Punjab College (Shalamar Campus)

Trigonometric Formulas

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$1^\circ = 60'$ (degree to minute) $\quad$ $1^\circ = 3600''$ (degree to second)

$1' = 60''$ (minute to second) $\quad$ $1' = \left(\frac{1}{60}\right)^\circ$ (minute to degree)

$1'' = \left(\frac{1}{3600}\right)^\circ$ (second to degree)

$l = r\theta$

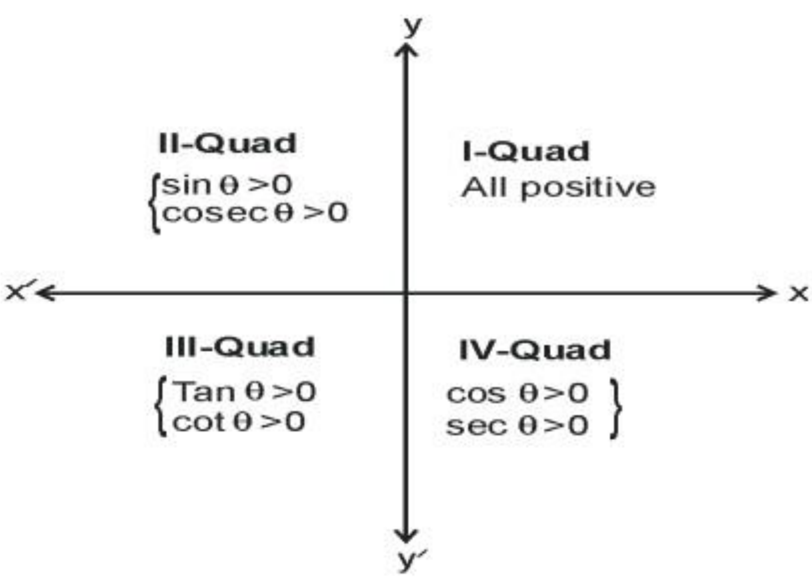
$1^\circ = \frac{\pi}{180} rad \approx 0.01745 rad$ (degree to radian)

$1 rad = \frac{180^\circ}{\pi} \approx 57.296^\circ$ (radian to degree)

Trigonometric Ratios

$$\sin\theta = \frac{P}{H}, \quad \cos\theta = \frac{B}{H}, \quad \tan\theta = \frac{P}{B}$$

$$\csc\theta = \frac{H}{P}, \quad \sec\theta = \frac{H}{B}, \quad \cot\theta = \frac{B}{P}$$



Trigonometric Ratios and their values:

θ	0°	30°	45°	60°	90°	120°	180°	270°	360°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	0	∞	0

Relation among Trigonometric Functions :

(a) $\sin^2\theta + \cos^2\theta = 1$

(b) $\sec^2\theta = 1 + \tan^2\theta$

(c) $\mathrm{cosec}^2\theta = 1 + \cot^2\theta$

(d) $\csc\theta = \frac{1}{sin\theta}$

(e) $\sec\theta = \frac{1}{cos\theta}$

(d) $\cot\theta = \frac{1}{tan\theta}$

<u>The Fundamental Law of Trigonometry:</u>

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ is called fundamental law of trigonometry.

<u>Addition formulas:</u>

i. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

ii. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

iii. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

iv. $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

v. $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$

Trigonometric Ratios of Allied Angles:

The angle associated with basic angles of measure θ to a right angle or its multiple are called allied angles.

Sine	Cosine	Tangent
$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$	$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$	$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$
$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$	$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$	$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$
$\sin(\pi-\theta)=\sin\theta$	$\cos(\pi-\theta)=-\cos\theta$	$\tan(\pi-\theta)=-\tan\theta$
$\sin(\pi+\theta)=-\sin\theta$	$\cos(\pi+\theta)=-\cos\theta$	$\tan(\pi+\theta)=\tan\theta$
$\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$	$\cos\left(\frac{3\pi}{2}-\theta\right)=-\sin\theta$	$\tan\left(\frac{3\pi}{2}-\theta\right)=\cot\theta$
$\sin\left(\frac{3\pi}{2}+\theta\right)=-\cos\theta$	$\cos\left(\frac{3\pi}{2}+\theta\right)=\sin\theta$	$\tan\left(\frac{3\pi}{2}+\theta\right)=-\cot\theta$
$\sin(2\pi-\theta)=-\sin\theta$	$\cos(2\pi-\theta)=\cos\theta$	$\tan(2\pi-\theta)=-\tan\theta$
$\sin(2\pi+\theta)=\sin\theta$	$\cos(2\pi+\theta)=\cos\theta$	$\tan(2\pi+\theta)=\tan\theta$

Double Angle Identities	Half Angle Identities	Triple Angle Identities
$\sin 2\alpha=2\sin\alpha\cos\alpha$	$\sin\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{2}}$	$\sin 3\alpha=3\sin\alpha-4\sin^3\alpha$
$\cos 2\alpha=\begin{cases}\cos^2\alpha-\sin^2\alpha\\ 1-2\sin^2\alpha\\ 2\cos^2\alpha-1\end{cases}$	$\cos\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{2}}$	$\cos 3\alpha=4\cos^3\alpha-3\cos\alpha$
$\tan 2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}$	$\tan\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$	$\tan 3\alpha=\frac{3\tan\alpha-\tan^3\alpha}{1-3\tan^2\alpha}$

Sum/Difference to Product formulas	Products to Sum/Difference formulas
$\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$	$2\sin\alpha cos\beta=\sin(\alpha+\beta)+\sin(\alpha-\beta)$
$\sin\alpha-\sin\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	$2\cos\alpha\sin\beta=\sin(\alpha+\beta)-\sin(\alpha-\beta)$
$\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$	$2\cos\alpha cos\beta=\cos(\alpha+\beta)+\cos(\alpha-\beta)$
$\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$	$-2\sin\alpha\sin\beta=\cos(\alpha+\beta)-\cos(\alpha-\beta)$

The law of sines	The law of cosine	The law of tangents
$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}$	$a^2=b^2+c^2-2bc\cos\alpha$ $b^2=a^2+c^2-2ac\cos\beta$ $c^2=a^2+b^2-2ab\cos\gamma$ OR $\cos\alpha=\frac{b^2+c^2-a^2}{2bc}$ $\cos\beta=\frac{a^2+c^2-b^2}{2ac}$ $\cos\gamma=\frac{b^2+a^2-c^2}{2ab}$	$\frac{\tan\frac{\alpha-\beta}{2}}{\tan\frac{\alpha+\beta}{2}}=\frac{a-b}{a+b}$ $\frac{\tan\frac{\gamma-\alpha}{2}}{\tan\frac{\gamma+\alpha}{2}}=\frac{c-a}{c+a}$ $\frac{\tan\frac{\beta-\gamma}{2}}{\tan\frac{\beta+\gamma}{2}}=\frac{b-c}{b+c}$

Area of Triangles:

Two sides and their included angle are given	One side and two angles are given	Three sides are given
$\Delta = \frac{1}{2}$ ab sin γ $\Delta = \frac{1}{2}$ bc sin α $\Delta = \frac{1}{2}$ ac sin β	$\Delta = \frac{a^2 \sin\beta \sin\gamma}{2\sin\alpha}$ $\Delta = \frac{b^2 \sin\alpha \sin\gamma}{2\sin\beta}$ $\Delta = \frac{c^2 \sin\beta \sin\alpha}{2\sin\gamma}$	$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ $s = \frac{a+b+c}{2}$

$R = \frac{abc}{4\Delta}$, $R = \frac{a}{2Sin\alpha} = \frac{b}{2Sin\beta} = \frac{c}{2Sin\gamma}$ $\qquad$ $r = \frac{\Delta}{s}$, $r_1 = \frac{\Delta}{s-a}$, $r_2 = \frac{\Delta}{s-b}$, $r_3 = \frac{\Delta}{s-c}$

Half Angles Formulas:

Sine	Cosine	Tangent
$\sin\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$ $\sin\frac{\beta}{2} = \sqrt{\frac{(s-a)(s-c)}{ac}}$ $\sin\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$	$\cos\frac{\alpha}{2} = \sqrt{\frac{s(s-a)}{bc}}$ $\cos\frac{\beta}{2} = \sqrt{\frac{s(s-b)}{ac}}$ $\cos\frac{\gamma}{2} = \sqrt{\frac{s(s-c)}{ab}}$	$\tan\frac{\alpha}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ $\tan\frac{\beta}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$ $\tan\frac{\gamma}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

Addition and Subtraction Formulas:

- $sin^{-1}A + sin^{-1}B = sin^{-1}\left(A\sqrt{1-B^2} + B\sqrt{1-A^2}\right)$
- $sin^{-1}A - sin^{-1}B = sin^{-1}\left(A\sqrt{1-B^2} - B\sqrt{1-A^2}\right)$
- $cos^{-1}A + cos^{-1}B = cos^{-1}\left(AB - \left(\sqrt{1-A^2}\right)\left(\sqrt{1-B^2}\right)\right)$
- $cos^{-1}A - cos^{-1}B = cos^{-1}\left(AB + \sqrt{\left(1-A^2\right)\left(1-B^2\right)}\right)$
- $tan^{-1}A + tan^{-1}B = tan^{-1}\frac{A+B}{1-AB}$
- $tan^{-1}A - tan^{-1}B = tan^{-1}\frac{A-B}{1+AB}$
- $2tan^{-1}A = tan^{-1}\left(\frac{2A}{1-A^2}\right)$