Compartment model for epidemiology

Abstract

This paper looks at a compartment model for epidemiology and analyzes the different relationships between the variables in the model. Finally, it draws conclusions of the impact the variables have on the model.

Model

The population is divided into compartments, with the assumption that every individual has the same characteristics. There is a total of five compartments: susceptible (S), exposed (E), infected(I), recovered (R), and diseased (D).

$$S \rightarrow E \rightarrow I \rightarrow R \,/\, D$$

The model is described as ordinary differential equations with the following variables:

- β : how often a susceptible-infected result in exposure.
- σ : the rate an exposed becomes infected.
- γ : the rate an infected recover.
- δ : the rate that a case that recovers dies.
- ν : vaccination ($\nu = 0$).
- S : initial susceptible.
- E : initial exposed.
- I : initial infected.
- R : initial recovered.
- D : initial deaths.

These variables are related through the following ordinary differential equations:

$$\frac{dS}{dt} = -\beta \frac{SI}{N} - \nu S$$

$$\frac{dE}{dt} = \beta \frac{SI}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I + \nu S - \gamma \delta I$$

$$\frac{dD}{dt} = \gamma \delta I$$

$$N = S + E + I + R + D$$

Analysis

In the analysis the initial population size is $N = 100.000$, and will start with one patient zero for all simulations.

In the first simulation a susceptible-infected exposure has the probability of 80% to become exposed. Once exposed, at any unit of time the case has a chance of 15% to become infected. This given an average time until infection of 6.66 days. There is a probability of 10% to recover per unit of time, whereas 5% of those cases result in death. This means it takes an average of 10 days to recover.

This gives the following initial conditions for the system: $\beta = 0.8, \sigma = 0.15, \gamma = 0.1, \delta = 0.05$ at $t = 0$.

Running this simulation for 120 days gives the following result:

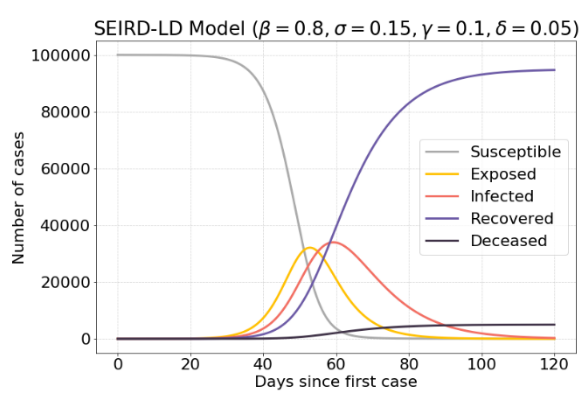

Figure 1) Normal initial condition.

With these initial conditions there are a maximum of around 37.000 infected.

If the value of σ is increased, meaning that exposed people will get infected faster, there is an increase in maximum infected cases. There are about 39.000 infected cases.

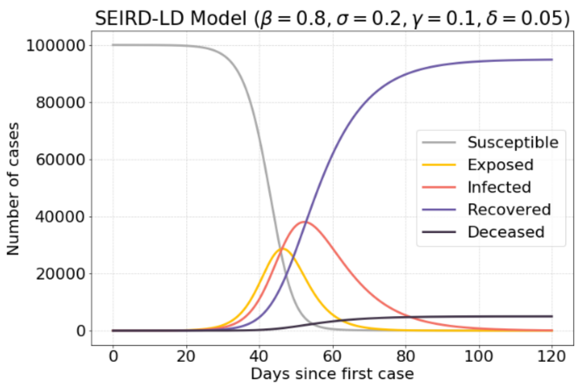

Figure 2) Increased infection rate σ.

If a lower value of σ is used, the resulting infections are spread out a lot more over time. There are about 19.500 infected cases.

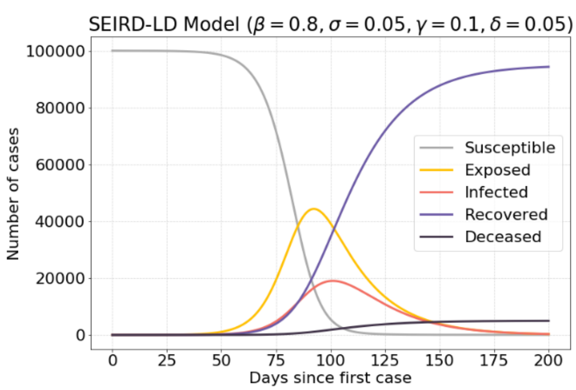

Figure 3) Decreased infection rate σ.

Another interesting factor is the recovery rate γ. If the recovery rate increased, there will be a lot less people infected at any given moment. There are about 21.000 infected cases.

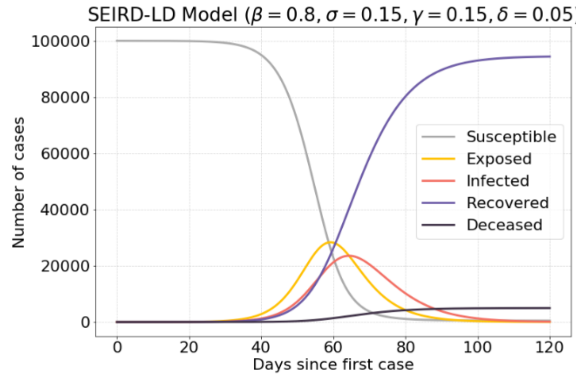

Figure 4) Increased recovery rate γ.

If it takes longer to recover, a massive surge is seen in the infected cases. There are about 50.000 infected cases, which is half of the population.

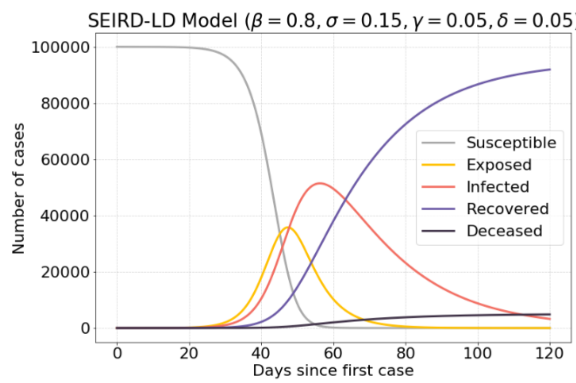

Figure 5) Decreased recovery rate γ.

The following model has an increased infection rate β, which means that exposed case are infected faster. This results in 41.000 infected cases.

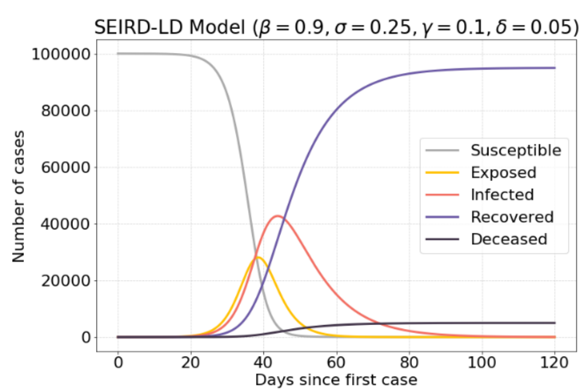

Figure 6) Increased infectious rate β.

Lockdown

The following model is the same model as in the previous section. However, after a period of time a lockdown will be in effect. This is modeled by settings β to a very low value during a brief period of time t.

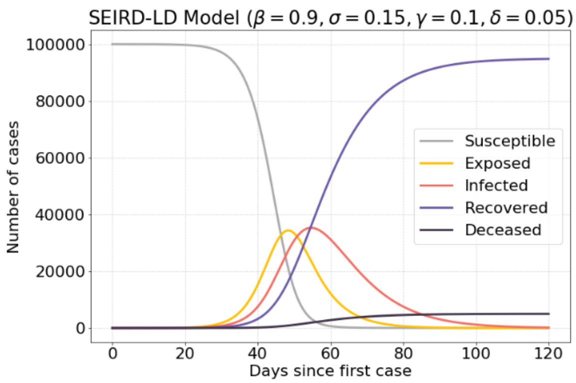

Figure 7) No lockdown

Without the lockdown there is a maximum of 38.000 infected cases, while with a good timed lockdown, there are 20.000 infected cases. This is a reduction of 47.37%.

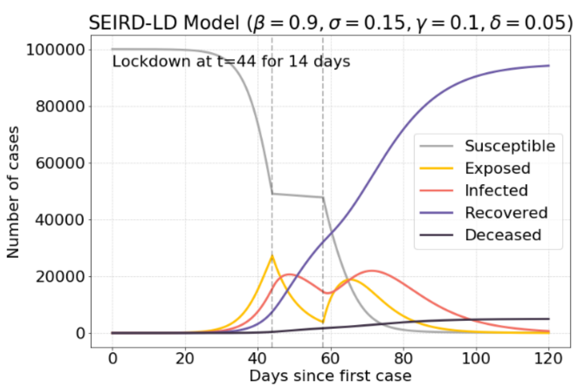

Figure 8) Lockdown