Ch #05:

Circular Motion

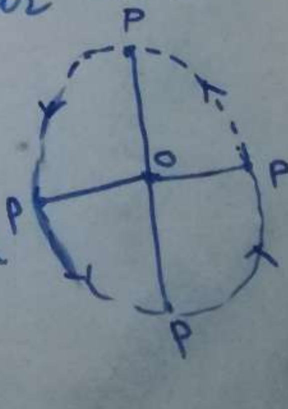
M.C.Q's	2 ⇒ 2	
SIQ	4 ⇒ 8	
LIQ	1 ⇒ 5/3	
	15/13	

Circular Motion:

If a body moves in a circular path, than the motion of the body is called circular Motion.

OR

When a body moves in such a way that its distance from a fixed point remains constant than the motion of the body is circular motion



e.g:

- Motion of earth around Sun

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Angular displacement:

The angle through which a particle moves in an interval of time while moving along a circle is called angular displacement.

OR.

The angle which is made at the centre of the circle, while moving a particle in a circular path is called angular displacement

. It is denoted by 'θ'

Explanation:

Consider a particle "p" moving in a circular path having radius 'r' as shown.

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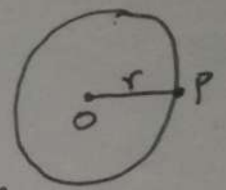
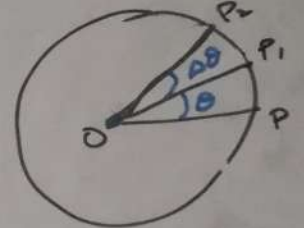


Suppose a particle moves through Point 'P' to 'P₁' in time 't' making an angle 'θ' as shown.

• So 'θ' is the angular displacement of point 'P' in time 't'



• Now if the particle moves again 'P₁' to 'P₂' in time 'Δt' making an angle 'Δθ'

• So 'Δθ' is the angular displacement of Point 'P₁' in time Δt

• Hence from Point 'P' to 'P₂', total angular displacement

will be
$t + \Delta t$

Convention:.

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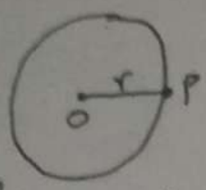

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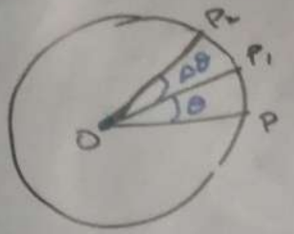

again 'P₁' to
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will be $\theta + \Delta\theta$, in time $t + \Delta t$

Convention:-

If a particle moves in an anticlock wise direction than angular displacement 'θ' is consider positive.

OR If a particle moves in clockwise direction, than angular displacement 'θ' is consider negative

- For a very Small value of '$\Delta\theta$' The angular displacement is a vector quantity & its direction is along the axis of rotation and can be find by right hand rule.

Right hand rul

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Right hand rule:-

Grasp the axis of rotation in right hand with curling the fingers in the direction of rotation than the erecting thumb points the direction of angular displacement.

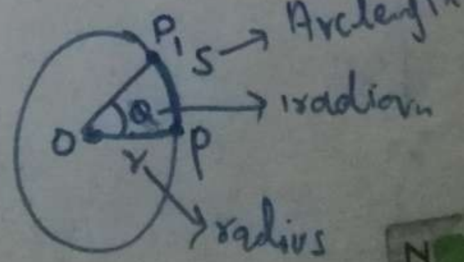
Units:-

There are three units of angular displacement

1) Degree
2) Revolution
3) Radian (S.I Unit)

Radian:-

The angle which is made at the centre of the circle, If the arc length is equal to the radius of the circle. than angle made at centre will be one radian.



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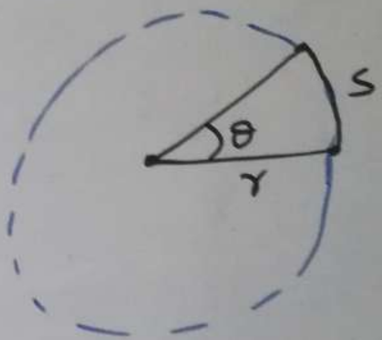
SIR

Prove $S = r\theta$

Consider an arc length 'S' making an angle 'θ' at the Centre of the circle having radius 'r' as shown.



. The value of 'θ' is in radian

We Know that

$$\frac{\text{Arc length}}{\text{Circumference of a Circle}} = \frac{\text{Angle Subtended}}{\text{total Angle}}$$

$$\frac{S}{\cancel{2\pi} r} = \frac{\theta}{\cancel{2\pi}}$$

$$S = r\theta \quad (\theta \text{ is in radian})$$

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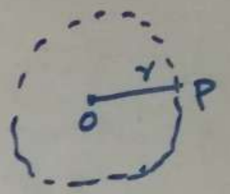
cceleration:-

S/R

Prove $1\,rad = 57.3^\circ$

we Know That when point 'p' covers distance along The circumference of the circle $S = 2\pi r$



$$S = 2\pi r$$

divided both side with 'r'.

$$\frac{S}{r} = \frac{2\pi r}{r} \qquad \frac{S}{r} = \theta$$

$$\theta = 2\pi$$

$\therefore S = 1$ revolution

$\therefore 2\pi = 360^\circ$

So

1 revolution $= 2\pi$ radian $= 360^\circ$

Now

$$2\pi\, rad = 360^\circ$$

$$\pi\, rad = \frac{360}{2}$$

$$\pi(1\,rad) = 180^\circ$$

$$1\,rad = \frac{180}{\pi}$$

$$1\,rad = \frac{180}{3.14}$$

$$1\,rad = 57.$$

S/R

Angular Velocity:

Rate of

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$1\,rad = \frac{180}{3.14}$ $\quad \pi = 3.14$

$1\,rad = 57.3^\circ$

Angular Velocity:-

Rate of change of angular displacement is called angular velocity.

- If $\Delta\theta$ is the angular displacement in time interval Δt so angular velocity will be

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$$

- It is denoted by 'ω'
- It is a vector quantity & its direction is along the axis of rotation, can be find by right hand rule.

3

$\frac{180}{3.14}$ $\pi = 3.14$

57.3°

ocity:-

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Δt is the angular ement in time interval o angular velocity

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$\omega_{avg} = \frac{\Delta\theta}{\Delta t}$

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ight hand rule.

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• Its unit is radian per second ($rad\ s^{-1}$), also measures in revolution per minute ($rev\ m^{-1}$).

$$1 rev = 2\pi\ rad$$

$$\frac{1 rev}{1 min} = \frac{2\pi\ rad}{60\ sec} \quad \therefore 1min = 60sec$$

$$\boxed{1 rev\ m^{-1} = \frac{\pi}{30}\ rad\ s^{-1}} \quad (m.c.R)$$

Instantaneous Angular Velocity:

The limiting value of $\frac{\Delta\theta}{\Delta t}$, where $\Delta t \to 0$ is called instantaneous angular velocity.

$$\omega_{ins} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

S/R:

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Proof:

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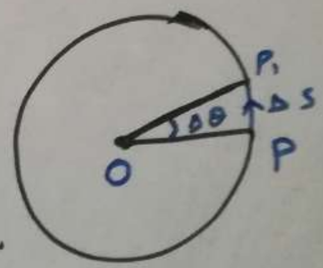
SIR:

Relation b/w linear velocity & angular velocity?

Proof:

Consider a point 'p' moves from 'p' to 'P_1', such that it covers the distance $PP_1 = \Delta S$ in a time interval Δt with the reference line 'OP' has an angular displacement '$\Delta\theta$' as shown.



we know that

$$\Delta S = r\,\Delta\theta$$

dividing bothside with 'Δt'

$$\frac{\Delta S}{\Delta t} = r\frac{\Delta\theta}{\Delta t}$$

Taking $\lim_{\Delta t \to 0}$ on both side

$$\lim_{\Delta t \to 0}\frac{\Delta S}{\Delta t} = r\lim_{\Delta t \to 0}\frac{\Delta\theta}{\Delta t}$$

c

we Know that $\lim\limits_{\Delta t \to 0} \frac{\Delta S}{\Delta t} = v$

in an interval Δt

& $\lim\limits_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \omega$

So

$$V = r\omega$$

M.C.R

$$\boxed{\bar{V} = \bar{r} \times \bar{\omega}}$$

Angular Accelration:

Rate of change of angular velocity is called angular accelration.

- It is denoted by alpha (α).

- If ω_i & ω_f is The initial & final angular velocity at time t_1 & t_2 respectively so angular accelration will be

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{t_2 - t_1}$$

- Its unit is radian per second square ($rad\,s^{-2}$).

Instantaneous angular accelration:-

The limiting value of $\frac{\Delta\omega}{\Delta t}$ where $\Delta t \to 0$, is called instantaneous angular accelation.

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$$\alpha_{ins} = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

- It is a vector quantity & its direction is along the axis of rotation, can be find by right hand rule.

S/Q Relation b/w linear acceleration & angular acceleration?

Proof:-

we know that

$$\Delta V = r \Delta \omega$$

divided both side with Δt

$$\frac{\Delta V}{\Delta t} = r \frac{\Delta \omega}{\Delta t}$$

Applying $\lim_{\Delta t \to 0}$ on both side

$$\lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = r \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

$$\lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = a ...$$

&

$$\lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = ...$$

So

$$a = r\alpha$$

This is the rel... acceleration & an...

Equation of ...

Linear Motion

$$V_f = V_i + at$$

$$S = V_i t + \frac{1}{2} a t^2$$

$$2aS = V_f^2 - V_i^2$$

$$\lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = a$$

&

$$\lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \alpha$$

So

$$\boxed{\bar{a} = \bar{r} \times \bar{\alpha}}$$ M.C.R

$$a = r\alpha$$

This is the relation b/w linear acceleration & angular acceleration.

Equation of angular Motion:-

Linear Motion	Angular Motion
$V_f = V_i + at$	$\omega_f = \omega_i + \alpha t$
$S = V_i t + \frac{1}{2} a t^2$	$\theta = \omega_i t + \frac{1}{2} \alpha t^2$
$2aS = V_f^2 - V_i^2$	$2\alpha\theta = \omega_f^2 - \omega_i^2$

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Centripetal force:

Defination:

The force which keep the body to move in a circular path is called centripetal force OR

The force needed to bend the normally straight path of the particle into circular path is called centripetal force.

- It is denoted by $\bar{F}_c$.

$$F_c = \frac{mv^2}{r}$$

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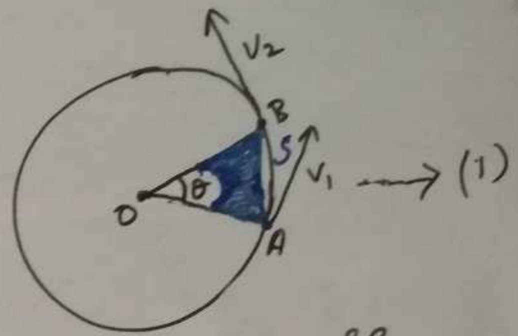
Explanation:

Suppose a particle is moving in a circular path with a constant speed say 'v', The velocity of the particle is changing at every point because its direction changes as shown from point 'A' & 'B', hence accelration of the particle is



$$a = \frac{\Delta V}{\Delta t} \rightarrow (1)$$

As we know that, particle is moving with constant speed say 'v' & covers the distance 'S' from point A to point B. in time Δt so

$$S = V\Delta t$$

$$\Delta t = \frac{S}{V} \rightarrow (2)$$

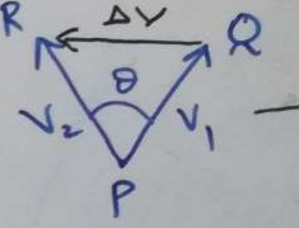
put eq (2) in (1) we get

$$a = \frac{\Delta V}{\frac{S}{V}}$$

$$a = V\frac{\Delta V}{S} \rightarrow (3)$$

Here 'V' is the Constant speed & ΔV is the change in velocity.

Now let us Consider a triangle PQR, such that 'PQ' is parallel and equal to 'V_1' & 'PR' is parallel and equal to 'V_2'



$\rightarrow (2)$

. As we Know that radius of the circle is perpendicular to the tangent drawn on circle

So from figure (1), OA is perpendicular to 'V_1' & 'OB' is perpendicular to 'V_2'

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& OA

So $\frac{\Delta V}{AB} =$

(3)

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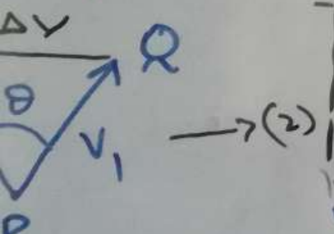

...t

...is perpendicular,

...n circle

OA is perpendicular

...endicular to 'V_2'

• So the angle AOB is equal to the angle QPR, b/w 'V_1' & 'V_2'

• we also observe that from figure '1' & '2', both triangles are same So we Know that

"Two Isosceles triangles are equal, if the angle b/w their equal arms are equal"

Hence we can say that both triangles AOB & PQR are Similar, Further we Know that

$$|V_1| = |V_2| = V \quad ; AB = S$$

$$\& \quad OA = OB = r$$

So

$$\frac{\Delta V}{AB} = \frac{V}{r} \qquad \boxed{\tan\theta = \tan\theta}$$

$$\frac{\Delta V}{S} = \frac{V}{r} \ ...$$

put equation...
get

$$a = V(\ ...$$

$$a_c = \frac{V}{r} \ ...$$

'a_c' is the c...
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To find the ...
centripetal ...

$$F_c = ma \ ...$$

F_c = cent...

put eq (5) ...
get

$$F_c = m \ ...$$

$$F_c = \frac{m}{\ ...} \ ...$$

In angular m...

$$F_c = \frac{m}{r} \times \ ...$$

$$= \frac{m}{r} \ ...$$

$$F_c = m \ ...$$

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$\boxed{\ldots n\theta = \tan\theta}$

$$\frac{\Delta V}{S} = \frac{V}{r} \rightarrow (4)$$

put equation (4) in (3) we get

$$a = V\left(\frac{V}{r}\right)$$

$$a_c = \frac{V}{r} \rightarrow (5)$$

'a_c' is the centripetal accelration called instantaneous accelration.

To find the expression of centripetal force consider

$$F_c = m a_c$$

F_c = centripetal force

put eq (5) in above eq we get

$$F_c = m\left(\frac{v^2}{r}\right)$$

$$F_c = \frac{mv^2}{r} \rightarrow (6)$$

In angular motion put $v = r\omega$

$$F_c = \frac{m}{r} \times (r\omega)^2$$

$$= \frac{m}{r} r^2 \omega^2$$

$$F_c = m r \omega^2$$

M.C.R

vector form

$$\bar{F}_c = \frac{mv^2}{r}$$

$$\bar{F}_c = mr\omega \ldots$$

$$\bar{F}_c = m(\bar{v} \times \ldots$$

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put $v = r\omega$

M.C.R

vector form

$$\bar{F}_c = \frac{mv^2}{r}\hat{r}$$

$$\bar{F}_c = mr\omega^2\hat{r}$$

$$\bar{F}_c = m(\bar{v} \times \bar{\omega})$$

Explanation:.

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