

am230hw4

jxue16

March 2020

## 1 P1

1.

$$1 < \tau < 2 \\ \|\tilde{p}(\tau)\|' = \|p^U\| > 0$$

$$\begin{aligned} & 1 < \tau < 2; \text{ We define } 0 < \alpha < 1 \\ & \text{Because } \|\tilde{p}(1+\alpha)\| \geq 0 \\ & \frac{d\|\tilde{p}(1+\alpha)\|}{d\alpha} \\ &= \frac{1}{\|\tilde{p}(1+\alpha)\|} \frac{d\frac{1}{2}\|\tilde{p}(1+\alpha)\|^2}{d\alpha} \\ & h(\alpha) = \frac{1}{2}\|\tilde{p}(1+\alpha)\|^2 = \frac{1}{2}\|p^U\|^2 + \alpha(p^U)^T(p^B - p^U) + \frac{1}{2}\alpha^2\|(p^B - p^U)\|^2 \\ & h'(\alpha) = (p^U)^T(p^B - p^U) + \alpha\|(p^B - p^U)\|^2 \\ & \geq (p^U)^T(p^B - p^U) \\ &= \frac{g^T g \cdot g^T B^{-1} g}{g^T B g} \left(1 - \frac{(g^T g)^2}{(g^T B g)(g^T B^{-1} g)}\right) \\ & \text{by Cauchy Schwartz's inequality} \\ & (g^T B^{-1} g)(g^T B g) \\ &= \|g^T B^{-1} g\| \|g^T B g\| \\ &= \|g\|^4 \|B\| \|B^{-1}\| \\ &\geq \|g\|^4 \|B B^{-1}\| \\ & \text{So } (g^T B^{-1} g)(g^T B g) \geq (g^T g)^2 \\ & \text{Thus } h'(\alpha) \geq 0 \end{aligned}$$

$$\begin{aligned} & 2. \\ & 0 < \tau < 1 \\ & m'(\tilde{p}(\tau)) = (\tilde{p}(\tau))' g + (\tilde{p}(\tau))' B \tilde{p}(\tau) \\ &= (p^U)^T g + (p^U)^T B \cdot \frac{1}{\tau} p^U \\ &= (p^U)^T \left(g + \frac{1}{\tau} B p^U\right) \\ &= (p^U)^T \left(g - \frac{1}{\tau} B \frac{g^T g}{g^T B g} g\right) \leq 0 \end{aligned}$$

$$\begin{aligned} & 1 < \tau < 2 \\ & z(\alpha) = m(\tilde{p}(1+\alpha)) \text{ for } 0 < \alpha < 1 \\ & z'(\alpha) = (p^B - p^U)^T (g + B p^U) + \alpha(p^B - p^U)^T B (p^B - p^U) \end{aligned}$$

$$\begin{aligned}
&\leq (p^B - p^U)^T (g + Bp^U + B(p^B - p^U)) \\
&= (p^B - p^U)^T (g + Bp^B) = 0
\end{aligned}$$

## 2 P2

Cauchy point method used 0.241 s, with iterations 293 times.

dogleg method used 2.399 s, with more than 10000 times.

And Cauchy point has better accuracy in this question.

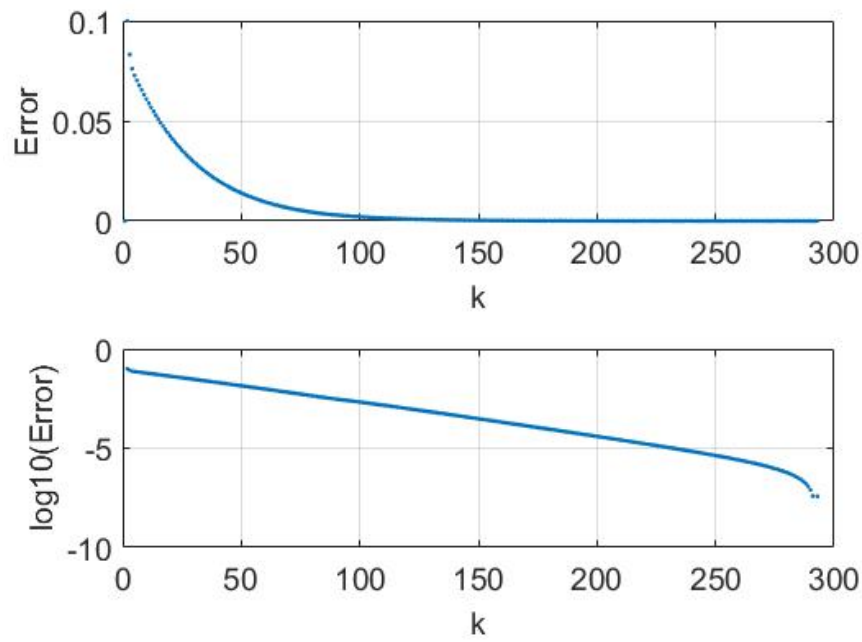


Figure 1: Cauchy point

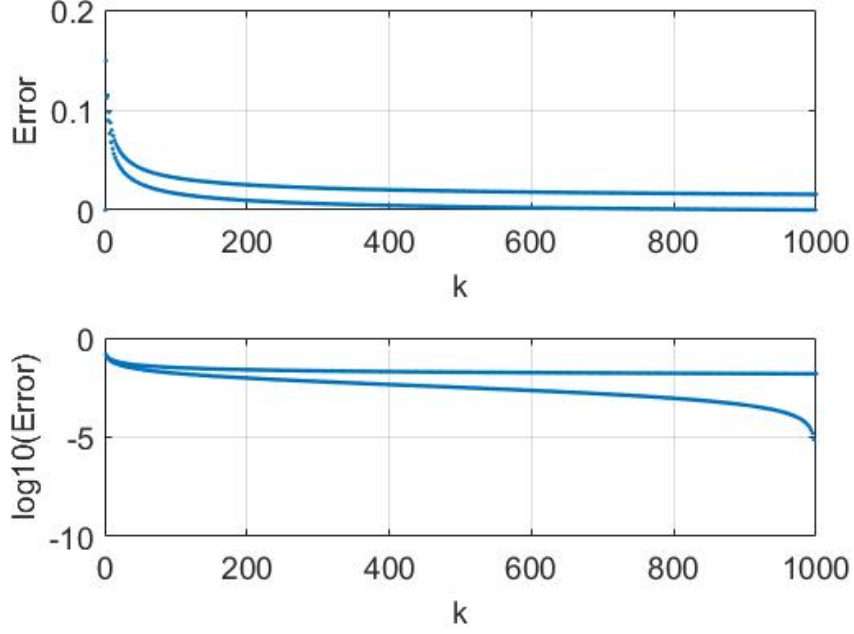


Figure 2: dogleg

### 3 P3

$$\begin{aligned}
& \text{define } u = (y_k - B_k s_k), B_+ = B_{k+1}, B = B_k \\
& H_{k+1} = B_+^{-1} \\
& = B^{-1} - B^{-1} u (1 + u^T B^{-1} u)^{-1} u^T B^{-1} \\
& = B^{-1} - \frac{B^{-1} u (B^{-1} u)^T}{1 + u^T B^{-1} u} \quad (B \text{ is symmetric}) \\
& = B^{-1} - \frac{B^{-1} u (B^{-1} u)^T}{u^T s_k + u^T B^{-1} u} \\
& = B^{-1} - \frac{B^{-1} u (B^{-1} u)^T}{u^T B^{-1} (B s_k + u)} \\
& = B^{-1} - \frac{B^{-1} u (B^{-1} u)^T}{u^T B^{-1} y_k} \\
& = B^{-1} - \frac{B^{-1} u (B^{-1} u)^T}{(B^{-1} u)^T y_k} \\
& = B^{-1} + \frac{B^{-1} u (B^{-1} u)^T}{(s_k - B^{-1} y_k)^T y_k} \\
& = H_k + \frac{(s_k - H_k y_k)(s_k - H_k y_k)^T}{(s_k - H_k y_k)^T y_k}
\end{aligned}$$

#### 4 P4

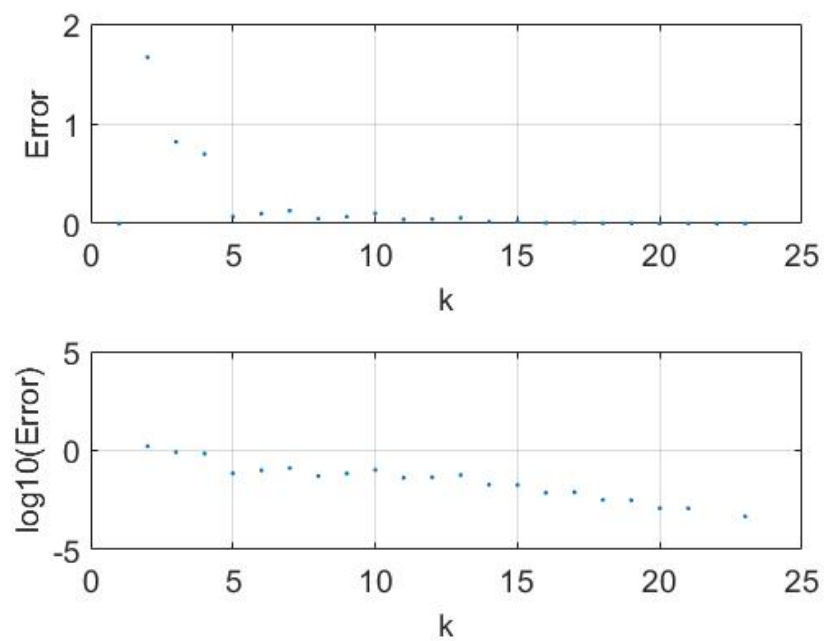
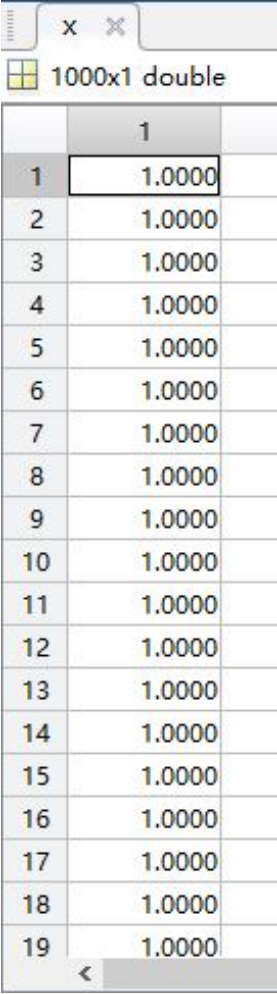


Figure 3:  $n = 1000$

and the result



A screenshot of a MATLAB window titled 'x'. The window shows a 1000x1 double array. The first 19 elements are displayed in a list view, each with a value of 1.0000. The window has a standard MATLAB interface with a title bar, a toolbar, and a scroll bar at the bottom.

	1
1	1.0000
2	1.0000
3	1.0000
4	1.0000
5	1.0000
6	1.0000
7	1.0000
8	1.0000
9	1.0000
10	1.0000
11	1.0000
12	1.0000
13	1.0000
14	1.0000
15	1.0000
16	1.0000
17	1.0000
18	1.0000
19	1.0000

Figure 4: result

Total running time is 2.712 s