Work

The product of force and displacement is called work don

OR

The Scalar product of force and displacement is called work done.

Mathematically

$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd\cos\theta$$

- It is a scalar quantity
- Its unit is Nm (Joule)
- If 1N force is applied on a body and it covers the distance of 1m in the direction of force than the body is doing 1 Joule work.

If

$\theta < 90°$ work is +ve

$\theta = 90°$ work is zero

$\theta > 90°$ work is -ve

$\theta = 0°$ work is maximum

Note:

Work done by Variable Force

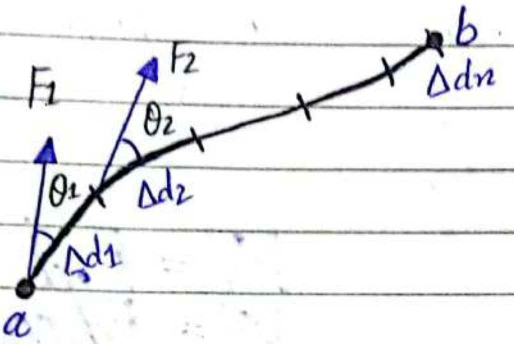
Variable force: If the direction or magnitude of the force changes than the force is called variable force

e.g.

A rocket moves away from the earth.

Explanation:

Consider a path AB in xy plane as shown



We have to calculate the total work done from path A to B which is.

$$W_T = W_1 + W_2 + \_\_\_\_\_ + W_n$$

We also observe that path is not linear So we divide the path into 'n' number of small distances as shown

$$\Delta d_1, \Delta d_2, \Delta d_3, \_\_\_\_\_\_, \Delta d_n$$

The force applied on each path will be

$$F_1, F_2, F_3, \_\_\_\_\_\_, F_n$$

Now

$$W_1 = F_1 \cdot \Delta d_1$$
$$= F_1 \Delta d_1 \cos\theta_1$$
$$W_2 = F_2 \cdot \Delta d_2$$
$$= F_2 \Delta d_2 \cos\theta_2$$
$$W_3 = F_3 \cdot \Delta d_3$$
$$F_3 \Delta d_3 \cos\theta_3$$

•

$$W_n = F_n \cdot \Delta d_n$$

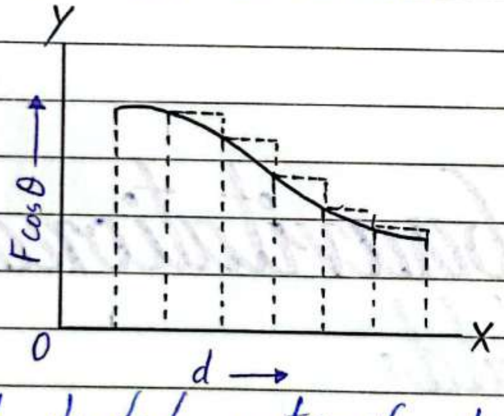
$$W_n = F_n \Delta d_n \cos\theta_n$$

$$W_T = F_1 \Delta d_1 \cos\theta_1 + F_2 \Delta d_2 \cos\theta_2 + \dots + F_n \Delta d_n \cos\theta_n$$

$$W_T = \sum_{i=1}^{n} F_i \Delta d_i \cos\theta_i$$

Graphically:

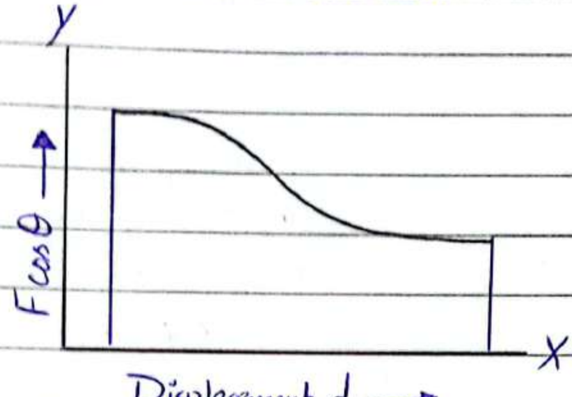
We can examine this graphically by plotting $F\cos\theta$ verses d.
The displacement d has been subdivided into the same n equal intervals. The value of $F\cos\theta$ at the begining of each interval is indicated as shown



Now the ith shaded rectangle has an area $F_i \cos\theta_i \Delta d_i$ which is the work done during the ith interval.
The work done equals the sum of the areas of all the rectangles
If we subdivided the distance into a large number of intervals so that each Δd becomes very small, the work done becomes more accurate
If we let each Δd to approach zero then we obtain an exact result for the work done, such as

$$W_{Total} = \lim_{\Delta d \to 0} F_i \cos\theta_i \Delta d_i$$

In this limit Δd approaches zero, the total area of the rectangles approaches the area between the $F\cos\theta$ curve and from a to b as shown.



Thus, the work done by a variable force in moving a particle between two points is equal to the area under the $F\cos\theta$ verses d curve between the two points a and b.

Work done by Gravitational Feild

Gravitational Feild:

The space or region around the earth with in which gravitational force acts on bodies is called gravitational field and the force acts on a body called gravitational force.

- Work done along the gravitational force is taken as **+ve** and work done against the gravitational force is taken as **-ve**

Explanation:

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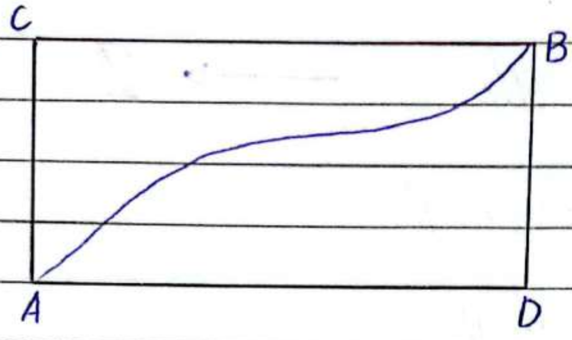
Consider two points **A** and **B** in gravitational field **as shown**



We have to calculate the workdone along all the paths as shown.

Workdone along path ACB:

We observe

$$W_{ACB} = W_{AC} + W_{CB}$$

$$W_{AC} = F \cdot d$$
$$= F d \cos\theta$$
$$= Fd \cos(90)$$
$$W_{AC} = 0 \longrightarrow 1$$

We observe that the angle between the gravitational force and displacement is 90°

Now

$$W_{CB} = F \cdot d$$
$$= Fd \cos\theta$$

Now here we observe that while moving upward. The angle between the gravitational field and displacement is 180°

than

$$W = Fd\cos(180)$$

$$W = -Fd$$

$\therefore F = w = mg$

$d = h$

$$W_{CB} = -mgh \longrightarrow 2$$

Now

$$W_{ACB} = 0 - mgh$$

$$W_{ACB} = -mgh$$

Work done along path ADB:

Now here we observe

$$W_{ADB} = W_{AD} + W_{DB}$$

Now

$$W_{AD} = F \cdot d$$
$$= Fd\cos\theta$$

We observe that the path 'AD' is similar to path 'CB' So

$$\theta = 180^\circ$$
$$F = w$$
$$d = h$$

$$W_{AD} = -mgh$$

Now

$$W_{DB} = F \cdot d$$
$$= Fd\cos\theta$$

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here path 'DB' is similar to 'AC'
So

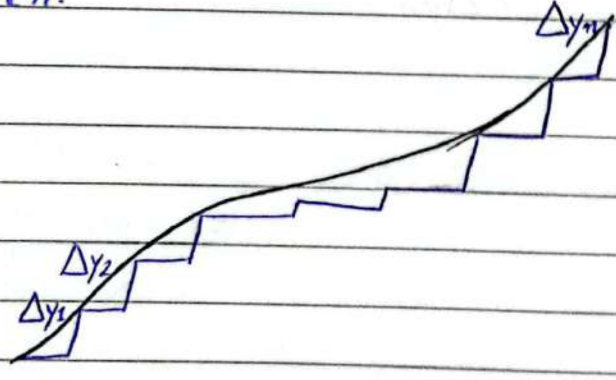
$$\theta = 90°$$

$$W_{DB} = Fd\cos(90)$$
$$W_{DB} = 0$$

$$W_{ADB} = -mgh + 0$$
$$W_{ADB} = -mgh$$

Work done along path AB:

Now, for work done along path "AB", divided the total path into 'n' number of horizontal and vertical steps as shown.



Work done along all horizontal steps will be zero because in each horizontal step the angle between gravitational force and displacement is 90°

So the only work done is because of all vertical steps

So

$$W_1 = F \cdot \Delta y_1$$
$$= F \Delta y \cos(180)$$
$$W_1 = -F \Delta y$$

Similarly

$$W_2 = -F \Delta y_2$$
$$W_3 = -F \Delta y_3$$

$$W_n = -F \Delta y_n$$

$$W_T = W_1 + W_2 + W_3 + \dots\dots\dots + W_n$$
$$= -F\Delta y_1 - F\Delta y_2 - F\Delta y_3 - \dots\dots - F\Delta y_n$$
$$W_T = -F(\Delta y_1 + \Delta y_2 + \Delta y_3 + \dots\dots + \Delta y_n)$$
$$h = \Delta y_1 + \Delta y_2 + \Delta y_3 + \dots + \Delta y_n$$

So

$$W_T = -Fh$$

$\therefore F = W = mg$

$$W_T = -mgh$$

Now from equ and we observe that

Conclusion: Workdone is a gravitational field between two points is independent of the path followed

(OR)

Workdone along a closed path is zero

Conservative field:

The field in which workdone is independent of the path followed.

(OR)

The field in which workdone along a closed path is zero.

The force act in a conservative field is called conservative force.

e.g.

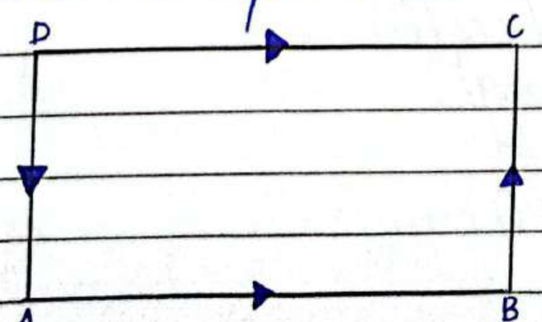
gravitational field, electric field.

Why the frictional force is non-conservative force?

The frictional force is a non-conservative force, because if an object is moving over a rough surface between two points along different paths, the work done against the frictional force certainly depends on the path followed.

Why workdone along a closed path is zero?

Proof: Consider a path ABCDA



We have to calculate the workdone along total path

So

$$W_{AB} = F \cdot d$$
$$= Fd\cos\theta$$

Angle between F_g and d is $90°$

$$= Fd\cos(90)$$
$$W_{AB} = 0$$

Now

$$W_{BC} = F \cdot d$$
$$= Fd\cos\theta$$

Angle between F_g and d is $180°$

$$= Fd\cos(180)$$
$$= -Fd$$
$$W_{BC} = -mgh$$

Now

$$W_{CD} = F \cdot d$$
$$= Fd\cos\theta$$

Angle between F_g and d is $90°$

$$= Fd\cos(90)$$
$$W_{CD} = 0$$

Now

$$W_{DA} = F \cdot d$$
$$= Fd\cos\theta$$

Here $\theta = 0°$

$$= Fd\cos(0)$$
$$W = +mgh$$

$$W = 0 - mgh + 0 + mgh$$
$$W = 0$$

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Power: Rate of doing work is called power.

The rate of change of workdone is called power.

$$P_{avg} = \frac{\Delta W}{\Delta t}$$

Instantaneous power:

The limiting value of $\frac{\Delta W}{\Delta t}$ where $\Delta t \rightarrow 0$ is called instantaneous power.

$$P_{inst} = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}$$

Imp*

Prove $P = F \cdot V$

Consider a body of mass 'm' having workdone ΔW
So consider the instantaneous power

$$P_{ins} = \lim_{\Delta t \to 0} \frac{F \cdot \Delta d}{\Delta t}$$

$$P_{ins} = F \cdot \lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t}$$

Now

$$\lim_{\Delta t \to 0} \frac{\Delta d}{\Delta t} = V$$

$$P = F \cdot V$$

It shows that the dot product of force and velocity is called power

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S/Q:- **1kWh = 36MJ**

$$1kWh = 1 \times 1000 \times 3600 sec \times W$$
$$= 3600000\ Ws$$
$$= 3.6 \times 10^6\ Ws$$
$$1kWh = 3.6MJ$$

kWh is a commercial unit of electrical energy.
1kWh is a workdone in one hour by angle whose power is 1kWh.

$P = \frac{W}{t}$

$W = \frac{J}{s}$

$Ws = J$

MCQs:-
Ws, Wh > energy workdone units.

Energy

Capacity of a body to do work (OR)
The ability of a body to do work is called energy

Types: Mechanically it has two types:-

- Kinetic energy.
- Potential energy.

Kinetic energy:

Energy possessed by a body due to its motion is called kinetic energy

$$K.E. = \frac{1}{2} mv^2$$

S/Q: **Is K.E. a scalar quantity or a vector quantity?**

The K.E. is a scalar quantity we can write K.E. as

Date:

$$K.E. = \frac{1}{2} mv^2 (\bar{V}.\bar{V})$$

- Here we use the dot product of velocity. So it is a scalar quantity
- The unit of K.E. is Joule which is also the unit of workdone. Workdone is a scalar quantity. So K.E. is a scalar quantity

Potential energy: The energy possessed by a body due to its position

$$P.E. = mgh$$

It is also called gravitational potential energy.

Elastic potential energy The energy stored in the spring due to its compression or stretching state is called elastic P.E.

Note:- **Work energy Principle**

Statement: Workdone is equal to change in kinetic energy

Proof: Consider a body of mass 'm' is moving with initial velocity 'V_i' by applying some force 'F' on it, its velocity becomes 'V_f'

So according to 3rd equation of motion

$$2aS = V_f^2 - V_i^2$$

From Newton's 2nd law of motion

$$F = ma \rightarrow a = \frac{F}{m}$$

$$2\left(\frac{F}{m}\right)S = V_f^2 - V_i^2$$

$$2FS = m\,(V_f^2 - V_i^2)$$

$$FS = \frac{1}{2} m\,(V_f^2 - V_i^2)$$

$$FS = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2$$

MCQs:-

$$W = P.E_f. - P.E_i.$$

$$= mgh_1 - mgh_2$$

$$W = mg\,(h_1 - h_2)$$

Work energy Principle (in Case of Potential energy)

$$FS = W, \quad K.E_f = \frac{1}{2} m V_f^2, \quad K.E_i = \frac{1}{2} m V_i^2$$

$$W = K.E_f. - K.E_i.$$

$$W = \Delta K.E.$$

Problem #1:

Data:

Force $= F = 40N$

$\theta = 20°$

$d = 20m$

To Find: $W = ?$

Solve:

$$W = F.d$$

$$W = Fd\cos\theta$$

$$W = (40)(20)\cos(20)$$

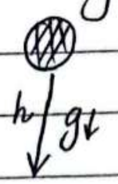
$$W = 751.7\ J$$

$$W = 7.51 \times 10^2\ J$$

Problem #2:

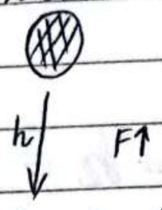
Data:

Gravity



So, $\theta = 0°$

Friction



So, $\theta = 180°$

$m = 3.35 \times 10^{-5}$ kg

$h = 100$ m

$g = 9.8\ ms^{-1}$

To Find:

Workdone by the gravity $= W = ?$

Workdone by friction $= W = ?$

Slove:

Workdone by the gravity

$W = F \cdot d$

$W = Fd\cos\theta$

$= Fd\cos(0)$

$= Fd$

$W = mgh$

$W = (3.35 \times 10^{-5})(9.8)(100)$

$W = 0.0328$ J

Workdone by friction

$W = F \cdot d$

$W = Fd\cos\theta$

$= Fd\cos(180)$

$= -Fd$

$W = -mgh$

$W = -(3.35 \times 10^{-5})(9.8)(100)$

$W = -0.0328$ J

Problem #3: Imp

Data:

Thickness $= h = 6cm$

$= 6/100$

$= 0.06m$

$m = 1.5kg$

To Find: $W = ?$

$$W_T = 0mgh + 1mgh + 2mgh + 3mgh + 4mgh + 5mgh + 6mgh + 7mgh + 8mgh + 9mgh$$

$$= 45\,mgh$$

$$W_T = 45 \times 1.5 \times 9.8 \times 0.06$$

$$W_T = 39.9 \approx 40J$$

Problem #4:

Data:

$m = 800kg$

$v_i = 54kmh^{-1}$

$= \frac{54 \times 1000}{3600} = 15ms^{-1}$

$v_f = 0$

$d = 60m$

To Find: $F = ?$

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$$W = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2$$

$$Fd = \frac{1}{2} m (V_f^2 - V_i^2)$$

$$Fd = \frac{1}{2} (800) \left[(0)^2 - (15)^2\right]$$

$$Fd = 400(-225)$$

$$Fd = -90000$$

$$F = \frac{-90000}{60}$$

$$F = \cancel{-}1500N$$

The original K.E. is used to do work against the friction.

Problem #6: Imp

Data:

$v = 100 m^3$

$h = 10 m$

$t = 20 min$

$20 \times 60 = 1200 sec$

$\rho = 1000 kg m^{-3}$

To Find: P.E. = ?

P = ?

Solve: P.E. = mgh

$$\rho = \frac{m}{v}$$

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$m = \rho V$

$P.E. = \rho V g h$

$P.E = (1000)(100)(9.8)(10)$

$= 9.8 \times 10^6 J$

Now $P = \frac{P.E}{t}$

$P = \frac{9.8 \times 10^6}{1200}$

$P = 8166.67$

$P = 8.2 kW$

Problem #7: Imp

Data:

$F = 400N$

$V = 800 km h^{-1}$

$= \frac{800 \times 1000}{3600} = 22.22 ms^{-1}$

To Find: $P = ?$

$P = F.V$

$P = F V \cos\theta$

$P = (400)(22.22)\cos(0)$

$P = 8888$

$P = 8.9 kW$

Problem #8:

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Data:

$m_e = 9.1 \times 10^{-31} kg$

$V_i = 0$

$V_f = 2 \times 10^7 ms^{-1}$

$d = 5cm$

$= 5 \times 10^{-2} m$

(MCQs)

$P = \frac{W}{t} = \frac{P.E}{t} = \frac{K.E}{t} = \frac{E.P.E}{t}$

$P = \frac{Fd}{t} = \frac{1/2 mv^2}{t} = \frac{mgh}{t} = \frac{1/2 kx^2}{t}$

To Find: F = ?

Solve:

$$W = \frac{1}{2} m V_f^2 - \frac{1}{2} m V_i^2$$

$$W = \frac{1}{2} m (V_f^2 - V_i^2)$$

$$Fd = \frac{1}{2} \times 9.1 \times 10^{-31} [(2 \times 10^7)^2 - (0)^2]$$

$$Fd = 1.82 \times 10^{-16}$$

$$F = \frac{1.82 \times 10^{-16}}{d}$$

$$F = \frac{1.82 \times 10^{-16}}{5 \times 10^{-2}}$$

$$F = 3.64 \times 10^{-15} N$$

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