Q No. 5.1: Explain the difference between tangential velocity and the angular velocity. Write a formula which relates them. (GUJ-2017)

Difference between Tangential Velocity and Angular Velocity

Tangential velocity gives the limiting rate of change in linear displacement of a body moving on circular path and is directed along tangent to circle. It is denoted by v and is given by

$$v=\frac{\Delta S}{\Delta t}$$

Its S.I unit is ms^{-1} Whereas, angular velocity gives the limiting rate of change in angular displacement of a body moving on circular path and is directed along axis of rotation. It is denoted by ω and is given by

$$\omega=\frac{\Delta\theta}{\Delta t}$$

its S.I unit is $rads^{-1}$.

The tangential velocity and angular velocity are related as

$$v = \omega r$$

Q No.5.2: Explain what is meant by centripetal force and why it must be furnished to an object if the object is to follow a circular path? (BWP-2019) (DGK-2016)

Centripetal Force

The force needed to bend normally the straight path of the object into a circular path is called centripetal force. Mathematically, centripetal force can be written as:

$$F_c = m\frac{v^2}{r}$$

Centripetal force must be furnished to an object if the object is to follow a circular path because without it, the object would be moving along tangent to circle. It is centripetal force which keeps the body moving along circular path.

Q No. 5.3: What is meant by moment of inertia? Explain its significance. (FSB-2021) (FSB-2021) (SHW-2021) (GUJ-2021) (FSB-2019) (DGK-2019) (SHW-2019) (SGD-2019) (MUL-2019) (MUL-2019) (RWP-2019) (FED-2018) (GUJ-2018) (FSB-2017) (LHR-2017) (RWP-2017) (FSB-2016) (FSB-2016) (DGK-2016)

Moment of inertia of a moving particle of mass (m) with constant angular acceleration (α) in a circle of radius (r).

$$\tau=(mr^2)\alpha \qquad ...(1)$$

Significant of Moment of Inertia

The quantity mr^2, is the rotational analogue of mass m in linear motion and is known as moment of inertia. Moment of inertia plays the same role in rotational motion as the mass in linear motion. It is denoted by I. or

$$I = mr^2$$

Putting $I = mr^2$ in equation (1) we get

$$\tau=I\alpha$$

Which is rotational analogue of Newton's second law of motion F = ma. Here F is replaced by τ, a by α and m by mr^2.

Q No.5.4: What is meant by angular momentum? Explain the law of conservation of angular momentum.

Angular Momentum

Quantity of angular motion is called angular momentum. It is also defined as cross product between position vector and moment of linear momentum. Angular momentum is vector quantity, directed along axis of rotation and is denoted by L .

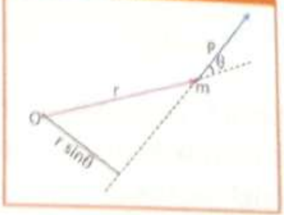
Angular momentum of a particle

The angular momentum $\vec{L}$ of a particle of mass m moving with velocity $\vec{v}$ and momentum $\vec{p}$ (Figure) relative to the origin O is defined as

$$\vec{L} = \vec{r} \times \vec{p}$$

Where r is the position vector of the particle at that instant relative to the origin O. Angular momentum is a vector quantity.

Figure



State Law of Conservation of Angular momentum

It is fundamental principle of physics which states that angular momentum of an isolated system of a rotating body remains constant. Or mathematically

$$\vec{L} = constant$$

Q No. 5.5: Show that orbital angular momentum L_o = mvr.

Consider a particle of mass m moving on circular path of radius r with its centre at O then its angular momentum is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} = rp\sin\theta n$$

The magnitude of angular momentum is

$$L = rp\sin\theta$$

Since

$$\vec{r} \perp \vec{p}$$

So $\theta = 90^\circ$

Hence

$$L_o = rp\sin 90^\circ$$

$$L_o = rp$$

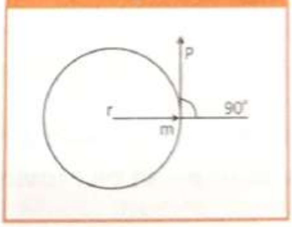
But

$$p = mv$$

$$L_o = rmv$$

$$L_o = mvr$$

Figure



Q No.5.6: Describe what should be the minimum velocity, for a satellite, to orbit close to the Earth around it.

Consider a satellite orbiting around earth in an orbit closer to the surface of Earth i.e. r = R, then centripetal acceleration of the satellite is given by

$$a_c = \frac{v^2}{r} = g$$

$$v = \sqrt{gR}$$

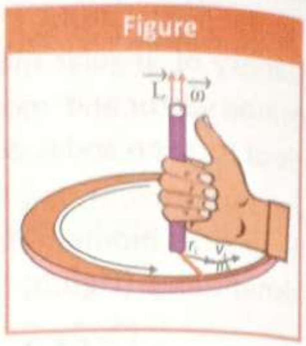
$$v = \sqrt{9.8\ ms^2 \times 6400 \times 10^3\ m}$$

$$v = 7.9 \times 10^3\ m\ s^{-1}$$

$$v = 7.9\ km\ s^{-1}$$

This is the minimum velocity necessary to put a satellite into the orbit around the earth and is called critical velocity

Q No.5.7: State the direction of the following vectors in simple situations; angular momentum and angular velocity.



In simple situation the direction of vectors; angular momentum and angular velocity is along axis of rotation as shown in figure and can be determined by right hand rule stated as: Grasp axis of rotation in right hand with fingers curling in the direction of rotation; the thumb points in the direction of angular momentum and angular velocity.

Q No.5.8: Explain why an object, orbiting the Earth, is said to be freely falling. Use your explanation to point out why objects appear weightless under certain circumstances.

Weightlessness: When a satellite is falling freely in space, everything within this freely falling system will appear to be weightless.

Freely falling body: An Earth's satellite is freely falling object. An object orbiting around the Earth is said to be freely falling body, because it is instantly accelerating toward center of the Earth with radial acceleration a=g, the free fall acceleration. If the object is thrown fast enough parallel to the Earth, the curvature of its path will match the curvature of the Earth. In this case the spaceship will simply circle round the Earth.

Q No.5.9: When mud flies off the tyre of a moving bicycle, in what direction does it fly? Explain. (BWP-2021) (DGK-2021) (LHR-2021) (RWP-2021) (SHW-2021) (LHR-2018) (LHR-2017) (LHR- 2017) (SGD-2017) (LHR-2017) (SGD-2016)

Reason

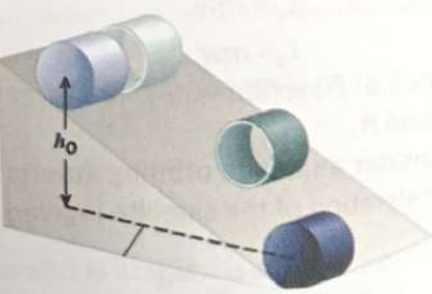
When mud flies off the tyre of moving bicycle it flies off along tangent to the tyre because linear velocity of each point on the tyre is tangential due to its inertia. Thus, every particle on the tyre tends to move along straight path along tangent to the tyre. It is centripetal force which bend the straight path of the particles on the tyre into circular path. But when mud splits up the tyre, it experiences no centripetal force and follows a straight path along tangent to the tyre.

Q No.5.10: A disc and a hoop start moving down from the top of an inclined plane at the same time. Which one will be moving faster on reaching the bottom? (MUL-2021) (MUL-2019) (RWP-2018) (FED-2017) (FSB-2016) (LHR-2016) (RWP-2016)

When disc and hoop are rolling down a frictionless incline plane, their P.E at top are converted into their rotational kinetic energies and translational kinetic energies at bottom. If h be the height of incline plane, then according to law of conservation of energy.

For Disc

$$P.E_d=(K.E_t)_d+(K.E_{rot})_d$$



$$mgh=\frac{1}{2}mv_d^2+\frac{1}{4}mv_d^2$$

$$gh=\frac{1}{2}v_d^2+\frac{1}{4}v_d^2$$

$$gh=\frac{3}{4}v_d^2$$

$$v_d=\sqrt{\frac{4}{3}gh} \quad ...(1)$$

For hoop

$$P.E_h=(K.E_t)_h+(K.E_{rot})_h$$

$$mgh=\frac{1}{2}mv_h^2+\frac{1}{2}mv_h^2$$

$$gh=\frac{1}{2}v_h^2+\frac{1}{2}v_h^2$$

$$gh = v_h^2$$

$$v_h = \sqrt{gh} \qquad \ldots(2)$$

From equation (1) and (2), we see that

$$v_d > v_h$$

This shows that disc will be moving faster than hoop on reaching the bottom of the inclined plane.

Q No.5.11: Why does a diver change his body position before and after diving in the pool? (DGK-2021) (MUL-2021) (SGD-2021) (BWP-2019) (SGD-2019) (MUL-2019) (GUJ-2019) (DGK-2019) (FSB-2018) (DGK-2018) (RWP-2018) (SHW-2018) (SGD-2018) (MUL-2018) (LHR-2017) (FED-2016) (RWP-2016) (RWP-2016) (GUJ-2016)

According to law of conservation of momentum

$$I_1\omega_1 = I_2\omega_2 = \text{constant}$$

$$I\omega = \text{constant}$$

$$\omega \propto \frac{1}{I}$$

$$\omega \propto \frac{1}{r^2}$$

So, the driver change his body position before and after diving in the pool in order to conserve his angular momentum. When diver pushes off the board with small angular velocity ω_1 about a horizontal axis through its centre of gravity. Upon lifting off from the board, the diver's arms and legs are fully extended which means that the diver has large moment of inertia I_1 about this axis. The moment of inertia is considerably reduced to a new value I_2 when the legs and arms are drawn into closed tuck position and diver must spin faster with speed ω_2.

Q No. 5.12: A student holds two dumb-bells with stretched arms while sitting on a turn table. He is given a push until he is rotating at certain angular velocity. The student then pulls the dumb bells towards his chest. What will be the effect on rate of rotation?

According to law of conservation of momentum

$$I_1\omega_1 = I_2\omega_2 = \text{constant}$$

$$I\omega = \text{constant}$$

$$\omega \propto \frac{1}{I}$$

Or $$\omega \propto \frac{1}{r^2}$$

Figure

So, the student change his arm position in order to conserve his angular momentum. Initially, the arms of the students are fully extended, so he has large moment of inertia but angular velocity is small. When the student curls his body, the moment of inertia reduces. In order to conserve the angular momentum, the value of angular velocity increases to such that

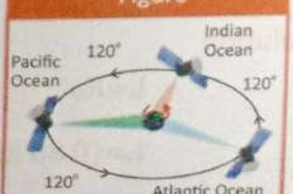
$$I_1\omega_1 = I_2\omega_2 = \text{constant}$$

Thus, the rate of rotation will increase.

Q No. 5.13: Explain how many minimum numbers of geo-stationary satellites are required for global coverage of T.V transmission. (SGD-2019) (GUJ-2019) (GUJ-2019) (LHR-2018) (SGD-2018) (BWP-2017) (DGK-2017) (RWP-2017) (SGD-2017) (FSB-2016)

Three correctly positioned geostationary satellites are required for global coverage of T.V transmission because one satellite covers 120° of longitude. Three correctly positioned geostationary satellites cover whole palpated earth.

Figure



$v = 98.99\ ms^{-1}$

Q No. 5.8: The Moon orbits the Earth so that the same side always faces the Earth determine the ratio of its spin angular momentum (about its own axis) and its orbital angular momentum. (in this case treat the Moon as the particle the earth) distance between the earth and the moon is 3.85×10^8 m. Radius of the Moon is 1.74×10^6 m.

Given Data

Spin angular velocity = Orbital Angular velocity $\Rightarrow \omega_s = \omega_o$

Distance between earth and moon = R= 3.85×10^8 m

Radius of moon = r= 1.74×10^6 m

To Find

$$\text{Ratio} = \frac{L_s}{L_o} = ?$$

Solution

$$\frac{L_s}{L_o} = \frac{I_s\omega_s}{I_o\omega_o} = \frac{I_s}{I_o}$$

$\because$ Spin angular velocity = Orbital Angular velocity $\Rightarrow \omega_s = \omega_o$

$$= \frac{\frac{2}{5}mr^2}{mR^2}$$

$$= \frac{0.4 \times r^2}{R^2}$$

$$= \frac{0.4 \times (1.74 \times 10^6)^2}{(3.84 \times 10^8)^2}$$

$$= 8.2 \times 10^{-6}$$

Q No. 5.9: The Earth rotates on its axis once a day. Suppose by some process the Earth contracts so that its radius is only half as large as at present. How fast will it be rotating then?

Given Data

Let

Initial Radius = r_i = R

Final Radius = r_f = R/2

Initial Period = T_i = 24 H

To Find

Final Period = T_f = ?

Solution

By Law of conservation of angular momentum

$$I_i\omega_i = I_f\omega_f$$

$$(\frac{2}{5}mr_i^2)(\frac{2\pi}{T_i}) = (\frac{2}{5}mr^2{}_f)(\frac{2\pi}{T_f})$$

$$\frac{r^2{}_i}{T_i} = \frac{r^2{}_f}{T_f}$$

$$\frac{R_2}{24} = \frac{(R/2)^3}{T_1}$$

$$\frac{R^3}{24} = \frac{R^3}{4T_1}$$

$$\frac{1}{6} = \frac{1}{T_1}$$

$$T_1 = 6h$$

Q No. 5.10: What should be the orbiting speed to launch a satellite in circular orbit 900 km above the surface of the Earth? (Take mass of the Earth as 6.0×10^{24} and its radius as 6400 km). (SHW-2019) (FED-2018) (DGK-2018) (LHR-2018) (SGD-2016) (BWP-2016)

Given Data

Height = h = 900km = 900×10^3 m

Radius of Earth = R_e = 6400 km = 6400×10^3 m

Mass of Earth = M_e = 6.0×10^{24} kg

To Find

Speed = v = ?

Solution

Distance from center of Earth to satellite $r = R_e + h$

$= 6400\text{ km} + 900\text{ km} = 7300\text{ km} = 7300 \times 10^3\text{ m}$

Now orbital velocity is given below

$$v = \sqrt{\frac{Gm_e}{r}}$$

$$v = \sqrt{\frac{(6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2})(6.0\text{x}10^{24}\text{kg})}{7300\times10^3\text{m}}}$$

$$v = 7404.18\text{ ms}^{-1}$$

$$v = 7.40 \times 10^3\text{ ms}^{-1}$$

$$v = 7.40\text{kms}^{-1}$$