Homework 1

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Section 3.2

Let $m : \mathcal{A} \to [0, \infty)$ be a set function where $\mathcal{A}$ is a σ-algebra. Assume m is countably additive over countable disjoint collections of sets in $\mathcal{A}$.

Problem 1

Given sets A, B, and C, if $A \subset B$ and $B \subset C$, then $A \subset C$.

Proof. Other symbols you can use for set notation are

- $A \supset B \supseteq C \subset D \subseteq E$. Also $\varnothing$vs$\emptyset$
- $\cup$ and $\cup_{k=1}^{\infty} E_k$
- $\cap$ and $\cap_{x \in \mathbb{N}} \{\frac{1}{\sqrt[3]{x}}\}$
- $\bigcup$ and $\bigcap\limits_{k=0}^{n}$ and $\bigcap$
- most Greek letters $\sigma\pi\theta\lambda_i e^{i\pi}$
- $\int_0^2 ln(2) x^2 sin(x) dx$
- $\leq < \geq > = \neq$

If you want centered math on its own line, you can use a slash and square bracket.

$$\left\{ \sum_{k=1}^{\infty} l(I_k) : A \subseteq \bigcup_{k=1}^{\infty} \{I_k\} \right\}$$

The left and right commands make the brackets get as big as we need them to be. $\square$

Problem 2

Prove equation (3.19) states:

$$\lg(n!) = \Theta(n \lg n) \tag{1}$$

Proof.

We use Stirling's approximation for this proof
With large values of n, $\Theta(\frac{1}{n})$is very smaller than 1.
So we can write n! as follows:

$$\begin{aligned}
\lg(n!) &\approx \lg\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right) \\
&= \lg(\sqrt{2\pi n}) + \lg\left(\frac{n}{e}\right)^n \\
&= \lg\sqrt{2\pi} + \lg\sqrt{n} + n\lg\left(\frac{n}{e}\right) \\
&= \lg\sqrt{2\pi} + \frac{1}{2}\lg n + n\lg n - n\lg e \\
&= \Theta(1) + \Theta(\lg n) + \Theta(n\lg n) - \Theta(n) \\
&= \Theta(n\lg n)
\end{aligned} \tag{2}$$

□

Problem 3

Proof. □

Section 2.2

Problem 6

Blah

Problem 7

Blah

Problem 10

Blah