UEMA151- ENGINEERING MATHEMATICS

UNIT 3- LAPLACE TRANSFORMS

CLASS WORK PROBLEMS

1	Laplace transforms, Applications, Advantages and sufficient conditions for Existence of Laplace transform
2-3	Laplace transform of standard functions
4-5	General properties of Laplace transforms: Linearity, change of scale, First shift theorem, Laplace transform of derivatives, Laplace transform of integrals, Multiplication by t^n and Division by t.
6-7	Problems based on the properties of Laplace transforms.
8	Laplace transform of periodic function: Statement and problems.
9	Laplace transform of Unit step function
10	Second shifting property
11	Laplace transform of unit impulse function

1. Discuss the existence of the Laplace transform for the following functions.
   a) e^{t^2} b) $\frac{1}{t^2}, t \in (-1,1)/\{0\}$
2. Find the Laplace transform of the following piecewise continuous function,
   $$\begin{cases} cost, & 0 \le t < 2\pi \\ 0 & t \ge 2\pi \end{cases}$$
3. Find L$\{t^3 + 2 + 2e^{-4t} + 5\, sin6t\}$
4. Find L$\{ sin2t\, cos2t\}$
5. Find L$\{ sin^3 2t\}$
6. Find L$\{cos(at + b)\}$
7. Find L$\{sinh^2 2t + 5\, cosh2t\}$
8. Find L$\{3\sqrt[3]{t} - \frac{3}{\sqrt{t}} - 2t^{3/2}\}$
9. If $L\{f(t)\} = \frac{s^2-s+1}{(2s+1)^2(s-1)}$, find $L\{f(2t)\}$
10. Find L$\{e^{-3t}\, sin4t\}$
11. Find L$\{coshat\, cosat\}$
12. Find L$\{t^2\, e^t\, sin4t\}$
13. Find L$\{t^3\, cos2t\}$
14. Find L$\{\frac{sinat}{t}\}$
15. Find L$\{\frac{1-cost}{t^2}\}$
16. Prove that L$\{\int_0^t \frac{sinu}{u}\, du\} = \frac{1}{s}\, tan^{-1}(\frac{1}{s})$
17. Using Laplace transform evaluate $\int_0^\infty \frac{cos6t - cos4t}{t}\, dt.$
18. Evaluate $\int_0^\infty t^3\, e^{-t}\, sint\, dt.$
19. Find L$\{sin\sqrt{t}\}. Hence\ evaluate\ L\{\frac{cos\sqrt{t}}{\sqrt{t}}\}.$
20. Find $L\{\frac{sint}{t}\}$ and hence, show that $\int_0^\infty \frac{sint}{t} = \frac{\pi}{2}.$

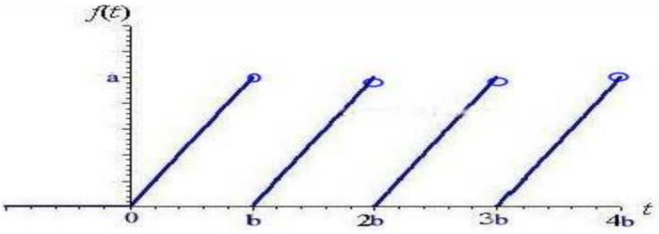
Problems on Laplace transform of Periodic function, Unit- Step and Unit- Impulse function

1\.

Determine the Laplace transform of the function

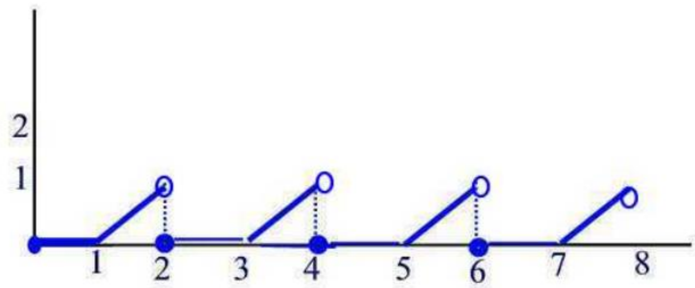
$$f(t) = \begin{cases} 1, & 0 \le t \le \frac{T}{2} \\ 0, & \frac{T}{2} < t < T \end{cases} \qquad f(t+T) = f(t),\ t \ge 0.$$

2\. Find the Laplace transform of the saw tooth wave



3\.

Find the Laplace transform of the periodic function whose graph is shown.



4\. Evaluate $L\{t^2 u(t-3)\}$

5\. Evaluate $L\{t\, u(t-1) + t^2 \delta(t-1)\}$

6\. Evaluate $L\{\, e^{-3t} \cos 5t\, \delta(t+\pi)\}$

Express the following in terms of Unit step function and hence find its Laplace transform

1\. $f(t) = \begin{cases} t-1, & when\ 1 < t < 2 \\ 3-t, & when\ 2 < t < 3 \end{cases}$

2\. $f(t) = \begin{cases} sint, 0 < t < \pi \\ sin2t, \pi < t < 2\pi \\ sin3t,\ t > 2\pi \end{cases}$

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UNIT 3 : LAPLACE TRANSFOMS

CLASSWORK PROBLEMS

1) Discuss the existence of the Laplace transform for the following functions.

a) e^{t^2} b) $1/t^2$, $t \in (-1,1)/\{0\}$.

Solution :

a) e^{t^2} is not of exponential order since

$$\lim_{t \to \infty} \frac{e^{t^2}}{e^{at}} = \lim_{t \to \infty} e^{t(t-a)} = \infty .$$

One of the conditions of the Theorem is not satisfied.

$\therefore$, The Laplace transform may or may not exist.

(since the conditions are only sufficient and not necessary for existence of Laplace transform.

b) $1/t^2$, $t \in (-1,1)/\{0\}$

$f(t) = \frac{1}{t^2}$ is not piecewise continuous function.

One of the conditions of the Theorem is not satisfied

$\therefore$ The Laplace transform may or may not exist

Moreover.

$$L\{\frac{1}{t^2}\} = \int_0^\infty e^{-st} \cdot \frac{1}{t^2} \cdot dt$$ doesn't ~~exist~~ converge.

$\therefore$ Therefore laplace of $1/t^2$ doesn't exist.

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2) Find the Laplace transform of the following piecewise continuous function.

$$f(t) = \begin{cases} \cos t & 0 \leq t < 2\pi \\ 0 & t \geqslant 2\pi \end{cases}$$

By definition of Laplace transform, we have

$$L\{f(t)\} = \int_0^{\infty} e^{-st} \cdot f(t)\, dt$$

$$= \int_0^{2\pi} e^{-st} \cdot \cos t\, dt + \int_{2\pi}^{\infty} e^{-st} \cdot 0\, dt$$

We know that

$$\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$$

$$L\{f(t)\} = \left[\frac{e^{-st}}{s^2+1}(-s\cos t + \sin t)\right]_0^{2\pi} + 0.$$

$$= \left[\frac{e^{-2\pi s}}{s^2+1}(-s\cos 2\pi + \sin 2\pi) - \frac{e^{-s(0)}}{s^2+1}(-s\cos 0 + \sin 0)\right]$$

$$= \frac{-e^{-2\pi s}}{s^2+1} + \frac{1}{s^2+1} = \frac{1-e^{-2\pi s}}{s^2+1}.$$

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3) Find $L\{t^3 + 2 + 2e^{-4t} + 5\sin 6t\}$.

$L\{t^3 + 2 + 2e^{-4t} + 5\sin 6t\}$

$= L\{t^3\} + L\{2\} + L\{2e^{-4t}\} + L\{5\sin 6t\}$.

$= \frac{6}{s^4} + \frac{2}{s} + \frac{2}{s+4} + \frac{5.(6)}{s^2+36}$.

$= \frac{6}{s^4} + \frac{2}{s} + \frac{2}{s+4} + \frac{30}{s^2+36}$

4) Find $L\{\sin 2t \cdot \cos 2t\}$.

We know that

$\sin A \cdot \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$L\{\sin 2t \cdot \cos 2t\} = \frac{1}{2} L\{\sin 4t + \sin 0t\}$.

$= \frac{1}{2} \cdot \frac{4}{s^2+16} = \frac{2}{s^2+16}$.

5) Find $L\{\sin^3 2t\}$

We know That

$\sin^3 t = \frac{1}{4}(3\sin t - \sin 3t)$.

$\therefore \sin^3 2t = \frac{1}{4}(3\sin 2t - \sin 6t)$.

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$$L\{\sin^3 2t\} = \frac{1}{4}\left[L\{3\sin 2t\} - L\{\sin 6t\}\right]$$

$$= \frac{1}{4}\left[\frac{3\cdot(2)}{s^2+4} - \frac{6}{s^2+36}\right]$$

$$= \frac{1}{4}\left[\frac{6}{s^2+4} - \frac{6}{s^2+36}\right]$$

6) Find $L\{\cos(at+b)\}$

We know That

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B.$$

$$L\{\cos(at+b)\} = L\{\cos at \cos b - \sin at \sin b\}$$

$$= L\{\cos at \cdot \cos b\} - L\{\sin at \sin b\}$$

$$= \cos b \cdot \frac{s}{s^2+a^2} - \sin b \cdot \frac{a}{s^2+a^2}$$

$$= \frac{s\cos b - a\sin b}{s^2+a^2}$$

7) Find $L\{\sinh^2 at + 5\cosh at\}$

We know That

$$\sinh at = \frac{e^{at} - e^{-at}}{2}.$$

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$$\sinh 2t = \frac{e^{2t} - e^{-2t}}{2}$$

$$\sinh^2 2t = \frac{1}{4}\left[e^{2t} - e^{-2t}\right]^2$$

$$= \frac{1}{4}\left[e^{4t} - 2e^{2t}e^{-2t} + e^{-4t}\right]$$

$$= \frac{1}{4}\left[e^{4t} - 2 + e^{-4t}\right]$$

$$L\{\sinh^2 2t + 5\cosh 2t\}$$

$$= L\left\{\frac{1}{4}(e^{4t} - 2 + e^{-4t})\right\} + 5L\{\cosh 2t\}$$

$$= \frac{1}{4}\left\{\frac{1}{s-4} - \frac{2}{s} + \frac{1}{s+4}\right\} + \frac{5 \cdot s}{s^2-4}$$

$$= \frac{1}{4}\left\{\frac{s(s+4) - 2(s^2-16) + s(s-4)}{s(s^2-16)}\right\} + \frac{5s}{s^2-4}$$

$$= \frac{1}{4}\left\{\frac{\cancel{s^2} + \cancel{4s} - \cancel{2s^2} + 32 + \cancel{s^2} - \cancel{4s}}{s(s^2-16)}\right\} + \frac{5s}{s^2-4}$$

$$= \frac{8}{s(s^2-16)} + \frac{5s}{s^2-4}.$$

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8) Find

$$L\left\{3\sqrt[3]{t} - \frac{3}{\sqrt{t}} - 2t^{3/2}\right\}.$$

$$L\left\{3\sqrt[3]{t} - \frac{3}{\sqrt{t}} - 2t^{3/2}\right\}$$

$$L\left\{3\sqrt[3]{t}\right\} - 3L\left\{\frac{1}{\sqrt{t}}\right\} - 2L\left\{t^{3/2}\right\}.$$

$$3 \cdot \frac{\Gamma 4/3}{s^{4/3}} - 3 \cdot \frac{\Gamma 1/2}{s^{1/2}} - 2 \cdot \frac{\Gamma 5/2}{s^{5/2}}$$

$$= \frac{\cancel{3} \cdot \frac{1}{\cancel{3}} \cdot \Gamma 1/3}{s^{4/3}} - 3\sqrt{\frac{\pi}{s}} - \frac{\cancel{2} \cdot \frac{3}{\cancel{2}} \cdot \frac{1}{2} \cdot \Gamma 1/2}{s^{5/2}}$$

$$= \frac{\Gamma 1/3}{s^{4/3}} - 3\sqrt{\frac{\pi}{s}} - \frac{3\sqrt{\pi}}{2s^{5/2}}$$

9) If $L\{f(t)\} = \dfrac{s^2 - s + 1}{(2s+1)^2(s-1)}$, find $L\{f(2t)\}$.

We know that

If $L\{f(t)\} = F(s)$

then $L\{f(at)\} = \frac{1}{a} F\left(\frac{s}{a}\right)$.

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$$L\{f(t)\} = \frac{s^2-s+1}{(2s+1)^2(s-1)} = F(s).$$

$$L\{f(2t)\} = \frac{1}{2} F\left(\frac{s}{2}\right)$$

$$= \frac{1}{2}\left\{\frac{\left(\frac{s}{2}\right)^2-\left(\frac{s}{2}\right)+1}{\left[\cancel{2}\left(\frac{s}{\cancel{2}}\right)+1\right]^2\left[\frac{s}{2}-1\right]}\right\}.$$

$$= \frac{1}{\cancel{2}}\left\{\frac{\frac{s^2}{4}-\frac{s}{2}+1}{(s^2+2s+1)(s-2)} \times \cancel{2}\right\}.$$

$$= \frac{(s^2-2s+4)}{4(s^2+2s+1)(s-2)}.$$

10). Find $L\{e^{-3t}\sin 4t\}$.

$$L\{\sin 4t\} = \frac{4}{s^2+16}$$

$$L\{e^{-3t}\cdot\sin 4t\} = \left\{\frac{4}{s^2+16}\right\}_{s\to s+3}$$

$$= \frac{4}{(s+3)^2+16} = \frac{4}{s^2+6s+9+16}$$

$$= \frac{4}{s^2+6s+25}.$$

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11) Find $L\{\cosh at \cdot \cos at\}$.

We know that $\cosh at = \dfrac{e^{at} + e^{-at}}{2}$

$$\cosh at \cdot \cos at = \left(\frac{e^{at} + e^{-at}}{2}\right) \cos at$$

$$= \frac{1}{2}\{ e^{at} \cos at + e^{-at} \cos at\}.$$

$$L\{\cosh at \cdot \cos at\} = \frac{1}{2}\left[L\{e^{at} \cos at\} + L\{e^{-at} \cos at\}\right]$$

$$= \frac{1}{2}\left[\left\{\frac{s}{s^2+a^2}\right\}_{s \to s-a} + \left(\frac{s}{s^2+a^2}\right)_{s \to s+a}\right]$$

$$= \frac{1}{2}\left[\frac{s-a}{(s-a)^2+a^2} + \frac{s+a}{(s+a)^2+a^2}\right]$$

$$= \frac{1}{2}\left[\frac{s-a}{s^2-2as+a^2+a^2} + \frac{s+a}{s^2+2as+a^2+a^2}\right\}$$

$$= \frac{1}{2}\left[\frac{s-a}{s^2+2a^2-2as} + \frac{s+a}{s^2+2a^2+2as}\right\}$$

$$= \frac{1}{2}\left[\frac{s-a(s^2+2a^2+2as) + (s+a)(s^2+2a^2-2as)}{(s^2+2a^2)^2 - 4a^2s^2}\right]$$

$$= \frac{1}{2}\left[\frac{2s^3}{s^4+4a^4}\right] = \frac{s^3}{s^4+4a^4}$$

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12) Find $L\{t^2 e^t \sin 4t\}$.

$$L\{\sin 4t\} = \frac{4}{s^2+16}.$$

$$L\{t^2 \cdot \sin 4t\} = (-1)^2 \cdot \frac{d^2}{ds^2}\left\{\frac{4}{s^2+16}\right\}.$$

$$= 4\frac{d}{ds}\left\{\frac{-1}{(s^2+16)^2} \cdot 2s\right\}$$

$$= -8\,\frac{d}{ds}\left\{\frac{s}{(s^2+16)^2}\right\}.$$

$$= -8 \cdot \left\{\frac{(s^2+16)^2(1) - s \cdot 2(s^2+16)(2s)}{(s^2+16)^4}\right\}.$$

$$= -8\left\{\frac{(s^2+16)\left[s^2+16-4s^2\right]}{(s^2+16)^4}\right\}$$

$$= -8\left[\frac{16-3s^2}{(s^2+16)^3}\right] = \frac{8(3s^2-16)}{(s^2+16)^3}.$$

$$L \cdot \{t^2 \cdot e^t \sin 4t\} = \left\{\frac{8(3s^2-16)}{(s^2+16)^3}\right\}_{s \to s-1}$$

$$= 8\left\{\frac{3(s-1)^2-16}{\left((s-1)^2+16\right)^3}\right\}.$$

$$= \frac{8(3s^2-6s-13)}{(s^2-2s+17)^3}.$$

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13) Find $L\{t^3 \cos 2t\}$

$$L\{\cos 2t\} = \frac{s}{s^2+4}.$$

$$L\{t^3 \cos 2t\} = (-1)^3 \cdot \frac{d^3}{ds^3}\left\{\frac{s}{s^2+4}\right\}.$$

$s^2 - 2s$

$$= -1 \cdot \frac{d^2}{ds^2}\left\{\frac{(s^2+4)(1) - s(2s)}{(s^2+4)^2}\right\}.$$

$$= (-1)\frac{d^2}{ds^2}\left\{\frac{-s^2+4}{(s^2+4)^2}\right\} = \frac{d^2}{ds^2}\left\{\frac{s^2-4}{(s^2+4)^2}\right\}$$

$$= \frac{d}{ds}\left\{\frac{(s^2+4)^2(2s) - (s^2-4)\,2(s^2+4)\,2s}{(s^2+4)^4}\right\}.$$

$$= \frac{d}{ds}\left\{\frac{(s^2+4)\{2s^3+8s-4s^3+16s\}}{(s^2+4)^4}\right\}.$$

$$= \frac{d}{ds}\left\{\frac{-2s^3+8s+16s}{(s^2+4)^3}\right\}$$

$$= \frac{(s^2+4)^3\{-6s^2+24\} - (-2s^3+24s)\,3(s^2+4)^2(2s)}{(s^2+4)^6}$$

$$= \frac{(s^2+4)^2 \cdot \left[(s^2+4)(-6s^2+24) - 6s(-2s^3+24s) \right]}{(s^2+4)^6}$$

$$= \frac{-6s^4 - \cancel{24s^2} + \cancel{24s^2} + 96 + 12s^4 - 144s^2}{(s^2+4)^4}$$

$$= \frac{6s^4 - 144s^2 + 96}{(s^2+4)^4}$$

14) Find $L \cdot \left\{ \frac{\sin at}{t} \right\}$.

$$L\{\sin at\} = \frac{a}{s^2+a^2}$$

$$L\left\{ \frac{\sin at}{t} \right\} = \int_s^\infty \frac{a}{s^2+a^2} \cdot ds$$

$$= \cancel{a} \cdot \frac{1}{\cancel{a}} \cdot \tan^{-1} \frac{s}{a} \Big]_s^\infty$$

$$= \tan^{-1}\infty - \tan^{-1}\frac{s}{a}$$

$$= \pi/2 - \tan^{-1}\frac{s}{a}$$

$$= \cot^{-1}\left(\frac{s}{a}\right).$$

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15) Find $L\left\{\frac{1-\cos t}{t^2}\right\}$.

$$L\{1-\cos t\} = L\{1\} - L\{\cos t\}$$

$$= \frac{1}{s} - \frac{s}{s^2+1}$$

$$L\left\{\frac{1-\cos t}{t}\right\} = \int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+1}\right) ds.$$

$$= \log s - \frac{1}{2}\log(s^2+1)\Big]_s^\infty.$$

$$= \left[\log \frac{s}{\sqrt{s^2+1}}\right]_s^\infty.$$

$$= \log \frac{1}{\sqrt{1+1/s^2}}\Big]_s^\infty.$$

$$= 0 - \log\sqrt{\frac{s^2}{s^2+1}}$$

$$= \log\sqrt{\frac{s^2+1}{s^2}}.$$

$$L\left\{\frac{1-\cos t}{t^2}\right\} = \int_s^\infty \log\sqrt{\frac{s^2+1}{s^2}}\,.\,ds.$$

$$= \int_s^\infty \left(\log\sqrt{s^2+1} - \log s\right) ds.$$

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$$= \frac{1}{2}\left[\int_s^{\infty}\left(\log(s^2+1) - 2\log s.\right) ds\right]$$

$$\int u . dv = uv - \int v . du.$$

$$u = \log(s^2+1) - \log s^2 \qquad dv = ds$$

$$du = \left(\frac{2s}{s^2+1} - \frac{2\cancel{s}}{s^2}\right) ds. \qquad v = s$$

$$= \frac{1}{2}\left[\left[s\left(\log(s^2+1) - \log s^2\right)\right]_s^{\infty} - \int_s^{\infty} s\left(\frac{2s}{s^2+1} - \frac{2\cancel{s}}{s^{\cancel{2}}}\right) ds\right.$$

$$= \frac{1}{2}\left[\left[s . \log\left(\frac{s^2+1}{s^2}\right)\right]_s^{\infty} - \int_s^{\infty} \frac{\cancel{s}\left(2s^2 - 2(s^2+1)\right)}{\cancel{s}(s^2+1)} ds\right]$$

$$= \frac{1}{2}\left[s . \log \frac{\cancel{s^2}(1+1/s^2)}{\cancel{s^2}}\right]_s^{\infty.} + \int_s^{\infty} \frac{2}{s^2+1} . ds.$$

$$= \frac{1}{2}\left[-s \log \frac{s^2+1}{s^2.} + 2 . \left[\tan^{-1}(s)\right]_s^{\infty}\right]$$

$$= -\frac{1}{2} s \log\left(\frac{s^2+1}{s^2}\right) + \pi/2 - \tan^{-1} s.$$

$$= \cot^{-1} s - \frac{1}{2} s \log\left(\frac{s^2+1}{s^2}\right)$$

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16) Prove That

$$L\left\{\int_0^t \frac{\sin u}{u}\cdot du\right\} = \frac{1}{s}\tan^{-1}\left(\frac{1}{s}\right).$$

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{ds}{s^2+1}$$

$$= \tan^{-1}(s)\Big]_s^\infty$$

$$= \pi/_2 - \tan^{-1}(s) = \cot^{-1}(s)$$

We know That

$$L\left\{\int_0^t f(t)\,dt\right\} = \frac{F(s)}{s}.$$

$$L\cdot\left\{\int_0^t \frac{\sin u}{u}\cdot du\right\} = \frac{\cot^{-1}(s)}{s}.$$

$$= \frac{1}{s}\tan^{-1}\left(\frac{1}{s}\right).$$

17) Using Laplace transform evaluate $\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\,dt$

$$L\{\cos 6t - \cos 4t\} = \frac{s}{s^2+36} - \frac{s}{s^2+16}.$$

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$$L\left\{\frac{\cos 6t-\cos 4t}{t}\right\} = \int_s^\infty \left(\frac{s}{s^2+36}-\frac{s}{s^2+16}\right)ds$$

$$= \frac{1}{2}\log(s^2+36)-\frac{1}{2}\log(s^2+16)\Big]_s^\infty$$

$$= \left[\frac{1}{2}\log\frac{s^2+36}{s^2+16}\right]_s^\infty$$

$$= \frac{1}{2}\log\frac{\cancel{s^2}\left(1+\frac{36}{s^2}\right)}{\cancel{s^2}\left(1+\frac{16}{s^2}\right)}\Bigg]_s^\infty$$

$$= \frac{1}{2}\log 1 - \frac{1}{2}\log\left(\frac{1+\frac{36}{s^2}}{1+\frac{16}{s^2}}\right)$$

$$= 0-\frac{1}{2}\log\frac{s^2+36}{s^2+16} = \frac{1}{2}\log\frac{s^2+16}{s^2+36}.$$

We know that

$$\int_0^\infty e^{-st} f(t)\,dt = L\{f(t)\}.$$

$$\int_0^\infty e^{-0t}\left(\frac{\cos 6t-\cos 4t}{t}\right)dt = L\left\{\frac{\cos 6t-\cos 4t}{t}\right\}@ s=0.$$

$$= \left\{\frac{1}{2}\log\frac{s^2+16}{s^2+36}\right\}@ s=0$$

$$= \frac{1}{2}\log\frac{16}{36} = \log\frac{4}{6}$$

$$= \log\frac{2}{3}.$$

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18) Evaluate $\int_0^\infty t^3 \cdot e^{-t} \sin t \, dt$.

$$L\cdot\{\sin t\} = \frac{1}{s^2+1}.$$

$$L\cdot\{t^3\cdot \sin t\} = (-1)^3 \cdot \frac{d^3}{ds^3}\left\{\frac{1}{s^2+1}\right\}.$$

$$= -1\,\frac{d^2}{ds^2}\left\{\frac{-1}{(s^2+1)^2}\cdot 2s\right\}.$$

$$= 2\,\frac{d^2}{ds^2}\left\{\frac{s}{(s^2+1)^2}\right\}$$

$$= 2\cdot\frac{d}{ds}\left\{\frac{(s^2+1)^2\cdot(1) - s\cdot 2(s^2+1)\cdot 2s}{(s^2+1)^4}\right\}.$$

$$= 2\cdot\frac{d}{ds}\cdot\left\{\frac{(s^2+1)\left[s^2+1-4s^2\right]}{(s^2+1)^4}\right\}.$$

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$$= \frac{d}{ds} \cdot \left\{ \frac{-6s^2+2}{(s^2+1)^3} \right\} = 2 \cdot \frac{d}{ds} \left\{ \frac{-3s^2+1}{(s^2+1)^3} \right\}$$

$$= 2 \cdot \left\{ \frac{(s^2+1)^3 \cdot \{-6s\} - (-3s^2+1)\,3(s^2+1)^2\,2s}{(s^2+1)^6} \right\}$$

$$= 2 \left\{ \frac{(s^2+1)^2 \left[-6s^3 - 6s + 18s^3 - 6s\right]}{(s^2+1)^6} \right\}$$

$$= 2 \cdot \left\{ \frac{12s^3 - 12s}{(s^2+1)^4} \right\}$$

$$= \frac{24s(s^2-1)}{(s^2+1)^4}.$$

$$\int_0^\infty e^{-st} \cdot f(t)\, dt = L\{f(t)\}$$

$$\int_0^\infty e^{-t} \cdot t^3 \sin t\, dt = L\{t^3 \sin t\} @ s=1.$$

$$= \left\{ \frac{24s(s^2-1)}{(s^2+1)^4} \right\} @ s=1$$

$$= 0.$$

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19) Find $L\{\sin\sqrt{t}\}$ Hence evaluate $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\}$.

We know that $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

$$L\{\sin\sqrt{t}\} = L\left\{\sqrt{t} - \frac{(\sqrt{t})^3}{3!} + \frac{(\sqrt{t})^5}{5!} + \cdots\right\}.$$

$$= L\left\{t^{1/2} - \frac{t^{3/2}}{3!} + \frac{t^{5/2}}{5!} - \frac{t^{7/2}}{7!} + \cdots\right\}.$$

$$= \frac{\Gamma 3/2}{s^{3/2}} - \frac{1}{3!}\frac{\Gamma 5/2}{s^{5/2}} + \frac{1}{5!}\frac{\Gamma 7/2}{s^{7/2}} - \frac{1}{7!}\frac{\Gamma 9/2}{s^{9/2}} + \cdots$$

$$= \frac{1/2 \cdot \sqrt{\pi}}{s^{3/2}} - \frac{1}{6}\cdot\frac{\frac{3}{2}\cdot\frac{1}{2}\cdot\sqrt{\pi}}{s^{5/2}} + \frac{1}{120}\frac{5/3 \cdot 3/2 \cdot 1/2\sqrt{\pi}}{s^{7/2}} - \cdots$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}}\left[1 - \left(\frac{1}{4s}\right) + \frac{1}{2!}\left(\frac{1}{4s}\right)^2 - \cdots\right]$$

$$= \frac{\sqrt{\pi}}{2s^{3/2}} \cdot e^{-1/4s}.$$

Since

$$\left\{e^{-x} = 1 - x + \frac{x^2}{2!} - \cdots\right\}!!$$

Therefore

$$L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}} \cdot e^{-1/4s}.$$

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Take $f'(t) = \dfrac{\cos\sqrt{t}}{2\sqrt{t}}$

Since $\dfrac{\cos\sqrt{t}}{2\sqrt{t}}$ is the derivative of $\sin\sqrt{t}$,

Choose $f(t) = \sin\sqrt{t}$, Then $f'(t) = \dfrac{\cos\sqrt{t}}{2\sqrt{t}}$, $f(0) = 0$

Using Theorem

$L\cdot\{f'(t)\} = s\cdot F(s) - f(0)$, we have

$$L\cdot\left\{\frac{\cos\sqrt{t}}{2\sqrt{t}}\right\} = s\cdot L\{\sin\sqrt{t}\} - 0.$$

$$\frac{1}{2} L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = s\cdot \frac{\sqrt{\pi}\cdot e^{-1/4s}}{2s^{3/2}}.$$

$$L\cdot\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \left(\frac{\pi}{s}\right)^{1/2}\cdot e^{-1/4s}.$$

26) Find $L\left\{\dfrac{\sin t}{t}\right\}$ and hence S.T $\displaystyle\int_0^\infty \frac{\sin t}{t}dt = \frac{\pi}{2}$.

$$L\{\sin t\} = \frac{1}{s^2+1}$$

$$L\left\{\frac{\sin t}{t}\right\} = \int_s^\infty \frac{ds}{s^2+1} = \tan^{-1}(s)\Big]_s^\infty$$

$$= \pi/2 - \tan^{-1}s$$

$$= \cot^{-1}(s).$$

$$\int_0^\infty e^{-st}\cdot f(t)\,dt = L\{f(t)\}$$

$$\int_0^\infty e^{-0t}\cdot\frac{\sin t}{t}\,dt = L\cdot\left\{\frac{\sin t}{t}\right\}\Big]_{@s=0} = \cot^{-1}(s)\Big]_{@s=0}$$

$$= \pi/2.$$

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