am230hw3

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1 P1

If $[p_0, p_1 ... p_n]$ is linear dependent, then there is ${}_0p_0 + ... + \alpha_n p_n = 0$.
for i=0,1...n
$0 = p_i^T A({}_0p_0 + ... + \alpha_n p_n) = \alpha_i p_i^T A p_i$
$\alpha_i p_i^T A p_i \neq 0$, so $\alpha_i = 0$
It is controversial, so $[p_0, p_1 ... p_n]$ is linear independent

2 P2

$\beta_{k+1} = \frac{r_{k+1}^T A p_k}{p_k^T A p_k}$
$r_{k+1} - r_k = \alpha_k A p_k$
$\therefore r_{k+1}^T A p_k = r_{k+1}^T (r_{k+1} - r_k)/\alpha_k = r_{k+1}^T r_{k+1}/\alpha_k$
$p_k = -r_k + \beta_{k+1} p_{k-1}$
$r_k^T p_k = -r_k^T r_k$
$p_k^T A p_k = -p_k^T r_k$
$p_k^T A p_k = -p_k^T r_k/\alpha_k = r_k^T r_k/\alpha_k$
$\therefore \beta_{k+1} = \frac{r_{k+1}^T r_{k+1}}{r_k^T r_k}$

3 P3

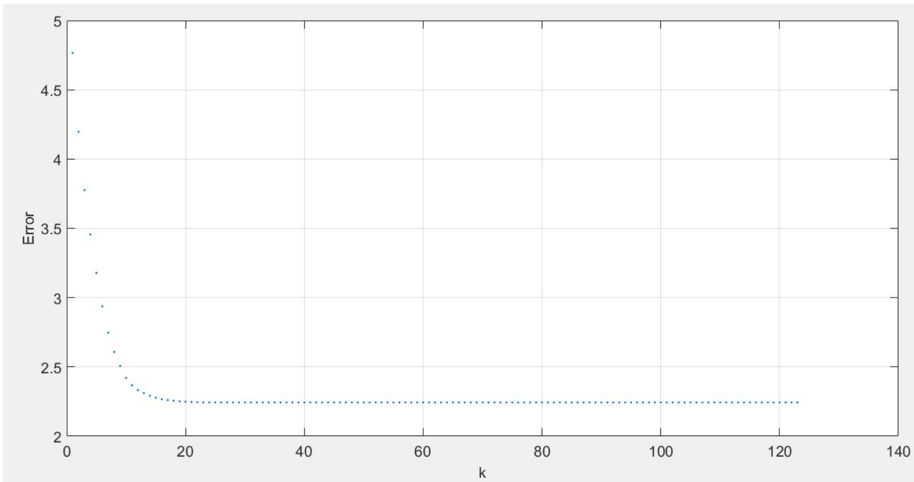

Figure 1

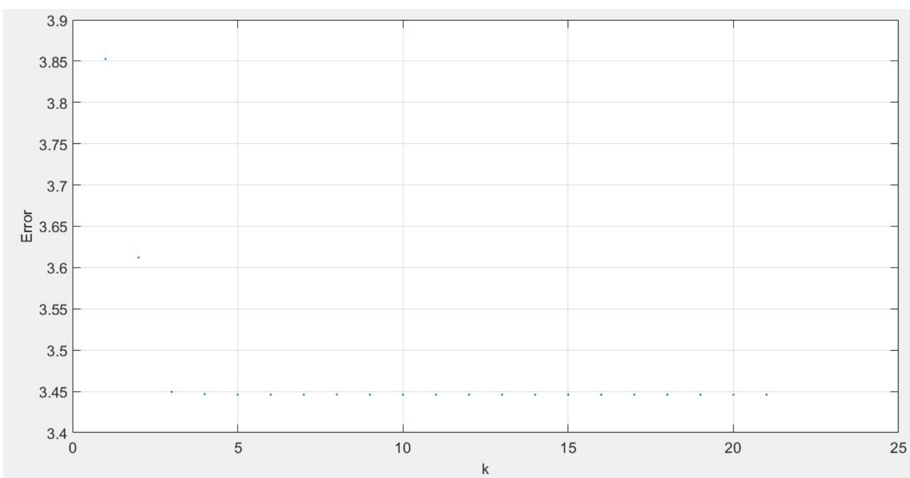

Figure 2

choose n=100

Figure 1 shows that without clustered eigenvalues, the convergence will be uniform. Figure 2 shows that the error of matrix A with two clustered eigenvalues will drop sharply after the some iterations.

4 P4

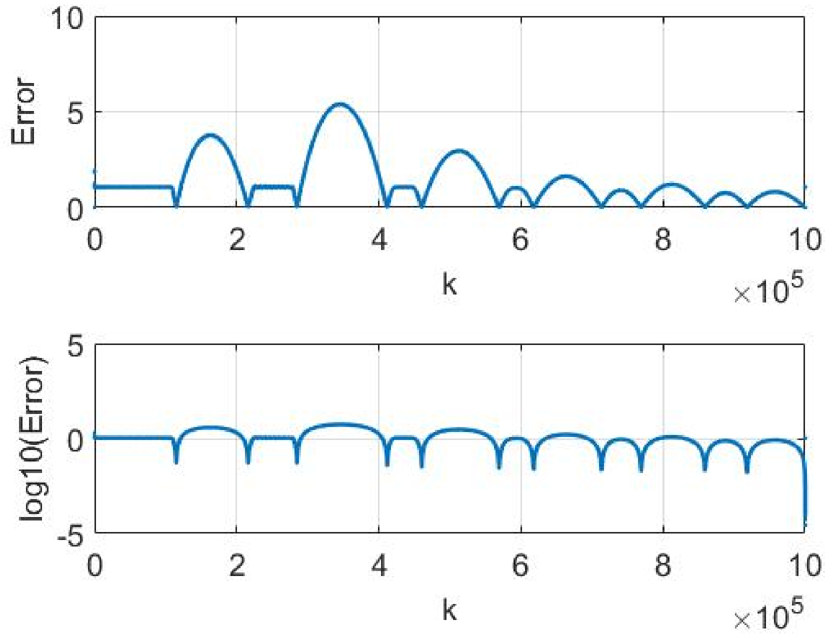

Figure 3: FR

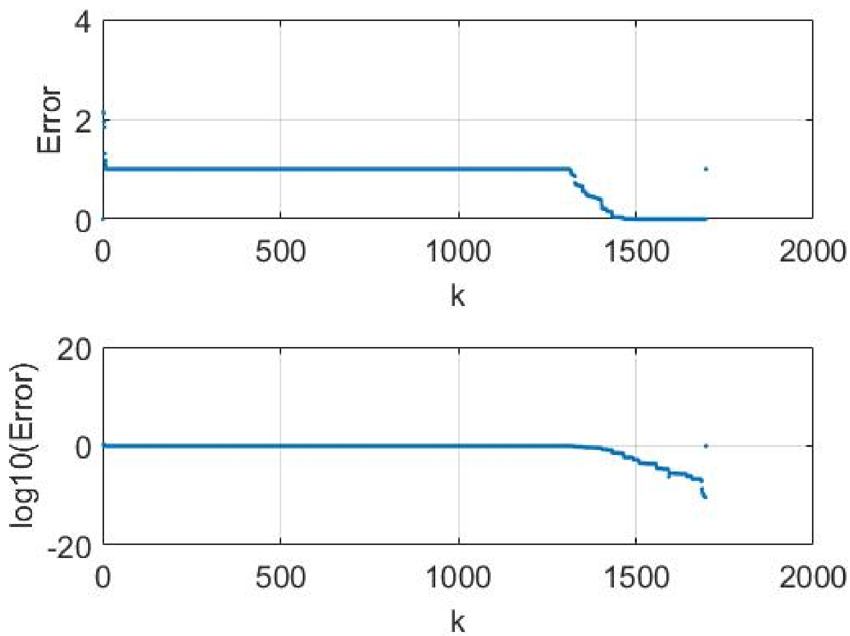

Figure 4: FR with restart

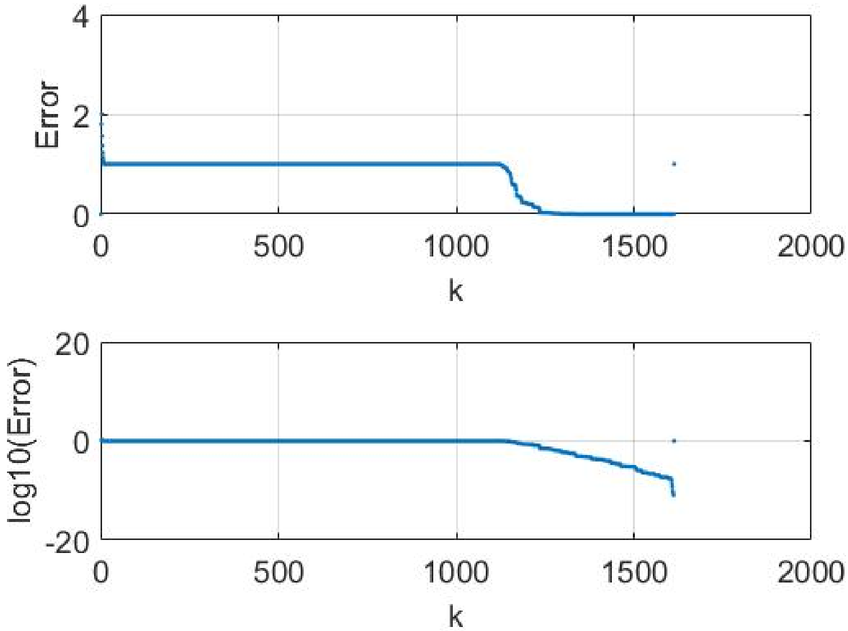

Figure 5: PR

Figure 1 shows that FR method doesn't converge well, it costs long time and the error plot oscillates.
Figure 2 shows that FR method can work well with restart. After 1600 iterations x can converge to $x = [1, 1, 1...1]^T$
Figure 3 shows that PR method can also work well without restart. So it is more robust and efficient than traditional FR method.