Homework 4 (due on Wednesday 03/04/2020)

Problem 1. Consider the dog-leg path

$$\tilde{p}(\tau) = \begin{cases} \tau p^U, & 0 \le \tau \le 1 \\ p^U + (\tau - 1)(p^B - p^U), & 1 \le \tau \le 2 \end{cases}$$

where $p^U = -\frac{\|g\|^2}{g^\intercal B g} g$ and $Bp^B = -g$. Suppose the symmetric matrix B and the vector g satisfy: 1)$g^T B g > 0$; 2)$(p^U)^T(p^B - p^U) > 0$. Prove that

1. $\|\tilde{p}(\tau)\|$ is an increasing function of τ;
2. $m(\tilde{p}(\tau))$ is a decreasing function of τ,

where $m(p) = g^T p + \frac{1}{2} p^T B p$.

Problem 2. Code Algorithm 4.1 in the textbook with 1). the Cauchy point method for the subproblem, 2) the dog-leg method based on the results from Problem 1. Test and compare the performance of the methods on the following problem:

$$\min_{x \in R^n} f(x) = \log(1 + x^\intercal Q x),$$

where Q is a symmetric and positive definite matrix.

Problem 3. Exercise 6.4 in the textbook.

Problem 4. Exercise 7.1 in the textbook. Choose $\alpha = 100$ and $n \ge 1000$ in your simulation. You can code Limited Memory BFGS using either Algorithm 7.4 or the following recursive function call as we discussed in the lecture.

% Algorithm for Limited Memory BFGS. H_k^0 is set to be γI

function $d = \text{LBFGSrec}(S, Y, d, \gamma, k)$

if $k > 0$

$\alpha \leftarrow \dfrac{s_k^\intercal d}{y_k^\intercal s_k}$;

$d \leftarrow d - \alpha y_k$;

$d \leftarrow \text{LBFGSrec}(S, Y, d, \gamma, k-1)$;

$d \leftarrow d + (\alpha - \dfrac{y_k^\intercal d}{y_k^\intercal s_k}) s_k$;

else

$d \leftarrow \gamma d$;

end