ENGINEERING MATHEMATICS II
UE20MA151
UNIT 1: INTEGRAL CALCULUS

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Problems on double integral

1. Sketch the region 'R' over which we would evaluate the integral $\int_{y=0}^{y=1}\int_{x=0}^{x=2-2y} f(x,y)dx\,dy$.
2. Evaluate $\int_0^4\int_0^4 12\,x^2y^3\ dx\,dy$. **Ans: $\mathbf{4^7}$**
3. Evaluate $\int_0^1\int_0^{\sqrt{1+x^2}} \frac{dydx}{1+x^2+y^2}$. **Ans: $\frac{\Pi}{4}\log_e(\sqrt{2}+1)$.**
4. Find the volume of the solid bounded above by $f(x,y) = x^2$, over the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x, y = 0$ and $x = 8$. **Ans: 448**
5. $\iint_A r^3\,dr\,d\theta$, where A is the area included between the circles r=2sinΘ and r=4sinΘ. **Ans: 22.5 Π**
6. Find the average value of the function $\sqrt{xy-y^2}$, over the triangle with vertex (0,0), (10, 1), (1,1). **Ans: 27**
7. Find the smaller of the areas bounded by $y = 2 - x$ and $x^2 + y^2 = 4$. Ans: $(\Pi - 4)Unit^2$.

CHANGE OF VARIABLES IN DOUBLE INTEGRALS:

1. Transform each of the given integrals to one or more iterated integrals in polar coordinates.

   a) $\int_0^1 dx \int_0^1 f(x,y)dy,$

   b) $\int_0^1 \left[\int_0^{x^2} f(x,y)dy\right] dx.$

   Ans: a) $\boldsymbol{\int_0^{\Pi/4}\left[\int_0^{sec\theta} f(rcos\theta, rsin\theta)rdr\right]d\theta,}$

   b) $\boldsymbol{\int_0^{\Pi/4}\left[\int_0^{sec\theta} f(rcos\theta, rsin\theta)rdr\right]d\,\theta +}$
   $\boldsymbol{\int_{\Pi/4}^{\Pi/4}\left[\int_0^{cosec\theta} f(rcos\theta, rsin\theta)rdr\right]d\,\theta,}$

2. Change into polar co-ordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dy\, dx$.
   Ans: $\boldsymbol{\frac{\Pi}{4}}$ **.**

3. Evaluate double integral $\int_R \int e^{x^2}\, dy\, dx$ where the region R is given by R:$2y \le x \le 2$ and $0 \le y \le 1$. **Ans:** $\boldsymbol{\frac{1}{4}(e^4 - 1)}$ **.**

4. Compute the following integrals by changing to polar coordinates.
   $\int_0^1 \int_x^{\sqrt{2x-x^2}} (x^2+y^2)\, dy\, dx.$ **Ans:** $\boldsymbol{\frac{3}{8}\Pi - 1}$

5. Express $\int_0^{a/\sqrt{2}} \int_0^x x\, dy\, dx + \int_{a/\sqrt{2}}^a \int_0^{\sqrt{a^2-x^2}} x\, dy\, dx$, as a single integral and then evaluate it.
   Ans: $\boldsymbol{\int_0^{\Pi/4} \int_0^a r^2 \cos\theta dr\, d\theta, \frac{a^3}{3\sqrt{2}}}$

6. Evaluate the following integrals by changing to polar coordinates.

   $\int_{x=0}^a \int_{y=0}^{\sqrt{a^2-x^2}} y^2\sqrt{x^2+y^2}\, dy\, dx,$

   Ans: a)$\boldsymbol{\frac{a^5\Pi}{20}}$

7. Find the area inside the circle r=2a cosΘ and outside the circle r=a. Ans: $2a^2\left(\frac{\Pi}{6}+\frac{\sqrt{3}}{4}\right)$ square units.

CHANGING THE ORDER OF INTEGRATION:

1. Show that $\int_0^1 \int_0^1 \frac{x-y}{(x+y)^3} dy\, dx \neq \int_0^1 \int_0^1 \frac{x-y}{(x+y)^3} dx\, dy$.
2. Evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dy\, dx$ by changing the order of integration.

   Ans: 1
3. Evaluate $\iint (3x^2 + y^2) dA$ over the region bounded by $-2 \le y \le 3, y^2 - 3 \le x \le y + 3$. **Ans:2375/7**.
4. Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate. **Ans:** $\mathbf{\frac{3}{8}}$.

TRIPLE INTEGRALS

1. Evaluate $\int_2^3 \int_1^2 \int_2^5 xy^2 dz\, dy\, dx$. Ans: $\frac{35}{2}$
2. Evaluate $I = \int_{x=0}^{x=1} \int_{y=0}^{y=1} \int_{z=0}^{z=1} x + y + z\, dz\, dy\, dx$. Ans: $\frac{3}{2}$.
3. Evaluate the following triple integral.
   $\int_{-1}^{1} dz \int_0^z dx \int_{x-z}^{x+z} (x + y + z) dy$ Ans: 0
4. Find the volume of the solid bounded by the surfaces $z = 0, z = 1 - x^2 - y^2, y = 0, y = 1 - x, x = 0 \text{ and } x = 1$. Ans: $\frac{1}{3}$
5. The temperature at a point (x,y,z) of a solid E bounded by the planes $x = 0, y = 0, z = 0$ and the plane $x + y + z = 1$ is $\frac{1}{(x+y+z)^3}$ degree Celsius . Find the average temperature over the solid.
   Ans: $(\frac{\ln 2}{2} - \frac{5}{16})/6$.
6. Evaluate the triple integral $\iiint_E \sqrt{x^2 + z^2} dxdydz$, where E is the region bounded by the paraboloid $y = x^2 + z^2$ and the plane $y = 4$. Ans: $\frac{128\pi}{15}$

CHANGE OF VARIABLES IN TRIPLE INTEGRALS: CYLINDRICAL AND SPHERICAL

1. Use cylindrical co-ordinates, to evaluate $\iiint_V (x^2+y^2)dxdydz$ taken over the region V bounded by the paraboloid $z = 9 - x^2 - y^2$ and the plane z=0 Ans: $\frac{243\pi}{2}$
2. By transforming into cylindrical co-ordinates evaluate the integral $\int\int\int (x^2+y^2+z^2)\,dx\,dy\,\,dz$ taken over the region

   $0 \le z \le x^2 + y^2 \le 1.$ Ans: $\frac{5\pi}{6}$
3. Calculate the volume of the solid bounded by the paraboloid $z = 2 - x^2 - y^2$ and the conic surface $z = \sqrt{x^2+y^2}$.

The region of integration is bounded from above by the paraboloid, and from below by the cone (Figure 11).

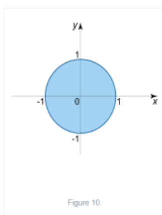

Figure 10

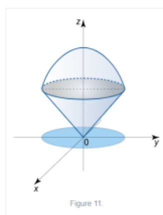

Figure 11

Ans: $2\pi\left(\frac{6\sqrt{6}-11}{3}\right)$

4. Evaluate $\iiint_V xyz(x^2+y^2+z^2)^{\frac{n}{2}}dxdydz$ taken through the positive octant of the sphere $x^2+y^2+z^2 = b^2$ provided $n+5>0$. Ans: $\frac{b^{n+6}}{8(n+6)}$
5. Evaluate $\int\int\int z^2\,dx\,dy\,dz$ taken over the volume bounded by the surfaces $x^2+y^2 = z$ and z=0. Ans: $\frac{\pi a^8}{12}$.

CENTER OF MASS AND MOMENT OF INERTIA

1. Find the total mass of the region in the cube.
   a. $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$ with density at any point given by xyz. Ans: $\frac{1}{8}$
2. Compute the mass of a sphere of radius b if the density varies inversely as the square of the distance from the center. Ans: 4kπb
3. Compute the moment of inertia of a right circular cylinder of altitude 2h and radius b, relative to the diameter of its median section with density equals to k, a constant. Ans: k($\frac{2\pi h^3 b^2}{3}$ +$\frac{hb^4}{2}$).