Chapter NO 2

Vectors And Equilibrium

The branch of science which deals with the study of matter energy and their interactions is called **physics**

Physical Quantities

Those quantities which can be obserable and measurable are physical quantities.

Vector Quantities:

Those quantities which canbe compeletely described by its magnitude and direction (**number and unit**)

i.e. Force

Scalar Quantities:

Those quantities which can be completely described by its magnitude only is called scalar quantities

i.e. mass , distance

Representation Of Vectors

Symbolically

Graphically

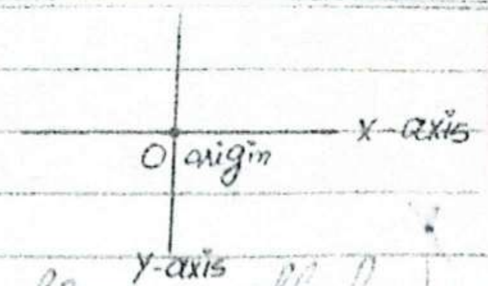
- It can be represented symbolically by a bold face character like **F**, **a** etc.
- It can also be represented by an arrow head or bar on it $\bar{F}, \bar{a}$ or $\vec{F}, \vec{a}$ etc.

A vector is represented b graphically by a directed line segment with an arrow head. The length of the line segment, according the the chosen scale, corresponds to the magnitude of the vector.

Rectangular Coordinate System

Two lines drawn at right angles to each other such that they concide with each other at point "O" named as origin as shown.

- One of the line is drawn horizontally named as x-axis and the other line is drawn vertically named as y-axis as shown.

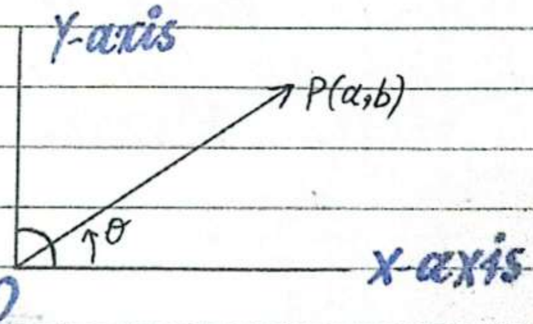


Short Question- What is called cartesian coordinate system?

this system of lines called rectangular coordinate system because both lines are prependicular to each other.
This is also called **Cartesian coordinate system.**

Inplane:

Now, consider a point p(a,b) having coordinates **a** and **b** making an angle **"θ"** with **x-axis** in an anticlock wise direction **as shown:**

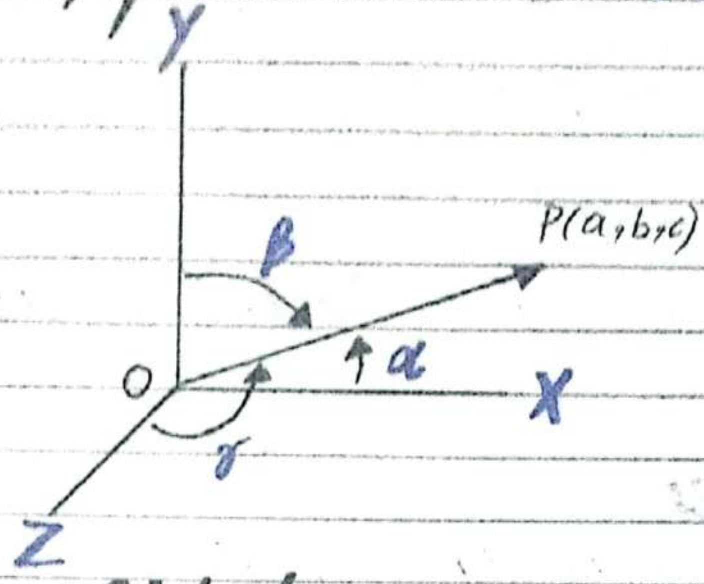


It means that if we start moving from origin we will move **'a'** distance on **x-axis** and **'b'** distance on **y-axis** to reach at point **p**

In space:

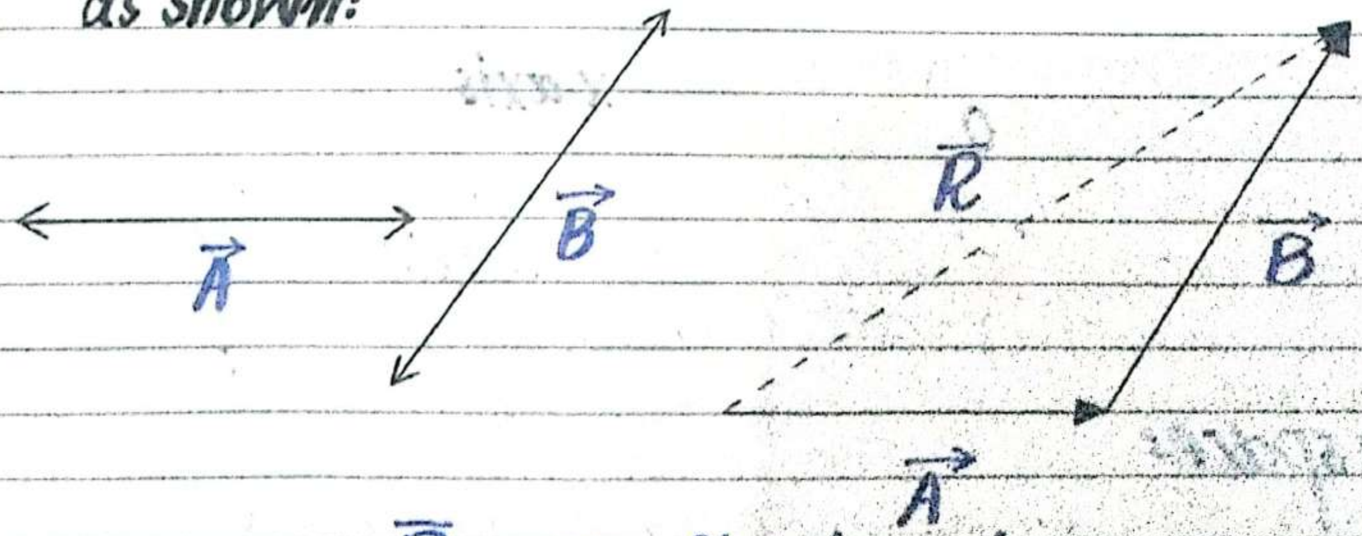
In sapace we required another axis which is prependicular to both of the axis named as **z-axis** **as shown:**

The point **p(a,b,c)** having coordinates (a, b and c) makes an angles (α, β, γ) with respect to **x-axis**, **y-axis** and **z-axis** respectively.



Addition of Vector:

Vector can be added by a rule called **"head to tail rule"**
In this rule joint the tail of the 2nd **vector** with the head of the 1st **vector** and so on **as shown:**

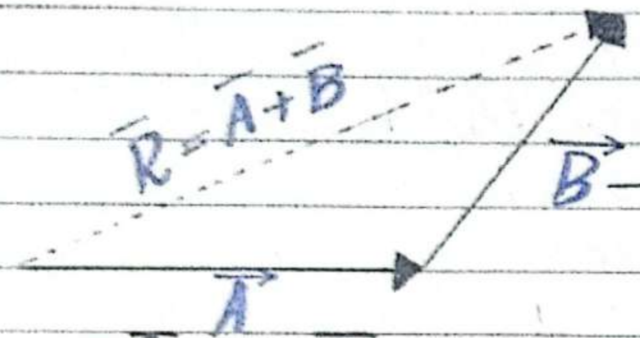


$\bar{R}$ = Resultant vector

Q: Prove that vector addition is commutative:

Proof: Consider two vectors $\vec{A}$ and $\vec{B}$ as shown:

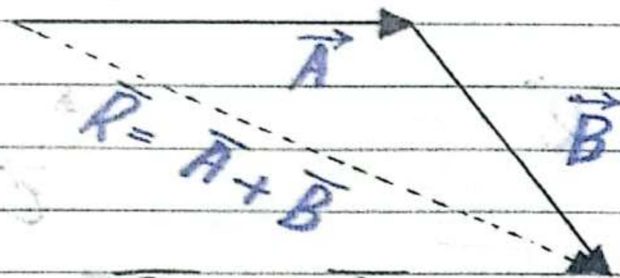
Add them by head to tail rule, such that we get a resultant vector $\vec{R}$ as shown:



$\vec{B} \longrightarrow$ I

$$\bar{R} = \bar{A} + \bar{B} \longrightarrow 1$$

Now add them in such a way that



$\longrightarrow$ II

$$\bar{R} = \bar{B} + \bar{A} \longrightarrow 2$$

From figure I and II we can draw,

From equation 1 and 2 we can write,

$$A + B = B + A$$

(MCQs)

1) Addition vector also obeys associative law

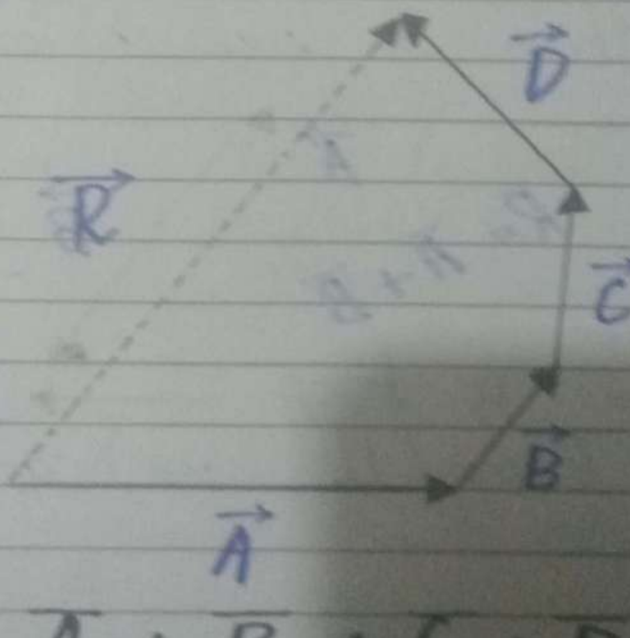
$$\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + \vec{C}$$

2) If $x = a + b$ and $y = b + a$ so what is the angle between x and y

0° degree angle

Resultant Vector:

Resultant vector is a single vector which is the same sum of all the vectors and it has the same effect of all the vectors to be added as shown:

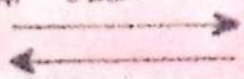


$$\bar{R} = \bar{A} + \bar{B} + \bar{C} + \bar{D}$$

Negative Vector:

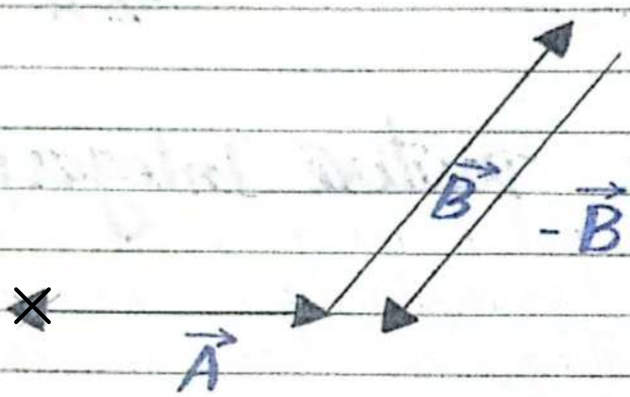
A vector is said to be negative if it have same length or magnitude with the other vector but opposite in direction as shown:

The angle between two negative vectors is 180° because they are antiparallel to each other



Vector Substraction:

Two vectors can be substract by adding a negative vector into other vector **as shown:**

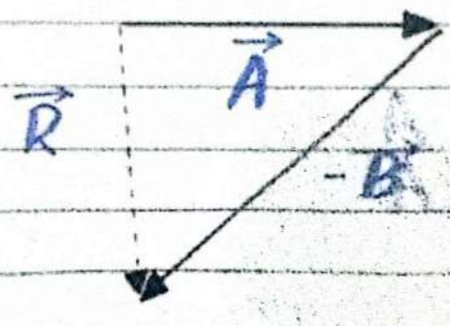


We can write it as

$$\bar{R} = \bar{A} - \bar{B}$$

$$\bar{A} + (-\bar{B})$$

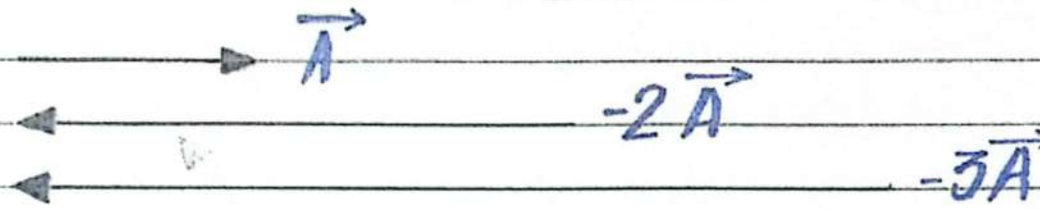
Make a vector B into negative and add it into. $\bar{A}$ **as shown:**



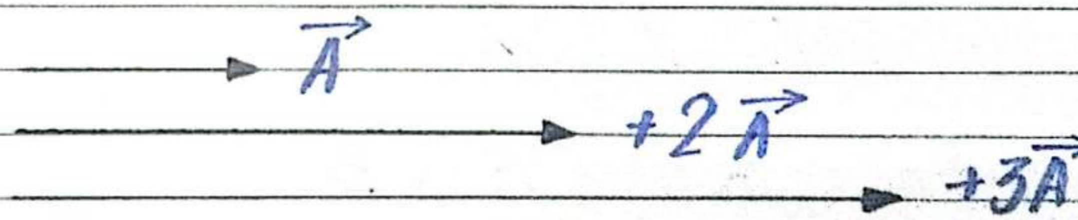
$$\bar{R} = \bar{A} - \bar{B}$$

Multiplication Of Vector:

If 'n' is a negative Integar: If a -ve integar multiplied with a vector 'A' then its length will increase upto that number but its direction is reversed **as shown:**



If 'n' is a positive Integar: If a +ve integar multiplied by a vector 'A' then its length will increase upto that number having same direction as of A **as shown:**



Unit Vector: A vector whose magnitude is one and gives **specified direction** is called unit vector.

It is dentod by a cap of a letter '$\hat{A}$'

$$\hat{A} = \frac{\vec{A}}{|A|}$$

Date:

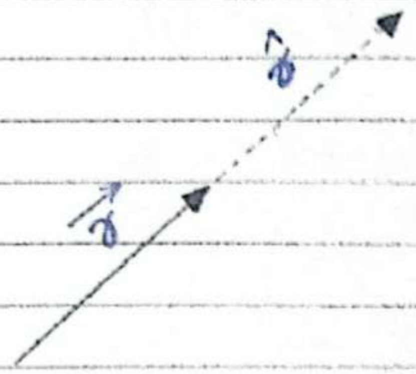
$\vec{A}$ = vector A

$|A|$ = magnitude of vector

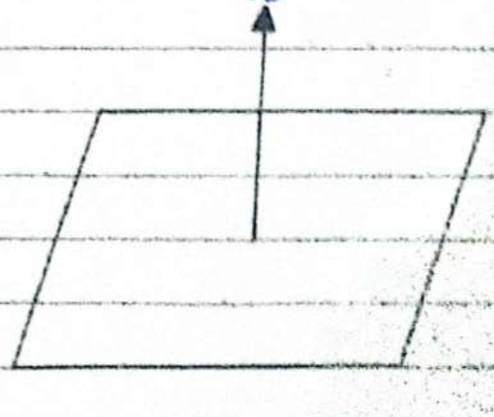
$\hat{A}$ = Unit vector A

$\hat{i}, \hat{j}, \hat{k}$ are the unit vectors of x-axis y-axis and z-axis respectively.

In general $\hat{r}$ is the unit vector used to represent the direction of any vector $\vec{r}$ as shown:



'$\hat{n}$' is the unit vector which is used to represent the direction of vector perpendicular to the plane as shown:



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Numerical:

2.3 What is the unit vector in the direction of the vector $\vec{A} = 4\hat{i} + 3\hat{j}$?

Data:

$$\vec{A} = 4\hat{i} + 3\hat{j}$$

To Find: $\hat{A} = ?$

Solution:

$$\hat{A} = \frac{\vec{A}}{|A|}$$

$$|A| = \sqrt{(4)^2 + (3)^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

$$\hat{A} = \frac{\vec{A}}{|A|}$$

$$\boxed{\hat{A} = \frac{4\hat{i} + 3\hat{j}}{5} \quad \textbf{Ans}}$$

Null Vector:
A vector whose magnitude is zero and gives arbitary direction.
It is denoted by $\vec{0}$.

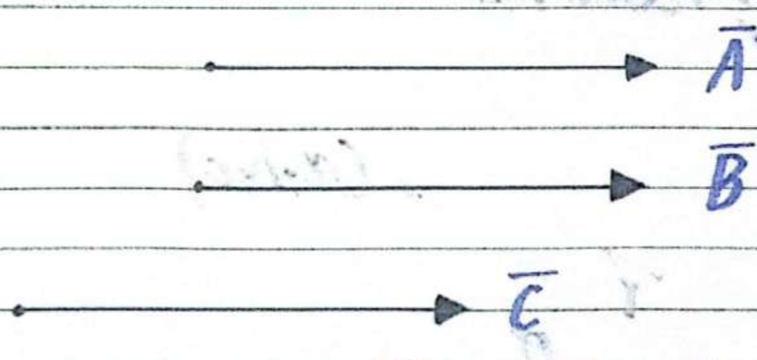
$$\bar{A} + (-\bar{A}) = 0$$

Arbitary:- Can not be drawn on page.
If we get a close shape after adding the vectors its resultant will be Null vector.

MCQs:- How many minimum number of vectors gives 0 resultant?
(a) 2 (b) 3 (c) 4 (d) 5

Equal Vector:
Two vectors are said to be equal if they have same magnitude and same direction regardless of their initial position.

The angle between two parallel vectors is 0 degree
All the equal vectors are parallel but all the parallel vectors may or may not be equal

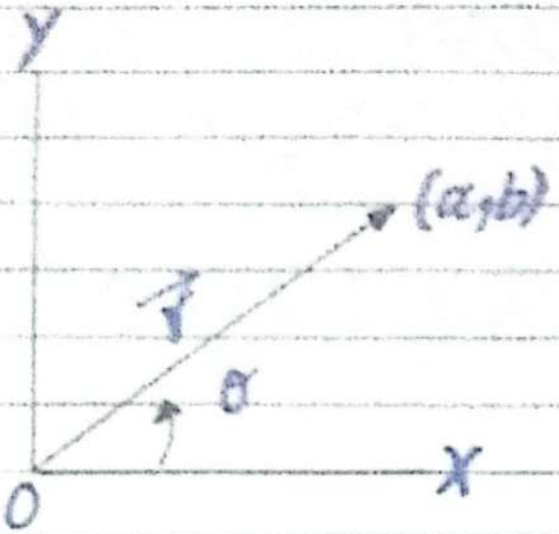


$$\bar{A} = \bar{B} = \bar{C}$$

Position Vector:
A vector which describe the location of an object with respect to origin.
• It is denoted by $\vec{r}$

In 2-Dimensions:

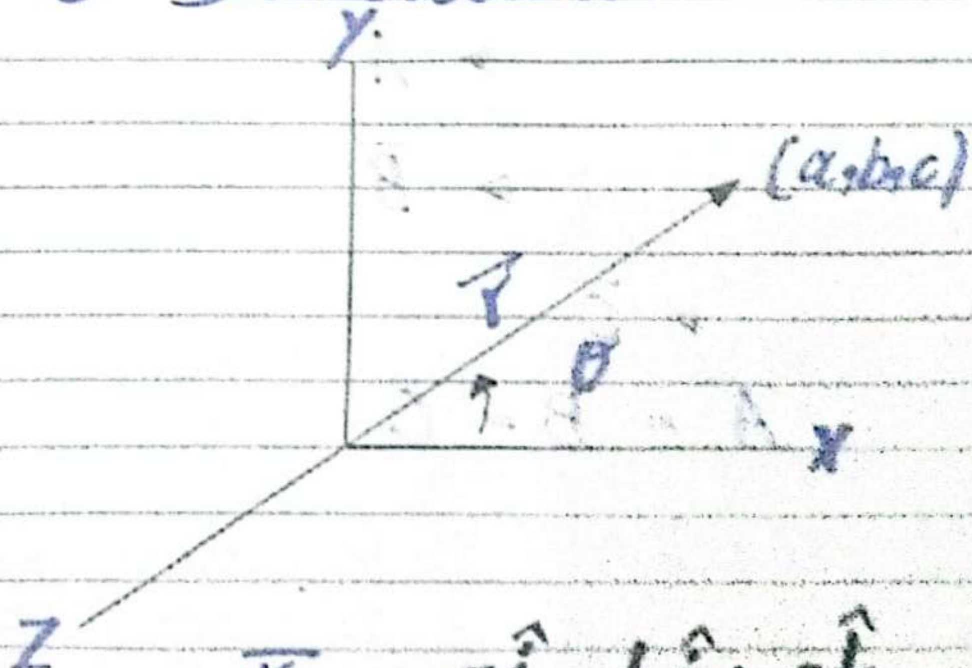
Origin is the point where position starts.



$$\bar{r} = a\hat{i} + b\hat{j}$$

$$|r| = \sqrt{(a)^2 + (b)^2}$$

In 3-Dimensions



$$\bar{r} = a\hat{i} + b\hat{j} + c\hat{k}$$

$$|r| = \sqrt{(a)^2 + (b)^2 + (c)^2}$$

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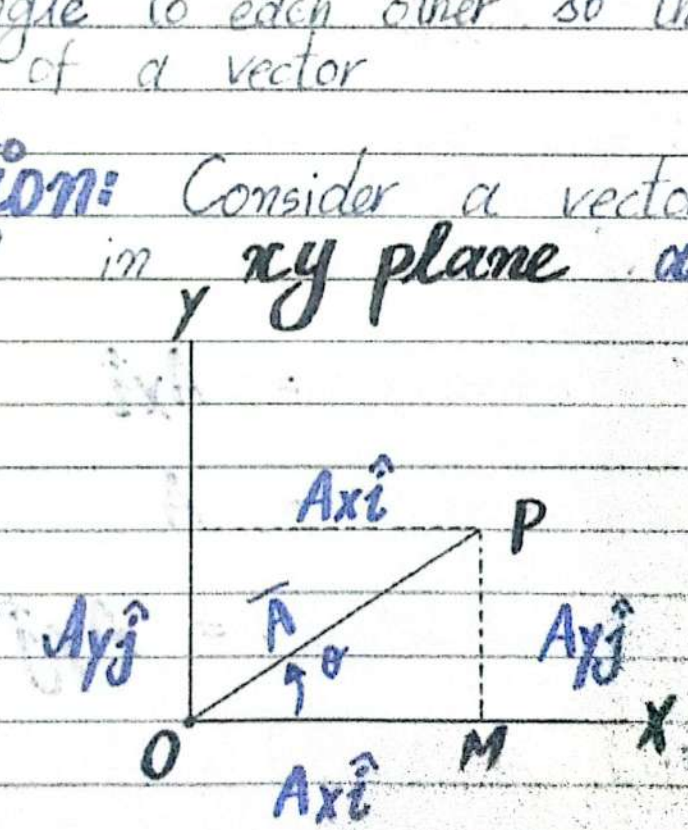
Rectangular Components Of A Vector:

The process of splitting a single vector into two or more than two vectors is called **resolution of vector**, usually a vector resolved into two vectors called **components of a vector**.

One of the component is along **x-axis** called horizontal component or x-component other component is along **y-axis** called vertical component or y-component.

These components are at right angle to each other so they rectangular components of a vector.

Explaination: Consider a vector $\vec{A}$ named as $\overline{OP}$ in **xy plane** as shown:



• Draw the projection of vector **A** along **x** and **y-axis** such that

$A_x\hat{i}$ = Projection of vector $\bar{A}$ along x-axis

$A_y\hat{j}$ = Projection of vector $\bar{A}$ along y-axis

We observe that

rectangular C.S Simple C.S

A vector may or may not be the resultant vector of their component

so, we can write

$$\bar{A} = A_x\hat{i} + A_y\hat{j}$$

MCQs

A vector have ___ maximum retangular components?

(a) 2 (b) 3 (c) 4 (d) 5

A vector have ___ component?

Ans:- Infinite.

To find the x and y- component of vector $\bar{A}$ we consider the triangle ΔOPM such that

OM = base = $A_x\hat{i}$

OP = hypot = A

MP = perpendicular = $A_y\hat{j}$

X-component:- To find the x-component of a vector we consider

$$\frac{\text{base}}{\text{hyp}} = \frac{OM}{OP} = \cos\theta$$

$$\frac{A_x}{A} = \cos\theta$$

$$A_x = A\cos\theta$$

Y-component:- To find the y-component again consider the same triangle ΔOPM such that:-

$$\frac{Perp}{hyp} = \frac{MP}{OP} = \sin\theta$$

$$\frac{A_y}{A} = \sin\theta$$

$$A_y = A\sin\theta$$

Magnitude:- To find the magnitude of vector $\overline{A}$ we apply **pathagouras theorem**

$$(hyp)^2 = (base)^2 + (prep)^2$$

$$(A)^2 = (A_x)^2 + (A_y)^2$$

$$A = \sqrt{(A_x)^2 + (A_y)^2}$$

Direction:- To find the direction of vector, consider projection so

$$\frac{Perp}{base} = \tan\theta$$

$$\tan\theta = \frac{A_y}{A_x}$$

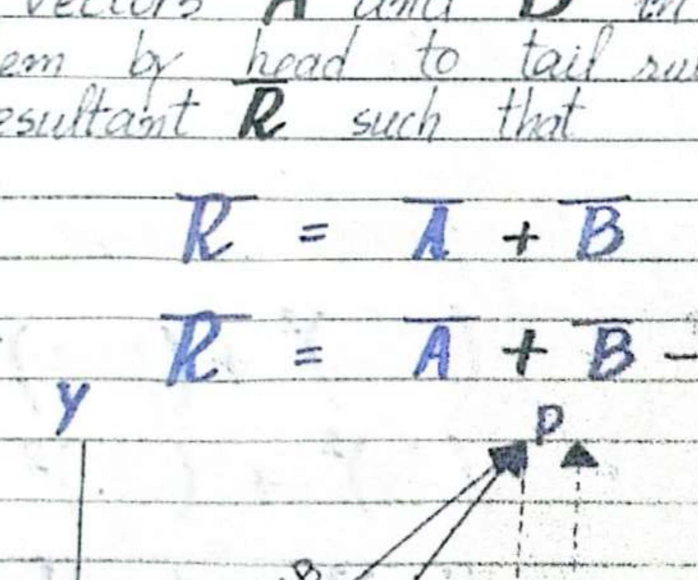
$$\theta = \tan^{-1}\frac{A_y}{A_x}$$

long Q: Vector Addition by rectangular Components:-

Two vectors $\overline{A}$ and $\overline{B}$ in xy plane Add them by head to tail rule and we get resultant $\overline{R}$ such that

$$\overline{R} = \overline{A} + \overline{B}$$

$$\overline{R} = \overline{A} + \overline{B} \longrightarrow (1)$$



Draw the projection of vector A, B and R

A_x, B_x, R_x be the x-component of vector $\overline{A}$, $\overline{B}$ and $\overline{R}$ and Similarly A_y, B_y, R_y be the y- component of vector $\overline{A}$, $\overline{B}$ and $\overline{R}$.

Fromfigure, we can write

$$\overline{OR} = \overline{OQ} + \overline{QR}$$

$$R_x = A_x + B_x \longrightarrow 2$$

$$R_x\hat{i} = A_x\hat{i} + B_x\hat{i}$$

$$R_x\hat{i} = (A_x + B_x)\,\hat{i}$$

Similarly,

$$\overline{RP} = \overline{RS} + \overline{SP}$$

$$R_y = A_y + B_y \longrightarrow 3$$

$$R_y\hat{j} = A_y\hat{j} + B_y\hat{j}$$

$$R_y\hat{j} = (A_y + B_y)\,\hat{j}$$

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Now we know that, A vector is the resultant vector of their components.

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

Put equation **2** and **3** in above equation.

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

Magnitude: To find the magnitude of the resultant vector $\vec{R}$ we use

$$|R| = \sqrt{(R_x)^2 + (R_y)^2}$$

$$|R| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

Direction: To find the direction of the resultant vector $\vec{R}$ we can write

$$\tan\theta = \frac{R_y}{R_x}$$

$$\tan\theta = \frac{A_y + B_y}{A_x + B_x}$$

$$\theta = \tan^{-1}\left[\frac{A_y + B_y}{A_x + B_x}\right]$$

For any number of coplanar vectors we can write

$$R = \sqrt{(A_x + B_x + C_x + \ldots)^2 + (A_y + B_y + C_y + \ldots)^2}$$

and $\theta = \tan^{-1} \sqrt{\frac{(A_y + B_y + C_y + \ldots)}{(A_x + B_x + C_x + \ldots)}}$

Page # 29 : Short Question

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Numericals

2.4: Two particles are located at $r_1 = 3\hat{i} + 7\hat{j}$ and $r_2 = -2\hat{i} + 3\hat{j}$ respectively. Find both the **magnitude** of the vector $(r_2 - r_1)$ and its **orientation** with respect to the x-axis.

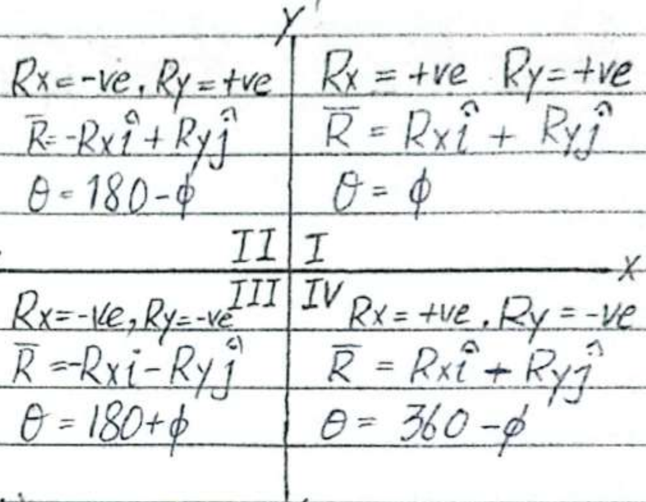
Data:

$$r_1 = 3\hat{i} + 7\hat{j}$$

$$r_2 = -2\hat{i} + 3\hat{j}$$

To Find:

$$r_2 - r_1 = ?$$



Solve:

$$r_2 - r_1 = (-2\hat{i} + 3\hat{j}) - (3\hat{i} + 7\hat{j})$$
$$= -2\hat{i} + 3\hat{j} - 3\hat{i} - 7\hat{j}$$
$$= -2\hat{i} - 3\hat{i} + 3\hat{j} - 7\hat{j}$$
$$= -5i - 4j$$

$$|r_2 - r_1| = \sqrt{(-5)^2 + (-4)^2}$$
$$= \sqrt{25 + 16}$$
$$= \sqrt{41}$$
$$= 6.4$$

$$\phi = \tan^{-1} \frac{r_y}{r_x}$$

$$\phi = \tan^{-1} \frac{4}{5}$$

$$\phi = \tan^{-1}(0.8)$$

$$\phi = 39^\circ$$

$$\theta = 180 + \phi$$

$$\theta = 180 + 39^\circ$$

$$\theta = 219^\circ$$