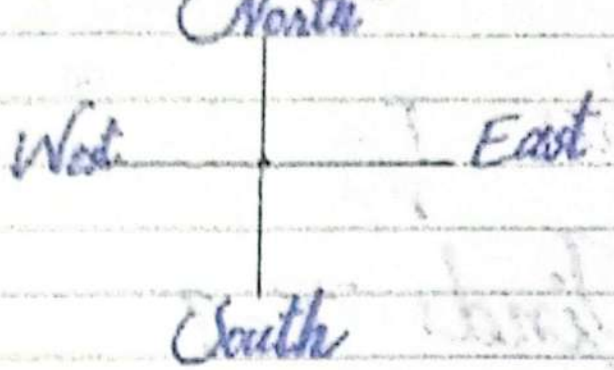
Numericals

2.11: Vectors A, B and C are 4 units north, 3 units west and 8 units east respectively. Describe carefully **(a)** $A \times B$ **(b)** $A \times C$ **(c)** $B \times C$

Data:

$\overline{A}$ = 4 units North
$\overline{B}$ = 3 units West
$\overline{C}$ = 8 units East



To Find:

$A \times B =$, $A \times C =$, $B \times C =$

Solve:

$\overline{A} \times \overline{C} = AC \sin\theta\, \hat{n}$
$= (4)(8) \sin(90)\, \hat{n}$
$= 32(1)\, \hat{n}$
$= 32\hat{n}$ (Its direction is perpendicular inward)

Now,

$\overline{A} \times \overline{B} = AB \sin\theta\, \hat{n}$
$= (4)(3) \sin(90)\, \hat{n}$
$= 12(1)\, \hat{n}$
$= 12\hat{n}$ (Its direction is perpendicular outward)

$\overline{B} \times \overline{C} = BC \sin\theta\, \hat{n}$
$= (3)(8) \sin(180)\, \hat{n}$
$= 24(0)\hat{n}$
$= 0$

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2.12: The torque or turning effect of force about a given point is given by $\mathbf{r} \times \mathbf{F}$ where $\mathbf{r}$ is the vector from the given point to the point of application of $\mathbf{F}$. Consider a force $\mathbf{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting on the point $7\hat{i} + 3\hat{j} + \hat{k}$. What is the torque in **Nm** about the origin?

Data:

$$\mathbf{F} = -3\hat{i} + \hat{j} + 5\hat{k}$$
$$\bar{r} = 7\hat{i} + 3\hat{j} + \hat{k}$$

To Find: $\bar{\tau} = ?$

Solve:

$$\bar{\tau} = \bar{r} \times F$$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$$\tau = \hat{i}(15-1) - \hat{j}(35+3) + \hat{k}(7+9)$$
$$\tau = 14\hat{i} - 38\hat{j} + 16\hat{k}$$

2.13: The line of action of force, $\mathbf{F} = \hat{i} - 2\hat{j}$, passes through the point whose position vector is $(-\hat{j} + \hat{k})$. Find **(a)** the moment of $\mathbf{F}$ about the origin, **(b)** the moment of $\mathbf{F}$ about the point of which the position vector is $\hat{i} + \hat{k}$

Data:

$$\mathbf{F} = \hat{i} - 2\hat{j}$$
$$\mathbf{r} = -\hat{j} + \hat{k}$$
$$\mathbf{r}' = \hat{i} + \hat{k}$$

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To find:

(a) $\overline{\tau} = r \times F = ?$

(b) $\overline{\tau} = r_1 \times F = ?$

Solve:

(a) $\overline{\tau} = \overline{r} \times F$

$$\tau = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

$\overline{\tau} = \hat{i}(0+2) - \hat{j}(0-1) + \hat{k}(0+1)$

$\overline{\tau} = 2\hat{i} + \hat{j} + k$

(b) $\overline{\tau} = \overline{r_1 \times F}$

$r_1 = r - r'$

$= (-\hat{j} + \hat{k}) - (\hat{i} + \hat{k})$

$= -\hat{j} + \hat{k} - \hat{i} - \hat{k}$

$= -\hat{j} - \hat{i}$

$= -\hat{i} - \hat{j}$

$\overline{\tau} = r_1 \times \overline{F}$

$$\overline{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 0 \\ 1 & -2 & 0 \end{vmatrix}$$

$\tau = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(2+1)$

$= +3\hat{k}$

2.14: Imp.
The magnitude of dot and cross products of two vectors are $6\sqrt{3}$ and 6 respectively. Find the angle between the vectors.

Data:

$|A.B| = 6\sqrt{3}$

$|A \times B| = 6$

To Find: $\theta = ?$

Solve:

$|A.B| = AB\cos\theta$

$|A \times B| = AB\sin\theta$

$6\sqrt{3} = AB\cos\theta \dashrightarrow 1$

$6 = AB\sin\theta \dashrightarrow 2$

$$\frac{AB\sin\theta}{AB\cos\theta} = \frac{6}{6\sqrt{3}}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

Equilibrium

"If a body is at rest or moving with uniform velocity then the body is said to be in equilibrium."

- When a body is at rest than the body is said to be in **static equilibrium**
- When a body is moving with uniform velocity then the body is in **dynamic equilibrium**

First Condition Of Equilibrium:

Vector sum of all the forces acting on a body is zero than the body satisfies the **1st condition of equilibrium**

$$\Sigma F = \Sigma F_x + \Sigma F_y$$

$$\Sigma F = 0$$

In case of coplanner forces:

- Vector sum of all **x-directed** forces is zero than it can be written as:

$$\Sigma F_x = 0$$

- Vector sum of all **y-directed** forces is zero than it can be written as:

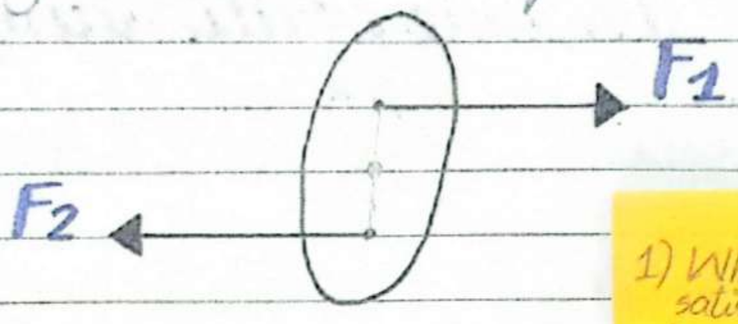
$$\Sigma F_y = 0$$

Second Condition Of Equilibrium

Vector sum of all the torques acting on a body is zero than the body satisfies **2nd condition of equilibrium.**

$\Sigma \tau = 0$

Explanation: If two forces acting on a same body, oppositely act but there line of action of points are different.
So, body rotate, its torque must be zero



MCQs

1) When 1st condition of equilibrium satisfies than the body is in translational equilibrium
2) When 2nd condition of equilibrium satisfies than the body is in rotational equilibrium
3) For complete equilibrium
$\Sigma F = 0$, $\Sigma \tau = 0$