

2.5: If a vector B is added to vector A , the result is $6\hat{i} + \hat{j}$. If a B is subtracted from A , the result is $-4\hat{i} + 7\hat{j}$. What is magnitude of vector A ?

Data:

$$A + B = 6\hat{i} + \hat{j} \quad \dots 1$$

$$A - B = -4\hat{i} + 7\hat{j} \quad \dots 2$$

To find: $|A| = ?$

Solve:

Adding equ 1 and 2

$$\begin{aligned} A + B &= 6\hat{i} + \hat{j} \\ A - B &= -4\hat{i} + 7\hat{j} \end{aligned}$$

$$2A = 2\hat{i} + 8\hat{j}$$

$$2A = 2(\hat{i} + 4\hat{j})$$

$$A = \hat{i} + 4\hat{j}$$

$$|A| = \sqrt{(1)^2 + (4)^2}$$

$$= \sqrt{1 + 16}$$

$$= \sqrt{17}$$

$$= 4.1$$

2.6: Given that $A = 2\hat{i} + 3\hat{j}$ and $B = 3\hat{i} - 4\hat{j}$, find the magnitude and angle of
(a) $C = A + B$ and (b) $D = 3A - 2B$

Data:

$$A = 2\hat{i} + 3\hat{j}$$

$$B = 3\hat{i} - 4\hat{j}$$

Solve:

(a) $C = A + B$

$$C = 2\hat{i} + 3\hat{j} + 3\hat{i} - 4\hat{j}$$
$$5\hat{i} - \hat{j}$$

$$\theta = 360 + \phi$$

$$\theta = 360 - 11.31^\circ$$

$$|C| = \sqrt{(5)^2 + (-1)^2}$$

$$\theta = 348.7^\circ$$

$$|C| = \sqrt{25 + 1}$$

$$|C| = \sqrt{26}$$

(b) $D = 3A - 2B$

$$D = 3(2\hat{i} + 3\hat{j}) - 2(3\hat{i} - 4\hat{j})$$

$$D = 6\hat{i} + 9\hat{j} - 6\hat{i} + 8\hat{j}$$

$$D = 0\hat{i} + 17\hat{j}$$

$$|D| = \sqrt{(0)^2 + (17)^2}$$

$$|D| = 17$$

$$|C| = 5.1$$

$$\phi = \tan^{-1} \frac{C_y}{C_x}$$

$$\phi = \tan^{-1} \frac{1}{5}$$

$$\phi = \tan^{-1}(0.2)$$

$$\phi = 11.31$$

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$$|D| = 17$$

$$\phi = \tan^{-1} \frac{D_y}{D_x}$$

$$\phi = \tan^{-1} \frac{17}{0}$$

$$\phi = \tan^{-1} \infty$$

$$\phi = 90^\circ$$

$$\theta = \phi$$

$$\theta = 90^\circ$$

2.1: Suppose in a rectangular coordinate system, a vector A has its tail at the point A $P(-2, -3)$ and its tip at $Q(3, 9)$. Determine the distance between these points.

Data: $P(-2, -3)$

$$r_1 = -2\hat{i} - 3\hat{j}$$

$$Q(3, 9)$$

$$r_2 = 3\hat{i} + 9\hat{j}$$

To Find: $|r_2 - r_1| = ?$

$$\text{Solve: } r_2 - r_1 = 3\hat{i} + 9\hat{j} - (-2\hat{i} - 3\hat{j})$$

$$= 3\hat{i} + 9\hat{j} + 2\hat{i} + 3\hat{j}$$

$$= 5\hat{i} + 12\hat{j}$$

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$$|r_2 - r_1| = \sqrt{(5)^2 + (12)^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13 \text{ units}$$

2.2: A certain corner of a room is selected as the origin of a rectangular coordinate system. If an insect is crawling on an adjacent wall at a point having coordinates (2, 1) where the units are in meters, what is distance of the object from the corner of the room?

Data:

$$\text{Origin} = O (0, 0)$$

$$r_1 = 0\hat{i} + 0\hat{j}$$

$$P = (2, 1)$$

$$r_2 = 2\hat{i} + \hat{j}$$

Solve:

$$r_2 - r_1 = 2\hat{i} + \hat{j} - (0\hat{i} + 0\hat{j})$$

$$= 2\hat{i} + \hat{j} - 0\hat{i} + 0\hat{j}$$

$$= 2\hat{i} + \hat{j}$$

$$|r_2 - r_1| = \sqrt{(2)^2 + (1)^2}$$

$$= \sqrt{4 + 1}$$

$$= \sqrt{5}$$

$$= 2.2 \text{ m}$$

Product of Vector:

There is two type of product of vector

i) Scalar product

ii) Vector product

Scalar Product:

The product of two vectors results into a scalar quantity is called scalar product

- It is also called dot product
- It is denoted by putting a dot between two vectors

Mathematically

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Where A and B are the magnitudes of vector A and B and θ be the angle between these vectors

Q:- Prove that scalar product obeys commutative law?

Physical Interpretation:

Consider two vectors A and B making an angle θ with each other

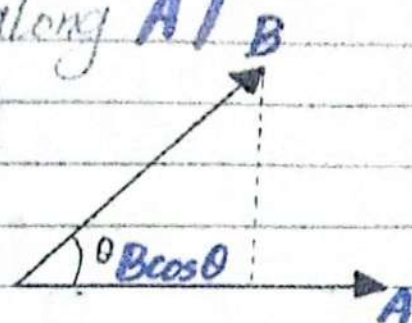
Draw the projection of vector B on A such that we can write

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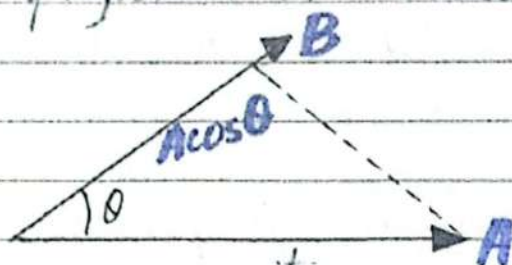
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$$\begin{aligned}\vec{A} \cdot \vec{B} &= |\vec{A}| \text{ (Projection of } \vec{B} \text{ on } \vec{A}) \\ &= |\vec{A}| \text{ (Projection of } \vec{B} \text{ along } \vec{A}) \\ &= |\vec{A}| (B \cos \theta)\end{aligned}$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta \quad \dots \quad 1$$



Now draw the projection of vector \vec{A} on \vec{B}



Such that we can write

$$\begin{aligned}\vec{B} \cdot \vec{A} &= |\vec{B}| \text{ (Projection of } \vec{A} \text{ on } \vec{B}) \\ &= |\vec{B}| \text{ (Projection of } \vec{A} \text{ along } \vec{B}) \\ &= |\vec{B}| (A \cos \theta)\end{aligned}$$

So we can write

$$\vec{B} \cdot \vec{A} = BA \cos \theta \quad \dots \quad 2$$

So we can write

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

It shows it obeys commutative law

Characteristics:

- It obeys commutative Law

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

It obeys Distributive Law

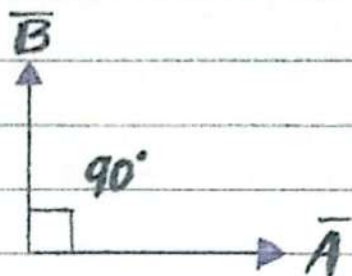
$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{C}$$

It doesn't obey Associated law

$$(\vec{A} \cdot \vec{B}) \cdot \vec{C} \neq \vec{A} \cdot (\vec{B} \cdot \vec{C})$$

- When $\theta = 90^\circ$

It means both vectors \vec{A} and \vec{B} are perpendicular

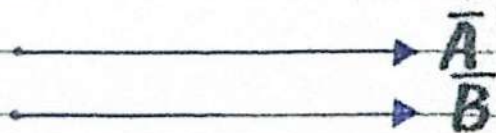


$$\begin{aligned}\vec{A} \cdot \vec{B} &= AB \cos \theta \\ &= AB \cos(90^\circ) \\ &= AB(0)\end{aligned}$$

$$\vec{A} \cdot \vec{B} = 0 \quad (\text{minimum product})$$

- When $\theta = 0$

It means both vectors \vec{A} and \vec{B} are parallel to each other



$$\begin{aligned}\vec{A} \cdot \vec{B} &= AB \cos \theta \\ &= AB \cos(0) \\ &= AB(1)\end{aligned}$$

$$\vec{A} \cdot \vec{B} = AB \quad (\text{max product})$$

- In case of unit vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

- In case of rectangular components we consider two vectors having components

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x B_x (\hat{i} \cdot \hat{i}) + A_y B_y (\hat{j} \cdot \hat{j}) + A_z B_z (\hat{k} \cdot \hat{k})$$

$$= A_x B_x (1) + A_y B_y (1) + A_z B_z (1)$$

$$= A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

Now,

$$\frac{A_x B_x + A_y B_y + A_z B_z}{AB} = \cos \theta$$

$\theta = 0$ (parallel)

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\hat{i} \cdot \hat{i} = 1$$

$$\hat{i} \cdot \hat{j} = 0$$

$$\hat{i} \cdot \hat{k} = 0$$

$$\hat{j} \cdot \hat{j} = 1$$

$$\hat{j} \cdot \hat{k} = 0$$

$$\hat{k} \cdot \hat{k} = 1$$

$\theta = 90^\circ$ (perpendicular)

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\hat{i} \cdot \hat{j} = 0$$

$$\hat{j} \cdot \hat{k} = 0$$

$$\hat{k} \cdot \hat{i} = 0$$

$$\hat{i} \cdot \hat{i} = 1$$

$$\hat{j} \cdot \hat{j} = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

$$A = |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$B = |\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

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• Self Product

When two equal vectors are multiplied with each other it means $\theta = 0^\circ$, $\vec{A} = \vec{B}$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{A} \cdot \vec{A} = AA \cos \theta$$

$$\vec{A} \cdot \vec{A} = AA \cos(0)$$

$$\vec{A} \cdot \vec{A} = AA (1)$$

$$\vec{A} \cdot \vec{A} = A^2$$

What is the self product of vector \vec{A} ?

Answer: A^2

Numericals

2.7: Find the angle between the two vectors $\vec{A} = 5\hat{i} + \hat{j}$ and $\vec{B} = 2\hat{i} + 4\hat{j}$

Data:

$$\vec{A} = 5\hat{i} + \hat{j}$$

$$\vec{B} = 2\hat{i} + 4\hat{j}$$

To Find:

$$\theta = ?$$

Formula:

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\vec{A} \cdot \vec{B} = (5\hat{i} + \hat{j}) \cdot (2\hat{i} + 4\hat{j})$$

$$= 10(\hat{i} \cdot \hat{i}) + 4(\hat{j} \cdot \hat{j})$$

$$= 10(1) + 4(1)$$

$$= 10 + 4$$

$$= 14$$

$$|\vec{A}| = \sqrt{(5)^2 + (1)^2}$$

$$= \sqrt{25+1}$$

$$= \sqrt{26}$$

$$|B| = \sqrt{(2)^2 + (4)^2}$$

$$= \sqrt{4+16}$$

$$= \sqrt{20}$$

$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\cos \theta = \frac{14}{\sqrt{25} \sqrt{20}}$$

$$\theta = \cos^{-1} \left[\frac{14}{\sqrt{25} \sqrt{20}} \right]$$

$$\theta = 52^\circ$$

2.8: Find the work done when the point of application of the force $3\hat{i} + 2\hat{j}$ moves in a straight line from the point $(2, -1)$ to the point $(6, 4)$

Data:

$$\vec{F} = 3\hat{i} + 2\hat{j}$$

$$\vec{r}_1 = (2, -1) = 2\hat{i} - 1\hat{j}$$

$$\vec{r}_2 = (6, 4) = 6\hat{i} + 4\hat{j}$$

To find: $W = ?$

Formula:

$$W = F \cdot \vec{d}$$

Solve:

$$\vec{d} = r_2 - r_1$$
$$\vec{d} = (6\hat{i} + 4\hat{j}) - (2\hat{i} - 1\hat{j})$$

$$= 6\hat{i} + 4\hat{j} - 2\hat{i} + 1\hat{j}$$
$$\vec{d} = 4\hat{i} + 5\hat{j}$$

$$W = F \cdot \vec{d}$$

$$= (3\hat{i} + 2\hat{j}) \cdot (4\hat{i} + 5\hat{j})$$

$$= 12(\hat{i} \cdot \hat{i}) + 10(\hat{j} \cdot \hat{j})$$

$$= 12(1) + 10(1)$$

$$= 12 + 10$$

$$W = 22 \text{ units}$$

2.9: Show that the three vectors $\hat{i} + \hat{j} + \hat{k}$, $2\hat{i} - 3\hat{j} + \hat{k}$ and $4\hat{i} + \hat{j} - 5\hat{k}$ are (multiplied), mutually perpendicular.

Data:

$$\vec{A} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{B} = 2\hat{i} - 3\hat{j} + \hat{k}$$

$$\vec{C} = 4\hat{i} + \hat{j} - 5\hat{k}$$

To Find:

$$\vec{A} \cdot \vec{B} = ?$$

$$\vec{A} \cdot \vec{C} = ?$$

$$\vec{B} \cdot \vec{C} = ?$$

$$\vec{A} \cdot \vec{B} = (\hat{i} + \hat{j} + \hat{k}) (2\hat{i} - 3\hat{j} + \hat{k})$$

$$\begin{aligned} &= 2(\hat{i} \cdot \hat{i}) + (-3)(\hat{j} \cdot \hat{j}) + 1(\hat{k} \cdot \hat{k}) \\ &= 2(1) - 3(1) + 1(1) \\ &= 2 - 3 + 1 \\ &= 0 \end{aligned}$$

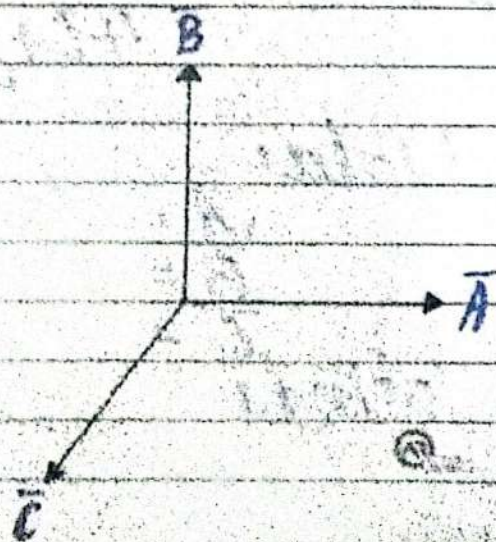
$$\vec{A} \cdot \vec{C} = (\hat{i} + \hat{j} + \hat{k}) (4\hat{i} + \hat{j} - 5\hat{k})$$

$$\begin{aligned} &= 4(\hat{i} \cdot \hat{i}) + (\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k}) \\ &= 4(1) + 1 - 5(1) \\ &= 4 + 1 - 5 \\ &= 0 \end{aligned}$$

$$\vec{B} \cdot \vec{C} = (2\hat{i} - 3\hat{j} + \hat{k}) (4\hat{i} + \hat{j} - 5\hat{k})$$

$$\begin{aligned} &= 8(\hat{i} \cdot \hat{i}) - 3(\hat{j} \cdot \hat{j}) - 5(\hat{k} \cdot \hat{k}) \\ &= 8(1) - 3(1) - 5(1) \\ &= 8 - 3 - 5 \\ &= 0 \end{aligned}$$

It means \vec{A} , \vec{B} and \vec{C} are mutually perpendicular.



2.10: Given that $A = \hat{i} - 2\hat{j} + 3\hat{k}$ and $B = 3\hat{i} - 4\hat{k}$ find the length of the projection of A on B

Data: $\bar{A} = \hat{i} - 2\hat{j} + 3\hat{k}$
 $\bar{B} = 3\hat{i} - 4\hat{k}$

Solve: Projection of \bar{A} on $\bar{B} = ?$

$$\text{Projection of } \bar{A} \text{ on } \bar{B} = \frac{\bar{B} \cdot \bar{A}}{|\bar{B}|}$$

$$\bar{B} \cdot \bar{A} = (\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (3\hat{i} - 4\hat{k})$$

$$3(\hat{i} \cdot \hat{i}) + (-12)(\hat{k} \cdot \hat{k})$$

$$3 + (-12)$$

$$\bar{B} \cdot \bar{A} = -9$$

$$|\bar{B}| = \sqrt{(3)^2 + (-4)^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$|\bar{B}| = 5$$

$$\text{Projection of } \bar{A} \text{ on } \bar{B} = \frac{-9}{5}$$

Vector Product: The product of two vectors result into a vector quantity is called vector product.

- It is called cross product.
- It is denoted by putting a cross between two vectors.

Mathematically, $\vec{A} \times \vec{B} = AB \sin \theta \hat{n}$

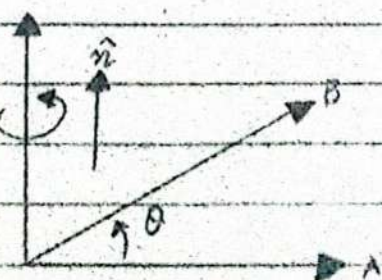
where A and B are the magnitudes of the vectors \vec{A} and \vec{B} and θ be the angle between the vectors.

' \hat{n} ' be the unit vector which is normal to the plane or perpendicular to the plane containing $\vec{A} \times \vec{B}$, can be find by **right hand rule**.

Explanation: Consider two vectors \vec{A} and \vec{B} making an angle ' θ ' with each other.

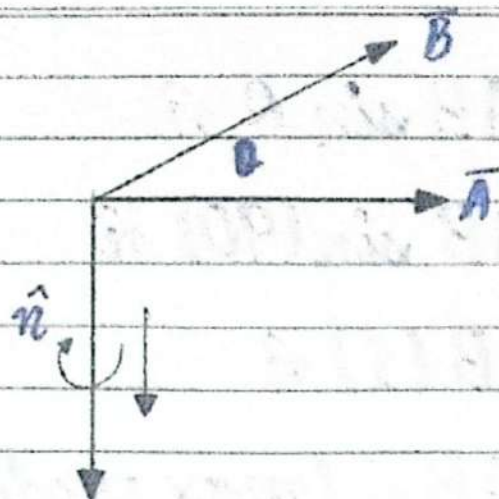
Rotate vector \vec{A} towards \vec{B} such that their product will be

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} \dots 1$$



Now rotate vector \vec{B} towards \vec{A} , so the product of $\vec{B} \times \vec{A}$ is

$$\vec{B} \times \vec{A} = BA \sin \theta \hat{n} \dots 2$$



It doesn't obey Associative Law

$$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$$

It obeys Distributive Law

$$\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$$

Now equ 1 and 2 we can write

$$AB \sin \theta \hat{n} = BA \sin \theta (-\hat{n})$$

here -ve sign shows that its direction is reversed
So

$$AB \sin \theta \hat{n} = -BA \sin \theta \hat{n}$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

It doesn't obey Commutative Law

Characteristics:

- It doesn't obey Commutative Law

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A}) \text{ (Anti Commutative)}$$

- When $\theta = 90^\circ$ (It means \vec{A} and \vec{B} are perpendicular)

$$\begin{aligned}\vec{A} \times \vec{B} &= AB \sin \theta \hat{n} \\ &= AB \sin(90) \hat{n} \\ &= AB(1) \hat{n}\end{aligned}$$

$$\vec{A} \times \vec{B} = AB \hat{n} \text{ (max product)}$$

- When $\theta = 0^\circ$ or 180° (It means \vec{A} and \vec{B} are parallel or anti-parallel)

$$\begin{aligned}\vec{A} \times \vec{B} &= AB \sin \theta \hat{n} \\ &= 0 \hat{n}\end{aligned}$$

$$\vec{A} \times \vec{B} = \vec{0} \text{ (Null vector)}$$

- In case of unit vectors

$$\begin{aligned}\hat{i} \times \hat{i} &= \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 \\ \hat{i} \times \hat{j} &= \hat{k}; \quad \hat{j} \times \hat{i} = -\hat{k} \\ \hat{j} \times \hat{k} &= \hat{i}; \quad \hat{k} \times \hat{j} = -\hat{i}\end{aligned}$$

$$\hat{k} \times \hat{i} = \hat{j}; \quad \hat{i} \times \hat{k} = -\hat{j}$$

$\hat{i} \times \hat{i} = 0$ (parallel means $\theta = 0$)

$$\begin{aligned}\vec{A} \times \vec{B} &= AB \sin \theta \hat{n} \\ \hat{i} \times \hat{i} &= |\hat{i}| |\hat{i}| \sin \theta \hat{n} \\ \hat{i} \times \hat{i} &= (1)(1) \sin(0) \hat{n} \\ \hat{i} \times \hat{i} &= 0 \hat{n} \\ \hat{i} \times \hat{i} &= \vec{0} \\ \hat{i} \times \hat{j} &= \hat{k} \text{ (perpendicular means } \theta = 90^\circ) \\ \vec{A} \times \vec{B} &= AB \sin \theta \hat{n} \\ \hat{i} \times \hat{j} &= |\hat{i}| |\hat{j}| \sin \theta \hat{n} \\ \hat{i} \times \hat{j} &= (1)(1) \sin(90) \hat{n} \\ \hat{i} \times \hat{j} &= 1(\hat{n}) \\ \hat{i} \times \hat{j} &= \hat{k}\end{aligned}$$

- In case of Rectangular Components:

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \times (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x B_x (\hat{i} \times \hat{i}) + A_x B_y (\hat{i} \times \hat{j}) + A_x B_z (\hat{i} \times \hat{k}) + A_y B_x (\hat{j} \times \hat{i}) + A_y B_y (\hat{j} \times \hat{j}) + A_y B_z (\hat{j} \times \hat{k}) + A_z B_x (\hat{k} \times \hat{i}) + A_z B_y (\hat{k} \times \hat{j}) + A_z B_z (\hat{k} \times \hat{k})$$

$$= A_x B_x (0) + A_x B_y (\hat{k}) + A_x B_z (-\hat{j}) + A_y B_x (-\hat{k}) + A_y B_y (0) + A_y B_z (\hat{i}) + A_z B_x (\hat{j}) + A_z B_y (-\hat{i}) + A_z B_z (0)$$

$$= A_y B_z (\hat{i}) + A_z B_y (-\hat{i}) + A_x B_z (-\hat{j}) + A_z B_x (\hat{j}) + A_x B_y (\hat{k}) + A_y B_x (-\hat{k})$$

$$= \hat{i} (A_y B_z + A_z B_y) + \hat{j} (A_x B_z + A_z B_x) + \hat{k} (A_x B_y + A_y B_x)$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= \hat{i} (A_y B_z - A_z B_y) - \hat{j} (A_x B_z - A_z B_x) + \hat{k} (A_x B_y - A_y B_x)$$

$$= \hat{i} (A_y B_z - A_z B_y) + \hat{j} (A_x B_z - A_z B_x) + \hat{k} (A_x B_y - A_y B_x)$$

- $|A \times B| = \text{Area of parallelogram}$

- Example:-

Torque is the example of cross product

$$\begin{aligned}\vec{\tau} &= \vec{r} \times \vec{F} \\ &= r F \sin \theta \hat{n}\end{aligned}$$

(MCQs)
 $\frac{1}{2} |A \times B| = \text{Area of Triangle}$

Torque (Moment of force)

"Turning effect of force produces in a body about an axis of rotation is called torque."

- It is denoted by tau τ

Convention:

- Anticlockwise torque is considered as +ve torque and clockwise torque is considered as -ve torque.

Mathematically it can be written as

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r F \sin \theta \hat{n}$$

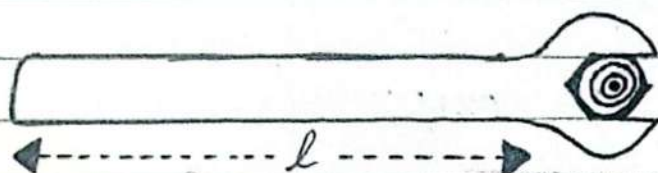
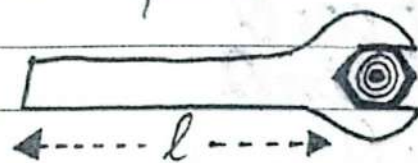
where $r F \sin \theta \hat{n}$ is the magnitude and \hat{n} be the unit vector which is always perpendicular to the plane can be found by right hand rule

- Its unit is "Nm."
- It is a vector quantity.

Moment Of Arm: The perpendicular distance between the line of action of force and the pivoted point or axis of rotation is called moment of arm.

Example:- Suppose a nut is tightened with the help of spanner as shown:-

- Length of the spanner is called moment of arm.



Greater the force will greater the torque and vice versa

- In this case torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

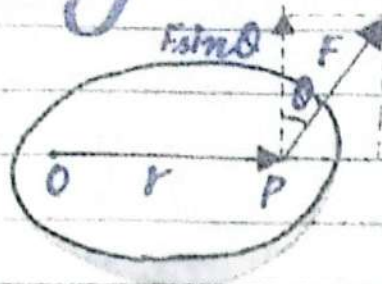
$$\tau = l F \sin \theta \hat{n}$$

- The line of acting of force acting on a fixed point
- then $l = 0$, So

$$\tau = (0) F \sin \theta$$

$$\tau = 0$$

Torque On a Rigid Body:



Consider a rigid body having position vector \vec{r} as shown, A force \vec{F} is applied to a point 'P'.

Resolve it into its components such that

$$F_x = F \cos \theta$$

$$F_y = F \sin \theta$$

Torque due to $F \cos \theta$ is zero because line of action of force is parallel to the displacement, so the only component which is responsible for torque is y -component which is

$$\vec{\tau} = r F \sin \theta \hat{n}$$

Short Q:- Write a note on torque and also write its significance?

Significance: Torque is the counter part of force for rotational. Just as force determines the linear acceleration produced in a body, the torque acting on a body determines its angular acceleration. If the body is at rest or rotating with uniform angular velocity, the angular acceleration will be zero. In this case the torque acting on the body will be zero.