L-IR

Moment of Inertia:

The product of mass & Square of the distance from the axis of rotation is called moment of Inertia.

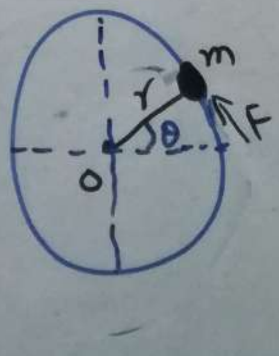
• It is denoted by 'I'.

$$I = mr^2$$

• Its unit is kgm^2

Explanation:

Consider a body of mass 'm', connected with a massless rod at point 'o' as shown.



A force
on a mass
it will acc
Know
F = m

• As force
will rotat
Circular pa
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accelation

$a_T = r\alpha$

put ② in

F = m

multiply

rF = m

as we kn

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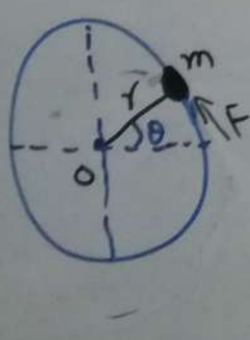
gm^2

ody of mass
with a massless
'o' as shown.



A force 'F' is applied on a mass 'm' so it will accelerate as we know

$$F = ma \rightarrow (1)$$

- As force applied it will rotate along a circular path as we know the tangential acceleration related with angular acceleration so

$$a_T = r\alpha \rightarrow (2)$$

put (2) in (1) we get

$$F = mr\alpha$$

multiply both side with 'r'

$$rF = mr^2\alpha$$

as we know that $rF = \tau$ & $mr^2 = I$

So $\tau = mr^2$...
$\tau = I\alpha$...

Significance:

Moment of ...
the same role i...
motion as the ...
in linear mo...
depend on ...
body & radius ...

Moment of In...

Consider a ...
made up of ...
of small m...
$m_1, m_2, m_3, \ldots$
distances 'r_1, r_2...
The axis of rot...
shown.

So $\tau = mr^2 a$

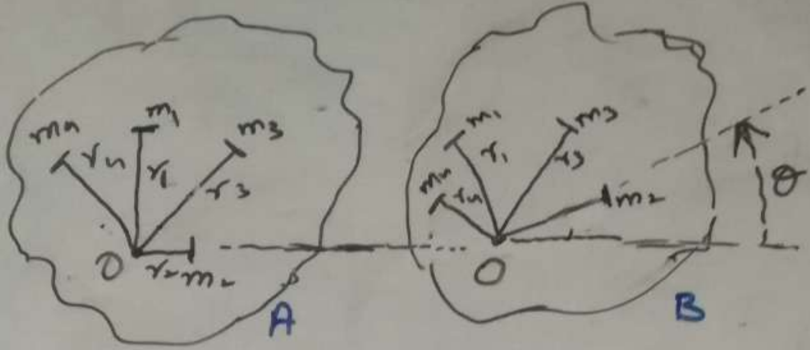
$\tau = I\alpha$

Significance:

Moment of Inertia plays the same role in angular motion as the mass plays in linear motion & its depend on mass of the body & radius

Moment of Inertia of a rigid body:

Consider a rigid body is made up of 'n' number of small masses say '$m_1, m_2, m_3, \ldots\ldots, m_n$', at distances '$r_1, r_2, r_3, \ldots, r_n$' from the axis of rotation say 'O' as shown.



let the rigid body be rotating with an angle 'θ' with an accelration 'a'

So the magnitude of the torque acting on each mass will be

$$\tau_1 = m_1 r_1^2 a_1$$

Similarly

$$\tau_2 = m_2 r_2^2 a_2$$

$$\vdots \qquad \vdots$$

$$\tau_n = m_n r_n^2 a_n$$

Total torque acting on a body will be

$$\tau_T = \tau_1 + \tau_2 + \tau_3 + \cdots + \tau_n$$

$$= m_1 r_1^2 a_1 + m_2 r_2^2 a_2 + \cdots + m_n r_n^2 a_n$$

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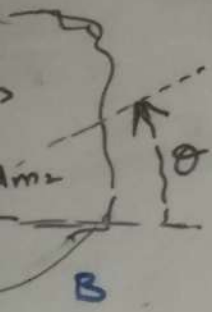
Section: B-01 Pre-Medical Part-I Boys Morning Roll Nos. (2005001-2005060)



be

gle 'θ'

a'

the

ach mass

a body

$+\tau_n$

$--+mr_i^2\alpha$

Σ

Since the body is rotating with same angular accelration 'α' so

$$\tau_T = m_1 r_1^2 \alpha + m_2 r_2^2 \alpha + \ldots\ldots + m_n r_n^2 \alpha$$

$$= \left[m_1 r_1^2 + m_2 r_2^2 + \ldots\ldots + m_n r_n^2\right]\alpha$$

$$\tau_T = \left[\sum_{i=1}^{n} m_i r_i^2\right]\alpha$$

$$\sum_{i=1}^{n} m_i r_i^2 = I$$

so

$$\tau_T = I\alpha$$

Angular Momentum:-

The cross product of linear momentum & the position vector '$\vec{r}$' about a reference axis is called angular momentum.

OR

The product of linear momentum & moment arm for momentum is called angular momentum.

· It is denoted by $\vec{L}$.

Mathmatically:-

The angular momentum $\vec{L}$ of a particle of mass 'm' moving in a circular path with velocity $\vec{v}$ is

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow rp\sin\theta\,\hat{n}$$

where $\vec{r}$ is
of the body w
origin 'O'.

· The magnitude
of the angula
mometum ca

$|L| = rp\sin$

we know th

$\vec{L} = rp$

· where 'θ' is
$\vec{r}$ & $\vec{P}$.

· $\hat{n}$ is the u
shows the dire
momentum $\vec{L}$,
to the plane c

duct of linear
position vector
erence axis
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of linear
moment arm
is called
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ted by $\vec{L}$.

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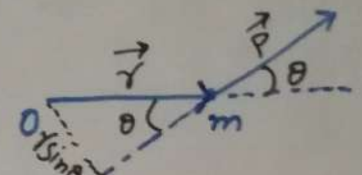
of mass 'm'
cular path with

$\vec{P} \Rightarrow rp\sin\theta\,\hat{n}$

Where $\vec{r}$ is the position vector of the body with respect to origin 'O'.



· The magnitude of the angular momentum can be find by

$$|L| = rp\sin\theta \rightarrow ①$$

we Know That

$$\vec{L} = rp\sin\theta\,\hat{n} \rightarrow ②$$

· where 'θ' is the angle b/w $\vec{r}$ & $\vec{P}$.

· $\hat{n}$ is the unit vector which shows the direction of angular momentum $\vec{L}$, which is perpendicular to the plane containing $\vec{r}$ & $\vec{P}$.

From eq ①

$L = rp\sin\theta$

$L = r(mv)\sin$

$L = mvr\sin$

where v is the
the particle

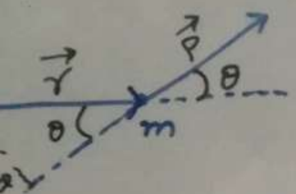
(S.I.) · If a partic
with uniform
velocity 'ω'
angle b/w
point will be
where $\vec{V}$ is th
tangential velo
So

$L = mvr\sin$

$= mvr$

$L = mvr$

...osition vector
...respect to



... find by

$\rightarrow$ ①

... $\hat{n}$ $\rightarrow$ ②

angle b/w

...vector which

...n of angular

...hich is perpendicular

...aining $\vec{r}$ & $\bar{p}$.

From eq ① we can write

$L = rp\sin\theta$

$p = mv$

$L = r(mv)\sin\theta$

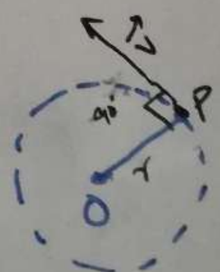
$L = mvr\sin\theta$

where v is the velocity of the particle

$\boxed{\text{Prove. } L = mvr,\ L = I\omega}$

(SQ) • If a particle is moving with uniform angular velocity 'ω'. Then the angle b/w $\vec{r}$ & $\vec{v}$ at every point will be 90°

where $\vec{v}$ is the tangential velocity



So

$L = mvr\sin\theta \qquad \theta = 90^\circ$

$= mvr\sin 90$

$L = mvr$

we know that

So

$L = m(r\omega$ ...

$L = mr^2\omega$ ...

$L = I\omega$

• Its unit is ...

Angular Mom...

• Consider a...
rotating a...
Through ce...

• Rigid b...
'n' number...
are also m...
axis so the ...
of each parti...

can write

mv

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$$\boxed{\text{rove. } L = mvr \\ L = I\omega}$$

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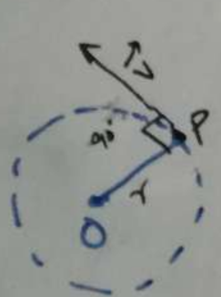
$\vec{r}$, $\vec{v}$ at every

90°



ty

θ

$\theta = 90°$

in 90

we Know That $V = r\omega$

So

$$L = m(r\omega)r$$

$$L = mr^2\omega$$

$$\therefore mr^2 = I$$

$$L = I\omega$$

• Its unit is $Kgm^2 s^{-1}$ or $J s$

Angular Momentum of a rigid body :-

• Consider a rigid body rotating about a fixed axis Through centre of mass.

• Rigid body Consist of 'n' number of particle, which are also moving about a fixed axis so the angular momentum of each particle would be

$$L_1 = m_1 r_1^2 \omega_1$$
$$L_2 = m_2 r_2^2 \omega_2$$
$$\vdots$$
$$L_n = m_n r_n^2 \omega$$

The total An

all particle

with angular

$$L_T = L_1 + L_2 + \ldots$$

$$= m_1 r_1^2 \omega + \ldots$$

$$= [m_1 r_1^2 + \ldots$$

$$L_T = \left[\sum_{i=}^{n} \ldots\right.$$

$$L_T = I\omega \ldots$$

$\underline{5}$

$L_1 = m_1 r_1^2 \omega_1$ Similarly

$L_2 = m_2 r_2^2 \omega_2$

$\vdots \quad \vdots$

$L_n = m_n r_n^2 \omega_n^2$

I

The total Angular Momentum of all particle which are moving with angular velocity 'ω'.

γs

gidbody:-

$$L_T = L_1 + L_2 + L_3 + \cdots + L_n$$

$$= m_1 r_1^2 \omega + m_2 r_2^2 \omega + \cdots\cdots + m_n r_n^2 \omega$$

$$= [m_1 r_1^2 + m_2 r_2^2 + \cdots\cdots + m_n r_n^2]\,\omega$$

$$L_T = \left(\sum_{i=1}^{n} m_i r_i^2\right)\omega$$

f

d axis

ss.

$\sum_{i=1}^{n} m_i r_i^2 = I$

Moment of Inertia

of

$$L_T = I\omega$$

which

a fixed

lentum

Pr#5.1

DATA

Diameter = length of an arc = $S = 2.50m$

Distance of moon from the earth $= r = 3.8 \times 10^{8} m$

To find:.

Divergence angle $= \theta = ?$

Sol:.

$$S = r\theta$$

$$\theta = \frac{S}{r}$$

$$\theta = \frac{2.5}{3.8 \times 10^{8}}$$

$$\theta = 0.657 \times 10^{-8}$$

$$\theta = 6.6 \times 10^{-1} \times 10^{-8}$$

$$\theta = 6.6 \times 10^{-9} \text{ rad}$$

Pr#5.2

DAT

Initial angu

Final angula

=

=

=

=

Time =

To find:.

Angula

Sol:.

$\alpha =$

=

α

$= S = 2.50m$

$r = 3.8 \times 10^{8} m$

$\theta = ?$

^{8}

7×10^{-8}

$\times 10^{-1} \times 10^{-8}$

$\times 10^{-9}$ rad

Pr#5.2 **DATA**

Initial angular velocity $= \omega_i = 0$

Final angular velocity $= 45$ rev/min

$= 45 \, rev \, m^{-1}$

$1 \, rev \, m^{-1} = \frac{\pi}{30} \, rad \, s^{-1}$

$= 45 \times \frac{\pi}{30} \, rad \, s^{-1}$

$= \frac{3\pi}{2} \, rad \, s^{-1}$

$= 1.5\pi \, rad \, s^{-1}$

Time $= \Delta t = 1.60$ sec

To find:

Angular Acc $= \alpha = ?$

Sol:

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t}$$

$$= \frac{1.5\pi - 0}{1.60}$$

$$\alpha = \frac{1.5 \times 3.14}{1.60} \quad \because \pi = 3.14$$

$$\alpha = 2.9 \, rad \, s^{-2}$$

Pr#5.3 **DATA**

Moment of Inerti

Angular velocity

Angular Accelerat

(because body is mo
with constant

To find:

Angular Momentu

Torque $= \tau = ?$

Sol:

$L = I\omega$

$= (0.80)$

$L = 80$

Now

$\tau = I\alpha$

$= (0.8$

$\tau = 0$

...locity = $\omega_i = 0$

...ocity = 45 rev/min

...ev m^{-1}

$1 \text{ rev } m^{-1} = \frac{\pi}{30} \text{ rad } s^{-1}$

$\frac{\pi}{30}$ rad s^{-1}

rad s^{-1}

π rad s

1.60 sec

...c = α = ?

$\frac{\omega_f - \omega_i}{\Delta t}$

$\frac{\pi - 0}{1.60}$

$\frac{1.5 \times 3.14}{1.60}$ $\because \pi = 3.14$

$2.9 \text{ rad } s^{-2}$

3

Pr#5.3 **DATA**

Moment of Inertia = $I = 0.80 \text{ kg} m^2$

Angular velocity = $\omega = 100 \text{ rad } s^{-1}$

Angular Accelaration = $\alpha = 0$

(because body is moving with Constant angular velocity)

To find:

Angular Momentum = $\vec{L}$ = ?

Torque = τ = ?

Sol:

$$L = I\omega$$
$$= (0.80)(100)$$
$$L = 80 \text{ Js}$$

Now

$$\tau = I\alpha$$
$$= (0.80)(0)$$
$$\tau = 0$$

4

Pr#5.6 **DATA**

Mass of Car = m...

Speed = v = 14...

= $\frac{14...}{3...}$

= 4...

radius = r = ...

To find:

centripetal ...

Sol:

$F_c = \frac{mv^2}{r}$

$= \frac{1000...}{10...}$

$F_c = 1.6...$

ia = $I = 0.80 kgm^2$
= ω = 100 rads^{-1}
ation = $\ell = 0$
noving
angular velocity)
tum = $\vec{L} = ?$
?
)
0)(100)
0 Js
= a
0.80)(0)
0

Pr#S.6. DATA

Mass of Car = $m = 1000 kg$

Speed = $v = 144 kmh^{-1}$

$= \frac{144 \times 1000}{3600}$

$= 40 ms^{-1}$

radius = $r = 100 m$

To find:

Centripetal Force = $F_c = ?$

Sol:

$F_c = \frac{mv^2}{r}$

$= \frac{1000 \times (40)^2}{100}$

$F_c = 1.6 \times 10^4 N$

Types Of An
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momen
·) S
·) O
Spin Angul
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Spinning
angula

1

Types Of Angular Momentum:-

There are two types of angular momentum.

- Spin Angular Momentum
- Orbital Angular Momentum

Spin Angular Momentum:-

The angular momentum of the spinning body is called spin angular Momentum.

- It is denoted by L_s

Orbital Angular Momentum:-

The momentum which is associated with the motion of a body along a circular path is called orbital angular momentum.

- It is denoted by L_o

2

LAW of conservation of angular Momentum:

From book

S/R, page # 112-113

M.C.Q

$L = I\omega$

$\frac{L}{\omega} = I$

If L = Constant

$\boxed{I \propto \frac{1}{\omega}}$

$I = mr^2$

$mr^2 \propto \frac{1}{\omega}$

m = Constant

$r^2 \propto \frac{1}{\omega}$

PT#5.2

DAT

Initial ang

Final angu

Time =

To find:

Angu

Sol: