# Elastic Collion In One-Dimension:
## (Uni-directional)

We know that, elastic collision momentum and Kinetic energy (K.E) of the system remains conserved. So we can write

Question:- Define elastic and Inelastic collision and also show that the speed of approach is equal and opposite to speed of separation?

For momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 v_1 - m_1 v_1' = m_2 v_2' - m_2 v_2$$

$$m_1 (v_1 - v_1') = m_2 (v_2' - v_2) \longrightarrow 1$$

Now for K.E

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$\frac{1}{2} (m_1 v_1^2 + m_2 v_2^2) = \frac{1}{2} (m_1 v_1'^2 + m_2 v_2'^2)$$

$$m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$m_1 v_1^2 - m_1 v_1'^2 = m_2 v_2'^2 + m_2 v_2^2$$

$$m_1 (v_1^2 - v_1'^2) = m_2 (v_2'^2 - v_2^2) \quad \because a^2-b^2=(a-b)(a+b)$$

$$m_1 (v_1 - v_1')(v_1 + v_1') = m_2 (v_2' - v_2)(v_2' + v_2) \longrightarrow 2$$

Now divide equ **1** and **2** we get

$$\frac{m_1 (V_1 - V_1')(V_1 + V_1')}{m_1 (V_1 - V_1')} = \frac{m_2 (V_2' - V_2)(V_2' + V_2)}{m_2 (V_2' - V_2)}$$

$$V_1 + V_1' = V_2' + V_2$$

$$V_1 \mp V_2 = V_2' - V_1'$$

$$\mathbf{V_1 \mp V_2 = -(V_1' - V_2')}$$

Speed of approach = Speed of separation

We note that, before collision $(V_1 - V_2)$ is the velocity of first ball relative to the second ball. Similarly $(V_1' - V_2')$ is the velocity of the first ball relative to the second ball after collision. It means that relative velocities before and after the collision have the same magnitude but are reversed after the collision. In orther words, the magnitude of relative velocity of approach is equal to the magnitude of relative velocity of separation.

# Calculations For $V_1'$ End $V_2'$

As we know that

$$V_1 + V_1' = V_2' + V_2 \longrightarrow 1$$

and for the equation of momentum.

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2' \longrightarrow 2$$

## For $V_1'$:

Now take equ 1

$$V_1 + V_1' = V_2' + V_2$$

$$V_1 + V_1' - V_2 = V_2' \longrightarrow 3$$

Put equ 3 in equ 2

$$m_1 V_1 + m_2 V_2 = m_1 V' + m_2 (V_1 + V_1' - V_2)$$

$$m_1 V_1 + m_2 V_2 = m_1 V' + m_2 V_1 + m_2 V_1' - m_2 V_2$$

$$m_1 V_1 - m_2 V_1 + m_2 V_2 + m_2 V_2 = m_1 V_1' + m_2 V_1'$$

$$V_1 (m_1 - m_2) + 2 m_2 V_2 = V_1' (m_1 + m_2)$$

$$\frac{V_1 (m_1 - m_2)}{m_1 + m_2} + \frac{2 m_2 V_2}{m_1 + m_2} = V_1'$$

## ① For $V_2'$

Now again take equ **1**

$$V_1 + V_1' = V_2' + V_2$$

$$V_1' = V_2' + V_2 - V_1 \longrightarrow 4$$

Now put equ **4** in equ **2**

$$m_1 v_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

$$m_1 V_1 + m_2 V_2 = m_1 (V_2' + V_2 - V_1) + m_2 V_2'$$

$$m_1 V_1 + m_2 V_2 = m_1 V_2' + m_1 V_2 - m_1 V_1 + m_2 V_2'$$

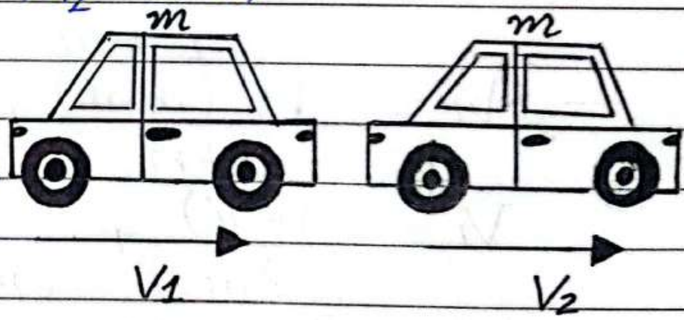
$$m_1 V_1 + m_1 V_1 + m_2 V_2 - m_1 V_2 = m_1 V_2' + m_2 V_2'$$

$$2 m_1 v_1 + V_2 (m_2 - m_1) = v_2' (m_1 + m_2)$$

$$\frac{2 m_1 v_1}{m_1 + m_2} + \frac{v_2 (m_2 - m_1)}{m_1 + m_2} = V_2'$$

# CASES

**Case I:** When $m_1 = m_2 = m$




$$V_1' = \frac{V_1 (m_1 - m_2)}{m_1 + m_2} + \frac{2 m_2 V_2}{m_1 + m_2}$$

$$V_1' = \frac{V_1 (m - m)}{m + m} + \frac{2 m V_2}{m + m}$$

$$V_1' = \frac{V_1 (0)}{2m} + \frac{2m V_2}{2m}$$

$V_1' = V_2$ $\qquad$ $V_2' = V_1$

$$V_1' = 0 + V_2$$

$$V_1' = V_2 \longrightarrow 1$$

$$V_2' = \frac{V_2(m_2 - m_1)}{m_1 + m_2} + \frac{2m_1 V_1}{m_1 + m_2}$$

$$V_2' = \frac{V_2(m - m)}{m + m} + \frac{2mV_1}{m + m}$$

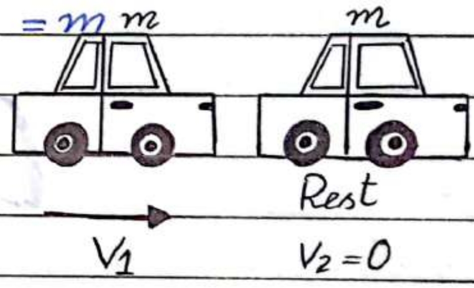
$$V_2' = \frac{V_2(0)}{2m} + \frac{2mV_1}{2m}$$

$$V_2' = 0 + V_1$$

$$V_2' = V_1 \longrightarrow 2$$

From equ **1** and **2** we observe that both the balls simply interchange their velocities.

## Case II: When $m_1 = m_2 = m$, $V_2 = 0$




Now

$$V_1' = \frac{V_1(m_1 - m_2)}{m_1 + m_2} + \frac{2m_2 V_2}{m_1 + m_2}$$

$$V_1' = \frac{V_1(m - m)}{m + m} + \frac{2m(0)}{m + m}$$

$$V_1' = \frac{V_1(0)}{2m} + \frac{0}{2m}$$

$$V_1' = 0 \longrightarrow 3$$

Q:- When two equal masses of bodies collide with each other where the second mass is at rest what will the affect on their velocity after striking

End Now

$$V_2' = \frac{V_2(m_2 - m_1)}{m_1 + m_2} + \frac{2m_1 V_1}{m_1 + m_2}$$

$$V_2' = \frac{(m-m)(0)}{m+m} + \frac{2mV_1}{m+m}$$

$$V_2' = 0 + \frac{2mV_1}{2m}$$

$$V_2' = V_1 \longrightarrow 4$$

Equ 3 and 4 Shows that both the balls will interchange their velocities.

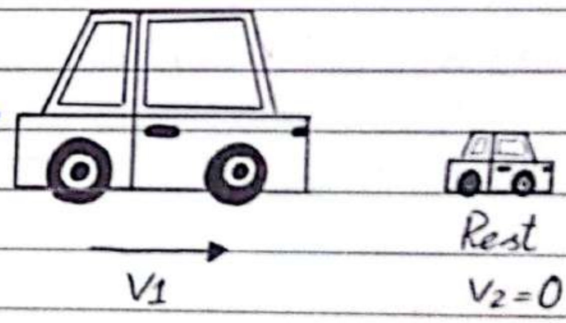
## Case III: When $m_1 >> m_2$

$$V_2 = 0$$

It observe that ball one is greater than ball two So $m_2 \approx 0$ (as compared to $m_1$)

Now

$$V_1' = \frac{V_1(m_1 - m_2)}{m_1 + m_2} + \frac{2m_2V_2}{m_1+m_2}$$




$$V_1' = \frac{V_1(m_1 - 0)}{m_1 + (0)} + \frac{2(0)(0)}{m_1 + (0)}$$

$$V_1' = \frac{m_1V_1}{m_1}$$

$$V_1' = V_1$$

End Now

$$= \frac{V_2(m_2 - m_1)}{m_1 + m_2} + \frac{2m_1V_1}{m_1+m_2}$$

$$= \frac{0(0-m_1)}{m_1+0} + \frac{2m_1V_1}{m_1+0}$$

$$= 0 + \frac{2m_1V_1}{m_1}$$

$$= 2V_1 \longrightarrow 5$$

Equ **5**: Shows that after the first ball will move with its 1st velocity but 2nd ball will move twice the velocity of 1st ball.

## Case IV:

When $m_1 << m_2$

$V_2 = 0$

It observe that the second ball is very large as compared to $m_1$ and is at rest. So $m_1 \approx 0$

Now

$$V_1' = \frac{(m_1 - m_2)V_1}{m_1+m_2} + \frac{2m_2V_2}{m_1+m_2}$$

$$V_1' = \frac{(0 - m_2)V_1}{0+m_2} + \frac{2m_2(0)}{0+m_2}$$

$$V_1' = \frac{-m_2V_1}{m_2} + 0$$

$$V_1' = -V_1 \longrightarrow 7$$

Now

$$V_2' = \frac{(m_2 - m_1)V_2}{m_1+m_2} + \frac{2m_1V_1}{m_1+m_2}$$

$$V_2' = \frac{(m_2 - 0)(0)}{0+m_2} + \frac{2(0)V_1}{0+m_2}$$

$$V_2' = 0 + 0$$

$$V_2' = 0 \longrightarrow 8$$

Q:- Wenever is small mass of body $\approx 0$ collide with a heavy mass of body which is at rest so what will the affect of their velocities after strike?

Equ 8 and 7: Shows that the 1st ball will bounce back with the same velocity and 2nd ball remains at rest.

# ① Force due to water flow:

Conside a horizontal pipe in which water flows and striking with the wall with velocity 'v' and after striking it comes to rest.

(MCQs)
$F = \frac{-mv}{t}$ If this will be the force exerted by the wall on the water. What is the force exerted by the water on the wall?
$F = \frac{mv}{t}$

So

$$V_i = V$$
$$V_f = 0$$

Now we know that
Impulse = Change in momentum

$$F \times t = mV_f - mV_i$$
$$F \times t = m(0) - mV$$

$$F = \frac{-mV}{t}$$

This is the force exerted by the wall on the water. Now according to Newton's third Law the force which exerted by the water on the wall will be

$$F = -\left(\frac{-mV}{t}\right)$$

$$F = \frac{mV}{t}$$

Formula Convertion:

$F = \frac{mV}{t}$ $\therefore \rho = \frac{m}{V}$ (volume)

$\boxed{F = \frac{\rho V v}{t}}$ $m = \rho V$

$\therefore \frac{V}{t}$ = Rate of flow

$F = \rho A v . v$ $\frac{V}{t} = A v$ (area of pipe) (Speed of water)

$\boxed{F = \rho A v^2}$

$\rho_{water} = 1000 kgm^3$

# Book Example 3.6:

## Data:

Speed = $v$ = $0.3 ms^{-1}$
Area = $A$ = $50 cm^2$

$= 50 \times (10^{-2})^2 m$
$= 50 \times 10^{-4} m$

## To Find: Force = ?

## Calculated:

$$F = \frac{mv}{t}$$

We also know that

$$F = \rho A v^2$$
$$F = (1000)(0.3)^2(50 \times 10^{-4})$$
$$F = 0.45N$$

# Numerical

**3.3:** A proton moving with speed of $1.0 \times 10^7 ms^{-1}$ passes through a 0.020cm thick sheet of paper with a speed of $2.0 \times 10^6 ms^{-1}$. Assuming uniform deceleration, find retardation and time taken to pass through the paper?

## Data:

Mass of proton $= m_p = 1.67 \times 10^{-27} kg$
Initial velocity $= V_i = 1.0 \times 10^7 ms^{-1}$
Distance $= S = 0.020cm = 0.02 \times 10^{-2} m$
Final velocity $= V_f = 2.0 \times 10^6 ms^{-1}$

## To Find:

$a = ?$
$t = ?$

Calculated:

$$2aS = V_f^2 - V_i^2$$

$$2a(0.02 \times 10^{-2}) = (2\times10^{6})^{2} - (1\times10^{7})^{2}$$

$$a = \frac{-9.6 \times 10^{13}}{2(0.02 \times 10^{-2})}$$

$$a = -2.4 \times 10^{17}\ ms^{-2}$$

Now

$$V_f = V_i + at$$

$$t = \frac{V_f - V_i}{a}$$

$$t = \frac{2\times10^{6} - 1\times10^{7}}{-2.4 \times 10^{17}}$$

$$t = 3.33 \times 10^{-11}\ sec.$$

**3.4:** Two masses $m_1$ and $m_2$ are initially at rest with a spring compressed between them. What will be the ratio of the magnitudes of their velocities after the spring has been released.

# Data:

Two masses $= m_1$ and $m_2$

Initially rest

$$V_1 = 0$$

$$V_2 = 0$$

# To Find:

$$\frac{V_1'}{V_2'} = ?$$

# Calculated:

We know that

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

$$m_1(0) + m_2(0) = m_1 V_1' + m_2 V_2'$$
$$0 = m_1 V_1' - m_2 V_2'$$
$$m_1 V_1' = m_2 V_2'$$

$$\frac{V_1'}{V_2'} = \frac{m_2}{m_1}$$

**3.5:** An ameoba of mass .......... so that the mass of the ameoba remains the same.

## Data:

Mass $= m = 1 \times 10^{-12}\, kg$

Velocity $= v = 1 \times 10^{-4}\, ms^{-1}$

$$\frac{m}{t} = 1 \times 10^{-13}\, kgs^{-1}$$

## To Find:

(a) $F = ?$ (b) $a = ?$

## Calculated:

We know that

$$F = ma$$
$$F = m\frac{v}{t}$$
$$F = \frac{m}{t} v$$
$$F = (1 \times 10^{-13})(1 \times 10^{-4})$$
$$F = 1 \times 10^{-17}\, N$$

Now,

$$F = ma$$
$$a = \frac{F}{m}$$
$$a = \frac{1 \times 10^{-17}}{1 \times 10^{-12}}$$

Imp

$a = 1 \times 10^{-5} ms^{-1}$

**3.6:** A boy places a fire cracker of negligible mass in an empty ............ will the can be going?

# Data:

Can $= m_1 = 40g = 0.040\ kg$

Wooden block $= m_2 = 200g = 0.20\ kg$

$V_1 = 0$

$V_2 = 0$

$V_2' = 3 ms^{-1}$

# To Find:

$V_1' = ?$

# Calculated:

We know that

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

$$m_1 (0) + m_2 (0) = (0.040)(V_1') + (0.20)(3)$$

$$0 = (0.040)(V_1') + 0.6$$

$$-0.6 = (0.040) V_1'$$

$$V_1' = -\frac{0.6}{0.040}$$

$$V_1' = -15 ms^{-1}$$

(-) Negative sign shows that both can and block moves oppositely.

**3.7:** An electron ......... undergoes a head on ... along a straight ............ velocity of hydrogen.

# Data:

$m_1 = 9.1 \times 10^{-31} kg$

$V_1 = 2 \times 10^7 ms^{-1}$

$m_2 = 1.67 \times 10^{-27} kg$

$V_2 = 0$

## To Find:
~~$V_2' = 0$~~

$V_2' = ?$

## Calculated:

$$V_2' = \frac{(m_2 - m_1)V_2}{m_1 + m_2} + \frac{2m_1 V_1}{m_1 + m_2}$$

$$V_2' = \frac{(1.67 \times 10^{-27})(0)}{(1.67 \times 10^{-27} + 9.1 \times 10^{-31})} + \frac{2(9.1 \times 10^{-31})(2 \times 10^{7})}{(1.67 \times 10^{-27} + 9.1 \times 10^{-31})}$$

$$V_2' = 0 + \frac{3.6 \times 10^{-23}}{1.67 \times 10^{-27}}$$

$$V_2' = 2.15 \times 10^{-23+27}$$

$$V_2' = 2.15 \times 10^{4}\ ms^{-1}$$

Imp

**3.8:** A truck weighing ......... with a velocity ...... a stationary car ......... Calculate their common velocity? (Common velocity) (In elastic collision case)

## Data:

Truck $m_1 = 2500kg$

$V_1 = 21ms^{-1}$

Car $m_2 = 1000g$

$V_2 = 0$

## To Find:
$V = ?$

## Calculated:

We know that

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

$$m_1 V_1 + m_2 V_2 = m_1 V + m_2 V$$
$$m_1 V_1 + (0) V_2 = V(m_1 + m_2)$$
$$m_1 V_1 = V(m_1 + m_2)$$
$$V = \frac{m_1 V_1}{m_1 + m_2}$$
$$V = \frac{(2500)(21)}{2500 + 1000}$$
$$V = \frac{52500}{3500}$$
$$V = 15 ms^{-1}$$

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