am230hw4

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1 P1

1.

$1 < \tau < 2$
$||\widetilde{p}(\tau)||^{'} = ||p^U|| > 0$

$1 < \tau < 2$; We define $0 < \alpha < 1$
Because $||\widetilde{p}(1+\alpha)|| \geq 0$
$\frac{d||\widetilde{p}(1+\alpha)||}{d\alpha}$
$= \frac{1}{||\widetilde{p}(1+\alpha)||}\frac{d1/2||\widetilde{p}(1+\alpha)||^2}{d\alpha}$
$h(\alpha) = \frac{1}{2}||\widetilde{p}(1+\alpha)||^2 = \frac{1}{2}||p^U||^2 + \alpha(p^U)^T(p^B - p^U) + \frac{1}{2}\alpha^2||(p^B - p^U)||^2$
$h^{'}(\alpha) = (p^U)^T(p^B - p^U) + \alpha||(p^B - p^U)||^2$
$\geqslant (p^U)^T(p^B - p^U)$
$= \frac{g^Tg \cdot g^TB^{-1}g}{g^TBg}(1 - \frac{(g^Tg)^2}{(g^TBg)(g^TB^{-1}g)})$
by Cauchy Schwartz's inequality
$(g^TB^{-1}g)(g^TBg)$
$= ||g^TB^{-1}g||||g^TBg||$
$= ||g||^4||B||||B^{-1}||$
$\geq ||g||^4||BB^{-1}||$
So $(g^TB^{-1}g)(g^TBg) \geq (g^Tg)^2$
Thus $h^{'}(\alpha) \geq 0$

2.
$0 < \tau < 1$
$m^{'}(\widetilde{p}(\tau) = (\widetilde{p}(\tau)^{'})^Tg + (\widetilde{p}(\tau)^{'})^TB\widetilde{p}(\tau)$
$= (p^U)^Tg + (p^U)^TB \cdot \frac{1}{\tau}p^U$
$= (p^U)^T(g + \frac{1}{\tau}Bp^U)$
$= (p^U)^T(g - \frac{1}{\tau}B\frac{g^Tg}{g^TBg}g) \leq 0$

$1 < \tau < 2$
$z(\alpha) = m(\widetilde{p}(1+\alpha)$ for $0 < \alpha < 1$
$z^{'}(\alpha) = (p^B - p^U)^T(g + Bp^U) + \alpha(p^B - p^U)^TB(p^B - p^U)$

$\leq (p^B - p^U)^T(g + Bp^U + B(p^B - p^U))$
$= (p^B - p^U)^T(g + Bp^B) = 0$

2 P2

Cauchy point method used 0.241 s, with iterations 293 times.
dogleg method used 2.399 s, with more than 10000 times.
And Cauchy point has better accuracy in this question.

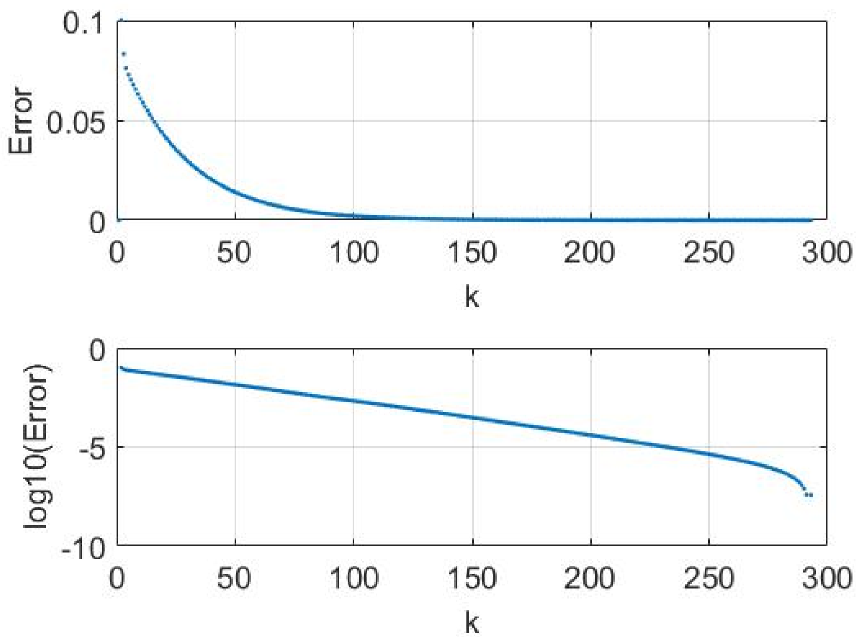


Figure 1: Cauchy point

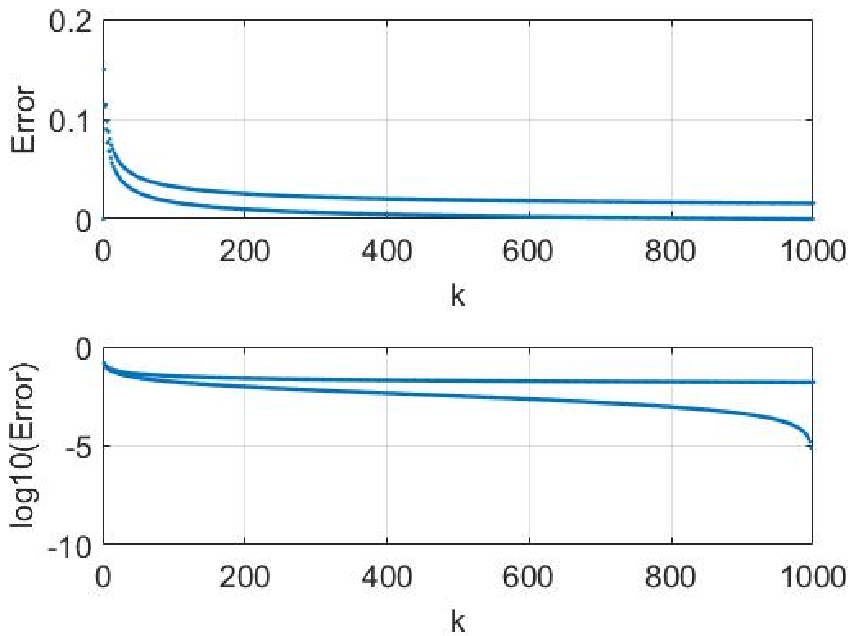


Figure 2: dogleg

3 P3

define $u = (y_k - B_k s_k)$,$B_+ = B_{k+1}$,$B = B_k$

$H_{k+1} = B_+^{-1}$

$= B^{-1} - B^{-1}u(1 + u^T B^{-1}u)^{-1}u^T B^{-1}$

$= B^{-1} - \frac{B^{-1}u(B^{-1}u)^T}{1+u^T B^{-1}u}$ (B is symmetric)

$= B^{-1} - \frac{B^{-1}u(B^{-1}u)^T}{u^T s_k + u^T B^{-1}u}$

$= B^{-1} - \frac{B^{-1}u(B^{-1}u)^T}{u^T B^{-1}(Bs_k + u)}$

$= B^{-1} - \frac{B^{-1}u(B^{-1}u)^T}{u^T B^{-1}y_k}$

$= B^{-1} - \frac{B^{-1}u(B^{-1}u)^T}{(B^{-1}u)^T y_k}$

$= B^{-1} + \frac{B^{-1}u(B^{-1}u)^T}{(s_k - B^{-1}y_k)^T y_k}$

$= H_k + \frac{(s_k - H_k y_k)(s_k - H_k y_k)^T}{(s_k - H_k y_k)^T y_k}$

4 P4

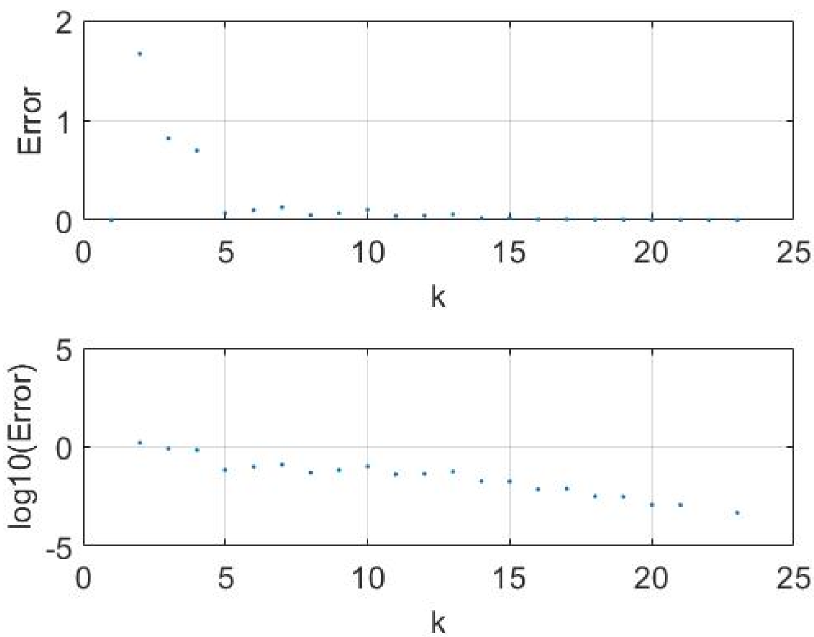
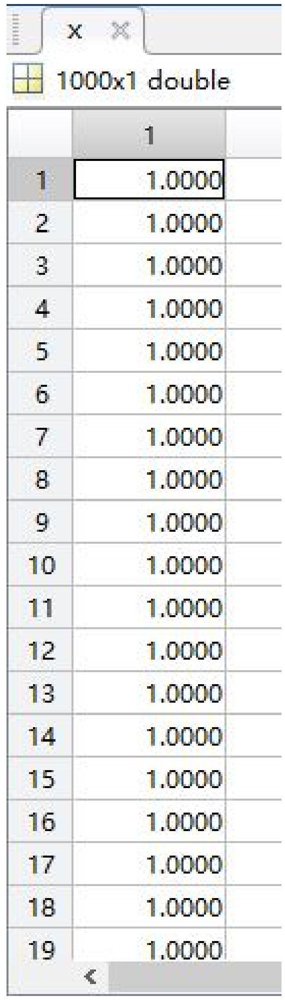


Figure 3: n = 1000

and the result



x

1000x1 double

	1
1	1.0000
2	1.0000
3	1.0000
4	1.0000
5	1.0000
6	1.0000
7	1.0000
8	1.0000
9	1.0000
10	1.0000
11	1.0000
12	1.0000
13	1.0000
14	1.0000
15	1.0000
16	1.0000
17	1.0000
18	1.0000
19	1.0000

Figure 4: result

Total running time is 2.712 s