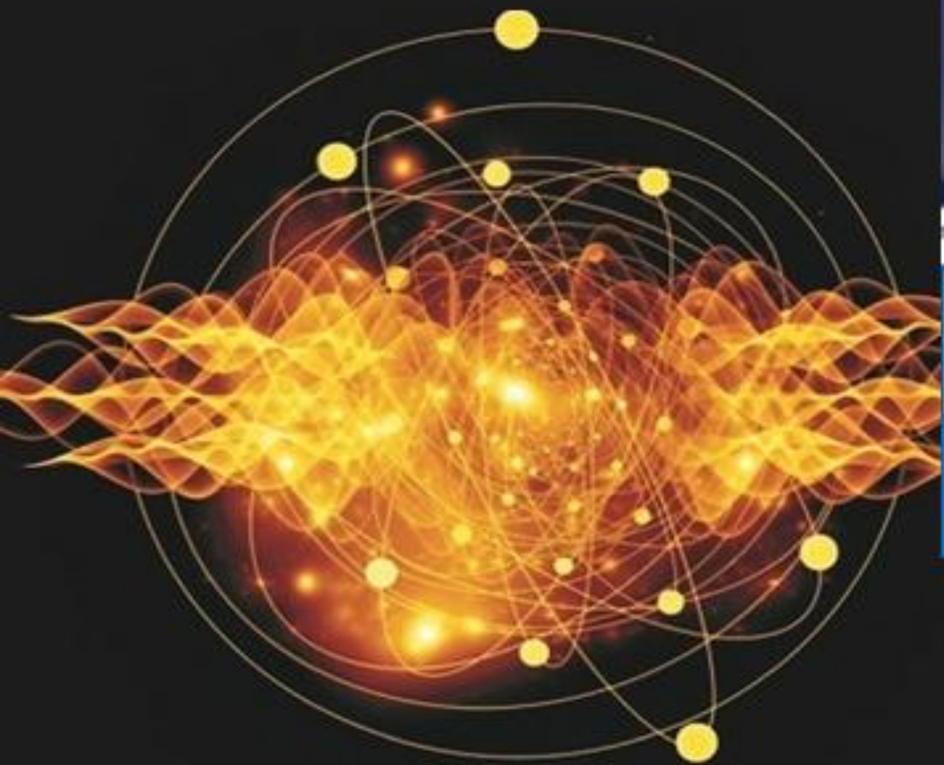
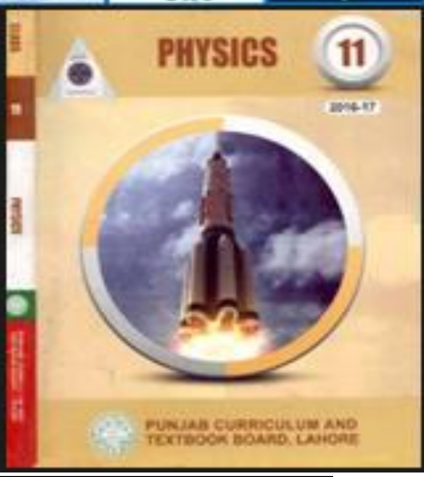
Numerical Problems of

F.Sc. Physics

Grade-11 (1st Year)

by Muhammad Ali Malik





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NUMERICAL PROBLEMS

F.Sc. Physics, Chapter # 1: Measurements

Q # 1. A light year is the distance light travels in one year. How many meters are there in one light year?

Given Data:	$\text{Time } t = 1 \text{ year} = 365 \text{ days} = 365 \times 24 \times 60 \times 60 \text{ s} = 3.154 \times 10^7 \text{ s}$ $\text{Speed of Light } v = 3 \times 10^8 \frac{\text{m}}{\text{s}}$
To Determine:	$\text{Distance } S = ?$
Calculations:	$S = vt = \left(3 \times 10^8 \frac{\text{m}}{\text{s}}\right)(3.154 \times 10^7 \text{ s}) = 9.5 \times 10^{15} \text{ m}$

Q # 2.

(i) How many seconds are there in 1 year?

(ii) How many nanoseconds in 1 year?

(iii) How many years in 1 second?

i	As $1 \text{ year} = 365 \text{ days}$, $1 \text{ day} = 24 \text{ hours}$, $1 \text{ hour} = 3600 \text{ s}$ Therefore $1 \text{ year} = 365 \text{ days} = 365 \times 24 \times 3600\text{s} = 3.1536 \times 10^7 \text{ s}$
ii	As $1 \text{ s} = 10^9 \text{ ns}$ Therefore $1 \text{ year} = 3.1536 \times 10^7 \text{ s} = 3.1536 \times 10^7 \times 10^9 \text{ ns} = 3.1536 \times 10^{16} \text{ ns}$
iii	As $1 \text{ year} = 3.1536 \times 10^7 \text{ s}$ $\Longrightarrow 1\text{s} = \frac{1}{3.1536 \times 10^7} \text{ years} = 3.17 \times 10^{-8} \text{ s}$

Q # 3. The length and width of a rectangular plate are measure to be 15.3 cm and 12.80 cm , respectively. Find the area of the plate.

Given Data:	$\text{Length of plate } l = 15.3 \text{ cm}$, $\text{Width of plate } w = 12.80 \text{ cm}$
To Determine:	$\text{Area } A = ?$
Calculations:	$A = l \times w = (15.3 \text{ cm})(12.80 \text{ cm}) = 195.84 \text{ cm}^2 \approx 196 \text{ cm}^2$ $\because$ The measurement 15.3 cm has least significant figure (three), therefore, the answer should be rounded off to 3 digits

Q # 4. Add the following masses given in kg upto appropriate precision. 2.189, 0.089, 11.8 and 5.32.

Given Data:	$\text{Masses } m_1 = 2.189 \text{ kg}, m_2 = 0.089 \text{ kg}, m_3 = 11.8 \text{ kg}, m_4 = 5.32 \text{ kg}$
To Determine:	$\text{Sum of Masses } S = ?$
Calculations:	$S = m_1 + m_2 + m_3 + m_4 = 2.189 \text{ kg} + 0.089 \text{ kg} + 11.8 \text{ kg} + 5.32 \text{ kg}$ $= 19.398 \text{ kg} \approx 19.4 \text{ kg}$ $\because$ The measurement 11.8 kg has least decimal places (one), therefore, the answer should be rounded off to 1 decimal place.

Q # 5. Find the value of g and its uncertainty using $T = 2\pi\sqrt{\frac{l}{g}}$ from the following measurements. Length of simple pendulum $l = 100$ cm, Time for 20 vibrations $t = 40.2$ s. Length was measured by the meter scale of accuracy 1 mm and time by the stop watch of accuracy up to 0.1 s.

Given Data: Length $l = 100\ \text{cm} = 1\text{m}$, Time for 20 vibrations $t = 40.2\ \text{s}$,

L.C of meter scale $= 1\ \text{mm} = 0.001\ \text{m}$, L.C of stop watch $= 0.1\ \text{s}$

To Determine: Gravitational Acceleration $g = ?$

Calculations: Time Period $T = \frac{t}{20} = \frac{40.2}{20} = 2.01\ \text{s}$

$$\text{As } T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow T^2 = 4\pi^2\left(\frac{l}{g}\right) \Rightarrow g = \frac{4\pi^2 l}{T^2} = \frac{4(3.14)^2(1)}{(2.01)^2} = 9.76\ \text{ms}^{-2}$$

<u>Uncertainty</u>: % Uncertainty in $l = \frac{0.001}{1} \times \frac{100}{100} = 0.1\ \%$,

$$\text{Uncertainty in time Period} = \frac{\text{Least Count}}{\text{Total Number of Vibratios}} = \frac{0.1}{20} = 0.005\ \text{s}$$

$$\%\ \text{Uncertainty in Time Period} = \frac{0.005}{2.01} \times \frac{100}{100} = 0.25\%$$

Total Uncertainty in $g = 2(\%\ \text{Uncertainty in T}) + \%\ \text{Uncertainty in } l = 2(0.25\ \%) + 0.1\ \% = 0.1\ \%$

Therefore $T = 9.76\ \text{ms}^{-2}$ with 0.6 % uncertainty OR $V = 9.76 \pm 0.06\ \text{ms}^{-2}$

Q # 6. What are the dimensions and units of gravitational constant G in the formula $F = G\frac{m_1 m_2}{r^2}$

Given Data: Formula $F = G\frac{m_1 m_2}{r^2}$

To Determine: Dimensions of G $[G] = ?$ Unit of G $= ?$

Calculation(i) $F = G\frac{m_1 m_2}{r^2} \Rightarrow G = \frac{F\ r^2}{m_1 m_2} \Rightarrow [G] = \left[\frac{F\ r^2}{m_1 m_2}\right] = \frac{[F]\ [r]^2}{[m_1][m_2]} = \frac{[MLT^{-2}]\ [L]^2}{[M][M]} = [M^{-1}L^3T^{-2}]$

(ii) As $G = \frac{F\ r^2}{m_1 m_2}$ So Unit of G: $\frac{N\ m^2}{kg^2}$

Q # 7. Show that the expression $v_f = v_i + at$ is dimensionally correct, where v_i is the velocity at $t = 0$, a is the acceleration and v_f is the velocity at time t.

Dimension of LHS $= [v_f] = [LT^{-1}]$

Dimension of RHS $= [v_i] + [a] \times [t] = [LT^{-1}] + [LT^{-2}] \times [T] = [LT^{-1}] + [LT^{-1}] = [LT^{-1}]$

Conclusion: As Dimension of LHS = Dimension of RHS

Hence, the equation $v_f = v_i + at$ is dimensionally correct.

Q # 8. The speed v of sound waves through a medium may be assumed to depend upon (a) the density ρ of the medium and (b) its modulus of elasticity E which is the ratio of stress to strain. Deduce by method of dimensions, the formula for speed of sound.

Given: $v \propto \rho$ & $v \propto E$

We have to find: Formula for Speed of Sound

Solution: $[E] = \frac{[Stress]}{[Strain]} = \frac{[F/A]}{1} = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$ & $[\rho] = \frac{[mass]}{[volume]} = \frac{[M]}{[L^3]} = [ML^{-3}]$

$$\text{Let } v \propto \rho^a E^b \Longrightarrow v = \text{constant } \rho^a E^b \quad ------ \quad (1)$$

Writing dimensions on both sides:

$$[v] = \text{constant } [\rho]^a [E]^b \Longrightarrow [LT^{-1}] = \text{constant } [ML^{-3}]^a [ML^{-1}T^{-2}]^b$$

$$\Longrightarrow [LT^{-1}] = \text{constant } \left[M^{a+b} L^{-3a-b} T^{-2b}\right]$$

Equating powers of T: $-1 = -2b \Longrightarrow b = \frac{1}{2}$

Equating powers of M: $0 = a + b \Longrightarrow a = -b \Longrightarrow a = -\frac{1}{2}$

Putting values in (1): $v = \text{constant } \rho^a E^b = \text{constant } \rho^{-\frac{1}{2}} E^{\frac{1}{2}} \Longrightarrow v = \text{constant } \sqrt{\frac{E}{\rho}}$

Q # 9. Show that the famous "Einstein's Equation" $E = mc^2$ is dimensionally consistent.

Dimension of LHS $= [E] = [W] = [F.d] = [F].[d] = \left[MLT^{-2}\right][L] = \left[ML^2T^{-2}\right]$

Dimension of RHS $= \left[mc^2\right] = [m][c]^2 = [M][LT^{-1}]^2 = [ML^2T^{-2}]$

Conclusion: As Dimension of LHS = Dimension of RHS

Hence, the equation $E = mc^2$ is dimensionally correct.

Q # 10. Suppose we are told that the acceleration of a particle moving in a circle of radius r with uniform speed v is proportional to some power of r, say r^n, and some power of v, say v^m, determine the powers of r and v.

Given: $a \propto r^n v^m$

To Determine: $m = ?, n = ?$

Calculations: $a \propto r^n v^m \Longrightarrow a = \text{constant } r^n v^m$

Writing dimensions on both sides:

$[a] = \text{constant } [r]^n [v]^m \Longrightarrow [LT^{-2}] = \text{constant } [L]^n [LT^{-1}]^m \Longrightarrow [LT^{-2}] = \text{constant } [L^{n+m}T^{-m}]$

Equating powers of T: $-2 = -m \Longrightarrow m = 2$

Equating powers of L: $1 = n + m \Longrightarrow n = 1 - m = 1 - 2 \Longrightarrow n = -1$

NUMERICAL PROBLEMS

F.Sc. Physics, Chapter # 2: Vectors and Equilibrium

Q # 1. Suppose, in rectangular coordinate system, a vector $\vec{A}$ has its tail at the point $P(-2,-3)$ and its tip at $Q(3,9)$. Determine the distance between these points.

Given Data: Initial Point $P(-2,-3)$, Final Point $Q(3,9)$

To Determine: Distance $|PQ| = ?$

Calculations: Position Vector of P $\vec{r_P} = -2\hat{\imath} - 3\hat{\jmath}$, Position Vector of Q $\vec{r_Q} = 3\hat{\imath} + 9\hat{\jmath}$

Displacement $\vec{PQ} = \vec{r_Q} - \vec{r_P} = 3\hat{\imath} + 9\hat{\jmath} - (-2\hat{\imath} - 3\hat{\jmath}) = 3\hat{\imath} + 9\hat{\jmath} + 2\hat{\imath} + 3\hat{\jmath} = 5\hat{\imath} + 12\hat{\jmath}$

Distance $|PQ| = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$ units

Q # 2. A certain corner of a room is selected as the origin of a rectangular coordinate system. If an insect is sitting on an adjacent wall at a point having coordinates $(2,1)$, where the units are in meters, what is the distance of the insect from this corner of the room.

Given Data: Initial Point $P(0,0)$, Final Point $Q(2,1)$

To Determine: Distance $|PQ| = ?$

Calculations: Position Vector of P $\vec{r_P} = 0\hat{\imath} + 0\hat{\jmath}$, Position Vector of Q $\vec{r_Q} = 2\hat{\imath} + \hat{\jmath}$

Displacement $\vec{PQ} = \vec{r_Q} - \vec{r_P} = 2\hat{\imath} + \hat{\jmath} - (0\hat{\imath} + 0\hat{\jmath}) = 2\hat{\imath} + \hat{\jmath}$

Distance $|PQ| = \sqrt{2^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5} = 2.2$ m

Q # 3. What is the unit vector in the direction of the vector $\vec{A} = 4\hat{\imath} + 3\hat{\jmath}$?

Given Data: Vector $\vec{A} = 4\hat{\imath} + 3\hat{\jmath}$

To Determine: Unit Vector $\hat{A} = ?$

Calculations: Magnitude of Vector $\vec{A} = |A| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$

Unit Vector $\hat{A} = \frac{\vec{A}}{|A|} = \frac{4\hat{\imath} + 3\hat{\jmath}}{5}$

Q # 4. Two particles are located at $\vec{r_1} = 3\hat{\imath} + 7\hat{\jmath}$ and $\vec{r_2} = -2\hat{\imath} + 3\hat{\jmath}$ respectively. Find both the magnitude of vector $(\vec{r_2} - \vec{r_1})$ and its orientation with respect to the x-axis.

Given Data: $\vec{r_1} = 3\hat{\imath} + 7\hat{\jmath}$, $\vec{r_2} = -2\hat{\imath} + 3\hat{\jmath}$

To Determine: $|\vec{r_2} - \vec{r_1}| = ?$, Orientation of $(\vec{r_2} - \vec{r_1})$: $\theta = ?$

Calculations: $\vec{r_2} - \vec{r_1} = -2\hat{\imath} + 3\hat{\jmath} - (3\hat{\imath} + 7\hat{\jmath}) = -2\hat{\imath} + 3\hat{\jmath} - 3\hat{\imath} - 7\hat{\jmath} = -5\hat{\imath} - 4\hat{\jmath}$

$|\vec{r_2} - \vec{r_1}| = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25 + 16} = \sqrt{41} = 6.4\ units$

$\theta = \tan^{-1}\left(\frac{-4}{-5}\right) = 180° + \tan^{-1}\left(\frac{4}{5}\right) = 180° + 39° = 219°$ $\qquad \because 3rd\ quadrent$

Q # 5. If a vector $\vec{B}$ is added to vector $\vec{A}$, the result is $6\hat{i}+\hat{j}$. If $\vec{B}$ is subtracted from $\vec{A}$, the result is $-4\hat{i}+7\hat{j}$. What is magnitude of vector $\vec{A}$?

Given Data: $\vec{A}+\vec{B}=6\hat{i}+\hat{j}\quad ---(1),\qquad \vec{A}-\vec{B}=-4\hat{i}+7\hat{j}\quad ---(2)$

To Determine: Magnitude of $\vec{A}=|\vec{A}|=?$

Calculations: Adding (1) & (2) $\vec{A}+\vec{B}+\vec{A}-\vec{B}=6\hat{i}+\hat{j}-4\hat{i}+7\hat{j}\Rightarrow 2\vec{A}=2\hat{i}+8\hat{j}\Rightarrow \vec{A}=\hat{i}+4\hat{j}$

Magnitude of $\vec{A}=|\vec{A}|=\sqrt{(1)^2+(4)^2}=\sqrt{1+16}=\sqrt{17}=4.1$

Q # 6. Given that $\vec{A}=2\hat{i}+3\hat{j}$ and $\vec{B}=3\hat{i}-4\hat{j}$, find the magnitude and angle of (a) $\vec{C}=\vec{A}+\vec{B}$, and (b) $\vec{D}=3\vec{A}-2\vec{B}$.

Given Data: $\vec{A}=2\hat{i}+3\hat{j},\qquad \vec{B}=3\hat{i}-4\hat{j}$

To Determine: **(i)** $|\vec{C}|=?$, Orientation of $\vec{C}$: $\theta=?$

(ii) $|\vec{D}|=?$, Orientation of $\vec{D}$: $\theta=?$

Calculations: (i) $\vec{C}=\vec{A}+\vec{B}=2\hat{i}+3\hat{j}+3\hat{i}-4\hat{j}=5\hat{i}-\hat{j}$

$$|\vec{C}|=\sqrt{(5)^2+(-1)^2}=\sqrt{25+1}=\sqrt{26}=5.1\ units$$

$$\theta=\tan^{-1}\left(\frac{-1}{5}\right)=360°-\tan^{-1}\left(\frac{4}{5}\right)=360°-11°=349°\ \because 4th\ quadrent$$

(ii) $\vec{D}=3\vec{A}-2\vec{B}=3(2\hat{i}+3\hat{j})-2(3\hat{i}-4\hat{j})=6\hat{i}+9\hat{j}-6\hat{i}+8\hat{j}=0\hat{i}+17\hat{j}$

$$|\vec{D}|=\sqrt{(0)^2+(17)^2}=\sqrt{0+289}=\sqrt{289}=17\ units$$

$$\theta=\tan^{-1}\left(\frac{17}{0}\right)=\tan^{-1}(\infty)=90°$$

Q # 7. Find the angle between two vectors, $\vec{A}=5\hat{i}+\hat{j}$ and $\vec{B}=2\hat{i}+4\hat{j}$.

Given Data: $\vec{A}=5\hat{i}+\hat{j},\qquad \vec{B}=2\hat{i}+4\hat{j}$

To Determine: $\theta=\cos^{-1}\left(\frac{A.B}{AB}\right)=?$

Calculations: $\vec{A}.\vec{B}=(5\hat{i}+\hat{j}).(2\hat{i}+4\hat{j})=10+4=14$

$$A=|\vec{A}|=\sqrt{5^2+1^2}=\sqrt{25+1}=\sqrt{26},\qquad B=|\vec{B}|=\sqrt{2^2+4^2}=\sqrt{4+16}=\sqrt{20}$$

So, $$\theta=\cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right)=\cos^{-1}\left(\frac{14}{\sqrt{26}\times\sqrt{20}}\right)=52°$$

Q # 8. Find the work done when the point of application of force $3\hat{i}+2\hat{j}$ moves in a straight line from point $(2,-1)$ to the point $(6,4)$.

Given Data: $\vec{F}=3\hat{i}+2\hat{j}$, Initial Point A$(2,-1)$, Final Point B$(6,4)$

To Determine: Work Done $W=?$

Calculations: Position Vector of Point A $\vec{r_A}=2\hat{i}-\hat{j}$, Position Vector of Point B $\vec{r_B}=6\hat{i}+4\hat{j}$

Displacement $\vec{d}=\vec{r_B}-\vec{r_A}=6\hat{i}+4\hat{j}-(2\hat{i}-\hat{j})=6\hat{i}+4\hat{j}-2\hat{i}+\hat{j}=4\hat{i}+5\hat{j}$

Work Done $W=\vec{F}.\vec{d}=(3\hat{i}+2\hat{j}).(4\hat{i}+5\hat{j})=12+10=22$ units

Q # 9. Show that the three vectors $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}-3\hat{j}+\hat{k}$ and $4\hat{i}+\hat{j}-5\hat{k}$ are mutually perpendicular.

Given Data: Let $\vec{A}=\hat{i}+\hat{j}+\hat{k},\ \vec{B}=2\hat{i}-3\hat{j}+\hat{k},\ \vec{C}=4\hat{i}+\hat{j}-5\hat{k}$

To Show: $\vec{A}.\vec{B}=0,\quad \vec{B}.\vec{C}=0,\quad \vec{C}.\vec{A}=0$

Calculations: $\vec{A}.\vec{B}=(\hat{i}+\hat{j}+\hat{k}).(2\hat{i}-3\hat{j}+\hat{k})=2-3+1=0,\quad$ So $\quad\vec{A}\perp\vec{B}$

$$\vec{B}.\vec{C}=(2\hat{i}-3\hat{j}+\hat{k}).(4\hat{i}+\hat{j}-5\hat{k})=8-3-5=0,\quad \text{So}\quad \vec{B}\perp\vec{C}$$

$$\vec{C}.\vec{A}=(4\hat{i}+\hat{j}-5\hat{k}).(\hat{i}+\hat{j}+\hat{k})=4+1-5=0,\quad \text{So}\quad \vec{C}.\perp\vec{A}$$

Q # 10. Given that $\vec{A}=\hat{i}-2\hat{j}+3\hat{k}$ and $\vec{B}=3\hat{i}-4\hat{k}$, find the projection of $\vec{A}$ on $\vec{B}$.

Given Data: $\vec{A}=\hat{i}-2\hat{j}+3\hat{k},\qquad \vec{B}=3\hat{i}-4\hat{k}$

To Determine: Projection of $\vec{A}$ on $\vec{B}=A\cos\theta=\frac{\vec{A}.\vec{B}}{B}=?$

Calculations: $\vec{A}.\vec{B}=(\hat{i}-2\hat{j}+3\hat{k}).(3\hat{i}-4\hat{k})=3-0-12=-9$

$$B=|\vec{B}|=\sqrt{3^2+0^2+(-4)^2}=\sqrt{9+16}=\sqrt{25}=5$$

So,
$$\text{Projection of } \vec{A} \text{ on } \vec{B}=A\cos\theta=\frac{\vec{A}.\vec{B}}{B}=\frac{-9}{5}$$

Q # 11. Vector $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 4 units north, 3 units west and 8 units east, respectively. Describe carefully (a) $\vec{A}\times\vec{B}$, (b) $\vec{A}\times\vec{C}$, (c) $\vec{B}\times\vec{C}$.

In coordinate plane, we consider (by convention) directions of east, west, north and south along positive $x-$ axis, negative $x-$ axis, positive $y-$ axis and negative $y-$ axis respectively. Hence,

Given Data: $\vec{A}=4$ units north $=4\hat{j}$, $\vec{B}=3$ units west $=-3\hat{i}$, $\vec{C}=8$ units east $=8\hat{i}$

To Determine: (a) $\vec{A}\times\vec{B}$, (b) $\vec{A}\times\vec{C}$, (c) $\vec{B}\times\vec{C}$

Calculations: (a) $\vec{A}\times\vec{B}=4\hat{j}\times(-3\hat{i})=-12(\hat{j}\times\hat{i})=-12(-\hat{k})=12\,\hat{k}$ i.e., 12 units upward

(b) $\vec{A}\times\vec{C}=4\hat{j}\times 8\hat{i}=32(\hat{j}\times\hat{i})=32(-\hat{k})=-32\,\hat{k}$ i.e., 32 unis downward

(c) $\vec{B}\times\vec{C}=(-3\hat{i})\times 8\hat{i}=-24(\hat{i}\times\hat{i})=-24(0)=0$

Q # 12. Consider a force $\vec{F}=-3\hat{i}+\hat{j}+5\hat{k}$ (newton) acting on the point $7\hat{i}+3\hat{j}+\hat{k}$ (m). What is the torque in N m about the origin?

Given Data: $\vec{F}=-3\hat{i}+\hat{j}+5\hat{k}$ (newton), Let Position Vector $\vec{r}=7\hat{i}+3\hat{j}+\hat{k}$ (m)

To Determine: $\vec{\tau}=\vec{r}\times\vec{F}=?$

Calculations:
$$\vec{\tau}=\vec{r}\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 7 & 3 & 1\\ -3 & 1 & 5\end{vmatrix}=\hat{i}\begin{vmatrix}3 & 1\\ 1 & 5\end{vmatrix}-\hat{j}\begin{vmatrix}7 & 1\\ -3 & 5\end{vmatrix}+\hat{k}\begin{vmatrix}7 & 3\\ -3 & 1\end{vmatrix}=14\hat{i}-38\hat{j}+16\hat{k}\ \text{Nm}$$

Q # 13. The line of action of force $\vec{F} = \hat{i} - 2\hat{j}$, passes through a point whose position vector is $-\hat{j} + \hat{k}$. Find (a) the moment of $\vec{F}$ about the origin, (b) the moment of $\vec{F}$ about the point of which the position vector is $\hat{i} + \hat{k}$.

Given Data: $\vec{F} = \hat{i} - 2\hat{j}$, Let $\vec{r_P} = -\hat{j} + \hat{k}$ & $\vec{r_A} = \hat{i} + \hat{k}$

To Determine: (a)Torque about origin $\vec{\tau} = \vec{r_P} \times \vec{F} = ?$ (b)Torque about Point A $\vec{\tau_A} = \vec{r_{AP}} \times \vec{F} = ?$

Calculations: (i) $\vec{\tau} = \vec{r_P} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 1 & -2 & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & 1 \\ -2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} 0 & -1 \\ 1 & -2 \end{vmatrix} = 2\hat{i} + \hat{j} + \hat{k}$

(ii) $\vec{r_{AP}} = \vec{r_P} - \vec{r_A} = -\hat{j} + \hat{k} - (\hat{i} + \hat{k}) = -\hat{j} + \hat{k} - \hat{i} - \hat{k} = -\hat{i} - \hat{j}$

$\vec{\tau} = \vec{r_{AP}} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 0 \\ 1 & -2 & 0 \end{vmatrix} = \hat{i}\begin{vmatrix} -1 & 0 \\ -2 & 0 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} + \hat{k}\begin{vmatrix} -1 & -1 \\ 1 & -2 \end{vmatrix} = 0\hat{i} + 0\hat{j} + 3\hat{k} = 3\hat{k}$

Q # 14. The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between vectors.

Given Data: For two vectors $\vec{A}$ & $\vec{B}$, $|\vec{A}.\vec{B}| = 6\sqrt{3}$, $|\vec{A} \times \vec{B}| = 6$

To Determine: Angle between vectors $\vec{A}$ & $\vec{B}$ $\theta = ?$

Calculations: $|\vec{A}.\vec{B}| = 6\sqrt{3} \Rightarrow AB\cos\theta = 6\sqrt{3}$ $-----(1)$

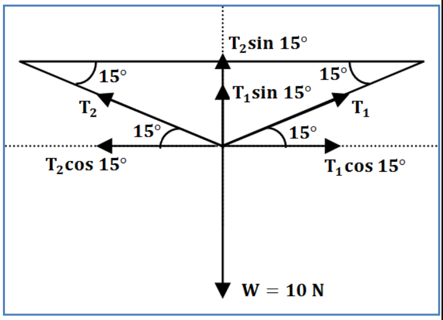
$|\vec{A} \times \vec{B}| = 6 \Rightarrow AB\sin\theta = 6$ $-----(2)$

Dividing (1) and (2): $\frac{AB\sin\theta}{AB\cos\theta} = \frac{6}{6\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$

Q # 15. A load of 10 N is suspended from a clothes line. This distorts the line so that it makes an angle of 15° with horizontal at each end. Find the tension in the clothes line.

Given Data: Load $W = 10\text{ N}$, $\theta = 15°$

To Determine: Tension in the String $T_1 = ?$, $T_2 = ?$,



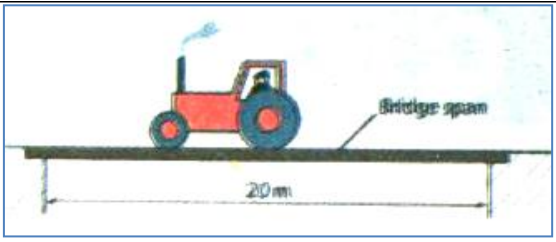
Calculations: As $\sum F_x = 0 \Rightarrow T_1\cos 15° - T_2\cos 15° = 0$

$\Rightarrow T_1\cos 15° = T_2\cos 15° \Rightarrow T_1 = T_2$

As $\sum F_Y = 0 \Rightarrow T_1\sin 15° + T_2\sin 15° - 10 = 0$

$\Rightarrow 2T_1\sin 15° = 10 \Rightarrow T_1 = \frac{10}{2\sin 15°} = 19.3\text{ N}$

Q # 16. A tractor of weight 15000 N crosses a single span bridge of weight 8000 N and of length 21 m. The bridge is supported half a meter from either end. The tractor's front wheels takes 1/3 of the total weight of the tractor, and the rear wheels are 3 m behinds the front wheels. Calculate the force on the bridge supports when the rear wheels are at the middle of the bridge span.



Given Data: Weight of Tractor $= 15000\text{N}$, Weight of Bridge $= 8000\text{N}$

Weight on Front Wheel $= 5000$ N, Weight on Rear Wheel $= 10000$ N

To Determine: Normal Reactions on Bridge $R_1 = ?$, $R_2 = ?$

Calculations: By first condition of Equilibrium $\sum \mathbf{F_Y} = 0$

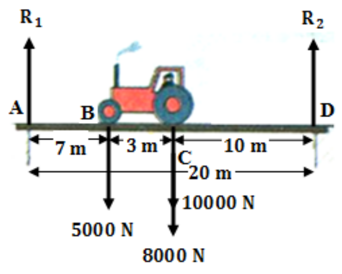
$$R_1 + R_2 - 5000 - 8000 - 10000 = 0 \Rightarrow R_1 + R_2 = 23000 \dots\dots (1)$$

By second condition of Equilibrium $\sum \boldsymbol{\tau} = 0$

(Taking Moments about Point A)

$$\Rightarrow R_2 \times 20 - 5000 \times 7 - 18000 \times 10 = 0 \Rightarrow R_2 = 10750 \text{ N}$$

<u>Putting values in (1)</u> $R_1 + 10750 = 23000 \Rightarrow R_1 = 12250$ N



Q # 17. A spherical ball of weight 50 N is to be lifted over the step as shown in the figure. Calculate the minimum force needed just to lift it above the floor.

Given Data: Weight of the ball $= 50\text{N}$, Radius of Sphere $= 20$ cm

To Determine: Minimum Force Needed to Lift it Above Floor $F = ?$

Calculations: By 2nd condition of Equilibrium $\sum \boldsymbol{\tau} = 0$

(Taking Moments about Point A)

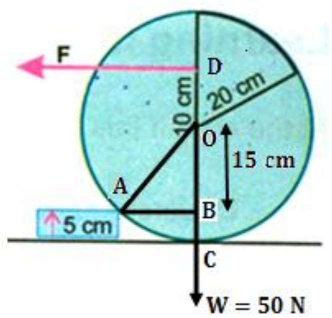
$$\Rightarrow F \times |BD| - W \times |AB| = 0 \quad ----(1)$$

<u>From Fig.</u> $|BD| = |OB| + |OD| = 15 + 10 = 25$ cm

<u>In Triangle OAB:</u> $|AB|^2 = |OA|^2 - |OB|^2 = (20)^2 - (15)^2 = 175$

$$\Rightarrow |AB| = \sqrt{175} \approx 13$$

<u>Putting values in (1)</u> $F \times 25 - 50 \times 13 = 0 \Rightarrow F = 26$ N



Q # 18. A uniform sphere of weight 10 N held by a string attached to a frictionless wall so that the string makes an angle of 30° with the wall as shown in the figure. Find the tension in the string and the force exerted on sphere by the wall.

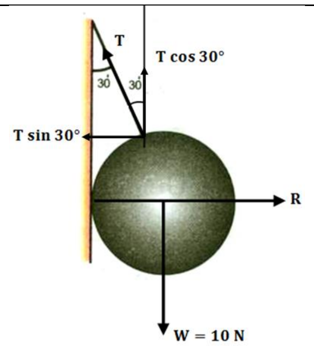
Given Data: Weight of Sphere $= 10$ N, Weight of Sphere $= 8000\text{N}$

To Determine: Normal Reactions $R = ?$, Tension in String $T = ?$

Calculations: By first condition of Equilibrium

$$\sum \mathbf{F_Y} = 0 \Rightarrow T\cos 30° - 10 = 0 \Rightarrow T = \frac{10}{\cos 30°} = 11.6 \text{ N}$$

$$\sum \mathbf{F_x} = 0 \Rightarrow R - T\sin 30° = 0 \Rightarrow R = 11.6 \times \sin 30° = 5.77 \text{ N}$$



NUMERICAL PROBLEMS

F.Sc. Physics, Chapter # 3: Motion and Force

Q # 1. A helicopter is ascending vertically at the rate of 19.6 ms^{-1}. When it is at the height of 156.8 m above the ground, a stone is dropped. How long does the stone take to reach the ground?

Initially, the dropped particle move in upward direction, attain the maximum height in time t_1 and then fall vertically downward under gravity and strike the ground after time t_2.

Given Data: Initial Velocity $v_i = 19.6\ ms^{-1}$, Height $h = 156.8\ m$

To Determine: Time taken by particle in air $t = t_1 + t_2 = ?$

Calculations: Upward Motion $v_i = 19.6\ ms^{-1}, v_f = 0\ ms^{-1}, a = -g$, time $= t_1$

By first equation of motion: $v_f = v_i - g\,t_1 \Rightarrow t_1 = \frac{v_f - v_i}{-g} = \frac{0-19.6}{-9.8} = 2\ s$

Distance covered during upward motion: $-2gS_1 = (v_f)^2 - (v_i)^2 \Rightarrow S_1 = \frac{0-(19.6)^2}{-2\times 9.8} = 19.6\ m$

Downward Motion $v'_i = 0\ ms^{-1}$, Distance Covered during downward motion $S_2 = S_1 + h = 176.4\ m$

By 2nd eq. of motion: $S_2 = v'_i \times t_2 + \frac{1}{2}g(t_2)^2 = 0 \times t_2 + \frac{1}{2} \times 9.8 \times (t_2)^2 \Rightarrow t_2 = \sqrt{\frac{2S_2}{9.8}} = \sqrt{\frac{2\times 176.4}{9.8}} = 6\ s$

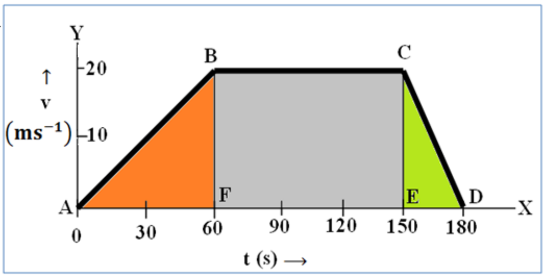
Time taken by particle in air $t = t_1 + t_2 = 2 + 6 = 8\ s$

Q # 2. Using the following data, draw a velocity-time graph for a short journey on a straight road of a motorbike.

Velocity (ms^{-1})	**0**	**10**	**20**	**20**	**20**	**20**	**0**
Time (s)	**0**	**30**	**60**	**90**	**120**	**150**	**180**

Use the graph to calculate

- **The initial acceleration**
- **The final acceleration and**
- **The total distance travelled by the motorcyclist.**



Ans. The graph tells us that the car starts from rest, and its velocity increases uniformly to 20 ms^{-1} in 60 seconds. Its average acceleration is:

$$\text{Initial Acceleration}\quad a_i = \frac{\Delta v}{\Delta t} = \frac{20\ ms^{-1}}{60\ s} = 0.33\ ms^{-2}$$

The graph further tells that the velocity of the car remains constant from 60th to 150th second and it then decreases uniformly to zero from 150th to 180th seconds. The acceleration of the car during last 30 seconds is:

$$\text{Final Acceleration}\quad a_f = \frac{\Delta v}{\Delta t} = \frac{-20\ ms^{-1}}{30\ s} = -0.67\ ms^{-2}$$

The negative sign indicates that the velocity of the car decreases during these 30 seconds.

Calculations: Distance Travelled $=$ Area of ΔABF + Area of rectangle BCEF + Area of ΔCDE

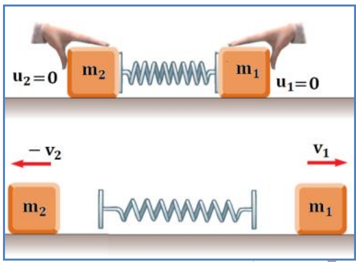
$$\Rightarrow \text{Distance Travelled} = \frac{1}{2} \times 20\ ms^{-1} \times 60\ s + 20\ ms^{-1} \times 90\ s + \frac{1}{2} \times 20\ ms^{-1} \times 30\ s$$

$$\Rightarrow \text{Distance Travelled} = 600\ m + 1800\ m + 300\ m = 2700\ m = 2.7\ km$$

Q # 3. A proton is moving with speed of $1.0\times10^{7}\ \text{ms}^{-1}$ passes through a 0.020 cm thick sheet of paper and emerges with a speed of $2.0\times10^{6}\ \text{ms}^{-1}$. Assuming uniform deceleration, find retardation and time taken to pass through the paper.

Given Data: Initial Velocity $v_i = 1.0\times10^{7}\ \text{ms}^{-1}$, Final Velocity $v_f = 2.0\times10^{6}\ \text{ms}^{-1}$, Thickness $S = 0.020\ \text{cm} = 0.020\times10^{-2}\ \text{m}$
To Determine: Retardation $a = ?$, Time taken by proton to pass through paper $t = ?$
Calculations: By 3rd eq.: $2aS = (v_f)^2-(v_i)^2 \Rightarrow a = \frac{(v_f)^2-(v_i)^2}{2S} = \frac{(2.0\times10^{6})^2-(10^{7})^2}{2\times0.020\times10^{-2}} = -2.4\times10^{17}\ \text{ms}^{-2}$ By first eq. of motion: $t = \frac{2.0\times10^{6}-1.0\times10^{7}}{-2.4\times10^{17}} = 3.3\times10^{-11}\ \text{s}$

Q # 4. Two masses m_1 and m_2 are initially at rest with a spring compressed between them. What is the ratio of the magnitude of their velocities after the spring has been released?



Given Data: Mass of First Object $= m_1$, Mass of Second Object $= m_2$ Initial Velocity of First Object $u_1 = 0$, Initial Velocity of Second Object $u_2 = 0$
To Determine: If v_1 & v_2 are the final velocities of First & *Second object respectively*: $\frac{v_1}{v_2} = ?$
Calculations: By Law of conservation of momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2(-v_2)$ $\Rightarrow 0 + 0 = m_1v_1 - m_2v_2 \Rightarrow m_1v_1 = m_2v_2 \Rightarrow \frac{v_1}{v_2} = \frac{m_2}{m_1}$

Q # 5. An amoeba of mass 1.0×10^{-12} kg propels itself through water by blowing a jet of water through tiny orifice. The amoeba ejects water with the speed of $1.0\times10^{-4}\ \text{ms}^{-1}$ at the rate of $1.0\times10^{-13}\ \text{kgs}^{-1}$. Assume the water is being continuously replenished so that mass of amoeba remains the same.

- **a. If there is no force on amoeba other than the reaction force caused by emerging jet, what would be the acceleration of the amoeba?**
- **b. If amoeba moves with constant velocity through water, what is the force of surrounding water (exclusively of jet) on the amoeba?**

Given Data: Mass of Amoeba $M = 1.0\times10^{-12}$ kg, Speed of Water $v = 1.0\times10^{-4}\ \text{ms}^{-1}$ Ejected mass per second $\left(\frac{m}{t}\right) = 1.0\times10^{-13}\ \text{kgs}^{-1}$
To Determine: (a) Acceleration of amoeba $a = ?$ **(b)** Force on amoeba due to water flow $F = ?$
Calculations: (b) Force on amoeba due to water flow $F = 1.0\times10^{-13} \times 1.0\times10^{-4} = 1.0\times10^{-17}$ N   **(a)** Acceleration of amoeba $a = \frac{F}{M} = \frac{1.0\times10^{-17}}{1.0\times10^{-12}} = 1.0\times10^{-5}\ \text{ms}^{-2}$

Q # 6. A boy places a fire cracker of negligible mass in an empty can of 40 g mass. He plugs the end with a wooden block of mass 200 g. After igniting the fire cracker, he throws the can straight up. It explodes at the top of its path. If the block shoots out with a speed of $3.0\ \text{ms}^{-1}$, how fast will the can be going?

Given Data: Mass of Can $m_1 = 40\text{ g} = 0.04\text{ kg}$, Mass of Wooden Block $m_2 = 200\text{ g} = 0.2\text{ kg}$ Velocity of Can before explosion (At Maximum Height) $u_1 = 0$, Velocity of Wooden Block before explosion (At Maximum Height) $u_2 = 0$ Velocity of Wooden Block After Explosion $v_2 = 3.0\text{ ms}^{-1}$
To Determine: Velocity of Can After Explosion $(-v_1) = ?$ $\therefore$ (−ve sign is due to motion in opposite direction)
Calculations: By Law of conservation of momentum: $m_1u_1 + m_2u_2 = m_1(-v_1) + m_2v_2$ $\Longrightarrow 0 + 0 = 0.04 \times (-v_1) + 0.2 \times 3 \Longrightarrow 0.04\ v_1 = 0.6 \Longrightarrow v_1 = 15\text{ ms}^{-1}$

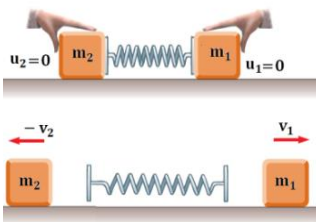
Q # 7. An electron $(m = 9.1 \times 10^{-31}\text{ kg})$ travelling at $2.0 \times 10^7\text{ms}^{-1}$ undergoes a head on collision with a hydrogen atom $(m = 1.67 \times 10^{-27}\text{ kg})$ which is initially at rest. Assuming the collision to be perfectly elastic and a motion to be along a straight line, find the velocity of the hydrogen atom.

Given Data: Mass of electron $m_1 = 9.1 \times 10^{-31}\text{ kg}$, Mass of H − atom $m_2 = 1.67 \times 10^{-27}\text{ kg}$ Initial Velocity of electron $v_1 = 2.0 \times 10^7\text{ms}^{-1}$, Initial Velocity of H − atom $v_2 = 0$
To Determine: Velocity of H-atom After Collision $v_2' = ?$
Calculations: As $v_2' = \frac{2m_1v_1}{(m_1+m_2)} + \frac{(m_2-m_1)v_2}{(m_1+m_2)} = \frac{2m_1v_1}{(m_1+m_2)} + 0 \Longrightarrow v_2' = \frac{2\times9.1\times10^{-31}\times2.0\times10^7}{(9.1\times10^{-31}+1.67\times10^{-27})} = 2.2 \times 10^4\text{ ms}^{-1}$

Q # 8. A truck weighing 2500 kg and moving with a velocity of 21 ms^{-1} collides with a stationary car weighing 1000 kg. The truck and the car moves together after the impact. Calculate their common velocity.

Given Data: Mass of Truck $m_1 = 2500\text{ kg}$, Mass of Car $m_2 = 1000\text{ kg}$ Initial Velocity of Truck $u_1 = 21\text{ ms}^{-1}$, Initial Velocity of Car $u_2 = 0\ \text{ms}^{-1}$
To Determine: Common Velocity of Truck and Car after impact $v_1 = v_2 = v = ?$
Calculations: By Law of conservation of momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ $\Longrightarrow m_1u_1 + m_2u_2 = (m_1 + m_2)v \Longrightarrow 2500 \times 21 + 1000 \times 0 = (2500 + 1000)v$ $\Longrightarrow v = \frac{2500 \times 21}{3500} = 15\text{ ms}^{-1}$

Q # 9. Two blocks of masses 2.0 kg and 0.50 kg are attached at the two ends of a compressed spring. The elastic potential energy stored in the spring is 10 J. Find the velocities of the blocks if the spring delivers its energy to the blocks when released.



Given Data: Mass of First Object $m_1 = 2\text{ kg}$, Mass of Second Object $m_2 = 0.5\text{ kg}$

Initial Velocity of First Object $u_1 = 0$, Initial Velocity of Second Object $u_2 = 0$

Elastic $\text{P.E.} = 10\text{ J}$

To Determine: If v_1 & v_2 are the final velocities of First & *Second object*: $v_1 = ?, v_2 = ?$

Calculations: By Law of conservation of momentum: $m_1u_1 + m_2u_2 = m_1v_1 + m_2(-v_2)$

$$\Rightarrow 0 + 0 = 2\times v_1 - 0.5\times v_2 \Rightarrow 0.5\times v_2 = 2\times v_1 \Rightarrow v_2 = 4\,v_1 ----(1)$$

In present case: Loss of $\text{P.E} = \text{Gain in K.E} \Rightarrow 10 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2$

$$\Rightarrow 10 = \frac{1}{2}m_1{v_1}^2 + \frac{1}{2}m_2{v_2}^2 \Rightarrow 20 = 2\times{v_1}^2 + 0.5\times(4\,v_1)^2 \Rightarrow 20 = 2\times{v_1}^2 + 8\times{v_1}^2$$

$$\Rightarrow 20 = 10\,{v_1}^2 \Rightarrow {v_1}^2 = 2 \Rightarrow v_1 = \sqrt{2} = 1.4\text{ ms}^{-1}$$

Put in (1): $v_2 = 4\,(1.4) = 5.6\text{ ms}^{-1}$

Q # 10. A football is thrown upward with an angle of 30° with respect to the horizontal. To throw a 40 m pass what must be the initial speed of the ball?

Given Data: Horizontal Range $R = 40\text{ m}$, Inclination $\theta = 30°$

To Determine: Initial Velocity $v_i = ?$

Calculations: $R = \frac{{v_i}^2}{g}\sin 2\theta \Rightarrow {v_i}^2 = \frac{gR}{\sin 2\theta} \Rightarrow v_i = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{9.8\times 40}{\sin 60°}} = 21\text{ ms}^{-1}$

Q # 11. A ball thrown horizontally from a height of 10 m with velocity of 21 ms^{-1}. How far off it hit the ground and with what velocity?

Given Data: Height $y = 10\text{ m}$, Horizontal Velocity $v_{ix} = 21\text{ ms}^{-1}$

To Determine: Horizontal Distance $x = v_{ix}\times t = ?$ Total Final Velocity $v = \sqrt{v_{fx}^2 + v_{fy}^2} = ?$

Calculations: Using Vertical Component of Motion $y = v_{iy}\times t + \frac{1}{2}a_y t^2 \Rightarrow 10 = 0\times t + \frac{1}{2}gt^2$

$$\Rightarrow t = \sqrt{\frac{10\times 2}{g}} = \sqrt{\frac{10\times 2}{9.8}} = 1.41\,s$$

Horizontal Distance $x = v_{ix}\times t = 21\times 1.41 = 30\text{ m}$

For Projectile $v_{ix} = v_{fx}$ and $v_{fy} = v_{iy} + a_y\times t = 0 + gt = 9.8\times 1.41 = 13.8\text{ ms}^{-1}$

Total Final Velocity $v = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(21)^2 + (13.8)^2} \approx 25\text{ ms}^{-1}$

Q # 12. A bomber dropped a bomb at a height of 490 m when its velocity along the horizontal was 300 kmh^{-1}.

(i) How long was it in air?

(ii) At what distance from the point vertically below the bomber at the instant the bomb was dropped, did it strike the ground?

Given Data: Height of Bomber $y = 490$ m, Horizontal Velocity $v_{ix} = 300\,\frac{km}{h} = \frac{300\times1000}{3600}\,\frac{m}{s} = 83.3\,\frac{m}{s}$

To Determine: Time taken by Bomb in air t =?, Horizontal Distance x =?

Calculations: Using Vertical Component of Motion $y = v_{iy}\times t + \frac{1}{2}a_y t^2 \Rightarrow 490 = 0\times t + \frac{1}{2}gt^2$

$$\Rightarrow t = \sqrt{\frac{490\times 2}{g}} = \sqrt{\frac{490\times 2}{9.8}} = 10\,s$$

Horizontal Distance $x = v_{ix}\times t = 83.3\times 10 = 833$ m

Q # 13. Find the angle of projection of a projectile for which its maximum height and horizontal range are equal.

Given Data: Horizontal Range R = Height of Projectile H

To Determine: Angle of projection =?

Calculations: As $R = H \Rightarrow \frac{{v_i}^2}{g}\sin 2\theta = \frac{v_i^2\sin^2\theta}{2g} \Rightarrow 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2} \Rightarrow 2\cos\theta = \frac{\sin\theta}{2}$

$$\Rightarrow \tan\theta = 4 \Rightarrow \theta = \tan^{-1}(4) = 76°$$

Q # 14. Prove that for angles of projection, which exceed of fall short of 45° by equal amounts, the ranges are equal.

To Determine: For Angle $\varphi < 45°$ Range for Angle $(45° - \varphi)$ = Range for Angle $(45° + \varphi)$

Calculations: <u>For $\theta = 45° - \varphi$</u> Horizontal Range $R = \frac{{v_i}^2}{g}\sin 2(45° - \varphi) = \frac{{v_i}^2}{g}\sin(90° - 2\varphi)$

$$\Rightarrow R = \frac{{v_i}^2}{g}\sin(90° - 2\varphi) = \frac{{v_i}^2}{g}\cos 2\varphi \qquad \because \text{By Allied Angle Concept } \sin(90° - 2\varphi) = \cos 2\varphi$$

: <u>For $\theta = 45° + \varphi$</u> Horizontal Range $R = \frac{{v_i}^2}{g}\sin 2(45° + \varphi) = \frac{{v_i}^2}{g}\sin(90° + 2\varphi)$

$$\Rightarrow R = \frac{{v_i}^2}{g}\sin(90° + 2\varphi) = \frac{{v_i}^2}{g}\cos 2\varphi \qquad \because \text{By Allied Angle Concept } \sin(90° + 2\varphi) = \cos 2\varphi$$

Hence Proved for angles of projection, which exceed of fall short of 45° by $\varphi < 45°$, the ranges are equal.

Q # 15. A SLBM (submarine launched ballistic missile) is fired from a distance of 3000 km. if the earth is considered flat and the angle of launch is 45° with horizontal, find the velocity with which the missile is fired and the time taken by SLBM to hit the target.

Given Data: Horizontal Range R = 3000 km = 3000000 m, Inclination $\theta = 45°$

To Determine: Initial Velocity v_i =?, Time of Flight t =?

Calculations: $R = \frac{{v_i}^2}{g}\sin 2\theta \Rightarrow {v_i}^2 = \frac{gR}{\sin 2\theta} \Rightarrow v_i = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{9.8\times3000000}{\sin 45°}} = 5420\ ms^{-1} = 5.42\ kms^{-1}$

$$t = \frac{2\,v_i\sin\theta}{g} = \frac{2\times 5420\times\sin 45°}{9.8} = 782\ s \approx 13\ min$$

NUMERICAL PROBLEMS

F.Sc. Physics, Chapter # 4: Work and Energy

Q # 1. A man pushes a lawn mover with a 40 N force directed at an angle of 20° downward from the horizontal. Find the work done by the man as he cuts a strip of grass 20 m long.

Given Data: Force $\mathrm{F = 40\ N}$, Inclination $\theta = 20°$, Displacement $\mathrm{d = 20\ m}$
To Determine: Work Done $\mathrm{W = ?}$
Calculations: As $\mathrm{W = Fd\cos\theta = 40 \times 20 \times \cos 20° = 750\ J = 7.5 \times 10^2\ J}$

Q # 2. A rain drop ($m = 3.35 \times 10^{-5}$kg) falls vertically at a constant speed under the influence of forces of gravity and friction. In falling through 100 m, how much work is done by (a) gravity and (b) friction.

Given Data: Mass of rain drop $m = 3.35 \times 10^{-5}\mathrm{kg}$, Height $\mathrm{h = 100\ m}$
To Determine: (a) Work Done by Gravity $\mathrm{W_G = ?}$ (b) Work Done by Friction $\mathrm{W_F = ?}$
Calculations: (a) $\mathrm{W_G = \vec{F}_g.\vec{h} = \lvert\vec{F}_g\rvert\lvert h\rvert\cos 0°}$ $\quad \therefore \vec{F}_g \parallel \vec{h}\ \&\ \lvert\vec{F}_g\rvert = \mathrm{mg}$ $\mathrm{= mgh = 3.35 \times 10^{-5} \times 9.8 \times 100 = 0.0328\ J}$ (b) As the rain drop falling with constant velocity, therefore force of friction will be equal but opposite to that of weight. $\mathrm{W_F = -mgh = -3.35 \times 10^{-5} \times 9.8 \times 100 = -0.0328\ J}$

Q # 3. Ten bricks, each 6.0 cm thick and mass 1.5 kg, lie flat on table. How much work is required to stack them one on the top of another?

Given Data: Mass of a brick $m = 1.5\ \mathrm{kg}$, Thickness of a brick $\mathrm{h = 6\ cm = 0.06\ m}$
To Determine: Work to stack them one on the top of another $\mathrm{W = ?}$
Calculations: Ten bricks lifted to the height $\mathrm{h_1 = 0\ m, h_2 = 0.06\ m, h_3 = 0.12\ m \ldots\ldots, h_{10} = 0.54\ m}$ respectively. Total Work Done $W_T = mg\mathrm{h_1} + mg\mathrm{h_2} + mg\mathrm{h_3} + \ldots\ldots + mg\mathrm{h_{10}}$ $\Longrightarrow W_T = mg(\mathrm{h_1 + h_2 + h_3 + h_4 + h_5 + h_6 + h_7 + h_8 + h_9 + h_{10}})$ $\Longrightarrow W_T = 1.5 \times 9.8 \times (0 + 0.06 + 0.12 + 0.18 + 0.24 + 0.30 + 0.36 + 0.42 + 0.48 + 0.54) = 40\ \mathrm{J}$

Q # 4. A car of mass 800 kg travelling 54 $\mathrm{kmh^{-1}}$ is brought to rest in 60 m. find the average retarding force on the car. What happened to original kinetic energy?

Given Data: Mass $\mathrm{m = 800\ kg}$, Initial Velocity $\mathrm{v_i = 54\ \frac{km}{h} = \frac{54 \times 1000\ m}{3600\ s} = 15\ ms^{-1}}$ Final Velocity $\mathrm{v_f = 0\ ms^{-1}}$, Distance Covered $\mathrm{d = 60\ m}$
To Determine: Average Retarding Force $\mathrm{F = ?}$
Calculations: By Work – Energy Principle: $\mathrm{F\ d} = \frac{1}{2}m\mathrm{v}_f^{\,2} - \frac{1}{2}m\mathrm{v}_i^{\,2} \Longrightarrow \mathrm{F} = \frac{m}{2\mathrm{d}}(\mathrm{v}_f^{\,2} - \mathrm{v}_i^{\,2})$ $\Longrightarrow \mathrm{F} = \frac{800}{2 \times 60}(0^2 - 15^2) = \frac{800}{2 \times 60}(0^2 - 15^2) = -1500\ \mathrm{N}$ The K. E. is dissipated in doing work against friction

Q # 5. A 1000 kg automobile at the tip of an incline 10 m high and 100 m long is released and rolls down the hill. What is its speed at the bottom of the incline if the average retarding force due to friction is 480 N?

Given Data: Mass $m = 1000\ \text{kg}$, Initial Height $h_i = 10\ \text{m}$, Final Height $h_f = 0\ \text{m}$ Initial Velocity $v_i = 0\ \text{ms}^{-1}$, Retarding Force $F = 480\ \text{N}$, Distance Covered $d = 100\ \text{m}$
To Determine: Final Velocity $v_f = ?$
Calculations: By Principle of Conservation of Mechanical Energy: Loss of P. E. = Gain in K. E. +Work against Friction $\Rightarrow mg(h_i - h_f) = \frac{1}{2}m(v_f^2 - v_i^2) + Fd$ $\Rightarrow 1000 \times 9.8 \times (10) = \frac{1}{2} \times 1000 \times (v_f^2 - 0) + 480 \times 100 \Rightarrow 500v_f^2 = 98000 - 48000$ $\Rightarrow 500v_f^2 = 50000 \Rightarrow v_f^2 = 100 \Rightarrow v_f = 10\ \text{ms}^{-1}$

Q # 6. 100 m^3 of water is pumped from a reservoir into a tank, 10 m higher than reservoir, in 20 minutes. If density of water is 1000 kg m^{-3}, find

(a) The increase in P.E.

(b) The power delivered by the pump.

Given Data: Volume $V = 100\ \text{m}^3$, Time $t = 20\ \text{min} = 20 \times 60\ \text{s} = 1200\ \text{s}$,   Height $h = 10\ \text{m}$, Density $\rho = 1000\ \text{kg m}^{-3}$
To Determine: (a)Increase in P. E =?, (b) Power =?
Calculations: (a) Increase in P. E $= mgh = \rho Vgh = (1000)(100)(9.8)(10) = 9.8 \times 10^6\ \text{J}$ (b)$P = \frac{Work}{Time} = \frac{\text{Increase in P. E}}{t} = \frac{9.8 \times 10^6}{1200} = 8200\ \text{W} = 8.2\ \text{kW}$

Q # 7. A force (thrust) of 400 N is required to overcome road friction and air resistance in propelling an automobile at 80 kmh^{-1}. What power (kW) must the engine developed?

Given Data: Force $F = 400\ \text{N}$, Velocity $v = 80\ \frac{\text{km}}{\text{h}} = \frac{80 \times 1000\ \text{m}}{3600\ \text{s}} = 22.22\ \text{ms}^{-1}$
To Determine: Power $P = ?$
Calculations: $P = \vec{F}.\vec{v} = Fv\cos 0°$ $\therefore \vec{F} \parallel \vec{v}$ $= 400 * 22.22 = 8888\ \text{W} \approx 8.9\ \text{kW}$

Q # 8. How large a force is required to accelerate an electron $(9.1 \times 10^{-31}\ \text{kg})$ from rest to a speed of $2 \times 10^7\ \text{ms}^{-1}$ through a distance of 5.0 cm?

Given Data: Mass $m = 9.1 \times 10^{-31}\ \text{kg}$, Initial Velocity $v_i = 0\ \text{ms}^{-1}$ Final Velocity $v_f = 2 \times 10^7\ \text{ms}^{-1}$, Distance Covered $d = 5\ \text{cm} = 0.05\ \text{m}$
To Determine: Accelerating Force $F = ?$
Calculations: By Work – Energy Principle: $F\,d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \Rightarrow F = \frac{m}{2d}(v_f^2 - v_i^2)$ $\Rightarrow F = \frac{9.1 \times 10^{-31}}{2 \times 0.05}[(2 \times 10^7)^2 - 0^2] = 3.6 \times 10^{-15}\ \text{N}$

Q # 9. A diver weighing 750 N dives from a board 10 m above the surface of a pool water. Use the principle of mechanical energy to find his speed at a point 5.0 m above the water surface, neglecting the air friction.

Given Data: Weight $w = mg = 750\text{ N}$, Initial Height $h_i = 10\text{ m}$, Final Height $h_f = 5\text{ m}$ Initial Velocity $v_i = 0\text{ ms}^{-1}$
To Determine: Final Velocity $v_f = ?$
Calculations: By Principle of Conservation of Mechanical Energy: $\text{Loss of P.E.} = \text{Gain in K.E.} \Rightarrow mg(h_i - h_f) = \frac{1}{2}m(v_f^2 - v_i^2) \Rightarrow g(h_i - h_f) = \frac{1}{2}(v_f^2 - v_i^2)$ $\Rightarrow 9.8 \times (10 - 5) = \frac{1}{2}(v_f^2 - 0) \Rightarrow v_f^2 = 2 \times 9.8 \times 5 \Rightarrow v_f^2 = 98 \Rightarrow v_f = 9.9\text{ ms}^{-1}$

Q # 10. A child starts from rest at the top of a slide of height 4.0 m. (a) What is his speed at the bottom if the slide is frictionless? (b) If he reaches the bottom, with a speed of 6 ms^{-1}, what percentage of his total energy at the top of the slide is lost as a result of friction?

Given Data: Initial Height $h_i = 4\text{ m}$, Final Height $h_f = 0\text{ m}$, Initial Velocity $v_i = 0\text{ ms}^{-1}$
To Determine: (**a**) Final Velocity $v_f = ?$, (**b**) % Loss of K.E. if $v_f' = 6\text{ ms}^{-1}$
Calculations: (**a**) By Principle of Conservation of Mechanical Energy: $\text{Loss of P.E.} = \text{Gain in K.E.} \Rightarrow mg(h_i - h_f) = \frac{1}{2}m(v_f^2 - v_i^2) \Rightarrow g(h_i - h_f) = \frac{1}{2}(v_f^2 - v_i^2)$ $\Rightarrow 9.8 \times (4) = \frac{1}{2} \times (v_f^2 - 0) \Rightarrow v_f^2 = 2 \times 9.8 \times 4 \Rightarrow v_f^2 = 78.4 \Rightarrow v_f = 8.8\text{ ms}^{-1}$ (**b**) $\%\text{ Loss of K.E.} = \frac{\text{Loss of K.E}}{\text{Maximum K.E}} \times 100 = \frac{\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_f'^2\right)}{\frac{1}{2}mv_f^2} \times 100 = \frac{\left(v_f^2 - v_f'^2\right)}{v_f^2} \times 100$ $= \frac{(8.8^2 - 6^2)}{8.8^2} \times 100 = 54\ \%$

NUMERICAL PROBLEMS

F.Sc. Physics, Chapter # 5: CIRCULAR MOTION

Q # 1. A tiny laser beam is directed from the Earth to the Moon. If the beam is to have a diameter of 2.50 m at the Moon, how small must divergence angle be for the beam? The distance of Moon from the Earth is 3.8×10^8 m.

Given Data: Diameter of Laser Beam = Arc Length $S = 2.50$ m Distance between Earth and Moon = Radius of Circular Arc $r = 3.8 \times 10^8$ m
To Determine: Divergence Angle $\theta = ?$
Calculations: As $S = r\theta \Longrightarrow \theta = \frac{S}{r} = \frac{2.50}{3.8\times10^8} = 6.6 \times 10^{-9}$ rad.

Q # 2. A gramophone record turntable accelerates from rest to an angular velocity of 45.0 rev/min in 1.60 s. What is average angular acceleration?

Given Data: Initial Angular Velocity $\omega_i = 0$ rad s^{-1}, Time $t = 1.60$ s Final Angular Velocity $\omega_f = 45\,\frac{\text{rev}}{\text{min}} = 45 \times \frac{2\pi\text{ rad}}{60\text{ s}} = \frac{3\pi}{2}$ rad $s^{-1} = 1.5\pi$ rad s^{-1}
To Determine: Average Angular Acceleration $\propto_{av} = ?$
Calculations: As $\propto_{av} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{1.5\pi - 0}{1.6} = 2.95$ rad s^{-2}

Q # 3. A body of moment of inertia $I = 0.80$ kg m^2 about a fixed axis, rotates with a constant angular velocity of 100 rad s^{-1}. Calculate its angular momentum L and the torque to sustain this motion.

Given Data: Moment of Inertia $I = 0.80$ kg m^2, Angular Velocity $\omega = 100$ rad s^{-1} (constant) As the object rotates with constant angular velocity, therefore Angular Acceleration $\propto = ?$
To Determine: (a) Angular Momentum $L = ?$ (b) Torque $\tau = ?$
Calculations: (a) $L = I\omega = 0.80 \times 100 = 80$ J s (b) $\tau = I \propto = 80 \times 0 = 0$

Q # 4. Calculate (a) the torque acting on the cylinder, (b) angular acceleration of the cylinder. (moment of inertia of cylinder $= \frac{1}{2}mr^2$)

Given Data: Mass $m = 5$ kg, Force $F = 0.60$ N, Radius of Cylinder $r = 0.20$ m
To Determine: (a) Torque $\tau = ?$ (b) Angular Acceleration $\propto = ?$
Calculations: As (a) Torque $\tau = rF\sin 90° = rF = 0.20 \times 0.60 = 0.12$ Nm $\therefore r \perp F$ (b) As $\tau = I \propto \Longrightarrow \propto = \frac{\tau}{I} = \frac{\tau}{\left(\frac{1}{2}mr^2\right)} = \frac{0.12}{0.5\times5\times(0.20)^2} = 1.2$ rad s^{-2}

Q # 5. Calculate the angular momentum of a star of mass 2.0×10^{30} kg and radius 7.0×10^{5} km. If it makes one complete rotation about its axis once in 20 days, what is its kinetic energy?

Given Data: Mass $m = 2.0 \times 10^{30}$ kg , Radius $r = 7.0 \times 10^{5}$ km $= 7.0 \times 10^{8}$ m Time Period $T = 20$ days $= 20 \times 24 \times 60 \times 60$ s $= 1728000$ s
To Determine: (**a**) Angular Momentum $L = ?$, (**b**) Rotational K. E. =?
Calculations: (a) $L = I\omega = \left(\frac{2}{5}mr^2\right)\left(\frac{2\pi}{T}\right) = [0.4 \times 2.0 \times 10^{30} \times (7.0 \times 10^{8})^2]\left(\frac{2\times3.14}{1728000}\right) = 1.4 \times 10^{42}\ \mathrm{J\,s}$ (**b**) Rotational K. E. $= \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{2\pi}{T}\right)^2 = \frac{1}{2}(0.4 \times 2.0 \times 10^{30} \times (7.0 \times 10^{8})^2)\left(\frac{2 \times 3.14}{1728000}\right)^2$ $= 2.5 \times 10^{36}$J

Q # 6. A 1000 kg car travelling with a speed of $144\ \mathrm{km\,h^{-1}}$ round the curve of radius 100 m. Find the necessary centripetal force.

Given Data: Mass $m = 1000$ kg, Velocity $v = 144\ \frac{\mathrm{km}}{\mathrm{h}} = 144 \times \frac{1000\ \mathrm{m}}{3600\ \mathrm{s}} = 40\ \frac{\mathrm{m}}{\mathrm{s}}$ Radius of Circle $r = 100$ m
To Determine: Centripetal Force $F_c = ?$
Calculations: As $F_C = \frac{m\,v^2}{r} = \frac{1000\times(40)^2}{100} = 1.6 \times 10^{4}$ N

Q # 7. What is the least speed at which an aero plane can execute a vertical loop of 1.0 km radius so that there will be no tendency for the pilot to fall down at the highest point?

Given Data: Radius $R = 1\ km = 1000\ m$
To Determine: *Least speed of aero − plane for execution of Vertical Loop* $v = ?$
Calculations: As $v = \sqrt{gR} = \sqrt{9.8 \times 1000} = 99\ ms^{-1}$

Q # 8. The Moon orbits the Earth so that the same side always faces the Earth. Determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum . (In this case treat the Moon as the particle orbiting the Earth). Distance between the Earth and the Moon is 3.85×10^{8} m. Radius of the Moon is 1.74×10^{6} m.

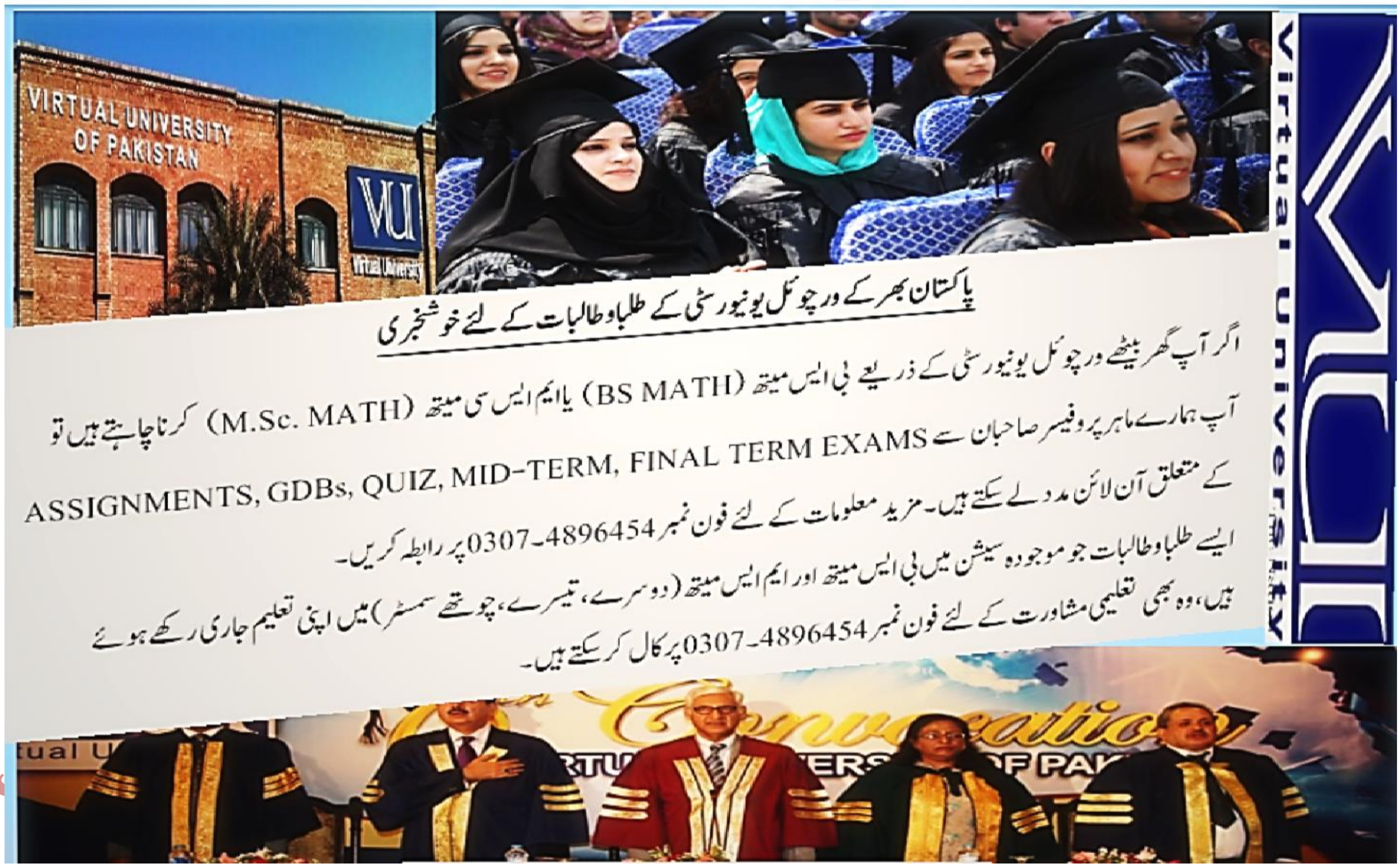
Given Data: Spin Angular Velocity = Orbital Angular Velocity $\Longrightarrow \omega_s = \omega_o$ Distance between Earth and Moon $R = 3.85 \times 10^{8}$ m, Radius of Moon $r = 1.74 \times 10^{6}$ m
To Determine: Ratio $\frac{L_s}{L_o} = ?$
Calculations: $\frac{L_s}{L_o} = \frac{I_s\,\omega_s}{I_o\,\omega_o} = \frac{I_s}{I_o} = \frac{\frac{2}{5}mr^2}{mR^2} = \frac{0.4\times r^2}{R^2} = \frac{0.4\times(1.74\times10^{6})^2}{(3.85\times10^{8})^2} = 8.2 \times 10^{-6}$

Q # 9. The earth rotates on its axis once a day. Suppose by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then?

Given Data: Let Initial Radius $r_i = R$, Final Radius $r_f = \frac{R}{2}$, Initial Period $T_i = 24\ h$	
To Determine: Final Period $T_f = ?$	
Calculations: <u>By Law of Conservation of Momentum</u>: $I_i\omega_i = I_f\omega_f \Rightarrow \left(\frac{2}{5}mr_i^2\right)\left(\frac{2\pi}{T_i}\right) = \left(\frac{2}{5}mr_f^2\right)\left(\frac{2\pi}{T_f}\right)$ $\Rightarrow \frac{r_i^2}{T_i} = \frac{r_f^2}{T_f} \Rightarrow \frac{R^2}{24} = \frac{\left(R/2\right)^2}{T_f} \Rightarrow \frac{R^2}{24} = \frac{R^2}{4\,T_f} \Rightarrow \frac{1}{6} = \frac{1}{T_f} \Rightarrow T_f = 6\ h$	

Q # 10. What should be the orbiting speed to launch a satellite in a circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as $6 \times 10^{24}\ kg$ and its radius as 6400 km).

Given Data: Height of Satellite above surface of earth $h = 900\ km = 900000\ m$ Radius of earth $R = 6400\ km = 6400000\ m$, Mass of Earth $M = 6 \times 10^{24}\ kg$	
To Determine: *Orbital Speed* $v = ?$	
Calculations: As $v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{6400000+900000}} = \sqrt{\frac{6.67\times10^{-11}\times6\times10^{24}}{7300000}} = 7400\ ms^{-1}$	



NUMERICAL PROBLEMS

F.Sc. Physics, Chapter # 6: FLUID DYNAMICS

Q # 1. Certain globular protein particle has a density of $1246\ \text{kg m}^{-3}$. It fall through pure water ($\eta = 8 \times 10^{-4}\ \text{Nm}^{-2}\text{s}$) with a terminal speed of $3\ \text{cm h}^{-1}$. Find the radius of the particle.

Given Data: $\rho = 1246\ \text{kg m}^{-3}, \eta = 8 \times 10^{-4}\ \text{Nm}^{-2}\text{s},\ \ v_t = 3\ \frac{\text{cm}}{\text{h}} = \frac{3\times10^{-2}\ \text{m}}{3600\ \text{s}} = 8.3 \times 10^{-6}\ \text{ms}^{-1}$
To Determine: Radius r =?
Calculations: As $v_t = \frac{2g\rho r^2}{9\eta} \Rightarrow r = \sqrt{\frac{9\eta v_t}{2g\rho}} = \sqrt{\frac{9\times8\times10^{-4}\times8.3\times10^{-6}}{2\times9.8\times1246}} = 5 \times 10^{-5}\ \text{m}$

Q # 2. Water flows through a hose, whose internal diameter is 1 cm at a speed of 1 ms^{-1}. What should be the diameter of the nozzle if the water is to emerge at $21\ \text{ms}^{-1}$?

Given Data: Diameter of hose $d_1 = 1\ \text{cm} = 10^{-2}\ \text{m}$, Velocity in hose $v_1 = 1\ \text{ms}^{-1}$ Velocity in nozzle $v_2 = 21\ \text{ms}^{-1}$
To Determine: Diameter of hose d_2 =?
Calculations: By equation of continuity: $A_1 v_1 = A_2 v_2 \Rightarrow \frac{\pi(d_1)^2 v_1}{4} = \frac{\pi(d_2)^2 v_2}{4} \Rightarrow d_2 = \frac{(d_1)^2 v_1}{v_2}$ $\Rightarrow d_2 = \frac{(10^{-2})^2 \times 1}{21} = 0.2\ \times 10^{-2} = 0.2\ \text{cm}$

Q # 3. The pipe near the lower end of a large water storage tank develops a small leak and a stream of water shoots from it. The top of water in the tank is 15 m above the point of leak.

i) With what speed does the water rush from the hole?

ii) If the hole has an area of $0.060\ \text{cm}^2$, how much water flows out in one second?

Given Data: Height of the top of water $h = 15\ \text{m}$, Area of hole $A = 0.060\ \text{cm}^2 = 6\ \times 10^{-6}\text{m}^2$
To Determine: Speed of Water v =?, Volume Flow Rate =?
Calculations: By Torricelli Theorem: $v = \sqrt{2g(h_2 - h_1)} = \sqrt{2 \times 9.8 \times 15} = 17.15 \approx 17\ \text{ms}^{-1}$ By Equation of Continuity: Flow Rate $= A \times v = 6\ \times 10^{-6} \times 17 = 102\ \text{m}^3\text{s}^{-1}$ Thus Volume Flow per Second $= 102\ \text{m}^3$

Q # 4. Water is flowing smoothly through a closed pipe system. At one point the speed of water is 3 ms^{-1}, while at another point 3 m higher, the speed is 4 ms^{-1}. If the pressure is 80 kPa at the lower point, what is pressure at the upper point?

Given Data: For Point 1: Velocity $v_1 = 3\ ms^{-1}$, Let Height $h_1 = 0$ m, Pressure $P_1 = 80$ kPa $= 80000$ Pa

For Point 2: Velocity $v_2 = 4\ ms^{-1}$, Height $h_2 = 3$ m

To Determine: Pressure at Point 2 $P_2 = ?$

Calculations: By Bernoulli's Equation: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$$\Rightarrow P_2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 - \frac{1}{2}\rho v_2^2 - \rho g h_2$$

$$= 80000 + \frac{1}{2} \times 1000 \times (3)^2 + 1000 \times 9.8 \times 0 - \frac{1}{2} \times 1000 \times (4)^2 - 1000 \times 9.8 \times 3 = 47000\ \text{Pa} = 47\ \text{k Pa}$$

Q # 5. An airplane wing is designed so that when the speed of the air across the top of the wing is 450 ms^{-1}, the speed of air below the wing is 410 ms^{-1}. What is pressure difference between the top and bottom of the wing? (Density of air = 1.29 kg m^{-3})

Given Data: Speed of air above the wing $v_1 = 450\ ms^{-1}$, Speed of air above the wing $v_2 = 410\ ms^{-1}$

Density of Air $\rho = 1.29\ kg\ m^{-3}$, For Present Case: $h_1 \approx h_2$

To Determine: Pressure Difference $P_2 - P_1 = ?$

Calculations: By Bernoulli's Equation: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2 \Rightarrow P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$$\Rightarrow P_2 - P_1 = \frac{1}{2}\rho v_1^2 - \frac{1}{2}\rho v_2^2 = \frac{1}{2}\rho(v_1^2 - v_2^2) = \frac{1}{2} \times 1.29 \times (450^2 - 410^2) = 22000\ \text{Pa} = 22\ \text{kPa}$$

Q # 6. The radius of the aorta is about 1 cm and blood flowing and blood flowing through it has a speed of about 30 cms^{-1}. Calculate the average speed of the blood in the capillaries using the fact that although each capillary has diameter of about 8×10^{-4} cm, there are literally millions of them so that their total cross section is about 2000 cm^2.

Given Data: Radius of aorta $r_1 = 1\ cm = 10^{-2}$ m, Velocity in aorta $v_1 = 30\ cms^{-1} = 30 \times 10^{-2}\ ms^{-1}$

Total Cross − sectional Area of all Capillaries $A_2 = 2000\ cm^2 = 2000 \times 10^{-4}\ m^2 = 0.2\ m^2$

To Determine: Average speed of blood in Capillaries $v_2 = ?$

Calculations: By equation of continuity: $A_1 v_1 = A_2 v_2 \Rightarrow \pi(r_1)^2 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{\pi(r_1)^2 v_1}{A_2}$

$$\Rightarrow v_2 = \frac{\pi(r_1)^2 v_1}{A_2} = \frac{3.14 \times (10^{-2})^2 \times 30 \times 10^{-2}}{0.2} = 4.7 \times 10^{-4} \approx 5 \times 10^{-4}\ ms^{-1}$$

Q # 7. How large must a heating duct be if air moving $3\ \text{ms}^{-1}$ along it can replenish the air in a room of $300\ \text{m}^3$ volume every 15 min? Assume the air's density remains constant.

Given Data: Speed of air $v = 3\ \text{ms}^{-1}$, Volume $V = 300\ \text{m}^3$, Time $t = 15\ \text{min} = 15 \times 60\ \text{s} = 900\ \text{s}$

To Determine: Radius of Duct $r = ?$

Calculations: By equ of continuity: $A \times v = \frac{V}{t} \Rightarrow \pi r^2 \times v = \frac{V}{t} \Rightarrow r = \sqrt{\frac{V}{\pi v t}} = \sqrt{\frac{300}{3.14 \times 3 \times 900}} = 0.19\ \text{m}$

Q # 8. An airplane design call for a "lift" due to the net force of the moving air on the wing of about $1000\ \text{Nm}^{-2}$ of wing area. Assume that air flows past the wing of an aircraft with streamline flow. If the speed of flow past the lower wing surface is $160\ \text{ms}^{-1}$, what is the required speed over the upper surface to give a "lift" of $1000\ \text{Nm}^{-2}$? The density of air is $1.29\ \text{kg m}^{-3}$ and assume maximum thickness of wing to be one meter.

Given Data: Pressure Difference between wings $P_2 - P_1 = 1000\ \text{Nm}^{-2}$, Speed below wing $v_2 = 160\ \text{ms}^{-1}$

Density of Air $\rho = 1.29\ \text{kg m}^{-3}$, Thickness of Wings $h_1 - h_2 = 1\ \text{m}$

To Determine: Speed of air above wings $v_1 = ?$

Calculations: By Bernoulli's Equation: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$$\Rightarrow v_1^2 = \frac{2}{\rho}\left(P_2 - P_1 + \frac{1}{2}\rho v_2^2 + \rho g h_2 - \rho g h_1\right) \Rightarrow v_1 = \sqrt{\frac{2}{\rho}\left(P_2 - P_1 + \frac{1}{2}\rho v_2^2 - \rho g (h_1 - h_2)\right)}$$

$$\Rightarrow v_1 = \sqrt{\frac{2}{1.29}\left(1000 + \frac{1}{2} \times 1.29 \times (160)^2 - 1.29 \times 9.8 \times 1\right)} = \sqrt{27130.78} = 164.71 \approx 165\ \text{ms}^{-1}$$

Q # 9. What gauge pressure is required in the city mains for a stream from a fire hose connected to the mains to reach a vertical height of 15 m?

Given Data: Height $h_1 - h_2 = 15\ \text{m}$, Consider $v_1 = v_2$

To Determine: Pressure Difference $(P_2 - P_1) = ?$

Calculations: By Bernoulli's Equation: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$$\Rightarrow P_1 + \rho g h_1 = P_2 + \rho g h_2 \Rightarrow P_2 - P_1 = \rho g h_1 - \rho g h_2 = \rho g (h_1 - h_2)$$

$$= 1.29 \times 9.8 \times 15 = 1.47 \times 10^5\ \text{Pa}$$

NUMERICAL PROBLEMS

F.Sc. Physics, Chapter # 7: OSCILLATION

Q # 1. A 100.0 g body hung on the spring elongates the spring by 4.0 cm. When a certain object is hung on the spring and set vibrating, its period is 0.568 s. What is the mass of the object pulling the spring?

Given Data:	Mass of First Object $m_1 = 100\text{ g} = 0.1\text{ kg}$, Elongation $x = 4\,cm = 0.04\,m$ Time Period $T = 0.568\text{ s}$
To Determine:	Mass of Second Object $m_2 = ?$, For m_2, we have to find the Spring Constant k
Calculations:	By Hook's Law $F = kx \Rightarrow m_1 g = kx \Rightarrow k = \frac{m_1 g}{x} = \frac{0.1\times 9.8}{0.04} = 24.5\text{ Nm}^{-1}$ As $T = 2\pi\sqrt{\frac{m_2}{k}} \Rightarrow T^2 = 4\pi^2\left(\frac{m_2}{k}\right) \Rightarrow m_2 = \frac{T^2\times k}{4\pi^2} = \frac{(0.568)^2\times 24.5}{4\times(3.14)^2} = 0.20\text{ kg}$

Q # 2. A load of 15.0 g elongates a spring by 2.0 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10.0 cm, what will be the (i) period (ii) Spring Constant (iii) maximum speed of its vibration.

Given Data:	Mass of First Object $m_1 = 15\text{ g} = 0.015\text{ kg}$, Elongation $x = 2\,cm = 0.02\,m$ Mass of Second Object $m_2 = 294\text{ g} = 0.294\text{ kg}$, Amplitude $x_0 = 10\text{ cm} = 0.01\text{ m}$
To Determine:	(i) Period $T = ?$ (ii) Spring Constant $k = ?$ (iii) Maximum Speed $v_0 = ?$
Calculations:	By Hook's Law (**ii**) $F = kx \Rightarrow m_1 g = kx \Rightarrow k = \frac{m_1 g}{x} = \frac{4\times 9.8}{0.02} = 7.35\text{ Nm}^{-1}$ (**i**) As $T = 2\pi\sqrt{\frac{m_2}{k}} = 2\times 3.14\times\sqrt{\frac{0.294}{7.35}} = 1.26\text{ s}$ (**iii**) As Maximum Speed $v_0 = x_0\sqrt{\frac{k}{m_2}} = 0.01\times\sqrt{\frac{7.35}{0.294}} = 0.05\text{ ms}^{-1}$

Q # 3. An 8.0 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find (i) Period (ii) Acceleration, speed, kinetic energy and potential energy when the displacement is 12 cm.

Given Data: Restoring Force $|F_r| = 60\text{ N}$, when Displacement $x_1 = 30\text{ cm} = 0.3\text{ m}$

Mass $m = 8\text{ kg}$, Amplitude $x_0 = 30\text{ cm} = 0.3\text{ m}$

To Determine: (**i**) Time Period $T = ?$

(**ii**) Acceleration $a = ?$, Speed $v = ?$, $K.E. = ?$, $P.E. = ?$ when displacement $x = 12\text{ cm} = 0.12\text{ m}$

Calculations: (**i**) As <u>Restoring Force</u> $|F_r| = kx_1 \Longrightarrow k = \frac{|F_r|}{x_1} = \frac{60}{0.3} = 200\text{ Nm}^{-1}$

- Time Period $T = 2\pi\sqrt{\frac{m}{k}} = 2 \times 3.14 \times \sqrt{\frac{8}{200}} = 1.3\text{ s}$
- (**ii**) Acceleration $a = -\frac{k}{m}x = -\frac{200}{8} \times 0.12 = -3\text{ ms}^{-2}$
- Speed $v = \sqrt{\frac{k}{m}(x_0^2 - x^2)} = \sqrt{\frac{200}{8}[(0.3)^2 - (0.12)^2]} = 1.33\text{ ms}^{-1}$
- $K.E. = \frac{1}{2}k(x_0^2 - x^2) = \frac{1}{2} \times 200 \times [(0.3)^2 - (0.12)^2] = 7.6\text{ J}$
- $P.E. = \frac{1}{2}kx^2 = \frac{1}{2} \times 200 \times (0.12)^2 = 1.44\text{ J}$

Q # 4. A block of mass 4.0 kg is dropped from a height of 0.80 m on to a spring of spring constant $k = 1960\text{ Nm}^{-1}$. Find the maximum distance through which the spring will be compressed.

Given Data: Spring Constant $k = 1960\text{ Nm}^{-1}$, Mass $m = 4.0\text{ kg}$, Height $h = 0.80\text{ ms}^{-1}$

To Determine: Maximum Distance $x_0 = ?$,

Calculation: For Present Case: Elastic P.E. = Gravitational P.E. $\Longrightarrow \frac{1}{2}kx_0^2 = mgh$

$$\Longrightarrow \frac{1}{2}kx_0^2 = mgh \Longrightarrow x_0 = \frac{2mgh}{k} = \frac{2 \times 4 \times 9.8 \times 0.80}{1960} = 0.42\text{ m}$$

Q # 5. A simple pendulum is 50.0 cm long. What will be its frequency of vibration at the place where $g = 9.8\ ms^{-2}$?

Given Data: Length of Pendulum $l = 50\text{ cm} = 0.5\text{ m}$, Gravitational Acceleration $g = 9.8\text{ ms}^{-2}$

To Determine: Frequency $f = ?$

Calculations: For Simple Pendulum $f = \frac{1}{T} = \frac{1}{\left(2\pi\sqrt{\frac{l}{g}}\right)} = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\times3.14} \times \sqrt{\frac{9.8}{0.5}} = 0.51\text{ Hz}$

Q # 6. A block of mass 1.6 kg is attached to a spring with spring constant $1000\ \text{Nm}^{-1}$. The spring is compressed through a distance of 2.0 cm and the block is released from rest. Calculate the velocity of the block as it passes through equilibrium position, $x = 0$, if the surface is frictionless.

Given Data: Spring Constant $k = 1000\ \text{Nm}^{-1}$, Mass $m = 1.6$ kg, Amplitude $x_0 = 2$ cm $= 0.02$ m
To Determine: Maximum Velocity $v_0 = ?$,
Calculation: $v_0 = x_0\sqrt{\frac{k}{m}} = 0.02 \times \sqrt{\frac{1000}{1.6}} = 0.5\ \text{ms}^{-1}$

Q # 7. A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant $20000\ \text{Nm}^{-1}$. If two people are riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

Given Data: Mass of Car $m_1 = 1300$ kg, Mass of Two Persons $m_2 = 160$ kg, Total Mass $m = m_1 + m_2 = 1300 + 1600 = 1460$ kg Spring Constant of One Spring $k' = 20000\ \text{Nm}^{-1}$, Total Spring Constant $k = 4k' = 8000\ \text{Nm}^{-1}$
To Determine: Frequency $f = ?$,
Calculation: As $f = \frac{1}{T} = \frac{1}{\left(2\pi\sqrt{\frac{m}{k}}\right)} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\times3.14} \times \sqrt{\frac{8000}{1460}} = 1.18$ Hz

Q # 8. Find the amplitude, frequency and period of an object vibrating at the end of a spring, if the equation from its position, as a function of time is $x = 0.25\ \cos\left(\frac{\pi}{8}\right)t$. What is the displacement of the object after 2.0 s?

Given Data: Equation of Motion of body executing SHM $x = 0.25\ \cos\left(\frac{\pi}{8}\right)t$ $---(1)$
To Determine: Amplitude $x_0 = ?$, Frequency $f = ?$, Time Period $T = ?$ Displacement $x = ?$ at $t = 2s$
Calculation: General Equation of Motion of body executing SHM $x = x_0 \cos \omega t$ $---(2)$ **(i)** *<u>Comparing (1) and (2)</u>*: $x_0 = 0.25$ $\omega = \frac{\pi}{8} \Rightarrow 2\pi f = \frac{\pi}{8} \Rightarrow f = \frac{1}{16}$ Hz $T = \frac{1}{f} = \frac{1}{(1/16)} = 16$ s **(ii)** *<u>Putting t=2s in equation (1)</u>*: $x = 0.25\ \cos\left[\left(\frac{\pi}{8}\right)\times 2\right] = 0.18$ m

NUMERICAL PROBLEM

F.Sc. Physics, Chapter # 8: WAVES

Q # 1. The wavelength of the signals from a radio transmitter is 1500 m and the frequency is 200 kHz. What is the wavelength for a transmitter operating at 1000 kHz and with what speed the radio waves travel?

Given Data: Case # 1 Wavelength $\lambda_1 = 1500\ m$, Frequency $f_1 = 200\ kHz = 200 \times 10^3\ Hz$

Case # 2 Frequency $f_2 = 100\ kHz = 1000 \times 10^3\ Hz$

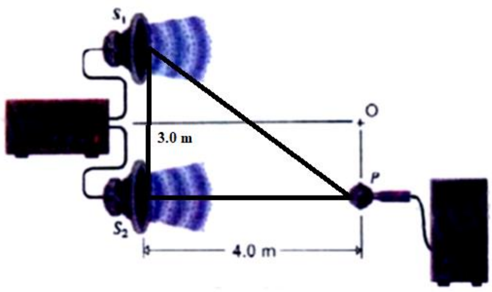
To Determine: (a) Velocity $v = ?$, **(b)** Wavelength $\lambda_2 = ?$

Calculations: (a) For Case # 1: $v = f_1\lambda_1 = 200 \times 10^3 \times 1500 = 3 \times 10^8 \frac{m}{s} \quad ----(1)$

(b) For Case # 2: $v = f_2\lambda_2 = 1000 \times 10^3 \lambda_2 \quad ----(2)$

Comparing (1) and (2): $1000 \times 10^3 \lambda_2 = 3 \times 10^8 \Longrightarrow \lambda_2 = 3 \times 10^2 = 300\ m$

Q # 2. Two speakers are arranged as shown in Fig. The distance between them is 3 m and they emit a constant tone of 344 Hz. A microphone P is moved along a line parallel to and 4.00 m from the line connecting the two speakers. It is found that tone of maximum loudness is heard and displayed on the CRO when microphone is on the center of the line and directly opposite each speakers. Calculate the speed of sound.



Given Data: Distance between speakers $S_1S_2 = 3\ m$, Frequency $f = 344\ Hz$,

Distance between speakers and microphone $S_2P = 4\ m$

To Determine: Speed of sound $v = ?$

Calculations: As $v = f\lambda \quad ---- \quad (1)$, Here $\lambda = ?$

From condition of constructive interference: Path Difference = Integral Multiple of λ

$\Longrightarrow |S_1P| - |S_2P| = \lambda \quad ---- \quad (2)$, For First Maxima $n = 1$, Here $S_1P = ?$

In right triangle S_1S_2P: $|S_1P|^2 = |S_1S_2|^2 + |S_2P|^2 \Longrightarrow |S_1P| = \sqrt{|S_1S_2|^2 + |S_2P|^2}$

$\Longrightarrow |S_1P| = \sqrt{(3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

Equation (2) becomes: $\lambda = |S_1P| - |S_2P| = 5 - 4 = 1\ m$

Putting values in (1): $v = f\lambda = 344 \times 1 = 344\ ms^{-1}$

Q # 3. A stationary wave is established in a string which is 120 cm long and fixed at both ends. The string vibrates in four segments, at a frequency of 120 Hz. Determine its wavelength and the fundamental frequency?

Given Data: Length of String $l = 120\ cm = 1.2\ m$, No. of loops $n = 4$ Frequency of Stationary waves with 4 segments $f_4 = 120\ Hz$
To Determine: (a) Wavelength with 4 segments $\lambda_4 = ?$, **(b)** Fundamental Frequency $f_1 = ?$
Calculations: (a) From General Equation of Wavelength of Stationary Waves: $\lambda_n = \frac{2l}{n}$ For present case: $\lambda_4 = \frac{2l}{4} = \frac{2\times1.2}{4} = 0.6\ m$ **(b)** As for Stationary Waves $f_n = nf_1$ For Present Case: $f_4 = 4f_1 \Rightarrow f_1 = \frac{f_4}{4} = \frac{120}{4} = 30\ Hz$

Q # 4. The frequency of the note emitted by a stretched string is 300 Hz. What will be the frequency of this note when; (a) the length of the wave is reduced by one-third without changing the tension, (b) the tension is increased by one-third without changing the length of the wire.

Given Data: Frequency $f = 300\ Hz$
To Determine: (a) <u>**Case 1**</u>Frequency $f' = ?$, When wavelength is reduced by one-third **(b)** <u>**Case 2**</u> Frequency $f'' = ?$, When tension is increased by one-third
Calculations: (a) When wavelength is reduced by one-third, then $\lambda' = \lambda - \frac{\lambda}{3} = \frac{2\lambda}{3}$ As tension in string is constant, so velocity will remain constant. Therefore, $v = v' \Rightarrow f\lambda = f'\lambda'$ $\Rightarrow f\lambda = f'\left(\frac{2\lambda}{3}\right) \Rightarrow f = \frac{2}{3}f' \Rightarrow f' = \frac{3}{2}f = 1.5\times300 = 450\ Hz$ **(b)** Initial Frequency of Stationary Waves: $f = \frac{1}{2l}\sqrt{\frac{F}{m}} \quad ---- \quad (1)$ When tension is increased by one-third, then new tension $F'' = F + \frac{F}{3} = \frac{4f}{3}$ Frequency of Wave for <u>case 2</u>: $f'' = \frac{1}{2l}\sqrt{\frac{F''}{m}} = \frac{1}{2l}\sqrt{\frac{\left(\frac{4f}{3}\right)}{m}} = \frac{1}{2l}\sqrt{\frac{4f}{3m}} \quad ---- \quad (2)$ Dividing Eq (1) and (2): $\frac{f''}{f} = \frac{\left(\frac{1}{2l}\sqrt{\frac{4f}{3m}}\right)}{\left(\frac{1}{2l}\sqrt{\frac{F}{m}}\right)} \Rightarrow \frac{f''}{f} = \sqrt{\frac{4}{3}} \Rightarrow f'' = \sqrt{\frac{4}{3}}f \Rightarrow f'' = \sqrt{\frac{4}{3}}\times300 = 346\ Hz$

Q # 5. An organ pipe has a length of 50 cm. Find the frequency of its fundamental note and the next harmonic when it is (a) open at both ends, (b) closed at one end.

Given Data: Length of organ pipe $l = 50\ cm = 0.5\ m$, Speed of Sound $v = 350\frac{m}{s}$
To Determine: (a) For Open Organ Pipe: Fundamental Frequency $f_1 =?$ and $f_2 =?$ **(b)** For Close Organ Pipe: Fundamental Frequency $f_1' =?$ and $f_2' =?$
Calculations: (a) For Open Organ Pipe: Fundamental Frequency $f_1 = \frac{v}{2l} = \frac{350}{2\times0.5} = 350\ Hz$ $f_2 = 2f_1 = 2 \times 350 = 700\ Hz$ **(b)** For Close Organ Pipe: Fundamental Frequency $f_1' = \frac{v}{4l} = \frac{350}{4\times0.5} = 175\ Hz$ $f_2' = 3f_1' = 3 \times 175 = 525\ Hz$

Q # 6. A church organ consists of pipes, each open at one end, of different lengths. The minimum length is 30 mm and the longest is 4 m. Calculate the frequency range of the fundamental notes. (Speed of sound = 340 ms^{-1})

Given Data: Speed of Sound $v = 340\frac{m}{s}$, **(a)** Minimum Length $l = 30\ mm = 0.03\ m$, **(b)** Maximum Length $l' = 4\ m$
To Determine: Frequency Range
Calculations: (a) Fundamental Frequency for Length l: $f_1 = \frac{v}{4l} = \frac{340}{4\times0.03} = 2833\ Hz$ **(b)** Fundamental Frequency for Length l': $f_1' = \frac{v}{4l'} = \frac{340}{4\times4} = 21\ Hz$ So the Frequency Range is from 21 Hz to 2833 Hz

Q # 7. Two tuning forks exhibit beats at a beat frequency of 3 Hz. The frequency of one fork is 256 Hz. Its frequency is then lowered slightly by adding a bit of wax to one of its prong. The two forks then exhibit a beat frequency of 1 Hz. Determine the frequency of the second tuning fork.

Given Data: (a) Let $f_B = 256\ Hz$, Without Wax: $f_{beat} = 3\ Hz \Rightarrow f_A - f_B = \pm3\ ---\ (1)$, **(b)** Without Wax: $f'_{beat} = 1\ Hz \Rightarrow f_A - f_B' = \pm1\ ---\ (2)$
To Determine: $f_A =?$
Calculations: Eq (1) can be written as: $f_A - 256 = \pm3 \Rightarrow f_A = 256 \pm 3 = 259, 253$ So Possible Unknown Frequency might be 259 Hz or 253 Hz. We Check both cases. **For $f_A = 259$**, Eq (2) becomes: $f_A - f_B' = \pm1 \Rightarrow f_B' = f_A - (\pm1) = 259 - (\pm1) = 258\ \&\ 260$ Increase of frequency f_B' of Fork loaded with Wax is **Not possible** **For $f_A = 253$**, Eq (2) becomes: $f_A - f_B' = \pm1 \Rightarrow f_B' = f_A - (\pm1) = 253 - (\pm1) = 252\ \&\ 254$ (Decreasing Trend) So Unknown Frequency $f_A = 253\ Hz$

Q # 8. Two cars P and Q are travelling along a motorway in the same direction. The leading car P travels at a steady speed of 12 ms^{-1}; the other car Q, travelling at a steady speed of 20 ms^{-1}, sound its horn to emit a steady note which P's driver estimates, has a frequency of 830 Hz. What frequency does Q's own driver hear?

(Speed of sound = 340 ms^{-1})

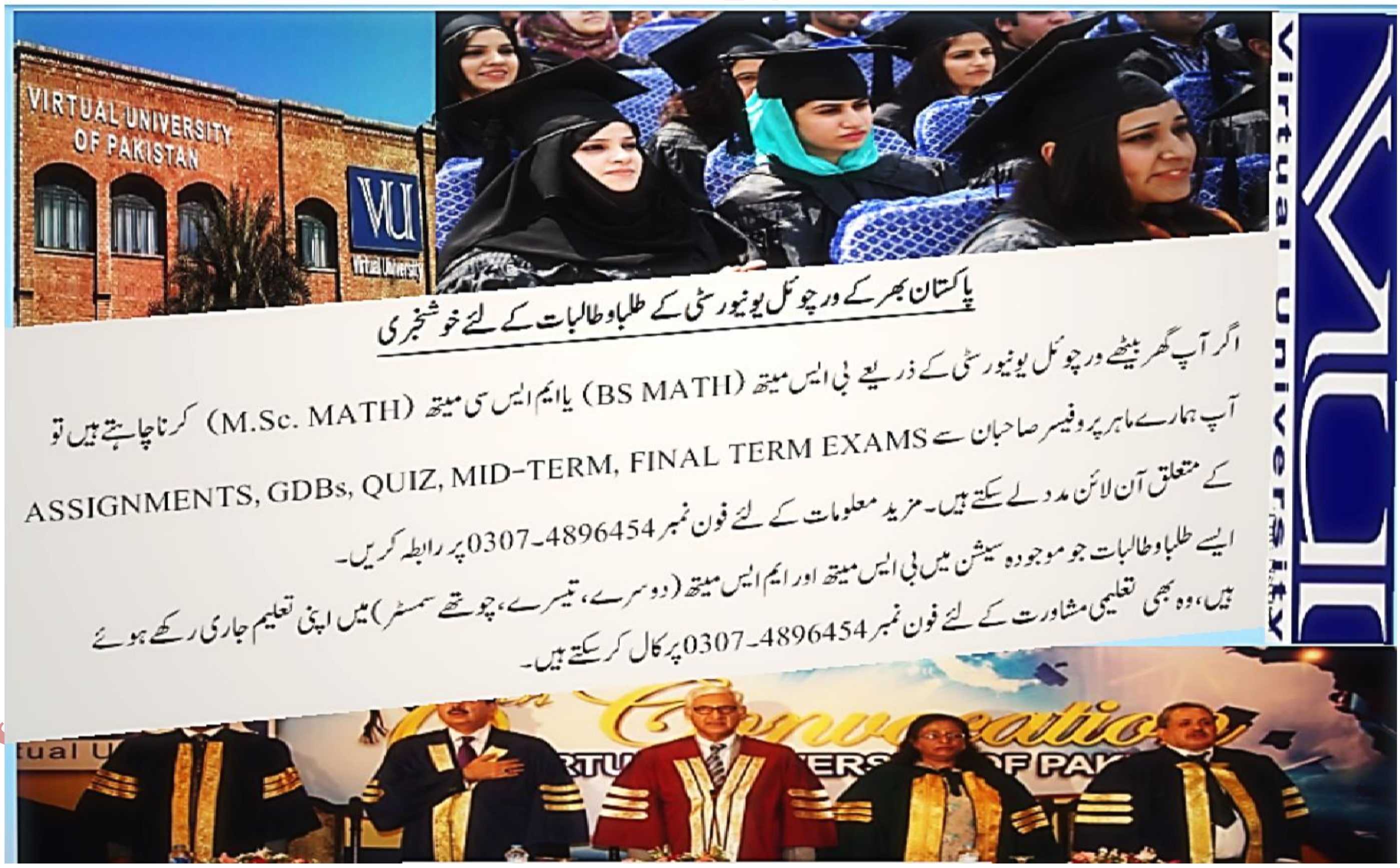
Given Data: Speed of Car P: $v_P = 12\,ms^{-1}$, Speed of Car Q: $v_Q = 20\,ms^{-1}$, Apparent Frequency: $f' = 830\,Hz$, Speed of Sound $v = 340\,\frac{m}{s}$
To Determine: Frequency emitted by Q: $f_Q = ?$
Calculations: Relative Velocity $v_S = v_Q - v_P = 20 - 12 = 8\,ms^{-1}$ As source is approaching observer, therefore $f' = \left(\frac{v}{v - v_S}\right) f_Q \Rightarrow f_Q = \left(\frac{v - v_S}{v}\right) f'$ $\Rightarrow f_Q = \left(\frac{340 - 8}{340}\right) \times 830 = 810\,Hz$

Q # 9. A train sounds its horn before it sets off from the station and an observer waiting on the platform estimates its frequency at 1200 Hz. The train then moves off and accelerates steadily. Fifty seconds after departure, the driver sounds the horn again and the platform observer estimates the frequency at 1140 Hz. Calculate the train speed 50 s after departure. How far from the station is the train after 50 s? (Speed of sound = 340 ms^{-1})

Given Data: Speed of Sound $v = 340\,\frac{m}{s}$, Time $t = 50\,s$, Initial Velocity $v_i = 0\,ms^{-1}$ Frequency when Source is at rest $f = 1200\,Hz$, Apparent Frequency $f' = 1140\,Hz$
To Determine: (a) Speed of Source after 50 s: $v_S = ?$, **(b)** Distance Covered $d = ?$
Calculations: (a) As source is moving away from observer, therefore: $f' = \left(\frac{v}{v + v_S}\right) f$ $\Rightarrow v + v_S = \left(\frac{v}{f'}\right) f \Rightarrow v_S = \left(\frac{v}{f'}\right) f - v = \left(\frac{340}{1140}\right) \times 1200 - 340 = 17.9\,ms^{-1}$ **(b)** Average Velocity $v_{av} = \frac{v_i + v_S}{2} = \frac{0 + 17.9}{2} = 8.95\,ms^{-1}$ Distance Covered $d = v_{av} \times t = 8.95 \times 50 = 447.5\,m \approx 448\,m$

Q # 10. The absorption spectrum of faint galaxy is measured and the wavelength of one of the lines identified as the Calcium α-line is found to be 478 nm. The same line has a wavelength of 397 nm when measured in a laboratory. (a) Is the galaxy moving towards or away from the Earth? (b) Calculate the speed of the galaxy relative to Earth. (Speed of light = 3.0 x10^{8} ms^{-1})

Given Data: Apparent Wavelength $\lambda' = 478\, nm = 478 \times 10^{-9} m$, Actual Wavelength $\lambda = 397\, nm = 397 \times 10^{-9} m$, Speed of light $v = c = 3 \times 10^{8} ms^{-1}$
To Determine: (a) Is the galaxy moving towards or away from the Earth? **(b)** Speed of the galaxy relative to Earth $v_S = ?$
Calculations: (a) Actual Frequency $f = \frac{v}{\lambda} = \frac{3\times10^{8}}{397\times10^{-9}} = 7.56 \times 10^{14}\, Hz$ Apparent Frequency $f' = \frac{v}{\lambda'} = \frac{3\times10^{8}}{478\times10^{-9}} = 6.28 \times 10^{14}\, Hz$ As $f' < f$, so galaxy is moving away from earth   **(b)** As source (galaxy) is moving away from earth, therefore: $f' = \left(\frac{v}{v+v_S}\right) f$ $\Rightarrow v + v_S = \left(\frac{v}{f'}\right) f \Rightarrow v_S = \left(\frac{3\times10^{8}}{6.28\times10^{14}}\right) \times 7.56 \times 10^{14} - 3 \times 10^{8}$ $\Rightarrow v_S = 3.611 \times 10^{8} - 3 \times 10^{8} = 0.611 \times 10^{8} ms^{-1} = 6.11 \times 10^{7} ms^{-1}$



NUMERICAL PROBLEM

F.Sc. Physics, Chapter # 9: PHYSICAL OPTICS

Q # 1: Light of wavelength 546 nm is allowed to illuminate the slits of Young's experiment. The separation between the slits is 0.10 mm and the distance of the screen from the slits where interference effects are observed is 20 cm. at what angle the first minimum will fall? What will be the linear distance on the screen between adjacent maxima?

Given Data: Wavelength $\lambda = 546\ nm = 546 \times 10^{-9}\ m$, Distance between Slits $d = 0.10\ mm = 10^{-4}\ m$, Inter-Screen Distance $L = 20\ cm = 0.20\ m$, $For\ first\ Dark\ Fringe \quad m = 0$

To Determine: **(i)** Angle for first minima $\theta = ?$, **(ii)** Fringe Width $\Delta y = ?$

Calculations: (i) For dark fringe $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \Rightarrow \sin\theta = \left(0 + \frac{1}{2}\right)\frac{\lambda}{d} = 0.5 \times \frac{546\times10^{-9}}{10^{-4}} = 0.00273$

$\Rightarrow \theta = \sin^{-1}(0.00273) = 0.16°$

(ii) Fringe Width $\Delta y = \frac{\lambda L}{d} = \frac{546\times10^{-9}\times0.20}{10^{-4}} = 1.1 \times 10^{-3}\ m = 1.1\ mm$

Q # 2: Calculate the wavelength of light, which illuminates two slits 0.5 mm apart and produces an interference pattern on a screen placed 200 cm away from the slits. The first bright fringe is observed at a distance of 2.40 mm from the central bright image.

Given Data: Distance between Slits $d = 0.5\ mm = 0.5 \times 10^{-3}\ m$, Inter-Screen Distance $L = 200\ cm = 2\ m$

For the 1st bright fringe $m = 1$, Distance of 1st bright fringe from central maxima $y = 2.40\ mm = 2.40 \times 10^{-3}\ m$

To Determine: Wavelength $\lambda = ?$

Calculations: For dark fringe $y = m\frac{\lambda L}{d} \Rightarrow 2.40 \times 10^{-3} = 1 \times \frac{\lambda\times2}{0.5\times10^{-3}} \Rightarrow \lambda = 0.6 \times 10^{-6}\ m = 600\ nm$

Q # 3: In a double slit experiment the second order maximum occurs at $\theta = 0.25°$. The wavelength is 650 nm. Determine the slit separation.

Given Data: Wavelength $\lambda = 650\ nm = 650 \times 10^{-9}\ m$, $For\ second\ bright\ fringe \quad m = 2$, Angle $\theta = 0.16°$

To Determine: Slit Separation $d = ?$

Calculations: For dark fringe $d \sin\theta = m\lambda \Rightarrow d = \frac{m\lambda}{\sin\theta} \Rightarrow d = \frac{2\times650\times10^{-9}}{\sin(0.16°)} = 0.3 \times 10^{-3} = 0.3\ mm$

Q # 4: A monochromatic light of $\lambda = 588\ nm$ is allowed to fall on the half silvered glass plate G_1, in the Michelson Interferometer. If mirror M_1 is moved through 0.233 mm, how many fringes will be observed to shift?

Given Data: Wavelength $\lambda = 588\ nm = 588 \times 10^{-9}\ m$, Distance covered by mirror $L = 0.233 \times 10^{-3}\ m$

To Determine: Number of Shifted Fringes $m = ?$

Calculations: For Michelson's Interferometer $L = \frac{m\lambda}{2} \Rightarrow m = \frac{2L}{\lambda} = \frac{2 \times 0.233 \times 10^{-3}}{588 \times 10^{-9}} = 792$

Q # 5: A second order spectrum formed at an angle of $38.0°$ when light falls normally on diffraction grating having 5400 lines per centimeter determine wavelength of the light used.

Given Data: For 2nd order Spectrum $m = 2$, Angle of Diffraction $\theta = 38°$

Number of lines per cm $N = 5400\ \text{cm}^{-1} = 540000\ \text{m}^{-1}$, $d = \frac{1}{N} = \frac{1}{540000} = 1.852 \times 10^{-6}\ m$

To Determine: Wavelength $\lambda = ?$

Calculations: $d \sin\theta = m\lambda \Rightarrow \lambda = \frac{d \sin\theta}{m} = \frac{1.852 \times 10^{-6} \times \sin 38°}{2} = 5.70 \times 10^{-7} = 570\ nm$

Q # 6: A light is incident normally on a grating which has 2500 lines per centimeter. Compute the wavelength of spectral line for which the deviation in second order is $15.0°$.

Given Data: For 2nd order Spectrum $m = 2$, Angle of Diffraction $\theta = 15°$

Number of lines per cm $N = 2500\ \text{cm}^{-1} = 250000\ \text{m}^{-1}$, $d = \frac{1}{N} = \frac{1}{250000} = 4 \times 10^{-6}\ m$

To Determine: Wavelength $\lambda = ?$

Calculations: $d \sin\theta = m\lambda \Rightarrow \lambda = \frac{d \sin\theta}{m} = \frac{4 \times 10^{-6} \times \sin 15°}{2} = 5.18 \times 10^{-7} = 518\ nm$

Q # 7: Sodium light ($\lambda = 589\ nm$) is incident normally on a grating having 3000 lines per centimeter. What is the highest order of the spectrum obtained with this grating?

Given Data: Wavelength $\lambda = 589\ nm = 589 \times 10^{-9}\ m$ Angle $\theta = 90°$

Number of lines per cm $N = 3000\ \text{cm}^{-1} = 300000\ \text{m}^{-1}$, $d = \frac{1}{N} = \frac{1}{300000} = 3.33 \times 10^{-6}\ m$

To Determine: Order of Spectra $m = ?$

Calculations: $d \sin\theta = m\lambda \Rightarrow m = \frac{d \sin\theta}{\lambda} = \frac{3.33 \times 10^{-6} \times \sin 90°}{589 \times 10^{-9}} = 5.66$

Hence, the highest order of spectrum is 5

Q # 8: Blue light of wavelength 480 nm illuminates a diffraction grating. The second order image is formed at an angle of 30° from the central image. How many lines in a centimeter of the grating have been ruled out?

Given Data: Wavelength $\lambda = 480\ nm = 480 \times 10^{-9}\ m$ Angle $\theta = 30°$, Order of Spectra $m = 2$
To Determine: Number of lines per cm $N = ?$, $where\ \ N = \frac{1}{d}$
Calculations: $d\sin\theta = m\lambda \Longrightarrow \frac{1}{N}\sin\theta = m\lambda \Longrightarrow N = \frac{\sin\theta}{m\lambda} = \frac{\sin 30°}{2 \times 480 \times 10^{-9}} = 5.2\ \times 10^{5}\ m^{-1} = 5200\ cm^{-1}$ Hence, the highest order of spectrum is 5th

Q # 9: X-rays of wavelength 0.150 nm are observed to undergo a first order reflection at a Bragg's angle of 13.3° from a quartz (SiO_2) crystal. What is the inter-planner spacing of the reflecting planes in the crystal?

Given Data: Wavelength $\lambda = 0.15\ nm = 0.15 \times 10^{-9}\ m$ Angle $\theta = 13.3°$, Order of Spectra $m = 1$
To Determine: Inter-Planner Spacing $d = ?$
Calculations: $2d\sin\theta = m\lambda \Longrightarrow d = \frac{m\lambda}{2\sin\theta} = \frac{1 \times 0.15 \times 10^{-9}}{2 \times \sin 13.3°} = 0.326 \times 10^{-9} m = 0.326\ nm$

Q # 10: An X-ray beam of wavelength λ undergoes a first order reflection from a crystal when its angle of incidence to a crystal face is 26.5°, and an X-ray beam of wavelength 0.097 nm undergoes a third order reflection when its angle of incidence to that face is 60.0°. Assuming that the two beams reflect from the same family of planes, calculate (a) the interplaner spacing of the planes and (b) the wavelength λ

Given Data: For 1st X − ray Beam: Order of Reflection $m_1 = 1$, Angle of incidence $\theta_1 = 26.5°$, $wavelength\ \ \lambda_1 = \lambda$ For 2nd X − ray Beam: Order of Reflection $m_2 = 3$, Angle of incidence $\theta_2 = 60.0°$, $wavelength\ \ \lambda_2 = 0.097 \times 10^{-9}\ m$
To Determine: (i) Inter-Planner Spacing $d = ?$ **(ii)** Wavelength of 1st beam $\lambda = ?$
Calculations: For 2nd Beam $2d\sin 60° = 3 \times 0.097 \times 10^{-9} \Longrightarrow d = \frac{3 \times 0.097 \times 10^{-9}}{2\sin 60°} = 0.168 \times 10^{-9} m = 0.168\ nm$ **For 1st Beam** $2d\sin 26.5° = 1 \times \lambda \Longrightarrow \lambda = 2d\sin 26.5° = 2 \times 0.168 \times 10^{-9} \times \sin 26.5° = 0.150 \times 10^{-9} = 0.150\ nm$

NUMERICAL PROBLEMS

F.Sc. Physics, Chapter # 10: OPTICAL INSTRUMENTS

Q # 1. A converging lens of focal length 5.0 cm is used as a magnifying glass. If the near point of the observer is 25 cm and the lens is -held close to the eye, calculate (i) the distance of the object from the lens (ii) the angular magnification. What is the angular magnification when the final image is formed at infinity?

Given Data: Focal Length $f = 5\ cm$, Distance of near point $d = 25\ cm$

To Determine: (i) Distance of object from lens p =?, (ii) Angular Magnification M =?,
(iii) Angular Magnification when image formed at infinity M' =?

Calculations: (i) Lens formula for virtual image $\frac{1}{f} = \frac{1}{p} - \frac{1}{q}$, For present case: $q = d = 25\ cm$

$\Rightarrow \frac{1}{p} = \frac{1}{f} + \frac{1}{q} = \frac{1}{5} + \frac{1}{25} = \frac{5+1}{25} \Rightarrow \frac{1}{p} = \frac{6}{25} \Rightarrow p = \frac{25}{6} = 4.2\ cm$

(ii) For lens, Angular Magnification $M = 1 + \frac{d}{f} = 1 + \frac{25}{5} = 1 + 5 = 6$

(iii) The image formed at infinity when object is placed at focus i.e, $p = f$.

Therefore, $M' = \frac{d}{p} = \frac{d}{f} = \frac{25}{5} = 5$

Q # 2. A telescope objective has focal length 96 cm and diameter 12 cm. Calculate the focal length and minimum diameter of a simple eye piece lens for use with the telescope, if the linear magnification required is 24 times and all the light transmitted by the objective from a distant point on the telescope axis is to fall on the eye piece.

Given Data: Focal Length of Objective $f_o = 96\ cm$, Diameter of Objective $D_o = 12\ cm$,
Magnification $M = 24$

To Determine: (i) Focal Length of eye-piece f_e =?, (ii) Diameter of Eye-Piece D_e =?

Calculations: (i) As Magnification $M = \frac{f_o}{f_e} \Rightarrow f_e = \frac{f_o}{M} = \frac{96}{24} = 4$

(ii) As the ratio of diameters of lenses (Objective and Eye-piece) is equal to the focal lengths of lenses, therefore, $\frac{D_e}{D_o} = \frac{f_e}{f_o} \Rightarrow D_e = \frac{f_e}{f_o} \times D_o = \frac{4}{96} \times 12 = 0.5\ cm$

Q # 3. A telescope is made of an objective of focal length 20 cm and an eye piece of 5.0 cm, both convex lenses. Find the angular magnification.

Given Data: Focal Length of Objective $f_o = 20\ cm$, Focal Length of Eye-Piece $f_e = 5\ cm$

To Determine: Angular Magnification M =?

Calculations: As Magnification $M = \frac{f_o}{f_e} = \frac{20}{5} = 4$

Q # 4. A simple astronomical telescope in normal adjustment has an objective of focal length 100 cm and an eye piece of focal length 5.0 cm. (i) Where is the final image formed ? (ii) Calculate the angular magnification.

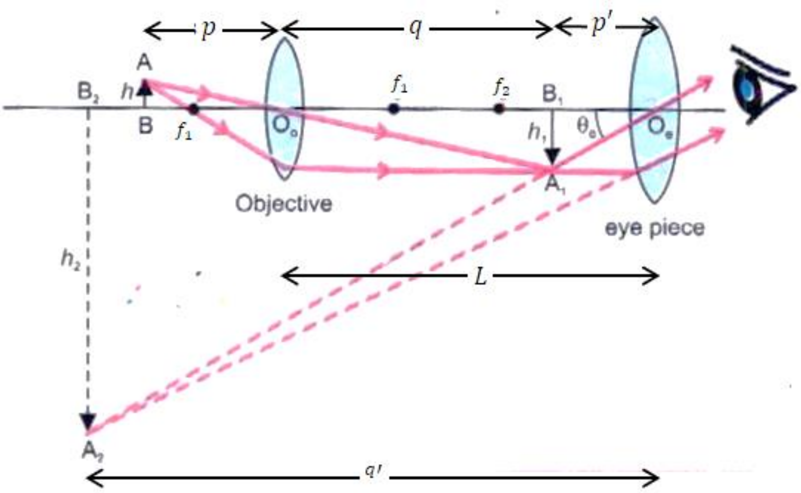
Given Data: Focal Length of Objective $f_o = 100\ cm$, Focal Length of Eye-Piece $f_e = 5\ cm$

To Determine: (i) Distance of Image $q = ?$, (ii) Angular Magnification $M = ?$

Calculations: (i) For Normal Adjustment, the image of OBJECTIVE will be formed at focus of EYE-PIECE. This image act as an object for EYE-PIECE. So image of EYE-PIECE will be formed at INFINITY i.e., $q = \infty$.

(ii) As Magnification $M = \frac{f_o}{f_e} = \frac{100}{5} = 20$

Q # 5 A point object is placed on the axis of and 3.6 cm from a thin convex lens of focal length 3.0 cm. A second thin convex lens of focal length 16.0 cm is placed coaxial with the first and 26.0 cm from it on the side away from the object. Find the position of the final image produced by the two lenses.



Given Data: Distance of Object for 1st lens $p = 3.6\ cm$, Focal Length of 1st Lens $f_1 = 3.0$ cm, Focal Length of 2nd Lens $f_2 = 16.0$ cm, Distance between Lenses $L = 26.0\ cm$

To Determine: Position of Final Image (image of 2nd lens) $q' = ?$

Calculations: **For Lens 1** $\frac{1}{f_1} = \frac{1}{p} + \frac{1}{q} \Rightarrow \frac{1}{q} = \frac{1}{f_1} - \frac{1}{p} = \frac{1}{3} - \frac{1}{3.6} = \frac{1}{3} - \frac{10}{36} = \frac{12-10}{36} = \frac{2}{36} = \frac{1}{18}$

$\Rightarrow \frac{1}{q} = \frac{1}{18} \Rightarrow q = 18\ cm$

The IMAGE of 1st lens will act as OBJECT for 2nd lens. If p' distance of image from 2nd lens, then $L = q + p' \Rightarrow p' = L - q = 26 - 18 = 8\ cm$

For Lens 2 $\frac{1}{f_2} = \frac{1}{p'} + \frac{1}{q'} \Rightarrow \frac{1}{q'} = \frac{1}{f_2} - \frac{1}{p'} = \frac{1}{16} - \frac{1}{8} = \frac{1-2}{16} = \frac{-1}{16}$

$\Rightarrow \frac{1}{q'} = \frac{-1}{16} \Rightarrow q' = -16\ cm$ (-ve sign show that image is virtual)

Q # 6. A compound microscope has lenses of focal length 1.0 cm and 3.0 cm. An object is placed 1.2 cm from the object lens. If a virtual image is formed, 25 cm from the eye, calculate the separation of the lenses and the magnification of the instrument.

Given Data: Focal Length of Objective $f_o = 1.0$ cm, Focal Length of Eyepiece $f_e = 3.0$ cm, Distance of Object for Objective $p = 1.2\ cm$, Distance of image from Eyepiece $q' = 25\ cm$

To Determine: (a) Separation of Lenses L =?, (b) Magnification of Compound Microscope M =?

Calculations: (a) If q and p' denote Distance of image from objective and Distance of image from eyepiece respectively, then $L = q + p' \quad ---- \quad (1)$

For Objective $\frac{1}{f_o} = \frac{1}{p} + \frac{1}{q} \Rightarrow \frac{1}{q} = \frac{1}{f_o} - \frac{1}{p} = \frac{1}{1} - \frac{1}{1.2} = 1 - \frac{10}{12} = \frac{12-10}{12} = \frac{2}{12} = \frac{1}{6}$

$\Rightarrow \frac{1}{q} = \frac{1}{6} \Rightarrow q = 6\ cm$

For Eyepiece $\frac{1}{f_e} = \frac{1}{p'} - \frac{1}{q'} \Rightarrow \frac{1}{p'} = \frac{1}{f_e} + \frac{1}{q'} = \frac{1}{3} + \frac{1}{25} = \frac{25+3}{75} = \frac{28}{75}$ (q' is negative due to virtual image)

$\Rightarrow \frac{1}{q} = \frac{28}{75} \Rightarrow q = \frac{75}{28} = 2.7\ cm$

Putting values in (1), we have: $L = 6 + 2.7 = 8.7\ cm$

(b) Magnification of Compound Microscope $M = \frac{q}{p}\left(1 + \frac{d}{f_e}\right) = \frac{6}{1.2}\left(1 + \frac{25}{3}\right) = \frac{60}{12}\left(\frac{3+25}{3}\right) = 5 \times \frac{28}{3} = 47$

Q # 7. Sodium light of wavelength 589 nm is used to view an object under a microscope. If the aperture of the objective is 0.90 cm, (i) find the limiting angle of resolution,(ii) using visible light of any wavelength, what is the maximum limit of resolution for this microscope.

Given Data: Wavelength $\lambda = 589\ nm = 589 \times 10^{-9}\ m$, Aperture of Objective $D = 0.90\ cm = 0.90 \times 10^{-2}\ m$

To Determine: (a) Limiting angle of resolution α =?, (b) Maximum Limit of resolution α' =?,

Calculations: (a) Limiting angle of resolution $\alpha = 1.22\frac{\lambda}{D} = 1.22 \times \frac{589\times10^{-9}}{0.90\times10^{-2}} = 8 \times 10^{-5}\ rad.$

(b)For Maximum Limit of Resolution, $\lambda' = 400\ nm = 400 \times 10^{-9}\ m$ (Minimum wavelength of visible spectrum)

Therefore, $\alpha' = 1.22\frac{\lambda'}{D} = 1.22 \times \frac{400\times10^{-9}}{0.90\times10^{-2}} = 5.4 \times 10^{-5}\ rad.$

Q # 8 An astronomical telescope having magnifying power of 5 consist of two thin lenses 24 cm apart. Find the focal lengths of the lenses.

Given Data: Magnifying power of astronomical telescope $M = 5$, Distance between lenses $L = 24\ cm$

To Determine: Focal Length of Lenses: Focal length of Eyepiece f_e =?, Focal length of Objective f_o =?

Calculations: For astronomical telescope: $L = f_e + f_o \quad ---- \quad (1)$

Also, Magnifying power of astronomical telescope $M = \frac{f_o}{f_e} \Rightarrow 5 = \frac{f_o}{f_e} \Rightarrow f_o = 5f_e \quad ---- \quad (2)$

Putting values in (1): $24 = f_e + 5f_e \Rightarrow 6f_e = 24 \Rightarrow f_e = 4\ cm$

Equation (2) becomes: $f_o = 5f_e = 5 \times 4 = 20\ cm$

Q # 9 A glass light pipe in air will totally internally reflect a light ray if its angle of incidence is at least 39°. What is the minimum angle for total internal reflection if pipe is in water? (Refractive Index of water = 1.33)

Given Data: Angle of incidence for total internal reflection for glass in air $\theta_c = 39°$,

Refractive Index of water $n_2 = 1.33$,

To Determine: Minimum Angle of incidence for total internal reflection for glass in water $\theta'_c = ?$

Calculations: Refractive Index of Glass $n_1 = \frac{1}{\sin\theta_c} = \frac{1}{\sin 39°} = 1.59$

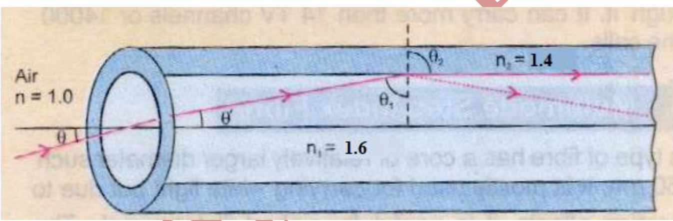
<u>**Snell's law For Glass-Water Interface**</u> $n_1 \sin\theta_1 = n_2 \sin\theta_2 \; ---- \quad (1)$

For Total Internal Reflection $\theta_2 = 90°$, Here $\theta_1 = \theta'_c$

Putting values in (1): $1.59 \times \sin\theta'_c = 1.33 \times \sin 90° \Rightarrow \sin\theta'_c = \frac{1.33 \times \sin 90°}{1.59} = \frac{1.33 \times 1}{1.59}$

$\Rightarrow \sin\theta'_c = \frac{1.33}{1.59} \Rightarrow \theta'_c = \sin^{-1}\left(\frac{1.33}{1.59}\right) = 57°$

Q # 10. In The refractive index of the core and cladding of an optical fibre are 1.6 and 1.4 respectively. Calculate (i) the critical angle for the interface (ii) the maximum angle of incidence in the air of a ray which enters the fibre and is incident at the critical angle on the interface.



Given Data: Refractive index of core $n_1 = 1.6$, Refractive index of cladding $n_2 = 1.4$

To Determine: (i) Critical Angle $\theta_c = ?$

(ii) Maximum Angle of incidence $\theta = ?$

Calculations: (i) For Core-Cladding Interface, Snell's Law $n_1 \sin\theta_1 = n_2 \sin\theta_2$, Here $\theta_1 = \theta_c$ and $\theta_2 = 90°$

Therefore, $1.6 \times \sin\theta_c = 1.4 \times \sin 90° \Rightarrow \sin\theta_c = \frac{1.4 \times \sin 90°}{1.6} = \frac{1.4}{1.6} \Rightarrow \theta_c = \sin^{-1}\left(\frac{1.4}{1.6}\right) = 61°$

(ii) From Figure, $\theta' = 90° - 61° = 29°$. For air $n = 1$

For Air-Core Interface, Snell's Law $n \sin\theta = n_1 \sin\theta' \Rightarrow \sin\theta = \frac{n_1 \sin\theta'}{n} = \frac{1.6 \times \sin 29°}{1} = 0.776$

$\Rightarrow \sin\theta = 0.776 \Rightarrow \theta = \sin^{-1}(0.776) = 51°$

NUMERICAL PROBLEM

F.Sc. Physics, Chapter # 11: THERMODYNAMICS

Q # 1. Estimate the average speed of nitrogen molecules in air under standard conditions of pressure and temperature.

Given Data: Under Standard Conditions: Temperature $T = 0°C = 0 + 273 = 273\ K$

To Determine: Average Speed of Nitrogen Molecules $\langle v \rangle = ?$

Calculations: $T = \frac{2}{3k}\langle K.E \rangle = \frac{2}{3k}\langle \frac{1}{2}mv^2 \rangle = \frac{m}{3k}\langle v^2 \rangle \Rightarrow \langle v^2 \rangle = \frac{3kT}{m} \Rightarrow \langle v \rangle = \sqrt{\frac{3kT}{m}} - - - (1)$

As Mass of Nitrogen Atom $m = \frac{\text{Molar Mass}}{N_A} = \frac{28\ g}{6.023\times10^{23}} = \frac{28\ kg}{6.023\times10^{26}} = 4.65 \times 10^{-26}\ kg$

Equation (1) becomes: $\langle v \rangle = \sqrt{\frac{3\times1.38\times10^{-23}\times273}{4.65\times10^{-26}}} = 493\ ms^{-1}$

Q # 2. Show that ratio of the root mean square speed of molecules of two different gases at certain temperature is equal to square root of the inverse ratio of their masses.

To Determine: Ratio of Root Mean Speed of Molecules of Different Gases $\frac{\langle v_1 \rangle}{\langle v_2 \rangle} = ?$

Calculations: Temperature of First Gas $T = \frac{2}{3k}\langle \frac{1}{2}m_1v_1^2 \rangle - - - (1)$

Temperature of Second Gas $T = \frac{2}{3k}\langle \frac{1}{2}m_2v_2^2 \rangle - - - (2)$

Equating Equation (1) and (2): $\frac{2}{3k}\langle \frac{1}{2}m_1v_1^2 \rangle = \frac{2}{3k}\langle \frac{1}{2}m_2v_2^2 \rangle \Rightarrow m_1\langle v_1^2 \rangle = m_2\langle v_2^2 \rangle$

$$\Rightarrow \frac{\langle v_1^2 \rangle}{\langle v_2^2 \rangle} = \frac{m_2}{m_1} \Rightarrow \frac{\langle v_1 \rangle}{\langle v_2 \rangle} = \sqrt{\frac{m_2}{m_1}}$$

Q # 3. A sample of the gas is compressed to one half of its initial volume at constant pressure of $1.25 \times 10^5\ Nm^{-2}$. During compression, 100 J of work is done on the gas. Determine the final volume of the gas.

Given Data: Pressure $P = 1.25 \times 10^5\ Nm^{-2}$, Work $W = -100\ J$ (Compression)

Initial Volume $V_i = V$, Final Volume $V_f = \frac{V}{2}$, Change in Volume $\Delta V = V_f - V_i = \frac{V}{2} - V = -\frac{V}{2}$

To Determine: Final Volume $V_f = \frac{V}{2} = ?$

Calculations: Work $W = P\,\Delta V \Rightarrow -100 = 1.25 \times 10^5 \times \left(-\frac{V}{2}\right) \Rightarrow V = \frac{2\times100}{1.25\times10^5} = 1.6 \times 10^{-3}\ m^3$

Now Final Volume $V_f = \frac{V}{2} = \frac{1.6\times10^{-3}}{2} = 8 \times 10^{-4}\ m^3$

Q # 4. A thermodynamic system undergoes a process in which its internal energy decreases by 300 J. If at the same time 120 J of work is done on the system, fine the heat lost by the system.

Given Data: Change in Internal Energy $\Delta U = -300\ \mathrm{J}$ (−ve sign indicate the decrease in Internal Energy) Work Done on the System $W = -120\ \mathrm{J}$	
To Determine: Heat Lost $Q = ?$	
Calculations: By First Law of Thermodynamics $Q = \Delta U + W = -300 - 120 = -420\ \mathrm{J}$	

Q # 5. A Carnot engine utilizes an ideal gas. The source temperature is 7°C , and the sink temperature is $127°\mathrm{C}$. Find the efficiency of the engine. Also find the heat input from source and heat rejected to sink when 10000 J of work is done.

Given Data: Temperature of Hot Reservoir $T_1 = 227°\mathrm{C} = 227 + 273 = 500\ \mathrm{K}$ Temperature of Cold Reservoir $T_2 = 127°\mathrm{C} = 127 + 273 = 400\ \mathrm{K}$, Work Done $W = 10000\ \mathrm{J}$	
To Determine: Efficiency $\eta = ?$, Heat Input $Q_1 = ?$, Heat Rejected $Q_2 = ?$	
Calculations: As $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{500} = 1 - 0.8 = 0.2 = 20\ \%$ Also $\eta = \frac{\mathrm{Output}}{\mathrm{Input}} = \frac{W}{Q_1} \Longrightarrow Q_1 = \frac{W}{\eta} = \frac{10000}{0.2} = 50000\ \mathrm{J}$ As $W = Q_1 - Q_2 \Longrightarrow Q_2 = Q_1 - W = 50000 - 10000 = 40000\ \mathrm{J}$	

Q # 6. A reversible engine works between two temperatures whose difference is 100°C. If it absorbs 746 J of heat from the source and rejects 546 J to the sink, calculate the temperature of the source and the sink.

Given Data: Temperature Difference $= T_1 - T_2 = 100°\mathrm{C} = 100\ \mathrm{K} \Longrightarrow T_1 = T_2 + 100\ \mathrm{K}\ \ ---(1)$ Absorbed Heat $Q_1 = 746\ \mathrm{J}$, Heat Rejected $Q_2 = 546\ \mathrm{J}$	
To Determine: Temperature of Source $T_1 = ?$, Temperature of Sink $T_2 = ?$	
Calculations: As $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{546}{746} = 0.268$ Also $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{T_2}{T_2+100} = \frac{T_2+100-T_2}{T_2+100} = \frac{100}{T_2+100} \Longrightarrow 0.268 = \frac{100}{T_2+100} \Longrightarrow T_2 + 100 = \frac{100}{0.268}$ $\Longrightarrow T_2 = \frac{100}{0.268} - 100 = 273\ \mathrm{K}$ *Putting Values in (1)*: $T_1 = 273 + 100 = 373\ K$	

Q # 7. A mechanical engineer develops an engine, working between 327°C and 27°C and claims to have an efficiency of 52%. Does he claim correctly? Explain.

Given Data: Temperature of Hot Reservoir $T_1 = 327°C = 327 + 273 = 600\text{ K}$

Temperature of Cold Reservoir $T_2 = 27°C = 27 + 273 = 300\text{ K}$

To Determine: Efficiency $\eta = ?$

Calculations: As $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = 1 - 0.5 = 0.5 = 50\ \%$

His claim is not correct.

Q # 8. A heat engine performs 100 J of work and at the same time rejects 400 J of heat energy to the cold reservoirs. What is the efficiency of the engine?

Given Data: Work Done $W = 100\text{ J}$, Heat Rejected $Q_2 = 400\text{ J}$, Let Heat Absorbed $= Q_1$

To Determine: Efficiency $\eta = ?$

Calculations: As $W = Q_1 - Q_2 \Longrightarrow Q_1 = W + Q_2 = 100 + 400 = 500\text{ J}$

Now $\eta = \frac{\text{Output}}{\text{Input}} = \frac{W}{Q_1} = \frac{100}{500} = 0.2 = 20\ \%$

Q # 9. A Carnot engine whose low temperature reservoir at 7°C has an efficiency of 50%. It is desired to increase the efficiency to 70%. By how many degrees the temperature of the source be increased?

Given Data: Temperature of Cold Reservoir $T_2 = 7°C = 7 + 273 = 280\text{ K}$

Let Temperature of Hot Reservoir for 50% Efficiency $= T_1$

And Temperature of Hot Reservoir for 70% Efficiency $= T'_1$

To Determine: Increase of Temperature of Hot Reserovoir $= T'_1 - T_1 = ?$

Calculations: For 50% Efficiency $\eta = 1 - \frac{T_2}{T_1} \Longrightarrow 0.5 = 1 - \frac{280}{T_1} \Longrightarrow \frac{280}{T_1} = 1 - 0.5 = 0.5 \Longrightarrow T_1 = \frac{280}{0.5} = 560\text{ K}$

For 70% Efficiency $\eta' = 1 - \frac{T_2}{T'_1} \Longrightarrow 0.7 = 1 - \frac{280}{T'_1} \Longrightarrow \frac{280}{T'_1} = 1 - 0.7 = 0.3 \Longrightarrow T'_1 = \frac{280}{0.3} = 933\text{ K}$

So Increase of Temperature of Hot Reserovoir $= T'_1 - T_1 = 933 - 560 = 373\text{ K}$

Q # 10. A steam engine has boiler that operates at 450 K. the heat changes water to steam, which drives the piston. The exhaust temperature of the outside air is about 300 K. What is the maximum efficiency of this steam engine?

Given Data: Temperature of Hot Reservoir $T_1 = 450\ \text{K}$ Temperature of Cold Reservoir $T_2 = 300\ \text{K}$
To Determine: Efficiency $\eta = ?$
Calculations: As $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{450} = 1 - 0.667 = 0.333 = 33.3\ \%$

Q # 11. 336 J of energy is required to melt 1 g of ice at ℃ . What is the change in entropy of 30 g of water at 0°C as it is changed to ice at 0°C by a refrigerator.

Given Data: Heat Required per unit mass $= 336\ \text{J g}^{-1} = 336000\ \text{J kg}^{-1}$ Mass of Ice $m = 30\ \text{g} = 0.03\ \text{kg}$, Temperature $T = 0℃ = 0 + 273 = 273\ \text{K}$
To Determine: Entropy $\Delta S = ?$
Calculations: As Total Heat Absorbed $\Delta Q = 336000\ \text{kg}^{-1} \times 0.03\ \text{kg} = 10080\ \text{J}$ So $\Delta S = \frac{\Delta Q}{T} = \frac{10080}{273} = 36.92\ \text{J K}^{-1}$

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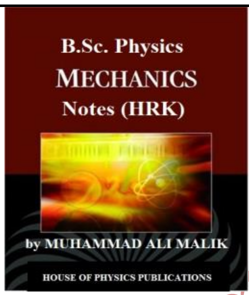

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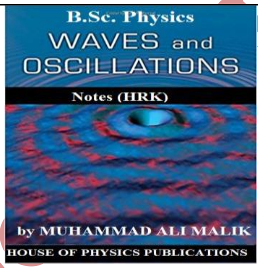

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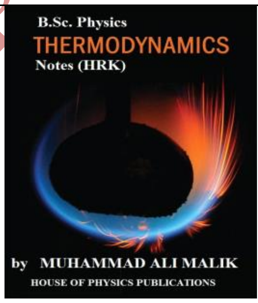

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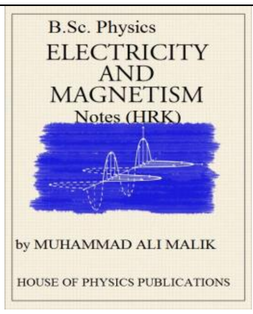

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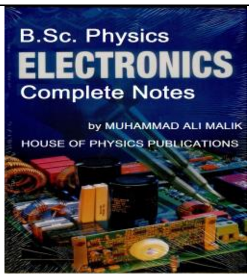

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