PES UNIVERSITY

PES University, Bangalore

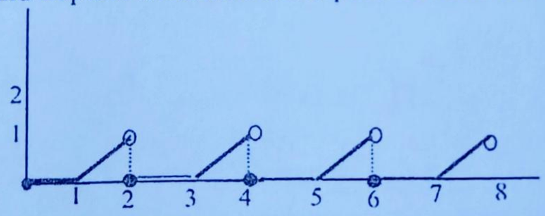
(Established under Karnataka Act No. 16 of 2013)

UE20MA151

END SEMESTER ASSESSMENT (ESA), B.TECH. II- SEMESTER- JULY- 2021

UE20MA151- ENGINEERING MATHEMATICS-II

Time: 3 Hrs | Answer All Questions | Max Marks: 100

1.	a)	Evaluate $\int_0^{\frac{a}{\sqrt{2}}}\int_0^x x\,dy\,dx + \int_{\frac{a}{\sqrt{2}}}^{a}\int_0^{\sqrt{a^2-x^2}} x\,dy\,dx$ by changing the order of integration. Also sketch the region of integration.	6
	b)	Find the mass of the lemniscate $(x^2+y^2)^2 = x^2-y^2$, with density function given by  $\frac{1}{(1+x^2+y^2)^2}$.	7
	c)	The temperature at a point (x,y,z) of a solid E bounded by the planes $x=0, y=0, z=0$ and the plane $x+y+z=1$ is $\frac{1}{(1+x+y+z)^3}$ degree Celsius. Find the average temperature over the solid.	7
2.	a)	Find the Directional derivative of $\vec{V}^2$, where $\vec{V} = xy^2i + zy^2j + xz^2k$ at the point (2,0,3) in the direction of the outward normal to the sphere $x^2+y^2+z^2 = 14$ at the point (3,2,1).	6
	b)	Use Stoke's theorem to evaluate $\oint_C \vec{F}\cdot\overrightarrow{dr}$, where $\vec{F} = y^2i + xyj + xzk$ and C is the bounding curve of the hemisphere $x^2+y^2+z^2 = 9, z > 0$, oriented in a positive direction.	7
	c)	Use Gauss's Divergence theorem to evaluate $\int\int \vec{v}.\hat{n}\,dA$, where $\vec{v} = x^2zi + yj - xz^2k$ over the boundary of the region bounded by the paraboloid $z = x^2+y^2$ and the plane $z = 4y$.	7
3.	a)	Find Laplace transform of the periodic function whose graph is given below. 	6
	b)	Find $L\{coshat\ sinbt + 2^t + sin2t\ cos3t + t^{3/2}\}$.	7
	c)	Find $L\{\int_0^t \frac{sint}{t} + \delta(t-a)u(t-a) + t\,e^{-t}sin2t\}$.	7
4.	a)	Find $L^{-1}\left\{\frac{1-3s}{s^2+8s+21} + \frac{s^2-1}{(s^2+1)^2} + \cot^{-1}s\right\}$	6
	b)	Apply Convolution theorem to evaluate the inverse Laplace transform of $L^{-1}\left\{\frac{s^2}{s^4+4a^4}\right\}$.	7
	c)	The deflection of a beam of length L, clamped, horizontally at both ends and loaded at $x = \frac{L}{4}$ by a weight W is given by $EI\frac{d^4y}{dx^4} = W\,\delta(x-\frac{L}{4})$. Find the deflection curve, given that $y = \frac{dy}{dx} = 0$ when $x = 0\ and\ x = L$.	7
5.	a)	Find the complex form of the Fourier Series of the function $f(x) = e^x$ in $-\pi \le x \le \pi$	6
	b)	Find the Fourier series of $f(x) = x^2$in $(-\pi,\pi)$. Hence deduce that $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$.	7

c) The following table gives displacement 'u' (in min) of a sliding piece from a fixed reference point for every 30 degrees of rotation of the crank. 7

θ	0	30	60	90	120	150	180	210	240	270	300	330
u	298	356	373	337	254	155	80	51	60	93	147	221

Find the constant term and coefficients of the first and second harmonics in the Fourier series expansion of 'u'. (Leave the answer rounded off to two decimal places)