Absolute Potential Energy:

Workdone by gravitational force in displacing an object from that point to infinity where force of gravity becomes equal to zero is called absolute potential energy.

Explanation:

Workdone by gravitational force is only true constant near to earth surface where the value of g becomes constant or force of gravity becomes constant but we know from **Newton's gravitational law**

$$F_g = \frac{GMm}{r^2}$$

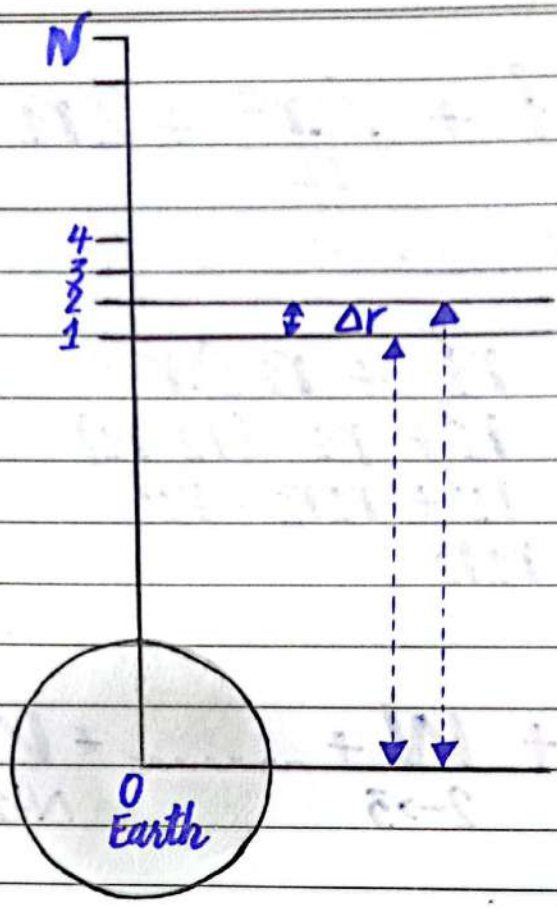
$$F_g \propto \frac{1}{r^2}$$

It means gravitational force varies as we move upward because value of g become smaller.

Thus we divide the total distance into **'n'** number of small steps as shown:-

Δr is the difference between the two steps and the value of **'g'** or gravitational force became constant in this step

Date:



We have to calculate the total workdone in all steps which is equal to the absolute potential energy

We consider distance So, $r = \frac{r_1 + r_2}{2}$

$$\Delta r = r_2 - r_1$$

$$r_2 = \Delta r + r_1$$

$$r = \frac{r_1 + \Delta r + r_1}{2}$$

$$r = \frac{2r_1 + \Delta r}{2}$$

$$r = r_1 + \frac{\Delta r}{2}$$

Taking square on both the sides

$$r^2 = \left(r_1 + \frac{\Delta r}{2}\right)^2$$

$$r^2 = r_1^2 + \frac{\Delta r^2}{4} + 2r_1 \frac{\Delta r}{2}$$

$\frac{\Delta r^2}{4} \approx 0$ (because it is very small value)

$$r^2 = r_1^2 + r_1 \Delta r$$
$$= r_1^2 + r_1 (r_2 - r_1)$$
$$= r_1^2 + r_1 r_2 - r_1^2$$
$$r^2 = r_1 r_2$$

Now the workdone which want to be calculated is

$$W_T = W_{1\to 2} + W_{2\to 3} + \ldots\ldots + W_{N-1\to N}$$

$$W_{1\to 2} = F_y . \Delta r$$
$$= F \Delta r \cos\theta$$
$$\theta = 180^\circ$$
$$= F \Delta r \cos(180)$$
$$= -F\Delta r$$
$$= -\frac{GMm}{r_1 r_2} (r_2 - r_1)$$

$$= -GMm \left[\frac{r_2 - r_1}{r_1 r_2}\right]$$

$$W_{1\to 2} = -GMm \left[\frac{r_2}{r_1 r_2} - \frac{r_1}{r_1 r_2}\right]$$

$$W_{1\to 2} = -GMm \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$W_{3\to 4} = -GMm \left[\frac{1}{r_3} - \frac{1}{r_4}\right]$$

Date:

$$W_{N-1 \to N} = -GMm\left(\frac{1}{r_{N-1}} - \frac{1}{r_N}\right)$$

Now the total workdone will be

$$W_T = W_{1\to 2} + W_{2} + \ldots + W_{N-1\to N}$$

$$= -GMm\left(\frac{1}{r_1} - \cancel{\frac{1}{r_2}} + \cancel{\frac{1}{r_2}} - \cancel{\frac{1}{r_3}} + \cancel{\frac{1}{r_3}} \ldots - \cancel{\frac{1}{r_{N-1}}} + \cancel{\frac{1}{r_{N-1}}} - \frac{1}{r_N}\right)$$

$$= -GMm\left(\frac{1}{r_1} - \frac{1}{r_N}\right)$$

$r_N = \infty$ Infinity

$$W_T = -GMm\left(\frac{1}{r_1} - \frac{1}{\infty}\right) \qquad \therefore \frac{1}{\infty} = 0$$

$$W_T = \frac{-GMm}{r_1}$$

Absolute potential energy is denoted by "U_g" so

$$U_g = \frac{-GMm}{r_1}$$

Note that when r increases, U becomes less negative ie., U increases. It means when we rise a body

Date: ______ Mon Tue Wed Thu Fri Sat Sun

above the surface of the Earth its P.E. increases.
The choice of zero point is arbitrary and difference of P.E. from the one point to another is significant, wether we consider the surface or the Earth or the point at infinity as zero P.E. reference, the change in P.E. as we move a body above the surface of the Earth, we always be positive.
Now the absolute potential energy on the surface of the Earth is found by putting $r = R$ (Radius of the Earth)

$$\text{Absolute potential energy} = U_g = -\frac{GMm}{R}$$

The negative sign shows that the Earth's gravitational field for mass m is attractive.
The above expression gives the work or the energy required to take the body out of the Earth's gravitational field, where its potential energy with respect to Earth is zero.

Date:

Note

Escape Velocity:

The minimum initial velocity provided to the body from the surface of earth from which it moves out the earth's gravitational field is called escape velocity.

- It is denoted by V_{esc}

Explanations:

We know that, the energy provided from the surface is the K.E. which is

$$K.E. = \frac{1}{2} m V^2 \longrightarrow 1$$

We also know that to reach the infinity point the workdone on a body is equal to the Absolute P.E. So increase in potential energy will be

$$\text{increase in P.E.} = 0 - \left(-\frac{GMm}{r}\right)$$

$$\text{increase in P.E.} = \frac{GMm}{r} \longrightarrow 2$$

From equ 1 and 2, we want to determine the escape velocity it must be equal

$$\frac{1}{2} m V_{esc}^2 = \frac{GMm}{r}$$

Date:

$$V_{esc}^2 = \frac{2GM}{r}$$

$$V_{esc} = \sqrt{\frac{2GM}{r}} \longrightarrow 3$$

By replacing $r = R$ (consider it the infinity point)

$$V_{esc} = \sqrt{\frac{2GM}{R} \times \frac{R}{R}}$$

$\therefore Fg = \frac{GMm}{R^2}$

$$V_{esc} = \sqrt{\frac{2GMR}{R^2}}$$

$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$

So, $V_{esc} = \sqrt{2gR}$

$g = \frac{GM}{R^2}$

As we know that

$$g = 9.8ms^{-1}$$

$$R = 6400 km$$

the escape velocity will be

$$11.2 kms^{-1}$$

From equ 3 we oberve

$$r = R + h$$

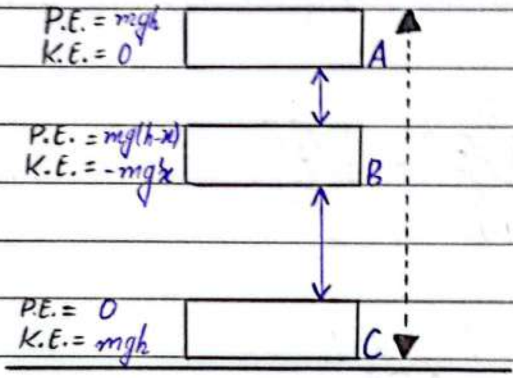
$$V_{esc} = \sqrt{\frac{2GM}{R+h}}$$

Law Of Conservation Of Energy:

Statement: Energy can be produced nor be destroyed but it can be change from one form to another but total energy remains constant.

Interconversions Of Energy:

Consider a body of mass placed at height '**h**' at point '**A**' from surface of earth as shown:



By ignoring air friction we observe that the body is falling downward.
So we have to calculate the energy at point **A, B and C** total energy as well.

At point A:

Now we observe at point '**A**'

$$P.E. = mgh \ (\textbf{Maximum})$$
$$K.E. = 0$$

Now

Date:

$$T.E. = P.E. + K.E$$
$$mgh + 0$$
$$T.E. = mgh.$$

At point B:

Now we have to calculate the energies at point 'B'
The body is falling downward.
So

$$V_i = 0$$
$$V_f = V_B \text{ (Velocity at point B)}$$
$$S = x \text{ (Distance covered from point A to B)}$$
$$a = g$$

Now 3rd equation of motion.

$$2aS = V_f^2 - V_i^2$$
$$2gx = (V_B)^2 - (0)^2$$
$$2gx = V_B^2$$

Now at point 'B' the K.E will be

$$K.E. = \frac{1}{2} m V_B^2$$
$$K.E. = \frac{1}{2} m (2gx)$$
$$K.E. = mgx$$

Now at point 'B' the

$$P.E. = mg(h-x)$$

Now to calculate:

$$T.E. = P.E. + K.E$$
$$= mg(h-x) + mgx$$

Date:

Mon Tue Wed Thu Fri Sat Sun

$= mgh - mgx + mgx$
$= mgh.$

At point C:

Now we have to calculate the energies at point 'C' Just before striking the ground.

$V_i = 0$ (Initial velocity)
$V_f = V_c$ (at point C the velocity is V_c)
$S = h$
$a = g$

$$2aS = V_f^2 - V_i^2$$
$$2gh = V_c^2 - (0)^2$$
$$2gh = V_c^2$$

Now at point 'C'

$$K.E. = \frac{1}{2} mV^2$$

Now put equ 'd' in above equation.

$$K.E. = \frac{1}{2} m(2gh)$$

$$K.E = mgh$$

Now at point 'C'

$$P.E. = 0$$
$$T.E = P.E + K.E$$
$$= 0 + mgh$$
$$T.E. = mgh$$

Now we observe from equ 1, 2 and 3 T.E. are same and energies are convert

Now observe while diping downward

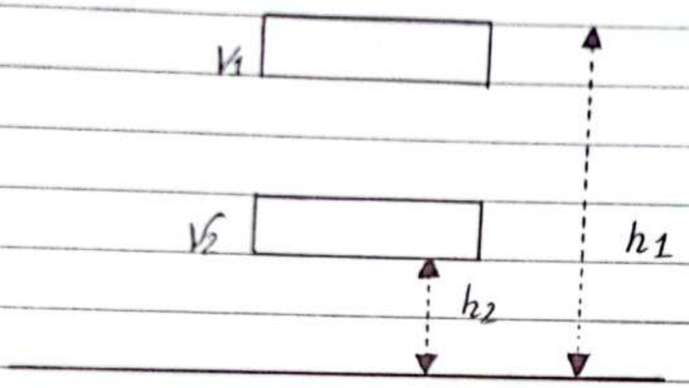
Date:

Mon Tue Wed Thu Fri Sat Sun

From height h_1 to h_2 we can write

Decrease in P.E. = Increase in K.E.

$$mg(h_1 - h_2) = \frac{1}{2} m (V_2^2 - V_1^2)$$



Date:

Mon Tue Wed Thu Fri Sat Sun

Workdone against Air Friction:

By considering air friction falling downward
Some part of potential energy is used to do some work against air friction.

So

Loss in P.E − Workdone against air friction = Gain in K.E

$$mgh - fh = \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv^2 + fh$$

Loss In P.E. = Gain in K.E. + Workdone against Air friction

Date:______ Mon Tue Wed Thu Fri Sat

Problem #5:

Data:

$m = 1000kg$
$h = 10m$
$S = 100m$
$F = 480N$

To Find: $v = ?$

Solve:

Loss In P.E. - Workdone against air friction = Gain In K.E.

$$mgh - fS = \frac{1}{2} mv^2$$

$$(1000)(9.8) - (480)(100) = \frac{1}{2}(1000) v^2$$

$$50000 = 500 v^2$$

$$v^2 = \frac{50000}{500}$$

$$v^2 = 100$$

$$v = \sqrt{100}$$

$$v = 10 ms^{-1}$$

Problem # 9:

Data:

Weight = $W = F = 60N$

$h_1 = 10m$

$h_2 = 5m$

$h = h_1 - h_2$

$h = 10 - 5 = 5m$

To Find: $V = ?$

Solve:

Loss In P.E. = Gain In K.E.

$$mgh = \frac{1}{2}mv^2$$

$$2gh = v^2$$

$$v = \sqrt{2gh}$$

$$v = \sqrt{(2)(9.8)(5)}$$

$$v = 9.9ms^{-1}$$

Now conservational energy servise
Page 92- 95
(S/Q + MCQs)