Q No. 2.1: Define the terms (i) unit vector (ii) position vector and (iii) components of a vector

(i) Unit Vector

A vector whose magnitude is equal to one in the direction of a given vector is called a unit vector.

A unit vector in the direction of a vector A is denoted by $\hat{A}$.

Thus,

$$\vec{A} = A\hat{A}$$

$$\hat{A} = \frac{\vec{A}}{A}$$

It is used to represent the direction of a vector.

(ii) Position Vector

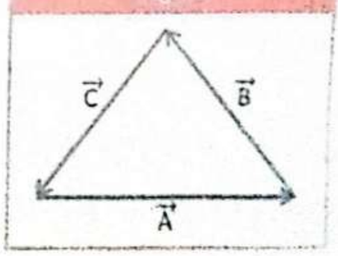
A vector that describes location of a point with respect to origin, is called position vector. It is denoted by $\hat{r}$.

(iii) Components of a Vector

Effective values of a vector along specified directions, is called its components.

Q No. 2.2: The vector sum of three vectors gives a zero resultant. What can be the orientation of the vectors?

Figure



If three vectors are adjacent along the sides of a triangle, then their resultant sum is equal to zero.

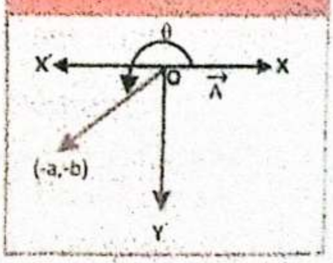
Let three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ as shown in figure.

$$\vec{A} + \vec{B} + \vec{C} = \vec{0}$$

So, vector sum is equal to null vector.

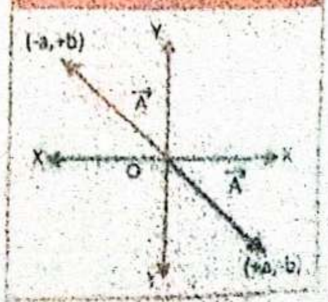
Q No. 2.3: Vector A lies in xy-plane. For what orientation will both of its rectangular components be negative? For what orientation will its components have opposite sign? (RWP-2019) (DGK-2019)

Figure



i) When a vector $\vec{A}$ lies in third quadrant of xy-plane, both of its components will be negative as shown in Figure.

Figure



ii) When the vector $\vec{A}$ lies in second or fourth quadrant of xy-plane, its components will have opposite signs as shown in Figure.

Q No. 2.4 : If one of the rectangular components of a vector is not zero, can its magnitude be zero? Explain. [illegible]

Magnitude of a vector having one of its rectangular components zero, cannot be zero as explained below
Consider a vector $\vec{A}$ lying in xy-plane then in terms of rectangular components

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$$

The magnitude of $\vec{A}$ is given by

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

This shows that if either $A_x = 0$ or $A_y = 0$ then A is not equal to zero.
Let $A_y = 0$ then

$$|\vec{A}| = \sqrt{A_x^2 + 0}$$

$$|\vec{A}| = A_x \neq 0$$

Q No. 2.5: Can a vector have a component greater than the vector 's magnitude? (RWP-2017) (FSD 2016)

No, a vector cannot have its rectangular component greater than the vector 's magnitude because a rectangular component of a vector is fractional part of the vector.
Consider a vector **A** lying in xy-plane then in terms of its rectangular components it can be written as

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$$

The magnitude of $\vec{A}$ is given by

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

This shows that either $A > A_x$ or $A > A_y$

Q No. 2.6: Can the magnitude of a vector ever be negative? Explain. OR Can the magnitude of a vector have a negative value? (MUL-2019) (FSD-2019) (BWP-2019) (DGK-2018) (LHR-2018) (SGD-2018) (BWP 2016)

No, the magnitude of a vector cannot have a negative value because magnitude of a quantity is scalar and scalar has no negative sense physically.
Consider a vector **A** lying in xy-plane then in terms of its rectangular components it can be written as

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j}$$

The magnitude of $\vec{A}$ is given by

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

Let its horizontal component is negative

$$|\vec{A}| = \sqrt{(-A_x)^2 + A_y^2}$$

After taking the square of A, then it becomes positive as given below.

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

Q No. 2.7: If $\vec{A} + \vec{B} = 0$, what can you say about components of the two vectors?

For $\vec{A} = -\vec{B}$, we can say that rectangular components of $\vec{A}$ and $\vec{B}$ must be equal in magnitude and opposite in direction in plane.

Let

$$\vec{A} = A_x\,\hat{i} + A_y\,\hat{j} + A_z\,\hat{k}$$

$$\vec{B} = (B_x\,\hat{i} + B_y\,\hat{j} + B_z\,\hat{k})$$

From given condition, we have

$$\vec{A} = -\vec{B}$$

$$A_x\hat{i} + A_y\hat{j} + A_z\hat{k} = -(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

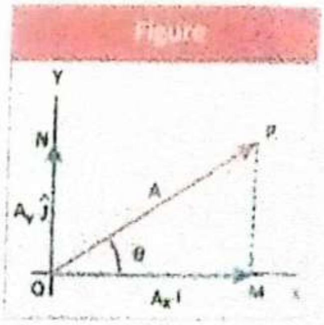
This shows that

$$A_x\hat{i} = -B_x\hat{i},\ A_y\hat{j} = -B_y\hat{j} \text{ and } A_z\hat{k} = -B_z\hat{k}$$

Q No.2.8: Under what circumstances would a vector have components that are equal in magnitude? (SGD-2019) (DGK-2019) (SWL-2017) (GRW-2016)

A vector making an angle of $\theta = 45^\circ$ with reference axis could have rectangular components that are equal in magnitude.

Consider a vector $\vec{A}$ inclined at an angle θ with x-axis then



$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

As given $\quad A_y = A_x$

So $$\theta = \tan^{-1}\left(\frac{A_x}{A_x}\right)$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45^\circ$$

Q No.2.9: Can you add zero to a null vector? (BWP-2021) (MUL-2021) (SGD-2021) (FSD-2019) (SGD-2018) (LHR-2017)

No, we cannot add zero into a null vector.

Reason

1. Because zero is a number and null vector is a vector and algebraically a number cannot be added into a vector.

2. Numbers can be added with the help of arithmetic rules and vectors can be added by using the head to tail rule.

Q No.2.10: Is it possible to add a vector quantity to a scalar quantity? Explain. (BWP-2019) (DGK-2021) (SWL-2019) (MUL-2019) (GRW-2019) (MUL-2017) (RWP-2017) (SGD-2017) (LHR-2016) (RWP-2016)

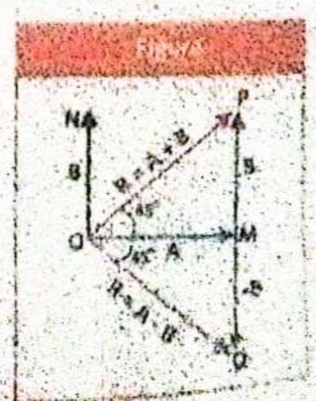
No, it is not possible to add a vector quantity to a scalar quantity because algebraically only like quantities can be added. Scalars can be added to scalars of like nature and vectors can be added to vectors of like nature.

Q No. 2.11: Two vectors have unequal magnitudes. Can their sum be zero? Explain. (LHR-2021) (SGD-2017) (GRW-2017) (FSD-2016) (DGK-2016) (LHR-2016)

When two vectors have unequal magnitudes, their sum cannot be zero because sum of two vectors can be zero when they have equal magnitudes and opposite in directions. Two vectors of unequal magnitudes and opposite in direction have minimum resultant in the direction of the vector of greater magnitude.

Q No. 2.12 : If two perpendicular vectors have same magnitude, find the angle between their sum and difference. (RWP-2019)

Consider two perpendicular vectors $\vec{A}$ and $\vec{B}$ taken along X-axis and Y-axis respectively represented by $\overline{OM}$ and $\overline{ON}$ such that $\overline{OM} = \overline{ON}$ as shown in Fig. Adding $\vec{A}$ and $\vec{B}$ by head to tail rule, their sum is $\vec{A} + \vec{B} = \vec{R}$ represented by $\overline{OP}$ which makes an angle θ with X-axis.



Using Pythagorean theorem, we have from right angled triangle OMP

$$OP^2 = OM^2 + MP^2$$

$$OP^2 = OM^2 + ON^2$$

$$OP = \sqrt{OM^2 + MP^2}$$

$$|R| = \sqrt{A^2 + B^2}$$

$$|A+B| = \sqrt{A^2 + B^2} \qquad \ldots (1)$$

Subtracting $\vec{B}$ from $\vec{A}$ by head to tail rule their difference is $\vec{A} - \vec{B} = \vec{R'}$ represented by OQ which makes an angle θ with x-axis.

Using Pythagorean theorem, we have from right angled triangle OMQ

$$OQ^2 = OM^2 + MQ^2$$

$$OQ = \sqrt{OM^2 + MQ^2}$$

$$R' = \sqrt{A^2 + (-B)^2}$$

$$|A - B| = \sqrt{A^2 + (-B)^2} \quad = \sqrt{A^2 + B^2} \qquad \ldots (2)$$

From equation (1) and equation (2), we get

$$|A + B| = |A - B|$$

Since OM=MP so $\angle MOP = 45^\circ$

Since OM=MQ so $\angle MOQ = 45^\circ$

Thus, $\angle QOP = \angle MOP + \angle MOQ = 45^\circ + 45^\circ = 90^\circ$

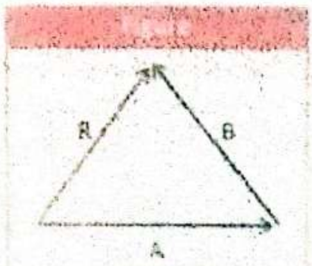
$$\Rightarrow \vec{R} \perp \vec{R'}$$

Q No.2.13 How would the two vectors of same magnitude have to be oriented, if they were to be combined to give a resultant equal to a vector of the same magnitude? (... 2014) (... 2017)

Consider two vectors $\vec{A}$ and $\vec{B}$ inclined at an angle θ such that A=B=R then magnitude of their resultant is given by

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Put given condition A=B=R



B/R/ $A = \sqrt{A^2 + A^2 + 2AA\cos\theta}$

$$A = \sqrt{2A^2 + 2A^2\cos\theta}$$

$$A = \sqrt{2A^2(1 + \cos\theta)}$$

$$A = \sqrt{2}A\sqrt{1 + \cos\theta}$$

$$\frac{1}{\sqrt{2}} = \sqrt{1 + \cos\theta}$$

Taking square on both sides

$$\frac{1}{2} = 1 + \cos\theta$$

$$\cos\theta = 1 - \frac{1}{2}$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = 120^\circ$$

Two vectors of same magnitude have to be oriented at $\theta = 120^\circ$ with each other so that if they were to be combined to give a resultant R equal to a vector of the same magnitude.

Q No.2.14: The two vectors to be combined have magnitudes 60N and 35 N. Pick the correct answer from those given below and tell why it is the only one of the three that is correct.
i) 100 N   ii) 70 N   iii) 20 N

When 60N and 35N forces are in same direction then both forces are parallel i.e. A + B = R.

So,

Max resultant = 60 + 35 = 95N

When 60N and 35N forces are in opposite direction then both forces are antiparallel i.e. A - B = Ŕ.

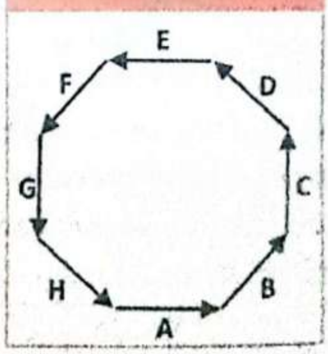
Minimum resultant = 60 – 35 = 20N

Hence answer should lie between 25N to 95N.

So, correct option is 70 N.

Q No.2.15: Suppose the sides of a closed polygon represent vectors arranged head to tail. What is the sum of these vectors? (FSD-2021) (LHR-2021) (GRW-2021) (DGK-2021)

Figure



If we suppose the sides of a closed polygon represent vectors arranged head to tail, then sum of these vectors will be a null vector because the vectors arranged along the sides of a closed figure by head to tail add up to give a resultant of zero magnitude.

Consider vectors $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}, \vec{F}, \vec{G}$ and $\vec{H}$ arranged head to tail along the sides of an octagon as shown in Figure and can be written as:

$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} + \vec{F} + \vec{G} + \vec{H} = \vec{O}$$

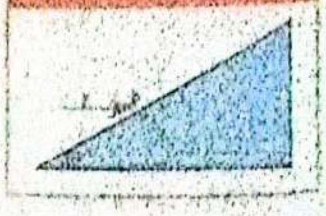
Q No.2.16: Identify the correct answer.

(i) Two ships X and Y are traveling in different directions at equal speeds. The actual direction of motion of X is due north but to an observer on Y, the apparent direction of motion of X is north-east. The actual direction of motion of Y as observed from the shore will be:

(A) East   **(B)** West   **(c)** West-east   **(D)** South-west

(ii) A horizontal force F is applied to a small object P of mass m at rest on a smooth plane inclined at an angle θ to the horizontal as shown in Figure. The magnitude of the resultant force acting up and along the surface of the plane, on the object is:

Figure



a) $F\cos\theta - mg\sin\theta$

b) $F\sin\theta - mg\cos\theta$

c) $F\cos\theta + mg\cos\theta$

d) $F\sin\theta + mg\sin\theta$

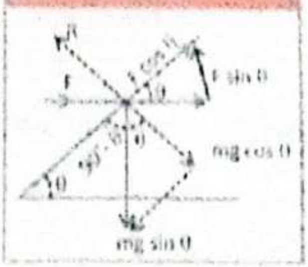
e) $mg\tan\theta$

(a) We know that the ship x is moving towards North from shore and according to observer on ship y, the ship x is moving towards north-east direction. So, ship y is approaching towards the line of motion of ship x. Thus the motion of ship y is towards west so (b) is correct.

(b) Now the horizontal force F and weight mg of the body can be resolved into its rectangular components as shown in Figure. The force acting up along the plane of the surface is:

$$= F\cos\theta - mg\sin\theta$$

Figure



Q No.2.17 : If all the components of the vectors, A_1 and A_2 were reversed, how would this alter $A_1 \times A_2$? (RWP 2021) (BWP 2021) (MUL 2021) (SWL 2021)

If all the components of the vectors A_1 and A_2 were reversed then this would not alter $A_1 \times A_2$ because on reversing all the components of A_1 and A_2, then A_1 and A_2 become $-A_1$ and $-A_2$ respectively and then cross product, resultant vector remains same as shown in Figure.

$$\vec{A}_1 = A_{1x}\hat{i} + A_{1y}\hat{j} + A_{1z}\hat{k}$$

$$\vec{A}_2 = A_{2x}\hat{i} + A_{2y}\hat{i} + A_{2z}\hat{k}$$

On reversing the components A_1 and A_2

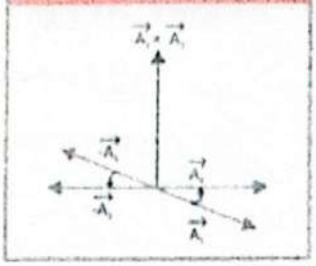
$$-\vec{A}_1 = -A_{1x}\hat{i} - A_{1y}\hat{j} - A_{1z}\hat{k}$$

$$-\vec{A}_2 = -A_{2x}\hat{i} - A_{2y}\hat{j} - A_{2z}\hat{k}$$

On taking cross product of $-A_1$ and $-A_2$, we get

$$-\vec{A}_1 \times -\vec{A}_2 = \vec{A}_1 \times \vec{A}_2$$

Figure



Q No. 2.18: Name the three different conditions that could make $A_1 \times A_2 = 0$. (LHR 2019) (SGD 2019) (FSD 2018) (MUL 2017) (LHR 2017) (RWP 2017) (SWL-2017) (GRW-2017) (FSD-2017) (FSD-2016) (BWP-2016) (RWP-2016) (SGD-2016)

Three different conditions that could make $\vec{A}_1 \times \vec{A}_2 = \vec{0}$ are:

i. $\vec{A}_1$ and $\vec{A}_2$ are parallel — i.e $\theta = \vec{0}$, $\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin\theta\,\hat{n} = 0\hat{n} = \vec{0}$

ii. Either $\vec{A}_1$ or $\vec{A}_2$ is null vector. — i.e . $\vec{A}_1 = \vec{0}$ or $\vec{A}_2 = \vec{0}$

iii. $\vec{A}_1$ and $\vec{A}_2$ are anti-parallel — i.e $\theta = 180^\circ$, $\vec{A}_1 \times \vec{A}_2 = A_1 A_2 \sin 180\,\hat{n} = \vec{0}$

Q No. 2.19: Identify true or false statements and explain the reason.

(a) A body in equilibrium implies that it is neither moving nor rotating.

(b) If coplanar forces acting on a body from a closed polygon, then the body is said to be in equilibrium.

Explanation

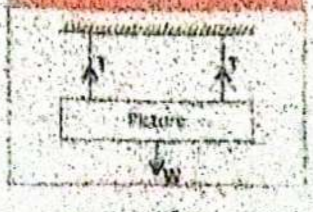
(a) This statement is false because the body is said to be in equilibrium if it is moving with constant velocity or rotating with uniform angular velocity.

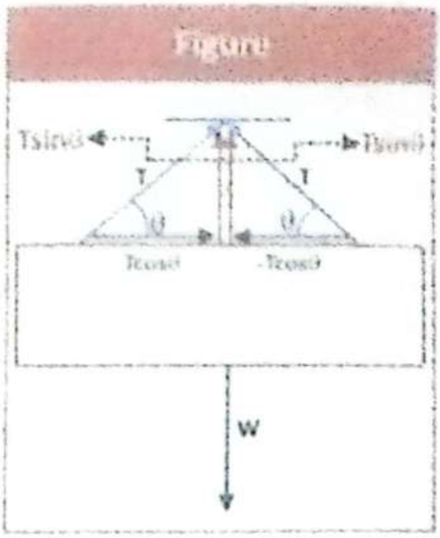
(b) This statement is true because when coplanar forces acting on a body in the form of a closed polygon then $\Sigma F = 0$. i.e , 1st condition of equilibrium, is satisfied so the body is in translational equilibrium.

Q No. 2.20 : A picture is suspended from a wall by two strings. Show by diagram the configuration of the strings for which the tension in the strings will be minimum. (SWL 2016) (SGD 2017) (LHR 2016)

To suspend a picture from a wall by two strings, the configuration of the strings for which the tension in the strings will be minimum is shown in Figure below. Consider a picture suspended from a wall by two strings which make an angle θ with horizontal as shown in Figure then applying first condition of equilibrium along vertical axis.

Figure





$$\sum F_y = 0$$

$$T\sin\theta + T\sin\theta + (-W) = 0$$

$$2T\sin\theta = W$$

$$T = \frac{W}{2\sin\theta}$$

T will be minimum if $\sin\theta$ should have maximum value

For $T = T_{min}$, $\sin\theta = 1(max)$

$$\theta = \sin^{-1}(1)$$

$$\theta = 90^\circ$$

So $$T = \frac{W}{2}$$

So, only in this case, tension in both strings will be minimum

Q No.2.21 . Can a body rotate about its center of gravity under the action of its weight? (RWP-2019) (DGK-2019) (FSD-2019) (GRW-2019) (FSD-2018) (DGK-2018) (RWP-2018) (RWP-2017) (LHR-2016) (SGD-2016) (GRW-2016)

No, a body cannot rotate about its center of gravity under the action of its weight because line of action of force passes through center of gravity and moment arm is zero. i.e. $r_\perp = 0$

$$\tau = r_\perp F = 0 \times W = 0$$

So torque is zero