Projectile Motion

Two dimentional motion under constant acceleration due to gravity is called projectile motion

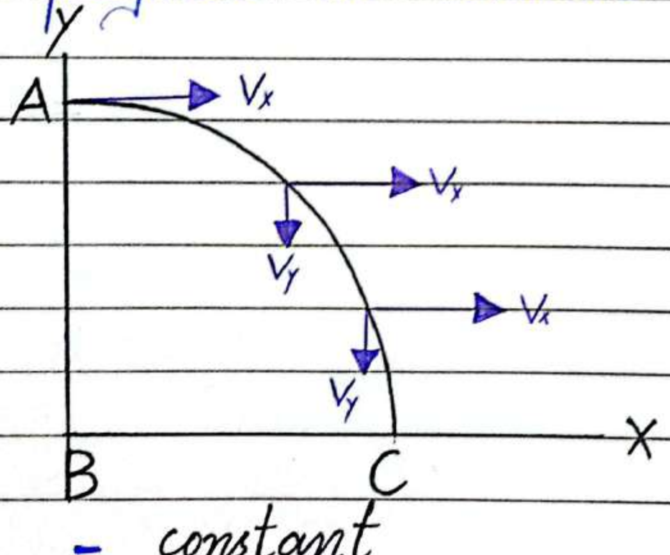
Trajectory: Path followed by the projectile is called trajectory

e.g. **A football motion and a rocket motion.**

Explanation:

Consider a projectile which is throw horizontally from a certain height A as shown:

Ignoring air friction we observe that the speed of the projectile is constant throughout its motion.



Now

V_x = constant

a_x = (horizontal acceleration)

So

Horizonal force

$$F_x = ma_x$$
$$= m(0)$$
$$F_x = 0$$

We also observe that the projectile is moving forward and as well as downward.

The only force which acts on a body is its weight which is equal to vertical force.

So

$$F_y = W$$
$$m a_y = mg$$
$$a_y = g$$
$$a_y = g \text{ (gravitational acceleration)}$$

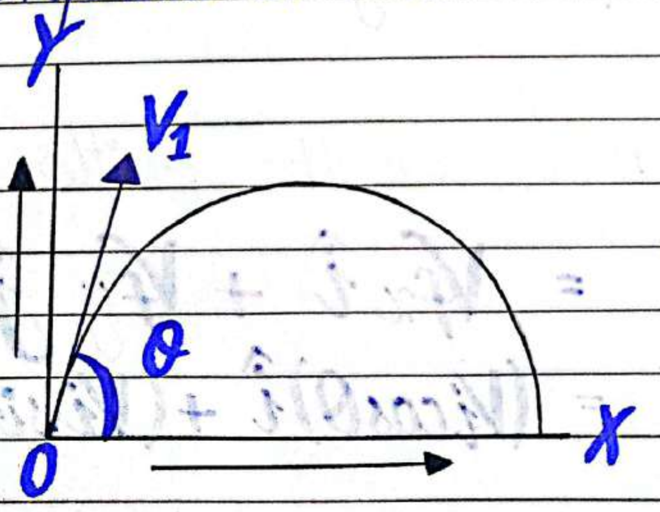
So it is the combined motion, in horizontal direction with constant velocity and in vertical direction with constant acceleration due to gravity.

(Horizontal and vertical distance)

Instantaneous Velocity:

Consider a projectile which is projected at an angle 'θ' with **x-axis** as shown the velocity 'V_i'

We observe that the velocity of the projectile having two components as shown



$$V_{ix} = V_i \cos\theta \text{ (x-component)} \text{ ——— } (a)$$

$$V_{iy} = V_i \sin\theta \text{ (y-component)} \text{ ——— } (b)$$

The x-component is called horizontal component and Y-component is called vertical component.

Now we have to calculate the final component of the velocity. So for this we consider.

$$V_{fx} = V_{ix} + a_x t$$
$$V_{fx} = V_i \cos\theta + (0)t$$
$$V_{fx} = V_i \cos\theta \text{ ——— } (1)$$

Equation (1) and (a) show that the horizonal component of the velocity of a projectile is same throughout the motion

Now

$$V_{fy} = V_{iy} + a_y t$$
$$V_{fy} = V_i \sin\theta + a_y t$$
$$V_{fy} = V_i \sin\theta - g t$$

Vector form:

Vector form can be written as

$$\vec{V} = V_{fx}\hat{i} + V_{fy}\hat{j}$$

$$\vec{V} = (V_i \cos\theta)\hat{i} + (V_i \sin\theta - gt)\hat{j}$$

Magnitude:

$$|V| = \sqrt{(V_{fx})^2 + (V_{fy})^2}$$

$$|V| = \sqrt{(V_i \cos\theta)^2 + (V_i \sin\theta - gt)^2}$$

Direction:

To determine the direction

$$\tan\theta = \frac{V_{fy}}{V_{fx}}$$

$$\tan\theta = \frac{V_i \sin\theta - gt}{V_i \cos\theta}$$

Horizontal Distance:

To determine the horizontal distance

$$V_{ix} = V_x$$

$$a_x = 0$$

$$S = X = ?$$

$$S = V_{ix} t + \frac{1}{2} a_x t^2$$

$$X = V_x t + \frac{1}{2}(0) t^2$$

$$X = V_x t$$

If a projectile is projected from a certain height with a speed of $10ms^{-1}$ and reaches a ground in 3sec what will be the total horizontal distance is covered

$X = V_x t$

$= (3)10$

$= 30$

Vertical Distance

To determine the vertical distance

$V_{iY} = 0$ (initial vertical component of velocity)

$a_Y = g$

$S = Y$

$$S = V_{iy}t + \frac{1}{2}a_y t^2$$

$$Y = (0)t + \frac{1}{2}gt^2$$

$$Y = \frac{1}{2}gt^2$$

(MCQs)

If a body drops from a certain height after 1sec how much distance it covered?

$Y = \frac{1}{2}gt^2$

$Y = \frac{1}{2}(10)(1)^2$

$Y = 5(1)$

$Y = 5$

If a body drops from 500m height after what time will be the ground

$Y = \frac{1}{2}gt^2$

$\sqrt{\frac{2H}{g}} = t$

$t = \sqrt{\frac{H}{5}}$

Height of the Projectile:

To determine the height of the projectile

$$2aS = V_f^2 - V_i^2$$

$$S = H = ?$$

$$S = H$$

$$a_y = -g$$

$$V_{fy} = 0$$

$$V_{iy} = V_i \sin\theta$$

$$2a_yH = V_{fy}^2 - V_{iy}^2$$

$$2(-g)H = (0)^2 - (V_i \sin\theta)^2$$

$$-2gH = -V_i^2 \sin^2\theta$$

$$H = \frac{V_i^2 \sin^2\theta}{2g}$$

$\sin^2\theta = 1$

$\sin\theta = 1$

$\theta = \sin^{-1}(1)$

$\theta = 90^\circ$

$H = \dfrac{V_i^2}{2g}\sin^2\theta$

$H_{max} = \dfrac{V_i^2}{2g}(\sin\theta)^2$

$H_{max} = \dfrac{V_i^2}{2g}$

$H = H_{max}\sin^2\theta$

Time To Flight:

To determine the projectile we use

$$S = v_i t + \frac{1}{2}at^2$$

$S = 0$ (total distance covered)

$$V_{iy} = V_i \sin\theta$$

$$a_y = -g$$

Date:

Now

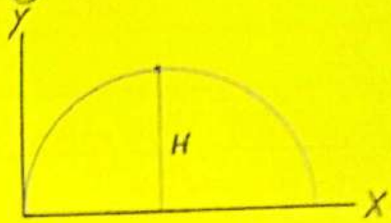
$$S = V_{iy}\, t + \frac{1}{2} a_y t^2$$

$$0 = (V_i \sin\theta)\, t + \frac{1}{2}(-g)\, t^2$$

$$\frac{1}{2} g\, t^{\cancel{2}} = (V_i \sin\theta)\, \cancel{t}$$

$$t = \frac{2 V_i \sin\theta}{g}$$

Time to reach maximum height



$$T = \frac{V_i \sin\theta}{g}$$

Range of projectile:

Maximum horizontal distance co projectile is called range of the projectile

$$S = R = ?$$

$$S = vt$$

Maximum range of a projectile is at angle of 45°

$$V_{ix} = V_i \cos\theta$$

$$T = \frac{2 V_i \sin\theta}{g}$$

Now

$$S = vt$$

$$R = V_{ix}\, T$$

$$R = V_i \cos\theta \times \frac{2 V_i \sin\theta}{g}$$

$$R = \frac{V_i^2}{g}\, 2 \sin\theta \cos\theta$$

$$R = \frac{V_i^2}{g} \sin 2\theta$$

Date:

Maximum Range:

To determine the range of the projectile.
We observe

$$\sin 2\theta = 1$$
$$2\theta = \sin^{-1}(1)$$
$$2\theta = 90^\circ$$
$$\theta = \frac{90}{2}$$
$$\theta = 45^\circ$$

Now

$$R_{max} = \frac{V_i^2}{g} \sin 2(45)$$
$$= \frac{V_i^2}{g} \sin 90^\circ$$
$$= \frac{V_i^2}{g}(1)$$
$$R_{max} = \frac{V_i^2}{g}$$

If the ranges of two projectiles are same so what will be the right option
(a) 60, 30 (b) 40, 60
(c) 45, 65 (d) 70, 70

$\theta_1 + \theta_2 = 90^\circ$
then ranges are equal.

$$R = R_{max} \sin 2\theta$$

Numericals

3.9: Two blocks of masses .... are attched ..... in the spring ... Find the velocities ... when released.

Data: Masses

$m_1 = 2kg$

$m_2 = 0.50kg$

$E.P.E = 10J$

$V_1 = 0$

$V_2 = 0$

To Find:

$V_1' = ?$

$V_2' = ?$

Calculated: Law of conservation of momentum

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

$$m_1(0) + m_2(0) = (2)V_1' + (0.50)V_2'$$

$$0 = 2V_1' + 0.50V_2'$$

$$-0.50V_2' = 2V_1'$$

$$V_1' = -\frac{0.50V_2'}{2}$$

$$V_1' = -0.25V_2'$$

Law of conservation of energy

$$E.P.E = K.E$$

$$10 = \frac{1}{2} m_1 V_1'^2 + \frac{1}{2} m_2 V_2'^2$$

$$10 = \frac{1}{2}(2)V_1'^2 + \frac{1}{2}(0.50)V_2'^2$$

$$10 = V_1'^2 + 0.25\, V_2'^2$$
$$10 = (-0.25\, V_2')^2 + 0.25\, V_2'^2$$
$$10 = 0.0625\, V_2'^2 + 0.25\, V_2'^2$$
$$10 = V_2'^2 (0.0625 + 0.25)$$
$$10 = V_2'^2 (0.3125)$$
$$V_2'^2 = \frac{10}{0.3125}$$
$$V_2'^2 = 32$$
$$V_2' = \sqrt{32}$$
$$V_2' = 5.6\,ms^{-1}$$

Now put in equ 1

$$V_1' = -0.25\, V_2'$$
$$V_1' = -0.25(5.6)$$
$$V_1' = -1.4\,ms^{-1}$$

(-ve) sign shows that it moves in opposite direction

3.10: A foot ball is thrown upward with ------- respect ---- speed of the ball?

Data:

Angle $= \theta = 30°$

Range $= R = 40m$

To Find: $V_i = ?$

Calculated: $R = \frac{V_i^2}{g} \sin 2\theta$

$$\frac{R \times g}{\sin 2\theta} = V_i^2$$

$$V_i = \sqrt{\frac{40 \times 9.8}{\sin 2(30)}}$$

$$V_i = 21.27\, ms^{-1}$$

3.11: A ball is thrown horizontally ..... velocity of $21\, ms^{-1}$ .... How .... that velocity?

Data:

$h = 10m$

$V_{ix} = 21\, ms^{-1}$

$V_{fx} = 21\, ms^{-1}$

$V_{iy} = 0\, ms^{-1}$

$g = 9.8\, ms^{-1}$

To Find:

$X = ?$

$V = ?$

Calculated:

$$Y = \frac{1}{2} g t^2 \qquad \therefore Y = H$$

$$t = \sqrt{\frac{2h}{g}}$$

$$t = \sqrt{\frac{2(10)}{9.8}}$$

$$t = 1.41\, sec$$

$$X = V_{ix}\, t$$

$X = 21 \times 1.41$
$X = 29.61 m$

Now

$V_{fy} = V_{iy} + a_y t$

$V_{fy} = 0 + (9.8)(1.41)$
$= 13.8 ms^{-1}$

$V = \sqrt{(V_{fx})^2 + (V_{fy})^2}$

$V = \sqrt{(21)^2 + (13.8)^2}$

$V = 25.12 ms^{-1}$

3.13:

Data:

$$H = \frac{V_i^2 \sin^2\theta}{2g}$$

$$R = \frac{V_i^2}{g} \sin 2\theta$$

To Find: $\theta =$

Solve: $H = R$

$$\frac{V_i^2 \sin^2\theta}{2g} = \frac{V_i^2}{g} \sin 2\theta$$

$$\frac{V_i^2 \sin^2\theta}{2g} = \frac{V_i^2}{g} 2\sin\theta\cos\theta$$

$$\frac{\sin^2\theta}{2} = 2\sin\theta\cos\theta$$

$$\frac{\sin\theta}{2} = 2\cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = 4$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1}(4)$$

$$\theta = 75.9 \approx 76$$

3.14:

Data:

$$R = \frac{V_i^2}{g} \sin 2\theta$$

Let $\theta = 15^\circ$ (which exceeds or fall)

To Find:

$R_1 = ?$

$R_2 = ?$

Solve:

$$R_1 = \frac{V_i^2}{g} \sin 2\theta_1$$

$\theta_1 = 45 + 15$ (exceed)

$\theta_1 = 60^\circ$

$$R_1 = \frac{V_i^2}{g} \sin 2(60)$$

$$R_1 = \frac{V_i^2}{g} (0.866) \longrightarrow 1$$

Now

$$R_2 = \frac{V_i^2}{g} \sin 2\theta_2$$

$\theta_2 = 45 - 15$ (fall)

$\theta_2 = 30^\circ$

$$R_2 = \frac{V_i^2}{g} \sin 2(30)$$

$$R_2 = \frac{V_i^2}{g} \sin 60$$

$$R_2 = \frac{V_i^2}{g} 0.866 \longrightarrow 2$$

From 1 and 2 we can write

$$R_1 = R_2$$

3.15:

Data: $\theta = 45°$

$$R = 3000 km$$
$$= 3000 \times 10^3 m$$
$$= 3 \times 10^6 m$$

To Find: $V_i = ?$

$T = ?$

Solve:

$$R = \frac{V_i^2}{g} \sin 2\theta$$

$$V_i = \sqrt{\frac{R \times g}{\sin 2\theta}}$$

$$V_i = \sqrt{\frac{(3 \times 10^6)(9.8)}{\sin 2(45)}}$$

$$V_i = 5422.17 ms^{-1}$$

$V_i = 5.4 kms^{-1}$

$$T = \frac{2 V_i \sin\theta}{g}$$

$$T = \frac{2(5.4 \times 10^3)\sin(45)}{9.8}$$

$T = 779.26$

$T = 13 min$

Problem 3.1:

Data: Initial velocity $= V_i = 19.6 ms^{-1}$

$h = -156.8$ (Asscending vertically upward)

$a = -g$

$a = -9.8 ms^{-1}$

To Find: $t = ?$

Solve: $S = V_i t + \frac{1}{2} a t^2$

$$-156.8 = (19.6)t + \frac{1}{2}(-9.8)t^2$$

$$-156.8 = (19.6)t - (4.9)t^2$$

$$\frac{-156.8}{4.9} = \frac{19.6}{4.9}t - \frac{4.9}{4.9} \cdot t^2$$

$$-32 = 4t - t^2$$

$$t^2 - 4t - 32 = 0$$

$$t^2 - 8t + 4t - 32 = 0$$

$$t(t-8) + 4(t-8) = 0$$

$$(t-8)(t+4) = 0$$

$t - 8 = 0$, $t + 4 = 0$

$t = 8$ sec , $t = -4$ sec

Answer   discard