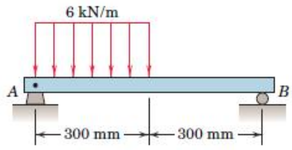
Chapter 5, Problem 101P

Problem

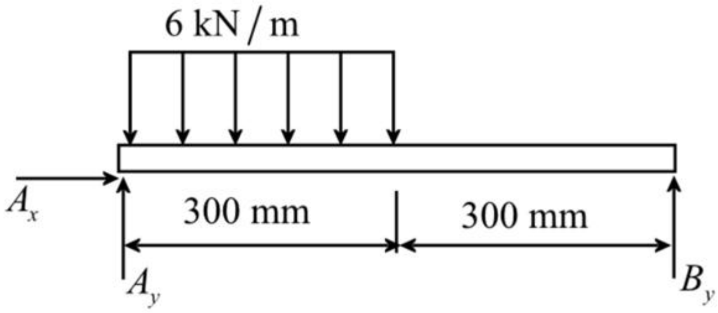
Determine the reactions at *A* and *B* for the beam subjected to the uniform load distribution.



Step-by-step solution

Step 1 of 3

Draw the free body diagram for the beam as follows:



Step 2 of 3

Consider the moment equilibrium condition about *A*:

$$\Sigma M_A = 0$$

$$B_y(0.6\text{ m}) - 6(0.3)\left(\frac{0.3}{2}\right) = 0$$

$$B_y = 0.45\text{ kN}(\uparrow)$$

Therefore, the support reaction at *B* is $\boxed{0.45\text{ kN}(\uparrow)}$.

Comments (2)

Anonymous

you converted the 300mm to 0.3m for both then added them. Afterwards you subtracted 6kN times 0.3m times 0.3/2. However, why are we dividing by 2?

Anonymous

Just need more of an explanation than showing steps.

Step 3 of 3

Consider the horizontal force equilibrium condition:

$$\Sigma F_x = 0$$

$$A_x = 0$$

Consider the vertical force equilibrium condition:

$$\Sigma F_y = 0$$

$$A_y + B_y - 6(0.3\text{ m}) = 0$$

Substitute 0.45 kN for B_y.

$$A_y + 0.45 - 6(0.3\text{ m}) = 0$$

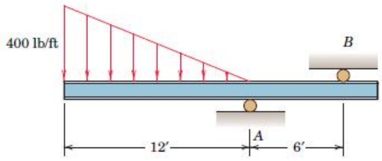
$$A_y = 1.35\text{ kN}(\uparrow)$$

Therefore, the support reaction at *A* is $\boxed{1.35\text{ kN}(\uparrow)}$.

Chapter 5, Problem 102P

Problem

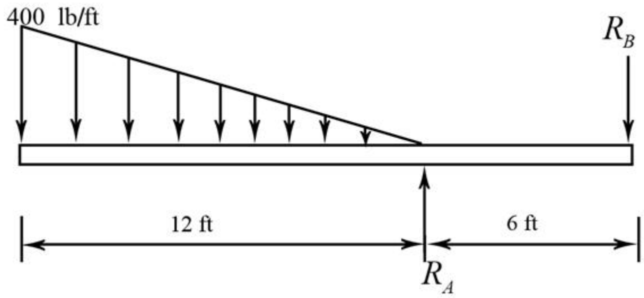
Calculate the reactions at *A* and *B* for the beam loaded as shown.



Step-by-step solution

Step 1 of 4

Draw the free body diagram of the beam and indicate the forces acting on it.



Step 2 of 4

Calculate the force acting by the distributed load acting on the beam.

$$F = \frac{Wl}{2}$$

Here, *W* is the distributed load acting on the beam.

Substitute $400\ \text{lb/ft}$ for *W* and $12\ \text{ft}$ for *l*.

$$F = \frac{400 \times 12}{2}$$
$$= 2400\ \text{lb}$$

Step 3 of 4

Take moment about *A*.

$$\Sigma M_A = 0$$
$$F\left(\frac{2}{3} \times 12\right) - R_B\left(L_{AB}\right) = 0$$

Here, R_B is the reaction force at *B* and L_{AB} is length of the beam *AB*.

Substitute $2400\ \text{lb}$ for *F* and $6\ \text{ft}$ for L_{AB}.

$$2400\left(\frac{2}{3} \times 12\right) - R_B(6) = 0$$
$$19200 - 6R_B = 0$$
$$R_B = 3200\ \text{lb}$$

Therefore, the reaction force at *B* is $\boxed{3200\ \text{lb}}$.

Step 4 of 4

Calculate the reaction force at *A*.

Consider the vertical force equilibrium of the beam.

$$\Sigma F = 0$$
$$R_A - F - R_B = 0$$

Substitute $2400\ \text{lb}$ for *F* and $3200\ \text{lb}$ for R_B.

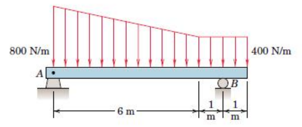
$$R_A - 2400 - 3200 = 0$$
$$R_A = 5600\ \text{lb}$$

Therefore, the reaction force at *A* is $\boxed{5600\ \text{lb}}$.

Chapter 5, Problem 103P

Problem

Determine the reactions at the supports of the beam which is loaded as shown.

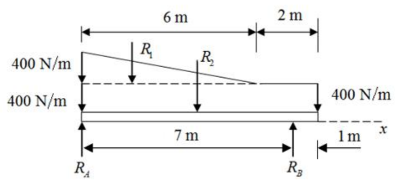


Step-by-step solution

Step 1 of 5

Draw the free body diagram and indicate the forces acting on it.

Distributed load area is divided into triangular and rectangular area and the corresponding resultant forces R_1, R_2 act at the centroid of the areas.



Step 2 of 5

Calculate the resultant load R_1.

$$R_1 = \frac{Wl}{2}$$

Here, W is the load acting on the triangular section and l is the length of triangular section.

Substitute $400\ \text{N/m}$ for W and $6\ \text{m}$ for l.

$$R_1 = \frac{400 \times 6}{2} = 1200\ \text{N}$$

Comments (3)

Anonymous
Where is the 1/2 coming from?

Anonymous
The resultant R1 is the area of the triangle. Thus we use the area for a right triangle formula -> A = (1/2) x base x height. For the base we have 7m, and for the height we have 400 N/m. This leaves us will 1200 (Nm/m) which simplifies to 1200 N.

Anonymous
My apologies, for base we have 6m.

Step 3 of 5

Calculate the reaction forces R_2.

$$R_2 = Wl$$

Here, W is the load acting in rectangular section and l is the length upon which load acts.

Substitute $400\ \text{N/m}$ for W and $8\ \text{m}$ for l.

$$R_2 = (400 \times 8) = 3200\ \text{N}$$

Step 4 of 5

Let R_A, R_B be the reaction forces acting at *A*, *B*.

Take moments about *A*.

$$\sum M_A = 0:$$
$$R_B L_{AB} - R_1\bar{x}_1 - R_2\bar{x}_2 = 0$$

Here, L_{AB} is the length *AB* of the beam, R_1 is the resultant load in triangular section, R_2 is the resultant load in rectangular section, $\bar{x}_1$ is the distance from the left end at which R_1 acts, $\bar{x}_2$ is the distance from the left end at which R_2 acts.

Substitute $1200\ \text{N}$ for R_1, $3200\ \text{N}$ for R_2, $2\ \text{m}$ for $\bar{x}_1$, and $4\ \text{m}$ for $\bar{x}_2$.

$$R_B \times (7) - (1200 \times 2) - (3200 \times 4) = 0$$
$$R_B = 2170\ \text{N}$$

Therefore, reaction force acting at *B* is $\boxed{2170\ \text{N}}$.

Step 5 of 5

Consider equilibrium of horizontal forces acting on the beam.

$$\sum F_y = 0:$$
$$R_A - R_1 - R_2 + R_B = 0$$

Substitute $1200\ \text{N}$ for R_1, $3200\ \text{N}$ for R_2, and $2170\ \text{N}$ for R_B.

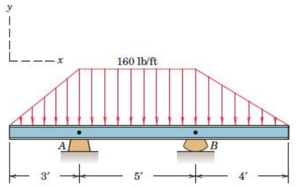
$$R_A - 1200 - 3200 + 2170 = 0$$
$$R_A = 2230\ \text{N}$$

Therefore, reaction force acting at *A* is $\boxed{2230\ \text{N}}$.

Chapter 5, Problem 104P

Problem

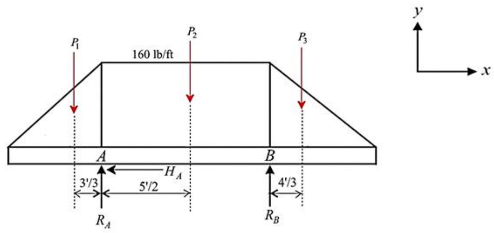
Determine the reactions at *A* and *B* for the loaded beam.



Step-by-step solution

Step 1 of 5

Draw free body diagram of the loaded beam:



Step 2 of 5

Calculate the force P_1.

$$P_1 = \frac{1}{2} \times 160 \times 3$$
$$= 240 \text{ lb}$$

The force P_1 acts at a distance of $3'/3$ from point *A*.

Calculate the force P_2.

$$P_2 = 160 \times 5$$
$$= 800 \text{ lb}$$

The force P_2 acts at midpoint that is at $5'/2$.

Calculate the force P_3.

$$P_3 = \frac{1}{2} \times 160 \times 4$$
$$= 320 \text{ lb}$$

The force P_1 acts at a distance of $4'/3$ from point *B*.

Step 3 of 5

Calculate the vertical reaction force at point *B*.

Take moments about *A*.

$$\sum M_A = 0$$
$$(R_B \times 5) + \left(P_1 \times \frac{3}{3}\right) = \left(P_2 \times \frac{5}{2}\right) + \left(P_3 \times \left(5 + \frac{4}{3}\right)\right)$$

Here, R_B is vertical the reaction force at support *B*.

Substitute 240 lb for P_1, 800 lb for P_2 and 320 lb for P_3.

$$(R_B \times 5) + \left(240 \times \frac{3}{3}\right) = \left(800 \times \frac{5}{2}\right) + \left(320 \times \left(5 + \frac{4}{3}\right)\right)$$
$$(R_B \times 5) + 240 = 2000 + 2026.67$$
$$R_B = 757.33 \text{ lbs}$$

Therefore, vertical reaction force at support *B* is $\boxed{757.33 \text{ lbs}}$.

Step 4 of 5

Calculate the vertical reaction force at support *A*.

Apply force equilibrium equation along vertical direction.

$$\sum F_y = 0$$
$$R_A + R_B = P_1 + P_2 + P_3$$

Substitute 240 lb for P_1, 800 lb for P_2, 757.33 lbs for R_B and 320 lb for P_3.

$$R_A + 757.33 = 240 + 800 + 320$$
$$R_A = 1360 - 757.33$$
$$R_A = 602.67 \text{ lbs}$$

Therefore, vertical reaction force at support *A* is $\boxed{602.67 \text{ lbs}}$.

Step 5 of 5

Calculate the horizontal reaction force at support *A*.

Apply force equilibrium equation along horizontal direction.

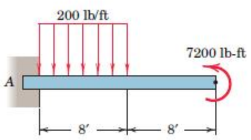
$$\sum F_x = 0$$
$$H_A = 0$$

Therefore, horizontal reaction force at support *A* is $\boxed{0}$.

Chapter 5, Problem 105P

Problem

Find the reaction at *A* due to the uniform loading and the applied couple.



Step-by-step solution

Step 1 of 6

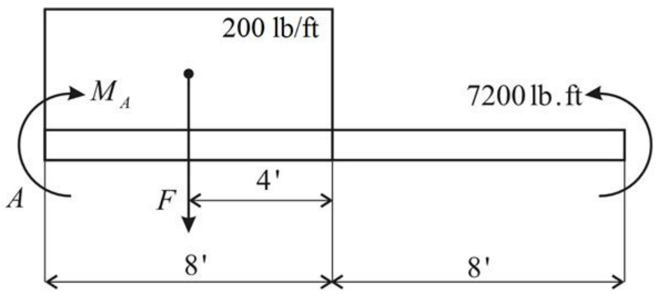
679-5-99P SA: 4540

SR: 7385

Step 2 of 6

Let R_A and M_A be the reacting forces and reacting moment about '*A*'

Step 3 of 6



Step 4 of 6

$F = 200 \times 8$

$= 1600\text{lb}$

Step 5 of 6

Taking moment about *A*

$\Sigma M_A = 0$

$7200 - M_A - 1600 \times 4 = 0$

$M_A = 7200 - 6400$

$M_A = 800$

$\therefore \boxed{M_A = 800\text{lb} \cdot \text{ft}}$

Step 6 of 6

Considering forces along y-axis

$\Sigma F_y = 0$

$R_A - F = 0$

$R_A - 1600 = 0$

$R_A = 1600$

$\therefore \boxed{R_A = 1600\text{lb}}$

Comments (1)

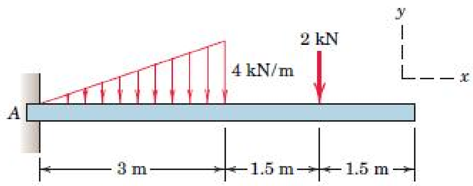
Anonymous

RsubA is almost correct since it is opposite force it is negative.

Chapter 5, Problem 106P

Problem

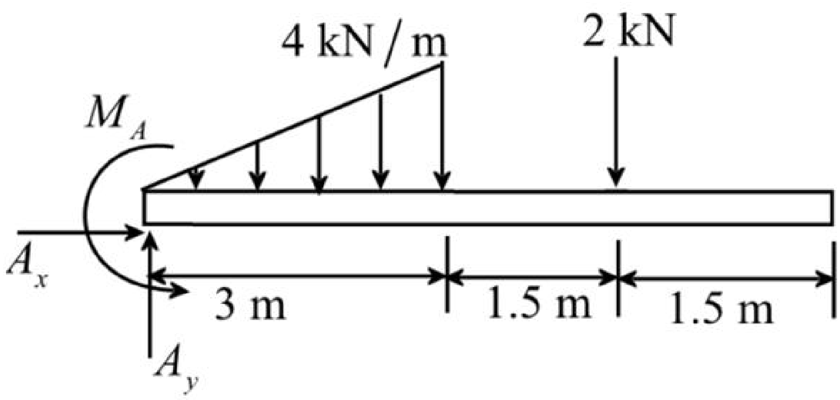
Determine the reactions at *A* for the cantilever beam subjected to the distributed and concentrated loads.



Step-by-step solution

Step 1 of 3

Draw the free body diagram for the beam as follows:



Step 2 of 3

Consider the horizontal force equilibrium condition:

$$\Sigma F_x = 0$$
$$A_x = 0$$

Consider the vertical force equilibrium condition:

$$\Sigma F_y = 0$$
$$A_y - \left(\frac{1}{2} \times 4 \times 3\right) - 2 = 0$$
$$A_y = 8 \text{ kN}(\uparrow)$$

Therefore, the horizontal and vertical forces at *A* are $\boxed{0}$ and $\boxed{8 \text{ kN}(\uparrow)}$.

Step 3 of 3

Consider the moment equilibrium condition about *A*.

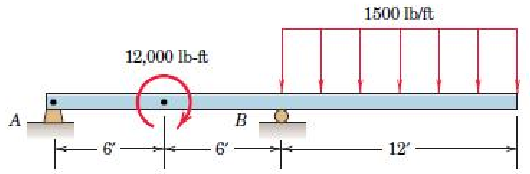
$$\Sigma M_A = 0$$
$$M_A - \left(\frac{1}{2} \times 4 \times 3\right)\left(\frac{2}{3} \times 3\right) - 2(4.5) = 0$$
$$M_A = 21 \text{ kN} \cdot \text{m}$$

Therefore, the moment about point *A* is $\boxed{21 \text{ kN} \cdot \text{m}}$ in counter clockwise direction.

Chapter 5, Problem 107P

Problem

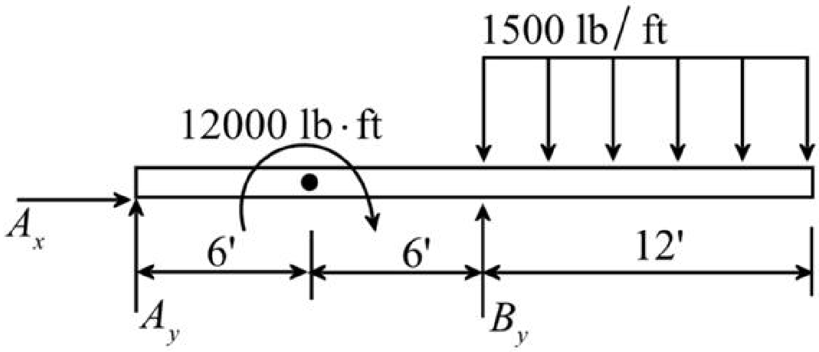
Determine the reactions at *A* and *B* for the beam loaded as shown.



Step-by-step solution

Step 1 of 3

Draw the free body diagram for the beam as follows:



Step 2 of 3

Consider the moment equilibrium condition about *A*.

$$\Sigma M_A = 0$$

$$B_y(12) - 1500(12)\left(12 + \frac{12}{2}\right) - 12000 = 0$$

$$B_y = 28000\ \text{lb}(\uparrow)$$

Therefore, the support reaction at point *B* is $\boxed{28000\ \text{lb}(\uparrow)}$.

Step 3 of 3

Consider the horizontal force equilibrium condition:

$$\Sigma F_x = 0$$

$$A_x = 0$$

Consider the vertical force equilibrium condition:

$$\Sigma F_y = 0$$

$$A_y + B_y - 1500 \times 12 = 0$$

Substitute 28000 lb for B_y.

$$A_y + 28000 - 1500 \times 12 = 0$$

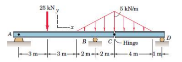
$$A_y = -10000\ \text{lb}$$

Therefore, the reaction at *A* is $\boxed{10000\ \text{lb}(\downarrow)}$.

Chapter 5, Problem 109P

Problem

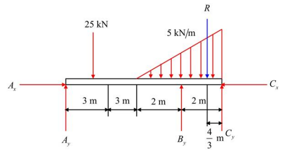
Determine the reactions at *A*, *B*, and *D* for the pair of beams connected by the ideal pin at *C* and subjected to the concentrated and distributed loads.



Step-by-step solution

Step 1 of 10

Draw the free body diagram of the beam *AC* and indicate the forces acting on it.



Comments (1)

Anonymous
why 4/3

Step 2 of 10

Calculate the reaction force *R* due to the varying load.

$$R = \frac{wl}{2}$$

Here, w is the load acting on the beam and *l* is the length upon which load acts.

Substitute $5\ \text{kN/m}$ for *w* and $4\ \text{m}$ for *l*.

$$R = \frac{5 \times 4}{2} = 10\ \text{kN}$$

Comments (2)

Anonymous
why 4

Anonymous
2m+2m =4m

Step 3 of 10

Consider the equilibrium of forces acting along the horizontal direction.

$$\Sigma F_x = 0$$
$$A_x - C_x = 0 \quad \ldots\ldots (1)$$

Here, A_x is the horizontal component of force at *A* and C_x is the horizontal component of force at *C*.

Consider the equilibrium of forces acting along the vertical direction.

$$\Sigma F_y = 0$$
$$A_y + B_y - C_y - 25 - R = 0 \quad \ldots\ldots (2)$$

Here, A_y is the vertical reaction force at *A*, B_y is the normal reaction force at *B*, C_y is the vertical reaction force at *C*, and R is the reaction force.

Step 4 of 10

Consider the moments acting on beam about *A*.

$$\Sigma M_A = 0$$
$$-25L_{AE} + B_y L_{AB} - C_y L_{AC} - \left(10 - \frac{4}{3}\right)R = 0$$

Here, L_{AC} is the length *AC* of the beam, L_{AB} is the length *AB* of the beam, and L_{AE} is the length *AE* of the beam.

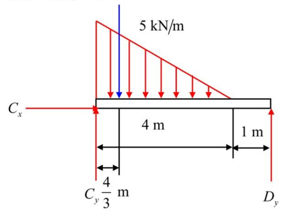
Substitute $3\ \text{m}$ for L_{AD}, $8\ \text{m}$ for L_{AB}, $10\ \text{m}$ for L_{AC}, and $10\ \text{kN}$ for R.

$$-25L_{AD} + B_y L_{AB} - C_y L_{AC} - \left(10 - \frac{4}{3}\right)R = 0$$
$$-25 \times 3 + B_y \times 8 - C_y \times 10 - \left(10 - \frac{4}{3}\right) \times 10 = 0$$
$$8B_y - 10C_y = 162 \quad \ldots\ldots (3)$$

Step 5 of 10

Consider the beam *CD*.

Draw the free body diagram of the beam.



Step 6 of 10

Consider the equilibrium of forces acting in *x*-direction.

$$\Sigma F_x = 0$$
$$C_x = 0$$

Substitute 0 for C_x in equation (1).

$$A_x - C_x = 0$$
$$A_x - 0 = 0$$
$$A_x = 0$$

Therefore, horizontal component of force at *A*, A_x, is $\boxed{0}$.

Step 7 of 10

Consider the moments acting on beam about *C*.

$$\Sigma M_C = 0$$
$$D_y L_{CD} - \left(\frac{4}{3}\right)R = 0$$

Here, L_{CD} is the length *CD* of the beam.

Substitute $5\ \text{m}$ for L_{CD} and $10\ \text{kN}$ for R.

$$L_{CD}D_y - \left(\frac{4}{3}\right)R = 0$$
$$5 \times D_y - \left(\frac{4}{3}\right) \times 10 = 0$$
$$D_y = 2.67\ \text{kN}$$

Therefore, vertical component of force at *D*, D_y, is $\boxed{2.67\ \text{kN}}$.

Step 8 of 10

Consider the equilibrium of forces acting in *y*-direction.

$$\Sigma F_y = 0$$
$$C_y + D_y - R = 0$$

Substitute $10\ \text{kN}$ for R and $2.67\ \text{kN}$ for D_y.

$$C_y + D_y - R = 0$$
$$C_y + 2.67 - 10 = 0$$
$$C_y = 7.33\ \text{kN}$$

Therefore, vertical component of force at *C*, C_y, is $\boxed{7.33\ \text{kN}}$.

Step 9 of 10

Calculate the vertical reaction force at *B*.

$$8B_y - 10C_y = 162$$

Substitute $7.33\ \text{kN}$ for C_y.

$$8B_y - 10C_y = 162$$
$$8B_y + 10 \times 7.33 = 162$$
$$B_y = 29.4\ \text{kN}$$

Therefore, vertical component of force at *B*, B_y, is $\boxed{29.4\ \text{kN}}$.

Step 10 of 10

Calculate the vertical reaction force at *A*.

$$A_y + N_B - C_y - 25 - R = 0$$

Substitute $7.33\ \text{kN}$ for C_y, $29.4\ \text{kN}$ for B_y, and $10\ \text{kN}$ for R.

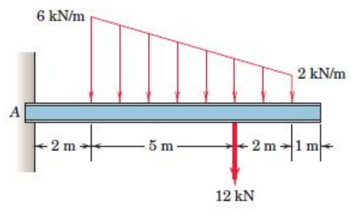
$$A_y + N_B - C_y - 25 - R = 0$$
$$A_y + 29.4 - 7.33 - 25 - 10 = 0$$
$$A_y = 12.93\ \text{kN}$$

Therefore, vertical component of force at *A*, A_y, is $\boxed{12.93\ \text{kN}}$.

Chapter 5, Problem 110P

Problem

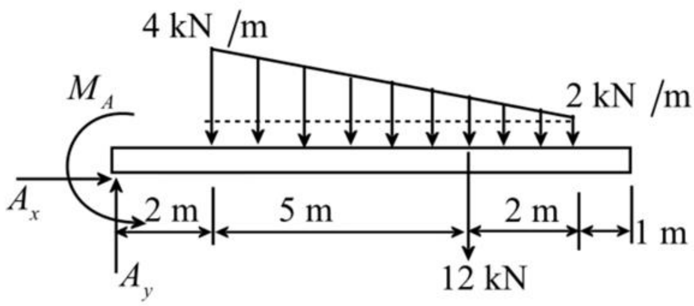
Determine the force and moment reactions at *A* for the cantilever beam subjected to the loading shown.



Step-by-step solution

Step 1 of 3

Draw the free body diagram for the beam as follows:



Comments (2)

Anonymous

why is it 4Kn/m and not 6Kn/m

Anonymous

^ The shape is split into a triangle and a square. 4Kn/m is the height of the triangle while 2Kn/m is the height of the rectangle.

Step 2 of 3

Consider the horizontal force equilibrium condition:

$$\Sigma F_x = 0$$
$$A_x = 0$$

Consider the vertical force equilibrium condition:

$$\Sigma F_y = 0$$
$$A_y - \left(\frac{1}{2}\times 4\times 7\right) - (2\times 7) - 12 = 0$$
$$A_y = 40\ \text{kN}(\uparrow)$$

Therefore, the horizontal and vertical forces at *A* are $\boxed{0}$ and $\boxed{40\ \text{kN}(\uparrow)}$.

Step 3 of 3

Consider the moment equilibrium condition about *A*.

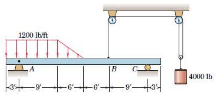
$$\Sigma M_A = 0$$
$$M_A - \left(\frac{1}{2}\times 4\times 7\right)\left(2+\frac{1}{3}\times 7\right) - (2\times 7)\left(2+\frac{7}{2}\right) - 12(7) = 0$$
$$M_A - 60.667 - 77 - 84 = 0$$
$$M_A = 221.667\ \text{kN}\cdot\text{m}$$
$$M_A \approx 222\ \text{kN}\cdot\text{m}$$

Therefore, the moment about point *A* is $\boxed{222\ \text{kN}\cdot\text{m}}$ in counter clockwise direction.

Chapter 5, Problem 111P

Problem

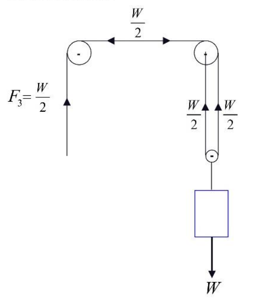
Determine the reactions at *A* and *C* for the beam subjected to the combination of point and distributed loads.

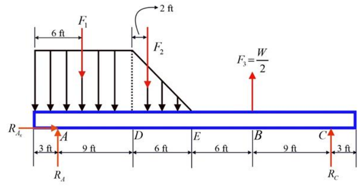


Step-by-step solution

Step 1 of 6

Draw the free body diagram:





Load area is divided into triangular and rectangular areas and the corresponding resultant forces F_1 and F_2 act at the centroid of the areas.

Comments (1)

Anonymous
why f3 w/2

Step 2 of 6

Consider the rectangular area enclosed between points *A* and *D* of the distributed load.

For the rectangular area, the centroid is at a distance of 6 ft from *A*.

The resultant force is given by the area of the rectangle.

$$F_1 = 1200 \times 12$$
$$F_1 = 14400\text{ lb}$$

Consider the triangular area enclosed between points *D* and *E* of the distributed load.

For the triangular area, the centroid is at a distance of 2 ft from *D*.

(Since, for a right angled triangle centroid is at distance of one third of the base, from the right angle)

The resultant force is given by the area of the triangle.

$$F_2 = \frac{1}{2} 1200 \times 6$$
$$F_2 = 3600\text{ lb}$$

Comments (2)

Anonymous
f1 is wronggggggggggggggggg

Anonymous
why f2 * 1/2?????????????

Step 3 of 6

Consider equilibrium of forces in horizontal direction.

$$\Sigma F_x = 0$$
$$R_{A_x} = 0$$

Here, R_{A_x} is the horizontal reaction at *A*.

Therefore, the horizontal reaction acting at *A* is $\boxed{0}$.

Step 4 of 6

Consider equilibrium of forces in vertical direction.

$$\Sigma F_y = 0$$
$$R_A - F_1 - F_2 + \frac{W}{2} + R_C = 0$$

Here, *W* is the weight acting on the pulley, R_A is the vertical reaction at *A* and R_C is the reaction at *C*.

Substitute 14400 lb for F_1, 3600 lb for F_2 and 4000 lb for *W*.

$$R_A - 14400 - 3600 + \frac{4000}{2} + R_C = 0$$
$$R_A + R_C = 16000\text{ lb} \qquad \text{......(1)}$$

Step 5 of 6

Calculate the vertical reaction force at *C* as follows:

Take moments about *A* from the sketch.

$$\Sigma M_A = 0$$
$$-3F_1 - 11F_2 + 21\left(\frac{W}{2}\right) + 30R_C = 0$$

Substitute 14400 lb for F_1, 3600 lb for F_2 and 4000 lb for *W*.

$$-3 \times 14400 - 11 \times 3600 + 21 \times \left(\frac{4000}{2}\right) + 30R_C = 0$$
$$R_C = 1360\text{ lb}$$

Therefore, vertical reaction force acting at *C* is $\boxed{1360\text{ lb up}}$.

Comments (1)

Anonymous
Where did 11 and 21 and 0 come from?

Step 6 of 6

Calculate the vertical reaction force acting at *A* as follows:

Substitute 1360 lb for R_C in equation (1).

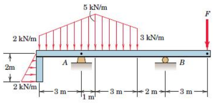
$$R_A + R_C = 16000\text{ lb}$$
$$R_A + 1360 = 16000\text{ lb}$$
$$R_A = 14640\text{ lb}$$

Therefore, vertical reaction force acting at *A* is $\boxed{14640\text{ lb up}}$.

Chapter 5, Problem 116P

Problem

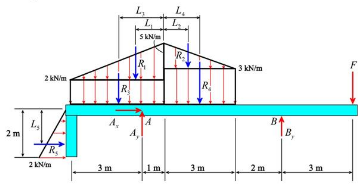
For the beam and loading shown, determine the magnitude of the force *F* for which the vertical reactions at *A* and *B* are equal. With this value of *F*, compute the magnitude of the pin reaction at *A*.



Step-by-step solution

Step 1 of 7

Draw the free body diagram of the beam dividing the distributed load area into a combination of rectangular and triangular areas showing the resultants of the areas and location as shown in the figure,



Step 2 of 7

Find the resultant load of the triangular area 1.

$$R_1 = \frac{1}{2} \times 4\,\text{m} \times (5\,\text{kN/m} - 2\,\text{kN/m})$$
$$= 6\,\text{kN}$$

Find the resultant load of the triangular area 2.

$$R_2 = \frac{1}{2} \times 3\,\text{m} \times (5\,\text{kN/m} - 3\,\text{kN/m})$$
$$= 3\,\text{kN}$$

Find the resultant load of the rectangular area 3.

$$R_3 = 4\,\text{m} \times 2\,\text{kN/m}$$
$$= 8\,\text{kN}$$

Find the resultant load of the rectangular area 4.

$$R_4 = 3\,\text{m} \times 3\,\text{kN/m}$$
$$= 9\,\text{kN}$$

Find the resultant load of the triangular area 5.

$$R_5 = \frac{1}{2} \times 2\,\text{m} \times 2\,\text{kN/m}$$
$$= 2\,\text{kN}$$

Step 3 of 7

Find the distance L_1.

$$L_1 = \frac{1}{3}(4\text{m})$$
$$= \frac{4}{3}\text{m}$$

Find the distance L_2.

$$L_2 = \frac{1}{3}(3\,\text{m})$$
$$= 1\,\text{m}$$

Find the distance L_3.

$$L_3 = \frac{1}{2}(4\,\text{m})$$
$$= 2\,\text{m}$$

Find the distance L_4.

$$L_4 = \frac{1}{2}(3\,\text{m})$$
$$= 1.5\,\text{m}$$

Find the distance L_5.

$$L_5 = \frac{2}{3}(2\,\text{m})$$
$$= \frac{4}{3}\text{m}$$

Step 4 of 7

Apply force equilibrium along horizontal direction to find the horizontal component of pin reaction force at *A*.

$$\sum F_x = 0;$$
$$A_x + R_5 = 0$$
$$A_x + 2 = 0$$
$$A_x = -2\,\text{kN}$$

Step 5 of 7

Apply force equilibrium along vertical direction.

$$\sum F_y = 0;$$
$$A_y + B_y - F - R_1 - R_2 - R_3 - R_4 = 0$$
$$A_y + B_y - F - 6 - 3 - 8 - 9 = 0$$
$$A_y + B_y - F = 26$$

Substitute A_y for B_y because the vertical reaction at *A* and *B* are equal according to the problem statement.

$$A_y + A_y - F = 26$$
$$A_y = 0.5F + 13$$

Step 6 of 7

Apply moment equilibrium about point *B*.

$$\sum M_B = 0;$$
$$R_1(5 + L_1) + R_2(5 - L_2) + R_3(5 + L_3) + R_4(5 - L_4) + R_5(L_5) - A_y(6) - F(3) = 0$$
$$6\left(5 + \frac{4}{3}\right) + 3(5-1) + 8(5+2) + 9(5-1.5) + 2\left(\frac{4}{3}\right) - A_y(6) - F(3) = 0$$
$$140.17 - 6A_y - 3F = 0$$

Substitute $0.5F + 13$ for A_y.

$$140.17 - 6(0.5F + 13) - 3F = 0$$
$$F = 10.36\,\text{kN}$$

Therefore, the magnitude of the force F is $\boxed{10.36\,\text{kN}}$.

Step 7 of 7

Find the vertical component of pin reaction force at *A* using the relation,

$$A_y = 0.5F + 13$$
$$= 0.5(10.36) + 13$$
$$= 18.18\,\text{kN}$$

Find the magnitude of the pin reaction at *A* using the formula,

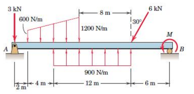
$$R_A = \sqrt{A_x^2 + A_y^2}$$
$$= \sqrt{(-2)^2 + (18.18)^2}$$
$$= 18.29\,\text{kN}$$

Therefore, the magnitude of the pin reaction at *A* is $\boxed{18.29\,\text{kN}}$.

Chapter 5, Problem 122P

Problem

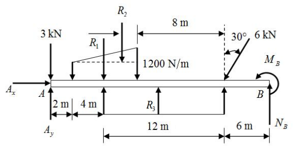
Determine the magnitude of the moment *M* which will cause the beam to just begin to lift off the roller at *B*. For this value of *M*, determine the magnitude of the pin reaction at *A*.



Step-by-step solution

Step 1 of 9

Draw the free body diagram of the beam.



4 m 6 m 12 m 6 kN 30° M_B 8 m 1200 N/m
3 kN 600 N/m R_3

Distributed load area on the upper portion is divided into triangular and rectangular areas. The corresponding resultant forces R_1 and R_2 act at centroid of their respective areas.

Step 2 of 9

Reaction forces due to distributed load on the upper portion of the beam act at the centroid of the area.

For the triangular area, reaction force R_2 acts at a distance of $\frac{8}{3}\text{ m}$ towards left.

For the rectangular area, reaction force R_1 acts at a distance of 4 m.

Step 3 of 9

Calculate the reaction forces R_1.

$$R_1 = Wl$$

Here *W* is the load acting on the beam and *l* is the length of the rectangular section.

Substitute 600 N/m for W, 8 m for *l*.

$$\begin{aligned} R_1 &= 600 \times (8) \\ &= 4800\text{ N} \end{aligned}$$

Comments (1)

Anonymous
Why 8m and not 6m?

Step 4 of 9

Calculate the reaction force R_2.

$$R_2 = \frac{(W_1 - W_2)l}{2}$$

Here W_1 is the load at one end, W_2 is the load at other end, and *l* is the length upon which the load acts.

Substitute 1200 N/m for W_1, 600 N/m for W_2, and 8 m for *l*.

$$\begin{aligned} R_2 &= \frac{(W_1 - W_2)l}{2} \\ &= \frac{1}{2} \times (1200 - 600) \times (8) \\ &= 2400\text{ N} \end{aligned}$$

Step 5 of 9

Uniformly distributed load acts at the bottom portion of the beam.

Reaction force R_3 acts at the bottom of the beam.

Calculate the reaction force R_3.

$$R_3 = Wl$$

Here *W* is the load acting on the beam and *l* is the length upon which the load acts.

Substitute 900 N/m for W_2 and 12 m for *l*.

$$\begin{aligned} R_3 &= 900 \times (12) \\ &= 10800\text{ N} \end{aligned}$$

Step 6 of 9

Consider that A_x and A_y be the horizontal and vertical forces acting at *A*.

Consider the equilibrium of horizontal forces acting on the beam.

$$\begin{aligned} &\Sigma F_x = 0 \\ &A_x - 6 \times (1000) \sin 30^\circ \\ &A_x = 3000\text{ N} \end{aligned}$$

Comments (2)

Anonymous
Where does the 1000 come from?

Anonymous
They made the 6KN go to 6000N. If you want it could be just -6sin(30) which will give you 3 KN = 3000 N

Step 7 of 9

Consider the equilibrium of vertical forces acting on the beam.

$$\begin{aligned} &\Sigma F_y = 0 \\ &3000 + R_1 + R_2 + 6000 \cos 30^\circ = R_3 + N_B + A_y \end{aligned}$$

When the beam just begins to lift, $N_B = 0$.

Substitute 4800 N for R_1, 2400 N for R_2, and 10800 N for R_3.

$$\begin{aligned} &3000 + 4800 + 2400 + 6000 \cos 30^\circ = 10800 + A_y \\ &A_y = 4596.152\text{ N} \end{aligned}$$

Step 8 of 9

Calculate the moment reaction at *A*.

$$\begin{aligned} &\Sigma M_A = 0 \\ &-6R_1 - \left(2 + \frac{2}{3} \times (8)\right) R_2 + 12R_3 - 18 \times (6) \times (1000) \cos 30^\circ + 24N_B + M_A = 0 \end{aligned}$$

Substitute 2400 N for R_2, 10800 N for R_3, 4800 N for R_1, 0 for N_B.

$$\begin{aligned} &-6 \times 4800 - \left(2 + \frac{2}{3} \times (8)\right) \times 2400 + 12 \times 10800 - 18 \times (6) \times (1000) \cos 30^\circ + M_A = 0 \\ &M_A = 10330.74\text{ N} \cdot \text{m} \end{aligned}$$

Therefore, moment about at *A* is $\boxed{10330.74\text{ N} \cdot \text{m counter clockwise}}$.

Comments (3)

Anonymous
why didnt you have to solve for M

Anonymous
This is wrong, as you didnt solve for M, rather you solved for Ma, which is not what the question required

Anonymous
They just stated it wrong. It should just say M

Step 9 of 9

Calculate the resultant reaction force at A.

$$R_A = \sqrt{A_x^2 + A_y^2}$$

Substitute 3000 N for A_x, 4596.152 N for A_y

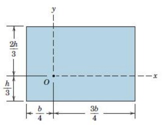
$$\begin{aligned} R_A &= \sqrt{A_x^2 + A_y^2} \\ &= \sqrt{3000^2 + 4596.152^2} \\ &= 5488.58\text{ N} \\ &\simeq 5489\text{ N} \end{aligned}$$

Therefore, the resultant reaction force at *A* is $\boxed{5489\text{ N}}$.

Chapter A, Problem 1P

Problem

Determine the moments of inertia of the rectangular area about the *x*- and *y*-axes and find the polar moment of inertia about point *O*.

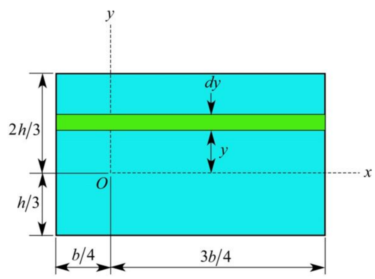


Step-by-step solution

Step 1 of 6

Consider a small horizontal strip on the rectangular area as shown:

Step 2 of 6



Step 3 of 6

Calculate the area of the horizontal strip.

$$dA = b(dy)$$

Here, *b* is with of the rectangle.

Calculate the moment of inertia of the rectangular area about *x* axis as follows:

$$I_x = \int y^2 dA$$

Substitute *bdy* for *dA*.

$$\begin{aligned} I_x &= \int_{-h/3}^{2h/3} y^2 (bdy) \\ &= b\left[\frac{y^3}{3}\right]_{-h/3}^{2h/3} \\ &= b\left[\frac{(2h/3)^3}{3} - \frac{(-h/3)^3}{3}\right] \\ &= b\left(\frac{8h^3}{81} + \frac{h^3}{81}\right) \\ &= \frac{bh^3}{9} \end{aligned}$$

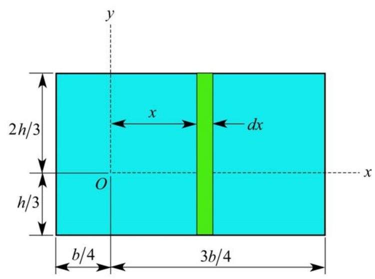
Hence, the moment of inertia of the rectangular area about the *x*-axis is $\boxed{\frac{1}{9}bh^3}$.

Comments (1)

Anonymous
why is y squared

Step 4 of 6

Consider a small vertical strip on the rectangular area as shown:



Step 5 of 6

Calculate the area of the vertical strip.

$$dA = h(dx)$$

Calculate the moment of inertia of the rectangular area about *y* axis as follows:

$$I_y = \int x^2 dA$$

Substitute *hdx* for *dA*.

$$\begin{aligned} I_y &= \int_{-b/4}^{3b/4} x^2 (hdx) \\ &= h\left[\frac{x^3}{3}\right]_{-b/4}^{3b/4} \\ &= h\left[\frac{(3b/4)^3}{3} - \frac{(-b/4)^3}{3}\right] \\ &= h\left(\frac{27b^3}{192} + \frac{b^3}{192}\right) \\ &= \frac{7hb^3}{48} \end{aligned}$$

Hence, the moment of inertia of the rectangular area about the *y*-axis is $\boxed{\frac{7hb^3}{48}}$.

Step 6 of 6

Calculate the polar moment of inertia of the rectangular area about point *O* as follows:

$$I_O = I_x + I_y$$

Substitute $\frac{1}{9}bh^3$ for I_x, and $\frac{7}{48}hb^3$ for I_y.

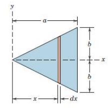
$$\begin{aligned} I_O &= \frac{1}{9}bh^3 + \frac{7}{48}hb^3 \\ &= bh\left(\frac{h^2}{9} + \frac{7b^2}{48}\right) \end{aligned}$$

Hence, the polar moment of inertia of the rectangular area about point *O* is $\boxed{bh\left(\frac{h^2}{9} + \frac{7b^2}{48}\right)}$.

Chapter A, Problem 2P

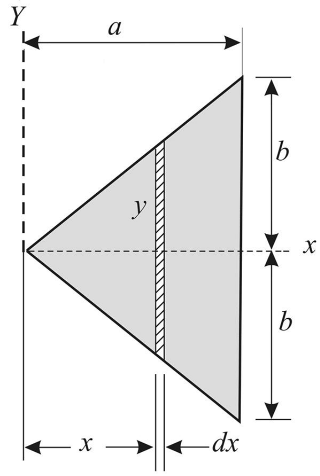
Problem

Use the differential element shown to determine the moment of inertia of the triangular area about the *x*-axis and about the *y*-axis.



Step-by-step solution

Step 1 of 6



Step 2 of 6

For the calculation of the moment of inertia dI_x a triangular strip of area $y \cdot dx$ is chosen so that all elements of the strip have the same $y-\text{coordinate}$

$$\therefore dI_x = \int_{-\frac{y}{2}}^{\frac{y}{2}} y^2 \cdot y\,dx$$

The moment of inertia about the centroidal $x-axis$

$$dI_x = \frac{1}{12}(2y)^3 \cdot dx$$

$$= \frac{1}{12} \times 8y^3 \cdot dx$$

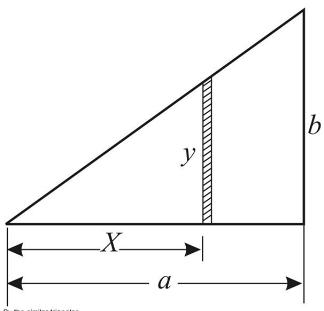
$$= \frac{2}{3}y^3 \cdot dx$$

Comments (1)

Anonymous

where does the 1/12 come from?

Step 3 of 6



By the similar triangles

$$\frac{b}{a} = \frac{y}{x}$$

$$y = \frac{b}{a} \cdot x$$

Step 4 of 6

Substituting the expression in the above equation and integrating we get

$$dI_x = \frac{2}{3}y^2 \cdot dx$$

$$dI_x = \frac{2}{3}\left(\frac{b}{a}x\right)^3 \cdot dx$$

$$\int dI_x = \frac{2b^3}{3a^3}\int_0^a x^3 \cdot dx$$

$$I_x = \frac{2}{3}\frac{b^3}{a^3}\left(\frac{x^4}{4}\right)_0^a$$

$$I_x = \frac{2}{3}\frac{b^3}{a^3} \cdot \frac{a^4}{4}$$

$\therefore$ Moment of inertia about *x*-axis $\boxed{I_x = \frac{ab^3}{6}}$

Step 5 of 6

Similarly the moment of inertia about the $y-axis$

$$dI_y = \int x^2 \cdot dA$$

$$dI_y = x^2(2ydx)$$

Substituting $y-\text{expression}$

$$dI_y = x^2\left(\frac{2b}{a}x \cdot dx\right)$$

$$= \frac{2b}{a} \cdot x^3 \cdot dx$$

By integrating

$$\int dI_y = \frac{2b}{a}\int_0^a x^3 dx$$

$$I_y = \frac{2b}{a}\left[\frac{x^4}{4}\right]_0^a$$

$$= \frac{2b}{a} \cdot \frac{a^4}{4}$$

$$= \frac{2a^3b}{4}$$

$$= \frac{1}{2}ba^3$$

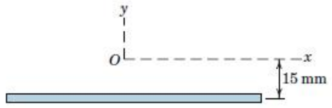
$\therefore$ Moment of inertia about *y*-axis $\boxed{I_y = \frac{1}{2}ba^3}$

Step 6 of 6

Chapter A, Problem 3P

Problem

The narrow rectangular strip has an area of 300 mm2, and its moment of inertia about the *y*-axis is 35(103) mm4. Obtain a close approximation to the polar radius of gyration about point *O*.



Step-by-step solution

Step 1 of 4

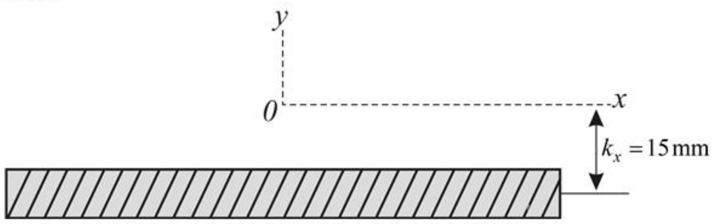
Given data:

The area of the Rectangular strip, $A = 300\text{mm}^2$

Moment of inertia about y-axis, $I_y = 35\times10^3\text{mm}^4$

Step 2 of 4

Figure:



Step 3 of 4

We know that

The moment of inertia about the $x-\text{axis}$ will be $I_x = k_x^2 A$

The distance is k_x is called the radius of gyration of the area about the $x-\text{axis}$

Therefore,

$$I_x = A k_x^2$$
$$I_x = 300\times(15)^2$$
$$I_x = 67500\,\text{mm}^4$$

Moment of inertia about *y*-axis, $I_y = 35\times10^3\,\text{mm}^4$

Now we know that

Polar moment of inertia,

$$I_0 = I_x + I_y$$
$$I_0 = 67500 + 35000$$
$$I_0 = 102500\,\text{mm}^4$$

Step 4 of 4

Polar radius of gyration about point *O*

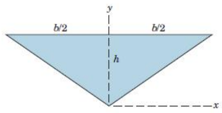
$$I_o = Ak_o^2$$
$$k_o^2 = I_o / A$$
$$k_o = \sqrt{\frac{I_o}{A}}$$
$$k_o = \sqrt{\frac{102500}{300}}$$
$$\boxed{k_o = 18.48\,\text{mm}}$$

Polar radius of gyration is $k_0 = 18.48\,\text{mm}$

Chapter A, Problem 4P

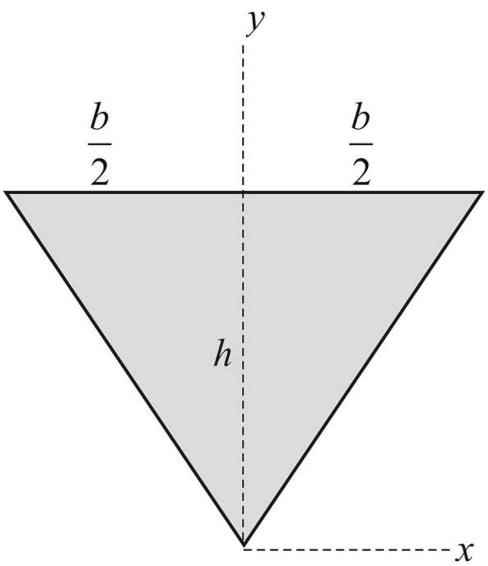
Problem

Determine the ratio *b/h* such that *Ix* = *Iy* for the area of the isosceles triangle.



Step-by-step solution

Step 1 of 4



Step 2 of 4

A transfer from the centroidal axis to the x-axis through the vertex gives

$$I_x = I + Ad^2$$
$$= \frac{bh^3}{36} + \left(\frac{bh}{2}\right)\left(\frac{2h}{3}\right)^2$$
$$= \frac{bh^3}{36} + \frac{bh}{2}\frac{4h^2}{9}$$
$$= \frac{bh^3}{36} + \frac{4bh^3}{18}$$
$$= \frac{bh^3 + 8bh^3}{36}$$
$$= \frac{9bh^3}{36}$$
$$= \frac{1}{4}bh^3$$

Step 3 of 4

Moment of inertia about *y*-axis

$$I_y = 2I_{y-y}$$
$$= 2\left[\frac{1}{12}h\left(\frac{b}{2}\right)^3\right]$$
$$= 2\left[\frac{1}{12}h\frac{b^3}{8}\right]$$
$$= 2\left[\frac{1}{12}h\frac{b^3}{8}\right]$$
$$= \frac{hb^3}{48}$$

Step 4 of 4

Therefore the given condition is $I_x = I_y$

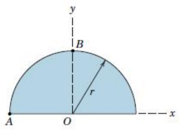
$$\frac{1}{4}bh^3 = \frac{hb^3}{48}$$
$$\frac{b}{h} = \frac{4}{48}\frac{b^3}{h^3}$$
$$\frac{b}{h} = \frac{1}{12}\frac{b^3}{h^3}$$
$$1 = \frac{1}{12}\frac{b^2}{h^2}$$
$$\frac{b^2}{h^2} = 12$$
$$\frac{b}{h} = \sqrt{12}$$
$$= \sqrt{4\times 3}$$
$$= 2\sqrt{3}$$

$\therefore$ The ratio is $\boxed{\frac{b}{h} = 2\sqrt{3}}$

Chapter A, Problem 6P

Problem

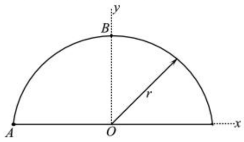
Determine the polar moments of inertia of the semicircular area about points *A* and *B*.



Step-by-step solution

Step 1 of 4

Draw the schematic diagram.



Step 2 of 4

Write the formula to calculate the polar moment of inertia about centroid.

$\bar{I}_z = \bar{I}_x + \bar{I}_y$ ...... (1)

Here, $\bar{I}_z$ is the polar moment of inertia about centroid, $\bar{I}_x$ is the inertia of the circular area about x_o- axis, and $\bar{I}_y$ is the inertia of the circular area about y_o- axis.

Obtain the relations from the table, "Properties of Plane Figures."

Write the relation for moment of inertia of the semi-circular about centroidal x_o- axis.

$$\bar{I}_x = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

Here, x_o is the centroid axis in horizontal direction, and r is the radius of the semi circle.

Write the relation for moment of inertia of the semi-circular about centroidal y_o- axis.

$$\bar{I}_y = \frac{\pi r^4}{8}$$

Substitute $\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$ for $\bar{I}_x$, and $\frac{\pi r^4}{8}$ for $\bar{I}_y$ in equation (1).

$$\begin{aligned}\bar{I}_z &= \bar{I}_x + \bar{I}_y \\ &= \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 + \frac{\pi r^4}{8}\end{aligned}$$

Step 3 of 4

Write the relation to calculate the moment of inertia about x-axis by using parallel axes theorem.

$$I_A = \bar{I}_z + A\left(r^2 + \bar{y}^2\right)$$

Here, I_A is the moment of inertia about x-axis, A is the area of the semi-circle, and $\bar{y}$ is the distance of the centroid C.

Obtain the relation from the table, "Properties of Plane Figures."

$$\bar{y} = \frac{4r}{3\pi}$$

Substitute $\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 + \frac{\pi r^4}{8}$ for $\bar{I}_z$, $\frac{\pi r^2}{2}$ for A, and $\frac{4r}{3\pi}$ for $\bar{y}$.

$$\begin{aligned}I_A &= \left(\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 + \frac{\pi r^4}{8}\right) + \frac{\pi r^2}{2}\left(r^2 + \left(\frac{4r}{3\pi}\right)^2\right) \\ &= \left(\frac{\pi}{4} - \frac{8}{9\pi} + \frac{8}{9\pi} + \frac{\pi}{2}\right) r^4 \\ &= \frac{3}{4}\pi r^4\end{aligned}$$

Hence, the polar moment of inertia I_A about point A is $\boxed{\frac{3}{4}\pi r^4}$.

Step 4 of 4

Write the relation to calculate the moment of inertia about y-axis by using parallel axes theorem.

$$I_B = \bar{I}_z + A\left(r - \bar{y}\right)^2$$

Here, I_B is the moment of inertia about y-axis, A is the area of the semi-circle, and $\bar{y}$ is the distance of the centroid C.

Obtain the relation from the table, "Properties of Plane Figures."

$$\bar{y} = \frac{4r}{3\pi}$$

Substitute $\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 + \frac{\pi r^4}{8}$ for $\bar{I}_z$, $\frac{\pi r^2}{2}$ for A, and $\frac{4r}{3\pi}$ for $\bar{y}$.

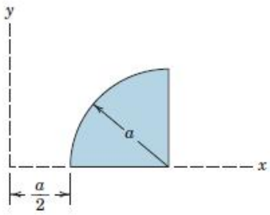
$$\begin{aligned}I_B &= \left(\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4 + \frac{\pi r^4}{8}\right) + \frac{\pi r^2}{2}\left(r - \left(\frac{4r}{3\pi}\right)\right)^2 \\ &= \left(\frac{\pi}{4} - \frac{8}{9\pi} + \frac{8}{9\pi} + \frac{\pi}{2} - \frac{4}{3}\right) r^4 \\ &= \left(\frac{3\pi}{4} - \frac{4}{3}\right) r^4\end{aligned}$$

Hence, the polar moment of inertia I_B about point B is $\boxed{\left(\frac{3\pi}{4} - \frac{4}{3}\right) r^4}$.

Chapter A, Problem 7P

Problem

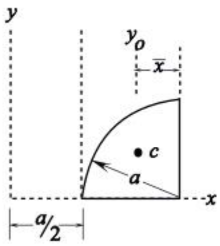
Determine the moment of inertia of the quarter-circular area about the *y*-axis.



Step-by-step solution

Step 1 of 3

Draw the schematic diagram.



Step 2 of 3

From the table $D/3$

$\bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4$

$\bar{x} = \frac{4a}{3\pi}$

Calculate the moment of inertia of the quarter circular area about the *y*-axis.

$I_y = \bar{I}_y + Ad_y^2$ ...... (1)

Here, *A* is the area, is $\frac{\pi a^2}{4}$

And,

$d_y = \frac{a}{2} + \left(a - \bar{x}\right)$

Substitute $\frac{4a}{3\pi}$ for $\bar{x}$.

$d_y = \frac{a}{2} + \left(a - \frac{4a}{3\pi}\right)$

Step 3 of 3

Substitute $\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4$ for $\bar{I}_y$, $\frac{\pi a^2}{4}$ for *A*, and $\frac{a}{2} + \left(a - \frac{4a}{3\pi}\right)$ for d_y in equation (1).

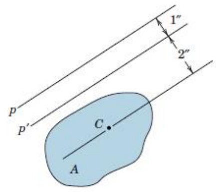
$$I_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right)a^4 + \frac{\pi a^2}{4}\left(\frac{a}{2} + \left(a - \frac{4a}{3\pi}\right)\right)^2$$
$$= \left(\frac{5\pi}{8} - 1\right)a^4$$

Hence, the moment of inertia of the quarter circular area about the *y*-axis is $\boxed{\left(\frac{5\pi}{8} - 1\right)a^4}$.

Chapter A, Problem 9P

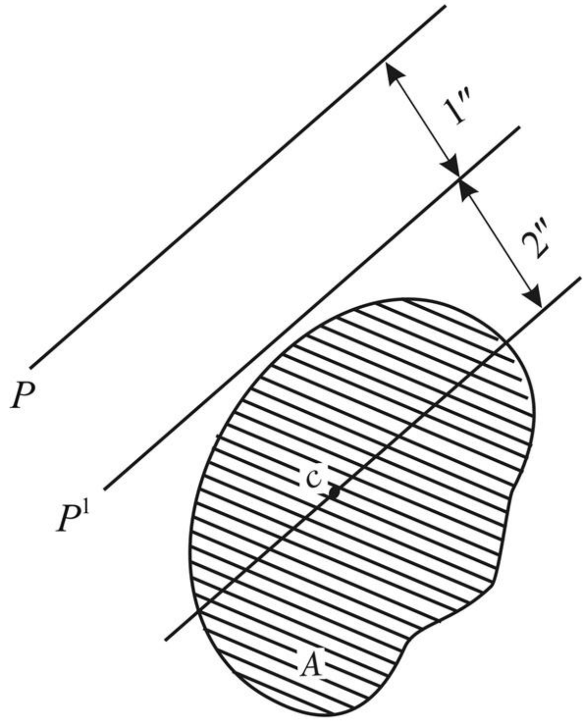
Problem

The moments of inertia of the area A about the parallel p- and p'-axes differ by 50 in.4
Compute the area A, which has its centroid at C.



Step-by-step solution

Step 1 of 4



$$I_P - I_{P^1} = 50\,\text{in}^4$$

Step 2 of 4

By the parallel axis- theorem the moment of inertia about the P-axis is

$$I_P = I_c + Ad^2$$
$$= I_c + A(3)^2$$
$$= I_c + 9A$$

Step 3 of 4

About $P^1-\text{axis is}$

$$I_{P'} = I_c + A(2)^2$$
$$= I_c + 4A$$

Step 4 of 4

By subtracting

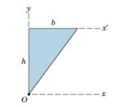
$$I_P - I_{P'} = 9A - 4A$$
$$I_P - I_{P'} = 5A$$
$$50 = 5\text{A}$$
$$A = \frac{50}{5}$$
$$= 10\,\text{in}^2$$

$\therefore$ The area A which has its centroid at C is $\boxed{A = 10\text{in}^2}$

Chapter A, Problem 14P

Problem

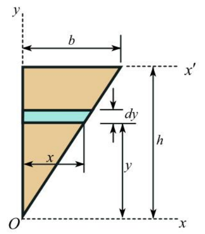
By direct integration, determine the moments of inertia of the triangular area about the x- and x'-axes.



Step-by-step solution

Step 1 of 6

Consider the elemental strip of width x and height dy at a distance of y from the x-axis.



Step 2 of 6

Calculate the length of elemental strip by using the geometry of the figure.

$$\frac{b}{h}=\frac{x}{y}$$
$$x=\frac{b}{h}y$$

Here, b is the width of the triangle, h is the height of the triangle, x is the width of the elemental strip, and y is the distance of elemental strip form x-axis.

Calculate the area of the elemental strip.

$$dA=xdy$$

Here, dA is the area of the elemental strip.

Substitute $\frac{b}{h}y$ for x.

$$dA=\frac{b}{h}ydy$$

Step 3 of 6

Calculate the moment of inertia of the triangle about x-axis by using the direct integration.

$$I_x=\int y^2dA$$

Here, I_x is the moment of inertia of the triangle about x-axis.

Substitute $\frac{b}{h}ydy$ for dA.

$$\begin{aligned}I_x&=\int_0^h y^2\left(\frac{b}{h}ydy\right)\\&=\frac{b}{h}\int_0^h y^3dy\\&=\frac{b}{h}\left[\frac{y^4}{4}\right]_0^h\\&=\frac{b}{h}\left[\frac{h^4-0^4}{4}\right]\\&=\frac{bh^3}{4}\end{aligned}$$

Therefore, the moment of inertia of the triangular area about the x-axis is, $\boxed{\frac{bh^3}{4}}$.

Step 4 of 6

The centroid of the triangle from the x-axis is, $d=\frac{2h}{3}$

Calculate the area of the triangle.

$$A=\frac{1}{2}bh$$

Here, A is the area of the triangle.

Calculate the moment of inertia of the triangle about its centroid by using the parallel axis theorem.

$$\bar{I}=I_x-Ad^2$$

Here, $\bar{I}$ is the moment of inertia of the triangle about its centroid.

Substitute $\frac{bh^3}{4}$ for I_x, $\frac{1}{2}bh$ for A, and $\frac{2h}{3}$ for d.

$$\begin{aligned}\bar{I}&=\frac{bh^3}{4}-\left(\frac{1}{2}bh\right)\left(\frac{2h}{3}\right)^2\\&=\frac{bh^3}{4}-\frac{2bh^3}{9}\\&=\frac{bh^3}{36}\end{aligned}$$

Step 5 of 6

Obtain the distance of centroid from the x'-axis.

$$d'=h-d$$

Here, d' is the distance of centroid from the x'-axis.

Substitute $\frac{2h}{3}$ for d.

$$\begin{aligned}d'&=h-\frac{2h}{3}\\&=\frac{h}{3}\end{aligned}$$

Step 6 of 6

Calculate the moment of inertia of the triangular area about the x'-axis by using the parallel axis theorem.

$$I_{x'}=\bar{I}+Ad'^2$$

Here, $I_{x'}$ is the moment of inertia of the triangular area about the x'-axis.

Substitute $\frac{bh^3}{36}$ for $\bar{I}$, $\frac{1}{2}bh$ for A, and $\frac{h}{3}$ for d'.

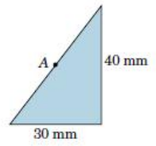
$$\begin{aligned}I_{x'}&=\frac{bh^3}{36}+\left(\frac{1}{2}bh\right)\left(\frac{h}{3}\right)^2\\&=\frac{bh^3}{36}+\frac{bh^3}{18}\\&=\frac{3bh^3}{36}\\&=\frac{bh^3}{12}\end{aligned}$$

Hence, the moment of inertia of the triangular area about the x'-axis is, $\boxed{\frac{bh^3}{12}}$.

Chapter A, Problem 16P

Problem

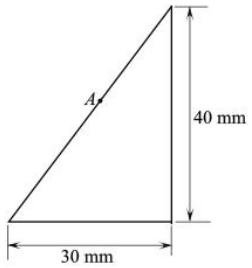
Determine the radius of gyration about a polar axis through the midpoint *A* of the hypotenuse of the right-triangular area. (*Hint:* Simplify your calculation by observing the results for a 30 × 40-mm rectangular area.)



Step-by-step solution

Step 1 of 3

Draw the schematic diagram.



Step 2 of 3

Obtain the relations from the, "Properties of Plane figures."

Write the relation to calculate the polar moment of inertia of the triangle.

$$I = \frac{1}{12}A\left(b^2 + h^2\right) \quad \ldots\ldots (1)$$

Here, I is the polar moment of inertia, b is the base of the triangle, b is the height of the triangle, and A is the area of the triangle.

Obtain the relations from the, "Properties of Plane figures."

Write the relation to calculate the area of the triangle.

$$A = \frac{bh}{2}$$

Substitute $30\,\text{mm}$ for b, and $40\,\text{mm}$ for h.

$$\begin{aligned} A &= \frac{30 \times 40}{2} \\ &= 600\,\text{mm}^2 \end{aligned}$$

Step 3 of 3

Substitute $600\,\text{mm}^2$ for A, $30\,\text{mm}$ for b, and $40\,\text{mm}$ for h in equation (1).

$$\begin{aligned} I &= \frac{1}{12}A\left(b^2 + h^2\right) \\ &= \frac{1}{12}\left(600\,\text{mm}^2 \times \left((30\,\text{mm})^2 + (40\,\text{mm})^2\right)\right) \\ &= 12.5\left(10^4\right)\text{mm}^4 \end{aligned}$$

Write the relation to calculate the polar radius of the about point A.

$$I = k_A{}^2 A$$

$$k_A = \sqrt{\frac{I}{A}} \quad \ldots\ldots (2)$$

Here, k_A is the polar radius of gyration about A.

Substitute $12.5\left(10^4\right)\text{mm}^4$ for I, and $600\,\text{mm}^2$ for A.

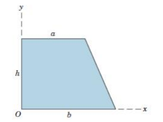
$$\begin{aligned} k_A &= \sqrt{\frac{12.5\left(10^4\right)\text{mm}^4}{600\,\text{mm}^2}} \\ &= 14.43\,\text{mm} \end{aligned}$$

Hence, the polar radius of gyration k_A about point A is $\boxed{14.43\,\text{mm}}$.

Chapter A, Problem 17P

Problem

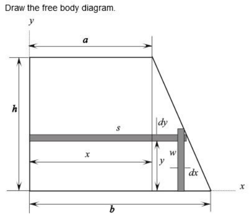
Determine by direct integration the moments of inertia of the trapezoidal area about the *x*- and *y*-axes. Find the polar moment of inertia about point *O*.



Step-by-step solution

Step 1 of 6

Draw the free body diagram.



Step 2 of 6

Write the formula to calculate moment of inertia of the rectangular area about x- axis.

$$I_x = \int_A y^2 dA \quad \text{...... (1)}$$

Here, dA is the elemental area, x is the centroid axis in horizontal direction, I_x is the inertia of the rectangular area about x- axis, and y is the distance from axis.

Consider area of horizontal strip.

$$dA = s(dy) \quad \text{...... (2)}$$

Here, s is the length of the elemental strip.

Step 3 of 6

Consider similar triangles in schematic diagram.

$$\frac{s-a}{b-a} = \frac{h-y}{h}$$
$$s = \frac{(b-a)(h-y)}{h} + a$$

Here, h is the height of the trapezoid, b is the one side base, and a is the other side base.

Substitute $\frac{(b-a)(h-y)}{h} + a$ for s in equation (2).

$$dA = \left(\frac{(b-a)(h-y)}{h} + a\right) dy$$

Substitute $\left(\frac{(b-a)(h-y)}{h} + a\right) dy$ for dA in equation (1).

$$\begin{aligned} I_x &= \int_A y^2 dA \\ &= \int_0^h y^2 \left(\frac{(b-a)(h-y)}{h} + a\right) dy \\ &= \frac{(b-a)}{h} \int_0^h y^2 (h-y) dy + a \int_0^h y^2 dy \\ &= \frac{(b-a)}{h} \left(\frac{hy^3}{3} - \frac{y^4}{4}\right)_0^h + a\left(\frac{y^3}{3}\right)_0^h \\ &= h^3\left(\frac{a}{4} + \frac{b}{12}\right) \end{aligned}$$

Hence, the moment of inertia of the trapezoid about x- axis is $\boxed{h^3\left(\frac{a}{4} + \frac{b}{12}\right)}$.

Step 4 of 6

Write the relation to calculate the moment of inertia of the trapezoidal area about y- axis.

$$I_y = \int_A x^2 dA \quad \text{...... (3)}$$

Here, I_y is the inertia of the rectangular area about y- axis, and x is the distance from axis.

Consider area of vertical strip.

$$dA = h(dx') + w dx \quad \text{...... (4)}$$

Here, h is the height of the trapezoidal section, and w is the width of the elemental strip.

Consider similar triangles in schematic diagram.

$$\frac{w}{b-x} = \frac{h}{b-a}$$
$$w = \frac{h}{b-a}(b-x)$$

Substitute $\frac{h}{b-a}(b-x)$ for w in equation (4).

$$dA = h(dx') + \frac{h}{b-a}(b-x)dx$$

Step 5 of 6

Substitute $h(dx') + \frac{h}{b-a}(b-x)dx$ for dA in equation (3).

$$\begin{aligned} I_y &= \int_A x^2 dA \\ &= h\int_a^b x^2\left(\frac{b-x}{b-a}\right)dx + h\int_0^a x'^2 dx' \\ &= \frac{h}{b-a}\left(\frac{bx^3}{3} - \frac{x^4}{4}\right)_a^b + h\left(\frac{x'^3}{3}\right)_0^a \\ &= \frac{h}{b-a}\left(\frac{b^4}{3} - \frac{b^3}{3} - \frac{ba^3}{3} + \frac{a^3}{3}\right) + \frac{ha^3}{3} \\ &= \frac{h}{12}\left(a^3 + a^2 b + ab^2 + b^3\right) \end{aligned}$$

Hence, the moment of inertia I_y about the y- axis is $\boxed{\frac{h}{12}\left(a^3 + a^2 b + ab^2 + b^3\right)}$.

Step 6 of 6

Calculate the polar moment of inertia of the trapezoidal area about point O.

Write the relation to calculate the polar moment of inertia of the rectangular area about point O.

$$I_z = I_x + I_y$$

Here, I_z is the polar moment of inertia.

Substitute $h^3\left(\frac{a}{4} + \frac{b}{12}\right)$ for I_x, and $\frac{h}{12}\left(a^3 + a^2 b + ab^2 + b^3\right)$ for I_y.

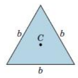
$$\begin{aligned} I_z &= h^3\left(\frac{a}{4} + \frac{b}{12}\right) + \frac{h}{12}\left(a^3 + a^2 b + ab^2 + b^3\right) \\ &= \frac{h}{12}\left(h^2(3a+b) + a^3 + a^2 b + ab^2 + b^3\right) \end{aligned}$$

Hence, the polar moment of inertia I_z about O is $\boxed{\frac{h}{12}\left(h^2(3a+b) + a^3 + a^2 b + ab^2 + b^3\right)}$.

Chapter A, Problem 18P

Problem

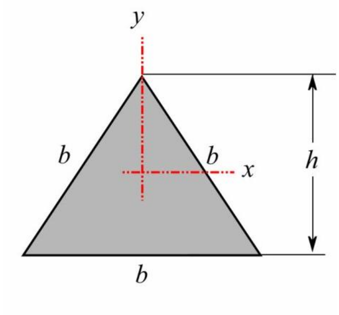
Determine the polar radius of gyration of the area of the equilateral triangle of side *b* about its centroid *C*.



Step-by-step solution

Step 1 of 5

Draw the schematic diagram of the triangle.



Step 2 of 5

Calculate the height h by using the Pythagoras relation:

$$h = \sqrt{\left(\frac{b}{2}\right)^2 + (b)^2}$$
$$= \frac{\sqrt{b^2 + 2b^2}}{2}$$
$$= \frac{\sqrt{3}b}{2}$$

Calculate the moment of inertia of the triangle about the x axis.

$$I_x = \frac{bh^3}{36}$$

Substitute $\frac{\sqrt{3}}{2}b$ for h.

$$I_x = \frac{b}{36}\left(\frac{\sqrt{3}}{2}b\right)^3$$
$$= \frac{3\sqrt{3}b^4}{36 \times 8}$$
$$= \frac{\sqrt{3}b^4}{96}$$

Step 3 of 5

Calculate the moment of inertia of the triangle about the y axis.

$$I_y = 2\left(\frac{b^3h}{12}\right)$$

Substitute $\frac{\sqrt{3}}{2}b$ for h and $\frac{b}{2}$ for b.

$$I_y = 2\left(\frac{\left(\frac{b}{2}\right)^3}{12}\right) \times \frac{\sqrt{3}}{2}b$$
$$= \frac{\sqrt{3}}{96}b^4$$

Step 4 of 5

Calculate the polar moment of inertia by using the following relation:

$$I = I_x + I_y$$

Substitute $\frac{\sqrt{3}b^4}{96}$ for I_x and $\frac{\sqrt{3}}{96}b^4$ for I_y.

$$I = \frac{\sqrt{3}b^4}{96} + \frac{\sqrt{3}b^4}{96}$$
$$= \frac{\sqrt{3}b^4}{48}$$

Calculate the area of cross section of the triangular plate.

$$A = \frac{1}{2}bh$$

Substitute $\frac{\sqrt{3}}{2}b$ for h.

$$A = \frac{1}{2}b \times \frac{\sqrt{3}}{2}b$$
$$= \frac{\sqrt{3}}{4}b^2$$

Step 5 of 5

Calculate the polar radius of gyration by using the following relation:

$$k = \sqrt{\frac{I}{A}}$$

Substitute $\frac{\sqrt{3}b^4}{48}$ for I and $\frac{\sqrt{3}}{4}b^2$ for A.

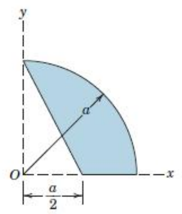
$$k = \sqrt{\frac{\left(\frac{\sqrt{3}b^4}{48}\right)}{\left(\frac{\sqrt{3}}{4}b^2\right)}}$$
$$= \sqrt{\frac{b^2}{12}}$$
$$= \frac{b}{2\sqrt{3}}$$

Therefore, the polar radius of gyration of the triangular plate is $\boxed{\frac{b}{2\sqrt{3}}}$.

Chapter A, Problem 19P

Problem

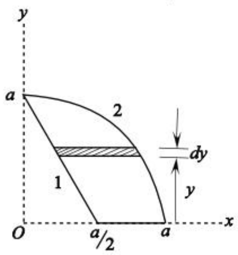
Determine the moment of inertia of the shaded area about the *x*-axis.



Step-by-step solution

Step 1 of 3

Draw the schematic diagram.



Step 2 of 3

Consider the diagram.

Part 1

$$y = -2x + a$$

$$x = \frac{a-y}{2}$$

Part 2

$$x^2 + y^2 = a^2$$

$$x = \sqrt{a^2 - y^2}$$

$$\begin{aligned} dA &= (x_2 - x_1)\,dy \\ &= \left[\sqrt{a^2 - y^2} - \frac{a-y}{2}\right]dy \end{aligned}$$

Here, *a* is the diameter, and *y* is the neutral axis about *y*-axis.

Calculate the moment of inertia of the shaded area about the *x*-axis.

$$I_x = \int y^2 dA$$

Step 3 of 3

Substitute $\left[\sqrt{a^2 - y^2} - \frac{a-y}{2}\right]dy$ for dA.

$$\begin{aligned} I_x &= \int_0^a y^2\left[\sqrt{a^2 - y^2} - \frac{a-y}{2}\right]dy \\ &= \int_0^a \left[y^2\sqrt{a^2 - y^2} - \frac{ay^2 - y^3}{2}\right]dy \\ &= \left[-\frac{y}{4}\sqrt{\left(a^2 - y^2\right)^3} + \frac{a^2}{8}\left(y\sqrt{a^2 - y^2} + a^2\sin^{-1}\frac{y}{a}\right) - \frac{ay^3}{6} + \frac{y^4}{8}\right]_0^a \end{aligned}$$

$$= \left[-\frac{a}{4}\sqrt{\left(a^2 - a^2\right)^3} + \frac{a^2}{8}\left(a\sqrt{a^2 - a^2} + a^2\sin^{-1}\frac{a}{a}\right) - \frac{a(a)^3}{6} + \frac{a^4}{8}\right]$$

$$= \left[\frac{a^4\pi}{16} - \frac{a^4}{24}\right]$$

$$= a^4\left[\frac{\pi}{16} - \frac{1}{24}\right]$$

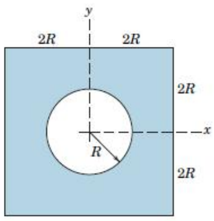
$$= \frac{a^4}{8}\left[\frac{\pi}{2} - \frac{1}{3}\right]$$

Hence, the moment of inertia of the shaded area about the *x*-axis is $\boxed{\frac{a^4}{8}\left[\frac{\pi}{2} - \frac{1}{3}\right]}$.

Chapter A, Problem 35P

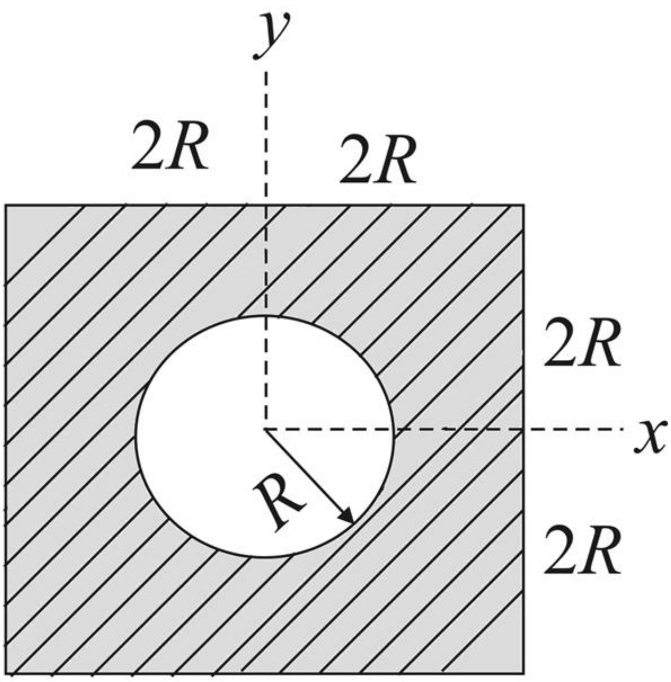
Problem

Determine the moment of inertia about the *x*-axis of the square area without and with the central circular hole.



Step-by-step solution

Step 1 of 3



Step 2 of 3

Considering whole square area without hole

The moment of inertia about x-axis

$I_x = \frac{1}{12}bh^3$

$= \frac{1}{12} \times 4R \times (4R)^3$

$= \frac{256}{12}R^4$

$= \frac{64R^4}{3}$

$= 21.3\text{R}^4$

$\therefore \boxed{I_x = 21.3\text{R}^4}$

Step 3 of 3

Considering whole p square area with hole

$I_x = \frac{64R^4}{3} - \frac{1}{4}(\pi R^2) \cdot R^2$

$= 21.3\text{R}^4 - 0.785\text{R}^4$

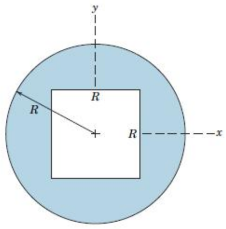
$= 20.52\ \text{R}^4$

$\therefore$ Moment of inertia of square area with hole $\boxed{I_x = 20.52\ \text{R}^4}$

Chapter A, Problem 36P

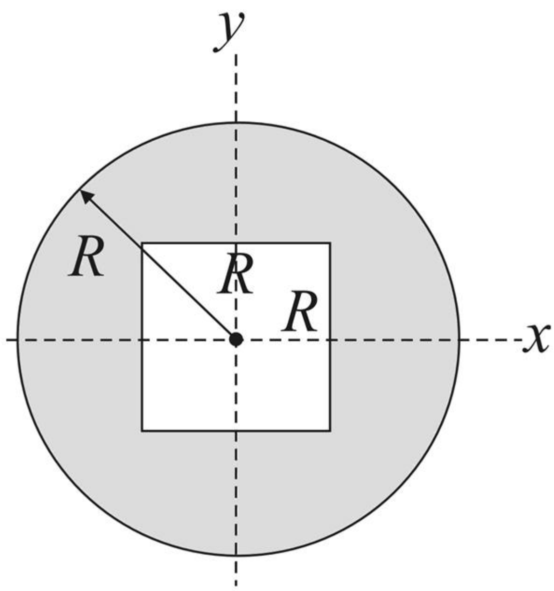
Problem

Determine the polar moment of inertia of the circular area without and with the central square hole.



Step-by-step solution

Step 1 of 4



Step 2 of 4

Considering whole circular area

By symmetry $I_x = I_y$

$\therefore$ The polar moment of inertia is given by I_z

$I_z = I_x + I_y$

$I_z = 2I_x$

$I_x = \frac{1}{4} Ar^2$

$= \frac{1}{4} AR^2$

$\therefore I_z = 2\left(\frac{1}{4} AR^2\right)$

$= \frac{1}{2} \pi R^2 \cdot R^2$

$= \frac{1}{2} \pi R^4$

$= 1.57079 \mathrm{R}^4$

Step 3 of 4

Considering circular area with the central square hole

$I_z = I_z$ Of circular area - I_z square are

$= 1.57079 R^4 - 2\left(\frac{1}{12} \cdot R \cdot R^3\right)$

$= 1.57079 R^4 - \frac{1}{6} R^4$

$= 1.57079 R^4 - 0.167 R^4$

$= 1.404 R^4$

Step 4 of 4

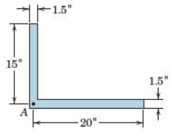
$\therefore$ The polar moment of inertia without cultural square hole is $\boxed{I_z = 1.57079 R^4}$

$\therefore$ The polar moment of inertia with central circular square hole is $\boxed{I_z = 1.404 R^4}$

Chapter A, Problem 37P

Problem

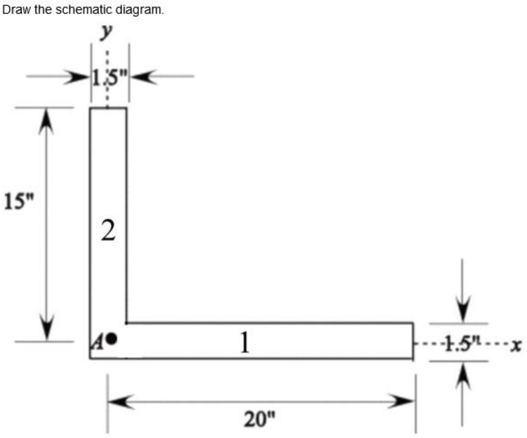
Calculate the polar radius of gyration of the area of the angle section about point *A*. Note that the width of the legs is small compared with the length of each leg.



Step-by-step solution

Step 1 of 5

Draw the schematic diagram.



Step 2 of 5

Consider b_1, h_1 to be the breadth and height of the rectangle 1 and b_2, h_2 to be the breadth and height of the rectangle 2 respectively.

Calculate the net moment of inertia of due to the rectangles 1 and 2 about *x*-axis.

$$I_x = \frac{b_1h_1^3}{12} + \left(\frac{b_2h_2^3}{12} + \left(b_2h_2 \times \frac{h_2}{2}\right)\right)$$

Substitute $20''$ for b_1, $1.5''$ for h_1, $1.5''$ for b_2, and $15''$ for h_2.

$$I_x = \frac{20\times1.5^3}{12} + \left(\frac{1.5\times15^3}{12}\right) + \left(1.5\times15\times\left(\frac{15}{2}\right)^2\right)$$
$$= 1693.125 \text{ in.}^4$$

Step 3 of 5

Calculate the net moment of inertia of due to the rectangles 1 and 2 about *x*-axis.

$$I_y = \frac{h_2b_2^3}{12} + \left(\frac{h_1b_1^3}{12} + \left(b_1h_1 \times \frac{b_1}{2}\right)\right)$$

Substitute $20''$ for b_1, $1.5''$ for h_1, $1.5''$ for b_2, and $15''$ for h_2.

$$I_y = \frac{15\times1.5^3}{12} + \left(\frac{1.5\times20^3}{12}\right) + \left(20\times1.5\times\left(\frac{20}{2}\right)^2\right)$$
$$= 4004.21875 \text{ in.}^4$$

Step 4 of 5

Calculate the net moment of inertia about the z axis, (point *A*) by using the following relation:

$$I_A = I_x + I_y$$

Substitute 1693.125 in.^4 for I_x and 4004.21875 in.^4 for I_y.

$$I_A = 1693.125 + 4004.21875$$
$$= 5697.34375 \text{ in.}^4$$

Step 5 of 5

Calculate the polar radius of gyration of the area of the angle section about point *A* by using the following relation:

$$k_A = \sqrt{\frac{I_A}{A_1 + A_2}}$$

Here, A_1 and A_2 are the areas of the rectangles 1 and 2 respectively.

Substitute 5697.34375 in.^4 for I_A, $(15''\times1.5'')$ for A_1, and $(20''\times1.5'')$ for A_2.

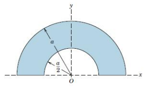
$$k_A = \sqrt{\frac{5697.34375}{(1.5\times15)+(20\times1.5)}}$$
$$= 10.417 \text{ in.}$$

Therefore, the polar radius of gyration of the area of the angle section about point *A* is $\boxed{10.417 \text{ in.}}$.

Chapter A, Problem 38P

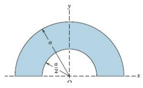
Problem

By the method of this article, determine the rectangular and polar radii of gyration of the shaded area, repeated here from Prob. A/33, about the axes shown.



Problem. A/33

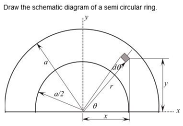
By the methods of this article, determine the rectangular and polar radii of gyration of the shaded area about the axes shown.



Step-by-step solution

Step 1 of 8

Draw the schematic diagram of a semi circular ring.



Step 2 of 8

Write the formula to calculate the radius of gyration about the x - axis.

$k_x = \sqrt{\frac{I_x}{A}}$ ...... (1)

Here, I_x is the moment of inertia about the x - axis, A is the area of the shaded section, and k_x is the radius of gyration about the x - axis.

Write the formula to calculate the moment of inertia of the shaded area about the x - axis.

$I_x = I_{xf} - I_{xr}$ ...... (2)

Here, I_{xf} is the moment of inertia of the semi circle before removal of the small semi circle, and I_{xr} is the moment of inertia of the small semi circle.

Step 3 of 8

Write the relation to calculate the moment of inertia of the semi circle before removal of the small semi circle.

Obtain the relations from the table D/3, "Properties of Plane figures" in the textbook.

$$I_{xf} = \frac{\pi r_1^4}{8}$$

Here, r_1 is the radius of the bigger semi circle.

Substitute a for r_1 from the diagram.

$$I_{xf} = \frac{\pi a^4}{8}$$

Step 4 of 8

Write the relation to calculate the moment of inertia of the small semi circle.

Obtain the relations from, "Properties of Plane Figures."

$$I_{xr} = \frac{\pi r_2^4}{8}$$

Here, r_2 is the radius of the smaller semi circle.

Substitute $\frac{a}{2}$ for r_2 from the diagram.

$$\begin{aligned} I_{xr} &= \frac{\pi\left(\frac{a}{2}\right)^4}{8} \\ &= \frac{\pi a^4}{128} \end{aligned}$$

Step 5 of 8

Substitute $\frac{\pi a^4}{128}$ for I_{xr}, and $\frac{\pi a^4}{8}$ for I_{xf} in equation (2).

$$\begin{aligned} I_x &= I_{xf} - I_{xr} \\ &= \frac{\pi a^4}{8} - \frac{\pi a^4}{128} \end{aligned}$$ Write the relation to calculate the total area of the shaded section.

$$= \frac{15\pi a^4}{128}$$

$$A = \frac{1}{2}\left(\pi r_1^2 - \pi r_2^2\right)$$

Here, A is the area of the shaded section, r_1 is the inner radius of the shaded section, and r_2 is the outer radius of the shaded section.

Write the radii in terms of a from the diagram.

$r_1 = a$

$r_2 = \frac{a}{2}$

Substitute a for r_1, and $\frac{a}{2}$ for r_2.

$$\begin{aligned} A &= \frac{1}{2}\left(\pi r_1^2 - \pi r_2^2\right) \\ &= \frac{\pi}{2}\left(a^2 - \left(\frac{a}{2}\right)^2\right) \\ &= \frac{\pi a^2}{2}\left(\frac{3}{4}\right) \\ &= \frac{3\pi a^2}{8} \end{aligned}$$

Step 6 of 8

Substitute $\frac{3\pi a^2}{8}$ for A, and $\frac{15\pi a^4}{128}$ for I_x in equation (1).

$$\begin{aligned} k_x &= \sqrt{\frac{I_x}{A}} \\ &= \sqrt{\frac{\frac{15\pi a^4}{128}}{\frac{3\pi a^2}{8}}} \\ &= \sqrt{\frac{5a^2}{16}} \\ &= \frac{\sqrt{5}a}{4} \end{aligned}$$

Hence, the radius of gyration of the shaded area about x - axis is $\boxed{\frac{\sqrt{5}a}{4}}$.

Step 7 of 8

Here, the moment of inertias about both axes are same due to similarity for the total shaded area.

Write the formula to calculate the moment of inertia of the shaded area about the y - axis.

$I_y = I_x$

Here, I_y is the moment of inertia about the y - axis.

Substitute $\frac{15\pi a^4}{128}$ for I_x.

$$I_y = \frac{15\pi a^4}{128}$$

Write the relation to calculate the polar moment of inertia about point O.

$I_z = I_x + I_y$

Here, I_z is the polar moment of inertia about point O.

Substitute $\frac{15\pi a^4}{128}$ for both I_x, and I_y.

$$\begin{aligned} I_z &= \frac{15\pi a^4}{128} + \frac{15\pi a^4}{128} \\ &= \frac{30\pi a^4}{128} \end{aligned}$$

Step 8 of 8

Write the relation to calculate the polar radius of the gyration about point O.

$$k_z = \sqrt{\frac{I_z}{A}}$$

Here, k_z is the polar radius of gyration about point O.

Substitute $\frac{30\pi a^4}{128}$ for I_z, and $\frac{3\pi a^2}{8}$ for A.

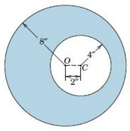
$$\begin{aligned} k_z &= \sqrt{\frac{\frac{30\pi a^4}{128}}{\frac{3\pi a^2}{8}}} \\ &= \sqrt{\frac{10a^4}{16}} \\ &= \frac{\sqrt{10}a}{4} \end{aligned}$$

Hence, the polar radius of gyration of the shaded area is $\boxed{\frac{\sqrt{10}a}{4}}$.

Chapter A, Problem 39P

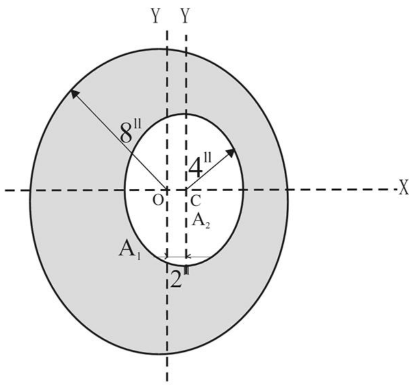
Problem

Calculate the polar radius of gyration of the shaded area about the center *O* of the larger circle.



Step-by-step solution

Step 1 of 7



Step 2 of 7

Area $= A_1 - A_2$

$= \pi R_1^2 - \pi R_2^2$

$= \pi\left(R_1^2 - R_2^2\right)$

$= \pi\left(8^2 - 4^2\right)$

$= 48\pi \text{ in}^2$

$= 150.796 \text{ in}^2$

Step 3 of 7

Polar moment of inertia about point '0' of area A_1

$I_{0_1} = I_x + I_y$

$= 2I_x$ (Symmetry)

$= 2.\frac{1}{4}AR^2 \left(\because I_x = \frac{1}{4}AR^2\right)$

$= \frac{1}{2}\pi R^2 \cdot R^2$

$= \frac{\pi R^4}{2}$

$= \frac{\pi \times 8^4}{2}$

$= 2048\pi \text{ in}^4$

$= 6433.98 \text{ in}^4$

Step 4 of 7

Moment of inertia about point 'c' of area A_2 is

As a center $I_{c_2} = 2I_x$ $(\because \text{ symmentry})$

$= 2\frac{1}{4}\pi R^2 \cdot R^2$

$= \frac{1}{2}\pi R^4$

$= \frac{1}{2}\pi (4)^4$

$= 128\pi \text{ in}^4$

$= 402.123 \text{ in}^4$

Step 5 of 7

$\therefore$ Polar moment of inertia of area A_2 is

$I_{0_2} = I_{c_2} + oc^2$

$= 402.123 + \pi R^2 \cdot oc^2$

$= 402.123 + \pi (4)^2 \times 2^2$

$= 603.186 \text{ in}^4$

Step 6 of 7

$\therefore$ The polar moment of inertia of whole area is

$I_0 = I_{01} - I_{02}$

$= 6433.98 - 603.186$

$= 5830.79 \text{ in}^4$

Step 7 of 7

Polar radius of gyration $k_0 = \sqrt{\frac{I_0}{A}}$

$k_0 = \sqrt{\frac{5830.79}{150.796}}$

$k_0 = 6.218 \text{ in}$

$\therefore$ Polar radius of gyration $\boxed{k_0 = 6.22 \text{ in}}$

Comments (1)

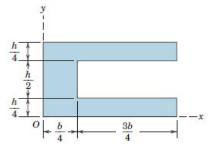
Anonymous

Hi team, in your correction if you can add theory would be perfect! ;)

Chapter A, Problem 40P

Problem

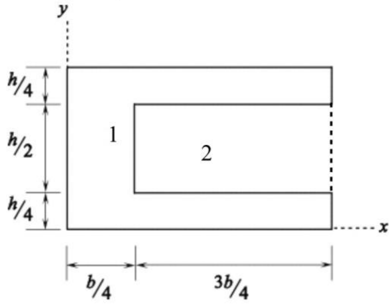
Determine the percent reductions in both area and area moment of inertia about the *y*-axis caused by removal of the rectangular cutout from the rectangular plate of base *b* and height *h*.



Step-by-step solution

Step 1 of 5

Draw the schematic diagram.



Step 2 of 5

Write the relation for area of the full rectangle 1.

$$A_1 = bh$$

Here, *b* is the breadth, and *h* is the height.

Calculate the area of the rectangle 2 after cut out.

$$\begin{aligned} A_2 &= bh - \frac{3b}{4}\left(\frac{h}{2}\right) \\ &= bh - \frac{3bh}{8} \\ &= \frac{5}{8}bh \end{aligned}$$

Step 3 of 5

Calculate the percent reduction in area.

$$n_A = \frac{A_1 - A_2}{A_1} \times 100$$

Substitute bh for A_1 and $\frac{5}{8}bh$ for A_2.

$$\begin{aligned} n_A &= \frac{bh - \frac{5}{8}bh}{bh} \times 100 \\ &= \frac{3}{8} \times 100 \\ &= 37.5\% \end{aligned}$$

Therefore, the percent reduction in area is $\boxed{37.5\%}$.

Step 4 of 5

Calculate the moment of inertia of the rectangle 1 about *y*-axis.

$$I_{y1} = \frac{1}{3}hb^3$$

Calculate the moment of inertia of the rectangle 2 after cut out about *y*-axis.

$$\begin{aligned} I_{y2} &= \frac{1}{3}hb^3 - \left(\frac{1}{12} \times \frac{h}{2} \times \left(\frac{3b}{4}\right)^3 + \frac{3}{4}b\frac{h}{2}\left(\frac{b}{4} + \frac{1}{2}\frac{3b}{4}\right)^2\right) \\ &= \frac{1}{3}hb^3 - \left(\frac{1}{12} \times \frac{h}{2} \times \left(\frac{3b}{4}\right)^3 + \frac{3}{8}bh\left(\frac{b}{4} + \frac{3b}{8}\right)^2\right) \\ &= \frac{1}{3}hb^3 - \left(\frac{9hb^3}{512} + \frac{75hb^3}{512}\right) \\ &= \left[\frac{1}{3} - \left(\frac{9}{512} + \frac{75}{512}\right)\right]hb^3 \end{aligned}$$

$$\begin{aligned} I_{y2} &= \left[\frac{1}{3} - \left(\frac{84}{512}\right)\right]hb^3 \\ &= \frac{260}{3 \times 512}hb^3 \\ &= \frac{65}{384}hb^3 \end{aligned}$$

Step 5 of 5

Calculate the percent reduction in area moment of inertia about the *y*-axis.

$$n_{I_y} = \frac{I_{y1} - I_{y2}}{I_{y1}} \times 100$$

Substitute $\frac{1}{3}hb^3$ for I_{y1} and $\frac{65}{384}hb^3$ for I_{y2}.

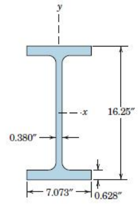
$$\begin{aligned} n_{I_y} &= \frac{\frac{1}{3}hb^3 - \frac{65}{384}hb^3}{\frac{1}{3}hb^3} \times 100 \\ &= \frac{\frac{1}{3} - \frac{65}{384}}{\frac{1}{3}} \times 100 \\ &= 49.22\% \end{aligned}$$

Therefore, the percent reduction in area moment of inertia about the *y*-axis is $\boxed{49.22\%}$.

Chapter A, Problem 41P

Problem

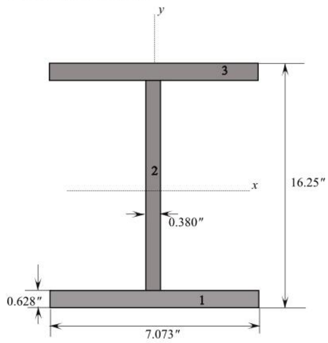
The cross-sectional area of a wide-flange I-beam has the dimensions shown. Obtain a close approximation to the handbook value of I_x = 657 in.4 by treating the section as being composed of three rectangles.



Step-by-step solution

Step 1 of 4

Draw the schematic diagram.



Step 2 of 4

Write the relation to calculate the moment of inertia of the I-beam about the x- axis.

$I_x = I_1 + I_2 + I_3$ ...... (1)

Here, I_3 is the moment of inertia of the I-beam, I_1, I_2,and I_3 are the moment of inertia of the corresponding parts.

Obtain the relations from the table D/3, "Properties of Plane figures" in the textbook.

Write the formula to calculate the moment of inertia of the I-beam section 2 about the x-axis.

$$I_2 = \frac{bh^3}{12}$$

Here, b is the width of the section 2, and h is the height of the section 2.

Substitute $0.380\,\text{in}$ for b, and $14.994\,\text{in}$ for h from the diagram.

$$\begin{aligned} I_2 &= \frac{(0.380\,\text{in})(14.994\,\text{in})^3}{12} \\ &= 106.7468\,\text{in}^4 \end{aligned}$$

Step 3 of 4

Write the formula to calculate the moment of inertia of the I-beam section 1 about the x-axis.

$$I_1 = \frac{b_1 h_1^3}{12} + A\left(\frac{h}{2} - \frac{h_1}{2}\right)^2 \text{ ...... (2)}$$

Here, b_1 is the width of the section 1, h is the height of the I-beam, h_1 is the height of the section 1, and A is the area of the section 1.

Calculate the area of the section 1.

$$A = b_1 h_1$$

Here, b_1 is the width of the section 1, and h_1 is the height of the section 1.

Substitute $7.073\,\text{in}$ for b_1, and $0.628\,\text{in}$ for h_1.

$$\begin{aligned} A &= (7.073\,\text{in})(0.628\,\text{in}) \\ &= 4.4418\,\text{in}^2 \end{aligned}$$

Substitute $7.073\,\text{in}$ for b_1, $0.628\,\text{in}$ for h_1, $16.25\,\text{in}$ for h, and $4.4418\,\text{in}^2$ for A in the equation (2).

$$\begin{aligned} I_1 &= \frac{b_1 h_1^3}{12} + A\left(\frac{h}{2} - \frac{h_1}{2}\right)^2 \\ &= \frac{(7.073\,\text{in})(0.628\,\text{in})^3}{12} + 4.4418\,\text{in}^2\left(\frac{16.25\,\text{in}}{2} - \frac{0.628}{2}\right)^2 \\ &= 0.14598\,\text{in}^4 + 271.00\,\text{in}^4 \\ &= 271.145\,\text{in}^4 \end{aligned}$$

Step 4 of 4

Write the formula to calculate the moment of inertia of the I-beam section 3 about the x-axis.

Here section 1 and section 3 are similar about x- axis.

$$\begin{aligned} I_1 &= I_3 \\ I_3 &= 271.145\,\text{in}^4 \end{aligned}$$

Substitute $106.7468\,\text{in}^4$ for I_2, $271.145\,\text{in}^4$ for I_3, and $271.145\,\text{in}^4$ for I_1 in equation (1).

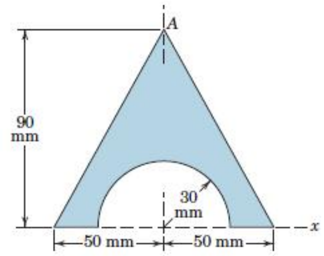
$$\begin{aligned} I_x &= I_1 + I_2 + I_3 \\ &= 271.145 + 106.7468 + 271.145 \\ &= 649.038\,\text{in}^4 \end{aligned}$$

Hence, the moment of inertia of the I-beam about the x- axis is $\boxed{649.038\,\text{in}^4}$.

Chapter A, Problem 42P

Problem

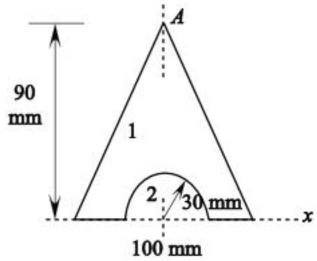
Calculate the moment of inertia of the shaded area about the *x*-axis.



Step-by-step solution

Step 1 of 3

Draw the schematic diagram.



Step 2 of 3

Calculate the moment of inertia of the triangle (Part 1) about *x*-axis.

$$I_{x_1} = \frac{1}{12}bh^3$$

Here, b is the breadth and h is the height.

Substitute 100 mm for b and 90 mm for h.

$$I_{x_1} = \frac{1}{12}(100)(90)^3$$
$$= 6.08 \times 10^6 \text{ mm}^4$$

Calculate the moment of inertia of the semi circle (Part 2) about *x*-axis.

$$I_{x_2} = -\frac{\pi}{8}r^4$$

Here, *r* is the radius.

Substitute 30 mm for *r*

$$I_{x_2} = \frac{\pi}{8}(30)^4$$
$$= 0.318 \times 10^6 \text{ mm}^4$$

Step 3 of 3

Calculate the total moment of inertia of the shaded Area about *x*-axis.

$$I_x = I_{x_1} - I_{x_2}$$

Substitute $6.08 \times 10^6 \text{ mm}^4$ for I_{x_1} and $0.318 \times 10^6 \text{ mm}^4$ for I_{x_2}.

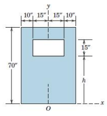
$$I_x = 6.08 \times 10^6 - 0.318 \times 10^6$$
$$= 5.76 \times 10^6 \text{ mm}^4$$

Therefore, the moment of inertia of the shaded Area about *x*-axis is $\boxed{5.76 \times 10^6 \text{ mm}^4}$.

Chapter A, Problem 43P

Problem

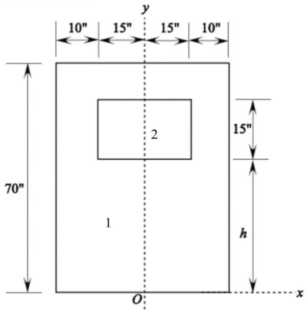
The variable *h* designates the arbitrary vertical location of the bottom of the rectangular cutout within the rectangular area. Determine the area moment of inertia about the *x*-axis for (*a*) *h* = 40 in. and (*P*) *h* = 60 in.



Step-by-step solution

Step 1 of 5

Draw the schematic diagram.



Step 2 of 5

(a)

For $h = 40\text{ in.}$:

Calculate the area moment of inertia of the rectangle 1 about the *x*-axis.

$$I_{x1} = \frac{1}{3} b_1 h_1^3$$

Here, b_1 is the breadth of rectangle 1 and h_1 is the height of rectangle 1.

Substitute 50 in. for *b* and 70 in. for h .

$$\begin{aligned} I_{x1} &= \frac{1}{3}(50)(70)^3 \\ &= 5.716666 \times 10^6\text{ in.}^4 \end{aligned}$$

Calculate the area moment of inertia of the small rectangle 2 about the *x*-axis.

$$I_{x2} = \frac{1}{3} b_2 h_2^3 + A_2 d^2$$

Here, b_2 is the breadth of rectangle 2, h_2 is the height of rectangle 2, A_2 is the area of rectangle 2, and d is the distance between the x axis and the centroid of rectangle 2.

Substitute $b_2 h_2$ for A_2 and $\left(h + \frac{h_2}{2}\right)$ in. for d.

$$I_{x2} = \frac{1}{12} b_2 h_2^3 + (b_2 h_2)\left(h + \frac{h_2}{2}\right)$$

Substitute 30 in. for b_2, 12 in. for h_2, and 40 in. for h

$$\begin{aligned} I_{x2} &= \left[\frac{1}{12}(30)(15)^3 + 30(15)\left(40 + \frac{15}{2}\right)^2\right] \\ &= 1.023750 \times 10^6\text{ in.}^4 \end{aligned}$$

Calculate the net moment of inertia about x axis for the schematic figure:

$$I_x = I_{x1} + I_{x2}$$

Substitute $5.716666 \times 10^6\text{ in.}^4$ for I_{x1} and $1.023750 \times 10^6\text{ in.}^4$ for I_{x2}.

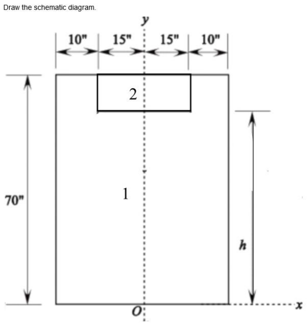
$$\begin{aligned} I_x &= (5.716666 \times 10^6) - (1.023750 \times 10^6) \\ &= 4.692916 \times 10^6 \\ &= 4.69 \times 10^6\text{ in.}^4 \end{aligned}$$

Therefore, the area moment of inertia about the *x*-axis for h equal to 40 in. is $\boxed{4.69 \times 10^6\text{ in.}^4}$.

Step 3 of 5

(b)

Draw the schematic diagram.



For $h = 60\text{ in.}$:

Calculate the area moment of inertia of the rectangle 1 about the *x*-axis.

$$I_{x1} = \frac{1}{3} b_1 h_1^3$$

Here, b_1 **is the breadth of rectangle 1 and** h_1 **is the height of rectangle 1.**

Substitute 50 in. **for *b* and** 70 in. **for** h .

$$\begin{aligned} I_{x1} &= \frac{1}{3}(50)(70)^3 \\ &= 5.716666 \times 10^6\text{ in.}^4 \end{aligned}$$

Comments (2)

Anonymous
why is I(x)=1/3bh^3 not 1/12bh^3

Anonymous
because your using the equation for the centroidal axis

Step 4 of 5

Calculate the area moment of inertia of the small rectangle 2 about the *x*-axis.

$$I_{x2} = \frac{1}{3} b_2 h_2^3 + A_2 d^2$$

Here, b_2 is the breadth of rectangle 2, h_2 is the height of rectangle 2, A_2 is the area of rectangle 2, and d is the distance between the x axis and the centroid of rectangle 2.

Substitute $b_2 h_2$ for A_2 and $\left(h + \frac{h_2}{2}\right)$ in. for d.

$$I_{x2} = \frac{1}{12} b_2 h_2^3 + (b_2 h_2)\left(h + \frac{h_2}{2}\right)$$

Substitute 30 in. for b_2, 12 in. for h_2, and 40 in. for h

$$\begin{aligned} I_{x2} &= \left[\frac{1}{12}(30)(10)^3 + 30(10)\left(60 + \frac{10}{2}\right)^2\right] \\ &= 1.27 \times 10^6\text{ in.}^4 \end{aligned}$$

Step 5 of 5

Calculate the net moment of inertia about x axis for the schematic figure:

$$I_x = I_{x1} + I_{x2}$$

Substitute $5.716666 \times 10^6\text{ in.}^4$ for I_{x1} and $1.27 \times 10^6\text{ in.}^4$ for I_{x2}.

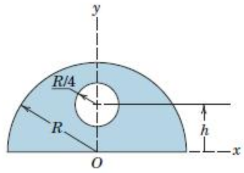
$$\begin{aligned} I_x &= (5.716666 \times 10^6) - (1.27 \times 10^6) \\ &= 4.446666 \times 10^6 \\ &= 4.45 \times 10^6\text{ in.}^4 \end{aligned}$$

Therefore, the area moment of inertia about the *x*-axis for h equal to 60 in. is $\boxed{4.45 \times 10^6\text{ in.}^4}$.

Chapter A, Problem 44P

Problem

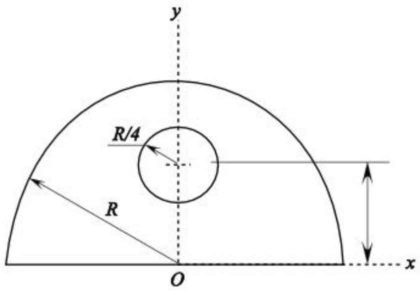
The variable *h* designates the arbitrary vertical location of the center of the circular cutout within the semicircular area. Determine the area moment of inertia about the *x*-axis for (*a*) *h* = 0 and (*b*) *h* = *R*/2.



Step-by-step solution

Step 1 of 3

Draw the schematic diagram.



Step 2 of 3

(a)

For $h=0$ **:**

Consider the moment of inertia about the x-axis for the schematic diagram for h equal to 0.

$$\begin{aligned}I_x &= \frac{\pi R^4}{8}-\frac{\pi\left(R/4\right)^4}{8}\\ &= \frac{\pi R^4}{8}-\frac{\pi R^4}{2048}\\ &= \frac{255}{2048}\pi R^4\\ &= 0.391\,R^4\end{aligned}$$

Therefore, the area moment of inertia about the *x*-axis for h equal to 0 is $\boxed{0.391R^4}$.

Step 3 of 3

(b)

For $h=R/2$ **:**

Consider the moment of inertia about the x-axis for the schematic diagram for h equal to $R/2$.

Consider the entire hole.

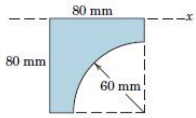
$$\begin{aligned}I_x &= \frac{\pi R^4}{8}-\left[\frac{\pi\left(R/4\right)^4}{4}+\pi\left(\frac{R}{4}\right)^2\left(\frac{R}{2}\right)^2\right]\\ &= \frac{\pi R^4}{8}-\left[\frac{\pi R^4}{1024}+\frac{\pi R^4}{64}\right]\\ &= \frac{111}{1024}\pi R^4\\ &= 0.341R^4\end{aligned}$$

Therefore, the area moment of inertia about the *x*-axis for h equal to $R/2$ is $\boxed{0.341R^4}$.

Chapter A, Problem 45P

Problem

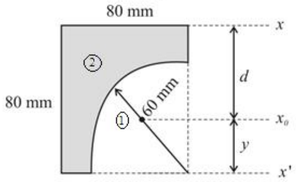
Calculate the moment of inertia of the shaded area about the *x*-axis.



Step-by-step solution

Step 1 of 3

Draw the schematic diagram.



Step 2 of 3

Calculate the moment of inertia of the square about its base x-axis.

$$(I_x)_1 = \frac{1}{3}bh^3$$

Here, b is the breath of rectangle and h is the height of a rectangle.

Substitute $80\,\text{mm}$ for b and $80\,\text{mm}$ for h.

$$\begin{aligned}(I_x)_1 &= \frac{1}{3}(80)^4 \\ &= 13.65\times10^6\ \text{mm}^4\end{aligned}$$

Calculate the centroid of the quarter-circle.

$$(\bar{y})_2 = \frac{4r}{3\pi}$$

Here, r is the radius of the circle.

Substitute $60\,\text{mm}$ for r.

$$\begin{aligned}(\bar{y})_2 &= \frac{4\times60}{3\pi} \\ &= 25.46\,\text{mm}\end{aligned}$$

Calculate the centriodal distance

$$d = h - (\bar{y})_2$$

Substitute $80\,\text{mm}$ for h and $25.46\,\text{mm}$ for $(\bar{y})_2$.

$$\begin{aligned}d &= 80 - 25.46 \\ &= 54.54\ \text{mm}\end{aligned}$$

Step 3 of 3

Calculate the moment of inertia about x-axis.

$$\begin{aligned}(I_x)_2 &= I_{x_0} + Ad^2 \\ &= I_{x'} + Ad^2 - A(\bar{y}_2)^2 \\ &= I_{x'} + A\left(d^2 - (\bar{y}_2)^2\right) \\ &= -\frac{1}{4}\frac{Ar^2}{4} - \frac{1}{4}\pi r^2\left(d^2 - (\bar{y}_2)^2\right) \\ &= \frac{-1}{4}\frac{\pi r^2}{4}\cdot r^2 - \frac{1}{4}\pi r^2\left(d^2 - (\bar{y}_2)^2\right) \\ &= \frac{-1}{4}\pi r^2\left(\frac{r^2}{4} + d^2 - (\bar{y}_2)^2\right) \\ &= \frac{-1}{4}\pi(60)^2\times\left(\frac{60^2}{4} + (54.54)^2 - (25.46)^2\right) \\ &= -9.122\times10^6\ \text{mm}^4\end{aligned}$$

Calculate the Moment of inertia of the whole area.

$$I_x = (I_x)_1 + (I_x)_2$$

Substitute $13.65\times10^6\ \text{mm}^4$ for $(I_x)_1$ and $-9.122\times10^6\ \text{mm}^4$ for $(I_x)_2$.

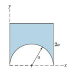
$$\begin{aligned}I_x &= 13.65\times10^6 - 9.122\times10^6 \\ &= 4.527\times10^6\ \text{mm}^4\end{aligned}$$

Therefore, the moment of inertia of shaded area about x-axis is $\boxed{4.527\times10^6\ \text{mm}^4}$.

Chapter A, Problem 46P

Problem

Determine the moments of inertia of the shaded area about the *x*- and *y*-axes.



Step-by-step solution

Step 1 of 13

Calculate the moment of inertia about x - axis for the shaded area.

Write the relation to calculate the moment of inertia of the shaded section about the x - axis.

$I_x = I_{x1} - I_{x2}$ ...... (1)

Here, I_x is the moment of inertia of the shaded area, I_{x1} is the moment of inertia about x - axis for square section, and I_{x2} is the moment of inertia of the semi circular section.

Write the formula to calculate the moment of inertia of the square section about the x - axis.

$I_{x1} = \overline{I_{x1}} + A_1 d_{x1}^2$ ...... (2)

Here, $\overline{I_{x1}}$ is the centroidal moment of inertia of the square section, A_1 is the area of the square section, and d_{x1} distance from centroid to the axis in horizontal direction.

Step 2 of 13

Write the relation to calculate the centroidal moment of inertia about the x - axis.

Obtain the relations from the, "Properties of Plane figures."

$$\overline{I_{x1}} = \frac{b_1 h_1^3}{12}$$

Here, b_1 is the width of the square, and h_1 is the height of the square section.

Substitute $2a$ for b_1, and $2a$ for h_1 from the diagram.

$$\overline{I_{x1}} = \frac{2a(2a)^3}{12} = \frac{4a^4}{3}$$

Step 3 of 13

Calculate the area of the square section.

$A_1 = b_1 h_1$

Substitute $2a$ for b_1, and $2a$ for h_1 from the diagram.

$$A_1 = (2a)(2a) = 4a^2$$

Write the relation for d_{x1} from the diagram.

$$d_{x1} = \frac{h_1}{2}$$

Substitute $2a$ for h_1.

$$d_{x1} = \frac{2a}{2} = a$$

Step 4 of 13

Substitute $\frac{4a^4}{3}$ for $\overline{I_{x1}}$, $4a^2$ for A_1, and a for d_{x1} in the equation (2).

$$\begin{aligned} I_{x1} &= \overline{I_{x1}} + A_1 d_{x1}^2 \\ &= \frac{4a^4}{3} + 4a^2 (a)^2 \\ &= \frac{16a^4}{3} \end{aligned}$$

Write the formula to calculate the moment of inertia of the semi circle section about the x - axis.

$I_{x2} = \overline{I_{x2}} + A_2 d_{x2}^2$ ...... (3)

Here, $\overline{I_{x2}}$ is the centroidal moment of inertia of the semi circular section, A_2 is the area of the semi circular section, and d_{x2} distance from centroid to the axis in horizontal direction.

Step 5 of 13

Write the relation to calculate centroidal moment of inertia of the semi circle about x - axis.

Obtain the relations from the, "Properties of Plane figures."

$$\overline{I_{x2}} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) r^4$$

Here, r is the radius of the semi circular section.

Substitute a for r.

$$\overline{I_{x2}} = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) a^4$$

Calculate the area of the semi circular section.

$$A_2 = \frac{\pi r^2}{2}$$

Substitute a for r.

$$A_2 = \frac{\pi a^2}{2}$$

Step 6 of 13

Write the relation for d_{x2} from the diagram.

Obtain the relations from the, "Properties of Plane figures."

$$d_{x2} = \frac{4r}{3\pi}$$

Substitute a for r.

$$d_{x2} = \frac{4a}{3\pi}$$

Substitute $\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) a^4$ for $\overline{I_{x2}}$, $\frac{\pi a^2}{2}$ for A_2, and $\frac{4a}{3\pi}$ for d_{x2} in the equation (3).

$$\begin{aligned} I_{x2} &= \overline{I_{x2}} + A_2 d_{x2}^2 \\ &= \left(\frac{\pi}{8} - \frac{8}{9\pi}\right) a^4 + \frac{\pi a^2}{2}\left(\frac{4a}{3\pi}\right)^2 \end{aligned}$$

Step 7 of 13

Substitute $\frac{16a^4}{3}$ for I_{x1}, and $\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) a^4 + \frac{\pi a^2}{2}\left(\frac{4a}{3\pi}\right)^2$ for I_{x2} in equation (1).

$$\begin{aligned} I_x &= I_{x1} - I_{x2} \\ &= \frac{16a^4}{3} - \left(\left(\frac{\pi}{8} - \frac{8}{9\pi}\right) a^4 + \frac{\pi a^2}{2}\left(\frac{4a}{3\pi}\right)^2\right) \\ &= 4.94a^4 \end{aligned}$$

Hence, the moment of inertia of the shaded area about the x - axis is $\boxed{4.94a^4}$.

Step 8 of 13

Calculate the moment of inertia about y - axis for the shaded area.

Write the relation to calculate the moment of inertia of the shaded section about the y - axis.

$I_y = I_{y1} - I_{y2}$ ...... (4)

Here, I_y is the moment of inertia of the shaded area, I_{y1} is the moment of inertia about y - axis for square section, and I_{y2} is the moment of inertia of the semi circular section.

Write the formula to calculate the moment of inertia of the square section about the y - axis.

$I_{y1} = \overline{I_{y1}} + A_1 d_{y1}^2$ ...... (5)

Here, $\overline{I_{y1}}$ is the centroidal moment of inertia of the square section, A_1 is the area of the square section, and d_{y1} distance from centroid to the axis in vertical direction.

Step 9 of 13

Write the relation to calculate the centroidal moment of inertia about the y - axis.

Obtain the relations from the, "Properties of Plane figures."

$$\overline{I_{y1}} = \frac{h_1 b_1^3}{12}$$

Here, b_1 is the width of the square, and h_1 is the height of the square section.

Substitute $2a$ for b_1, and $2a$ for h_1 from the diagram.

$$\overline{I_{y1}} = \frac{2a(2a)^3}{12} = \frac{4a^4}{3}$$

Step 10 of 13

Calculate the area of the square section.

$A_1 = b_1 h_1$

Substitute $2a$ for b_1, and $2a$ for h_1 from the diagram.

$$A_1 = (2a)(2a) = 4a^2$$

Write the relation for d_{y1} from the diagram.

$$d_{y1} = \frac{b_1}{2}$$

Substitute $2a$ for b_1.

$$d_{y1} = \frac{2a}{2} = a$$

Step 11 of 13

Substitute $\frac{4a^4}{3}$ for $\overline{I_{y1}}$, $4a^2$ for A_1, and a for d_{y1} in the equation (5).

$$\begin{aligned} I_{y1} &= \overline{I_{y1}} + A_1 d_{y1}^2 \\ &= \frac{4a^4}{3} + 4a^2 (a)^2 \\ &= \frac{16a^4}{3} \end{aligned}$$

Write the formula to calculate the moment of inertia of the semi circle section about the y - axis.

$I_{y2} = \overline{I_{y2}} + A_2 d_{y2}^2$ ...... (6)

Here, $\overline{I_{y2}}$ is the centroidal moment of inertia of the semi circular section, A_2 is the area of the semi circular section, and d_{y2} distance from centroid to the axis in vertical direction.

Step 12 of 13

Write the relation to calculate centroidal moment of inertia of the semi circle about y - axis.

Obtain the relations from the, "Properties of Plane figures."

$$\overline{I_{y2}} = \left(\frac{\pi}{8}\right) r^4$$

Here, r is the radius of the semi circular section.

Substitute a for r.

$$\overline{I_{y2}} = \left(\frac{\pi}{8}\right) a^4$$

Write the relation for d_{y2} from the diagram.

Obtain the relations from the, "Properties of Plane figures."

$d_{y2} = r$

Substitute a for r.

$d_{y2} = a$

Step 13 of 13

Substitute $\left(\frac{\pi}{8}\right) a^4$ for $\overline{I_{y2}}$, $\frac{\pi a^2}{2}$ for A_2, and a for d_{y2} in the equation (6).

$$\begin{aligned} I_{y2} &= \overline{I_{y2}} + A_2 d_{y2}^2 \\ &= \left(\frac{\pi}{8}\right) a^4 + \frac{\pi a^2}{2}(a)^2 \\ &= \frac{5\pi a^4}{8} \end{aligned}$$

Substitute $\frac{16a^4}{3}$ for I_{y1}, and $\frac{5\pi a^4}{8}$ for I_{y2} in equation (4).

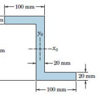
$$\begin{aligned} I_y &= I_{y1} - I_{y2} \\ &= \frac{16a^4}{3} - \frac{5\pi a^4}{8} \\ &= 3.369a^4 \end{aligned}$$

Hence, the moment of inertia of the shaded area about the y - axis is $\boxed{3.369a^4}$.

Chapter A, Problem 48P

Problem

Determine the moments of inertia of the Z-section about its centroidal x0- and y0-axes.



Step-by-step solution

Step 1 of 10

Calculate the moment of inertia about x - axis for the shaded area.

Write the relation to calculate the moment of inertia of the shaded section about the x_o - axis.

$I_{x_o} = I_{x1} + I_{x2} + I_{x2}$ ...... (1)

Here, I_x is the moment of inertia of the shaded area, I_{x1}, I_{x2}, and I_{x3} are the moment of inertia about x_o - axis for rectangle sections.

Write the formula to calculate the moment of inertia of the rectangle section 1 about the x_o - axis.

$I_{x1} = \overline{I_{x1}} + A_1\left(\overline{y_1}\right)^2$ ...... (2)

Here, $\overline{I_{x1}}$ is the centroidal moment of inertia of the rectangle section 1, A_1 is the area of the rectangle section 1, and $\overline{y_1}$ distance from centroid to the axis in horizontal direction.

Step 2 of 10

Write the relation to calculate the centroidal moment of inertia about the x_o - axis.

Obtain the relations from the, "Properties of Plane figures."

$$\overline{I_{x1}} = \frac{b_1 h_1^3}{12}$$

Here, b_1 is the width of the square, and h_1 is the height of the rectangle section 1.

Substitute $80\,\text{mm}$ for b_1, and $20\,\text{mm}$ for h_1 from the diagram.

$$\begin{aligned}\overline{I_{x1}} &= \frac{80\,\text{mm}(20\,\text{mm})^3}{12}\\ &= 53333.33\,\text{mm}^4\end{aligned}$$

Step 3 of 10

Calculate the area of the rectangle section 1.

$A_1 = b_1 h_1$

Substitute $80\,\text{mm}$ for b_1, and $20\,\text{mm}$ for h_1 from the diagram.

$$\begin{aligned}A_1 &= (80\,\text{mm})(20\,\text{mm})\\ &= 1600\,\text{mm}^2\end{aligned}$$

Write the relation for d_{x1} from the diagram.

$d_{x1} = \overline{y}$

Here, $\overline{y}$ is the centroid in vertical direction.

Substitute $140\,\text{mm}$ for $\overline{y}$.

$$\begin{aligned}d_{x1} &= \frac{140\,\text{mm}}{2}\\ &= 70\,\text{mm}\end{aligned}$$

Substitute $53333.33\,\text{mm}^4$ for $\overline{I_{x1}}$, $1600\,\text{mm}^2$ for A_1, and $70\,\text{mm}$ for d_{x1} in the equation (2).

$$\begin{aligned}I_{x1} &= \overline{I_{x1}} + A_1 d_{x1}^2\\ &= 53333.33\,\text{mm}^4 + 1600\,\text{mm}^2(70\,\text{mm})^2\\ &= 7.893\left(10^6\right)\text{mm}^4\end{aligned}$$

Step 4 of 10

Write the formula to calculate the moment of inertia of the rectangular section 2 about the x_o - axis.

Obtain the relations from the, "Properties of Plane figures."

$I_{x2} = \frac{b_2 h_2^3}{12}$ ...... (3)

Here, I_{x2} is the moment of inertia of the rectangular section 2, b_2 is the width of the rectangular section 2, and h_2 height of the rectangular section 2 from the axis in vertical direction.

Substitute $20\,\text{mm}$ for b_2, and $160\,\text{mm}$ for h_2 from the diagram.

$$\begin{aligned}I_{x2} &= \frac{20\,\text{mm}(160\,\text{mm})^3}{12}\\ &= 6.83\left(10^6\right)\text{mm}^4\end{aligned}$$

Step 5 of 10

Write the formula to calculate the moment of inertia of the rectangular section 3 about the x_o - axis.

Here, section 1 and section 3 are similar about axes.

$$\begin{aligned}I_{x3} &= I_{x1}\\ &= 7.893\left(10^6\right)\text{mm}^4\end{aligned}$$

Substitute $7.893\left(10^6\right)\text{mm}^4$ for I_{x1}, $6.83\left(10^6\right)\text{mm}^4$ for I_{x2}, and $7.893\left(10^6\right)\text{mm}^4$ for I_{x3} in the equation (1).

$$\begin{aligned}I_{x_o} &= I_{x1} + I_{x2} + I_{x2}\\ &= 7.893\left(10^6\right)\text{mm}^4 + 6.83\left(10^6\right)\text{mm}^4 + 7.893\left(10^6\right)\text{mm}^4\\ &= 22.616\left(10^6\right)\text{mm}^4\end{aligned}$$

Hence, moment of inertia of the shaded section about the x - axis is $\boxed{22.616\left(10^6\right)\text{mm}^4}$.

Step 6 of 10

Calculate the moment of inertia about y_o - axis for the shaded area.

Write the relation to calculate the moment of inertia of the shaded section about the - axis.

$I_{y_o} = I_{y1} + I_{y2} + I_{y3}$ ...... (4)

Here, I_{y_o} is the centroid moment of inertia of the shaded area, I_{y1} is the moment of inertia about y_o - axis for rectangular section 1, I_{y2} is the moment of inertia of the rectangular section 2, and I_{y3} is the moment of inertia of the rectangular section 3.

Write the formula to calculate the moment of inertia of the rectangular section 1 about the y_o - axis.

$I_{y1} = \overline{I_{y1}} + A_1 d_{y1}^2$ ...... (5)

Here, $\overline{I_{y1}}$ is the centroidal moment of inertia of the rectangular section 1, A_1 is the area of the rectangular section, and d_{y1} distance from centroid to the axis in vertical direction.

Step 7 of 10

Write the relation to calculate the centroidal moment of inertia about the y_o - axis.

Obtain the relations from the, "Properties of Plane figures."

$$\overline{I_{y1}} = \frac{h_1 b_1^3}{12}$$

Here, b_1 is the width of the square, and h_1 is the height of the rectangular section 1.

Substitute $80\,\text{mm}$ for b_1, and $20\,\text{mm}$ for h_1 from the diagram.

$$\begin{aligned}\overline{I_{y1}} &= \frac{20\,\text{mm}(80\,\text{mm})^3}{12}\\ &= 853333.33\,\text{mm}^4\end{aligned}$$

Step 8 of 10

Write the relation for d_{y1} from the diagram.

$d_{y1} = \overline{x}$

Here, $\overline{x}$ is the centroid in horizontal direction.

Substitute $50\,\text{mm}$ for $\overline{x}$ from the diagram.

$d_{y1} = 50\,\text{mm}$

Substitute $853333.33\,\text{mm}^4$ for $\overline{I_{y1}}$, $1600\,\text{mm}^2$ for A_1, and $50\,\text{mm}$ for d_{y1} in the equation (5).

$$\begin{aligned}I_{y1} &= \overline{I_{y1}} + A_1 d_{y1}^2\\ &= 853333.33\,\text{mm}^4 + 1600\,\text{mm}^2(50\,\text{mm})^2\\ &= 4.853\left(10^6\right)\text{mm}^4\end{aligned}$$

Step 9 of 10

Write the formula to calculate the moment of inertia of the rectangular section 2 about the y_o - axis.

$I_{y2} = \frac{h_2 b_2^3}{12}$ ...... (6)

Substitute $20\,\text{mm}$ for b_1, and $160\,\text{mm}$ for h_1 from the diagram.

$$\begin{aligned}I_{y2} &= \frac{160\,\text{mm}(20\,\text{mm})^3}{12}\\ &= 106666.667\,\text{mm}^4\end{aligned}$$

Write the formula to calculate the moment of inertia of the rectangular section 3 about the y_o - axis.

Here, section 1 nad section 3 are similar about the axis.

$I_{y3} = I_{y1}$

Substitute $4.853\left(10^6\right)\text{mm}^4$ for I_{y1}.

$I_{y3} = 4.853\left(10^6\right)\text{mm}^4$

Step 10 of 10

Substitute $4.853\left(10^6\right)\text{mm}^4$ for I_{y1}, $106666.667\,\text{mm}^4$ for I_{y2}, and $4.853\left(10^6\right)\text{mm}^4$ for I_{y3} in equation (4).

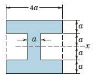
$$\begin{aligned}I_{y_o} &= I_{y1} + I_{y2} + I_{y3}\\ &= 4.853\left(10^6\right)\text{mm}^4 + 106666.667\,\text{mm}^4 + 4.853\left(10^6\right)\text{mm}^4\\ &= 9.812\left(10^6\right)\text{mm}^4\end{aligned}$$

Hence, the moment of inertia of the shaded area about the y_o - axis is $\boxed{9.812\left(10^6\right)\text{mm}^4}$.

Chapter A, Problem 49P

Problem

Determine the moment of inertia of the shaded area about the *x*-axis in two different ways.



Step-by-step solution

Step 1 of 8

Calculate the moment of inertia about in sections method.

Write the relation to calculate the moment of inertia of the shaded section about the x-axis.

$I_x = I_{x1} + I_{x2} + I_{x3}$ ...... (1)

Here, I_x is the moment of inertia of the shaded area, I_{x1}, I_{x2} and I_{x3} are the moment of inertia of the corresponding parts.

Obtain the relations from the, “Properties of Plane figures.”

Write the formula to calculate the moment of inertia of the shaded section 2 about the x-axis.

$I_{x2} = \left(\frac{b_2 h_2^3}{12}\right)$ ...... (2)

Here, b_2 is the width of the section 2, h_2 is the height of the section 2.

Substitute a for b_2, and $2a$ for h_2 from the diagram.

$$\begin{aligned} I_{x2} &= \frac{(a)(2a)^3}{12} \\ &= \frac{8a^4}{12} \end{aligned}$$

Step 2 of 8

Here there are section1, and section3 are similar about x- axis.

$I_{x1} = I_{x3}$

Write the formula to calculate the moment of inertia of the shaded section 1 about the x-axis.

$I_{x1} = \left(\frac{b_1 h_1^3}{12} + A d_{x1}^2\right)$ ...... (3)

Here, b_1 is the width of the section 1, h is the height of the shaded section, h_1 is the height of the section 1, and A is the area of the section 1.

Calculate the area of the section 1.

$A = b_1 h_1$

Substitute $4a$ for b_1, and a for h_1.

$$\begin{aligned} A &= (4a)(a) \\ &= 4a^2 \end{aligned}$$

Calculate d_{x1} from the figure.

$d_{x1} = a + \frac{a}{2}$

Step 3 of 8

Substitute $4a$ for b_1, and a for h_1, $a + \frac{a}{2}$ for d_{x1}, and $4a^2$ for A in equation (3).

$$\begin{aligned} I_{x1} &= \left(\frac{b_1 h_1^3}{12} + A\left(\frac{h}{2}\right)^2\right) \\ &= \left(\frac{4a(a)^3}{12} + (4a)^2\left(a + \frac{a}{2}\right)^2\right) \\ &= \frac{56a^4}{3} \end{aligned}$$

Write the value for moment of inertia of section 3 about x- axis.

$I_{x3} = I_{x1}$

Substitute $\frac{56a^4}{3}$ for I_{x1}.

$I_{x3} = \frac{56a^4}{3}$

Step 4 of 8

Substitute $\frac{56a^4}{3}$ for I_{x1}, and $\frac{8a^4}{12}$ for I_{x2}, and $\frac{56a^4}{3}$ for I_{x3} in equation (1).

$$\begin{aligned} I_x &= I_{x1} + I_{x2} + I_{x3} \\ &= \frac{56a^4}{3} + \frac{8a^4}{12} + \frac{56a^4}{3} \\ &= \frac{58a^4}{3} \end{aligned}$$

Hence, the moment of inertia of the shaded area about the x- axis is $\boxed{\frac{58a^4}{3}}$.

Step 5 of 8

Calculate the moment of inertia about x- axis by other method.

Write the relation to calculate the moment of inertia of the shaded section about the x-axis.

$I_x = I_{x1} - I_{x2}$ ...... (4)

Here, I_x is the moment of inertia of the shaded area, I_{x1}, and I_{x2} are the moment of inertia of the corresponding parts of outer rectangle and inner rectangle.

Step 6 of 8

Obtain the relations from the, “Properties of Plane figures.”

Write the formula to calculate the moment of inertia of the outer rectangle about the x-axis.

$I_{x1} = \left(\frac{b_1 h_1^3}{12}\right)$ ...... (5)

Here, b_1 is the width of the rectangular section 1, and h_1 is the height of the outer rectangle.

Substitute $4a$ for b_1, and $4a$ for h_1 from the diagram.

$$\begin{aligned} I_{x1} &= \frac{(4a)(4a)^3}{12} \\ &= \frac{64a^4}{3} \end{aligned}$$

Step 7 of 8

Obtain the relations from the, “Properties of Plane figures.”

Here there are two parts at inner parts which are similar about axes.

Write the formula to calculate the moment of inertia of the inner rectangle about the x-axis.

$I_{x2} = 2\left(\frac{b_2 h_2^3}{12}\right)$ ...... (6)

Here, b_2 is the width of the inner rectangle, and h_2 is the height of the inner rectangle.

Substitute $1.5a$ for b_2, and $2a$ for h_2 from the diagram.

$$\begin{aligned} I_{x2} &= \frac{2(1.5a)(2a)^3}{12} \\ &= 2a^4 \end{aligned}$$

Step 8 of 8

Substitute $\frac{64a^4}{3}$ for I_{x1}, and $2a^4$ for I_{x2} in equation (4).

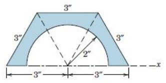
$$\begin{aligned} I_x &= I_{x1} - I_{x2} \\ &= \frac{64a^4}{3} - 2a^4 \\ &= \frac{58a^4}{3} \end{aligned}$$

Hence, the moment of inertia of the shaded area about the x- axis is $\boxed{\frac{58a^4}{3}}$.

Chapter A, Problem 51P

Problem

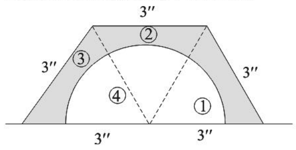
Calculate the moment of inertia of the shaded area about the *x*-axis.



Step-by-step solution

Step 1 of 9

The moment of inertia about Centroidal x – axis for triangle



Step 2 of 9

1) – 3) Triangle

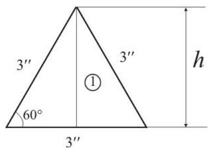
2) – Inverted triangle

4) –

Step 3 of 9

The moment of inertia about Centroidal x – axis for triangle $(I_x)=\frac{bh^3}{12}$

Step 4 of 9



Step 5 of 9

$\sin 60° = \frac{h}{3}$

$h = 3\times\frac{\sqrt{3}}{2}$

$(I_x) = \frac{(3)\times\left(\frac{3\sqrt{3}}{2}\right)^3}{12}$

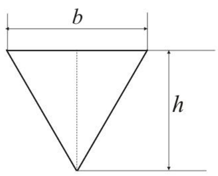
For (1) and (3), $\left(I_{x_{1,3}}\right) = 2\left[\frac{3\times\left(\frac{3\sqrt{3}}{2}\right)^3}{12}\right]$

$= 8.768\,\text{in}^4$

Step 6 of 9

The moment of inertia about Centroidal x – axis for inverted triangle $(I_x)=\frac{bh^3}{4}$

Step 7 of 9



Step 8 of 9

$\sin 60° = \frac{h}{3}$

$\frac{\sqrt{3}}{2} = \frac{h}{3}$

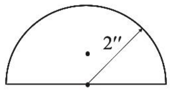
$\frac{3\sqrt{3}}{2} = h$

$(I_x) = \frac{3\times\left(\frac{3\sqrt{3}}{2}\right)^3}{4}$

$(I_x) = 13.152\,\text{in}^4$

Step 9 of 9

The moment of inertia about Centroidal x – axis for half circle $(I_x)=\frac{1}{8}\pi r^4$



$I_x = \frac{1}{8}\times\pi\times(2)^4$

$I_x = 6.283$

Total moment of inertia about Centroidal x – axis for shaded area of entire object.

$(I_x) = 8.768 + 13.152 - 6.283$

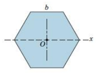
$= 15.637$

$\therefore \boxed{I_x = 15.637\,\text{in}^4}$

Chapter A, Problem 53P

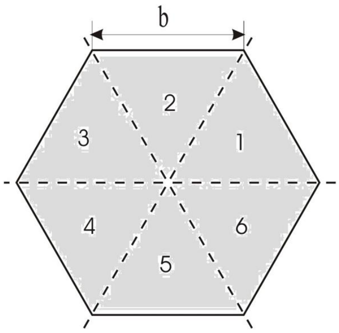
Problem

Develop a formula for the moment of inertia of the regular hexagonal area of side *b* about its central *x*-axis.



Step-by-step solution

Step 1 of 8



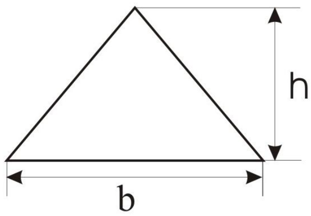
Step 2 of 8

Moment of inertia about Centroidal $x-$axis for triangle (1), (3), (4) & (6)

$$\left(I_{x_1}\right)=\frac{b\,h^3}{12}$$

$$=4\times\frac{b\,h^3}{12}$$

Step 3 of 8



Step 4 of 8

$$\sin 60^\circ=\frac{h}{b}$$

$$\frac{\sqrt{3}}{2}\times b=h$$

$$I_{x_1}=4\left[\frac{b\times\left(\frac{b\sqrt{3}}{2}\right)^3}{12}\right]$$

$$=\frac{4\times b\times b^3\times 3\times\sqrt{3}}{96}$$

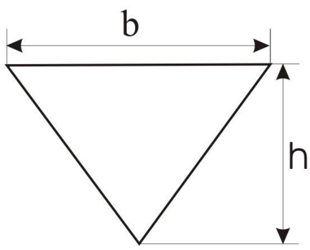
$$=\frac{b^4\sqrt{3}}{8}$$

Step 5 of 8

Moment of inertia about centroidal $x-$axis for inverted triangles (2) and (5)

$$\left(I_x\right)=\frac{1}{4}b\,h^3$$

Step 6 of 8



Step 7 of 8

$$\sin 60^\circ=\frac{h}{b}$$

$$\frac{\sqrt{3}}{2}\times b=h$$

$$\left(I_x\right)_2=\frac{1}{4}bh^3$$

$$\left(I_x\right)_2=2\left[\frac{1}{4}bh^3\right]$$

$$=2\left[\frac{1}{4}\times b\times\left(\frac{\sqrt{3}}{2}b\right)^3\right]$$

$$=2\left[\frac{1}{4}\times b\times\frac{3\times\sqrt{3}\times b^3}{8}\right]$$

$$=\frac{3\sqrt{3}b^4}{16}$$

Step 8 of 8

Total moment of inertia about Centroidal $x-$axis

$$I_x=\frac{b^4\sqrt{3}}{8}+\frac{3\sqrt{3}\,b^4}{16}$$

$$=\frac{2b^4\sqrt{3}+3\sqrt{3}\,b^4}{16}$$

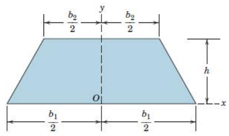
$$=\frac{5\sqrt{3}b^4}{16}$$

$$\therefore \boxed{I_x=\frac{5\sqrt{3}b^4}{16}}$$

Chapter A, Problem 54P

Problem

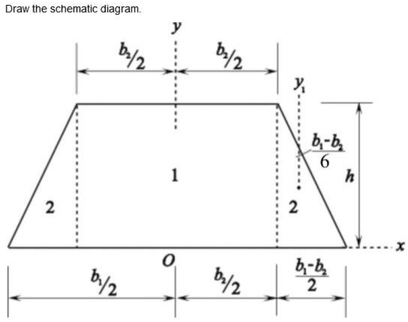
By the method of this article, determine the moments of inertia about the *x*- and *y*-axes of the trapezoidal area.



Step-by-step solution

Step 1 of 5

Draw the schematic diagram.



Step 2 of 5

Write the relation for moment of inertia of rectangular portion 1 about x axis.

$$I_{x1} = \frac{1}{3} b_2 h^3$$

Here, b_2 is the breadth and h is the height of the rectangle.

Write the relation for the moment of inertia of the two triangles 2 about x axis.

$$\begin{aligned} I_{x2} &= 2\left[\frac{1}{12}\left(\frac{b_1 - b_2}{2}\right)h^3\right] \\ &= \frac{b_1 h^3 - b_2 h^3}{12} \end{aligned}$$

Step 3 of 5

Write the net moment of inertia for the schematic diagram about x axis.

$$I_x = I_{x_1} + I_{x_2}$$

Substitute $\frac{1}{3} b_2 h^3$ for I_{x1} and $\frac{b_1 h^3 - b_2 h^3}{12}$ for I_{x2}.

$$\begin{aligned} I_x &= \frac{1}{3} b_2 h^3 + \left(\frac{b_1 h^3 - b_2 h^3}{12}\right) \\ &= \frac{1}{3} b_2 h^3 + \frac{b_1 h^3}{12} - \frac{b_2 h^3}{12} \\ &= \frac{b_1 h^3}{12} + \frac{b_2 h^3}{4} \\ &= h^3\left(\frac{b_1}{12} + \frac{b_2}{4}\right) \end{aligned}$$

Therefore, the area moment of inertia about the *x*-axis is $\boxed{h^3\left(\frac{b_1}{12} + \frac{b_2}{4}\right)}$.

Step 4 of 5

Write the relation for moment of inertia of rectangular portion 1 about y axis.

$$I_{y1} = \frac{1}{12} h b_2^3$$

Write the relation for the moment of inertia of the two triangles 2 about y axis.

$$\begin{aligned} I_{y2} &= 2\left[\frac{1}{36} h\left(\frac{b_1 - b_2}{2}\right)^3 + \frac{1}{2} h\left(\frac{b_1 - b_2}{2}\right)\left(\frac{b_2}{2} + \frac{1}{3}\left(\frac{b_1 - b_2}{2}\right)\right)^2\right] \\ &= 2\left[\frac{1}{36} h\left(\frac{b_1 - b_2}{2}\right)^3 + \frac{1}{2} h\left(\frac{b_1 - b_2}{2}\right)\left(\left(\frac{2b_2 + b_1}{6}\right)\right)^2\right] \\ &= 2h\left[\frac{1}{288}\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + \frac{1}{144}\left(b_1 - b_2\right)\left(4b_2^2 + b_1^2 + 4b_1 b_2\right)\right] \\ &= \frac{h}{144}\left[\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + 2\left(b_1 - b_2\right)\left(4b_2^2 + b_1^2 + 4b_1 b_2\right)\right] \end{aligned}$$

$$\begin{aligned} I_{y2} &= \frac{h}{144}\left[\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + 2\left(4b_2^2 b_1 + b_1^3 + 4b_1^2 b_2 - 4b_2^3 - b_1^2 b_2 - 4b_1 b_2^2\right)\right] \\ &= \frac{h}{144}\left[\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + \left(8b_2^2 b_1 + 2b_1^3 + 8b_1^2 b_2 - 8b_2^3 - 2b_1^2 b_2 - 8b_1 b_2^2\right)\right] \\ &= \frac{h}{144}\left[\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + \left(2b_1^3 + 6b_1^2 b_2 - 8b_2^3\right)\right] \\ &= \frac{h}{144}\left[\left(3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 - 9b_2^3\right)\right] \end{aligned}$$

Step 5 of 5

Write the net moment of inertia for the schematic diagram about y axis.

$$I_y = I_{y1} + I_{y2}$$

Substitute $\frac{1}{12} h b_2^3$ for I_{y1} and $\frac{h}{144}\left[\left(3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 - 9b_2^3\right)\right]$ for I_{y2}.

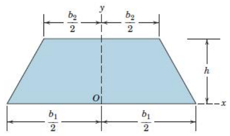
$$\begin{aligned} I_y &= \frac{1}{12} h b_2^3 + \frac{h}{144}\left[\left(3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 - 9b_2^3\right)\right] \\ &= \frac{h}{144}\left[\left(12b_2^3 + 3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 - 9b_2^3\right)\right] \\ &= \frac{h}{144}\left[\left(3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 + 3b_2^3\right)\right] \\ &= \frac{h}{48}\left[\left(b_1^3 + b_1^2 b_2 + b_2 b_1^2 + b_2^3\right)\right] \end{aligned}$$

Therefore, the area moment of inertia about the *y*-axis is $\boxed{\frac{h}{48}\left(b_1^3 + b_1^2 b_2 + b_1 b_2^2 + b_2^3\right)}$.

Chapter A, Problem 54P

Problem

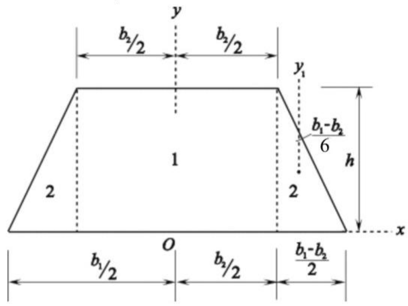
By the method of this article, determine the moments of inertia about the *x*- and *y*-axes of the trapezoidal area.



Step-by-step solution

Step 1 of 5

Draw the schematic diagram.



Step 2 of 5

Write the relation for moment of inertia of rectangular portion 1 about x axis.

$$I_{x1} = \frac{1}{3} b_2 h^3$$

Here, b_2 is the breadth and h is the height of the rectangle.

Write the relation for the moment of inertia of the two triangles 2 about x axis.

$$\begin{aligned} I_{x2} &= 2\left[\frac{1}{12}\left(\frac{b_1 - b_2}{2}\right)h^3\right] \\ &= \frac{b_1 h^3 - b_2 h^3}{12} \end{aligned}$$

Step 3 of 5

Write the net moment of inertia for the schematic diagram about x axis.

$$I_x = I_{x_1} + I_{x_2}$$

Substitute $\frac{1}{3} b_2 h^3$ for I_{x1} and $\frac{b_1 h^3 - b_2 h^3}{12}$ for I_{x2}.

$$\begin{aligned} I_x &= \frac{1}{3} b_2 h^3 + \left(\frac{b_1 h^3 - b_2 h^3}{12}\right) \\ &= \frac{1}{3} b_2 h^3 + \frac{b_1 h^3}{12} - \frac{b_2 h^3}{12} \\ &= \frac{b_1 h^3}{12} + \frac{b_2 h^3}{4} \\ &= h^3\left(\frac{b_1}{12} + \frac{b_2}{4}\right) \end{aligned}$$

Therefore, the area moment of inertia about the *x*-axis is $\boxed{h^3\left(\frac{b_1}{12} + \frac{b_2}{4}\right)}$.

Step 4 of 5

Write the relation for moment of inertia of rectangular portion 1 about y axis.

$$I_{y1} = \frac{1}{12} h b_2^{\,3}$$

Write the relation for the moment of inertia of the two triangles 2 about y axis.

$$\begin{aligned} I_{y2} &= 2\left[\frac{1}{36} h\left(\frac{b_1 - b_2}{2}\right)^3 + \frac{1}{2} h\left(\frac{b_1 - b_2}{2}\right)\left(\frac{b_2}{2} + \frac{1}{3}\left(\frac{b_1 - b_2}{2}\right)\right)^2\right] \\ &= 2\left[\frac{1}{36} h\left(\frac{b_1 - b_2}{2}\right)^3 + \frac{1}{2} h\left(\frac{b_1 - b_2}{2}\right)\left(\left(\frac{2b_2 + b_1}{6}\right)\right)^2\right] \\ &= 2h\left[\frac{1}{288}\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + \frac{1}{144}\left(b_1 - b_2\right)\left(4b_2^2 + b_1^{\,2} + 4b_1 b_2\right)\right] \\ &= \frac{h}{144}\left[\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + 2\left(b_1 - b_2\right)\left(4b_2^2 + b_1^{\,2} + 4b_1 b_2\right)\right] \end{aligned}$$

$$\begin{aligned} I_{y2} &= \frac{h}{144}\left[\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + 2\left(4b_2^2 b_1 + b_1^{\,3} + 4b_1^{\,2} b_2 - 4b_2^3 - b_1^{\,2} b_2 - 4b_1 b_2^{\,2}\right)\right] \\ &= \frac{h}{144}\left[\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + \left(8b_2^2 b_1 + 2b_1^{\,3} + 8b_1^{\,2} b_2 - 8b_2^3 - 2b_1^{\,2} b_2 - 8b_1 b_2^{\,2}\right)\right] \\ &= \frac{h}{144}\left[\left(b_1^3 - 3b_1^2 b_2 + 3b_2 b_1^2 - b_2^3\right) + \left(2b_1^{\,3} + 6b_1^{\,2} b_2 - 8b_2^3\right)\right] \\ &= \frac{h}{144}\left[\left(3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 - 9b_2^3\right)\right] \end{aligned}$$

Step 5 of 5

Write the net moment of inertia for the schematic diagram about y axis.

$$I_y = I_{y1} + I_{y2}$$

Substitute $\frac{1}{12} h b_2^{\,3}$ for I_{y1} and $\frac{h}{144}\left[\left(3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 - 9b_2^3\right)\right]$ for I_{y2}.

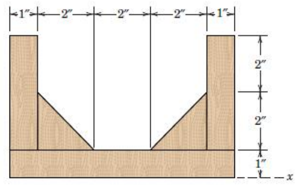
$$\begin{aligned} I_y &= \frac{1}{12} h b_2^{\,3} + \frac{h}{144}\left[\left(3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 - 9b_2^3\right)\right] \\ &= \frac{h}{144}\left[\left(12b_2^{\,3} + 3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 - 9b_2^3\right)\right] \\ &= \frac{h}{144}\left[\left(3b_1^3 + 3b_1^2 b_2 + 3b_2 b_1^2 + 3b_2^3\right)\right] \\ &= \frac{h}{48}\left[\left(b_1^3 + b_1^2 b_2 + b_2 b_1^2 + b_2^3\right)\right] \end{aligned}$$

Therefore, the area moment of inertia about the *y*-axis is $\boxed{\frac{h}{48}\left(b_1^{\,3} + b_1^{\,2} b_2 + b_1 b_2^{\,2} + b_2^{\,3}\right)}$.

Chapter A, Problem 55P

Problem

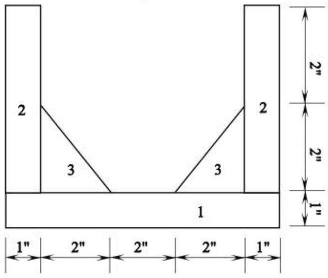
Determine the momentof inertia of the crosssectional area of the reinforced channel about the *x*-axis.



Step-by-step solution

Step 1 of 4

Draw the schematic diagram.



Same numbering to the sections are due to the symmetry of the figure.

Step 2 of 4

Calculate the moment of inertia of the cross sectional area of the reinforced channel about the *x*-axis by using the following relation:

$I_x = \Sigma \bar{I}_x + \Sigma A d_x^2$ …… (1)

Here, $\bar{I}_x$ is the centroidal moment of inertia about *x*-axis, *A* is the area, d_x is the distance from the centre about *x*-axis.

Consider b_1, h_1 to be the breadth and height of the rectangle 1, b_2, h_2 is the breadth and height of the rectangle 2, and b_3, h_3 be the base and height of the triangle 3.

Step 3 of 4

Tabulate the following terms.

Part	Area $A\ (\text{in.}^2)$	d_x (in.)	$\bar{I}_x$ (in.^4)	Ad_x^2 (in.^4)
1	b_1h_1 $=8(1)$ $=8$	$\frac{h_1}{2}$ $=\frac{1}{2}$ $=0.5$	$\frac{1}{12}b_1h_1$ $=\frac{1}{12}8(1)^3$ $=0.667$	2
2	$2[b_2h_2]$ $=2[1\times4]$ $=8$	$h_1+\frac{h_2}{2}$ $=1+\frac{4}{2}$ $=3$	$2\left[\frac{1}{12}b_2h_2^3\right]$ $=2\left[\frac{1}{12}(1)(4)^3\right]$ $=10.667$	72
3	$2\left[\frac{1}{2}b_3h_3\right]$ $=2\left[\frac{1}{2}(2)(2)\right]$ $=4$	$h_1+\frac{h_3}{3}$ $=\left(1+\frac{2}{3}\right)$ $=1.667$	$2\left(\frac{1}{36}b_3h_3^3\right)$ $=2\left(\frac{1}{36}(2)(2)^3\right)$ $=0.889$	11.11
	$\Sigma\bar{I}_x = 12.22\text{ in.}^4$	$\Sigma Ad_x^2 = 85.1\text{ in.}^4$		

Step 4 of 4

Substitute 12.22 in.^4 for $\Sigma\bar{I}_x$ and 85.1 in.^4 for ΣAd_x^2 in equation (1).

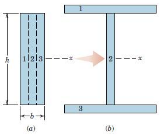
$$
\begin{aligned}
I_x &= \Sigma\bar{I}_x + \Sigma Ad_x^2 \\
&= 12.22 + 85.1 \\
&= 97.32\text{ in.}^4
\end{aligned}
$$

Therefore, the moment of inertia of the cross sectional area of the reinforced channel about the *x*-axis is $\boxed{97.32\text{ in.}^4}$.

Chapter A, Problem 56P

Problem

The rectangular area shown in part *a* of the figure is split into three equal areas which are then arranged as shown in part *b* of the figure. Determine an expression for the moment of inertia of the area in part *b* about the centroidal *x*-axis. What percent increase *n* over the moment of inertia for area *a* does this represent if *h* = 200 mm and *b* = 60 mm?



Step-by-step solution

Step 1 of 7

Calculate the moment of inertia of the figure (a) from the diagram.

$$I_{xr}=\frac{bh^3}{12} \quad \text{......}\ (1)$$

Here, I_{xr} is the moment of the inertia about the x- axis, b is the width of the rectangle section, and h is the height of the rectangle section.

Substitute $60\ \text{mm}$ for b and $200\ \text{mm}$ for h.

$$\begin{aligned} I_{xr} &= \frac{(60\,\text{mm})(200\,\text{mm})^3}{12} \\ &= 40(10^6)\,\text{mm}^4 \end{aligned}$$

Calculate the moment of inertia of the figure (b) from the diagram.

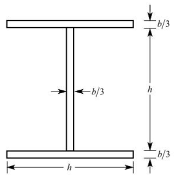
Write the relation to calculate the moment of inertia of the shaded section about the x- axis.

$$I_x = I_{x1}+I_{x2}+I_{x3} \quad \text{......}\ (2)$$

Here, I_x is the moment of inertia of the shaded area, I_{x1}, I_{x2} and I_{x3} are the moment of inertia of the corresponding parts.

Step 2 of 7

Redraw figure (b) with the dimensions mentioned in it.



Step 3 of 7

Obtain the relations from the, "Properties of Plane figures."

Write the formula to calculate the moment of inertia of the shaded section 2 about the x- axis.

$$\begin{aligned} I_{x2} &= \left[\frac{(b/3)h^3}{12}\right] \\ &= \frac{bh^3}{36} \end{aligned}$$

Substitute $60\ \text{mm}$ for b and $200\ \text{mm}$ for h.

$$\begin{aligned} I_{x2} &= \frac{(60/3)(200)^3}{12} \\ &= 13.33(10^6)\,\text{mm}^4 \end{aligned}$$

Here, section 1 and section 3 are similar about x- axis.

$$I_{x1}=I_{x3}$$

Step 4 of 7

Write the formula to calculate the moment of inertia of the shaded section 1 about the x- axis.

$$\begin{aligned} I_{x1} &= \left(\frac{h(b/3)^3}{12}+Ad_{x1}{}^2\right) \\ &= \left[\frac{hb^3}{324}+\left(\frac{b}{3}\times h\right)\left(\frac{h}{2}+\frac{b/3}{2}\right)^2\right] \\ &= \left[\frac{hb^3}{324}+\frac{bh}{3}\left(\frac{h^2}{4}+\frac{b^2}{36}+2\times\frac{h}{2}\times\frac{b}{6}\right)\right] \\ &= \frac{bh}{324}\left[27h^2+4b^2+18bh\right] \end{aligned}$$

Substitute $60\ \text{mm}$ for b and $200\ \text{mm}$ for h.

$$\begin{aligned} I_{x1} &= \frac{60\times 200}{324}\left[(27\times 200^2)+(4\times 60^2)+(18\times 60\times 200)\right] \\ &= 48.5333(10^6)\,\text{mm}^4 \end{aligned}$$

Step 5 of 7

Write the value for moment of inertia of section 3 about x- axis.

$$I_{x3}=I_{x1}$$

Substitute $48.5333(10^6)\,\text{mm}^4$ for I_{x1}.

$$I_{x3}=48.5333(10^6)\,\text{mm}^4$$

Substitute $48.5333(10^6)\,\text{mm}^4$ for I_{x1}, $13.33(10^6)\,\text{mm}^4$ for I_{x2}, and $48.5333(10^6)\,\text{mm}^4$ for I_{x3} in equation (2).

$$\begin{aligned} I_x &= I_{x1}+I_{x2}+I_{x3} \\ &= 48.5333(10^6)\,\text{mm}^4+13.33(10^6)\,\text{mm}^4+48.5333(10^6)\,\text{mm}^4 \\ &= 110.39(10^6)\,\text{mm}^4 \end{aligned}$$

Step 6 of 7

Substitute $\frac{bh}{324}\left[27h^2+4b^2+18bh\right]$ for I_{x1}, $\frac{bh^3}{36}$ for I_{x2}, $\frac{bh}{324}\left[27h^2+4b^2+18bh\right]$ for in equation (2).

$$\begin{aligned} I_x &= I_{x1}+I_{x2}+I_{x3} \\ &= \frac{bh}{324}\left[27h^2+4b^2+18bh\right]+\frac{bh^3}{36}+\frac{bh}{324}\left[27h^2+4b^2+18bh\right] \\ &= \frac{bh}{162}\left[27h^2+4b^2+18bh\right]+\frac{bh^3}{36} \\ &= \frac{bh}{324}\left[54h^2+8b^2+36bh+9h^2\right] \\ &= \frac{bh}{9}\left(\frac{7}{4}h^2+\frac{2}{9}b^2+bh\right) \end{aligned}$$

Therefore, the expresiion for moment of inertia for figure (b) is $\boxed{\frac{bh}{9}\left(\frac{7}{4}h^2+\frac{2}{9}b^2+bh\right)}$.

Step 7 of 7

Write the relation to calculate the percentage increase in moment of inertia.

$$n=\left(\frac{I_x-I_{xr}}{I_{xr}}\right)\times 100$$

Here, n is the percentage increase in moment of inertia.

Substitute $110.39(10^6)\,\text{mm}^4$ for I_x, and $40(10^6)\,\text{mm}^4$ for I_{xr}.

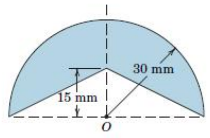
$$\begin{aligned} n &= \left(\frac{110.39(10^6)\,\text{mm}^4-40(10^6)\,\text{mm}^4}{40(10^6)\,\text{mm}^4}\right)\times 100 \\ &= 175.975\% \end{aligned}$$

Hence, the percentage increase in moment of inertia about x- axis is $\boxed{175.975\%}$.

Chapter A, Problem 57P

Problem

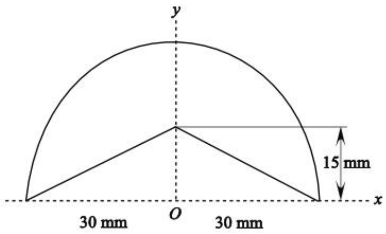
Calculate the polar moment of inertia of the shaded area about point *O*.



Step-by-step solution

Step 1 of 4

Draw the schematic diagram.



Step 2 of 4

Write the relation for moment of inertia for the semi-circle about the z axis (point O).

$$I_{O,\text{ semi-circle}} = \frac{1}{4}\pi r^4$$

Here, *r* is the radius of the semi-circle.

Substitute 30 mm for *r*

$$\begin{aligned} I_{O,\text{ semi-circle}} &= \frac{1}{4}\pi(30)^4 \\ &= 0.6362\times10^6\ \text{mm}^4 \end{aligned}$$

Step 3 of 4

Write the relation for moment of inertia for the two triangles about the *x*-axis.

$$I_{x,\text{triangle}} = 2\left(\frac{1}{12}bh^3\right)$$

Here, b is the breadth and h is the height.

Substitute 60 mm for *b* and 15 mm for *h*

$$\begin{aligned} I_{x,\text{triangle}} &= \frac{1}{12}(60)(15)^3 \\ &= 0.01688\times10^6\ \text{mm}^4 \end{aligned}$$

Calculate the moment of inertia for the two triangles about the *y*-axis.

$$I_{y,\text{triangle}} = 2\left(\frac{1}{12}hb^3\right)$$

Substitute 60 mm for *b* and 15 mm for *h*

$$\begin{aligned} I_{y,\text{triangle}} &= \frac{2}{12}\times15\times30^3 \\ &= 0.06750\times10^6\ \text{mm}^4 \end{aligned}$$

Calculate the moment of inertia about the z axis, (point *O*).

$$I_{O,\text{ triangle}} = I_{x,\text{triangle}} + I_{y,\text{triangle}}$$

Substitute $0.01688\times10^6\ \text{mm}^4$ for $I_{x,\text{triangle}}$ and $0.06750\times10^6\ \text{mm}^4$ for $I_{y,\text{triangle}}$.

$$\begin{aligned} I_{O,\text{ triangle}} &= 0.01688\times10^6 + 0.06750\times10^6 \\ &= 0.0844\times10^6\ \text{mm}^4 \end{aligned}$$

Step 4 of 4

Calculate the polar moment of inertia of the shaded area about point *O*.

$$I_O = I_{O,\text{ semi-circle}} - I_{O,\text{ triangle}}$$

Substitute $0.6362\times10^6\ \text{mm}^4$ for $I_{O,\text{ semi-circle}}$ and $0.0844\times10^6\ \text{mm}^4$ for $I_{O,\text{ triangle}}$.

$$\begin{aligned} I_O &= 0.6362\times10^6\ \text{mm}^4 - 0.0844\times10^6\ \text{mm}^4 \\ &= 0.552\times10^6\ \text{mm}^4 \end{aligned}$$

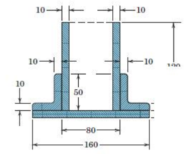
Therefore, the polar moment of inertia of the shaded area about the point *O* is

$\boxed{0.552\times10^6\ \text{mm}^4}$.

Chapter A, Problem 58P

Problem

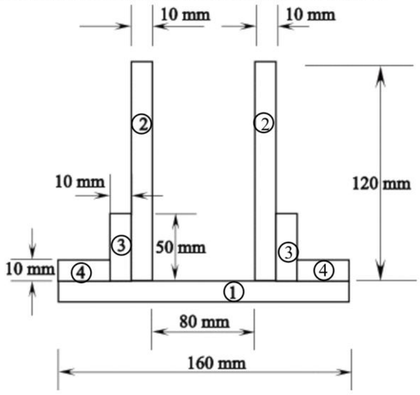
Calculate the area moment of inertia about the *x*-axis for the built-up structural section shown.



Step-by-step solution

Step 1 of 6

Divide the schematic diagram into different sections as indicated in the following figure.



Step 2 of 6

Same numbering to the sections are due to the symmetry of the figure.

Step 3 of 6

Calculate the moment of inertia of the cross sectional area of the reinforced channel about the *x*-axis by using the following relation:

$I_x = \Sigma \bar{I}_x + \Sigma A d_x^{\,2}$ ...... (1)

Here, $\bar{I}_x$ is the centroidal moment of inertia about *x*-axis, *A* is the area, d_x is the distance from the centre about *x*-axis.

Step 4 of 6

Consider b_1, b_2, b_3, and b_4 to be the breadths of the rectangles 1,2,3, and 4 respectively, and h_1, h_2, h_3, and h_4 to be the heights of the rectangles 1,2,3, and 4 respectively.

Step 5 of 6

Tabulate the following terms.

Part	Area $A\ (\text{mm}^2)$	d_x (mm)	$\bar{I}_x$ (mm^4)	$Ad_x^{\,2}$ (mm^4)
1	b_1h_1 $=160(10)$ $=1600$	$\frac{h_1}{2}$ $=\frac{10}{2}$ $=5$	$\frac{1}{12}b_1h_1^3$ $=\frac{1}{12}160(10)^3$ $=13333.33$	1600×5^2 $=40000$
2	$2(b_2)(h_2)$ $=2(120)(10)$ $=2400$	$h_1+\frac{h_2}{2}$ $=10+\frac{120}{2}$ $=10+60$ $=70$	$2\left[\frac{1}{12}(b_2)(h_2)^3\right]$ $=2\left[\frac{1}{12}(10)(120)^3\right]$ $=2.88\times10^6$	2400×70^2 $=1176000($
3	$2(b_3)(h_3)$ $=2(50)(10)$ $=1000$	$h_1+\frac{h_3}{2}$ $=10+\frac{50}{2}$ $=10+25$ $=35$	$2\left(\frac{1}{12}(b_3)(h_3)^3\right)$ $=2\left(\frac{1}{12}(10)(50)^3\right)$ $=208333.33$	1000×35^2 $=1225000$
4	$2(b_4)(h_4)$ $=2(30)(10)$ $=600$	$h_1+\frac{h_4}{2}$ $=10+\frac{10}{2}$ $=10+5$ $=15$	$2\left(\frac{1}{12}(b_4)(h_4)^3\right)$ $=2\left(\frac{1}{12}(30)(10)^3\right)$ $=5000$	600×15^2 $=135000$
	$\Sigma\bar{I}_x=3.11\times10^6\ \text{mm}^4$	$\Sigma Ad_x^{\,2}=13.16\times10^6\ \text{mm}^4$		

Comments (1)

Anonymous

so we just assume that the height of section 1 is 10mm?

Step 6 of 6

Substitute $3.11\times10^6\ \text{mm}^4$ for $\Sigma\bar{I}_x$ and $13.16\times10^6\ \text{mm}^4$ for $\Sigma Ad_x^{\,2}$ in equation (1).

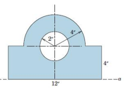
$$\begin{aligned} I_x &= \Sigma\bar{I}_x + \Sigma Ad_x^{\,2} \\ &= 3.11\times10^6 + 13.16\times10^6 \\ &= 16.27\times10^6\ \text{mm}^4 \end{aligned}$$

Therefore, the moment of inertia of the about the *x*-axis is $\boxed{16.27\times10^6\ \text{mm}^4}$.

Chapter A, Problem 59P

Problem

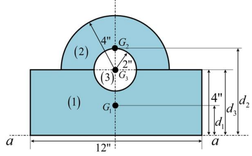
The cross section of a bearing block is shown in the figure by the shaded area. Calculate the moment of inertia of the section about its base *a-a*.



Step-by-step solution

Step 1 of 10

Divide the composite figure in to three sections as shown:



Here, G_1 is the centroid of the rectangle, G_2 is the centroid of the semicircle, and G_3 is the centroid of the circle.

Step 2 of 10

Determine the moment of inertia of the rectangle about the base *a-a*.

$I_{x1} = \overline{I}_{x1} + A_1 d_1^2$ ...... (1)

Here, $\overline{I}_{x1}$ is the moment of inertia of the rectangle, I_{x1} is the moment of inertia of the rectangle about the base *a-a*, A_1 is the area of the rectangle, and d_1 is the distance from the base *a-a* to the centroid of the rectangle.

Calculate the moment of inertia of the rectangle.

$$\overline{I}_{x1} = \frac{bh^3}{12}$$

Here, *b* is the base of the rectangle and *h* is the height of the rectangle.

Substitute $12"$ for *b* and $4"$ for *h*.

$$\begin{aligned}\overline{I}_{x1} &= \frac{12 \times 4^3}{12} \\ &= 64 \text{ in}^4\end{aligned}$$

Step 3 of 10

The distance from the base *a-a* to the centroid of the rectangle is,

$$\begin{aligned}d_1 &= \frac{4"}{2} \\ &= 2"\end{aligned}$$

The area of the rectangle is,

$$A_1 = b \times h$$

Substitute $12"$ for *b* and $4"$ for *h*.

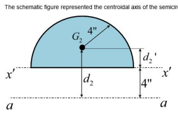
$$\begin{aligned}A_1 &= 12 \times 4 \\ &= 48 \text{ in}^2\end{aligned}$$

Substitute 64 in^4 for $\overline{I}_{x1}$, 48 in^2 for A_1, and $2"$ for d_1 in the equation (1).

$$\begin{aligned}I_{x1} &= 64 + 48(2)^2 \\ &= 64 + 192 \\ &= 256 \text{ in}^4\end{aligned}$$

Step 4 of 10

The schematic figure represented the centroidal axis of the semicircle is as follows:



Step 5 of 10

Calculate the moment of inertia of the semicircle about centroidal axis.

$\overline{I}_{x2'} = \overline{I}_{x2} - A_2 d_2'^2$ ...... (2)

Here, $\overline{I}_{x2'}$ is the moment of inertia of the semicircle about centroidal axis and d_2' is the distance from the base of the semicircle to the centroidal axis.

Calculate the moment of inertia of the semicircle.

$$\overline{I}_{x2} = \frac{\pi r^4}{8}$$

Here, *r* is the radius of the semicircle.

Substitute $4"$ for *r*.

$$\begin{aligned}\overline{I}_{x2} &= \frac{\pi \times 4^4}{8} \\ &= 100.53 \text{ in}^4\end{aligned}$$

Comments (1)

Anonymous
Why do you subtract??

Step 6 of 10

Calculate the distance from the base of the semicircle to the centroidal axis.

$$d_2' = \frac{4r}{3\pi}$$

Substitute $4"$ for *r*.

$$\begin{aligned}d_2' &= \frac{4(4)}{3\pi} \\ &= 1.69"\end{aligned}$$

The area of the semicircle is,

$$A_2 = \frac{\pi r^2}{2}$$

Substitute $4"$ for *r*.

$$\begin{aligned}A_2 &= \frac{\pi (4)^2}{2} \\ &= 25.13 \text{ in}^2\end{aligned}$$

Substitute 100.53 in^4 for $\overline{I}_{x2}$, 25.13 in^2 for A_2, and $1.69"$ for d_2' in the equation (2).

$$\begin{aligned}\overline{I}_{x2'} &= 100.53 - 25.13(1.69)^2 \\ &= 100.53 - 72.45 \\ &= 28.76 \text{ in}^4\end{aligned}$$

Step 7 of 10

Determine the moment of inertia of the semicircle about the base *a-a*.

$I_{x2} = \overline{I}_{x2'} + A_2 d_2^2$ ...... (3)

Here, I_{x2} is the moment of inertia of the semicircle about the base *a-a*, A_2 is the area of the semicircle, and d_2 is the distance from the base *a-a* to the centroid of the semicircle.

The distance from the base *a-a* to the centroid of the semicircle is,

$$d_2 = 4 + \frac{4r}{3\pi}$$

Substitute $4"$ for *r*.

$$\begin{aligned}d_2 &= 4 + \frac{4(4)}{3\pi} \\ &= 4 + 1.69 \\ &= 5.69"\end{aligned}$$

Substitute 28.76 in^4 for $\overline{I}_{x2'}$, 25.13 in^2 for A_2, and $5.69"$ for d_2 in the equation (3).

$$\begin{aligned}I_{x2} &= 28.76 + 25.13(5.69)^2 \\ &= 28.76 + 813.61 \\ &= 842.4 \text{ in}^4\end{aligned}$$

Step 8 of 10

Determine the moment of inertia of the circle about the base *a-a*.

$I_{x3} = \overline{I}_{x3} + A_3 d_3^2$ ...... (3)

Here, $\overline{I}_{x3}$ is the moment of inertia of the circle, I_{x3} is the moment of inertia of the circle about the base *a-a*, A_3 is the area of the circle, and d_3 is the distance from the base *a-a* to the centroid of the circle.

Calculate the moment of inertia of the circle.

$$\overline{I}_{x3} = \frac{\pi r^4}{4}$$

Here, *r* is the radius of the circle.

Substitute $2"$ for *r*.

$$\begin{aligned}\overline{I}_{x3} &= \frac{\pi \times 2^4}{4} \\ &= 12.57 \text{ in}^4\end{aligned}$$

Step 9 of 10

The distance from the base *a-a* to the centroid of the circle is,

$$d_3 = 4"$$

The area of the circle is,

$$A_3 = \pi r^2$$

Substitute $2"$ for *r*.

$$\begin{aligned}A_3 &= \pi (2)^2 \\ &= 12.57 \text{ in}^2\end{aligned}$$

Substitute 12.57 in^4 for $\overline{I}_{x3}$, 12.57 in^2 for A_3, and $4"$ for d_3 in the equation (3).

$$\begin{aligned}I_{x3} &= 12.57 + 12.57(4)^2 \\ &= 12.57 + 201.12 \\ &= 213.69 \text{ in}^4\end{aligned}$$

Step 10 of 10

Calculate the moment of inertia of the composite figure about the base *a-a*.

$$I_{a-a} = I_{x1} + I_{x2} - I_{x3}$$

Here, I_{a-a} is the moment of inertia of the composite figure about the base *a-a*.

Substitute 256 in^4 for I_{x1}, 842.4 in^4 for I_{x2}, and 213.69 in^4 for I_{x3}.

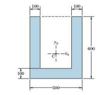
$$\begin{aligned}I_{a-a} &= 256 + 842.4 - 213.69 \\ &= 885 \text{ in}^4\end{aligned}$$

Therefore, the moment inertia of the section about the base *a-a* is $\boxed{885 \text{ in}^4}$.

Chapter A, Problem 60P

Problem

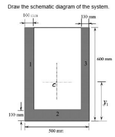
Calculate the polar radius of gyration of the shaded area about its centroid C.



Step-by-step solution

Step 1 of 12

Draw the schematic diagram of the system.



Step 2 of 12

Consider the shaded diagram as 3 rectangular strips.

Write the relation to calculate the moment of inertia of the shaded area about x - axis.

$$I_x = I_{x1} + I_{x2} + I_{x3} \quad \text{...... (1)}$$

Write the relation to calculate the moment of inertia about x - axis for the shaded area of strip 1.

$$I_{x1} = \overline{I_{x1}} + A_1\left(\frac{h_1}{2} - \bar{y}\right)^2 \quad \text{...... (2)}$$

Here, the moment of inertia about x - axis of strip 1 is I_{x1}, the centroidal moment of inertia of the rectangle section 1 is $\overline{I_{x1}}$, the height of the strip 1 is h_1, the area of the rectangle section 1 is A_1, and distance from centroid to the axis in horizontal direction is $\bar{y}$.

Step 3 of 12

Write the relation to calculate the centroidal moment of inertia about the x - axis.

Obtain the relations from the, "Properties of Plane figures."

$$\overline{I_{x1}} = \frac{b_1 h_1^3}{12}$$

Here, the width of the rectangle is b_1.

Substitute $100\,\text{mm}$ for b_1, and $500\,\text{mm}$ for h_1 from the diagram.

$$\begin{aligned}\overline{I_{x1}} &= \frac{100(500)^3}{12}\\ &= 10.41 \times 10^8\ \text{mm}^4\end{aligned}$$

Step 4 of 12

Calculate the area of the rectangle section 1.

$$A_1 = b_1 h_1$$

Substitute $100\,\text{mm}$ for b_1, and $500\,\text{mm}$ for h_1.

$$\begin{aligned}A_1 &= (100)(500)\\ &= 50000\,\text{mm}^2\end{aligned}$$

Write the relation to calculate the centroid in vertical direction when strip 1, and strip 3 are similar about their axes.

$$\begin{aligned}\bar{y} &= \frac{\sum A\bar{y}}{\sum A}\\ &= \frac{2(A_1 y_1) + A_3 y_3}{2A_1 + A_3}\end{aligned} \quad \text{...... (3)}$$

Here, the sum of the products of the area and centroid is $\sum A\bar{y}$, the distance of outer surface to axis of the strip 2 is y_2, and the area of the strip 2 is A_2.

Step 5 of 12

Calculate area of the rectangle strip 2.

$$A_2 = b_2 h_2$$

Here, the width of the rectangle is b_2, and the height of the rectangle section 2 is h_2.

Substitute $500\,\text{mm}$ for b_2, and $100\,\text{mm}$ for h_2.

$$\begin{aligned}A_3 &= (500)(100)\\ &= 50000\,\text{mm}^2\end{aligned}$$

Substitute $50000\,\text{mm}^2$ for A_1, $50000\,\text{mm}^2$ for A_3, $250\,\text{mm}$ for y_1, and $-50\,\text{mm}$ for y_3 in equation (3).

$$\begin{aligned}\bar{y} &= \frac{2(A_1 y_1) + A_3 y_3}{2A_1 + A_3}\\ &= \frac{2\left[(50000)(250)\right] + (50000)(-50)}{2 \times 50000 + 50000}\\ &= 150\,\text{mm}\end{aligned}$$

Step 6 of 12

Substitute $10.41(10^6)\,\text{mm}^4$ for $\overline{I_{x1}}$, $50000\,\text{mm}^2$ for A_1, $500\,\text{mm}$ for h_1, and $150\,\text{mm}$ for $\bar{y}$ in the equation (2).

$$\begin{aligned}I_{x1} &= \overline{I_{x1}} + A_1\left(\frac{h_1}{2} - \bar{y}_1\right)^2\\ &= 10.41 \times 10^8 + 50000\left(\frac{500}{2} - 150\right)^2\\ &= 10.41 \times 10^8\ \text{mm}^4\end{aligned}$$

Write the relation to calculate the moment of inertia of the strip 3 when strip 1 and strip 3 are similar about their axes.

$$I_{x3} = I_{x1}$$

Substitute $10.41 \times 10^8\ \text{mm}^4$ for I_{x1}.

$$I_{x3} = 10.41 \times 10^8\ \text{mm}^4$$

Step 7 of 12

Write the formula to calculate the moment of inertia of the rectangular section 2 about the x - axis.

$$I_{x2} = \frac{b_2 h_2^3}{12} + A_2\left(\frac{h_2}{2} - \bar{y}\right)^2$$

Here, the width of the rectangular section 2 is b_2, the area of the rectangular strip 2 is A_2, and height of the rectangular section 2 from the axis in vertical direction is h_2.

Substitute $500\,\text{mm}$ for b_2, $100\,\text{mm}$ for h_2, $50000\,\text{mm}^2$ for A_1, and $150\,\text{mm}$ for $\bar{y}$ from the diagram.

$$\begin{aligned}I_{x2} &= \frac{(500)(100)^3}{12} + 50000\left(\frac{(100)}{2} + (150)\right)^2\\ &= 41.667(10^6) + 20(10^8)\\ &= 20.416 \times 10^8\ \text{mm}^4\end{aligned}$$

Step 8 of 12

Substitute $20.416(10^8)\,\text{mm}^4$ for I_{x2}, $10.41 \times 10^8\ \text{mm}^4$ for I_{x1}, and $10.41 \times 10^8\ \text{mm}^4$ for I_{x3} in the equation (1).

$$\begin{aligned}I_x &= I_{x1} + I_{x2} + I_{x2}\\ &= 10.41 \times 10^8 + 20.416(10^8) + 10.41 \times 10^8\\ &= 41.23 \times 10^8\ \text{mm}^4\end{aligned}$$

Calculate the moment of inertia about y - axis for the shaded area.

$$I_y = I_{y1} + I_{y2} + I_{y3} \quad \text{...... (4)}$$

Here, the moment of inertia about y_o - axis for rectangular section 1 is I_{y1}, the moment of inertia of the rectangular section 2 is I_{y2}, and the moment of inertia of the rectangular section 3 is I_{y3}.

Step 9 of 12

Write the formula to calculate the moment of inertia of the rectangular section 1 about the y - axis.

$$I_{y1} = \frac{h_1 b_1^3}{12} + A_1\left(\frac{b_1}{2} + \bar{y}\right)^2$$

Substitute $500\,\text{mm}$ for h_1, $100\,\text{mm}$ for b_1, $50000\,\text{mm}^2$ for A_1, and $150\,\text{mm}$ for $\bar{y}$.

$$\begin{aligned}I_{y1} &= \frac{(500)(100)^3}{12} + 50000\left(\frac{(100)}{2} + (150)\right)^2\\ &= 41.667(10^6) + 20(10^8)\\ &= 20.416 \times 10^8\ \text{mm}^4\end{aligned}$$

Step 10 of 12

Calculate moment of inertia of the strip 3 about y - axis.

$$I_{y2} = I_{y1}$$

Substitute $20.416 \times 10^8\ \text{mm}^4$ for I_{y1}.

$$I_{y2} = 20.416(10^8)\,\text{mm}^4$$

Step 11 of 12

Write the relation to calculate the moment of inertia of the strip 2 about the y - axis.

Obtain the relations from the, "Properties of Plane figures."

$$I_{y2} = \frac{h_2 b_2^3}{12}$$

Substitute $500\,\text{mm}$ for b_2, and $100\,\text{mm}$ for h_2 from the diagram.

$$\begin{aligned}I_{y2} &= \frac{100(500)^3}{12}\\ &= 10.42 \times 10^8\ \text{mm}^4\end{aligned}$$

Substitute $20.416(10^8)\,\text{mm}^4$ for I_{y1}, $10.42(10^8)\,\text{mm}^4$ for I_{y2}, and $20.416(10^8)\,\text{mm}^4$ for I_{y3} in equation (4).

$$\begin{aligned}I_y &= I_{y1} + I_{y2} + I_{y3}\\ &= 20.416(10^8) + 10.42(10^8) + 20.416(10^8)\\ &= 51.236(10^8)\,\text{mm}^4\end{aligned}$$

Calculate the polar moment of inertia of the shaded area by using the relation.

$$I_z = I_x + I_y$$

Substitute $41.23 \times 10^8\ \text{mm}^4$ for I_x, and $51.236(10^8)\,\text{mm}^4$ for I_y.

$$\begin{aligned}I_z &= 41.23 \times 10^8 + 51.236 \times 10^8\\ &= 92.466 \times 10^8\ \text{mm}^4\end{aligned}$$

Write the relation to calculate the polar radius of gyration about centroid.

$$k_C = \sqrt{\frac{I_z}{A}} \quad \text{...... (5)}$$

Here, k_C is the polar radius of gyration of the shaded area about its centroid, and A is the total area of the shaded area.

Step 12 of 12

Calculate total area by using relation.

$$A = A_1 + A_2 + A_3$$

Substitute $50000\,\text{mm}^2$ for A_1, $50000\,\text{mm}^2$ for A_2, and $50000\,\text{mm}^2$ for A_3.

$$\begin{aligned}A &= 3(50000)\\ &= 150000\,\text{mm}^2\end{aligned}$$

Substitute $150000\,\text{mm}^2$ for A, and $92.466 \times 10^8\ \text{mm}^4$ for I_z in equation (5).

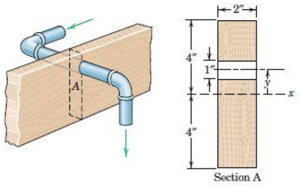
$$\begin{aligned}k_C &= \sqrt{\frac{I_z}{A}}\\ &= \sqrt{\frac{92.466 \times 10^8}{150000}}\\ &= 248.28\,\text{mm}\end{aligned}$$

Therefore, the polar radius of gyration of the shaded area about centroid is $\boxed{248.28\ \text{mm}}$.

Chapter A, Problem 50P

Problem

A floor joist which measures a full 2 in. by 8 in. has a 1-in. hole drilled through it for a water-pipe installation. Determine the percent reduction *n* in the moment of inertia of the cross-sectional area about the *x*-axis (compared with that of the undrilled joist) for hole locations in the range $0 \le y \le 3.5$ in. Evaluate your expression for $y = 2$ in.



Step-by-step solution

Step 1 of 4

Determine the percentage reduction in the moment of inertia about x- axis.

$$n = \left(\frac{I_x - I_{xh}}{I_x}\right) \times 100 \;\ldots\ldots\; (1)$$

Here, n is the percentage reduction in the moment of inertia about x- axis, I_x is the moment of inertia of the rectangle, and I_{xh} is the moment of inertia of the hole in the rectangle about the x- axis.

Obtain the relations from the, "Properties of Plane figures."

Write the relation to calculate the moment of inertia of the rectangle section about the x- axis.

$$I_x = \frac{bh^3}{12} \;\ldots\ldots\; (2)$$

Here, b is the width of the rectangular section, and h is the height of the rectangular section.

Substitute $2\,\text{in}$ for b, and $8\,\text{in}$ for h from the diagram.

$$\begin{aligned} I_x &= \frac{2\,\text{in}(8\,\text{in})^3}{12} \\ &= 85.33\,\text{in}^4 \end{aligned}$$

Step 2 of 4

Write the formula to calculate the moment of inertia of the hole in rectangular section about the x- axis.

$$I_{xh} = I_x - \left(\frac{b_h h_h^3}{12} + A_h y^2\right) \;\ldots\ldots\; (3)$$

Here, b_h is the width of the hole, h_h is the height of the hole, A_h is the area of the hole, and y is the centroid in vertical direction.

Calculate the area of the hole.

$$A_h = b_h h_h$$

Substitute $2\,\text{in}$ for b_h, and $1\,\text{in}$ for h_h from the diagram.

$$\begin{aligned} A_h &= (2\,\text{in})(1\,\text{in}) \\ &= 2\,\text{in}^2 \end{aligned}$$

Substitute $2\,\text{in}$ for b_h, $85.33\,\text{in}^4$ for I_x, $2\,\text{in}^2$ for A_h, and $1\,\text{in}$ for h_h from the diagram in the equation (3).

$$\begin{aligned} I_{xh} &= I_x - \left(\frac{b_h h_h^3}{12} + A_h y^2\right) \\ &= 85.33\,\text{in}^4 - \left(\frac{(2\,\text{in})(1\,\text{in})^3}{12} + (2\,\text{in}^2)y^2\right) \\ &= (85.163 - 2y^2)\,\text{in}^4 \end{aligned}$$

Step 3 of 4

Substitute $(85.163 - 2y^2)\,\text{in}^4$ for I_{xh}, and $85.33\,\text{in}^4$ for I_x in equation (1).

$$\begin{aligned} n &= \left(\frac{I_x - I_{xh}}{I_x}\right) \times 100 \\ &= \left(\frac{85.33\,\text{in}^4 - \left((85.163 - 2y^2)\,\text{in}^4\right)}{85.33\,\text{in}^4}\right) \times 100 \;\ldots\ldots\; (4) \\ &= 0.1957 + 2.34y^2 \end{aligned}$$

Therefore, the expression for *n* is $\boxed{0.1957 + 2.34y^2}$.

Step 4 of 4

Substitute $2\,\text{in}$ for y in equation (4).

$$\begin{aligned} n &= 0.1957 + 2.34y^2 \\ &= 0.1957 + (2.34 \times 2^2) \\ &= 9.57\% \end{aligned}$$

Hence, the percentage reduction in moment of inertia of the shaded section having hole is $\boxed{9.57\%}$.