), in is PhD thesis, extends previous works by considering further adaptive learning models and provides a more detailed explanation of the agent bidding formulation. His aim is to test market efficiency for two price mechanisms commonly adopted in the spot electricity market, that is, discriminatory and uniform double-auction. In particular, the author considers three learning algorithms tested with different parameters to investigate the generators' bidding behaviors. In addition to an own learning model similar to the one proposed in Visudhiphan and Ilic (2001), the author proposes two learning models derived from the multi-agent learning literature. In particular, the first model is inspired by the work of Auer et al. (2002), and three distinct implementations of such learning model are considered. These algorithms are based on the assumption that the agent knows the number of actions and the rewards of selected actions in previous trials. The last learning model considered is a simple reinforcement learning algorithm 3.3 that maintains the balancing between exploration and extraction in the action selection by implementing a softmax action selection using a Boltzmann distribution (Sutton and Barto 1998). They conclude that learning algorithms significantly affect simulated market results, in any case discriminatory price mechanism tends to exhibit higher market price, confirming other studies such as Bower and Bunn (2001a). Finally, discussing about the empirical validation issue, they highlight that, to carry out a correct validation, many important data such as marginal cost functions, bilateral contract obligations, plant outages are not publicly available and thus render difficult the validation procedure.

3.2.4 Model Based Upon Supply Function Optimization

Professor D.W. Bunn at the London Business School (LBS) has been very active in this research domain. He has proposed several ACE studies where models tailored on specific market assumptions have been adopted for studying several research issues about the England and Wales wholesale electricity market. In the following, we consider two papers (Day and Bunn 2001; Bunn and Day 2009), which share the same behavioral modeling approach based on a supply-function optimization heuristic. Both papers consider adaptive portfolio generating companies which, as profit maximizers, seek to optimize both their daily profits and their long-term financial contracts, typically contracts for differences (CFD). In the daily market session, they are modeled to submit piecewise linear supply functions. All such strategies are built on the same discrete price grid and thus differ only on quantity value assignments. The short-run marginal costs have been estimated on real data. Their strategic reasoning is based upon the conjecture that their competitors will submit the same supply functions as they did in the previous day. The optimization routine is conceived to modify iteration-after-iteration just one supply-function relative to only one generating unit, that is, the unit that increase the most objective function. The authors then study several market settings with a price-elastic demand. Both papers are theoretically validated in the sense that a simplified scenario where three symmetric firms are considered is run and the theoretical continuous supply-function equilibria is evaluated. Simulation results show, on average, a good fit with the theoretical solution. However, each simulation exhibits no convergence to a stationary supply function, instead a repeated cycle behavior is highlighted. These findings motivate the authors to extend such models to study more realistic market settings where asymmetric firms hedge market risk with contract cover. In particular, Day and Bunn (2001) study on the second divestiture plan of generating assets imposed by the electricity regulatory authority in 1999. The authors show that also this second round of divesture (40%) might result insufficient because results show prices substantially (about 20%) above short-run marginal costs for the proposed divestiture plan. In general, for the different divestiture plan scenarios tested (25% and 50%), the average bid significantly decreases. So the authors conclude that probably a further period of regulatory price management will be required.

Bunn and Day (2009) focus on market power issues. The authors remark that the England and Wales electricity market is not characterized by perfect competition, but on the contrary is a daily profit-maximizing oligopoly. Thus, they suggest and computationally prove that it is not correct to assess market power exertion by adopting marginal cost as a benchmark, that is, perfect competition. The results of their several tested market settings show that the simulated supply-functions lie always above marginal cost functions. The authors conclude that this is evidence of the market structures and thereby of the strategic opportunities offered to the market participants, and no collusive aspects are modeled. Thus, the authors propose the outcomes of their simulations as realistic baselines for the identification of market power abuse. These results compared to real market supply functions according to such hypothesis show that a clear market power abuse occur in the England and Wales wholesale electricity market.

Minoia et al. (2004) propose an original model for wholesale electricity markets providing incentives for transmission investments. The transmission owner is supposed to strategically compete in the spot market, thus ISO objective function takes into account not only the power producers' bids but also the transmission owners' bids. In this model, the authors consider a simplified two node grid, two constant price-inelastic loads, and asymmetric single-unit generators, which have fixed marginal cost. Each generator submits the minimum price willing to be produced and the transmission owner submits the price she asks per power quantity (MW) flowing through the line. The total expenditure is a function of generators' produced power quantity and the power quantity flowing into the line. The market clearing mechanism is built with the aim to minimize the total expenditure function subject to some technological constraints, that is, the balancing between supply and demand, generators' maximum capacities, and line constraints. Throughout the simulation, each agent maximizes its profits conjecturing that all other agents keep fixed their previous strategies. The authors discuss four different market settings with two and five generators and two different transmission capacities. They compare simulation results with a standard locational marginal price settlement rule. The results show that the market settlement rules remarkably affect generators, transmission owner, loads, and ISO rewards. However, the authors are not able to establish which settlement rule is more efficient.

3.3 Reinforcement Learning (RL) Models

The majority of wholesale electricity market ACE models adopt standard RL algorithms, such as classical Roth and Erev (Roth and Erev 1995; Erev and Roth 1998) and Q-learning (Watkins and Dayan 1992) algorithms or modifications (see Table 2).[10] These algorithms, and mainly all reinforcement learning models based on model-free approaches (Shoham et al. 2007), share some common modeling features. First, these algorithms assume that individual agents are endowed with minimal information about the evolution of the game, that is, they record only their own past history of plays and their associate instantaneous rewards. Moreover, each agent has no prior knowledge about the game structure or other players. Second, they assume no communication capabilities among the agents. The only permitted communication is from the environment/system to the agent. Third, these algorithms implement backward-looking stimulus and response learning, that is, they learn from past experience how to best react to the occurrence of a specific event without relying on forward-looking reasoning. Finally, these algorithms do not take into account opponents' strategies, and agents learn a strategy that does well against the opponents without considering the path of plays of their opponents.

In general, a standard feature of all RL algorithms consists on implementing a sort of exploitation and exploration mechanism by relying on a specific action selection policy, which map the strenght/Q-value/propensity values $S_t^i(a^i)$ at time t to a probability distribution function over actions $\pi_t^i(a^i)$. Standard models are a proportional rule (see (1)) and an exponential rule based on Gibbs–Boltzmann distribution (see (2)).

$$\pi_t^i(a^i) = \frac{S_t^i(a^i)}{\sum_{a^i} S_t^i(a^i))}. \tag{1}$$

$$\pi_t^i(a^i) = \frac{e^{\lambda S_t^i(a^i)}}{\sum_{a^i} e^{\lambda S_t^i(a^i)})}. \tag{2}$$

The idea is to increase the probability of selecting a specific action after probing the economic environment, that is, changing the relative values among the strenghts/

[10] An instructive special issue about "Foundations of multi-agent learning"(Vohra and Wellman 2007) has been published by the journal *Artificial Intelligence*. The special issue has been devoted to open a debate on the multiagent learning (MAL) agenda by bringing joint contributions of "machine learners" and "economists" to highlight different viewpoints and experiences in the field. The starting point of the discussion is the paper by Shoham et al. (2007), where they attempt to pinpoint the goal of the research on MAL and the properties of the online learning problem. The authoritative contribution of Fudenberg and Levine (2007) underlines that the theory of mechanism design can well benefit from development of computational techniques and with respect to learning models envisages: "these models may be useful for giving people advice about how to play in games, and they may also help us make better predictions. That is because learning rules for games have evolved over a long period of time, and there is some reason to think that rules that are good rules from a prescriptive point of view may in fact be good from a descriptive point of view" (Fudenberg and Levine 2007).

Q-values/propensities values. A widely adopted feature of the exponential model is that the λ parameter can be defined to increase with time. Thereby, as the learning phase proceeds, the action selection model becomes more responsive to propensity values differences, and so agents are more and more likely to select better than worse choices. At the final simulations stages, the very high values reached by the λ parameter lead to a peaked probability distribution function on the strategy with the highest propensity, that is, the best strategy.

A further method, commonly adopted for QL implementation, is the ϵ-greedy policy. It selects with probability $1-\epsilon$ the action that maximizes its expected reward and it chooses with probability ϵ one of the alternative actions. The ϵ parameter may vary with time.

3.3.1 Naive Reinforcement Learning Models

Some early models adopted own learning approaches based on elementary RL mechanisms. In the following we report some of them.

A second research activity at LBS has concerned the adoption of learning models to study electricity spot market. A common ACE modeling framework has been adopted for a series of papers (Bower and Bunn 2000, 2001b; Bower et al. 2001). In the first two papers, the authors address the issue of market efficiency by comparing different market mechanisms: pool day-ahead daily uniform price, pool day-ahead daily discriminatory (pay as bid) , bilateral hourly uniform price, and bilateral hourly discriminatory. Pool market determines a single price for each plant for a whole day, bilateral market 24 separate hourly prices for each plant. On the other hand, in the last paper, the authors adopt the same ACE model for studying the German electricity market focusing on the impact of different mergers operations proposed at that time. All papers consider a fixed and price-elastic aggregate demand and portfolio generating companies. The objectives of a generating company are twofold: first, to obtain at least the target rate of utilization for their whole plant portfolio, and second, to achieve a higher profit on their own plant portfolio, than for the previous trading day. Each GenCo has a set of four rules/actions to choose from conditioned to past market performances, that is, the achievement of target rate of utilization and the increase or decrease of profits. These actions implement the decision on diminishing, keeping or increasing prices for all or part of the power plants of the GenCo.

Bower and Bunn (2000) conclude that prices are higher under discriminatory mechanism for both pool and bilateral models. In particular, uniform settlement simulations show that most low cost plants bid at close to zero, whereas for discriminatory settlement they learn to bid prices close to the market clearing prices, intuitively for maximizing their profits. Bilateral models exhibit higher prices than pool ones. Another finding is that GenCo with few power plants perform better in the uniform than in the discriminatory settlement. The authors argue that under the uniform setting all GenCos receive the same information, that is, the unique market clearing price, whereas in the pay as bid mechanism there is also informational disadvantage among GenCos on the basis of their market share, that is, GenCos owning more power-plants receive more price signals from the market. Bower and Bunn (2001b) enrich this previous paper with a theoretical validation of the simulation model by means of classical model of perfect competition, monopoly, and duopoly. On average, the simulation results of perfect competition and monopoly are in great accordance with the unique equilibrium solutions, that is, same price for both settlement mechanisms. On the contrary, in the duopolistic case, the mean simulated market clearing price for the uniform settlement mechanism is equal with the theoretical value, but for the discriminatory settlement mechanisms is significantly lower than the theoretical value.

Finally, Bower et al. (2001) consider only the pay as bid mechanisms to study the German electricity market. In particular, they focus on potential mergers solutions. In particular, six market settings are studied for different demand level and different plant utilization rate objectives: marginal cost, no mergers, two mergers, four mergers, two mergers, and the closing of peaking oil plants by the two mergers, two mergers, and the closing of all nuclear plants. The authors conclude that prices raise considerably as an effect of mergers for all scenarios. In particular, for the two scenarios, considering the closing of power plants, the raise in prices is more significant.

A third ACE model has been proposed at LBS. Bunn and Oliveira (2001) and Bunn and Oliveira (2003) describe the model and adopt for studying pricing and strategic behavior of the proposed NETA (new electricity trading arrangements) for the UK electricity market and the presence of market power abuse by two specific generation companies, respectively. The ACE model is an extension of previous model in several directions. First, it actively models the demand side, by considering suppliers, that is, agents purchasing from wholesale market to "supply" end-use customers. Second, it models the interactions between two different markets the bilateral market (PX) and the balancing mechanism (BM) as sequential one-shot markets. Both are modeled in a simplified way, as single call markets, that is, sealed-bid multiple-unit auctions. As far as concerns market participants, both suppliers and portfolio generating companies are modeled. Suppliers are characterized by the following parameters: a retail market share, a BM exposure (percentage of forecast demand that they intend to purchase in the PX), retail price, mean average prediction error in forecasting the contracted load, and a search propensity parameter (how fast or slow the agents change their strategy with experience). Generators are characterized by the following parameters: plants portfolio, plant cycles, installed capacity, plant availability corresponding to outage rates for each plants, BM exposure, and search propensity parameter. Both types of agent seek to maximize total daily profits and to minimize the difference between its fixed objective for the BM exposure and the actual BM exposure. For PX, agents learn to set mark-ups relative to the PX price in the previous day, and for BM they learn to set mark-ups relative to the PX price in the same day. They update the propensities for each mark-up level considered according to a reinforcement learning model. Finally, some lower bounds of rationality through operational rules are considered, for example, peak plants never offer prices below their marginal costs.

In Bunn and Oliveira (2001), the authors model only a typical day, and they consider a realistic market scenario based on power plant data of the UK electricity market. Simulation results confirm the intuitive behavior that system buy price and sell price in the balancing mechanism are considerably distant and the average bilateral price is centrally located between them. In Bunn and Oliveira (2003), the authors study whether two specific generation companies in the UK electricity market can exert market power. They first propose a simplified model of a one-shot Bertrand oligopoly model with constraints to assess basic model performances. Then a realistic scenario of the UK electricity market in 2000 is studied. The authors find that only one generator can raise PX prices unilaterally, whereas BM prices are quite robust against manipulation by the two generating companies. The authors conclude that learning in repeated games is thus a suitable modeling framework for studying market behavior such as tacit collusion.

A fourth ACE model is developed at LBS to investigate how market performance depends on the different technological types of plant owned by the generators, and whether, through the strategic adaptation of their power plant portfolios, there is a tendency for the market to evolve into concentrations of specialized or diversified companies. Bunn and Oliveira (2007) consider a sequential market game. The first stage is the plant trading game where generators trade power generation assets among themselves. The second stage is the electricity market where generating companies trade electricity and is modeled as a Cournot game. In particular, two electricity market mechanisms are compared: a single-clearing (power pool) and a multi-clearing (bilateral market) mechanism. In the latter also baseload, shoulder, and peakload plants are traded in three different markets. The authors simulate two realistic scenarios using data from the England and the Wales electricity market and study both under the two market clearing mechanisms. In the first market setting (specialization scenario), three players own baseload, shoulder, and peakload plants, respectively. The pool model evolves towards a monopoly (maximum concentration), whereas in the bilateral market two companies (baseload and shoulder generating companies) at the end own equally most of the capacity. In the second market setting (diversification scenario), the three players are similar in their portfolio. In this case, the market structure does not converge on the monopolistic solution for both market types, and in the pool mechanism the structure converged to a more concentrated configuration than in the multi-clearing mechanism. The authors conclude that if the industry is at a state of great diversification it will tend to remain so, independently of the market-clearing mechanism.

3.3.2 Classical Reinforcement Learning Models

In the following, two reinforcement learning algorithms are described, which are far the most adopted by the wholesale electricity market literature. A list of agent-based model implementing such algorithms are also discussed.

Roth and Erev Reinforcement Learning Algorithm (RE). The original algorithm formulation is in Roth and Erev (1995) and Erev and Roth (1998). In this model, three psychological aspects of human learning in the context of decision-making behavior are considered: the power law of practice, that is, learning curves are initially steep and tend to progressively flatten out, the recency effect, that is, forgetting effect, and an experimentation effect, that is, not only experimented action but also similar strategies are reinforced. Nicolaisen et al. (2001) studied market power occurrence in the context of discriminatory auction, proposing some modifications to the original algorithm to play a game with zero and negative payoffs. In particular, they compare simulation results with the previous paper (Nicolaisen et al. 2000) adopting GA. The modified formulation is: for each strategy $a_i \in \mathcal{A}_i$, a propensity $S_{i,t}(a_i)$ is defined. At every round t, propensities $S_{i,t-1}(a_i)$ are updated according to a new set of propensities $S_{i,t}(a_i)$. The following formula holds:

$$S_{i,t}(a_i) = (1-r) \cdot S_{i,t-1}(a_i) + E_{i,t}(a_i) \tag{3}$$

where $r \in [0,1]$ is the recency parameters, which contributes to exponentially decrease the effect of past results. The second term of (3) is called experimentation function.

$$E_{i,t}(a_i) = \begin{cases} \Pi_{i,t}(\hat{a}_i) \cdot (1-e) & a_i = \hat{a}_i \\ S_{i,t-1}(a_i) \cdot \frac{e}{n-1} & a_i \neq \hat{a}_i; \end{cases} \tag{4}$$

where $e \in [0,1]$ is an experimentation parameter, which assigns different weight between played action and nonplayed actions and n is the number of sellers.

Propensities are then normalized to determine the mixed strategy or action selection policy $\pi_{i,t+1}(a_i)$ for next auction round $t+1$. The original formulation (Roth and Erev 1995) considered a proportional action selection model 1, but it has also been adopted for the exponential model 2 (Nicolaisen et al. 2001).

The group led by Prof. Tesfatsion at Iowa state has repeatedly considered this algorithm in its works (Nicolaisen et al. 2001; Koesrindartoto 2002; Sun and Tesfatsion 2007; Li et al. 2008; Somani and Tesfatsion 2008). The first two papers (Nicolaisen et al. 2001; Koesrindartoto 2002) aim to assess the effectiveness of the agent-based model proposed to study market efficiency of a wholesale electricity market with a discriminatory double-auction. The authors conclude that the approach provide useful insights into the study of market efficiency; in particular, the second paper shows that changes in the learning parameter have a substantial systematic effect on market efficiency. The last three papers are related to the large scale agent-based simulator project AMES (see 4.1), which is still in its development phase. They all describe the software framework and present simulation results for the same test case with a five node transmission grid and five learning single-unit generation companies. The market model considers a day-ahead market with locational marginal pricing. Two different scenarios are always simulated and compared, that is, a competitive scenario where the five generators submit their true linear marginal costs and a strategic scenario where the generators submit "reported" linear marginal cost functions. In the work of Sun and Tesfatsion (2007), the most detailed description of the features of AMES simulator is provided and simulation results are provided to assess the effectiveness of the agent-based model. Li et al. (2008) investigate how demand-bid price sensitivity, supply offer price caps, and generator learning affect dynamic wholesale power market performance. They discuss simulation results with respect to six different performance measures, for example, average locational marginal price and average Lerner index. They conclude that generators' learning affects significantly market outcomes and that the presence of a binding price cap level affects market price spiking and volatility, either increasing or decreasing the effect. Finally, Somani and Tesfatsion (2008) aim to study market efficiency under transmission congestions and to detect the exertion of market power by producers. They adopt five market performance indexes: the Herfindahl–Hirschman Index, the Lerner Index, the Residual Supply Index, the Relative Market Advantage Index, and the Operational Efficiency Index. A detailed analysis of simulation results is provided for each of the previous indexes to highlight shortcomings or virtues.

Another group of research very active in this research domain is leaded by Prof. Veit at the University of Mannheim in Germany. In two papers (Veit et al. 2006; Weidlich and Veit 2006), they study wholesale electricity markets focusing on two-settlement market mechanisms, a day-ahead market and a real-time balancing market. Generators act strategically on the day-ahead electricity market and on the balancing power by bidding both prices and quantities to maximize their individual profits. In Weidlich and Veit (2006), the day-ahead market is a uniform double-auction receiving bids by each seller as a couple of price and quantity, whereas the balancing market for managing minute reserve is played in two stages. In the first stage, occurring one day ahead, TSO selects the power plants that are held in reserve, in the second stage, both pay as bid and uniform double-auctions are considered. The learning model has been subdivided into two separate learning task, one for each market, but the reinforcement of each chosen action in both markets comprises the profit achieved on the associated market, but it also includes opportunity costs, which take into account profits potentially achieved in the other market. Four market scenarios are studied for different market sequence combinations: first day-ahead, second balancing with uniform; first day-ahead, second balancing with pay as bid; first balancing with uniform, second day-ahead; and first balancing with pay as bid, second day-ahead. Results show that prices attain a higher (lower) level if the day-ahead market is cleared after (before) the balancing power market, and average prices are higher under uniform price than under pay-as-bid, although agents bid at higher prices under pay-as-bid. In Veit et al. (2006), the two-settlement electricity market is modeled differently, based on the Allaz (1992) work. Thus, they define a day-ahead market as the forward market and they model it as a classical Cournot model. The real-time market is called spot market and is modeled with a locational marginal pricing. The generators are portfolio traders and they learn to strategically bid on both markets separately. In particular, the propensities for forward bid quantities are updated on the basis of the generators total profit, that is, these actions are likely updated enclosing potential future profits earned in the spot market. Action propensities for the spot market are updated only on the basis of their achieved spot market profits. The authors simulate the Belgian electricity network with five different demand scenarios for the spot market. The strategic behavior of two portfolio generating companies is studied. Simulation results show that the access to the forward market leads to more competitive behaviors of the suppliers in the spot market, and thus to lower spot energy prices. Furthermore, the authors computationally prove that the introduction of a forward market creates incentives for suppliers to engage in forward contracts. Such incentives determine lower energy prices and smaller price volatility as compared to a system without the forward market.

Bin et al. (2004) investigate pool-based electricity market characteristic under different settlement mechanisms, uniform, discriminatory (pay as bid), and the Electricity Value Equivalent (adopted in China) methods with portfolio generation companies bidding prices and demand in price-inelastic. The authors conclude that the EVE pricing method has many market characteristics better than other pricing methods, because it exists little room for a power supplier to raise the market price, whereas uniform and discriminatory settlements are not able to restrain generators' market power.

Banal-Estanol and Rupérez-Micola (2007) build an agent-based model to study how the diversification of electricity generation portfolios affects wholesale prices. The authors study a duopolistic competition with a uniform price double-auction, where the demand is price-inelastic and the portfolio generators submit single-price bids for each unit. The learning model is the classical Roth and Erev except for the inclusion of a mechanism called "extinction in finite time," where actions whose probability value falls below a certain threshold are removed from the action space. Simulation results show that technological diversification often leads to lower market prices. They test for different demand to supply ratio and they identify for each ratio value a diversification breaking point. Thus, the authors conclude that, up to the breaking point, more intense competition due to higher diversification always leads to lower prices. However, in high-demand cases, further diversification leads to higher prices, but prices remain lower or equal to those under perfect specialization.

Finally, Hailu and Thoyer (2007) aim to compare the performances of three common auction formats, that is, uniform, discriminatory (pay as bid), and generalized Vickrey auctions by means of an agent-based model of an electricity market. Sellers submit linear continuous bid supply functions and they employ the classical Roth and Erev algorithm to update their bidding strategies. Four different market scenarios are studied, where eight suppliers varies from a homogenous to a completely differentiated setting with respect to maximum capacity of production and marginal-cost functions. Simulation results enable to conclude that bidding behavior cannot be completely characterized by auction format because heterogeneity in cost structure and capacity may play an important role. In general, the authors conclude that in most cases the discriminatory auction is the most expensive format and Vickrey is the least expensive.

Q-Learning (QL). The QL algorithm was originally formulated by Watkins (1989) in his PhD thesis. Since then, QL algorithm is a popular algorithm in computer science. It presents a temporal-difference mechanism (Sutton and Barto 1998), which derives from considering the intertemporal discounted sum of expected rewards as utility measure, originally conceived to solve model-free dynamic programming problems in the context of single-agent. Unlike Roth and Erev RL algorithm, QL updates only the Q-value (propensity value for RE algorithm) of the last played action and implements state-action decision rules. In the standard modeling framework, states are identical and common knowledge among the agents.

The ith agent takes an action a_i in state s at time t and obtains a reward $R_{i,t}(a_i, a_{-i}, s)$, depending also on the actions played by the opponents a_{-i}. Both action and state spaces must be discrete sets. Then, it performs an update of the Q-function $Q_i(a_i, s)$ according to the following recursive formula:

$$Q_{i,t}(a_i, s) = \begin{cases} (1-\alpha_t) Q_{i,t-1}(a_i, s) + \alpha_t [R_{i,t} + \gamma \max_a Q_{i,t-1}(a, s')] \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } a_i = a_{i,t}, \\ Q_{i,t-1}(a_i, s) \qquad\qquad\qquad\qquad\qquad\qquad\quad \text{otherwise;} \end{cases} \tag{5}$$

where $0 \leq \gamma < 1$ is the discount factor of the discounted sum of expected rewards and $0 < \alpha_t \leq 1$ is the time-varying learning rate. γ determines the importance of future rewards, whereas α_t determines to what extent the newly acquired information will override the old information. The most common action selection policy is the ϵ-greedy selection rule (see Par. 3.3).

Several papers have adopted QL algorithm or extensions (Xiong et al. 2004; Krause et al. 2006; Krause and Andersson 2006; Naghibi-Sistani et al. 2006; Nanduri and Das 2007; Guerci et al. 2008a,b; Weidlich and Veit 2008a).

In particular, some papers (Krause et al. 2006; Krause and Andersson 2006; Guerci et al. 2008a,b) adopt the formulation with no state representation or equivalently with a single state.

Krause and Andersson (2006) compare different congestion management schemes, that is, locational marginal pricing (LMP), market splitting, and flow-based market coupling. In particular, the authors investigate two market regimes, a perfect competition and a strategic case, where single-unit producers bid linear cost functions and they face a price-elastic demand. To compare simulation results, the authors focus on some performance measures such as producer, consumer and overall surplus, congestion cost, and maximum and minimum nodal prices. The authors conclude that in both perfect competition and oligopolistic cases, strategic locational marginal pricing exhibits the highest overall welfare, followed by market splitting. In a second paper, Krause et al. (2006) adopt the same learning model to study network-constrained electricity market. Generation companies' action space is a discrete grid of intercept value of a linear cost function that is very shallow, that is, markup values. The demand is assumed price-inelastic. They aim of the work is to validate theoretically learning models by comparing the convergence properties of the coevolving market environment to Nash equilibria. Two case studies are considered, the first one where a unique equilibria is present and the second one where two equilibria are present. In the former situation, simulation results show that, in a robust way with respect to different parameters, there is high likelihood for the Q-learning algorithm to converge to the strategic solution, whereas in the latter a cyclic behaviors is observed.

Guerci et al. (2008a) address a similar issue in an extension of a previous paper (Guerci et al. 2007). The authors compare the discriminatory (pay as bid) and uniform market settlement rules with respect to two game-theoretical solution concepts, that is, Nash equilibria in pure strategies and Pareto optima. The authors aim to investigate the convergence to tacit collusive behavior by considering the QL algorithm, which implements the discounted sum of expected rewards. On this purpose, they also compare QL algorithm simulation results with a further RL algorithm proposed originally by Marimon and McGrattan (1995), which is supposed to converge to Nash equilibria. Three market settings are studied for each pricing mechanism, two duopolistic competition cases with low and high price-inelastic demand levels and one tripolistic case with the same low demand value. All these market settings are characterized by a multiplicity of Nash equilibria (up to 177 in the tripoly uniform case). The difference between payments to suppliers and total generation costs are estimated so as to measure the degree of market inefficiency. Results point out that collusive behaviors are penalized by the discriminatory auction mechanism in low demand scenarios, whereas in a high demand scenario the difference appears to be negligible.

In the following papers, a state space is defined and consequently agents learn action-state decision rules. In particular, Xiong et al. (2004) compare market performances of the two standard discriminatory (pay as bid) and uniform auctions under a price-inelastic and -elastic demand scenarios, respectively. Single-unit producers submit a price bid and they learn from experience with a classical Q-learning adopting as discrete state space a grid of market prices. Simulation results show that discriminatory prices are lower and less volatile than those in the uniform price auction, and furthermore discriminatory is less affected by demand side response. Naghibi-Sistani et al. (2006) consider a rather simplified pool-based electricity market settings where two single-unit producers compete in a uniform double-auction facing a price-inelastic demand. The state space is composed by two states relative to two different reference value of fuel costs. Simulation results show to converge to the unique strategic solution and to be robust to parameter variation. Nanduri and Das (2007) aim to investigate market power and market concentration issues in a network-constrained day-ahead wholesale electricity markets. Three different pricing settlement rules are considered discriminatory, uniform, and second-price uniform. The Herfindahl-Hirschmann and Lerner indexes and two own indexes, quantity modulated price index (QMPI), and revenue-based market power index (RMPI) are adopted to measure the exertion of market power. A state of the system is defined as the vector of loads and prices at each bus realized in the last auction round. A 12-bus electric power network with eight generators and price-inelastic loads is simulated. High and low demand scenarios are studied. The authors conclude that in most of the cases, discriminatory auction determines highest average prices followed by the uniform and second price uniform auctions, confirming computational results of other paper (Visudhiphan and Ilic 2001; Bower and Bunn 2000). In any case, generators in a discriminatory auction tend to bid higher as the network load increases; consequently, the Lerner Index and the QMPI values show that a consistently high bid markup is exhibited under this settlement mechanism. In the uniform and second price uniform auctions, the bids do not change appreciably.

Weidlich and Veit (2008a) is an original paper studying three interrelated markets reproducing the German electricity market: a day-ahead electricity market with uniform price, a market for balancing power, which is cleared before the day-ahead market, and a carbon exchange for CO_2 emission allowances modeled as a uniform double-auction. Agents learn to bid separately their generating capacities for the day-ahead and for the balancing power market. The prices for carbon dioxide emission allowances are also included into the reinforcement process as opportunity costs. The state space for each agent is represented by three levels of bid prices, that is, low, intermediate, and high with respect to maximum admissible bid price, and three levels of trading success defined as marginal, intra-marginal, and extra-marginal. The reward function for each market session used to reinforce specific actions considers the consequence of actions played on other markets as opportunity costs. A classical QL algorithm is thus implemented with ϵ-greedy action selection rule. The authors empirically validate at macrolevel simulation results by comparing prices with real market prices observed in the German electricity market in 2006. Simulated prices on both day-ahead and balancing market are in accordance to real-world prices in many months of the year 2006. Besides, influence of CO_2 emissions trading on electricity prices is comparable to observations from real-world markets.

Some papers have adopted variants of the original QL algorithm. Bakirtzis and Tellidou (2006) and Tellidou and Bakirtzis (2007) adopt the SA-Q learning formulation, which apply the Metropolis criterion, used in the Simulated Annealing (SA) algorithm, to determine the action-selection strategy instead of the classical ϵ-greedy rule. The former paper considers uniform and pay as bid pricing mechanisms to study the bidding behaviors of single-unit suppliers. The state space for each agent is represented by the previous market price and the action space by a grid of bid prices. Four market scenarios are studied under different market concentration conditions and number of players. The simulation results confirm the fact that the prices under uniform pricing are lower compared to the ones under pay-as-bid. This occurs if no supplier is market dominant; otherwise, the difference in prices is negligible due to the exertion of market power in both pricing settlement rules. Tellidou and Bakirtzis (2007) investigate tacit collusion in electricity spot markets using location marginal pricing. Unlike previous paper, they use no space representation and the action space is two-dimensional, that is, offer quantity and price. The authors aim to consider capacity withholding strategies. A simple two-node system is used for studying two market settings: a duopolistic test case and a case with eight asymmetric generators. Simulation results show that under high market concentration suppliers may learn to adopt capacity withholding strategies and that also in competitive settings, that is, eight generators, tacit collusion behaviors may arise.

3.4 Learning Classifier Systems (LCS)

The last class of agent-based adaptive behavioral models considered refers to Learning Classifier Systems (LCSs).[11] LCSs are complex adaptive[12] behavioral models linking GA and RL models. The basic element of such systems are classifiers defined as condition-action rules or state-action rules in a Markov-decision process framework. The algorithm's goal is to select the best classifier within a ruleset or action-space in those circumstances and in the future by altering the likelihood of taking that action according to past experience. They can be categorized as backward-looking learning models, even if anticipatory LCSs have been devised (Butz 2002). The standard formulation is to adopt GA to select the best condition-action rules by exploiting mutation and crossover operators. Moreover, fitness is based on future performance according to a reinforcement learning mechanism, that is, the rule that receives the most reward from the system's environment will be more reinforced, thus resulting in a higher fitness and consequently his "offspring" will likely be larger.

Bagnall and Smith (2000) and Bagnall and Smith (2005) aim to increase market realism by enhancing the modeling of the agent architecture. Their work has been widely presented and discussed in several papers (Bagnall 1999, 2000a,b, 2004; Bagnall and Smith 2005). In any case, Bagnall and Smith (2005) present aims and results of all previous works and extends them, enclosing also a detailed description of the common agent and market architectures. Their long-term project objectives which are clearly stated in their last paper are as follows: first to study if the agents are capable of learning real-world behavior, second to test for alternative market structures, and finally to find evidences for the evolution of cooperation. The market model focuses on a simplified pre-NETA UK electricity market and in particular on studying the supply side where 21 generator agents are considered, classified as one of the four types: nuclear, coal, gas, or oil/gas turbine. Generators differ also for fixed cost, start-up cost, and generation cost. Agents are grouped into three different groups: constrained on (required to run), or constrained off (forced not to generate), and unconstrained. In particular, three fixed groups of seven units are considered. The agents implement a multiobjective adaptive agent architecture based on two related objectives, that is, maximize profits avoiding losses. Both goals are represented by an own learning classifier system within the agent. The learning task is to set the bid price. An important part of the agent architecture is the state representation. Every day, each agent receives a common message from the environment to properly identify the state of the economic system. The environment state is randomly drawn every day. The state is represented by three

[11] Invented in 1975 by John Holland (Holland 1975), LCSs are less famous than GA though; GAs were originally invented as a sub-part of LCSs. At the origin of Hollands work, LCSs were seen as a model of the emergence of cognitive abilities, thanks to adaptive mechanisms, particularly evolutionary processes.

[12] In the sense that their ability to choose the best action improves with experience

variables, related to electricity demand level (four levels are considered, summer weekend, summer weekday, winter weekend, and winter weekday), the nature of the constraints, and information concerning capacity premium payments (an additional payment mechanism used in the UK market as an incentive to bid low in times of high demand).

In particular, Bagnall (2000b) examine how the agents react under alternative market mechanisms (uniform and discriminatory price mechanisms) and under what conditions cooperation is more likely to occur. The author also reports and discusses results obtained within previous works (Bagnall 1999, 2000a). Findings shows that the uniform mechanism determines higher total cost of generation, but agents tend to bid higher under the discriminatory one. Finally, they show that cooperation can emerge even if it is difficult to sustain the cooperation pattern for longer period. They impute this to the greedy behavior of agents always striving to find new ways of acting in the environment.

Bagnall and Smith (2005) address all three original research issues. As regards the first question of interest, the authors show that nuclear units bid at or close to zero and are fairly unresponsive to demand, gas and coal units bid close to the level required for profitability by increasing generally their bid-prices in times of high demand, and finally oil/GT units are bidding high to capture peak generation. They conclude that the agents' behavior broadly corresponds to real-word strategies. As far as concerns the comparison of alternatives market structures, the authors confirm previous findings that, on average, pay-as-bid price mechanism determines for each type of agent to bid higher than in the uniform one. Furthermore, they show that uniform settlement method is more expensive overall than payment at bid price even if the nuclear units perform worse while coal units perform better. Finally, they discuss about the third research issue, that is, evolution of cooperation. They consider cooperation as the situation in which two or more players make the same high bid. They find only situations where for a limited number of days this cooperation criterion is met. They ascribe this difficulty in maintaining for long-term cooperation patterns to the large number of available actions, the exploration/exploitation policy, and the potential incorrect generalization over environments.

4 Market Modeling

The wholesale electricity market agent-based models widely discussed in previous section have shown the great variability of modeling approaches adopted for both the agent architecture and the market model. Several market models have been proposed from rather simplified market settings, such as duopolistic competitions in network-unconstrained market settings, to very complex market mechanisms aiming to replicate real market features. In particular, as far as concerns latter models, large-scale agent-based models comprising multiple sequential markets and considering different time scales have been recently developed with the aim to fully exploit the flexibility and descriptive power of the ACE approach to set up decision support systems for the practitioners of this industrial sector. In this section, an overview of the major features of some of these large-scale ACE models is presented. Furthermore, Table 3 reports a schematic representation of the market modeling approach for the considered literature. Highlighted aspects are the considered market environment, that is, single or multi-settlement markets. In particular, day-ahead market (DAM), forward market (FM) corresponding to purely financial markets, and real-time balancing market for minute reserve (RT) are considered. However, it is often difficult to classify exactly all contributions within one of the considered classes. Some proposed market mechanisms may differ with respect to the standard one. Moreover, a list of pricing settlement rules are highlighted from column 2 to 7: discriminatory settlement (DA), such as midpoint or pay as bid pricing; uniform settlement (UA); classical theoretical model (Class.), such as Cournot and Bertrand oligopoly models; locational marginal pricing (LMP), such as DCOPF or ACOPF; and finally other less common market mechanisms (Other), such as EVE (Bin et al. 2004). Finally, a further column is added to indicate if network constraints are considered in the market settlement. Two adopted notations are the following: first, if one paper adopts both network-constrained and -unconstrained market mechanisms, the table reports network-constrained, and second, if two or more market clearing mechanisms or market architectures are considered, these are highlighted in all relevant columns.

4.1 Large Scale

Several large scale agent-based models have been developed. The common philosophy of these projects is to replicate national electricity market by considering multiple markets and time scales. ACE simulators for the US are EMCAS (Conzelmann et al. 2005), Marketecture (Atkins et al. 2004), N-ABLE (Ehlen and Scholand 2005), MAIS (Sueyoshi and Tadiparthi 2008b), AMES (Sun and Tesfatsion 2007); for Australia NEMSIM (Batten et al. 2005); and for Germany PowerACE (Weidlich et al. 2005; Genoese et al. 2005). Some of them are commercial, for example, EMCAS and MAA, whereas AMES results to be the first open-source software project. In the following four of them are discussed to introduce some modeling trends.

The Electricity Market Complex Adaptive System (EMCAS) developed at Argonne National Laboratory is an outgrowth of SMART II+, which originally was a project for integrating long-term-model of the electric power and natural gas markets North (2001). EMCAS is a large scale agent-based modeling simulator aiming to be used as a decision support system tool for concrete policy making as well as to simulate decisions on six different time scales from real time to multi-year planning. It comprises several kind of agents such as generating, transmission, and distribution companies to capture the heterogeneity of real markets (Conzelmann et al. 2005). Different markets are implemented, such as spot markets either adopting uniform or discriminatory mechanism, bilateral markets or four markets for the grid regulation

Table 3 Categorization of market models for each paper

Paper	Market clearing rule						Single markets	Multi-settlement	
	DA	*UA*	*Class.*	*LMP*	*Other*	*N-const.*	*DAM*	*DAM & FW*	*DAM & RT*
Curzon Price (1997)	✓		✓				✓		
Richter and Sheblé (1998)	✓						✓		
Visudhiphan and Ilic (1999)		✓					✓		
Bagnall and Smith (2000)	✓	✓					✓		
Bower and Bunn (2000)	✓	✓						✓	
Lane et al. (2000)	✓					✓	✓		
Nicolaisen et al. (2000)	✓					✓	✓		
Bower et al. (2001)	✓							✓	
Bower and Bunn (2001b)	✓	✓						✓	
Bunn and Oliveira (2001)	✓								✓
Day and Bunn (2001)		✓						✓	
Nicolaisen et al. (2001)	✓						✓		
Visudhiphan and Ilic (2001)		✓					✓		
Cau and Anderson (2002)				✓[13]			✓		
Koesrindartoto (2002)	✓						✓		
Bunn and Oliveira (2003)	✓								✓
Cau (2003)				✓			✓	✓	
Visudhiphan (2003)	✓	✓					✓		
Atkins et al. (2004)		✓	✓			✓		✓	
Bin et al. (2004)	✓	✓			✓		✓		
Minoia et al. (2004)				✓		✓	✓		
Xiong et al. (2004)	✓	✓					✓		
Bagnall and Smith (2005)	✓	✓					✓		
Bunn and Martoccia (2005)		✓					✓		
Bakirtzis and Tellidou (2006)			✓				✓		

(Continued)

Table 3 (Continued)

Paper	Market clearing rule						Single markets	Multi-settlement	
	DA	*UA*	*Class.*	*LMP*	*Other*	*N-const.*	*DAM*	*DAM & FW*	*DAM & RT*
Botterud et al. (2006)	✓	✓					✓		
Chen et al. (2006)		✓		✓		✓	✓		✓
Krause et al. (2006)				✓		✓	✓		
Krause and Andersson (2006)				✓	✓	✓	✓		
Ma et al. (2006)				✓		✓	✓		
Naghibi-Sistani et al. (2006)		✓					✓		
Veit et al. (2006)				✓		✓			✓
Weidlich and Veit (2006)	✓	✓							✓
Banal-Estanol and Rupérez-Micola (2007)		✓					✓		
Bunn and Oliveira (2007)	✓	✓	✓				✓[14]		
Guerci et al. (2007)	✓	✓					✓		
Hailu and Thoyer (2007)	✓	✓	✓				✓		
Ikeda and Tokinaga (2007)	✓						✓		
Nanduri and Das (2007)				✓	✓	✓	✓		
Sueyoshi and Tadiparthi (2007)				✓		✓			✓
Sun and Tesfatsion (2007)				✓		✓	✓		
Tellidou and Bakirtzis (2007)				✓		✓	✓		
Guerci et al. (2008a)	✓	✓					✓		
Guerci et al. (2008b)				✓		✓	✓		

Li et al. (2008)				✓		✓	✓		✓
Somani and Tesfatsion (2008)				✓		✓	✓		
Sueyoshi and Tadiparthi (2008a)				✓		✓			✓
Weidlich and Veit (2008a)		✓							✓[15]
Bunn and Day (2009)		✓						✓	
Total occurrences	22	23	5	15	3	17	35	6	8

[13] They do not consider transmission constraints, so it is equivalent to a uniform pricing settlement rule.
[14] A second market is considered where generators trade power generation assets among themselves.
[15] A third interrelated market is also considered for CO_2 emission allowances.

From columns 2–6 market, clearing price mechanisms are reported: discriminatory settlement (DA), such as midpoint or pay as bid pricing; uniform settlement (UA); classical theoretical model (Class.), such as Cournot and Bertrand oligopoly models; locational marginal pricing (LMP), such as DCOPF or ACOPF; and finally other less common market mechanisms (Other). Column 7 lists the paper where transmission network constraints are considered in the market clearing rule (N-const.). Finally, single or multi-settlement markets are highlighted in columns 8–10. In particular, day-ahead market (DAM), forward market (FM) corresponding to purely financial markets, and real-time balancing market for minute reserve (RT) are considered. A ✓notation is adopted for paper presenting the column feature, otherwise an empty field means that the negation is valid.

used for managing reserves. The demand side is populated of retail consumers supplied by demand companies, and on the supply side portfolio generating companies are considered. A complex agent architecture is developed to enable researchers to reproduce complicated decision-making patterns. Agents are endowed with a multiobjective utility function, where configurable objectives may be selected such as minimum profit or minimum market. Furthermore, they have risk-preferences and they are also able to forecast market prices. An application of the EMCAS platform is described in Conzelmann et al. (2005), which shows the great modeling power of the software platform. The future power market configuration for a part of the Midwestern United States is studied. This work reproduces a market setting including about 240 generators, about 2,000 buses, over 30 generation companies, and multiple transmission and distribution companies. Batten et al. (2005) developed the first large scale agent-based simulation model (NEMSIM), which represents Australia's National Electricity Market (NEM). This simulator enables researchers to automatically retrieve a huge amount of historical data and other information to simulate market structure from long time decisions to real time dispatch. Unlike EMCAS, NEMSIM comprises also a greenhouse gas emissions market module. Batten et al. (2005) describe the adaptive agent architecture, which implements both backward- and forward-looking learning models, and enables agents to learn from other participants strategies. Agents may have different goals at different timescales, and a specific multi-objective utility function characterizes each type of agent. Since 2004, at the University of Karlsruhe and at the Fraunhofer Institute of Karlsruhe PowerACE, an agent-based model of electric power and CO2-certificate markets has been developed (Weidlich et al. 2005; Genoese et al. 2005). The main objectives of the simulator is to promote renewable energy and to study CO_2-emission trading schemes. The impacts of renewable power energy sources in the wholesale market and their CO_2 savings can be simulated as well as long term analysis, such as investment in power plant capacities development. Suppliers can participate in the integrated market environment (comprising spot, forward, balancing, and CO_2 markets) as plant dispatchers or traders, whereas, in the demand side, consumers negotiate with the supplier agents the purchase of electric power and suppliers in their turn purchase the required electricity on the power markets. Several papers adopt PowerACE to study the German liberalized electricity market (Sensfuß and Genoese 2006; Sensfuß et al. 2008; Genoese et al. 2008). In particular, Sensfuß and Genoese (2006) and Sensfuß et al. (2008) investigate the price effects of renewable electricity generation on the CO_2 emissions, and Genoese et al. (2008) study how several emission allocation schemes affect investment decision based on both expected electricity prices and CO_2 certificate prices. Koesrindartoto and Tesfatsion (2004) report of a collaborative project between Iowa State University and the Los Alamos National Laboratory for the development of an agent-based modeling of electricity systems simulator (AMES) for testing the economic reliability of the US wholesale power market platform. The leader of the project is Prof. Leigh Tesfatsion at Iowa State University who has already published several interesting research works (see Koesrindartoto et al. (2005), Sun and Tesfatsion (2007), Li et al. (2008), and Somani and Tesfatsion (2008)). One remarkable feature of this project is that it is the first noncommercial open source software project. It is entirely developed in JAVA to be machine-independent and it enables the computational study of the US Wholesale Power Market Platform design. The AMES market architecture includes several sequential markets such as forward and financial transmission rights markets, a day-ahead and realtime market session, and finally a re-bid period. DC optimal power flow and AC optimal power flow locational pricing mechanisms for day-ahead and realtime market settlement are implemented. The AMES software package comprises a scalable software module of reinforcement learning algorithms Gieseler (2005).

5 Conclusions

Agent-based computational economics applied to wholesale electricity markets is an active and fast-growing branch of the power system literature. This paper has documented the great interest of the international scientific community in this research domain by reviewing a large number of papers that appeared throughout the last decade in computer science, power system engineering, and economics journals. Reviewed papers deal exclusively with wholesale electricity markets models. In any case, space reasons as well as the fact that this topic has been the major strand of research in electricity market ACE motivated our choice to focus on it. Nonetheless, several research issues can be highlighted within this scientific literature and a comprehensive outlook of them is reported in Table 1. For the sake of comparison, three summary tables (Tables 1, 2, and 3) are proposed, providing a complete outlook on all considered publications from a chronological viewpoint. The reported tables enable to assess the evolution of the research in the specific area during more or less one decade of research in the field.

As far as concerns research issues (Table 1), it is not possible to highlight a clear tendency by the ACE research community to progressively focus on some topics. Since the early works, the addressed issues have regarded mainly the study of market performance and efficiency and of market mechanisms comparison, which were, and still are, among the major issues also for theoretical and experimental economics. This fact probably denotes the great confidence placed by ACE researchers in providing useful and complementary insights in the market functioning by a "more realistic" modeling approach. Conversely, it is possible to highlight a clear evolution in the selection of modeling assumptions and solutions for both agent's architecture and market properties. In particular, as far as concerns the former (Table 2), the early works experienced several adaptive behavioral models ranging from social to individual learning models and from naive rules of thumbs to very complex agent's architecture based on learning classifier systems. However, progressively researchers have focused their research activity on reinforcement learning models, such as Roth and Erev and Q-learning algorithms. Currently, these are by far the most frequently adopted models. As far as concerns wholesale market models (Table 3), classical discriminatory and uniform double-auction clearing mechanisms have been intensively studied and compared throughout all these years. Recently, network-constrained market models, such as locational marginal pricing, have been studied to better model some real wholesale power markets.

The heterogeneity of agent-based modeling approaches witnesses the great flexibility of such methodology, but it also suggests the risk of neglecting to assess the quality and relevance of all these diverse approaches. Moreover, it is often difficult to compare them because of different modeling assumptions and also because detailed and clear explanations of the agent architecture are not always provided. On this purpose, this survey of ACE works aims to suggest or raise awareness in ACE researchers about critical aspects of modeling approaches and unexplored or partially explored research topics. One of the mostly evaded research approach concerns the validation of simulation results. Two approaches, which have been rarely pursued, can be adopted. The first one regards a theoretical validation based on analytical benchmark models. The drawback is that the validation occurs with analytical models, which are commonly oversimplified models of wholesale electricity markets. This approach usually involves that behavioral models successfully validated on simplified market settings are automatically adopted to investigate more realistic market scenarios where the complexity of the market scenario may obviously arise. This aspect is rarely discussed or studied. A second fruitful approach is to address the task of an empirical validation. This latter approach is seldom adopted either at a macro or a micro level. The rationale is the unavailability of real datasets comprising historical market outcomes, and real features of market operators and transmission network. The few researchers who have performed an empirical validation at a macro-level, that is, they have compared simulated prices with market prices, have often limited their comparisons to verbal or graphic considerations. No paper has tackled a statistical analysis at an aggregate level to prove the statistical significance of the computational model results, to the best of the authors' knowledge. A micro-behavioral statistical validation is an intriguing perspective, which, however, is more demanding in terms of availability of real data and in terms of modeling assumptions. The building of a model to be validated at micro-level requires careful generalization of real world markets and agent's features to enable researchers to test for computational models of market operators' behavior. Indeed, the development of effective agent architecture is a major task also without aiming to empirically validate the model. This consideration underlines a further disregarded modeling issue, that is, the relevance of the model for the decision-making process of market operators. It is worth remembering that the literature shows that simulation results are considerably different by comparing performances of several learning algorithms in an identical market setting (e.g., Koesrindartoto (2002); Visudhiphan (2003); Krause et al. (2006); Guerci et al. (2008a)). Thus, some questions need an answer: is there a suitable minimum degree of rationality? to what extent very complicated agent architectures, such as the one based on learning classifier systems or coevolutionary genetic programming, enhance the description of real market participants? In general, a high degree of bounded rationality characterizes the selected behavioral models such as the Roth and Erev classical reinforcement learning model. The choice of selecting this behavioral model is implicitly justified by the fact that this adaptive model has been studied and then calibrated on human-laboratory experiments. However, market operators do not take their decisions uniquely on the basis of private information, but also they take advantage of a more or less public information shared among all or the majority of market operators. This common knowledge may comprise technological features and historical behavior of opponents.

Furthermore, all market operators are described with the same behavioral features, for example, reasoning and learning capabilities or risk aversion, irrespective of a great heterogeneity in real world market participants. Some market actors may correspond to departments of quantitative analysis and/or operations management dedicated to the optimization scheduling of a portfolio of power-plants on a long-term horizon, whereas others may correspond to purely financial traders speculating on a short-term horizon.

Last but not least, a challenging issue is to adopt adaptive models with the capability of enhancing the action and/or state space representation to continuous variables. In the current literature, often discrete action and state spaces with low cardinality are considered, whereas the bidding format, for example, continuous, step-, or piece-wise supply functions, may be reproduced by divers parameters. Thus, this simplified settings may preclude the relevance of simulation results.

Finally, the research efforts should be further directed in the study of markets interdependence. Indeed, the strong point of ACE methodology is to provide a research tool for addressing the complexity of these market scenarios by modeling highly interconnected economic environments. Electricity markets are very complex economic environments, where a highly interdependence exists among several markets. In particular, from a modeling point of view, multi-settlement markets and market coupling (with foreign countries) issues must be adequately investigated, because they may be prominent factors in determining market outcomes. Some papers have started to investigate the interdependence of day-ahead market sessions with forward or real-time market sessions, analogously other works have started to study how CO_2 emission allowances, transmission rights or other commodities markets, such as gas, may affect market outcomes. However, this great opportunity offered by ACE methodology to model the complexity of electricity markets must be taken not to the detriment of an empirical validation of simulation results. This is obviously the challenging issue and certainly the final goal of a successful ACE agenda in the study of wholesale electricity markets.

Nonetheless, the ACE methodology has been a fertile approach for successfully studying electricity market. The several ACE works presented in this paper witness the richness and validity of contributions brought to electricity market studies. The agent-based computational approach confirms to be a very promising approach, which can greatly complement theoretical and human-subject experiments research studies.

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Futures Market Trading for Electricity Producers and Retailers

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Abstract Within a yearly time framework this chapter describes stochastic programming models to derive the electricity market strategies of producers and retailers. Both a financial futures market and a day-ahead pool are considered. Uncertainties on hourly pool prices and on end-user demands are represented modeling these factors as stochastic processes. Decisions pertaining to the futures market are made at monthly/quarterly intervals while decisions involving the pool are made throughout the year. Risk on profit variability is modeled through the CVaR. The resulting decision-making problems are formulated and characterized as large-scale mixed-integer linear programming problems, which can be solved using commercially available software.

Keywords CVaR · Futures market · Power producer · Power retailer · Risk · Stochastic programming

1 Introduction: Futures Market Trading

This chapter provides stochastic programming decision-making models to identify the best trading strategies for producers and retailers within a yearly time horizon in an electricity market (Birge and Louveaux 1997; Conejo and Prieto 2001; Ilic et al. 1998; Kirschen and Strbac 2004; Shahidehpour et al. 2002; Sheblé 1999).

We consider that electric energy can be traded in two markets, a pool and a futures market. The pool consists in a day-ahead market while the futures market allows trading electricity up to several years ahead. The futures market may present a lower/higher average price for the seller/buyer than the pool but involves a reduced volatility. Thus, it allows hedging against the financial risk inherent to pool price volatility.

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Within a decision horizon of 1 year or longer, an appropriate bundling of hourly pool-price values and demand values is a requirement to achieve computational tractability. Therefore, hourly pool prices and demands are aggregated into a smaller number of values providing an appropriate trade-off between accuracy and computational tractability.

Pool price and end-user demand uncertainty are described through a set of *scenarios*. Each scenario contains a plausible realization of pool prices for a producer, and a plausible realization of pool prices and end-user demands for a retailer, with an occurrence probability. Scenarios are organized using scenario trees. Procedures on how to build efficiently scenario trees are explained, for instance, in Høyland and Wallace (2001) and Høyland et al. (2003).

Care should be exercised in constructing the scenario tree and in the actual generation of scenarios to achieve a comprehensive description of the diverse realizations of the involved stochastic processes.

Figure 1 depicts a typical two-stage scenario tree. A total number of n scenarios are represented by this tree. The tree includes branches corresponding to different realizations of uncertain parameters. We have only a futures market trading decision vector for the single black node of the tree. Pool trading decisions, made at the white nodes, depend on the scenario realizations.

For simplicity, a two-stage decision framework is considered throughout this chapter. However, note that a multi-stage decision framework is both possible and desirable for some applications. Nevertheless, care should be exercised to build multi-stage models since these models become easily intractable from a computational viewpoint.

Note that pool prices and forward contract prices are, in general, dependent stochastic processes. The joint treatment of these two stochastic processes to build scenario trees that embody the stochastic dependencies of these processes is a subject of open research. However, in two-stage stochastic programming models, forward contract prices are known to the decision maker and an appropriate forecasting procedure can be used to generate pool price scenarios.

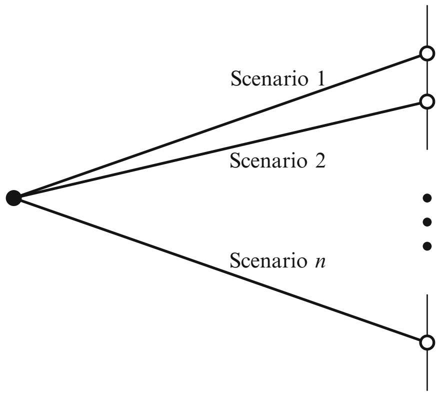

Fig. 1 Scenario tree example

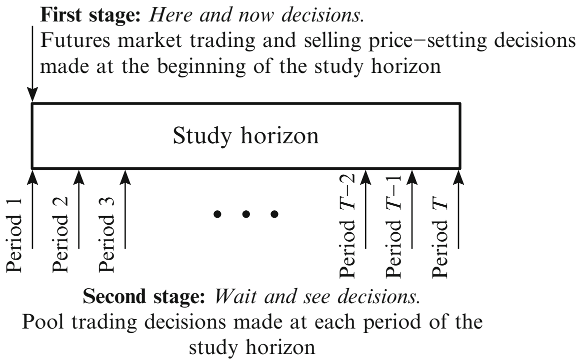

Fig. 2 Decision framework

Decisions pertaining to the futures market and to selling price determination, the latter only by a retailer, are made at monthly or quarterly intervals, while pool decisions are made throughout the year. Thus, futures market selling and price-setting decisions are made before knowing the realization of the stochastic processes and they are denoted as *here-and-now* decisions. In contrast, pool decisions are deferred in time with respect to the futures market decisions and we consider that they are made with perfect information and are referred to as *wait-and-see* decisions. This two-stage decision framework is illustrated in Fig. 2.

Specifically, a producer decides at monthly or quarterly intervals the energy to sell (and eventually to buy) in the futures market, and throughout the year the energy to sell (and eventually to buy) in the pool. A retailer decides at monthly or quarterly intervals the energy to buy (and eventually to sell) in the futures market and the selling price to be charged to its clients, and throughout the year the energy to buy (and eventually to sell) in the pool. The target of both the producer and the retailer is to maximize their respective expected profit subject to a level of risk on profit variability. The risk on profit variability refers to the volatility of that profit. The tradeoff expected profit vs. profit standard deviation is illustrated in Fig. 3. This figure represents two different probability distribution functions of the profit. The flatter one (right-hand side) has a high expected profit but a high volatility. The pointy one (left-hand side) has a low volatility as a result of a smaller expected profit.

The models presented in this chapter are to be used on a monthly basis within a rolling window information framework. At the beginning of each month decisions pertaining to the following 12 months involving futures markets and selling price-setting are made. During the considered month, trade is carried out in the pool until the beginning of the following month when futures market and selling price determination decisions spanning the following 12 months are made again, and so on. As a result of the procedure above, forward contracts and selling prices can be modified once a month. Observe that the above decision framework should be tailored to suit the trading preferences of any given producer or retailer. Note also that a

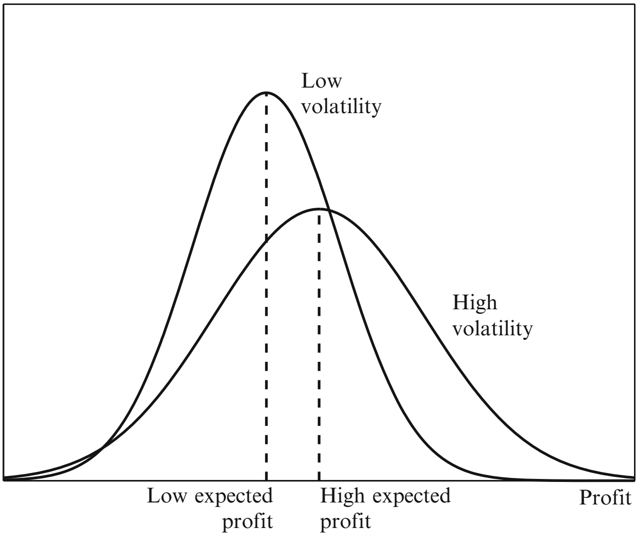

Fig. 3 Trade-off expected profit vs. profit standard deviation

multi-stage stochastic programming model suits properly the decision framework above, but the computational burden of such multi-stage approach leads more often than not to intractability.

The rest of this chapter is organized as follows. Sections 2 and 3 describe and characterize the decision-making problems of a producer and a retailer, respectively. Examples and case studies are presented and discussed. Section 4 provides some relevant conclusions. In the Appendix, the conditional value at risk (CVaR) metric is described.

2 Producer Trading

2.1 Producer: Trading

At monthly/quarterly intervals, the producer decides which forward contracts to sign in order to sell (or to buy) electric energy in the futures markets, and throughout the year it decides how much energy to sell (or to buy) in the pool.

In the derivations below, $\lambda(\xi)$ represents the different scenario realizations of the vector of electricity pool prices (stochastic process). Variable x represents the vector defining the energy traded (mostly sold) in the futures market, and $y(\xi)$ represents the variable vector defining the energy traded (mostly sold) in the pool. Note that pool decisions depend on price scenario realizations while futures market decisions do not. $R^F(\cdot)$ is the revenue associated with selling energy in the futures market, while $R^P(\cdot)$ is the revenue associate with selling in the pool. $C^O(\cdot)$ is the total production cost of the producer. $\mathcal{E}_\xi\{\cdot\}$ is the expectation operator over the stochastic process vector represented using scenarios ξ, while $\mathcal{R}_\xi\{\cdot\}$ is a risk measure over ξ, for example, the minus CVaR (Rockafellar and Uryasev 2000, 2002). If uncertainty is described via scenarios, then a linear expression for CVaR is provided in the Appendix. Sets Ω^F, Ω^P, Ω^O, and Ω^R represent, respectively, the feasibility region of futures market trading, the feasibility region of pool operation, the operating constraints of the producer, and the set of constraints needed to model the risk of profit variability.

The two-stage stochastic programming problem pertaining to a risk-neutral producer can be formulated as

$$\underset{x}{\text{maximize}} \quad R^F(x) + S(x) \tag{1}$$

subject to

$$x \in \Omega^F, \tag{2}$$

where

$$S(x) = \mathcal{E}_\xi \left\{ \underset{y(\xi)}{\text{maximize}} \quad \left(R^P(\lambda(\xi), y(\xi)) - C^O(x, y(\xi))\right) \right\} \tag{3}$$

subject to

$$y(\xi) \in \Omega^P, \forall \xi; \quad (x, y(\xi)) \in \Omega^O, \forall \xi. \tag{4}$$

Objective function (1) represents the expected profit of the producer, which is the sum of the revenue from selling in the futures market and, as expressed by (3), the expected revenue from selling in the pool minus the expected production cost. Constraint (2) imposes the feasibility conditions pertaining to the futures market, and constraints (4) enforce the feasibility conditions pertaining to the pool and the operating constraints of the producer.

Under rather general assumptions (Birge and Louveaux 1997), the maximization and expectation operators can be swapped in (3). Then, the two-stage stochastic programming problem (1)–(4) is conveniently formulated as the deterministic mathematical programming problem stated below,

$$\underset{x,\, y(\xi)}{\text{maximize}} \quad R^F(x) + \mathcal{E}_\xi \left\{R^P(\lambda(\xi), y(\xi)) - C^O(x, y(\xi))\right\} \tag{5}$$

subject to

$$x \in \Omega^F; \quad y(\xi) \in \Omega^P, \forall \xi; \quad (x, y(\xi)) \in \Omega^O, \forall \xi. \tag{6}$$

If risk is considered, the producer two-stage stochastic programming problem is formulated as the deterministic mathematical programming problem stated below,

$$\begin{aligned} \underset{x,\, y(\xi)}{\text{maximize}} \quad & R^F(x) + \mathcal{E}_\xi \left\{ R^P(\lambda(\xi), y(\xi)) - C^O(x, y(\xi)) \right\} \\ & -\beta\, \mathcal{R}_\xi \left\{ R^F(x) + R^P(\lambda(\xi), y(\xi)) - C^O(x, y(\xi)) \right\} \end{aligned} \tag{7}$$

subject to

$$x \in \Omega^F; \quad y(\xi) \in \Omega^P, \forall \xi; \quad (x, y(\xi)) \in \Omega^O, \forall \xi; \quad (x, y(\xi)) \in \Omega^R, \forall \xi. \tag{8}$$

Objective function (7) expresses the expected profit of the producer minus a risk measure of its profit. The expected profit is the sum of the revenue from selling in the futures market and the expected revenue from selling in the pool minus the expected production cost. Note that β is a weighting factor to realize the tradeoff expected profit vs. profit variability. Last constraints in (8) enforce conditions pertaining to the risk term. It should be noted that the risk term depends on both futures market and pool trading.

Relevant references related to the medium-term decision-making problem faced by an electricity producer include Collins (2002), Conejo et al. (2007), Conejo et al. (2008), Gedra (1994), Kaye et al. (1990), Niu et al. (2005), Shrestha et al. (2005), Tanlapco et al. (2002).

Results obtained solving the model described earlier for different values of the risk weighting factor (β) include the efficient frontier for the producer. Figure 4 illustrates how the expected profit of a producer increases as its standard deviation (risk) increases. This information allows the decision-maker to make an informed decision on both futures market and pool involvement. Note that this curve is generally concave; that is, if the risk assumed by a producer is small, an increase in risk results in a significant increase in expected profit, but if the level of risk assumed by a producer is high, an increase in the level of risk results in a rather small increase in the expected profit.

For the risk-neutral case ($\beta = 0$), an appropriate measure of the advantage of a stochastic programming approach with respect to a deterministic one is the value of the stochastic solution (VSS) (Birge and Louveaux 1997).

The objective function value of the stochastic programming problem at its optimal solution is denoted by z^{SP} and represents the average value of profit over all scenarios for the optimal solution obtained solving the stochastic programming problem.

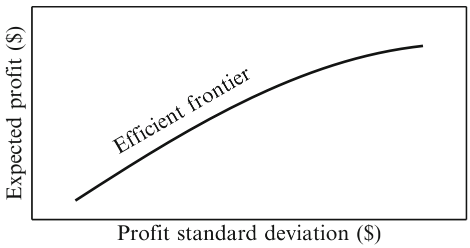

Fig. 4 Producer: expected profit vs. profit standard deviation

On the other hand, let us consider a deterministic problem in which the stochastic processes are replaced by their respective expected values. The solution of this deterministic problem provides optimal values for the futures-market decisions. The original stochastic programming problem is then solved fixing the values of the futures-market decisions to those obtained by solving the deterministic problem. The objective function value of the modified stochastic problem is denoted by z^{DP}.

The value of the stochastic solution is then computed as

$$\mathrm{VSS} = z^{\mathrm{SP}} - z^{\mathrm{DP}} \tag{9}$$

and provides a measure of the gain obtained from modeling stochastic processes as such, avoiding substituting them by average values.

2.1.1 Alternative Formulation

If the CVaR is used as the risk measure (see the Appendix), the risk can be considered in the producer problem using an alternative formulation to (7)–(8). Describing uncertainty via scenarios, the profit of the producer is a discrete random variable characterized by its probability density function (pdf). The value at risk (VaR) corresponds to the $(1-\alpha)$ percentile of this pdf, α being a parameter representing the confidence level. The CVaR represents the quantile of this pdf corresponding to the average profit conditioned by not exceeding the VaR. Therefore, the VaR at the α confidence level is the greatest value of profit such that, with probability α, the actual profit will not be below this value; and CVaR at the α confidence level is the expected profit below the profit represented by VaR.

Considering the definition of CVaR, an alternative formulation for problems (7)–(8) is the problem below whose objective is maximizing the quantile represented by the CVaR, that is,

$$\underset{x,\, y(\xi)}{\text{maximize}} \quad -\mathcal{R}_{\xi}\left\{R^F(x) + R^P(\lambda(\xi), y(\xi)) - C^O(x, y(\xi))\right\} \tag{10}$$

subject to

$$x \in \Omega^F; \quad y(\xi) \in \Omega^P, \forall \xi; \quad (x, y(\xi)) \in \Omega^O, \forall \xi; \quad (x, y(\xi)) \in \Omega^R, \forall \xi, \tag{11}$$

where the risk measure used, $\mathcal{R}_{\xi}\{\cdot\}$, is the minus CVaR.

By definition of CVaR, a confidence level $\alpha = 0$ in problem (10)–(11) corresponds to maximizing the expected profit of the producer. Increasing the confidence level α entails that the producer is willing to assume a lower risk of having low profits, which results in a decrease in the expected profit of the producer. A confidence level $\alpha = 1$ corresponds to maximizing the lowest possible profit.

The main difference between problems (7)–(8) and (10)–(11) is the number of required parameters. While weighting factor β and confidence level α need to be specified for solving problem (7)–(8), only the confidence level α is needed for solving problem (10)–(11). However, both formulations provide the same efficient frontier for the case study analyzed.

2.2 Producer: Model Characterization

Model (7)–(8) can be formulated as a large-scale linear programming problem of the form (Castillo et al. 2002)

$$\underset{v}{\text{maximize}} \quad z = d^{\mathrm{T}} v \tag{12}$$

subject to

$$Av = b, \tag{13}$$

where $v \in \mathcal{R}^n$ is the vector of optimization variables comprising first and second-stage variables, z is the objective function value, and A, b, and d are a matrix and two vectors, respectively, of appropriate dimensions.

The above problem involving several hundreds of thousands of both real variables and constraints can be readily solved using commercially available software (The FICO XPRESS Website 2010; The ILOG CPLEX Website 2010). However, a large number of scenarios may result in intractability. Note that the continuous variables and the constraints increase with the number of scenarios.

For example, consider a producer owning six thermal units, a decision horizon involving 72 price values describing price behavior throughout 1 year, 12 forward contracts, and 1,000 price scenarios. This model involves 505,013 continuous variables and 937,012 constraints.

To reduce the dimension of this problem while keeping as much as possible the stochastic information embodied in the scenario structure, scenario-reduction techniques can be used (Dupačova et al. 2000; Gröwe-Kuska 2003; Pflug 2001). These techniques trim down the scenario tree, reducing its size while keeping as intact as possible the stochastic information that the tree embodies. Appropriate decomposition techniques are also available to attack particularly large problems. These techniques include Benders decomposition, Lagrangian relaxation, and other specialized techniques (Conejo et al. 2006; Higle and Sen 1996).

2.3 Producer: Example

To clarify and illustrate the derivations above, a simple example is provided below. This simple example illustrates the decision-making process of a producer who can sell its energy either through forward contracts at stable prices or in the pool at uncertain prices. The objective of this producer is to maximize its profit while

Table 1 Producer example: pool price scenarios ($/MWh)

Period	Scenario				
	1	2	3	4	5
1	56	40	62	52	58
2	58	41	65	56	59

Table 2 Producer example: generator data

Capacity (MW)	Minimum power output (MW)	Linear production cost ($/MWh)
500	0	45

Table 3 Producer example: forward contract data

Contract	Price ($/MWh)	Power (MW)
1	52	100
2	50	50
3	46	25

Table 4 Producer example: power sold through forward contracts (MW)

Period	$\beta = 0$	$\beta = 1$	$\beta = 5$	$\beta = 100$
1	0	100	150	175
2	0	100	150	175

controlling the volatility risk of this profit. Forward contracts allow selling energy at stable prices, but they prevent selling the energy already committed in forward contracts to the pool during periods of high prices. For a given level of risk aversion, the producer should determine how much of its energy production should be sold through forward contract and how much in the pool. The example below illustrates this decision-making process.

A planning horizon of two periods is considered. The pool price is treated as a stochastic process using a tree of five equiprobable scenarios. Table 1 provides the pool prices for each period and scenario.

We consider a producer that owns a generator whose technical characteristics are provided in Table 2. This producer can sign the three forward contracts detailed in Table 3. Forward contracts are defined as a single block of energy at fixed price spanning the two periods. For instance, signing contract 1 implies selling 100 MW during the two considered time periods. For the sake of clarity, note that arbitrage between the pool and the futures market is explicitly excluded.

The risk measure used is the CVaR at 0.95 confidence level (see the Appendix).

This problem is solved for four different values of the weighting factor, $\beta = \{0, 1, 5, 100\}$. The power sold through forward contracts in each period is provided in Table 4. Table 5 provides the power generated and the power sold in the pool by the producer for each period and scenario, respectively.

Table 5 Producer example: power generated/sold in the pool (MW)

Scenario	$\beta = 0$		$\beta = 1$	
	t = 1	t = 2	t = 1	t = 2
1	500/500	500/500	500/400	500/400
2	0/0	0/0	100/0	100/0
3	500/500	500/500	500/400	500/400
4	500/500	500/500	500/400	500/400
5	500/500	500/500	500/400	500/400
Scenario	$\beta = 5$		$\beta = 100$	
	t = 1	t = 2	t = 1	t = 2
1	500/350	500/350	500/325	500/325
2	150/0	150/0	175/0	175/0
3	500/350	500/350	500/325	500/325
4	500/350	500/350	500/325	500/325
5	500/350	500/350	500/325	500/325

Table 6 Producer example: expected profit and profit standard deviation

	$\beta = 0$	$\beta = 1$	$\beta = 5$	$\beta = 100$
Expected profit ($)	10,600.0	9,880.0	9,320.0	8,840.0
Profit standard deviation ($)	6,850.2	5,480.1	4,795.1	4,452.6

Both the power generation and the power sold in the pool are dependent on the pool price scenarios. For example, the pool price in both periods of scenario 2 is lower than the production cost of the generator (see Tables 1 and 2, respectively). Thus, as Table 5 shows, the producer decides not to sell power in the pool in scenario 2 and only to produce the energy contracted by forward contracts (cases $\beta = 1$, $\beta = 5$ and $\beta = 100$).

The expected profit and the profit standard deviation obtained for the four values of β are provided in Table 6. Figure 5 depicts the efficient frontier, that is, the expected profit vs. the profit standard deviation for different values of β. For the risk-neutral case ($\beta = 0$), the expected profit is $10,600 with a standard deviation of $6,850.2. On the other hand, the risk-averse case ($\beta = 100$) results in a profit of $8,840 with a smaller standard deviation of $4,452.6.

2.4 *Producer Case Study*

The model corresponding to the producer is further illustrated through a realistic case study based on the electricity market of mainland Spain.

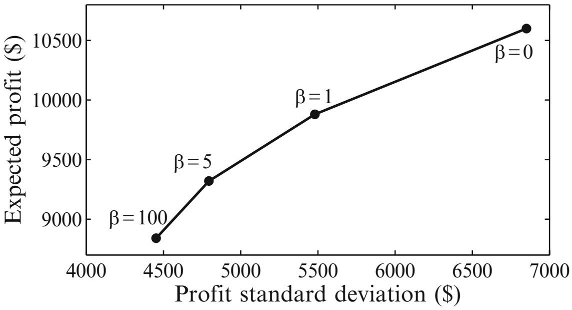

Fig. 5 Producer example: expected profit vs. profit standard deviation

Table 7 Producer case study: characteristics of the thermal units

Unit	Type	Cost ($/MWh)	Capacity (MW)	Peak/off-peak (%)
1	Coal_1	57	255	30
2	Coal_2	61	255	30
3	Oil_1	80	295	25
4	Oil_2	74	295	25
5	CCGT_1	50	300	25
6	CCGT_2	45	300	25

2.4.1 Data

The considered producer owns six thermal units whose characteristics are provided in Table 7. The last column of this table indicates the minimum percentage of the energy produced by each unit during peak periods that must be produced during off-peak periods. The minimum power output of all units is 0 MW. A decision horizon of 1 year is considered and hourly pool prices of the whole year are aggregated in 72 price values. Within this framework, 12 forward contracts are considered, one per month. Each forward contract consists of two selling blocks of 80 MW each. Table 8 lists the data of prices for both blocks of each forward contract. Initially 200 price scenarios are generated, and the fast forward scenario reduction algorithm explained in Gröwe-Kuska (2003) is used to reduce this number to 100, considering the profit obtained for each scenario as the scenario reduction metric. Figure 6 depicts the evolution of the price scenarios, the evolution of the average pool price and the forward contract prices throughout the year.

2.4.2 Results

The risk measure used is the CVaR at 0.95 confidence level. The resulting problem is solved for different values of the weighting factor β.

Figure 7 illustrates how the optimal objective function value changes as the number of scenarios increases for the risk-neutral case. The information provided by this figure allows selecting 100 as an appropriate number of scenarios, which yields an adequate tradeoff between tractability and accuracy.

Table 8 Producer case study: monthly forward contracts

Contract	Price ($/MWh)		Contract	Price ($/MWh)	
	First block	Second block		First block	Second block
1	76	73	7	64	57
2	68	66	8	64	62
3	59	56	9	61	58
4	59	57	10	61	58
5	63	58	11	64	61
6	60	57	12	62	60

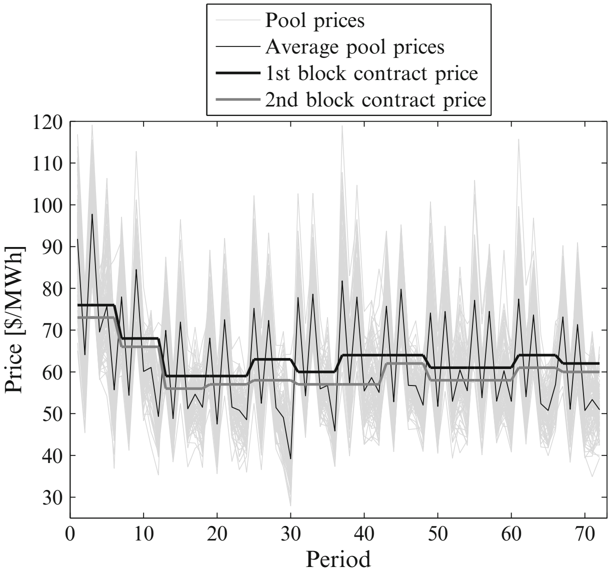

Fig. 6 Producer case study: pool prices and forward contract prices

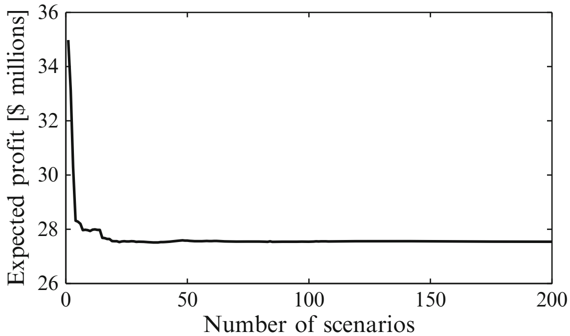

Fig. 7 Producer case study: expected profit vs. number of scenarios

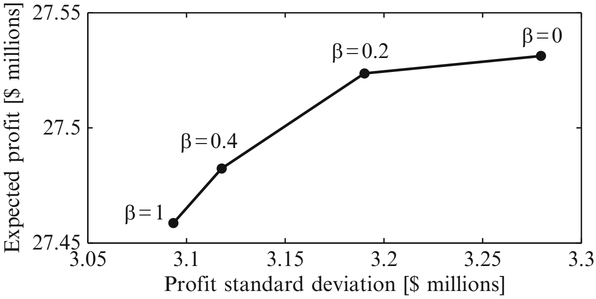

Fig. 8 Producer case study: evolution of expected profit vs. profit standard deviation

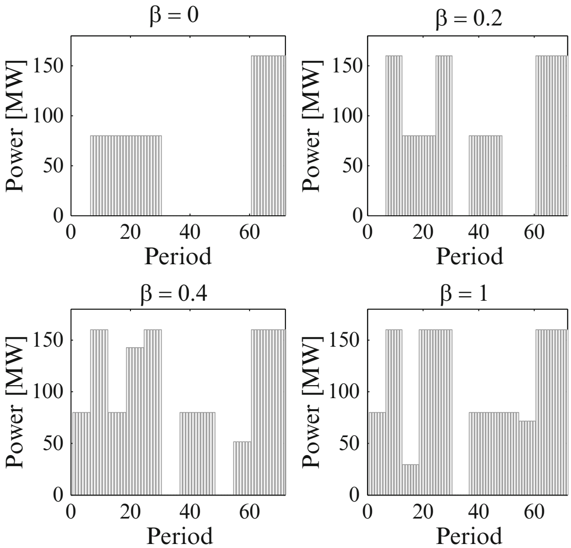

Fig. 9 Producer case study: evolution of power involved in futures market

Figure 8 shows the efficient frontier, that is, the expected profit as a function of the standard deviation. A risk-neutral producer ($\beta = 0$) expects to achieve a profit of \$27.531 million with a standard deviation of \$3.280 million. For a risk-averse producer ($\beta = 1$), the expected profit is \$27.463 million with a standard deviation of \$3.082 million. Observe that the expected profit increases as its corresponding standard deviation, which is related to risk, increases.

Figure 9 depicts the power traded in the futures market for different values of β. The y-axis represents power, not energy; since time periods include different number of hours, the above remark is necessary. We observe that as the concern on risk increases, the selling power in the futures market increases. This behavior is justified by the fact that forward contracts involve more stable prices than the pool. Therefore, selling energy in futures market generally results in lower risk and lower profit.

The value of the stochastic solution for the risk-neutral case is VSS $= \$27.531 - \$27.491 = \$0.04$ million, that is, 0.15%. Note that \$27.491 million is the average profit obtained solving the stochastic programming problem once forward contracting decisions are fixed to the optimal values obtained from the deterministic problem.

The considered problem, characterized by 108,124 constraints and 50,525 continuous variables, has been solved using CPLEX 10.0 (The ILOG CPLEX Website 2010) under GAMS (Rosenthal 2008) on a Linux-based server with two processors clocking at 2.4 GHz and 8 GB of RAM. CPU time required to solve it is less than 10 s.

3 Retailer Trading

3.1 Retailer: Trading

A retailer must procure energy in both the futures market and the pool to sell that energy to its clients. On a monthly/quarterly basis, the retailer must decide which forward contracts to sign and the optimal selling prices to be charged to its clients, and throughout the year it must determine the energy to be traded (mostly bought) in the pool.

Note that $\lambda(\xi)$ and $d(\lambda^C, \xi)$ represent different scenario realizations of the pool prices and demands (stochastic processes), respectively, and that client demands depend on the selling price offered by the retailer, λ^C.

The retailer decision-making problem is conveniently formulated as the deterministic mathematical programming problem stated below,

$$\underset{x,\, y(\xi),\, \lambda^C}{\text{maximize}} \quad \begin{array}{l} \mathcal{E}_\xi \left\{ R^C(\lambda^C, d(\lambda^C, \xi)) - C^P(\lambda(\xi), y(\xi)) \right\} - C^F(x) \\ \quad -\beta \mathcal{R}_\xi \left\{ R^C(\lambda^C, d(\lambda^C, \xi)) - C^P(\lambda(\xi), y(\xi)) - C^F(x) \right\} \end{array} \tag{14}$$

subject to

$$x + y(\xi) = d(\lambda^C, \xi), \forall \xi, \tag{15}$$

$$x \in \Omega^F; \quad y(\xi) \in \Omega^P, \forall \xi; \quad (x, y(\xi)) \in \Omega^R, \forall \xi. \tag{16}$$

where x represents the variable vector defining the energy traded (mostly bought) in the futures market, while $y(\xi)$ represents the variable vector defining the energy traded (mostly bought) in the pool. Pool decisions depend on scenario realizations while futures market decisions do not. λ^C is the variable vector of selling prices to clients, $R^C(\cdot)$ the revenue from selling energy to clients, $C^F(\cdot)$ the cost associated with buying energy in the futures market, and $C^P(\cdot)$ the cost associated with buying in the pool. Sets Ω^F, Ω^P, and Ω^R represent, respectively, the feasibility region of forward contracting, the feasibility region of pool operation, and the set of constraints needed to model the risk of profit variability.

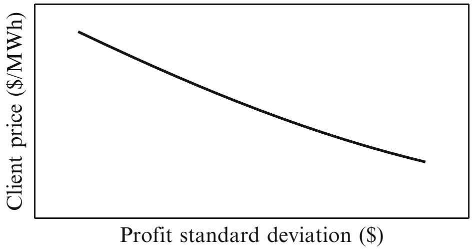

Fig. 10 Retailer: client price vs. profit standard deviation

Objective function (14) expresses the expected profit of the producer minus a risk measure of it. The expected profit is obtained as the expected revenue from selling to the clients minus the expected cost from buying from the pool minus the cost of buying through forward contracts. Constraints (15) impose that the demand of the clients should be supplied. Constraints (16) enforce the feasibility conditions pertaining to the futures market and the pool and the conditions pertaining to the risk term. Note that the risk term depends on both futures market and pool trading decisions.

Results obtained solving this model for different values of the risk weighting factor (β) include the efficient frontier for the retailer, which is similar to the one obtained for a producer (Fig. 4), and how the prices set by a retailer to its clients change with the standard deviation of the profit of the retailer. Figure 10 shows how the selling price to clients (fixed by the retailer) decreases with the standard deviation (risk) of the profit of the retailer. As the retailer relies more on its purchases from the pool (higher profit standard deviation), the selling price offered to its clients decreases to attract as much consumption as possible. Conversely, as the retailer relies more on its purchases from forward contracts (lower profit standard deviation), the selling price offered to its clients increases as the energy from forward contracts is more expensive than that from the pool.

Relevant references on the medium-term decision-making problem faced by an electric energy retailer include Carrión et al. (2007), Conejo et al. (2007), Fleten and Pettersen (2005), Gabriel et al. (2002), Gabriel et al. (2006).

The value of the stochastic solution can also be obtained as explained in Sect. 2.1. This measure is an indication of the quality of the stochastic solution vs. the corresponding deterministic one.

3.1.1 Alternative Formulation

As explained in Sect. 2.1.1 for the producer case, the retailer problem (14)–(16) can also be formulated as

$$\underset{x,\, y(\xi),\, \lambda^C}{\text{maximize}} \quad -\mathcal{R}_\xi \left\{ R^C(\lambda^C, d(\lambda^C, \xi)) - C^P(\lambda(\xi), y(\xi)) - C^F(x) \right\} \tag{17}$$

subject to

$$x + y(\xi) = d(\lambda^C, \xi), \forall\xi, \tag{18}$$

$$x \in \Omega^F; \quad y(\xi) \in \Omega^P, \forall\xi; \quad (x, y(\xi)) \in \Omega^R, \forall\xi. \tag{19}$$

where the risk measured used, $\mathcal{R}_\xi\{\cdot\}$, is the minus CVaR.

Problems (14)–(16) and (17)–(19) provide the same efficient frontier for the case study analyzed.

3.2 Retailer: Model Characterization

The model presented for the retailers (14)–(16) can be formulated as large-scale mixed-integer linear programming problem of the form (Castillo et al. 2002)

$$\underset{v,u}{\text{maximize}} \quad z = d^{\mathrm{T}}v + e^{\mathrm{T}}u \tag{20}$$

subject to

$$Av + Bu = b, \tag{21}$$

where $v \in \mathcal{R}^n$ is a vector of real optimization variables comprising first- and second-stage decisions; $u \in \{0, 1\}^m$ is a vector of binary optimization variables pertaining to both first- and second-stage decisions; and A, B, d, e, and b are two matrices and three vectors, respectively, of appropriate dimensions.

If the selling price is modeled through a stepwise price-quota curve, binary variables are needed to identify the interval of the curve corresponding to the selling price. The price-quota curve provides the relationship between the retail price and the end-user demand supplied by the retailer.

The above problem involving a few hundreds of binary variables and several hundreds of thousands of both real variables and constraints can be readily solved using commercially available software (The FICO XPRESS Website 2010; The ILOG CPLEX Website 2010). However, a very large number of scenarios may result in intractability.

For example, consider a retailer that provides the electricity demand of three end-user groups throughout 1 year, which is divided into 72 time periods. The relationship between the selling price and the demand provided by the retailer is modeled by price-quota curves with 100 steps. The retailer has the possibility of signing 12 contracts in the futures market. A 1,000-scenario tree is used to take into account the uncertainty of pool prices and end-user demands. The formulation of this problem requires 650,930 constraints, 577,508 continuous variables, and 300 binary variables.

As in the producer case, if a very large number of scenarios results in intractability, scenario-reduction and decomposition techniques can be used to achieve tractability.

3.3 Retailer: Example

This simple example illustrates the decision-making process of a retailer that, to supply its contracting energy obligations can buy energy either through forward contacts at stable prices or in the pool at uncertain prices. Forward contracts allow buying energy at stable prices, but they prevent buying the energy already committed in these contracts from the pool during periods of low prices. The objective of the retailer is to maximize its profits from selling energy to its clients while controlling the volatility risk of this profit. Note that the amount of energy to be supplied to clients is uncertain and, additionally, depends on the selling price offered to these clients by the retailer. For a given level of risk aversion, the retailer should determine the energy selling price to its clients, as well as how much of the energy to be supplied should be bought from forward contacts and how much from the pool. The example below illustrates this decision-making process.

In this example we consider a retailer that buys energy in the futures market through forward contracts and in the pool to provide electricity to its clients. The demand of the clients is modeled as a stochastic process with three equiprobable scenarios. Table 9 provides the demand of the clients for each period and scenario. Note that Table 9 provides the maximum demand values that the retailer may supply to its clients and correspond to 100% of demand supplied in Fig. 11. These maximum values are supplied only for selling prices not above 60 $/MWh, as illustrated in Fig. 11.

The response of the clients to the price offered by the retailer is represented through the price-quota curve shown in Fig. 11, which expresses how the demand decreases as the price increases. A single price-quota curve is used for all periods and scenarios.

The retailer can purchase energy in the futures market through two forward contracts. The characteristics of these forward contracts are provided in Table 10.

The problem is solved for three different values of the weighting factor, $\beta = \{0, 0.5, 5\}$. The power purchased through forward contracts in each period is provided in Table 11. The price offered by the retailer to its clients is given in Table 12 for the different values of β. The price offered increases from 70 $/MWh ($\beta = \{0, 0.5\}$) to 82 $/MWh ($\beta = 5$). A higher price originates a lower demand to be supplied but also a smaller risk of incurring a financial loss. Table 13 provides the power purchased in the pool for each period and scenario.

Table 9 Retailer example: client demands (MW)

Scenario	Period 1	Period 2
1	450	412
2	482	395
3	421	381

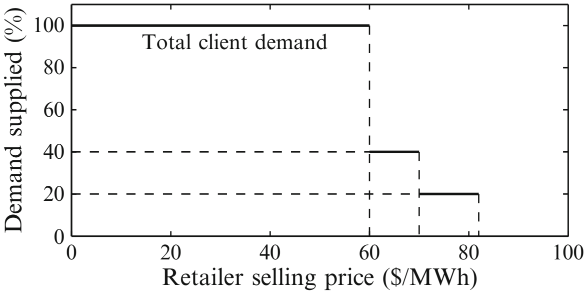

Fig. 11 Retailer example: price-quota curve

Table 10 Retailer example: forward contracts

Contract	Price ($/MWh)	Power (MW)
1	57	50
2	59	25

Table 11 Retailer example: power purchased through forward contracts (MW)

Period	$\beta = 0$	$\beta = 0.5$	$\beta = 5$
1	0	50	75
2	0	50	75

Table 12 Retailer example: price offered by the retailer to its clients ($/MWh)

$\beta = 0$	$\beta = 0.5$	$\beta = 5$
70	70	82

Table 13 Retailer example: power purchased in the pool (MW)

Scenario	$\beta = 0$		$\beta = 0.5$		$\beta = 5$	
	t = 1	t = 2	t = 1	t = 2	t = 1	t = 2
1	180.00	164.80	130.00	114.80	15.00	7.40
2	192.80	158.00	142.80	108.00	21.40	4.00
3	168.40	152.40	118.40	102.40	9.20	1.20

The expected profit and the profit standard deviation obtained for the three values of β are provided in Table 14, while Fig. 12 depicts the efficient frontier. The risk-neutral case ($\beta = 0$) results in a high expected profit of $4,507.3 involving a high profit standard deviation of $1,359.8. On the other hand, a highly risk-averse case ($\beta = 5$) results in a low profit, $4,136.5, involving a low profit standard deviation, $249.7. Finally, Fig. 13 depicts the selling price offered to its clients by the retailer vs. the profit standard deviation. Observe that as the standard deviation of the profit increases, the selling price decreases.

Table 15 provides the power sold by the retailer to its client in each scenario and each value of the weighting factor β.

Table 14 Retailer example: expected profit and profit standard deviation

	$\beta = 0$	$\beta = 0.5$	$\beta = 5$
Expected profit ($)	4,507.3	4,474.0	4,136.5
Profit standard deviation ($)	1,359.8	1,009.2	249.7

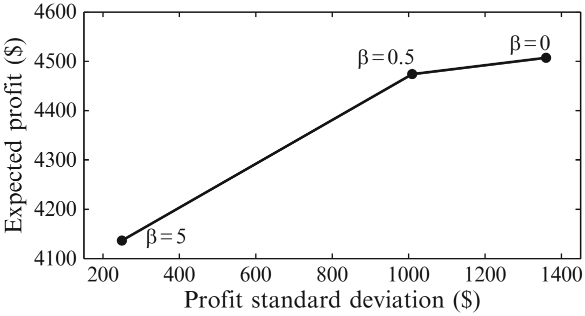

Fig. 12 Retailer example: expected profit vs. profit standard deviation

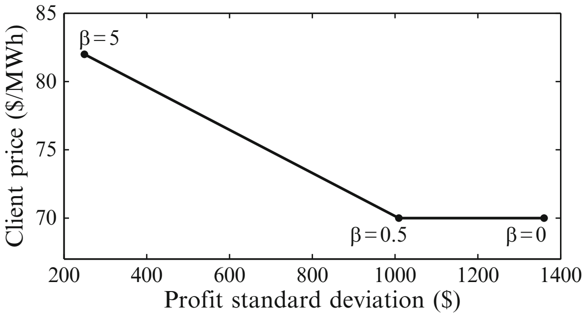

Fig. 13 Retailer example: client price vs. profit standard deviation

Table 15 Retailer example: power sold to the clients (MW)

Scenario	$\beta = 0$		$\beta = 0.5$		$\beta = 5$	
	t = 1	t = 2	t = 1	t = 2	t = 1	t = 2
1	180.00	164.80	180.00	164.80	90.00	82.40
2	192.80	158.00	192.80	158.00	96.40	79.00
3	168.40	152.40	168.40	152.40	84.20	76.20

3.4 Retailer Case Study

Below, the model of the retailer is analyzed and tested using a realistic case study based on the electricity market of mainland Spain.

Table 16 Retailer case study: forward contracts

Contract	Price ($/MWh)	
	First block	Second block
1	76	82
2	68	71
3	59	62
4	59	61
5	63	68
6	60	67
7	64	71
8	64	68
9	61	65
10	61	64
11	64	67
12	62	64

3.4.1 Data

A planning horizon of 1 year is considered. The year is divided into 72 periods. The retailer participates in a futures market where 12 forward contracts are available, one per month. Each forward contract consists of two selling blocks of 80 MW each. Table 16 lists the data of prices for both blocks of each forward contract. Three groups of clients are considered according to different consumption patterns, namely residential, commercial, and industrial. The response of each group of client to the price offered by the retailer is modeled through the 100-block price-quota curves depicted in Fig. 14. Note that a single price-quota curve is used for each client group in all periods and scenarios. An initial scenario tree of 200 pool price scenarios and client demands is generated. Taking into account the profit associated to each scenario, this scenario tree is trimmed down using the fast forward scenario reduction algorithm provided in Gröwe-Kuska (2003), considering the profit obtained for each scenario as the scenario reduction metric, yielding a final tree comprising 100 scenarios. The evolution of the pool price scenarios, the average pool price and the forward contract prices are plotted in Fig. 15. The client demand for each group is depicted in Fig. 16.

3.4.2 Results

The risk measure used is the CVaR at 0.95 confidence level. The resulting problem is solved for different values of the weighting factor β.

The evolution of the expected profit with the number of scenarios in provided in Fig. 17. This figure shows that the selection of 100 scenarios is adequate in terms of tractability and accuracy.

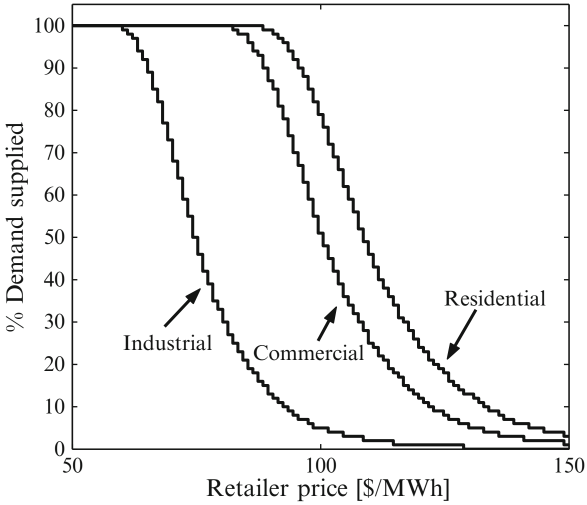

Fig. 14 Retailer case study: price-quota curve

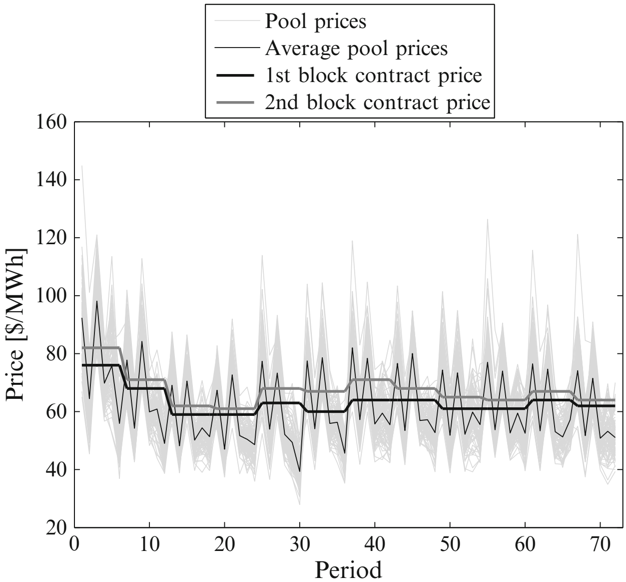

Fig. 15 Retailer case study: pool prices and forward contract prices

Figure 18 provides the efficient frontier, that is, the expected profit vs. the profit standard deviation for different values of the weighting factor β. This figure shows that the expected profit of a risk-neutral retailer is equal to \$595.99 million with a standard deviation of \$52.29 million. A risk-averse retailer ($\beta = 100$) expects to achieve a profit of \$577.70 million with a standard deviation equal to \$37.62 million.

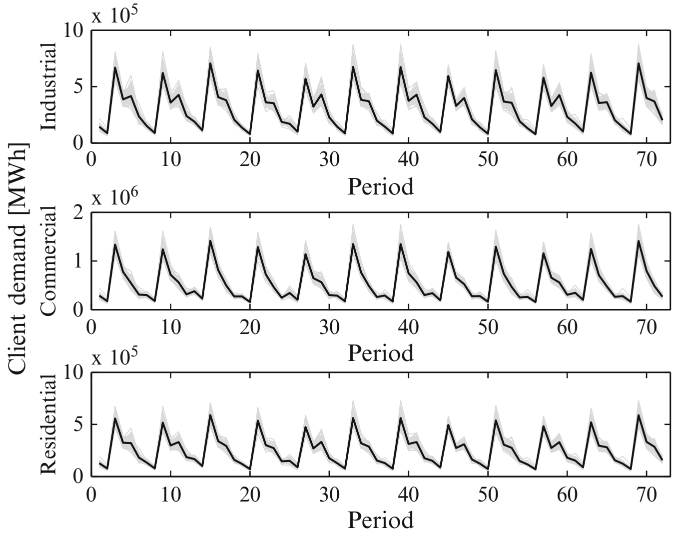

Fig. 16 Retailer case study: client demand

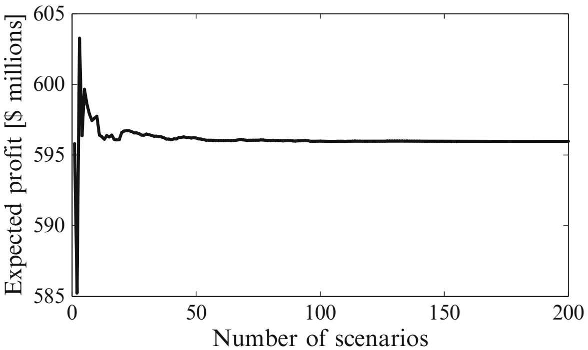

Fig. 17 Retailer case study: expected profit vs. number of scenarios

Then, we can conclude that a retailer who desires to achieve large expected profits must assume high risk.

The value of the stochastic solution (risk-neutral case) is VSS = \$591.991 – \$591.961 = \$0.03 million, that is, 0.005%. Note that \$591.961 million is the expected profit obtained if the optimal forward contract decisions derived using the deterministic problem are fixed for solving the stochastic problem.

Figure 19 shows the resulting power purchased in the futures market for different values of β. Note that the y-axis represents the power purchased from each contract for all hours within the delivery period of the contract. It can be observed that the quantity of power purchased increases as the risk-aversion of the retailer becomes

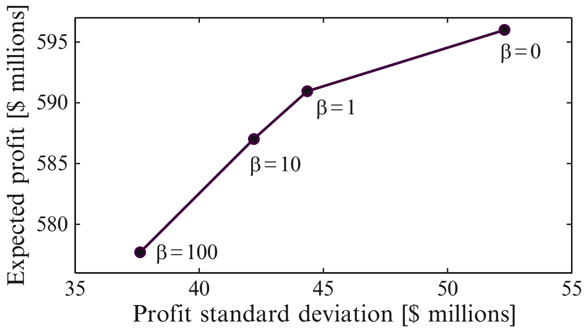

Fig. 18 Retailer case study: evolution of expected profit vs. profit standard deviation

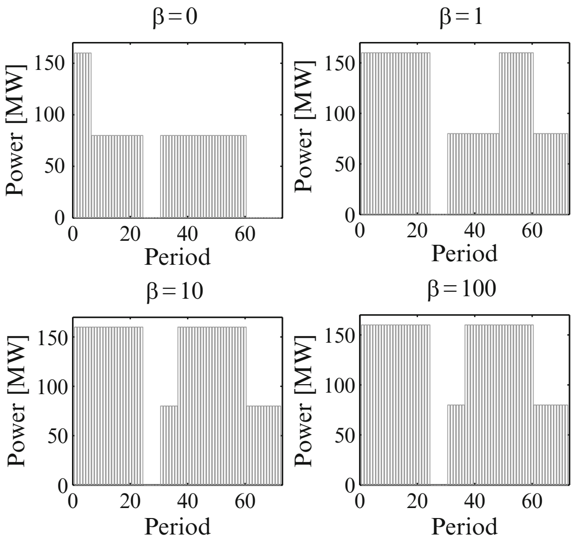

Fig. 19 Retailer case study: evolution of power involved in futures market

higher (smaller standard deviation of profit). This result is coherent since the prices of the forward contracts are more stable than the pool prices.

The prices offered to each clients group for different values of β are depicted in Fig. 20. As the risk-aversion increases (decreasing standard deviation of profit), observe that the price offered also increases. An increase in the selling price causes a reduction of both quantity and volatility of the demand supplied to the clients. In addition, the rise of the selling price is motivated by an increase of the forward contract purchases, because forward contracting is a more expensive electricity source than the pool.

The above problem, characterized by 7,925 constraints, 7,926 continuous variables, and 300 binary variables is solved using CPLEX 10.0 (The ILOG CPLEX Website 2010) under GAMS (Rosenthal 2008) on a Linux-based server with two

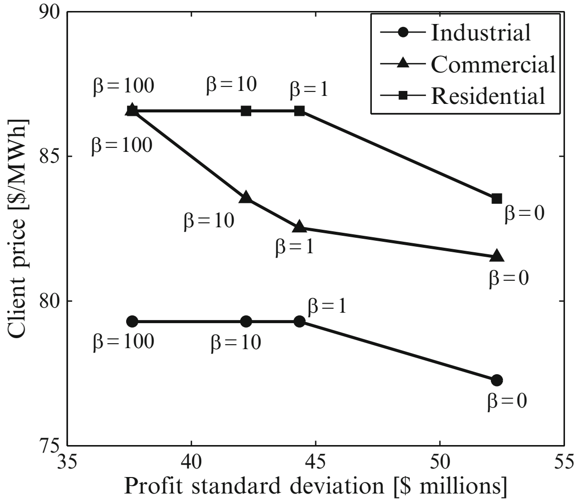

Fig. 20 Retailer case study: client price vs. profit standard deviation

processors clocking at 2.4 GHz and 8 GB of RAM. All cases has been solved in less than 13 s.

4 Conclusions

Stochastic programming is an appropriate methodology to formulate the medium-term (e.g., 1 year) trading problems faced by producers and retailers in an electricity market. This chapter provides models that allow a power producer/retailer engaging in forward contracting to maximize its expected profit for a given risk-level.

A power producer facing the possibility of signing forward contracts has to deal with uncertain pool prices. For the case of a power retailer, it copes with not only uncertain pool prices, but also with end-user demands dependent on the selling price offered by the retailer. Pool prices and demands are stochastic processes that can be characterized through a scenario set that properly represents the diverse realization of these stochastic processes. Care should be exercised in the generation of scenarios so that the solution attained is independent of any specific scenario set.

The resulting mathematical programming models are generally large-scale mixed-integer linear programming problems that can be solved using commercially available software. If these problems become intractable, then scenario-reduction or decomposition techniques make them tractable.

The results provided by these models include the efficient frontier giving expected profit vs. profit standard deviation (risk). Efficient frontier curves are used by decision-makers to achieve informed decisions on futures market, pool involvement, and client prices.

Acknowledgements We are thankful to A. Canoyra, Á. Caballero, and A. de Andrés from UNION FENOSA Generación for insightful comments and relevant observations linking our models to the actual world.

Appendix: Conditional Value at Risk

The conditional value at risk (CVaR) is a measure of risk of particular importance in applications where uncertainty is modeled through scenarios (Rockafellar and Uryasev 2002).

Given the probability distribution function of the profit (see Fig. 21), the CVaR is the expected profit not exceeding a value called value at risk (VaR), that is,

$$\mathrm{CVaR} = \mathcal{E}\{\mathrm{profit}|\mathrm{profit} \leq \mathrm{VaR}\}, \tag{22}$$

where $\mathcal{E}$ is the expectation operator.

The VaR is a measure computed as

$$\mathrm{VaR} = \max\{\mathrm{x}|\mathrm{p}\{\mathrm{profit} \leq \mathrm{x}\} \leq 1-\alpha\}, \tag{23}$$

where confidence level α usually lies between 0.9 and 0.99.

If profit is described via scenarios, CVaR can be computed as the solution of Rockafellar and Uryasev (2000):

$$\underset{\zeta,\,\eta(\omega)}{\text{maximize}} \quad \zeta - \frac{1}{(1-\alpha)} \sum_{\omega=1}^{N_\omega} p(\omega)\eta(\omega) \tag{24}$$

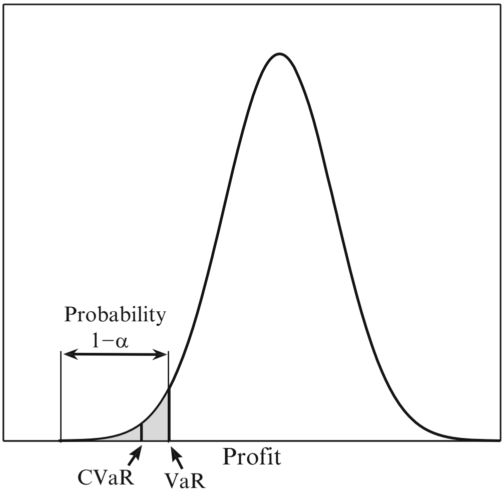

Fig. 21 Conditional value at risk (CVaR)

subject to

$$-\text{profit}(\omega) + \zeta - \eta(\omega) \leq 0 \quad \forall \omega \;, \tag{25}$$

$$\eta(\omega) \geq 0 \quad \forall \omega \;, \tag{26}$$

where ω is the scenario index, ζ is the VaR at the optimum, $p(\omega)$ is the probability of scenario ω, profit(ω) is the profit for scenario ω, and $\eta(\omega)$ is a variable whose value at the optimum is equal to zero if the scenario ω has a profit greater than VaR. For the rest of scenarios, $\eta(\omega)$ is equal to the difference of VaR and the corresponding profit. N_ω is the total number of considered scenarios.

To consider risk, the objective function (24) is added to the original objective function of the considered producer/retailer problem using the weighting factor β (Rockafellar and Uryasev 2000, 2002), while constraints (25)–(26) are incorporated as additional constraints to that problem. Note that constraints (25) couple together the variables related to risk and the variables pertaining to the producer/retailer problem through the "profit(ω)" term, which is the profit obtained by the producer/retailer in scenario ω.

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A Decision Support System for Generation Planning and Operation in Electricity Markets

Andres Ramos, Santiago Cerisola, and Jesus M. Latorre

Abstract This chapter presents a comprehensive decision support system for addressing the generation planning and operation. It is hierarchically divided into three planning horizons: long, medium, and short term. This functional hierarchy requires that decisions taken by the upper level model will be internalized by the lower level model. With this approach, the position of the company is globally optimized. This set of models presented is specially suited for hydrothermal systems. The models described correspond to long-term stochastic market planning, medium-term stochastic hydrothermal coordination, medium-term stochastic hydro simulation, and short-term unit commitment and bidding. In the chapter it is provided a condensed description of each model formulation and their main characteristics regarding modeling detail of each subsystem. The mathematical methods used by these models are mixed complementarity problem, multistage stochastic linear programming, Monte Carlo simulation, and multistage stochastic mixed integer programming. The algorithms used to solve them are Benders decomposition for mixed complementarity problems, stochastic dual dynamic programming, and Benders decomposition for SMIP problems.

Keywords Electricity competition · Market models · Planning tools · Power generation scheduling

1 Introduction

Since market deregulation was introduced in the electric industry, the generation companies have shifted from a cost minimization decision framework to a new one where the objective is the maximization of their expected profit (revenues minus costs). For this reason, electric companies manage their own generation resources

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and need detailed operation planning tools. Nowadays, planning and operation of generation units rely upon mathematical models whose complexity depends on the detail of the model that needs to be solved.

These operation planning functions and decisions that companies address are complex and are usually split into *very-long-term*, *long-term*, *medium-term*, and *short-term* horizons (Wood and Wollenberg 1996). The very long-term horizon decisions are mainly investment decisions and their description and solution exceed the decision support system (DSS) that we are presenting in this chapter. The long-term horizon deals with risk management decisions, such as electricity contracts and fuel acquisition, and the level of risk that the company is willing to assume. The medium-term horizon decisions comprise those economic decisions such as market share or price targets and budget planning. Also operational planning decisions like fuel, storage hydro, and maintenance scheduling must be determined. In the short term, the final objective is to bid to the different markets, based on energy, power reserve, and other ancillary services, for the various clearing sessions. As a result, and from the system operation point of view, the company determines first the unit commitment (UC) and economic dispatch of its generating units, the water releases, and the storage and pumped-storage hydro system operation.

In hydrothermal systems and in particular in this DSS, special emphasis is paid to the representation of the hydro system operation and to the hydro scheduling for several reasons (Wood and Wollenberg 1996):

- Hydro plants constitute an energy source with very low variable costs and this is the main reason for using them. Operating costs of hydro plants are due to operation and maintenance and, in many models, they are neglected with respect to thermal units' variable costs.
- Hydro plants provide a greater regulation capability than other generating technologies, because they can quickly change their power output. Consequently, they are suitable to guarantee the system stability against contingencies.
- Electricity is difficult to store, even more when considering the amount needed in electric energy systems. However, hydro reservoirs and pumped-storage hydro plants give the possibility of accumulating energy, as a volume of water. Although in some cases the low efficiency of pumped-storage hydro units may be a disadvantage, usually this water storage increases the flexibility of the daily operation and guarantees the long-term electricity supply.

In this chapter we present a proposal for covering these three hierarchical horizons with some planning models, which constitute a DSS for planning and operating a company in an electricity market. The functional hierarchy requires that decisions taken by the long-term level will be internalized by the medium-term level and that decisions taken by the medium-term level will be internalized by the short-term level. With this approach, the position of the company is globally optimized. At an upper level, a stochastic market equilibrium model (Cabero et al. 2005) with monthly periods is run to determine the hydro basin production, as well as fuel and electricity contracts, while satisfying a certain risk level. At an intermediate level, a medium-term hydrothermal coordination problem obtains weekly water release tables for large reservoirs and weekly decisions for thermal units. At a lower level, a stochastic simulation model (Latorre et al. 2007a) with daily periods incorporates those water release tables and details each hydro unit power output. Finally, a detailed strategic UC and bidding model defines the commitment and bids to be offered to the energy market (Baillo et al. 2004; Cerisola et al. 2009). In Fig. 1 the hierarchy of these four models is represented. Different mathematical methods are used for modeling the electric system: mixed complementarity problem (MCP) (Cottle et al. 1992), multistage stochastic programming (Birge and Louveaux 1997), Monte Carlo simulation (Law and Kelton 2000), and stochastic mixed integer programming. With the purpose of solving realistic-sized problems, those models are combined with special purpose algorithms such as Benders decomposition and stochastic dual dynamic programming to achieve the solution of the proposed models.

For using the DSS in real applications, it is important to validate the consistency of the system results. As it can be observed from Fig. 1, several feedback loops are included to check the coherence of the DSS results. As the upper level models pass target productions to lower level models, a check is introduced to adjust the system results to those targets with some tolerance.

Although the DSS conception is general, it has been applied to the Spanish electric system, as can be seen in the references mentioned for each model. In the following Table 1 we present the summary of the demand balance and the installed capacity by technologies of the mainland Spanish system in 2008, taken from Red Electrica de Espana (2008).

2 Long-term Stochastic Market Planning Model

In a liberalized framework, market risk management is a relevant function for generating companies (GENCOs). In the long term they have to determine a risk management strategy in an oligopolistic environment. Risk is defined as the probability of a certain event times the impact of the event in the company's objective of an expected return. Some of the risks that the GENCOs face are operational risk, market risk, credit risk, liquidity risk, and regulatory risk. Market risk accounts for the risk that the value of an investment will decrease due to market factors' movements. It can be further divided into equity risk, interest rate risk, currency risk, and commodity risk. For a GENCO the commodity risk is mainly due to the volatility in electricity and fuel prices, in unregulated hydro inflows and in the demand level.

The purpose of the long-term model of the DSS is to represent the generation operation by a market equilibrium model based on a conjectural variation approach, which represents the implicit elasticity of the residual demand function. The model decides the total production for the considered periods (months) and the position in futures so as to achieve the acceptable risk for its profit distribution function. Stochasticity of random variables are represented by a scenario tree that is computed by clustering techniques (Latorre et al. 2007b). Traditionally, models that represent

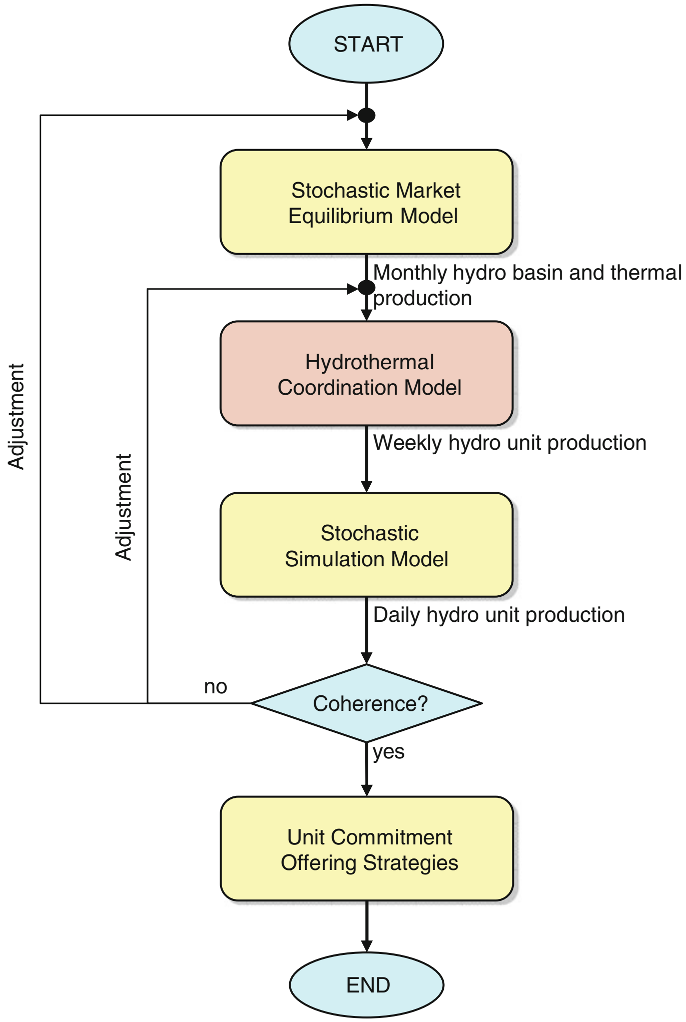

Fig. 1 Hierarchy of operation planning models

market equilibrium problems are based on linear or mixed complementarity problem (Hobbs et al. 2001a; Rivier et al. 2001), equivalent quadratic problem (Barquin et al. 2004), variational inequalities (Hobbs and Pang 2007), and equilibrium problem with equilibrium constraints (EPEC) (Yao et al. 2007). The formulation of this problem is based on mixed complementarity problem (MCP), that is, the combination of Karush–Kuhn–Tucker (KKT) optimality conditions and complementary slackness conditions, and extends those techniques that traditionally are used to represent the market equilibrium problem to a combined situation that simultaneously

Table 1 Demand balance and installed capacity of mainland Spanish electric system

	GWh	MW
Nuclear	58,756	7,716
Coal	46,346	11,359
Combined cycle	91,821	21,667
Oil/Gas	2,454	4,418
Hydro	21,175	16,657
Wind	31,102	15,576
Other renewable generation	35,434	12,552
Pumped storage hydro consumption	3,494	
International export	11,221	
Demand	263,961	

considers the market equilibrium and the risk management decisions, in a so-called integrated risk management approach.

2.1 Model Description

The market equilibrium model is stated as the profit maximization problem of each GENCO subject to the constraint that determines the electricity price as a function of the demand, which is the sum of all the power produced by the companies. Each company profit maximization problem includes all the operational constraints that the generating units must satisfy. The objective function is schematically represented by (Fig. 2).

In the long term the demand is represented by a load duration curve divided into peak, shoulder, and off-peak levels by period, the period being a month. For each load level the price p is a linear function of the demand d,

$$p = p_0 - p_0' \sum_c q_c \tag{1}$$

$$d = \sum_c q_c = q_c + q_c^- \tag{2}$$

p_0, p_0' being the intercept and slope of the inverse demand function, q_c the production of company c, and q_c^- the production of the remaining companies different from c. Then, the profit of each company becomes quadratic with respect to the quantities offered by companies. It accounts for those revenues that depend on the spot price and those that depend on long-term electricity contracts or take-or-pay fuel contracts. Schematically it can be stated as

$$\max \sum_\omega pr^\omega \sum_{t \in c} \left[p^\omega q_t^\omega - c^\omega (q_t^\omega) \right] \tag{3}$$

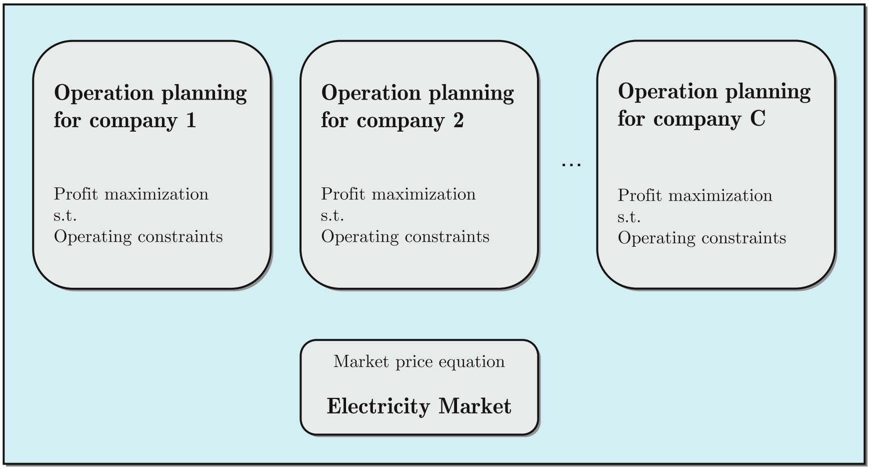

Fig. 2 Market equilibrium problem

ω being any scenario of the random variates, pr^{ω} the corresponding probability, p^{ω} the price, q_t^{ω} the energy produced by thermal unit t belonging to company c in scenario ω, and $c^{\omega}(q_t^{\omega})$ the thermal variable costs depending quadratically on the output of thermal units.

When considering the Cournot's approach, the decision variable for each company is its total output, while the output from competitors is considered constant. In the conjectural variation approach the reaction from competitors is included into the model by a function that defines the sensitivity of the electricity price with respect to the output of the company. This function may be different for each company:

$$\frac{\partial p}{\partial q_c} = -p_0' \left(1 + \frac{\partial q_c^-}{\partial q_c}\right). \tag{4}$$

Operating constraints include fuel scheduling of the power plants, hydro reservoir management for storage and pumped-storage hydro plants, run-of-the-river hydro plants, and operation limits of all the generating units.

We incorporate in the model several sources of uncertainty that are relevant in the long term, such as water inflows, fuel prices, demand, electricity prices, and output of each company sold to the market. We do this by classifying historical data into a multivariate scenario tree. The introduction of uncertainty extends the model to a stochastic equilibrium problem and gives the company the possibility of finding a hedging strategy to manage its market risk. With this intention, we force currently future prices to coincide with the expected value of future spot prices that the equilibrium returns for each node of the scenario tree. Future's revenues are calculated as gain and losses of future contracts that are canceled at the difference between future and spot price at maturity. Transition costs are associated to contracts and computed when signed.

The risk measure used is the *Conditional Value at Risk* (CVaR), which computes the expected value of losses for all the scenarios in which the loss exceeds the *Value at Risk* (VaR) with a certain probability α, see (5).

$$CVaR_X(\alpha) = \mathbb{E}\left[X | X \geq VaR_X(\alpha)\right], \tag{5}$$

where X are the losses, $CVaR_X(\alpha)$ and $VaR_X(\alpha)$ are the CVaR and VaR of α quantile.

All these components set up the mathematical programming problem for each company, which maximizes the expected revenues from the spot and the futures market minus the expected thermal variable costs and minus the expected contract transaction costs. The operating constraints deal with fuel scheduling, hydro reservoir management, operating limits of the units for each scenario, while the financial constraints compute the CVaR for the company for the set of scenarios. Linking constraints for the optimization problems of the companies are the spot price equation and the relation of future price as the expectation of future spot prices.

The KKT optimality conditions of the profit maximization problem of each company together with the linear function for the price define a *mixed linear complementarity problem*. Thus the market equilibrium problem is created with the set of KKT conditions of each GENCO plus the price equation of the system, see Rivier et al. (2001). The problem is linear if the terms of the original profit maximization problem are quadratic and, therefore, the derivatives of the KKT conditions become linear.

The results of this model are the output of each production technology for each period and each scenario, the market share of each company, and the resulting electricity spot price for each load level in each period and each scenario. Monthly hydro system and thermal plant production are the magnitudes passed to the medium-term hydrothermal coordination model, explained below.

3 Medium-term Stochastic Hydrothermal Coordination Model

By nature, the medium-term stochastic hydrothermal coordination models are high-dimensional, dynamic, nonlinear, stochastic, and multiobjective. Solving these models is still a challenging task for large-scale systems (Labadie 2004). One key question for them is to obtain a feasible operation for each hydro plant, which is very difficult because models require a huge amount of data, due to complexity of hydro systems and by the need to evaluate multiple hydrological scenarios. A recent review of the state of the art of hydro scheduling models is done in Labadie (2004).

According to the treatment of stochasticity hydrothermal coordination models are classified into deterministic and stochastic ones.

- *Deterministic models* are based on network flows, linear programming (LP), nonlinear programming (NLP), or mixed integer linear programming (MILP), where binary variables come from commitment decisions of thermal or hydro units or from piecewise linear approximation of nonlinear and nonconvex water head effects. For taking into account these nonlinear effects, successive LP solutions are often used. This process does not converge necessarily to the optimal solution, see Bazaraa et al. (1993).

- *Stochastic models* are represented by stochastic dynamic programming (SDP), stochastic linear programming (SLP) (Seifi and Hipel 2001), and stochastic nonlinear programming (SNLP). For SLP problems decomposition techniques like Benders (Jacobs et al. 1995), Lagrangian relaxation, or stochastic dual dynamic programming (SDDP) (Pereira and Pinto 1991) can be used.

In this medium-term model, the aggregation of all the hydro plants of the same basin in an equivalent hydro unit (as done for the long-term model) is no longer kept. We deal with hydro plants and reservoir represented individually, as well as we include a cascade representation of their physical connections. Besides, thermal power units are considered individually. Thus, rich marginal cost information is used for guiding hydro scheduling.

The hydrothermal model determines the optimal yearly operation of all the thermal and hydro power plants for a complete year divided into decision periods of 1 week. The objective function is usually based on cost minimization because the main goal is the medium-term hydro operation, and the hydro releases have been determined by the upper level market equilibrium model. Nevertheless, the objective function can be easily modified to consider profit maximization if marginal prices are known (Stein-Erik and Trine Krogh 2008), which is a common assumption for fringe companies.

This model has a 1 year long scope beginning in October and ending in September, which models a hydraulic year, with special emphasis in large reservoirs (usually with annual or even hyperannual management capability). Final reserve levels for large reservoirs are given to the model to avoid the initial and terminal effects on the planning horizon. Uncertainty is introduced in natural inflows and the model is used for obtaining optimal and "feasible" water release tables for different stochastic inflows and reservoir volumes (Fig. 3).

The demand is modeled in a weekly basis with constant load levels (peak and off-peak hours, e.g.). Thermal units are treated individually and commitment decisions are considered as continuous variables, given that the model is used for medium-term analysis. For hydro reservoirs a different modeling approach is followed depending on the following:

- *Owner company*: Own reservoirs are modeled in water units (volume in hm^3 and inflow in $m^3\ s^{-1}$) while reservoirs belonging to other companies are modeled in energy units as equivalent and independent power plants with one reservoir each, given that the reservoir characteristics of the competitors are generally ignored.
- *Relevance of the reservoir*: Important large reservoirs are modeled with nonlinear water head effects while smaller reservoirs are represented with a linear dependency; therefore, the model does not become complex unnecessarily.

Unregulated hydro inflows are assumed to be the dominant source of uncertainty in a hydrothermal electric system. Temporal changes in reservoir reserves

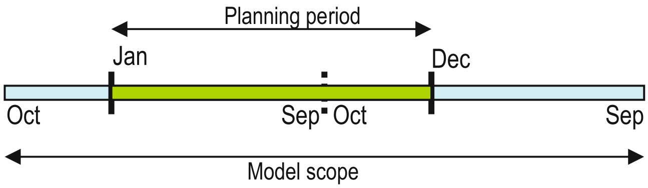

Fig. 3 Model scope for yearly operation planning

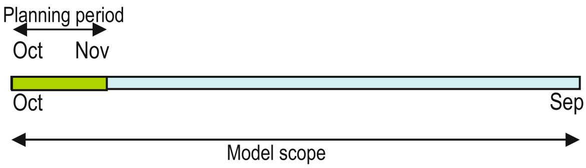

Fig. 4 Model scope for next future decisions under uncertain inflows

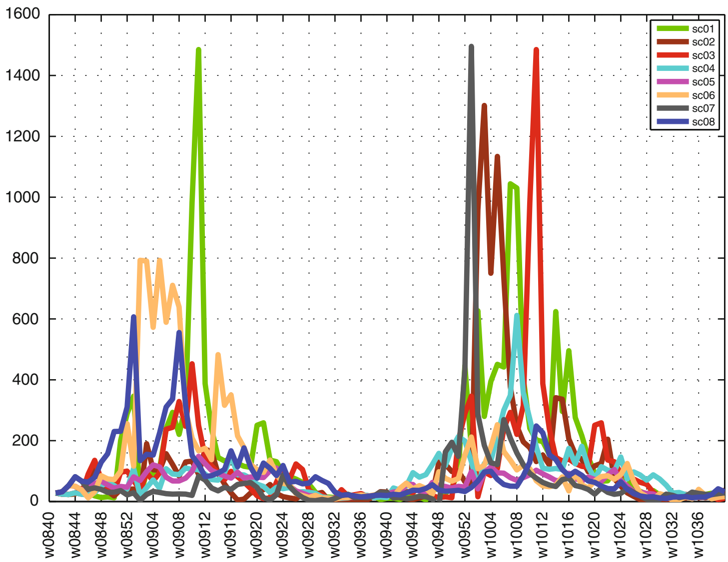

Fig. 5 Scenario tree with eight hydro inflows' scenarios from week 40 of year 2008 to week 39 of year 2010 expressed in ($m^3\,s^{-1}$)

are significant because of stochasticity in hydro inflows, highly seasonal pattern of inflows, and capacity of each reservoir with respect to its own inflow (Fig. 4).

Stochasticity in hydro inflows is represented for the optimization problem by means of a multivariate scenario tree, see Fig. 5 as a real case corresponding to a specific location. This tree is generated by a neural gas clustering technique (Latorre et al. 2007b) that simultaneously takes into account the main stochastic inflow series and their spatial and temporal dependencies. The algorithm can take historical or synthetic series of hydro inflows as input data. Very extreme scenarios can be artificially introduced with a very low probability. The number of scenarios generated is enough for medium-term hydrothermal operation planning.

3.1 Model Description

The *constraints* introduced into the model are the following:

- Balance between generation and demand including pumping: Generation of thermal and storage hydro units minus consumption of pumped-storage hydro units is equal to the demand for each scenario, period (week), and subperiod (load level).
- Minimum and maximum yearly operating hours for each thermal unit for each scenario: These constraints are relaxed by introducing slack and surplus variables that are penalized in the objective function. Those variables can be strictly necessary in the case of many scenarios of stochasticity. This type of constraints is introduced to account for some aspects that are not explicitly modeled into this model like unavailability of thermal units, domestic coal subsidies, CO_2 emission allowances, capacity payments, etc.
- Minimum and maximum yearly operating hours for each thermal unit for the set of scenarios.
- Monthly production by thermal technology and hydro basin: These constraints establish the long-term objectives to achieve by this medium-term model.
- Water inventory balance for large reservoirs modeled in water units: Reservoir volume at the beginning of the period plus unregulated inflows plus spillage from upstream reservoirs minus spillage from this reservoir plus turbined water from upstream storage hydro plants plus pumped water from downstream pumped-storage hydro plants minus turbined and pumped water from this reservoir is equal to reservoir volume at the end of the period. An artificial inflow is allowed and penalized in the objective function. Hydro plant takes water from a reservoir and releases it to another reservoir. The initial value of reservoir volume is assumed known. No lags are considered in water releases because 1 week is the time period unit.
- Energy inventory balance for reservoirs modeled in energy: Reservoir volume at the beginning of the period plus unregulated inflows minus spillage from this reservoir minus turbined water from this reservoir is equal to reservoir volume at the end of the period. An artificial inflow is allowed and penalized in the objective function. The initial value of reservoir volume is assumed known.
- Hydro plant generation is the product of the water release and the production function variable (also called efficiency): This is a nonlinear nonconvex constraint that considers the long-term effects of reservoir management.
- Total reservoir release is equal to the sum of reservoir releases from each downstream hydro plant.

- Pumping from a reservoir: Pumped water is equal to the pumped-storage hydro plant consumption divided by the production function.
- Achievement of a given final reservoir volume with slack and surplus variables: This final reserve is determined by the upper level long-term stochastic market equilibrium model of the DSS. The reserve levels at the end of each month of the problem are also forced to coincide with those levels proposed by the stochastic market equilibrium model.
- Minimum and maximum reservoir volume per period with slack and surplus variables: Those bounds are included to consider flood control curve, dead storage, and other plan operation concerns. The slack variables will be strictly necessary in the case of many scenarios.
- Computation of the plant water head and the production function variable as a linear function: Production function variable is a linear function of the water head of the plant that is determined as the forebay height of the reservoir minus the tailrace height of the plant. Tailrace height of the plant is the maximum of the forebay height of downstream reservoir and the tailrace height of the plant.
- Computation of the reservoir water head and the reservoir volume as a nonlinear function: Reservoir water head is determined as the forebay height minus the reference height. Reserve volume is a quadratic function of the reservoir water head.
- Variable bounds, that is, reservoir volumes between limits for each hydro reservoir and power operation between limits for each unit.

The *multiobjective function* minimizes:

- Thermal variable costs plus

$$\min \sum_{t\omega} pr^{\omega} c_t q_t^{\omega} \tag{6}$$

q_t^{ω} the energy produced by thermal unit t in scenario ω and c_t the variable cost of the unit.
- Penalty terms for deviations from the proposed equilibrium model reservoir levels, that is, slack or surplus of final reservoir volumes, exceeding minimum and maximum operational rule curves, artificial inflows, etc.
- Penalty terms for relaxing constraints like minimum and maximum yearly operation hours of thermal units.

It is important for this model to obtain not only optimal solutions but also feasible solutions that can be implemented. Different solutions and trade-offs can be obtained by changing these penalties.

The main results for each load level of each period and scenario are storage hydro, pumped-storage hydro and thermal plant operation, reservoir management, basin and river production, and marginal costs. As a byproduct the optimal water release tables for different stochastic inflows and reservoir volumes are obtained. They are computed by stochastic nested Benders' decomposition technique (Birge and Louveaux 1997) of a linear approximation of the stochastic nonlinear optimization problem. These release tables are also used by the lower level daily stochastic simulation model, as explained in the next section.

4 Medium-term Stochastic Simulation Model

Simulation is the most suitable technique when the main objective is to analyze complex management strategies of hydro plants and reservoirs and their stochastic behavior. Simulation of hydrothermal systems has been used in the past for two main purposes:

- *Reliability analysis of electric power systems.* An example of this is given in Roman and Allan (1994), where a complete hydrothermal system is simulated. The merit order among all the reservoirs to supply the demand is determined as a function of their reserve level. Simulated natural hydro inflows and transmission network are considered. The goal is to determine the service reliability in thermal, hydro, or hydrothermal systems.
- *Hydrothermal operation.* In De Cuadra (1998) a simulation scheme for hydrothermal systems is proposed, where medium and long-term goals (maintenance, yearly hydro scheduling) are established. The system is simulated with stochastic demand and hydro inflows. For each day an optimization problem is solved to achieve the goals obtained from long-term models.

The hydro simulation model presented in this section takes into account the detailed topology of each basin and the stochasticity in hydro inflows, see Latorre et al. (2007a). It is directly related to the previous medium-term hydrothermal model, based on optimization. The stochastic optimization model guides the simulation model through a collection of hydro weekly production objectives that should be attained in different weeks for the largest hydro reservoirs. Once these guidelines are provided, the simulation model checks the feasibility of these goals, may test the simplifications made by the optimization model, and determines the energy output of hydro plants, the reserve evolution of the reservoirs and, therefore, a much more detailed daily operation. This double hierarchical relation among different planning models to determine the detailed hydro plants operation has also been found in Turgeon and Charbonneau (1998). It is a dynamic model, whose input data are series of historical inflows in certain basins' spots. Historical or synthetic series (obtained by forecasting methods) can be used for simulation. For this reason, it is also a stochastic model. Finally, system state changes take place once a day. These events can be changes of inflows, scheduled outages, etc. Consequently, the model is discrete with 1 day as time step. This is a reasonable time step because the usual model scope is 1 year and no hourly information is needed. The simulation model deals with plausible inflow scenarios and generates statistics of the hydro operation. Its main applications are the following:

- Comparison of several different reservoir management strategies
- Anticipation of the impact of hydro plant unavailabilities for preventing and diminishing the influence of floods
- Increment of hydro production by reducing the spillage

The model is based on the object-oriented paradigm and defines five classes that are able to represent any element of a hydro basin. The object oriented programming (OOP) paradigm (Wirfs-Brock et al. 1990) becomes very attractive for simulation because it allows to encapsulate the basic behavior of the elements and permits the independent simulation of each system element, which simply needs to gather information from incoming water flows from the closest elements to it. The model incorporates a simulation algorithm in three phases that decides the production of the hydro plants following several strategies about reservoir management. These management strategies and a description of the five classes are presented next.

4.1 Data Representation

A natural way to represent the hydro basin topology is by means of a graph of nodes, each one symbolizing a basin element. Those nodes represent reservoirs, plants, inflow spots, and river junctions. Nodes are connected among them by arcs representing water streams (rivers, channels, etc). Each node is independently managed, although it may require information about the state of other upstream basin elements. As a result of this data structure, object-oriented programming is a suitable approach to solve the problem of the simulation of a hydro basin (Fig. 6).

Analyzing real hydro basin patterns, we have concluded that five classes are enough to represent adequately every possible case. These object types are described in the next section. Additionally, different reserve management strategies can be pursued in a reservoir element. These nodes represent reservoirs, channels, plants, inflows, and river junctions, which are now described.

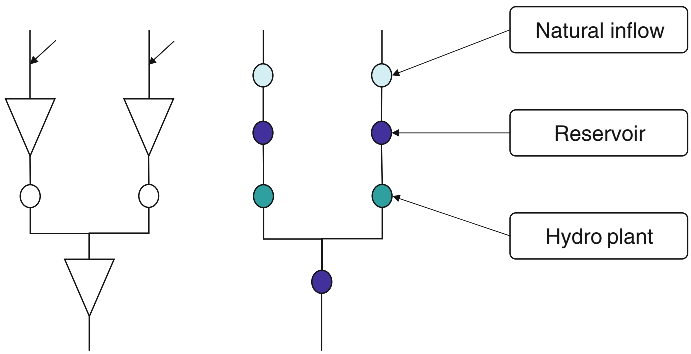

Fig. 6 Basin topology represented by a graph of nodes

4.1.1 Reservoirs

The objects representing reservoirs have one or more incoming water streams and only one outgoing. Apart from other technical limitations, they may have a minimum outflow release, regarding irrigation needs, sporting activities, or other environmental issues. Besides, they may have rule volume curves guiding their management. Examples are minimum and maximum rule curves, which avoid running out of water for irrigation or spillways risk.

Reservoirs are the key elements where water management is done. The chosen strategy decides its outflow, taking into account minimum and maximum guiding curves, absolute minimum and maximum volume levels, and water release tables. The different management strategies are described in Sect. 4.2.

4.1.2 Channels

These elements carry water between other basin elements, like real water streams do. They do not perform any water management: they just transport water from their origin to their end. However, they impose an upper limit to the transported water flow, which is the reason to consider them.

4.1.3 Plants

Kinetic energy from the water flow that goes through the turbine is transformed into electricity in the plant. In electric energy systems, hydro plants are important elements to consider due to their flexibility and low production costs. However, in this simulation model water management is decided by the reservoir located upstream. Hence, from this point of view they are managed in the same fashion as channels: they impose an upper limit to the transported flow.

As a simulation result, electric output is a function of the water flow through the plant. This conversion is done by a production function depending on the water head, which is approximated linearly. Water head is the height between the reservoir level and the maximum between the drain level and the level of the downstream element. In hydro plants, once the water flow has been decided, daily production is divided between peak and off-peak hours, trying to allocate as much energy as possible in peak hours where expensive thermal units are producing.

In addition, some plants may have pumped-storage hydro units, which may take water from downstream elements and store it in upstream elements (generally, both elements will be reservoirs). It is important to emphasize that, in this simulation model, pumping is not carried out with an economic criterion, as it does not consider the thermal units, but with the purpose of avoiding spillage.

4.1.4 Natural Inflows

These objects introduce water into the system. They represent streamflow records where water flow is measured, coming from precipitation, snowmelt, or tributaries. These elements have no other upstream elements. The outflow corresponds to the day been simulated in the series. These series may come from historic measures or from synthetic series obtained from forecasting models based on time series analysis.

4.1.5 River Junctions

This object groups other basin elements where several rivers meet. An upper bound limits the simultaneous flow of all the junction elements. An example of this object appears when two reservoirs drain to the same hydro plant. As both reservoirs share the penstock, this element has to coordinate both reservoirs' behavior.

4.2 Reservoir Management Strategies

Reservoir management is the main purpose of the simulation; the rest of the process is an automatic consequence of this. Different strategies represent all possible real alternatives to manage reservoirs with diverse characteristics. These strategies combine the implicit optimization of the upper level models with operational rules imposed by the river regulatory bodies to the electric companies. They are discussed in the following paragraphs.

4.2.1 Water Release Table

This strategy is used for large reservoirs that control the overall basin operation. Typically, those reservoirs are located at the basin head. The water release table is determined by the long-term optimization model and gives the optimal reservoir outflow as a multidimensional function of the week of the year of the simulated day, the inflows of the reservoir, the volume of the reservoir being simulated, and the volume of another reservoir of the same basin, if that exists, that may serve as a reference of the basin hydrological situation. The reservoir outflow is computed by performing a multidimensional interpolation among the corner values read from the table.

4.2.2 Production of the Incoming Inflow

This strategy is specially indicated for small reservoirs. Given that they do not have much manageable volume, they must behave as run-of-the-river plants and drain the incoming inflow.

4.2.3 Minimum Operational Level

In this strategy, the objective is to produce as much flow as possible. Of course, when the reservoir level is below this operational level no production is done. When the volume is above this minimum operational level, the maximum available flow must be turbined to produce electricity. This strategy can be suitable for medium-sized reservoirs when little energy is available in the basin.

4.2.4 Maximum Operational Level

With this strategy, the volume is guided to the maximum operational level curve. This is a curve that prevents flooding or spillage at the reservoirs. The reason behind this operation is that when the water head is larger, the production will be higher. However, in case of extreme heavy rain it can be dangerous to keep the reservoir at this level. This strategy can be suitable for medium-sized reservoirs when enough energy is available in the basin.

4.3 Simulation Method

Simulating a hydro basin allows to observe its evolution for different possible hydro inflows. Operation of hydro units follows the management goals of the reservoirs and the limitations of the other river basin objects. However, other factors can force changes in previous decisions, for example, avoiding superfluous spillage and assurance of minimum outflows. To achieve this purpose we propose a three-phase simulation method consisting in these phases:

1. Decide an initial reservoir management, considering each reservoir independently. It also computes the ability of each reservoir to modify its outflow without reaching too low or high water volumes.
2. Modify the previous management to avoid spillage or to supply irrigation and ecological needs. This uses the modification limits computed in the previous step.
3. Determine the hydro units' output with the final outflows decided in the previous step.

Results are obtained for each series, both in the form of detailed daily series and mean values, and mean and quantiles of the weekly values are also calculated. This permits general inspection of the results for each reservoir as well as a more thorough analysis of the daily evolution of each element of the river basin.

5 Short-term Unit Commitment and Bidding Strategies

The last step in the decision process is faced once the weekly production decisions for the thermal units and the daily hydro production are obtained with the medium-term stochastic optimization and simulation models, respectively. The optimal unit commitment schedule for the next day will comply with those decisions taken by upper level models of the DSS.

We assume an electricity market where agents have to indicate, by means of an offering curve, the amount of energy they are willing to sell for different prices for each of the 24 h of the next day. In the same manner, agents willing to purchase electricity may indicate the quantities they are willing to buy for different prices. An independent system operator (ISO) intersects both curves and sets the hourly price for the very next day. Those prices are denoted as market clearing prices. We focus our attention on a marginal pricing scheme where those offers with prices less than the market clearing price are accepted, while those whose price is higher than the market clearing price are rejected.

In this framework, companies are responsible of their offer and suffer the uncertainty of the disclosure of the market price, which is mainly induced by the uncertain behavior of the agents. Even more, if they have a large market share, their own offer may affect the final price. We model this uncertainty of the market clearing price by means of the residual demand function. The residual demand function is created for each company eliminating from the purchase offer curve the sell offer curves of the remaining agents. By doing this we obtain a function that relates the final price to the total amount of energy that the company may sell. Once this relation is available, the company may optimize its benefit determining the amount of energy that maximizes their profit, defined as the difference between the revenues earned in the market and the production costs.

The residual demand function so far commented is clearly unavailable before the market clearing process is done, and thus the company has to estimate it based on historical data (e.g., from the market operator). Nevertheless to say, for a relative small company that can be considered as a price taker, it is just enough to estimate the market prices for the next day.

With the purpose of deciding the committed units for the next day and with the intention of including the weekly thermal and hydro production decision taken by the DSS, we consider the problem of operating a diversified portfolio of generation units with a 1 week time horizon. This problem decides the hourly power output of each generation unit during the week of study, which implies choosing the generating units that must be operating at each hour. We introduce uncertainty by means of a weekly scenario tree of residual demand curves. The scenario tree branches at the beginning of each day and serial correlation is considered for the residual demand curves of the same day. This residual demand curves considered are neither convex nor concave, and we model them as well as the profit function by introducing binary variables.

So, the weekly unit commitment of the DSS is formulated as a large scale stochastic mixed integer programming problem. The problem decides the commitment schedule for the 7 days of the upcoming week, although just the solution of the very first day is typically the accepted solution. For the next day a new weekly unit commitment problem ought to be solved. For realistic large-scale problems a decomposition procedure may be used. The reader is referenced to Cerisola et al.

(2009), where a Benders-type algorithm for integer programming is applied for the resolution of a large-scale weekly unit commitment problem.

The objective function of the unit commitment problem maximizes the revenues of the company for the time scope:

$$\max \sum_h \Pi_h(q_h) = \sum_h p_h q_h = \sum_h R_h^{-1}(q_h) q_h, \tag{7}$$

where $\Pi_h(q_h)$ is the revenue of company in hour h, q_h the company output, p_h the spot price, $R_h(p_h)$ the residual demand faced by the company, and $R_h^{-1}(q_h)$ the inverse residual demand function.

In general, this revenue function is not concave but can be expressed as a piecewise-linear function by using auxiliary binary variables Cerisola et al. (2009).

We complete this section with a brief description of the *constraints* that constitute the mathematical problem of the unit commitment model:

- Market price is a function of total output and revenue is also a function of total output; both are modeled as piecewise linear equations using the δ-form as in Williams (1999).
- The production cost of each thermal unit is a linear function of its power output and its commitment state, which is modeled with a binary variable.
- The company profit is defined as the difference between the market revenue and the total operating cost.
- Maximum capacity and minimum output of thermal units are modeled together with the commitment state variable as usual. If the commitment state is off, the unit output will be zero.
- Ramp limits between hours are modeled linearly.
- Start-up and shut-down decisions are modeled as continuous variables, and their values decided by a dynamic relation between commitment states of consecutive hours.
- The power output for each hydro unit is decided by the higher level model of the DSS.
- The model forces the weekly thermal production of each plant to be equal to the decision of the medium-term hydrothermal coordination problem.

6 Conclusions

In this chapter we have presented a complete DSS that optimizes the decisions of a generation company by a hierarchy of models covering the long-term, medium-term, and short-term planning functions, see Fig. 7. The decisions taken from the highest model are passed to the lower level model and so on. These models are specially suited for representing a hydrothermal system with the complexities derived from

Table 2 Summary of characteristics of the models

	Long-term stochastic market planning	Medium-term stochastic hydrothermal coordination	Medium-term stochastic hydro simulation	Short-term unit commitment and bidding strategies
Scope	1 year	1 year	1 year	1 week
Time step	1 month	1 week	1 day	1 h
Stochastic variables	Fuel prices Hydro inflows aggregated monthly Demand	Hydro inflows aggregated weekly	Daily inflows	Residual demand curves
Stochastic representation	Monthly scenario tree	Weekly scenario tree	Series with given initial point	Daily scenario tree
Aggregation level	Hydro system aggregated by basins Thermal system aggregated by technologies	Hydro and thermal systems represented by units	Hydro units represented separately	Hydro units aggregated by offering unit Thermal units represented separately
Input data from higher level model	Risk management level (CVaR)	Hydro production for each month and basin Thermal production for each month and technology	Weekly water release tables for large reservoirs	Weekly production for thermal units Daily production for hydro units
Economic and operational decisions	Long-term fuel and electricity contracts Hydro production for each month and basin Thermal production for each month and technology	Budget planning Weekly water release tables for large reservoirs Weekly production for thermal plants	Daily production for hydro units	Bidding strategy for the next 24 h
Mathematical method	Mixed complementarity problem	Multistage stochastic linear programming	Monte Carlo simulation	Multistage stochastic mixed integer programming
Algorithm	Benders decomposition for mixed complementarity problems	Stochastic dual dynamic programming		Benders decomposition for MIP problems

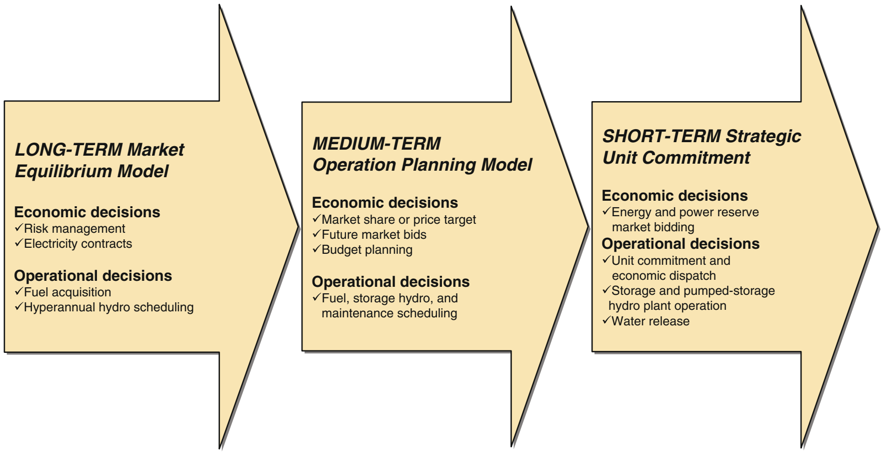

Fig. 7 Hierarchy of operation planning functions

hydro topology and stochastic hydro inflows that are conveniently incorporated into the models.

Table 2 summarizes the main characteristics of the models that are solved hierarchically passing the operation decisions to achieve the optimality of the planning process.

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A Partitioning Method that Generates Interpretable Prices for Integer Programming Problems

Mette Bjørndal and Kurt Jörnsten

Abstract Benders' partitioning method is a classical method for solving mixed integer programming problems. The basic idea is to partition the problem by dividing the variables into complicating variables, normally the variables that are constrained to be integer valued, and easy variables, normally the continuous variables. By fixing the complicating variables, a convex sub-problem is generated. Solving this convex sub-problem and its dual generates cutting planes that are used to create a master problem, which when solved generates new values for the complicating variables, which have a potential to give a better solution. In this work, we assume that the optimal solution is given, and we present a way in which the partitioning idea can be used to generate a valid inequality that supports the optimal solution. By adding some of the continuous variables to the complicating variables, we generate a valid inequality, which is a supporting hyperplane to the convex hull of the mixed integer program. The relaxed original programming problem with this supporting valid inequality added will produce interpretable prices for the original mixed integer programming problem. The method developed can be used to generate economically interpretable prices for markets with non-convexities. This is an important issue in many of the deregulated electricity markets in which the non-convexities comes from large start up costs or block bids. The generated prices are, in the case when the sub-problem generated by fixing the integer variables to their optimal values has the integrality property, also supported by nonlinear price functions that are the basis for integer programming duality.

Keywords Bender's partitioning method · Indivisibilities · Integer programming · Non-convexities · Pricing

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1 Introduction

One of the main reasons for the success of optimization formulations in economics is the existence of shadow prices, dual variables, which can be given economically meaningful interpretations. The dual variables are used to analyze the economic model under study, and to find out if a certain solution is optimal or can be further improved. The dual variables are also used in comparative static analyses, etc. As many economic situations involve non-convexities in the form of economies of scale, start up costs and indivisibilities, there have been several attempts to generate interpretable prices also for non-convex optimization problems. In this paper, we suggest a new way to generate prices for non-convex optimization models. It should be noted that the prices suggested are not intended as a means for finding the optimal solutions of the production problems, but rather as a means for communicating the optimality of the equilibrium to the various market participants. An excellent article on the relations between microeconomics and mathematical programming has been written by Scarf, Scarf (1990).

In the article "The Allocation of Resources in the Presence of Indivisibilities", Scarf (1994) states that, "The major problem presented to economic theory in the presence of indivisibilities in production is the impossibility of detecting optimality at the level of the firm, or for the economy as a whole, using the criterion of profitability based on competitive prices." Scarf illustrates the importance of the existence of competitive prices and their use in the economic evaluation of alternatives by considering a hypothetical economic situation which is in equilibrium in the purest Walrasian sense. It is assumed that the production possibility set exhibits constant returns to scale so that there is a profit of zero at the equilibrium prices. Each consumer evaluates personal income at these prices, and market demand functions are obtained by the aggregation of individual utility maximizing demands. As the system is assumed to be in equilibrium, the market is clearing, and thus, supply equals demand for each of the goods and services in the economy. Because of technological advance, a new manufacturing technology has been presented. This activity also possesses constant returns to scale. The question is if this new activity is to be used at a positive level or not? In this setting the answer to the question is simple and straightforward. If the new activity is profitable at the old equilibrium prices, then there is a way to use the activity at a positive level, so that with suitable income redistributions, the welfare of every member of society will increase. Also, if the new activity makes a negative profit at the old equilibrium prices, then there is no way in which it can be used to improve the utility of all consumers, even allowing the most extraordinary scheme for income redistribution. This shows the strength of the pricing test in evaluating alternatives. However, the existence of the pricing test relies on the assumptions made, that is the production possibility set displays constant or decreasing returns to scale. When we have increasing returns to scale, the pricing test for optimality might fail. It is easy to construct examples showing this, for instance by introducing activities with start up costs. In the failure of a pricing test, Scarf introduces as an alternative a quantity test for optimality. Although elegant and based on a theory that has lead to the development of algorithms for parametric integer programming problems, we find it hard to believe that the quantity test suggested by Scarf will have the same impact in economic theory as the pricing test has had in the case where non-convexities do not exist.

Over time, a number of suggestions have been made to address the problem of finding prices for problems with non-convexities, especially for the case where the non-convexities are modelled using discrete variables. The aim has been to find dual prices and interpretations of dual prices in integer programming problems and mixed integer programming problems. The seminal work of Gomory and Baumol (1960) addresses this issue. The ideas in the Gomory and Baumol's work were later used to create a duality theory for integer programming problems, and Wolsey (1981) gives a good description of this theory, showing that, in the pure integer programming case, we need to expand our view of prices to price functions to achieve interpretable and computable duals. However, these dual price functions are rather complex, and other researchers have suggested approximate alternatives. Alternative prices and price interpretations for integer programming problems have been proposed by Alcaly and Klevorik (1966) and Williams (1996). None of these suggestions have so far, to our knowledge, been used successfully to analyze equilibrium prices in markets with non-convexities.

More recently, O'Neill et al. (2005) presented a method for calculating discriminatory prices, IP-prices, based on a reformulation of the non-convex optimization problem. The method is aimed at generating market clearing prices and assumes that the optimal production plan is known. The reason for this renewed interest in interpretable prices in markets with non-convexities is that most deregulated electricity markets have inherent non-convexities in the form of start-up costs or the allowance of block bidding. The papers by Gribik et al. (2007), Bjørndal and Jørnsten (2004, 2008) and Muratore (2008) are all focusing on interpretable prices in electricity markets.

In this paper, we show that a related idea, which we call modified IP-prices, has the properties that we are seeking. The modified IP-prices are in fact equivalent to the coefficients that are generated from the Benders' sub-problem when the complicating variables are held fixed at their optimal values. These prices are derived using a minor modification of the idea of O'Neill et al., and are based on the generation of a valid inequality that supports the optimal solution and can be viewed as a mixed integer programming version of the separating hyperplane that supports the linear price system in the convex case. The modified IP-prices generated are, for the pure integer case, based on integer programming duality theory, and it can be shown that in this case, there exist a non-linear non-discriminatory price function that supports the modified IP-prices. In the mixed integer programming case, further research is needed to find the corresponding non-discriminatory price function that supports the modified IP-prices. However, the modified IP- prices yield a non-linear price structure consisting of linear commodity prices and a fixed uplift fee.

2 Benders' Partitioning Method

Benders' decomposition, or Benders' partitioning method Benders (1962), was developed in the 1960s as a computational method to calculate an optimal solution for mathematical programming problems in which the variables can be partitioned into complicating and non-complicating variables. In its original version, the method was developed as a means of solving linear mixed integer programming problems. However, the method has been used also to generate computational methods for general mathematical programming problems, where some of the variables make the optimization problem non-convex, whereas the problem is a standard convex optimization problem when these complicating variables are held fixed. The theoretical basis of the method relies on duality theory for convex optimization problems. For the complicating variables held fixed, the remaining problem, Benders' subproblem, is a convex optimization problem. Solving this problem and its dual gives us information in the form of cutting planes that involve a constant and a functional expression containing the complicating variables only.

We present the Benders' decomposition method for a problem of the form

$$\begin{aligned} & Min\ c^T x + f^T y \\ \text{s.t.}\quad & Ax + Fy \geq b \\ & x \geq 0 \\ & y \in S = \{y \,\big|\, Cy \geq d,\ y \geq 0 \ and\ y \in Z^+\} \end{aligned}$$

where Z^+ is the set of non-negative integers. Let R be the set of feasible variable values for y, that is

$$\mathrm{R} = \{y \mid \text{there exists}\ x \geq 0\ \text{such that}\ Ax \geq b - Fy\ ,\ y \in S\}$$

By the use of Farkas' lemma, the set R can be rewritten as

$$\mathrm{R} = \{y \,|(b - Fy\,)^T u_i^r \leq 0,\ i = 1, \ldots\ldots, n_r,\ y \in S\}$$

where n_r denotes the number of extreme rays in the cone $\mathrm{C} = \left\{u | A^T\ u \leq 0, u \geq 0\right\}$.

It is clear that if the set R is empty, then the original problem is infeasible. Assume now that we choose a solution $y \in R$. We can now rewrite the original problem into

$$Min_{y \in R}\ f(y) + Min\left\{c^T\ x \,| Ax \geq b - Fy,\ x \geq 0\right\}$$

Using linear programming duality theory, the inner minimization problem can be rewritten as

$$Max\{(b - Fy)^T \,\Big|\, A^T u \leq c,\ u \geq 0\}$$

The polyhedron P $= \{u | A^T\ u \geq c, u \geq 0\}$ is independent of y and is according to the assumption that $y \in R$, nonempty. The dual optimum is then reached at an extreme point of P. Let u_i^p denote the extreme points of P, where i $= 1, \ldots, n_p$, and n_p denotes the number of extreme points of P.

Using this fact, the original optimization problem can be rewritten as

$$Min_{y \in R} \{f^T\ y + Max_{1 \leq i \leq n_p} (b - Fy)^T\ u_i^p\}$$

The solution of the complete master problem gives us the optimal solution and the corresponding optimal objective function value. Benders' partitioning method, when used to generate the optimal solution to the original problem, is an iterative solution method that starts off with a suggested solution for the complicating variables, evaluates this solution and uses the information from the sub-problem and its dual to construct a relaxation of the full master problem which, when it is solved, gives a new suggestion for the complicating variables, which have the potential to give a better objective function value than the current best.

3 Creating Interpretable Equilibrium Prices from Partitioning

O'Neill et al. (2005) presented a novel approach to generate interpretable equilibrium prices in markets with non-convexities. The basic idea is that, if we know the optimal solution, we can rewrite the original optimization problem by adding a set of constraints that stipulate that the integer variables should be held at their optimal values. This modified problem, which is convex, can then be solved, and the dual variables are interpretable as equilibrium prices. The dual variables for the demand constraints give the commodity prices, and the dual variables associated with the added set of constraints yield the necessary uplift fee.

O'Neill et al.'s modified optimization problem is

$$\begin{aligned} &Min\ c^T x + f^T y \\ &\text{s.t.}\ \ Ax + Fy \geq b \\ &\qquad x \geq 0 \\ &\qquad y = y^* \\ &\qquad y \in S \end{aligned}$$

where y^* consist of the optimal values for the complicating variables. Let π be the dual variables to the appended equality constraints. It is easily seen that the dual variables associated with the added equality constraints are in fact the dual variables generated from the Benders' sub-problem.

However the inequality $\pi^T\ y \geq \pi^T\ y^*$ is not necessarily a valid inequality. This is not surprising, since the Benders' objective function cut, derived from the suggested solution y^*, yields a direction for potential improvement, and was not intended to be used as a cutting plane inequality. In our case, however, we know that y^* is the optimal solution, and that no better solution exists. Hence, we derive a valid inequality that supports the optimal solution, and when added to the original problem, gives us the optimal objective function value when the original problem is solved as a linear programming problem. Since the valid inequality $\pi^T\ y \geq \pi^T\ y^*$ does not necessarily possess this property, the prices and uplift charges generated by this procedure can be very volatile to small changes in demand requirements. This is illustrated in the examples in Sect. 5.

4 Modified IP-prices

The modified IP-prices are based on the connection between the IP-prices and the dual information generated when Benders' decomposition is used to solve the resource allocation problem. Benders' decomposition is explained above as a computational method, which is often used to solve models in which a certain set of variables, the complicating variables, are fixed at certain values, and the remaining problem and its dual is solved to get bounds for the optimal value of the problem, and also generates information on how to fix the complicating variables to obtain a new solution with a potentially better objective function value. When Benders' decomposition is used to solve a nonlinear integer programming problem as the problem studied in this paper, the problem is partitioned into solving a sequence of "easy" convex optimization problems, where the complicating integer variables are held fixed, and their duals. These problems, often called the Benders' sub-problems, are generating lower bounds on the optimal objective function value and yield information that is added to the Benders' master problem, a problem involving only the complicating variables. The information generated takes the form of cutting planes.

Here, we are not interested in using Benders' decomposition to solve the optimization problem, as we assume that we already know the optimal solution. However, viewing the reformulation used by O'Neill et al. as solving a Benders' sub-problem, in which the complicating integer variables are held fixed to their optimal values, reveals useful information concerning the IP-prices generated in the reformulation.

In linear and convex programming, the existence of a linear price vector is based on the use of the separating hyperplane Theorem. Based on the convexity assumption, the equilibrium prices are the dual variables or Lagrange multipliers for the market clearing constraints. In the non-convex case, it is well known that not every efficient output can be achieved by simple centralized pricing decisions or by linear competitive market prices. However, there exists an equivalent to the separating hyperplane in the form of a separating valid inequality, that is an inequality that supports the optimal solution, and does not cut off any other feasible solutions. If a supporting valid inequality can be found, this inequality can be appended to the original problem, and when this problem is solved with the integrality requirements relaxed, the objective function value is equal to the optimal objective function value. The dual variables to this relaxed problem are then the equilibrium prices we are searching for.

For some problems, the coefficients in the Benders' cut, derived when the integer variables are held fixed to their optimal values, are in fact coefficients from a separating hyperplane for the mixed integer programming problem studied. For other problems they are not. This means that if we are looking at a class of problems, for which a supporting valid inequality does only include integer variables, the IP-prices generated will yield a supporting valid inequality in the sense that the inequality supports the optimal solution and is a separating hyperplane valid inequality that does not cut off any other feasible solution. In other problems, we need to regard other variables as "price" complicating, that is determine a subset of the integer variables and continuous variables that, when held fixed, yields dual variables that are coefficients in a supporting separating valid inequality.

This means that when using the partitioning idea in Benders' decomposition in order to calculate the optimal solution, it is clear that only the integer variables are complicating, where complicating is interpreted as complicating from a computational point of view. However, when our aim is to generate interpretable prices, another set of variables should be regarded as complicating. We need to find out those variables that should be held fixed at their optimal values, in order for the Benders' cutting plane derived by solving the Benders' sub-problem and its dual, to be a valid inequality that supports the optimal solution. When these variables are held fixed at their optimal values, this cutting plane will, when added to the problem, generate commodity prices and uplift charges that are compatible with market clearing prices, and the right hand side of the added valid inequality will give us the necessary uplift charge. In this way, we have generated a nonlinear price system consisting of linear commodity prices and a fixed uplift fee. We call the prices derived in this way the modified IP-prices. In the examples presented in Sect. 5, we show how these prices can be calculated.

It is clear that both the Walrasian interpretation that supports O'Neill et al.'s market clearing contracts and the partial equilibrium model that is used by Hogan and Ring (2003) can be reformulated for our modified IP-prices. The reformulated problem we need to solve in order to generate the modified IP-prices is

$$
\begin{aligned}
&Min\ c^T x + f^T y \\
&\text{s.t.}\quad Ax + Fy \geq b \\
&\qquad\ \ x \geq 0 \\
&\qquad\ \ y_i = y_i^* \quad i \in I \\
&\qquad\ \ x_k = x_k^* \quad k \in K \\
&\qquad\ \ y \in S
\end{aligned}
$$

where the sets I and K denote subsets of the set of all integer and continuous variables in the original problem, respectively.

How then should the variables to be held fixed be selected? The answer is found in the difference between, on the one hand, the structure of the linear programming solution of the original problem, and on the other hand, the structure of the optimal solution. Variables that appear in the optimal integer solution and not in the linear programming solution have to be forced into the solution. The examples in the next section illustrate this technique.

The price determining problem can then be stated as

$$
\begin{aligned}
&Min\ c^T x + f^T y \\
&\text{s.t.}\quad Ax + Fy \geq b \\
&\qquad x \geq 0 \\
&\qquad Cy \geq d \\
&\qquad \sum_{i \in I} \pi_i y_i + \sum_{k \in K} \lambda_k x_k \geq \sum_{i \in I} \pi_i y_i^* + \sum_{k \in K} \lambda_k x_k^* \\
&\qquad y \geq 0
\end{aligned}
$$

where π, λ are the dual variables of the equality constraints in the reformulated problem over.

When the appended inequality is a supporting valid inequality, the optimal dual variable for this inequality will be one, and the right hand side value will be the necessary uplift charge that is needed to support the prices that are given by the other optimal dual variables generated when the above problem is solved.

5 Examples

5.1 Example 1

We first use an example from Hogan and Ring (2003) to illustrate the generation of the modified IP prices that are supported by a nonlinear price function. The example consists of three technologies, Smokestack, High Tech and Med Tech, with the following production characteristics:

	Smokestack		High tech		Med tech
Capacity	16		7		6
Minimum output	0		0		2
Construction cost	53		30		0
Marginal cost	3		2		7
Average cost	6	0.3125	6	0.2857	7
Maximum number	6		5		5

We can formulate Hogan and Ring's allocation problem as a mixed integer programming problem as follows. Denote this problem P.

$$
\begin{aligned}
&Minimize\ 53z_1 + 30z_2 + 0z_3 + 3q_1 + 2q_2 + 7q_3 \\
&\text{s.t.} \\
&\qquad q_1 + q_2 + q_3 = D \\
&\qquad 16z_1 - q_1 \geq 0 \\
&\qquad 7z_2 - q_2 \geq 0 \\
&\qquad 6z_3 - q_3 \geq 0 \\
&\qquad -2z_3 + q_3 \geq 0
\end{aligned}
$$

$$-z_1 \geq -6$$
$$-z_2 \geq -5$$
$$-z_3 \geq -5$$
$$q_1, q_2, q_3 \geq 0$$
$$z_1, z_2, z_3 \geq 0, \text{ and integer}$$

D denotes the demand, and the construction variables for Smokestack, High Tech, and Med Tech are z_1, z_2 and z_3, respectively, while q_1, q_2 and q_3 denote the level of production using the Smokestack, High Tech, and Med Tech technologies, respectively. Note that, for fixed integer values of z_1, z_2 and z_3, the remaining problem in the continuous variables has the integrality property. Hence, the allocation problem is in fact a pure integer programming problem. The reason for this is the special form of the constraint matrix for this example.

The optimal solutions for demand levels 55 and 56 are given in the table below:

Demand	Smoke-stack		High tech		Med tech		Total cost
	Number	Output	Number	Output	Number	Output	
55	3	48	1	7	0	0	347
56	2	32	3	21	1	3	355

O'Neill et al.'s reformulation of problem P is

$$\textit{Minimize } 53z_1 + 30z_2 + 0z_3 + 3q_1 + 2q_2 + 7q_3$$
s.t.
$$q_1 + q_2 + q_3 = D$$
$$16z_1 - q_1 \geq 0$$
$$7z_2 - q_2 \geq 0$$
$$6z_3 - q_3 \geq 0$$
$$-2z_3 + q_3 \geq 0$$
$$-z_1 \geq -6$$
$$-z_2 \geq -5$$
$$-z_3 \geq -5$$
$$z_1 = z_1^*$$
$$z_2 = z_2^*$$
$$z_3 = z_3^*$$
$$q_1, q_2, q_3 \geq 0$$
$$z_1, z_2, z_3 \geq 0, \text{ and integer}$$

where z_1^*, z_2^* and z_3^* represent the optimal integer solution for the specified demand.

For demand equal to 55, the IP-prices generated are $p = 3$ for the demand constraint and the dual variables for the equality constraints are 53, 23 and 0, respectively. Here 53 and 23 are coefficients in a supporting valid inequality, namely the inequality $53z_1 + 23z_2 + 4q_3 \geq 182$. However, for demand equal to 56, the IP-prices that are generated are $p = 7$ for the demand constraint and $-11,\ -5$ and 0 for the other three equality constraints. It is obvious that these numbers cannot be part of a supporting valid inequality.

For the modified IP-prices, the reformulated problem to be solved is

$$
\begin{aligned}
&\textit{Minimize}\ 53z_1 + 30z_2 + 0z_3 + 3q_1 + 2q_2 + 7q_3\\
&\text{s.t.}\\
&\qquad q_1 + q_2 + q_3 = D\\
&\qquad 16z_1 - q_1 \geq 0\\
&\qquad 7z_2 - q_2 \geq 0\\
&\qquad 6z_3 - q_3 \geq 0\\
&\qquad -2z_3 + q_3 \geq 0\\
&\qquad -z_1 \geq -6\\
&\qquad -z_2 \geq -5\\
&\qquad -z_3 \geq -5\\
&\qquad z_1 = z_1^*\\
&\qquad z_2 = z_2^*\\
&\qquad q_3 = q_3^*\\
&\qquad q_1, q_2, q_3 \geq 0\\
&\qquad z_1, z_2, z_3 \geq 0, \text{ and integer}
\end{aligned}
$$

With this definition of complicating variables, $z_1,\ z_2$ and q_3, the dual variables generated are $p = 3$ and the prices 53, 23 and 4 for the other equality constraints. Hence, both for demands equal to 55 and 56, the supporting valid inequalities $53z_1 + 23z_2 + 4q_3 \geq 182$ and $53z_1 + 23z_2 + 4q_3 \geq 187$ are generated, respectively. The right hand side values 182 and 187 are the respective uplift charges.

5.2 *Example 2*

A second example is taken from a situation with hockey stick bidding in an electricity market with generators having start up costs. The example includes four generators A, B, C and D, each having a limited capacity, an energy price and a start up cost. The data for the four units are as follows

	Capacity	Energy price	Start up cost
Unit A	45	10	0
Unit B	45	20	0
Unit C	10	100	20
Unit D	80	30	2,000

The integer programming formulation for this problem is

$$
\begin{aligned}
Min\ & 20z_3 + 2000z_4 + 10q_1 + 20q_2 + 100q_3 + 30q_4 \\
s.t\ & q_1 + q_2 + q_3 + q_4 \geq 100 \\
& -q_1 + 45z_1 \geq 0 \\
& -q_2 + 45z_2 \geq 0 \\
& -q_3 + 10z_3 \geq 0 \\
& -q_4 + 89z_4 \geq 0 \\
& -z_1 \geq -1 \\
& -z_2 \geq -1 \\
& -z_3 \geq -1 \\
& -z_4 \geq -1 \\
& z_1, z_2, z_3, z_4, q_1, q_2, q_3, q_4 \geq 0 \\
& z_1, z_2, z_3, z_4 \text{ integer}
\end{aligned}
$$

The optimal solution is to use units A and B to their full capacity and committing unit C to produce 10 units. The total cost for this production is 2,370.

Regarding the integer, restricted variables as complicating variables yields the commodity price $p = 100$, that is the committed unit with the highest energy cost sets the market price. This means that the hockey stick bidder sets the market price. The shadow prices for the equality constraints are $-4,050$, $-3,600$, 20 and $-3,600$, which are not coefficients in a supporting valid inequality.

If we instead regard the variables q_3 and z_4 as complicating variables, the commodity price generated is $p = 20$, and the two equality constraints have dual variables equal to 80 and 2,000, corresponding to coefficients in the supporting valid inequality $80q_3 + 2000z_4 \geq 80$ or $q_3 + 25z_4 \geq 10$. This example illustrates that what should be regarded as complicating variables from a computational point of view and from a pricing point of view might be very different.

5.3 Example 3

A third example is a capacitated plant location problem, with three potential facilities and two customers. The mathematical programming formulation is

$$
\begin{aligned}
Min\ & 10y_1 + 8y_2 + 12y_3 + 5x_{11} + 6x_{12} + 6x_{21} + 7x_{22} + 7x_{31} + 3x_{32} \\
s.t.\ & x_{11} + x_{21} + x_{31} = 10 \\
& x_{12} + x_{22} + x_{32} = 2 \\
& -x_{11} - x_{12} + 8y_1 \geq 0 \\
& -x_{21} - x_{22} + 7y_2 \geq 0 \\
& -x_{31} - x_{32} + 6y_3 \geq 0 \\
& x_{ij} \geq 0 \quad i = 1, 2, 3\ j = 1, 2 \\
& y_i \in [0, 1] \quad i = 1, 2, 3
\end{aligned}
$$

The optimal solution to the capacitated facility location problem has objective function value 82. Plants 1 and 3 are to be opened, eight units of goods should be transported from plant 1 to customer 1, two units of goods should be transported from plant 2 to customer 1 and finally two units of goods are to go from plant 3 to customer 2.

Adding constraints $y_1 = 1,\ y_3 = 1,\ x_{31} = 2$ gives dual variables to these equality constraints of 2, 12, and 1, respectively. Based on this result, the price supporting valid inequality

$$2y_1 + 8y_2 + 12y_3 + 3x_{12} + 4x_{22} + 1x_{31} \geq 16$$

can be generated. If the linear relaxation of the original plant location problem is solved with this valid inequality appended, the prices of goods at customers 1 and 2 will be 6 and 3, respectively, and the uplift charge needed is 16, that is the right hand side of the added valid inequality, which in the optimal LP relaxation has an optimal dual price of 1.

Note, however, that when the constraints $y_1 = 1,\ y_3 = 1,\ x_{31} = 2$ are added to the original problem, an alternative dual solution to the equality constraints exists, with dual variables 10, 12, and 2. Using these dual variables leads to a price supporting valid inequality, which reads

$$10y_1 + 8y_2 + 12y_3 + 3x_{12} + 1x_{21} + 4x_{22} + 2x_{31} \geq 26$$

If this inequality is added to the original problem and its LP relaxation is solved, the price for goods at customer 1 is 5 and at customer 2 is 3. Note that with these lower commodity prices the uplift charge is substantially higher. Following Hogan and Ring (2003), it is obvious that a price structure with a lower uplift charge is better, and should be preferred in cases like the one in this example, where degeneracy leads to alternative price structures.

5.4 Example 4

The final example is a modification of the first one, in which the two units Smokestack and High Tech are located in node 1 of a three node meshed electricity network, and where the Med Tech unit is located in node 2. All the demand is located at node 3, and there is a capacity limit of 15 between node 1 and 2. The mathematical programming formulation of this problem is

$$\begin{aligned}
&\textit{Minimize}\ 53z_1 + 30z_2 + 0z_3 + 3q_1 + 2q_2 + 7q_3 \\
&\text{s.t.} \\
&\qquad q_1 + q_2 - f_{12} - f_{13} = 0 \\
&\qquad q_3 + f_{12} - f_{23} = 0
\end{aligned}$$

$$
\begin{aligned}
& f_{13} + f_{23} = 56 \\
& f_{12} - f_{13} + f_{23} = 0 \\
& 16z_1 - q_1 \geq 0 \\
& 7z_2 - q_2 \geq 0 \\
& 6z_3 - q_3 \geq 0 \\
& -2z_3 + q_3 \geq 0 \\
& -z_1 \geq -6 \\
& -z_2 \geq -5 \\
& -z_3 \geq -5 \\
& f_{12} \leq 15 \\
& -f_{12} \leq 15 \\
& q_1, q_2, q_3 \geq 0 \\
& z_1, z_2, z_3 \geq 0, \text{ and integer}
\end{aligned}
$$

The optimal solution has objective function value 358, and $z_1 = 1,\ z_2 = 5,\ z_3 = 1$ The production from the different producers in the optimal solution is 15.5, 35 and 5.5 respectively, and the flow on the links are as follows: link 1–3 flow is at the capacity limit of 35.3, link 2–3 flow at 20.5 and finally link 1–2 flow at 15. Solving the problem with the constraints $z_1 = 1,\ z_2 = 5,\ f_{12} = 15$ yields dual variables for these constraints, which have the values 53, 23 and −6, respectively. Apart from this, the prices in the three nodes, the dual variables of flow constraints, are 3, 7 and 5.

Using this information, the valid inequality is $53z_1 + 23z_2 - 6f_{12} \geq 78$. When appended to the original problem and solving the LP relaxation, it yields the node prices 3, 7, and 5, and the dual price for the appended inequality is equal to 1. It is notable that the coefficient −6 is the negative of the shadow price on the constrained link 1–2. The uplift charge required to support these commodity prices are as low as eight, since the congestion fee from the constrained link, which is six times $15 = 90$, is used to make the required uplift charge lower.

Note also that in this example the continuous variables are not integer-valued in the optimal solution; hence, we do not know if there exists a nonlinear price function that supports this price structure.

6 Conclusions and Issues for Future Research

In this paper, we have shown that it is possible to construct modified IP-prices as a nonlinear price structure with affine commodity prices, supplemented by a fixed uplift charge. The prices are derived by the use of knowledge of the optimal solution to generate a reformulation that generates coefficients in a supporting valid inequality to the resource allocation optimization problem. When this valid inequality is appended to the original resource allocation problem, and the integer constraints are relaxed, the resulting optimal dual variables are the modified IP-prices, and on the right hand side the uplift charge needed to support these commodity prices.

For resource allocation including demand constraints, it is possible to interpret the modified IP-prices as prices derived from a partial equilibrium model.

How to choose the variables that should be held fixed when deriving the modified IP-prices, in order to get the price structure with the minimum uplift charge, needs to be further investigated. Another interesting research question is to determine how the bidding format and a contract mechanism should be constructed in a market in which modified IP-prices are used. Other research questions that should be addressed are how elastic demand can be incorporated in markets with non-convexities. If and how the uplift charges should be collected among the customers, and how this cost allocation should influence the design of market clearing contracts.

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An Optimization-Based Conjectured Response Approach to Medium-term Electricity Markets Simulation

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Abstract Medium-term generation planning may be advantageously modeled through market equilibrium representation. There exist several methods to define and solve this kind of equilibrium. We focus on a particular technique based on conjectural variations. It is built on the idea that the equilibrium is equivalent to the solution of a quadratic minimization problem. We also show that this technique is suitable for complex system representation, including stochastic risk factors (i.e., hydro inflows) and network effects. We also elaborate on the use of the computed results for short-term operation.

Keywords Medium-term market simulation · Conjectured responses · Stochasticity · Network influence on equilibrium · Short and medium-term coordination

1 Introduction

Electricity deregulation has led to a considerable effort in analyzing the behavior of power markets. In particular, a wide range of research is devoted to the analysis of market behavior, which assumes a scope short enough to make unnecessary the study of the investment problem. Even with this simplification, the representation of markets by means of a single model is an extremely difficult task. Therefore, it is common practice to pay attention to only the most relevant phenomena involved in the situation under study. This point of view naturally leads to different modeling strategies depending on the horizon considered. In the short term, the most critical effects concern operational issues. The short-term behavior of the market will be highly influenced by the technical operation of power plants, as well as by short-term uncertainty. However, the modeling of strategic interaction between market

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players may be simplified if one assumes that strategic effects can be obtained by statistical estimation. The main assumption behind this approach is that, in the short term, the fundamental drivers that determine power prices do not change, and thus the complete formulation of the model of strategic interactions may not be required. This approach does not necessarily disregard market power effects in the short term, but it assumes that they can be estimated through recent past market data. A considerable amount of research has been devoted to the characterization and estimation of this short-term market representation, which can be classified under two broad headers. On the one hand, one can find models on the basis of describing the market using an aggregated representation, which is defined by the market price. Time series models, for example, Hamilton (1994) or Harvey (1989), and their financial versions, see Eydeland and Wolyniec (2003) for a recent review, belong to this category. Another useful alternative is based on considering competitors' decisions simply as another uncertain variable that the generator faces when optimizing its operation. The competitors' behavior is thus represented by the residual. Under this statement, the problem consists of an agent that is dealing with a residual demand and wishes to maximize its profit. A review can be found in Baillo et al. (2006).

In longer terms, however, the role of strategic issues becomes more important, because the behavior of market players may change, and consequently should be anticipated. We will refer to this time scope (typically up to 3 years) as medium-term, leaving the definition "long-term" for investment models. A wide range of models have been proposed for analyzing the interaction of competing power producers who price strategically. For instance, emission allowances management, fuel purchases, hydro resources allocation, and forward contracting are typically medium-term decisions, and consequently they will be affected by the strategic interaction between power producers. In addition to their applications as tools to aid the decision-making processes of generation companies, these models are useful for gaining insights into firms' strategic behavior as they allow for comparisons between different market scenarios, and they are therefore suitable for regulatory purposes as well (e.g., impacts of mergers or different market designs). In this context, Barquín (2006), Kiss et al. (2006), or Neuhoff (2003) are examples of regulatory applications.

Therefore, in the medium-term context, the modeling of market players' strategic behavior should be a central issue. When modeling electricity markets, three dimensions should be taken into account. First, the model for market equilibrium should represent the behavior of market players. In addition, an appropriate modeling of system constraints is required, so that it allows for the specification of the power producers' costs. Finally, the algorithms for finding the solution of the model must play a central role when developing the model, so that it allows for the resolution of realistic cases.

One popular alternative for describing power markets consists in using one-stage games to represent oligopolistic competition. The rationale behind this is to find an equilibrium point of the game to describe the market behavior; that is, a set of prices, generator outputs, consumptions, and other relevant numerical quantities, which no market agent can modify unilaterally without a decrease in its profits (Nash 1950).

Although alternative approaches have been proposed, such as agent-based models (Otero-Novas et al. 2000), we prefer to concentrate our review on game-theoretic models. Although the former simulation models may be very useful, especially for scenario generation, we think game-theoretic models are more able to respond to changes in the market structure and more appropriate for gaining insights into the market behavior.

1.1 Strategic Behavior of Market Players

The set of strategies that market agents can play defines the possible agents' behaviors (the actual behavior is determined by the outcome of the game). In fact, from Cournot (1838) and its criticism (Bertrand 1883), economists debate the merits of choosing prices or quantities as strategic variables. Price competition seems to be discarded in power markets, as capacity constraints play an important role in electricity markets, but a great number of models based on a quantity-game have been proposed to describe power-market equilibria, (Scott and Read 1996; Borenstein and Bushnell 1999; Hobbs 2001). These models are built on the idea that market players choose their quantities to maximize their profits, that is, the classic Cournot competition (Daughety 1988). Although their specification is not particularly challenging, they benefit from the fact that system constraints are relatively easy to represent. In addition, Cournot models make possible the representation of the worst possible case for oligopoly in the market, which may be of interest in some regulatory studies.

In power markets, a more realistic set of strategies would consist of a set of price–quantity pairs. Actually, in most pool-based markets, power producers bid a curve made up of the prices for each output. This is the idea behind the supply function equilibrium (Kemplerer and Meyer 1989). Examples of the application of these models to electricity markets can be found in Green and Newbery (1992) and Rudkevich (2005). However, their use in realistic settings is often difficult because simple operational constraints may pose problems in the definition of the supply function. For instance, in Baldick et al. (2004), it is shown that the relatively simple problem of specifying a linear supply function is not a closed question when upper capacity limits on power plants are considered. Nevertheless, there have also been recent promising advances (Anderson and Xu 2006).

Therefore, a compromise, the conjectured-supply-function approach (Vives 1999), has attracted considerable attention. The central idea of the methodology is to restrict the admissible set of supply functions to a much smaller parameterized set of curves. Very often it is assumed that market players bid linear supply functions with fixed and known slopes and decide on the intercept of the curve. There are several ways to define the slope of the conjectured supply function, which include the conjecture of the production response of competitors to market price variation (Day et al. 2002), the residual demand elasticity curve of each agent (García-Alcalde et al. 2002), and the total production variation of competitors with respect to the agent's production (Song et al. 2003). In this context, Bunn (2003) provides a wide range of techniques that can be used to obtain the conjectures. One of the key advantages of this methodology is that it can describe a wide range of market behaviors, from perfect to Cournot competition: zero-price response is, by definition, perfect competition; Cournot competition is represented by equal price response for every market player, given by the slope of the demand curve, see for instance Day et al. (2002). In addition, it is important to note that initially the game was proposed to avoid the estimation of a market state typical of the short-term models. However, when using the conjectured-response approach, the fundamental model is highly sensitive to the value of the conjecture, and so conjectured-supply-function models imply the estimation of the behavior of market players, placing them closer to short-term models.

1.2 Representation of the Power System

In addition, as electricity cannot be stored, operational constraints are of paramount importance even in medium-term analysis, and so there is a need for an adequate representation of technical characteristics of the system. The detail in the system constraints and in the complex operational characteristics of power plants may play a considerable role. In fact, the cost curve of power producers is typically made up of the costs of each power plant in their portfolio and of their maximum and minimum output, which results in a step-wise curve. Modeling, for example, a supply function equilibrium including these constraints is a very difficult task. An important instance of the operational constraints relevance is the representation of the power network. A popular approach to model oligopolistic markets is to disregard the transmission network. However, in some situations this approach may fail to represent adequately the strategic interaction between market players, mainly because the existence of the network can add opportunities for the exercising of market power. For example, congestion in one or more lines may isolate certain subsets of nodes so that supply to those nodes is effectively restricted to a relatively small subset of producers. Then, there will be increased market power, since it is relatively easy for these producers to increase prices in the isolated area by withholding power, irrespective of the competitiveness of the whole system. Furthermore, system variables assumed as input data when defining the game are often subject to uncertainty. In this context, market agents play a static game, and each of them faces a multiperiod optimization subject to uncertainty. Hence, the uncertainty of input data may be represented by means of a scenario tree to describe non-anticipatory decisions of market players.

The previous considerations are strongly linked to the problem of developing algorithms to find the equilibrium. Ideally, a detailed representation of the system would be needed to correctly represent the equilibrium of the game. However, it may result in computationally unaffordable problems. Therefore, the requisite level of simplification is ultimately determined by the algorithms available. Several methodologies have been proposed, but most of them share the characteristic of benefiting from the statement of complementarity conditions. Bushnell (1998) found the solution by means of heuristic methods. The mixed or linear complementarity problem was used by Hobbs (2001) and Rivier et al. (1999). Mixed complementary problem (MCP) has a structure that makes possible the formulation of equilibrium problems, including a higher level of complexity (e.g., network and production constraints or different competitor's behavior). Nevertheless, formulating the problem in this framework does not guarantee either the existence or the uniqueness of the solution. Commercial solvers exist (such as PATH or MILES) that allow this problem to be addressed, but they have some limitations: the size of the problem is restricted and, if a solution is not reached, no conclusions can be drawn regarding its existence. Moreover, if a solution is found, it may not be the only one. A particular situation arises when demand is considered to be affine, cost functions are linear or quadratic, and all the constraints considered are linear. In this case, MCP becomes a linear complementary problem (LCP) (Bushnell 2003; Day et al. 2002; García-Alcalde et al. 2002), and the existence and uniqueness of the solution can be guaranteed in most practical situations (Cottle et al. 1992). Closely related to MCP is the variational inequality (VI) problem (e.g. Yuan and Smeers 1999). MCP is a special case of this wider framework, which enables a more natural formulation of market equilibrium conditions (Daxhelet and Smeers 2001).

1.3 Dynamic Effects in Power Markets

Although we are essentially reviewing static games, there are several examples of dynamic games that are worthy of comment. For example, when dealing with power networks, several authors have proposed to describe the game as one of two stages: market players decide on their output in the first stage and the system operator clears the market in the second. This representation can be found, for instance, in Cardell et al. (1997). Regarding the algorithms developed to solve such equilibria, the equilibrium problem with equilibrium constraints (EPEC) is the most used algorithm to solve these two-stage games in power markets. For example, Hobbs et al. (2000) and Ehrenmann and Neuhoff (2003) use an EPEC formulation to solve the equilibrium, which takes into account the power network.

The strategic use of forward contracting is a typical application of dynamic games techniques. Actually, from Allaz and Villa (1993), there is an increasing body of literature that analyzes the use of forward positions as strategic variables. Yao et al. (2004) propose a similar model, but include a very detailed representation of the power system. In fact, they modeled a two-stage game using an EPEC formulation. However, they are being contested by some recent research. For instance, Liski and Montero (2006) and Le Coq (2004), among others, study the effect of collusive behavior in repeated games, while Zhang and Zwart (2006) is concerned with the effects of reputation.

1.4 The Proposed Methodology

The rest of the chapter is devoted to the description of an alternative methodology of dealing with equilibrium problems. The methodology is built on the idea that, under some reasonable conditions, such as monotonous increasing cost function, the solution of a conjectured-supply-function game is equivalent to the solution of a quadratic minimization problem. The approach will be applied, in the rest of the chapter, to different models of the power system, including the representation of the power network and the consideration of uncertainty. This methodology contributes some important advantages. First, the equilibrium existence is clearly stated, and it can be computed with very efficient optimization techniques. Second, the formulation has a structure that strongly resembles classical optimization of hydrothermal coordination, and thus, it allows for the inclusion of linear technical constraints in a direct way. Third, by using commercial optimization tools, large-size problems can be solved. This last point makes possible a more detailed representation of the power system.

The chapter continues with the introduction of a simplified deterministic case and the equivalent minimization problem. After that, the model is generalized to the multiperiod setting. The transmission network is included in the subsequent section, and the uncertainty is then considered in the market model. Finally, there is a description of how the additional information of the medium-term model can become signals for short-term operation. Most of the results described herein have been previously published in the literature by the authors.

2 Market Equilibrium Under Deterministic Conditions: The Single Period Case

This section starts by describing the market model in a simplified version, which is restricted to a single unitary time period and disregards technical constraints and forward contracting. In following sections, the model is generalized to consider a multiperiod setting, in which hydro operation and technical constraints are described. Finally, the effects of the transmission network in the strategic interaction of market agents are included in the model.

The formulation can be interpreted as a conjectured-price-response approach that represents the strategic behavior of a number of firms $u = 1, 2, \ldots, U$ competing in an oligopolistic market. The variation of the clearing price λ with respect to each generation-company production P_u is assumed to be known. Under the logical assumption that this variation is nonpositive, the non-negative parameter θ_u is defined as

$$\theta_u = -\frac{\partial \lambda}{\partial P_u} \geq 0 \tag{1}$$

Disregarding contracts, the profit B_u that generation company u obtains when remunerated at the marginal price is

$$B_u = \lambda \cdot P_u - C_u\left(P_u\right) \tag{2}$$

The first term $\lambda \cdot P_u$ represents the revenue function for firm u, and the second $C_u(P_u)$ its cost function. The equilibrium is obtained by expressing the first-order profit-maximization condition[1] for each generation company

$$\frac{\partial B_u}{\partial P_u} = 0 = \lambda + P_u \cdot \frac{\partial \lambda}{\partial P_u} - \frac{\partial C_u\left(P_u\right)}{\partial P_u} \tag{3}$$

The conjectured response of the clearing price with respect to each generation company's production leads to

$$\lambda = \frac{\partial C_u\left(P_u\right)}{\partial P_u} + P_u \cdot \theta_u, \; \forall u \tag{4}$$

To obtain the equilibrium conditions, it is necessary to define the equations that determine the market price, and consequently to take into account the market clearing process (the system operator, who clears the market, is the remaining player of the game). Thus, a generic market clearing process would imply the maximization of the demand utility subject to system constraints. Provided that D^0 is the constant term of the demand function, and α_0 represents the demand slope, the utility function of the demand $U(D)$ is defined as

$$U\left(D\right) = \int_0^D \lambda\left(D\right) \cdot dD = \frac{1}{\alpha_0} \cdot \left(D \cdot D^0 - \frac{D^2}{2}\right) \tag{5}$$

The only system constraint in this case is the power balance equation. Therefore, the system operator behavior is defined by the problem

$$\begin{aligned} &\min_D \; -U\left(D\right) \\ &\text{s.t.} \;\; \sum_{u=1}^{U} P_u - D = 0 \colon \lambda \end{aligned} \tag{6}$$

Furthermore, the corresponding first optimality conditions of the above problem, that is, the market-clearing conditions, are

$$D = D^0 - \alpha_0 \cdot \lambda \tag{7}$$

[1] This is a consequence of Nash equilibrium. None of the utilities could modify its production without a decrease in its profits. Therefore, the first derivative is null at the point of maximum profit.

$$\sum_{u=1}^{U} P_u - D = 0 \tag{8}$$

Therefore, equilibrium of the game is defined by (4), (7), and (8). This equilibrium point can be alternatively computed by solving an equivalent optimization problem:

$$\begin{aligned} &\min_{P_u, D} \sum_{u=1}^{U} \left[\bar{C}_u\left(P_u\right)\right] - U\left(D\right) \\ &\text{s.t.} \quad \sum_{u=1}^{U} P_u - D = 0 \quad : \quad \lambda \end{aligned} \tag{9}$$

where $\bar{C}_u\left(P_u\right)$ denotes a term called effective cost function[2]

$$\bar{C}_u\left(P_u\right) = C_u\left(P_u\right) + \frac{P_u^2 \cdot \theta_u}{2} \tag{10}$$

Under the hypothesis of continuous and convex cost functions, and single node market clearing as well as non-negative θ_u, it can be proved that this optimization problem (9) is equivalent to the market equilibrium problem defined by (4), (7), and (8). Note that the dual variable λ of the power-balance constraint is the system's marginal price that clears the market.

Furthermore, the model can be generalized to include forward contracts signed by the utilities (Barquín et al. 2004). Two kinds of contracts are considered:

1. *Forward contracts*. In these contracts the utility agrees to generate a certain power G_u^k at price λ_u^{Gk}.
2. *Contracts for differences*. In these financial instruments, the utility contracts a power H_u^l and it is paid the difference between the clearing price λ and an agreed one λ_u^{Hl}.

When physical contracts are being considered, generated power P_u is different from the tenders in the spot market P_u^S. Specifically,

$$P_u = P_u^S + \sum_k G_u^k \tag{11}$$

The utility profit is

$$B_u = \lambda \cdot P_u^S - C_u\left(P_u\right) + \sum_k \lambda_u^{Gk} \cdot G_u^k + \sum_l \left(\lambda_u^{Hl} - \lambda\right) H_u^l \tag{12}$$

[2] It is important to highlight that the effective cost function derivative is the effective marginal cost function $\overline{MC_u}\left(P_u\right) = MC_u\left(P_u\right) + \theta_u \cdot P_u$. Therefore, the solution of the minimization problem, where the effective cost functions sum is minimized, implies that utilities' effective marginal costs are equal to system marginal price.

The previous equation can be written as

$$B_u = \lambda \cdot P_u - C_u(P_u) + \sum_k \left(\lambda_u^{Gk} - \lambda\right) G_u^k + \sum_l \left(\lambda_u^{Hl} - \lambda\right) H_u^l \tag{13}$$

Profit maximization leads to the equilibrium equations

$$\lambda = \frac{\partial C_u(P_u)}{\partial P_u} + \theta_u \left(P_u - \sum_l H_u^l - \sum_k G_u^k \right), \forall u \tag{14}$$

These equations happen to be the optimality conditions of problem (9) if effective cost is defined as

$$\bar{C}_u(P_u) = C_u(P_u) + \frac{\theta_u}{2} \left(P_u - \sum_l H_u^l - \sum_k G_u^k \right)^2 \tag{15}$$

Finally, some common market situations are represented considering that demand is inelastic, that is, a constant known value. For example, it is usual to consider that the competitors' reaction comes before the demand's reaction, and that consequently the demand elasticity should be disregarded. In this situation, market equilibrium is obtained as the solution of an optimization problem, which does not include the utility function of the demand

$$\begin{aligned} \min_{P_u} \quad & \sum_{u=1}^{U} \bar{C}_u(P_u) \\ \text{s.t.} \quad & \sum_{u=1}^{U} P_u - D^0 = 0 : \lambda \end{aligned} \tag{16}$$

3 Conjectural Price Response's Specification

A key issue when adjusting the model is the adequate estimation of conjectured-price responses. Several papers have dealt with this substantial topic. The existing methodologies can be grouped into three categories: implicit methods, bid-based methods, and costs-based methods.

- Implicit methods take into account actual and past prices and companies' production to estimate the value of conjectures according to (4). Two applications of this methodology can be found in García-Alcalde et al. (2002) and López de Haro et al. (2007).
- Bid-based methods use historical bids made by companies to carry out the estimation. Three different applications can be found in Bunn (2003), where it is shown how conjectures can be estimated through different approximations of bid

functions combined with clustering analysis, time series analysis, or input–output hidden Markov models.

- Finally, cost-based methods are useful when dealing with new markets, or markets in which no information is provided from the auction process (Perry 1982).

There are, both from the theoretical and the practical points of view, important drawbacks to all these approaches. The first two (implicit and bid-based methods) are essentially historical approaches that ultimately rely on the assumption that the future will resemble the past. In other words, no dramatic changes in the economic or regulatory conditions of the system are to be expected in the study horizon. However, they should not be applied if these conditions are not met, for instance, when analyzing the impact of new significant generation capacity additions or that of sufficiently significant regulatory changes like large Virtual Power Plants auctions. On the other hand, they can be recommended for medium-term studies.

The last approach (cost-based methods) can be thought of as a way to determine the conjecture endogenously. These methods can conceivably overcome the drawbacks described in the previous paragraph, but we are hampered for our ignorance of some basic issues regarding the nature of oligopolistic competition. Actually, conjectural variations or responses techniques have been often looked upon with suspicion by a number of economists, as it can be argued that, being able to explain almost anything, they are not useful for predicting anything (Figuières et al. 2004). Our approach here is a pragmatic one. We assume that for medium-term studies the "historical methods" can provide sensible figures, and that for longer-term or regulatory studies the cost based techniques can provide plausible working guides. For instance, when analyzing competition enhancing reforms for the Spanish market in the Spanish White Paper on Electricity, analysts relied on a supply function equilibrium approach to obtain rough estimates of the effectiveness of some proposed methods (Pérez-Arriaga et al. 2005; Barquin et al. 2007). However, we think that it can be fairly stated that currently results from all these approaches are to be taken as suggestive of possible outcomes rather than as reliable quantitative forecasts.

As we move towards multiperiod, networked systems, the number of conjectural parameters are greater, and so, as just stated, they are of concern. Although these are important issues that merit close attention, they will not be pursued further on.

A closely related issue concerns the forward position specification. The assumption implicitly made to obtain (14) is that the effective production subject to oligopolistic behavior is not the actual output, but the output sold in the spot market. Although there is a considerable body of literature that addresses the issue of how forward equilibrium positions are determined, we have adopted the conclusion in Allaz and Villa (1993), which stated that the production previously contracted is not considered by power producers when deciding their output in the spot market. In other words, the contracted production is not an incentive to raise the spot price. Therefore, the production involved in the oligopolistic term of the costs is the output minus the contracted production. The conjectured price response can be estimated from historical data or predicted from the knowledge of the relevant cost functions and economic and regulatory constraints. There are advantages and drawbacks similar to those stated earlier. In the case of using the implicit method, it could be advisable to jointly estimate conjectures and forward positions. Cournot studies typically assume away forward positions and consider conjectured price responses much more elastic than those implied by short-term demand elasticity.

4 Market Equilibrium Under Deterministic Conditions: The Multiperiod Case

Production-cost functions are usually multivariable functions that relate different periods. The structure proposed in this model makes possible the introduction of additional variables and constraints to represent the cost functions and the associated technical constraints within the same optimization problem in a natural way. The characteristics of these cost functions are very dependent on the particular system to be represented. It is very common, in medium-term horizon studies, to represent cost as linear or quadratic cost functions. In any of these situations, the formulation yields a quadratic programming (QP) problem. This schema closely resembles classical optimization medium-term operation models and maintains its main advantageous features: there is a clear model structure (extended objective function and technical constraints), reasonable computing time, and dual information is easily obtained.

As an example of the proposed methodology, a hydrothermal power system constituted by a set of generation companies will be represented with the objective of obtaining a QP problem that could also be achieved with different cost function representations.

For the time representation, the model scope is divided into periods, weeks, or months, denoted as p. Each period is divided into subperiods, denoted as s. A subperiod consists of several load levels, denoted as b. Each load level b of a subperiod s and a period p is characterized by its duration l_{psb}.

The optimization model should yield as optimality conditions a generalization of equilibrium (14), namely

$$\lambda_{psb} = \left.\frac{\partial C_u(P_u)}{\partial P_u}\right|_{psb} + \theta_{upsb}\left(P_{upsb} - G_{upsb} - H_{upsb}\right), \ \forall u, p, s, b \qquad (17)$$

G_{upsb} and H_{upsb} are the total physical and financial contracted amounts, respectively. Additional decision variables are required to represent the power system, which consists of a set of thermal and hydro units' productions. All of them have to be replicated for each period p, subperiod s, and load level b.

- t_{jpsb} Power generation of thermal unit j (MW)
- h_{mpsb} Power generation of hydro unit m (MW)
- b_{mpsb} Power consumption of pumped-hydro unit m (MW)
- r_{mp} Energy reservoir level of hydro unit m at the end of period p. The initial value, as well as the value for the last period, is considered to be known (MWh)
- s_{mp} Energy spillage of hydro unit m at period p (MWh)

Generated power for each company does not require a specific variable, as will be shown later; it may be computed as a linear combination of other decision variables.

Some new parameters should also be considered to make possible the introduction of power system's technical characteristics. Each thermal unit j is defined by using its

$\bar{t}_j$ Maximum power generation (MW)
δ_{jp} Variable cost in period p (e/MWh)
o_j Owner generation company

Hydro units are represented as a single group with pumping capability. Every hydro unit m has an associated reservoir and may have pumping capability. Run-off-the-river production is considered separately for each utility. Hydro units' parameters include

$\bar{h}_{mp}$ Maximum power generation in period p (MW)
o_m Owner generation company
$\bar{b}_m$ Maximum pumping power consumption (MW)
ρ_m Performance of pumping (p.u.)
$\bar{r}_{mp}$ Maximum energy reservoir storage at the end of period p (MWh)
$\underline{r}_{mp}$ Minimum energy reservoir storage at the end of period p (MWh)
f_{upsb} Run-off-the-river hydro energy for the whole company u at load level b of subperiod s and period p (MW)
I_{mp} Hydro inflows, except run-off-the-river, in period p (MWh)
r_{m0} Initial energy reservoir level (MWh)
r_{mp^*} Final energy reservoir level in the last period p^* (MWh)

Each utility u may have signed bilateral (physical) contracts or financial contracts for each load level b of subperiod s and period p

G_{upsb} Amount of physical contracts (MW)
λ^G_{upsb} Price of physical contracts (€/MWh)
H_{upsb} Amount of financial contracts (MW)
λ^H_{upsb} Price of financial contracts (€/MWh)

From the previous definitions, generated power for utility u at load level b of subperiod s and period p is obtained as a linear combinations of decision variables and parameters

$$P_{upsb} = \sum_{j|o_j=u} t_{jpsb} + \sum_{m|o_m=u} \left(h_{mpsb} - b_{mpsb}\right) + f_{upsb} \tag{18}$$

Definition of cost functions requires the addition of the following technical constraints: bounds for the decision variables, power balances, and energy balances. The decision variable bounds correspond to the maximum power generation of thermal unit j

$$t_{jpsb} \le \bar{t}_j \tag{19}$$

The maximum power generation of hydro unit m is

$$h_{mpsb} \leq \bar{h}_{mp} \tag{20}$$

The maximum pumping power consumption of hydro unit m is

$$b_{mpsb} \leq \bar{b}_{m} \tag{21}$$

And, finally, the minimum- and maximum-level energy reservoir storage of hydro unit m is

$$\underline{r}_{mp} \leq r_{mp} \leq \bar{r}_{mp} \tag{22}$$

Power balance for each load level b of subperiod s and period p is shown next. The dual variable of each one of these constraints will be referred to as π_{psb}, which is necessary to compute the system's marginal price:

$$\sum_{j} t_{jpsb} + \sum_{m} h_{mpsb} + \sum_{u} f_{upsb} - D_{psb} - \sum_{m} b_{mpsb} = 0 \tag{23}$$

Energy balance for each load level b of subperiod s and period p, and hydro unit m is

$$r_{mp} - r_{m,p-1} = \sum_{s} \sum_{b} l_{psb} \left(h_{mpsb} - b_{mpsb} \right) + I_{mp} - s_{mp} \tag{24}$$

The cost function can be now expressed for each company as a linear function of decision variables and can be directly included in the objective function without modifying the QP structure:[3]

$$C_{upsb} = \sum_{j|o_j=u} \left(\delta_{jp} \cdot t_{jpsb} \right) \tag{25}$$

The objective function includes total system operation costs, the quadratic term of the effective cost, and demand utility function:

$$\min \sum_{p} \sum_{s} \sum_{b} l_{psb} \cdot \sum_{u=1}^{U} \left(C_{upsb} + \frac{\left(P_{upsb} - H_{upsb} - G_{upsb} \right)^2 \cdot \theta_{upsb}}{2} \right) - U\left(D\right) \tag{26}$$

where the demand utility function $U(D)$ is computed as

$$U\left(D\right) = \sum_{p} \sum_{s} \sum_{b} l_{psb} \cdot \frac{1}{\alpha_{0psb}} \left(D_{psb} \cdot D^{0}_{psb} - \frac{D^{2}_{psb}}{2} \right) \tag{27}$$

[3] The model can easily be extended in order to consider start-up and shut-down costs.

As mentioned, the final problem has a QP structure. Quadratic terms are included only in objective function since all restrictions are linear. Extremely efficient algorithms and solvers are available for this particular schema.

One of the main features of the proposed approach is that marginal price is directly obtained from the results of the optimization problem. Price is computed from the dual variable of the power balance constraint (23) as

$$\lambda_{psb} = \frac{\pi_{psb}}{l_{psb}} \tag{28}$$

In addition, the profit of each utility u is obtained as its income minus its costs:

$$B_u = \sum_p \sum_s \sum_b l_{psb} \cdot \Big[\lambda_{psb} \cdot P_{upsb} - C_{upsb} + \left(\lambda^G_{upsb} - \lambda_{psb}\right) \cdot G_{upsb} + \left(\lambda^H_{upsb} - \lambda_{psb}\right) \cdot H_{upsb}\Big] \tag{29}$$

5 Equilibrium in Power Networks

When firms decide their output, they must anticipate which lines will be congested after the market clearing, in order to anticipate how their production will affect prices. One way of tackling this problem is within the leader–follower framework. The problem can be stated as a two-stage game, where the firms first allocate their output and submit their bids to the central auctioneer (leaders) and then the central auctioneer clears the market, given the bids of the firms (follower). Hereinafter, we will assume a central auctioneer as a representation of an efficient market-clearing process, in spite of the fact that the clearing mechanism may not include a central auction, as in the case of transmission auctions. Unfortunately, when the transmission network is taken into account by means of a DC load flow, a pure strategies equilibrium for such a game may not exist or may be not unique (Ehrenmann 2004; Daxhelet and Smeers 2001), since optimality conditions of the auctioneer problem considered as constraints of the firms' profit maximization result in a nonconvex feasible region. An alternative proposal to cope with the problem is to assume some conjectured reaction of the firms to the decisions of the transmission system operator. This facilitates the convexity of the feasible region, while still playing a quantity game against other firms when deciding their output. This is the case of Metzler et al. (2003) and Hobbs and Rijkers (2004), which assume the reactions of the firms to the decisions of the transmission system operator to be exogenous parameters, regardless of the distribution of the flows in the network: in Hobbs and Rijkers (2004) the conjecture is no reaction, that is, firms are price-takers with respect to transmission prices.

In principle, including transmission constraints in the above model may be dealt with like any other technical constraint. However, the power network adds the spatial dimension to the problem, and consequently the strategic interaction between power producers is extended to the new dimension. The main problem when taking into account the power network is that price sensitivities, modeled above as conjectured variations, are not independent of the set of congested lines of the system. Therefore, power producers must anticipate the result of the market clearing to anticipate how their decisions will affect prices. We describe in this section a way to obtain the market equilibrium avoiding the statement of a two-stage game between power producers and the system operators. The rationale here is to model the decision-making process of the firms as a search for consistent assumptions: firms assume that the central auctioneer will decide that a certain set of transmission lines will be congested and, on that assumption, decide their output. If the market clearing considering these bids results in the assumed set of congested lines, then the assumption is consistent and the solution is a Nash equilibrium. The firms' assumption about congested lines gives their price response, so that any iteration can be thought of as their conjectured-transmission-price response (Metzler et al. 2003; Hobbs and Rijkers 2004). However, the iterative procedure may be viewed as a way of selecting the firms' reactions to the power-network constraints that is consistent with the central auctioneer behavior, and thus of foreseeing the auctioneer's reactions to the firms' bids.

The statement of the multiperiod problem with network constraints is close to the model described in previous sections. However, we choose to describe a simplified single-period model with the aim of highlighting network-related effects.[4] In the same spirit, we do not take into account any forward contract. Nevertheless, both inter-temporal constraints and forward positions can be included as in the sections above. The transmission network has a set of nodes $M = \{1, 2, \ldots, N\}$. Market agents will be characterized by their output decisions at each node of the network, denoted by P_u^i. Consequently, there is, in general, a different price at each node λ_i. Thus, firms decide by solving the following program:

$$B_u = \sum_{i \in M} \lambda_i P_u^i - C_u^i \left(P_u^i\right) \tag{30}$$

The above expression is just the multi-node generalization of the single period model, where the profit of a certain firm is the sum of its profit over every node. Again, the set of solutions to the problems for each firm characterizes the market equilibrium:

$$\lambda_i + \left[\sum_{j \in M} \frac{\partial \lambda_j}{\partial P_u^i} P_u^j - \frac{\partial C_u^i \left(P_u^i\right)}{\partial P_u^i} \right] = 0 \tag{31}$$

[4] However, interaction of inter-temporal and network constraints can lead to other concerns, as in Johnsen (2001) and Skaar and Sorgard (2006) in the case of hydro management in a system with network constraints.

Therefore, the conjectured price-response is

$$\theta_u^{ij} = -\frac{\partial \lambda_j}{\partial P_u^i} \tag{32}$$

Under reasonable assumptions, it can be shown that $\theta_u^{ij} = \theta_u^{ji}$ (Barquín 2008). Following the same rationale as in the single node case, we can define an effective cost function, given by the expression

$$\bar{C}_u\left(P_u^i\right) = C_u^i\left(P_u^i\right) + \frac{1}{2}\sum_j P_u^i \theta_u^{ij} P_u^j \tag{33}$$

Note that symmetry of conjectured responses θ_u^{ij} is required in order that optimality conditions be the same as equilibrium conditions.

After defining the effective cost function for market players, following the reasoning of previous sections implies defining the market-clearing conditions. In the single-period, single-node case, market clearing was defined by the demand curve and the power balance equation. When the transmission network is considered, demands and power flows at each node are determined to maximize the utility of demand. That is, the market clearing problem is the solution of an optimal power flow solved by the system operator. Then

$$\begin{gathered} \min_{D,f,\theta} -\sum_i U\left(D_i\right) \\ \text{s.t. } D_i + \sum_j m_{ij} f_j = \sum_u P_u^i \\ f_j = y_j\left(\theta_i - \theta_k\right) \\ -f_j^{\max} \le f_j \le f_j^{\max} \end{gathered} \tag{34}$$

The first constraint is the power balance equation: f_j is the flow through the line j. m_{ij} is 1 if the line j is leaving the node i, -1 if the line is arriving at the node i, and 0 otherwise. The second constraint represents the DC power flow equations: y_j is the admittance of the line j, and $\theta_i - \theta_k$ is the difference of voltage phases between the nodes of the line j. The last constraint is the maximum flow through transmission lines. The full equivalent optimization problem is

$$\begin{gathered} \min_{P_u^i D,f,\theta} \sum_{u,i}\left[\bar{C}_u\left(P_u^i\right)\right] - \sum_i U\left(D_i\right) \\ \text{s.t. } \quad D_i + \sum_j m_{ij} f_j = \sum_u P_u^i \ : \lambda_i \\ f_j = y_j\left(\theta_i - \theta_k\right) \\ f_j^{\max} \le f_j \le f_j^{\max} \end{gathered} \tag{35}$$

It is important to notice that the above problem represents the market-clearing process. Therefore, one may argue that many power markets have different mechanisms, and that the problem above represents the particular case of nodal-pricing.

However, it should be understood as a description of any efficient mechanism to clear the market. For example, the result of a market splitting design would be the same as the solution to the above problem.

The previous problem allows for the calculation of the conjectured-response equilibrium, as long as the conjectured price variations θ_u^{ij} are known. In principle, it is possible, as in the single-node model, to consider the variations as known data. However, when the power network is considered, this task is more delicate. The variations represent, by definition, the response of every player in the market. But the transmission constraints complicate the kind of reactions that can be found. Consider, for example, a two-node grid. If the unique transmission line is binding, changes in the production of units located at one of the nodes can be compensated only by units at the same node. Thus, the owner of the unit is competing only with firms owning units at the same node. In contrast, if the line is below its limits, the system may be considered to be a single-node network. In such a case, the previous unit is competing with the whole system. Therefore, in general, the conjectured responses depend on whether the line is binding or not, and hence the maximum flow constraint of the line may be used strategically. Therefore, it is necessary, in principle, to define a set of conjectures θ_u^{ij} for each possible set of constrained lines (hereinafter, we will refer to this set as network state). To take into account the strategic use of congestions, and thus to consider rational agents, one faces the problem of anticipating the changes in price responses with respect to the network state. In other words, rational agents would anticipate the auctioneer's decisions. The problem resulting from such a statement, in addition to being computationally expensive, may have infinite equilibria, or may have none.

As an alternative, we propose an algorithm that is designed to search for consistent price responses as an approximation to the equilibrium of the two-stage game. In the two-node example, consistent price responses imply that the producer is not willing to congest the line, as if he were, and in the next iteration he would produce so that the line is binding. Therefore, although at each step of the algorithm, power producers do not take into account congestions as strategic variables, the complete algorithm describes the market players' selection of the most lucrative set of congested lines. The iterative algorithm that we explain below can be thought of as a way of calculating endogenously the state-dependent responses.

In any case, in the market clearing problem, generated energy P_u^i is no longer a decision variable but an input data. Therefore, the network state as well as the relevant set of conjectured price responses (see below) can be computed as a function of the generated energy. So we state the condition that, as power producers must assume the network state to make their decisions, the network state assumed when deciding the output of each unit must be the same as the network state obtained after the market has been cleared. This is the rationale for the iterative algorithm, which works as follows. First, a network state is assumed. Then, with the corresponding conjectures θ_u^{ij} the optimization problem is solved, resulting in a new network state. If it coincides with the network state previously assumed, then the algorithm ends; if not, the profit-maximization problem is solved again with the new conjectures and the whole procedure iterates again.

The algorithm loops until either convergence is obtained or a limit cycle is detected. The algorithm convergence necessarily implies that a consistent Nash equilibrium has been computed, because price sensitivities assumed by the oligopolistic agents when bidding are the same as those resulting after the market has been cleared. However, the convergence of the algorithm is not assured for a number of reasons: the game may have no pure strategies equilibrium, and so the algorithm will not converge; in addition, the procedure does not consider points where any line is just changing from congested to non-congested, or vice versa, since market players, in each of the iterations, do not take into account that the lines may change their state, for example, from congested to non-congested, as a result of the production decisions. However, in such cases, multiple Nash equilibria (even a continuum of equilibria) may exist (Ehrenmann 2004). Further discussion can be found in Barquín and Vázquez (2008).

In addition, the model can also be written by using power distribution factors. However, we do think that one definite advantage of directly stating a DC load flow representation is that it facilitates, from the numerical point of view, the study of more complex power systems, because the admittance matrix used in the DC load flow formulation is a sparse one.

Even if the market clearing problem makes it possible to relate the production decisions to the network status, the problem of defining a complete set of conjectures θ_u^{ij} is far from having been solved. In particular, it is necessary to define which nodes react to output decisions at a certain node, and this problem is not obvious in complex networks. We will next show how to calculate conjectured responses based on the following Cournot-like assumption: the production decision at a certain node is not affected by the competitors' decisions at the rest of the nodes. However, more general conjectures can be considered. For instance, in Barquin (2008) the case of more general supply functions is analyzed. One of the most important results of all these analysis is a qualitative one: conjectured-price responses θ_u^{ij} are found to be symmetric $\left(\theta_u^{ij} = \theta_u^{ji}\right)$ under general circumstances. To obtain explicit values, the techniques considered in Sect. 3 can be used, although attention must be paid to the network status.

The Cournot-like case is a particularly important instance, as it is assumed in a number of studies with regulatory aims. So let us consider a net additional megawatt at a certain node (in general due to the combination of decisions of every producer located at the node). In this case, from the point of view of the auctioneer, there is an additional power injection in that bus. Therefore, given the new generation, the system operator will react to maximize the total consumer value by means of redistributing power flows and demands. More formally, consider a net additional megawatt at the ith node δP^i. The system operator will decide on the incremental variables δD_i, δf_j, and $\delta\theta_i$ to reoptimize the load flow. In addition, the flow through unconstrained lines may change, while the flow in constrained lines must remain constant, equal to its limit value. Therefore, the system operator solves the following program:

$$\begin{aligned} \min_{D,f,\theta} \quad & -\sum_i U\left(D_i + \delta D_i\right) \\ \text{s.t. } & \delta D_i + \sum_j m_{ij} \delta f_j = \delta P^i \\ & \delta f_j = y_{ij}\left(\delta\theta_i - \delta\theta_k\right) \\ & \delta f_{cong} = 0 \end{aligned} \tag{36}$$

The problem is the incremental version of the program previously described. The last constraint represents the fact that after a small change the network state will not change. This is not a harmless assumption, as the oligopolistic game outcome can result in lines being "critically congested." In these lines the flow is at its maximum, but arbitrarily small changes in the system conditions can result in the line being either congested or not congested (Cardell et al. 1997; Barquín 2006).Therefore, the solution to this problem provides δD_i for every node. Moreover, by solving the problem consecutively for an increment of production δP^i at each of the nodes, it is possible to obtain the derivatives $\frac{\partial D_j}{\partial P^i}$. The last step in calculating the conjectured response is to state the relationship between the previous derivative and the response. To do so, taking into account the definition of the demand curve given in the single-node case, we have that the conjectural response is

$$\frac{\partial \lambda_j}{\partial P_u^i} = -\frac{1}{\alpha_j} \frac{\partial D_j}{\partial P_u^i} \tag{37}$$

Thus, it is enough to obtain the derivative $\frac{\partial D_j}{\partial P_u^i}$. On the other hand, we have

$$\frac{\partial \lambda_j}{\partial P_u^i} = -\frac{1}{\alpha_j} \frac{\partial P^i}{\partial P_u^i} \frac{\partial D_j}{\partial P^i} \tag{38}$$

Finally, armed with the definition of the conjectured responses of the single-node case, we have that

$$\frac{\partial \lambda_j}{\partial P_u^i} = \theta_u^{ij} \frac{\partial D_j}{\partial P^i} \tag{39}$$

where θ_u^i is the variation as defined in the previous sections, that is, disregarding the effects of the rest of the nodes. Examination of the optimality conditions of the incremental program allows us to prove that $\theta_u^{ij} = \theta_u^{ji}$.

6 Consideration of Uncertainty: Stochastic Model

Some studies have dealt with market equilibrium under uncertainty. In Murto (2003), equilibrium with uncertain demand is analyzed and solved within the framework of game theory. Results are obtained analytically, which provides insight into market behavior but is restricted to two periods. The study in Kelman et al. (2001) addresses the representation of static market equilibrium where market players face a multiperiod optimization, and expands the concept of stochastic dynamic programming (SDP) with the objective of assessing market power in hydrothermal systems where inflows are subject to uncertainty. Nash–Cournot equilibrium is solved for each period for different reservoir levels, and a set of future-benefit functions that replace the classical single future-cost function is computed. Convexity of these future-benefit functions is not guaranteed, nor is the convergence of this SDP algorithm. The structure of the optimization problem is suitable for stochastic representations of the operation of hydrothermal generation systems and the consideration of uncertainty (Morton 1993; Gorenstin et al. 1992; Pereira and Pinto 1991). When it comes to medium-term generation operation, several major sources of uncertainty can be identified: hydro inflows, fuel prices, emissions costs, system demand, generating units' failures, and competitors' behavior. Any of these factors can be considered as stochastic with the presented method, which is based on a scenario-tree representation (Dupacova et al. 2000). The proposed model allows for the computation of non-anticipatory decisions for systems with numerous generation units, including a detailed description of the technical characteristics of generating plants that determine the shape of agents' cost functions. Moreover, scenario trees can be extended to a large number of periods. These two characteristics add special interest to the model from a practical point of view. In addition, we do not represent forward contracting to simplify the description of the model.

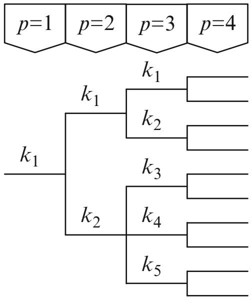
An extension of the deterministic market representation is required to consider uncertainty for some of the parameters included in it.[5] Scenario analysis arises as the first possibility. The number of alternative scenarios that can be considered has to be reduced due to the size of the problem. This approach consists of first finding market equilibrium separately for each alternative value of the uncertain parameters and then performing a simultaneous analysis of the results obtained, computing, for example, an average value. This method lacks robustness for short-term operation: independent equilibrium computation leads to different solutions for the first periods when a single decision would be desirable. Moreover, the reduced number of scenarios makes the analysis insufficiently accurate if treated as a Monte Carlo simulation.

A second and more sophisticated alternative for taking into account uncertainty, especially when the size of the problem prevents an extensive Monte Carlo analysis, is using a scenario tree, which allows the inclusion of stochastic variables and provides a single decision in the short-term. Figure 1 shows an example of the structure of a scenario tree. Each period p has been assigned a consecutive number, as well as each branch k within each period.

The set $B(p)$ comprises those branches that are defined for each period. The correspondence of precedence $a(p,k)$ establishes a relationship between the branch k of period p with the previous one. These two elements precisely define the tree structure. For example, in Fig. 1, $a(p_3, k_2) = k_1$ and $B(p_3) = \{k_1, k_2, k_3, k_4, k_5\}$.

[5] No transmission network is considered in this section.

Fig. 1 Example structure of a scenario tree



Each branch is assigned a probability w_{pk}. This assignment must satisfy two coherence conditions. First, the probability of the single branch in the first period must be one, and second, the subsequent branches of a single branch must add to the probability of the previous branch:

$$\sum_{k^*|a(p,k^*)=k} w_{pk^*} = w_{p-1,k} \ \forall p > 1, \ \forall k \in B(p-1) \tag{40}$$

The overall profit function that will be considered in the stochastic model is the expected profit, which will be maximized for every generation company u in the model:

$$B_u = \sum_{p} \sum_{k \in B(p)} \sum_{s} \sum_{b} \left[l_{psb} \cdot w_{pksb} \cdot \left[\lambda_{pksb} \cdot P_{upksb} - C_{upksb} \left(P_{upksb} \right) \right] \right] \tag{41}$$

This extension of profit definition includes different prices and companies' productions for each branch, which allows the inclusion of uncertainty in parameters that affect costs, such as fuel costs, hydro inflows, or the availability of units. Note that the cost function for every branch depends on the production levels in other branches, possibly including the present and all the previous ones. So, it is possible to accommodate hydro-management and other inter-temporal policies. On the other hand, costs of a given company are not dependent on other companies' productions. This excludes issues like joint management of common hydrobasins. Companies are linked only in the electricity market.

Some extensions are required to establish a stochastic formulation of the previous deterministic market equilibrium over a scenario tree structure. First, the conjectured responses of the clearing price with respect to each generation company production are different in each branch of the tree and represent different behaviors of the companies under different circumstances:

$$\theta_{upksb} = -\frac{\partial \lambda_{pksb}}{\partial P_{upksb}} \tag{42}$$

Additionally, demand is also different for each branch, which makes possible the introduction of uncertainty in its value:

$$D_{pksb} = D^0_{pksb} - \alpha_{0pksb} \cdot \lambda_{pksb} \tag{43}$$

Furthermore, system balance constraint must be satisfied for every branch:

$$\sum_{u=1}^{U} P_{upksb} - D_{pksb} = 0 \tag{44}$$

Finally, cost functions $C_{upksb}(P_{upksb})$ are also dependent on the branch. Maximization of the new profit expression for each generation company leads to

$$l_{psb} \cdot w_{pksb} \cdot \lambda_{pksb} = \frac{\partial \sum\limits_{upksb} C_{upksb}\left(P_{upksb}\right)}{\partial P_{upksb}} + l_{psb} \cdot w_{pksb} \cdot P_{upksb} \cdot \theta_{upksb} \tag{45}$$

Equations (43), (44), and (45) define the stochastic market equilibrium. The newly defined equilibrium is obtained from the solution of the following optimization problem:

$$\begin{aligned} \min_{P_{upksb}, D_{pksb}} \; & \sum_{p} \sum_{k \in B(k)} \sum_{s} \sum_{b} \left[l_{psb} \cdot w_{pksb} \cdot \left[\sum_{u=1}^{U} \left[\bar{C}_{upksb}\left(P_{upksb}\right) \right] - U\left(D_{pksb}\right) \right] \right] \\ \text{s.t.} \; & \sum_{u=1}^{U} P_{upksb} - D_{pksb} = 0 \qquad : \eta_{pksb} \end{aligned} \tag{46}$$

In this expression, effective cost functions $\bar{C}_{upksb}\left(P_{upksb}\right)$ have been introduced:

$$\bar{C}_{upksb}\left(P_{upksb}\right) = C_{upksb}\left(P_{upksb}\right) + \frac{P^2_{upksb} \cdot \theta_{upksb}}{2} \tag{47}$$

Utility function $U(D_{pksb})$ maintains its definition (5) in each branch

$$U\left(D_{pksb}\right) = \int_0^{D_{pksb}} \lambda_{pksb}\left(D_{pksb}\right) \cdot dD_{pksb} = \frac{1}{\alpha_{0pksb}} \cdot \left(D_{pksb} \cdot D^0_{pksb} - \frac{D^2_{pksb}}{2} \right) \tag{48}$$

The system's marginal clearing price λ_{pksb} for each period p, branch k, subperiod s, and load level b can be computed from the Lagrange multiplier of the constraint as follows:

$$\lambda_{pksb} = \frac{\eta_{pksb}}{l_{psb} \cdot w_{pksb}} \tag{49}$$

For the sake of simplicity, the complete formulation of the stochastic optimization problem is not shown. Nevertheless, the equivalent minimization problem for the deterministic case is easily extended to obtain an equivalent problem for the stochastic one (Centeno et al. 2007). In this study, the above modeling was applied to a medium-term system with significant hydro resources, which were the main uncertainty source and also the most significant reason for inter-temporal constraints. Natural final conditions of the reservoirs' levels were also available: those required at the end of the dry season. Therefore, there was a natural horizon to be analyzed.

7 Medium-Term Signals for Short-Term Operation

The operation of power generation systems has traditionally been organized following a hierarchical structure. In the new liberalized framework, this hierarchy is maintained. Planning decisions belong to a long-, medium-, and short-term level according to their horizon of influence (Pereira and Pinto 1983). Typically, the long-term decision level considers more than 3 years of operation, the medium-term level encompasses from a few months up to 2 years, and the short-term level includes at most the following week. The detail with which the power system and the time intervals are represented diminishes as the time horizon of interest increases.

Longer-term decision levels yield resource allocation requirements that must be incorporated into shorter-term decision levels. This coordination between different decision levels is particularly important to guarantee that certain aspects of the operation that arise in the medium-term level are explicitly taken into account. Therefore, when a constraint is defined over a long- or medium-term scope, it is necessary to apply a methodology, so that it will be considered properly in the short-term operation planning.

Traditional short-term operation-planning tools such as unit-commitment or economic-dispatch models include guidelines to direct their results towards the objectives previously identified by the medium-term models. It is important to highlight that the models adopted in practice to represent medium- and short-term operation differ significantly. Usually, medium-term models are equilibrium problems with linear constraints that represent oligolopolistic markets. However, short-term models represent in detail the problem for a company facing a residual demand function (Baillo et al. 2006). There are two main reasons for not adopting a market-equilibrium model in the short term: the model would become computationally unaffordable, and it is very rare for a company to own detailed data about its competitors' generation units.[6]

When addressing the coordination between its medium-term planning and its short-term operation, the company faces two kinds of situations.

[6] A company can estimate medium-term model data from its knowledge of its own units. Nevertheless, short-term model data include real-time characteristics, which can differ significantly between different units.

- The allocation of limited-energy resources throughout the whole medium-term horizon, for example, hydro resources or maximum production levels for thermal units with limited emission allowances.
- The allocation of obligatory-use resources throughout the whole medium-term horizon, such as minimum production levels for technical, economic, or strategic reasons. Two examples of this situation are a minimum-fuel-consumption requirement due to a take-or-pay contract or a minimum market share for the company to maintain its market position.

Three different approaches for coordinating medium- and short-term models are proposed: the primal-information approach, the dual-information approach, and the marginal resource-valuation function.

7.1 Primal-Information Approach

This has been mostly used in practice to send signals from medium-term generation-planning models to short-term operation models.

Once the market equilibrium is computed with the medium-term model, a resource production level will be obtained for each time period considered in the medium-term scope, either for limited-energy resources or for obligatory-use resources. The strategy followed by the primal-information approach is to strictly impose these production levels of resources, that is, primal signals, as constraints on the short-term operation of the company.

The main advantage of the primal coordination is that it is easily obtained from the medium-term models and is not difficult to implement in the short-term models. Furthermore, the signals provided with this approach are very easy to understand, since they are mere short-term-scope production levels of the resources. Finally, the primal-information approach ensures that the medium-term objective will be satisfied.

However, this methodology has important drawbacks. The main one is the lack of flexibility in the decisions that the company can make in the short term: the resource usage levels provided by the medium-term model may not be optimal, mainly due to the time aggregations in the models and to the presence of uncertainty. In addition, the results obtained with the medium-term model are based on a forecast of the future market conditions that, in the end, may not arise. Finally, the medium-term model deals with aggregated load levels that may distort short-term results.

Hence, when facing the short-term operation, the company may find a market situation considerably different from the one forecasted, and the primal signals may not be consistent or economically efficient.

7.2 Dual-Information Approach

The dual-information approach is based on valuing the company's resources in the medium-term horizon. The proposed medium-term model makes possible the computing of marginal valuations of the resources, either limited-energy resources or obligatory-use resources (Reneses et al. 2004). These valuations are given by the dual variables of the corresponding constraints. For a limited-energy resource, the valuation is the dual variable of the maximum-medium-term-production constraint. For an obligatory-use resource, the valuation is the dual variable of the minimum-medium-term-production constraint.

Once the medium-term planning provides these valuations, that is, dual signals, they can be incorporated into the short-term operation. The short-term model may incorporate the explicit valuation of the company's resources into its objective function, depending on the kind of resource. For a limited-energy resource, its use will be penalized. For an obligatory-use resource, it will be a bonus for its use.

Equation (50) illustrates an example of an objective function. It considers hydro production for a generating unit h valuated through the dual variable μ, and annual minimum production for a thermal unit t valuated through the dual variable π. Both of them are obtained from the medium-term model constraints:

$$B_u = \sum_n \left[\lambda_n \left(P_{un}\right) \cdot P_{un} - C_{un}\left(\dot{P}_{un}\right) + \mu \cdot \sum_{h \in u} P_{hn} + \pi \cdot \sum_{t \in u} P_{tn} \right] \tag{50}$$

where B_u is the profit of the company u and $C_{un}(P_{un})$ is the cost function of the company's generating portfolio in hour n.

Note that μ is nonpositive and π is non-negative. Thus, there is a penalty for the use of hydro production of unit h and a bonus for the use of thermal production of unit t. As a consequence of this, the sign of dual variables directly takes into account the penalty or bonus for different resources.

An interesting aspect of the dual-information approach is the interpretation of the dual signals. They should not be considered as a reflection of how much it *costs* the company to use the resource in the short-term scope, but rather of how much it *costs* in the rest of the medium-term scope.

The main disadvantage of dual coordination is the lack of robustness. A small change in the medium-term valuation can lead to important changes in the short-term operation. Another problem of the dual approach arises when the actual market conditions are considerably different from the forecasts used in the medium-term planning. In this situation, the valuation provided by the medium-term model may be incorrect. One way to avoid these undesirable effects is to combine the primal and dual approaches. The valuation of the resources is included, but the deviation from the medium-term results is limited to a selected range.

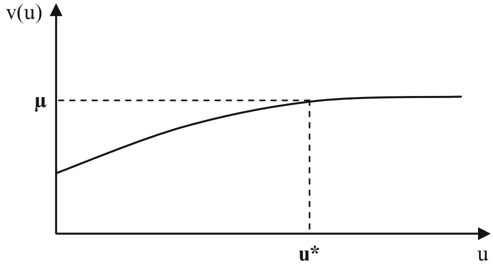


Fig. 2 Marginal valuation function for a limited-energy resource

7.3 Marginal Resource-Valuation Functions

A marginal resource-valuation function is a continuous valuation of a resource, either a limited-energy resource or an obligatory-use resource, for a range of operating points that the company could face. The valuation function is an extension of the dual-information approach, which provides only one point of the function. An example of the absolute value, because it is nonpositive, of a marginal valuation function $v(u)$ for a limited-energy resource u is shown in Fig. 2.

The independent variable of the function is the total use of the resource in the short-term scope. μ is the absolute value of the valuation for the resource used in the dual-information approach, and u^* is the resource level used in the primal-information approach.

Note that the absolute value of the function is nondecreasing, because it will be usually higher if the use of the resource is higher in the short-term operation, and, consequently, it will be lower in the rest of the medium-term scope. However, the marginal valuation for an obligatory-use resource is a nonincreasing function.

The coordination based on marginal-resource-valuation functions improves the dual-information approach, because it eliminates its two main disadvantages: on the one hand, the valuation function provides robustness, since significant changes in short-term operation are discouraged; on the other hand, the function provides information for all the values of the independent variable, even for those far from the medium-term forecast.

The marginal valuation functions are incorporated into the short-term operation through new terms in the objective function of the short-term model. Each term corresponds to the total valuation $V(u)$ of a resource, computed as the integral of the marginal valuation function:

$$V\left(u\right) = \int_{0}^{u} v\left(r\right) dr \tag{51}$$

The main drawback of this method is its practical implementation. The computation of the function may be difficult with the medium-term model, although it allows the obtaining of different points of the function through different executions of the model. The computation of this set of points leads to the incorporation of (51) as a piecewise-linear function (see Reneses et al. 2006 for a practical application).

Finally, it should be pointed out that marginal resource-valuation functions are actually multidimensional, that is, the valuation of every resource depends not only on its use, but also on the use of the rest of the resources. Hence, marginal valuation functions are really an approximation of these multidimensional functions. The computation and implementation of multidimensional valuation functions is an immediate extension of the proposed methodology. Nevertheless, the computer time requirements could not justify their use. An alternative is making use of approximate techniques that are well known in cost-minimization frameworks. For example, dynamic programming by successive approximations is used in Giles and Wunderlich (1981) in the long-term operation of a large multi-reservoir system.

8 Conclusion

Electricity market's equilibrium computing remains a fascinating area, with a large number of topics open to research. Some of these topics address concerns of interest in fundamental economics, such as the modeling of oligopolistic behavior. In fact, there is not a wide accepted economic theory for this kind of markets, as opposed to the perfect competition and monopoly situations. However, research in electricity markets has contributed some interesting insights in this regard.

The aim of this paper has been more modest: to contribute to the modeling of well-known oligopolistic competition paradigms (namely, Cournot and conjectural variations competition) for huge systems, including all the complexities that have traditionally been considered, especially medium-term uncertainties and network effects. Many topics remain open in both areas, such as risk aversion modeling or complex strategies based on network effects.

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