Topics Covered Today:

1. SES
2. DES/TES
3. Stationarity
4. DF Test
5. Converting Non-stationary TS to Stationary TS
6. Autocorrelation Basics

Agenda → Smoothing family

→ Stationary
→ DF Test
→ Auto Correlation

→ ARIMA family

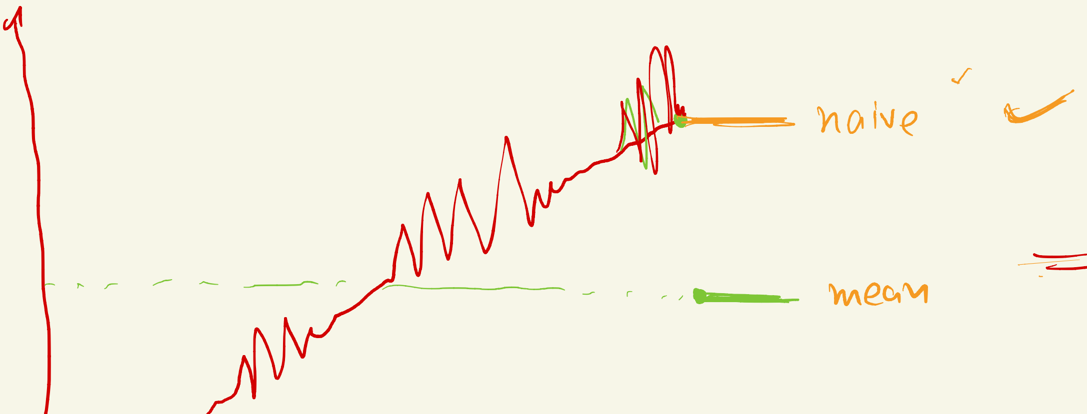
Start at 9:05

Recap → ① naive

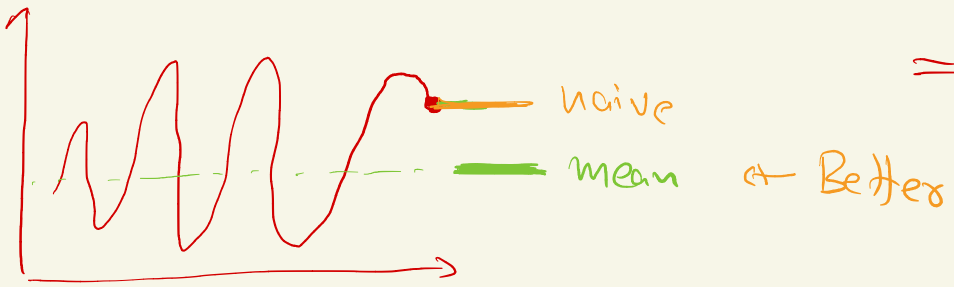
$$\hat{y}_{t+1} = y_t$$

② mean

$$\hat{y}_{t+1} = \frac{1}{t} y_t + \frac{1}{t} y_{t-1} + \frac{1}{t} y_{t-2} + \dots \frac{1}{t} y_0$$



⇒ For trended TS naive



⇒ mean is better

$$\hat{y}_{t+1} = \underline{\quad} \, y_t + \underline{\quad} \, y_{t-1} + \dots \, y_{t-0}$$

$\Rightarrow$ What is these weight going to be

① Can we use Static weights 0.65, 0.3, 0.005, . . .

300 past $\rightarrow$ 300 weights

12 months — 312 values $\rightarrow$ 312 weight

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$

$$\hat{y}_{t+1} = \underline{\alpha} \, y_t + \underline{\alpha(1-\alpha)} \, y_{t-1} + \underline{\alpha(1-\alpha)^2} \, y_{t-2} + \dots$$

$\Rightarrow$ Sum of an infinite GP =

$$a, ar, ar^2, \dots = \frac{a}{1-r}$$

$$\Rightarrow \frac{\alpha}{1-(1-\alpha)} = 1$$

Sum of all weights $= 1$

$0 \le \alpha \le 1$

$\alpha = \underline{0.9999} \rightarrow$ Naive

$\alpha = \underline{0.00001} \rightarrow$ mean

$$\alpha = \frac{1}{2 * \text{Seasonality}}$$

$\leftarrow$ general advice

Smoothing Parameter / Co-efficient

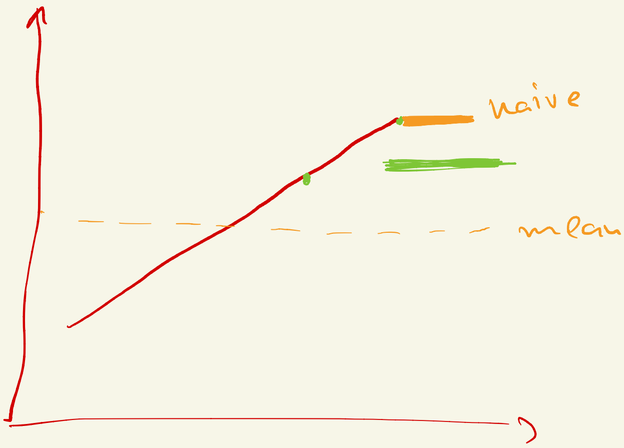
Same Algo → Recursive form → most books

$$\boxed{\hat{y}_{t+1} = \alpha y_t + (1-\alpha)\,\hat{y}_t}$$

$$(1-\alpha)\left(\alpha y_{t-1} + (1-\alpha)\,\hat{y}_{t-1}\right)$$

SES → reasonable estimate of the signal value.

↳



$$\alpha y_t + \alpha(1-\alpha) y_{t-1} + \ldots$$

SES → don't capture trend

↓

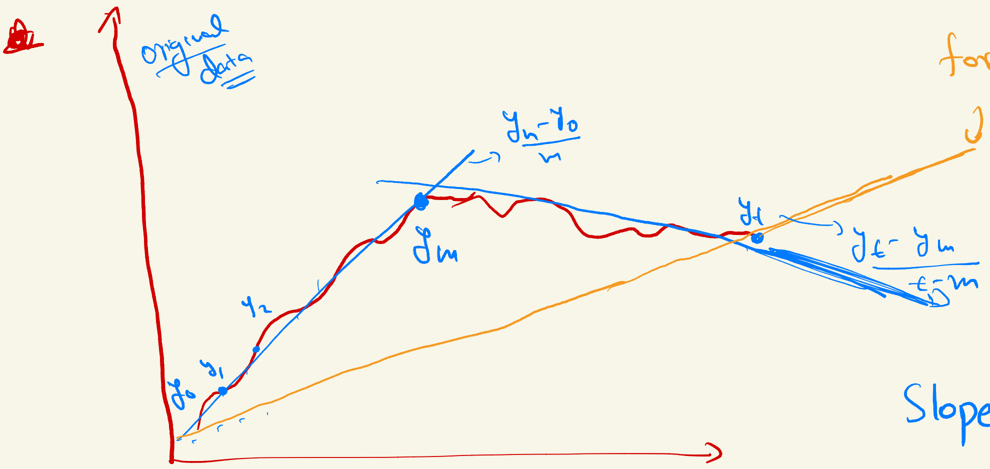
increment / decrement

Trend → • rate of change

$$\text{Slope} = \frac{y_t - y_0}{t} \Rightarrow \text{Drift method}$$

⇓

$$y_{t+1} = y_t + \text{slope} * 1$$

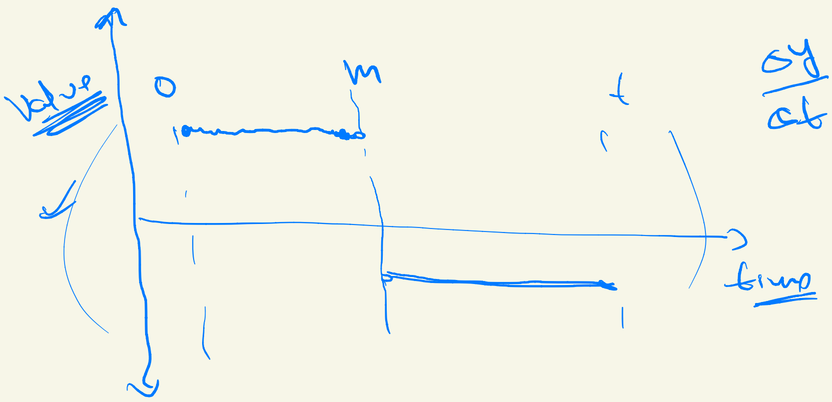


forecasting along this line using drift.

$$\text{Slope} = \frac{\Delta y}{\Delta t}$$

$\frac{y_1 - y_0}{1}$, $\frac{y_2 - y_1}{1}$, $\frac{y_3 - y_2}{1}$, ... $\Rightarrow$ Instaneous rate of change series

$\Rightarrow$ $\rightarrow$ trend ~~class~~ series, return series $\rightarrow$ Finance increment or decrement



$\frac{\Delta y}{\Delta t}$ graph

$\rightarrow$ trend time series

$\Rightarrow$

naive

$$\hat{b}_{t+1} = \hat{b}_t$$

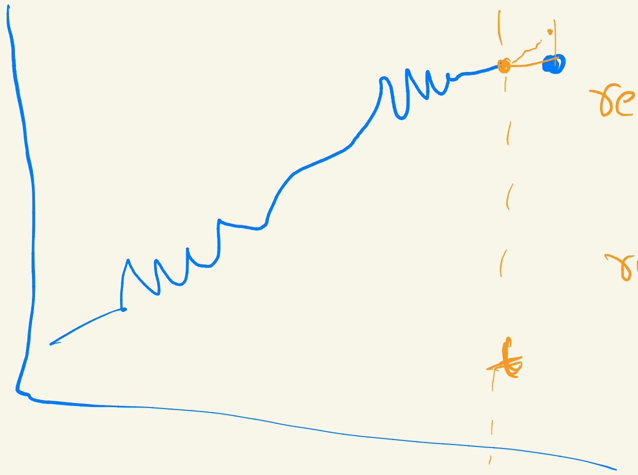
trend or rate

mean

$$\hat{b}_{t+1} =$$

trend

$\boxed{\text{SES}}$



reasonable base

+

reasonable trend / slope

$$\boxed{\hat{y}_{t+h} = \underline{l_t} + (h)\, \underline{b_t}}$$

= Current suitable base ; Suitable Slope

$\Rightarrow$ $\underline{\text{SES}}$ + Drift $\rightarrow$ $\underline{\text{predict}}$

l_t

Simple approach

$\Rightarrow$ SES $\rightarrow$ over all data $\rightarrow$ l_t $\rightarrow$

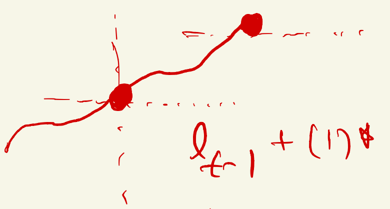
SES $\rightarrow$ $\frac{\Delta y}{\Delta t}$ series $\rightarrow$ b_t $\rightarrow$

actual data + trend

99.5%

$$\Rightarrow \boxed{\begin{aligned} l_t &= \alpha\, y_t + (1-\alpha)\left[l_{t-1} + b_{t-1}\right] \\ b_t &= \beta \times \underline{(l_t - l_{t-1})} + (1-\beta)\, b_{t-1} \end{aligned}}$$

(slope)



l_t =

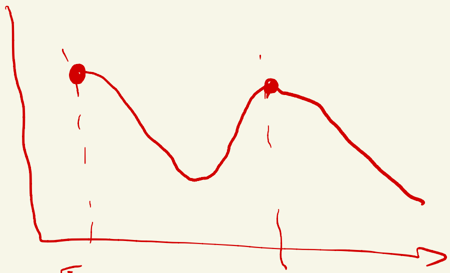
$\underline{l_{t-1} + (1) * b_{t-1}}$

⟹ SES → reasonable estimate of level

DES → reasonable estimates of level & trend

⟹ SES + Drift ~ DES

⟹ Seasonality ??



→ New TS with Just

1. Jan →
2. Feb →
⋮
12. Dec →

$$\underline{\text{Jan }23} = \underset{\text{overall data}}{DES} + \gamma(J22) + \gamma(1-\gamma)(J21) + \dots$$

$$\text{Feb}23 = DES + \gamma(F22) + \gamma(1-\gamma)(F21) + \dots$$

$$\boxed{\hat{y}_{t+h} = \underbrace{l_t}_{\text{reasonable base}} + h\underbrace{b_t}_{\text{reasonable grow}} + S_{t-m+h}}$$

m → seasonality time frame

31st Dec 22

Jan 23 ↓ Jan 22 →	Feb23 ↓ Feb 22

$$
\boxed{
\begin{aligned}
l_t &= \alpha \left[y_t - S_{t-m} \right] + (1-\alpha) \left[l_{t-1} + b_{t-1} \right] \\
b_t &= \beta \left[l_t - l_{t-1} \right] + (1-\beta)\, b_{t-1} \\
S_t &= \gamma \left[y_t - l_{t-1} - b_{t-1} \right] + (1-\gamma)\, S_{t-m}
\end{aligned}
}
$$

⇒ Smooting family of Models → Trended, Seasonal data.

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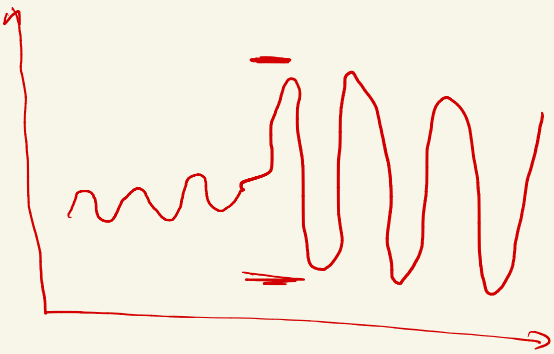
① <u>Stationary Signals</u>

↳ If its mean, variance, <u>amplitude</u>, frequency do not change with time.

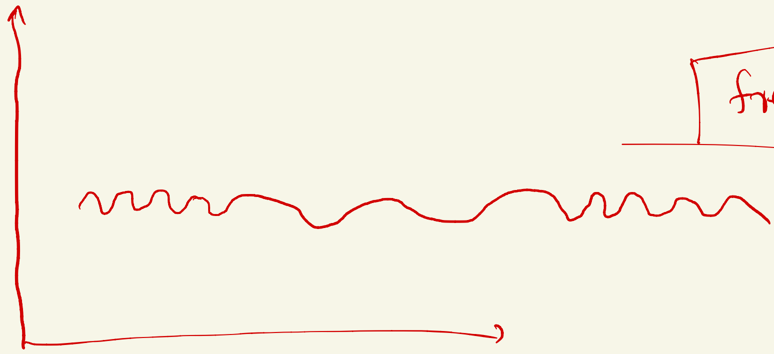
⇒ ✓



$y_{max} - y_{min}$ has change

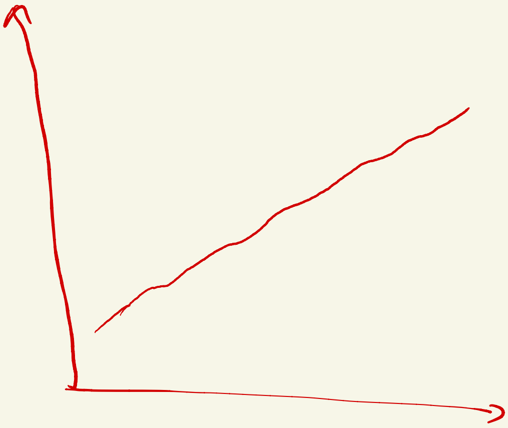
$\boxed{\text{Amplitude change}}$

⇒



frequency change

⇒



trend

nonstationary

mean is changing

⇒

Seasonal ⇒ are non stationary

⇒ **Dickey-Fuller Test** →

definition

↳ Hypothesis Testing →

↳ null hypothesis = the Data is non-stationary if P value < Significan level than Stationary

Python → Library

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why are we looking at Stationary TS

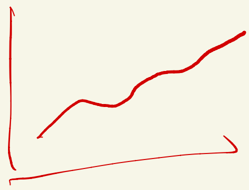
1. Predicting / Forecasting → TS doesn't seasonality / Trend
2. To predict Residue.

How to make something Stationary →

1. Decomposition

$$y \to b(t),\ s(t) \qquad r(t) = y(t) - b(t) - s(t)$$

2. Detrending →



$$y(t) = \underbrace{b(t)}_{y = mx + c} + z(t)$$

$$\Rightarrow b(t) = m(t) + c$$

$$y(t) = m(t) + c + z(t)$$

Differentiate

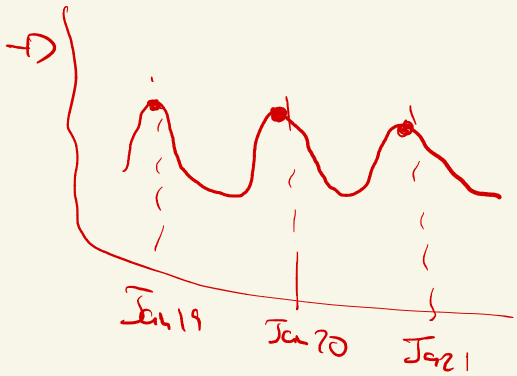
$$y'(t) = m + z'(t)$$

⇒ How do you differentiate (Calculus) in python.

$$\frac{dy}{dt} = \frac{y_t - y_{t-1}}{t-(t-1)} = \boxed{y_t - y_{t-1}}$$

③ De Seasonality →

Seasonality Period → Subtract last Period value.



$$\boxed{Y_{Jan22} - Y_{Jan21}}$$

↳ remove seasonality

⇒ Auto Correlation →

Q. Can I use past data to forecast future values??

Auto Correlation

Actual Data →	98	105	76	89	94	☐	
Past Data by 1 day		98	105	76	89	94	

2day	98	105	76	89	94	☐	
			98	105	76	89	94