I INTRODUCTION TO STATICS

1. Explain the terms: a) Space b) Time c) Mass d) Force e) Particle f) Rigid Body

a) Space is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system. For three-dimensional problems, three independent coordinates are needed. For two-dimensional problems, only two coordinates are required.

b) Time is the measure of the succession of events and is a basic quantity in dynamics. Time is not directly involved in the analysis of statics problems.

c) Mass is a measure of the inertia of a body, which is its resistance to a change of velocity. Mass can also be thought of as the quantity of matter in a body. The mass of a body affects the gravitational attraction force between it and other bodies. This force appears in many applications in statics.

d) Force is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its *magnitude,* by the *direction* of its action, and by its *point of application*. Thus force is a vector quantity.

e) *Particle* is a body of negligible dimensions. In the mathematical sense, a particle is a body whose dimensions are considered to be near zero so that we may analyze it as a mass concentrated at a point. We often choose a particle as a differential element of a body. We may treat a body as a particle when its dimensions are irrelevant to the description of its position or the action of forces applied to it.

f) Rigid body. A body is considered rigid when the change in distance between any two of its points is negligible for the purpose at hand. For instance, the calculation of the tension in the cable which supports the boom of a mobile crane under load is essentially unaffected by the small internal deformations in the structural members of the boom. For the purpose, then, of determining the external forces which act on the boom, we may treat it as a rigid body. Statics deals primarily with the calculation of external forces which act on rigid bodies in equilibrium. Determination of the internal deformations belongs to the study of the mechanics of deformable bodies, which normally follows statics in the curriculum.

2. Explain the difference between Scalars and Vectors.

Scalar quantities are those with which only a magnitude is associated.

Examples of scalar quantities are time, volume, density, speed, energy, and mass.

Vector quantities, on the other hand, possess direction as well as magnitude, and must obey the parallelogram law of addition.

Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum. Vectors representing physical quantities can be classified as free, sliding, or fixed.

3. Explain the terms a) Free Vector b) Sliding Vector c) Fixed Vector

a) ***Free Vector*** is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector. This vector describes equally well the direction and magnitude of the displacement of every point in the body. Thus, we may represent the displacement of such a body by a free vector.

b) ***Sliding Vector*** has a unique line of action in space but not a unique point of application. For example, when an external force acts on a rigid body, the force can be applied at any point along its line of action without changing its effect on the body as a whole *(principle of transmissibility)*, and thus it is a sliding vector.

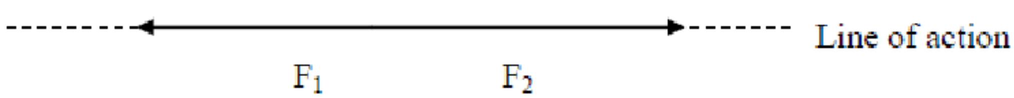
c) ***Fixed Vector*** is one for which a unique point of application is specified. The action of a force on a deformable or non rigid body must be specified by a fixed vector at the point of application of the force. In this instance the forces and deformations within the body depend on the point of application of the force, as well as on its magnitude and line of action.

II FORCES

4. Explain the terms (with sketches) a. Collinear Forces b. Coplanar Forces c. Concurrent Forces.

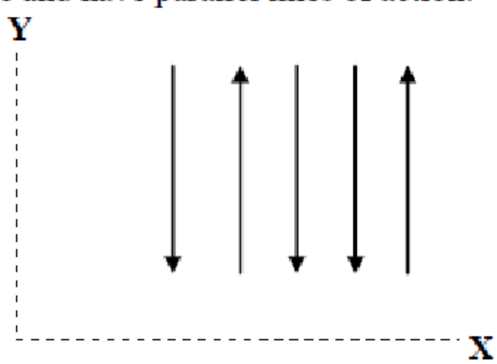
<u>Classification of force systems</u>: Depending upon their relative positions, points of applications and lines of actions, the different force systems can be classified as follows.

1) **<u>Collinear forces</u>**: It is a force system, in which all the forces have the same line of action.



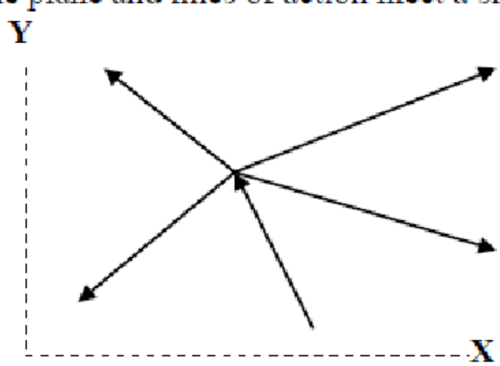
Ex.: Forces in a rope in a tug of war.

2) **<u>Coplanar parallel forces</u>**: It is a force system, in which all the forces are lying in the same plane and have parallel lines of action.



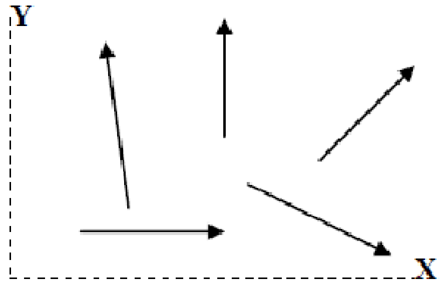
Ex.: The forces or loads and the support reactions in case of beams.

<u>3) Coplanar Concurrent forces:</u> It is a force system, in which all the forces are lying in the same plane and lines of action meet a single point.



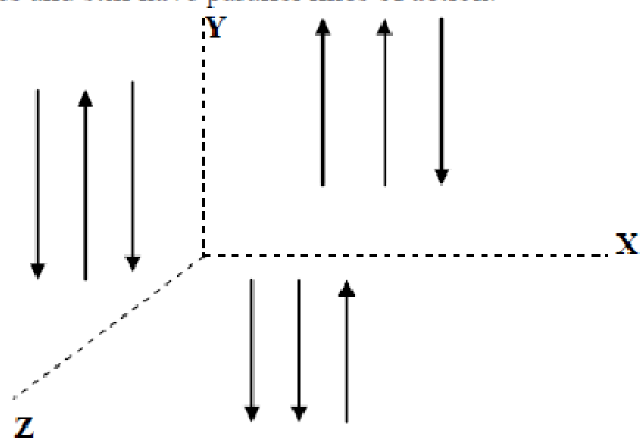
Ex.: The forces in the rope and pulley arrangement.

4) **<u>Coplanar non-concurrent forces:</u>** It is a force system, in which all the forces are lying in the same plane but lines of action do not meet a single point.



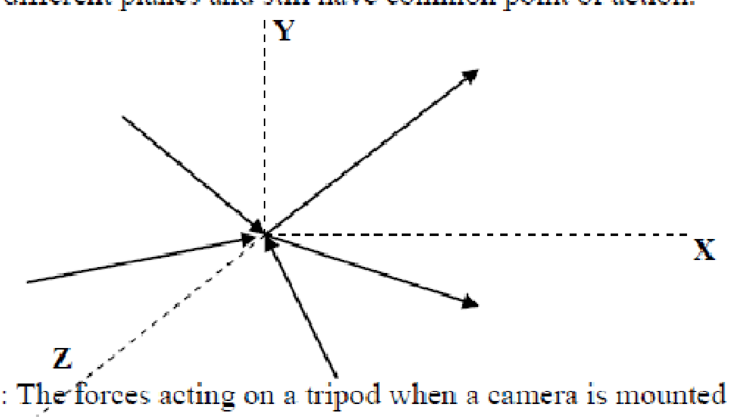
Ex.: Forces on a ladder and reactions from floor and wall, when a ladder rests on a floor and leans against a wall.

5) **<u>Non- coplanar parallel forces</u>**: It is a force system, in which all the forces are lying in the different planes and still have parallel lines of action.



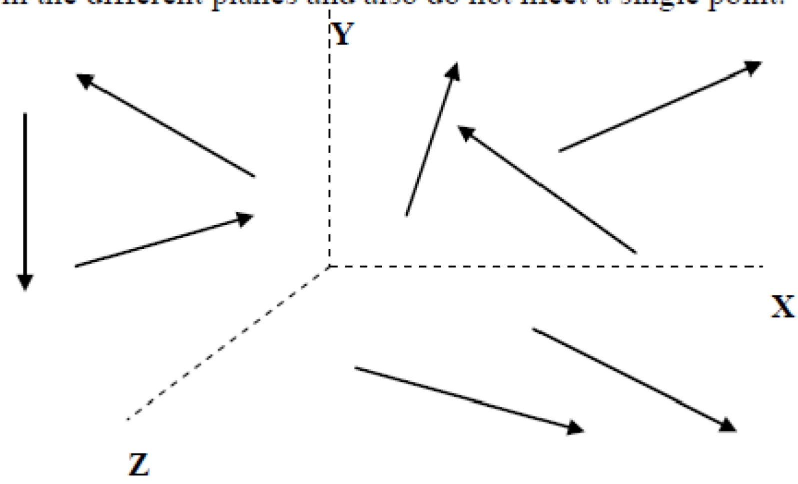
Ex: The forces acting and the reactions at the points of contact of bench with floor in a classroom.

6) <u>Non- coplanar concurrent forces</u> It is a force system, in which all the forces are lying in the different planes and still have common point of action.



Ex.: The forces acting on a tripod when a camera is mounted on a tripod.

7) **Non- coplanar non-concurrent forces**: It is a force system, in which all the forces are lying in the different planes and also do not meet a single point.



Ex.: Forces acting on a building frame.

5. What are (Explain with Sketches) a) Rectangular Components of a Force b) Components of a Force c) Projections of a Force

a) Rectangular Components of a force

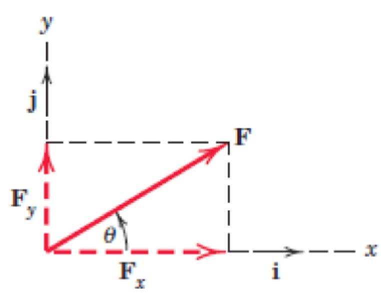


Fig.5 a)

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector **F** of Fig. may be written as

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y$$

where $\mathbf{F}_x$ and $\mathbf{F}_y$ are *vector components* of **F** in the *x*- and *y*-directions.

Each of the two vector components may be written as a scalar times the appropriate unit vector. In terms of the unit vectors **i** and **j** of Fig. 2/5, $\mathbf{F}_x = F_x\mathbf{i}$ and $\mathbf{F}_y = F_y\,\mathbf{j}$, and thus we may write

$$\mathbf{F} = F_x\mathbf{i} + F_y\,\mathbf{j}$$

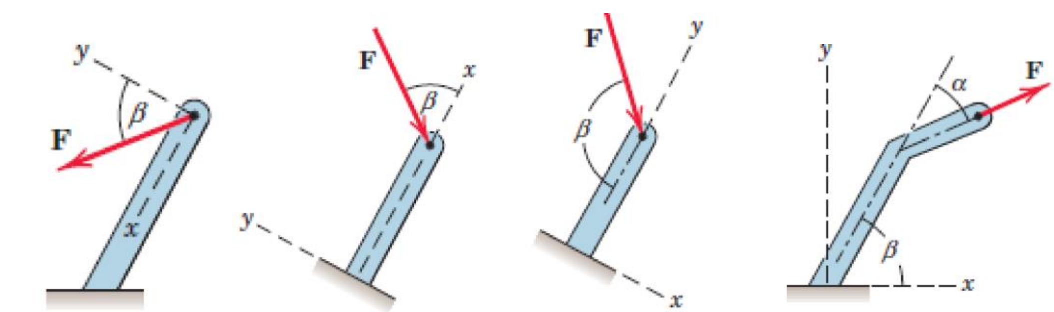
where the scalars F_x and F_y are the x and y *scalar components* of the vector **F**.

The scalar components can be positive or negative, depending on the quadrant into which **F** points. For the force vector of Fig.5 a), the x and y scalar components are both positive and are related to the magnitude and direction of **F** by

$$F_x = F\cos\theta \qquad F = \sqrt{F_x^{\,2} + F_y^{\,2}}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

b) Components of a Force



$F_x = F\sin\beta$ $\quad$ $F_y = F\cos\beta$

$F_x = -F\cos\beta$ $\quad$ $F_y = -F\sin\beta$

$F_x = F\sin(\pi-\beta)$ $\quad$ $F_y = -F\cos(\pi-\beta)$

$F_x = F\cos(\beta-\alpha)$ $\quad$ $F_y = F\sin(\beta-\alpha)$

Fig.5 b)

Dimensions are not always given in horizontal and vertical directions, angles need not be measured counterclockwise from the x-axis, and the origin of coordinates need not be on the line of action of a force. Therefore, it is essential that we be able to determine the correct components of a force no matter how the axes are oriented or how the angles are measured. Fig. 5b) suggests a few typical examples of vector resolution in two dimensions.

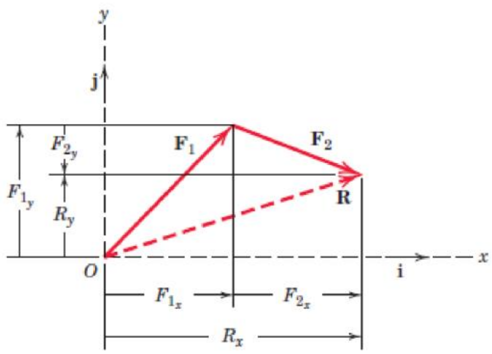


Fig.5 c)

Rectangular components are convenient for finding the sum or resultant **R** of two forces which are concurrent. Consider two forces $\mathbf{F}_1$and $\mathbf{F}_2$ which are originally concurrent at a point *O*. Fig.5 c) shows the line of action of $\mathbf{F}_2$ shifted from *O* to the tip of **F**1. In adding the force vectors **F**1 and **F**2, we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

Or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

from which we conclude that

$$R_x = F_{1_x} + F_{2_x} = \Sigma F_x$$
$$R_y = F_{1_y} + F_{2_y} = \Sigma F_y$$

The term ΣF_X means "the algebraic sum of the *x* scalar components".

c) Projections of a Force:

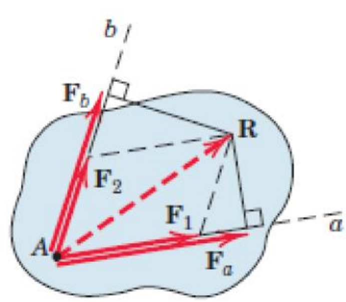


Fig.5 d)

Figure *5 d)* shows the perpendicular projections $\mathbf{F}_a$ and $\mathbf{F}_b$ of the given force **R** onto axes *a* and *b*, which are parallel to the vector components $\mathbf{F}_1$ and $\mathbf{F}_2$. Figure 5 d) shows that the components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore, the vector sum of the projections $\mathbf{F}_a$ and $\mathbf{F}_b$ is not the vector **R**, because the parallelogram law of vector addition must be used to form the sum. The components and projections of **R** are equal only when the axes *a* and *b* are perpendicular.

6. Explain the Transmissibility of a force with a neat sketch.

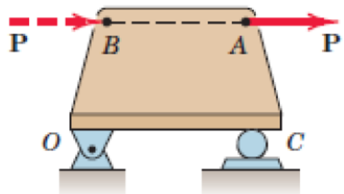


Fig.6 a)

When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force **P** acting on the rigid plate in Fig.6 a) may be applied at *A* or at *B* or at any other point on its line of action, and the net external effects of **P** on the bracket will not change. The external effects are the force exerted on the plate by the bearing support at *O* and the force exerted on the plate by the roller support at *C*.

This conclusion is summarized by the *principle of transmissibility*, which states that a force may be applied at any point on its given line of action without altering the resultant effects of the force *external* to the *rigid* body on which it acts. Thus, whenever we are interested in only the resultant external effects of a force, the force may be treated as a *sliding* vector, and we need specify only the *magnitude*, *direction*, and *line of action* of the force, and not its *point of application*.

7. Define Force and State its Characterstics.

Force is the action of one body on another. A force tends to move a body in the direction of its action. A force is a *vector quantity*, because its effect depends on the direction as well as on the magnitude of the action.

The action of a force is characterized by its *magnitude*, by the *direction* of its action, and by its *point of application*.

III Moment

8. Explain the term Moment of a force with neat sketch

The rotation effect caused by force about a point is called as moment of force about that point.

Moment of force= Force X Perpendicular distance

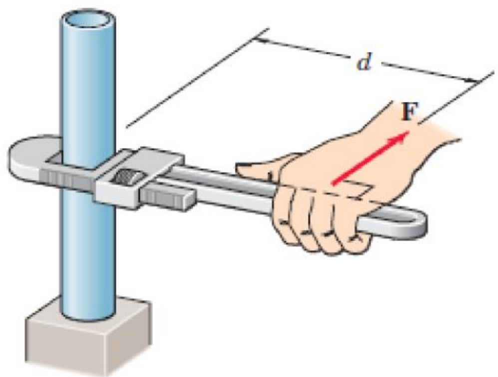


Fig.8 a)

As a familiar example of the concept of moment, consider the pipe wrench of Fig.8*a)*. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude *F* of the force and the effective length *d* of the wrench handle.

9. State and Prove the Varignon's theorem/Principle of Moments.

One of the most useful principles of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

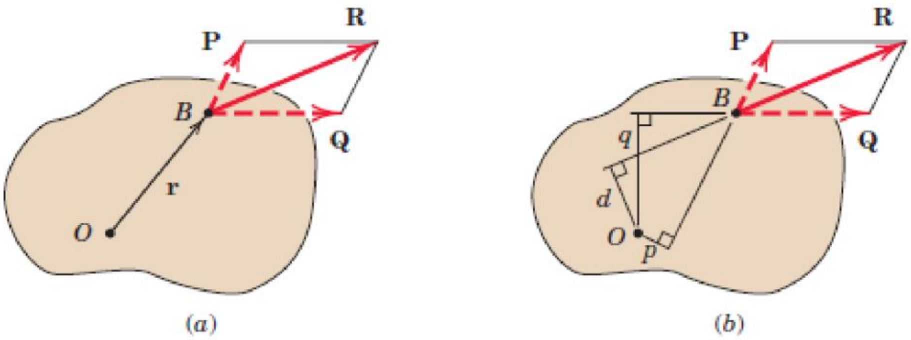


Fig.9

To prove this theorem, consider the force **R** acting in the plane of the body shown in Fig.9*a)*. The forces **P** and **Q** represent any two nonrectangular components of **R**. The moment of **R** about point *O* is

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

Because $\mathbf{R} = \mathbf{P} + \mathbf{Q}$, we may write

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

Using the distributive law for cross products, we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q}$$

which says that the moment of **R** about *O* equals the sum of the moments about *O* of its components **P** and **Q**. This proves the theorem.

Fig.9 *b)* illustrates the usefulness of Varignon's theorem. The moment of **R** about point *O* is *Rd*. However, if *d* is more difficult to determine than *p* and *q*, we can resolve **R** into the components **P** and **Q**, and compute the moment as

$$M_O = Rd = -pP + qQ$$

IV COUPLE

10. What is Couple?

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. The couple can produce clockwise rotation or anticlockwise rotation.

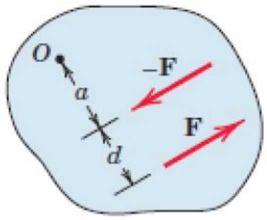


Fig.10 *a)*

Consider the action of two equal and opposite forces **F** and **-F** a distance *d* apart, as shown in Fig.10 *a)*. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as *O* in their plane is the couple **M**. This couple has a magnitude

$$M = F(a + d) - Fa$$

or

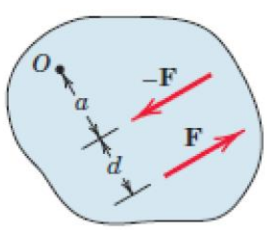
$$M = Fd$$

11) Define the term Couple and State its Characterstics.

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*.

Characterstics:

- The effect of couple on body is independent of its position
- The magnitude of both forces should be same
- They act in opposite direction separated by a small distance
- The algebraic sum of forces, constituting the couple is zero i.e F-F=0
- The algebraic sum of the moments of the forces, constituting the couple, about any point is equal to the moment of the couple itself



Taking moment of forces about the point O

$$M = F(a + d) - Fa$$

$$M = Fd \qquad \text{................(1)}$$

Moment of the Couple $M = Fd$ ....(2)

From (1) and (2) it is proved

- A couple can be balanced by an equal and opposite couple in the same plane.
- Any number of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.
- Any two couples whose moments are equal and of same sign are equivalent

12. Explain the term Force –Couple System with the help of neat sketch.

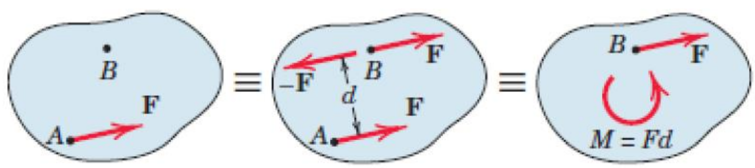


Fig.12

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

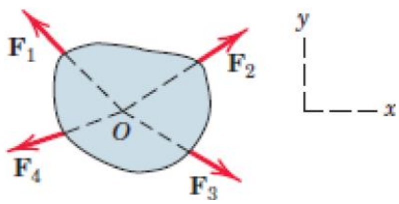
The replacement of a force by a force and a couple is illustrated in Fig.12 , where the given force **F** acting at point *A* is replaced by an equal force **F** at some point *B* and the counterclockwise couple $M = Fd$. The transfer is seen in the middle figure, where the equal and opposite forces **F** and **-F** are added at point *B* without introducing any net external effects on the body. We now see that the original force at *A* and the equal and opposite one at *B* constitute the couple $M = Fd$, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at *A* by the same force acting at a different point *B* and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig.12 is referred to as a *force–couple system*.

V EQUILIBRIUM

13. Define the term Equilibrium.

When a stationary body is subjected to external forces and if the body remains in the state of rest under the action of forces, it is said to be in equilibrium.

14. State and explain the conditions of equilibrium required for a system of coplanar, concurrent forces.

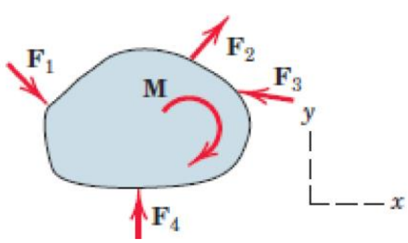


$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

Free Body Diagram   Independent Equations

Equilibrium of forces which lie in a plane (*x*-*y* plane) and are concurrent at a point *O*, requires the two force equations only, since the moment sum about *O*, that is, about a *z*-axis through *O*, is necessarily zero.

15. State and explain the conditions of equilibrium required for a system of coplanar, non-concurrent forces



$$\Sigma F_x = 0 \qquad \Sigma M_z = 0$$
$$\Sigma F_y = 0$$

Free Body Diagram   Independent Equations

Equilibrium of a general system of forces in a plane (*x*-*y*), requires the two force equations in the plane and one moment equation about an axis (*z*-axis) normal to the plane.

16. Explain the difference between statical determinancy and statical indeterminancy of a structure

Rigid bodies which are supported by the minimum number of constraints necessary to ensure an equilibrium configuration are called *statically determinate*, and for such bodies the equilibrium equations are sufficient to determine the unknown external forces.

A rigid body, or rigid combination of elements treated as a single body, which possesses more external supports or constraints than are necessary to maintain an equilibrium position is called *statically indeterminate*. Supports which can be removed without destroying the equilibrium condition of the body are said to be *redundant*. The number of redundant supporting elements present corresponds to the *degree of statical indeterminacy* and equals the total number of unknown external forces, minus the number of available independent equations of equilibrium.

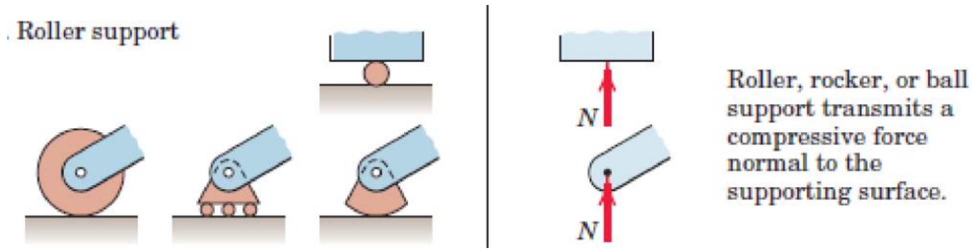
17. What is meant by Free Body Diagram and why are they important.

The diagrammatic representation of the body which is isolated or separated from the contact surface and the contact surfaces are replaced by support reaction is called Free Body Diagram.

The essential purpose of the free-body diagram is to develop a reliable picture of the physical action of all forces (and couples if any) acting on a body. So it is helpful in representing the forces in their correct physical sense whenever possible. In this way, the free-body diagram becomes a closer model to the actual physical problem than it would be if the forces were arbitrarily assigned or always assigned in the same mathematical sense as that of the assigned coordinate axis.

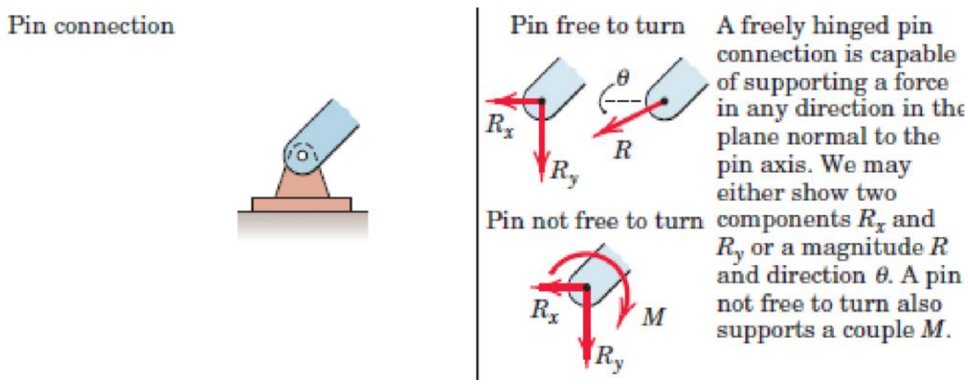
18. What do you understand by the terms 'Roller Support', 'Hinge Support', and 'Fixed Support.

Roller Support:
Roller supports are those which exert reactions perpendicular to the plane of the support. They restrict translation of the body along one direction only, and rotation is allowed.



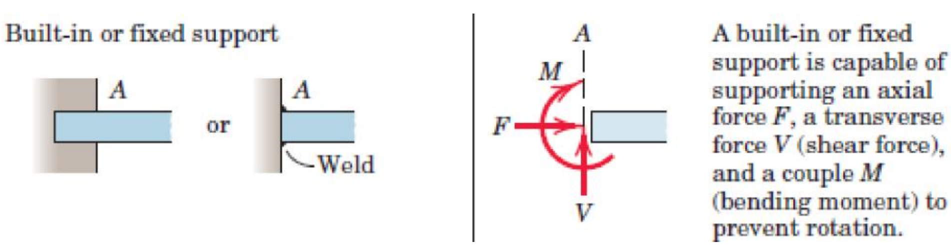
Hinged support:

Hinged supports are those which exert reactions in any direction, but from our convenient point of view we resolve these reactions into two components. Therefore, hinged supports restrict translation in both directions. But rotation is possible.



Fixed Support:

Fixed supports are those which restrict both translation and rotation of the body. Fixed supports develop an internal moment as restraint moment to prevent the rotation of the body.



VI CENTROID

19. Distinguish between Centroid, Centre of Mass and Centre of Gravity.

Centroid : It is a point through which total area of plane acts.

Centroid of Mass: It is a point at which entire mass of body acts.

Centre of Gravity: It is a point at which entire weight of body acts.

20. Determine the Centroid for an “area of a circular sector”

Solution. Choosing the axis of symmetry as the x-axis makes $\bar{y} = 0$. A differential element of arc has the length $dL = r\,d\theta$ expressed in polar coordinates, and the x-coordinate of the element is $r\cos\theta$.

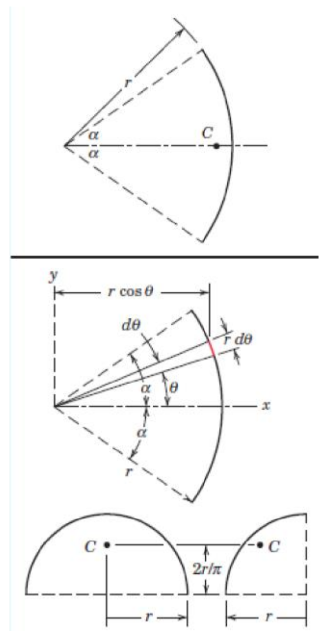
Applying the first of Eqs. 5/4 and substituting $L = 2\alpha r$ give

$$[L\bar{x} = \int x\,dL] \qquad (2\alpha r)\bar{x} = \int_{-\alpha}^{\alpha} (r\cos\theta)\,r\,d\theta$$

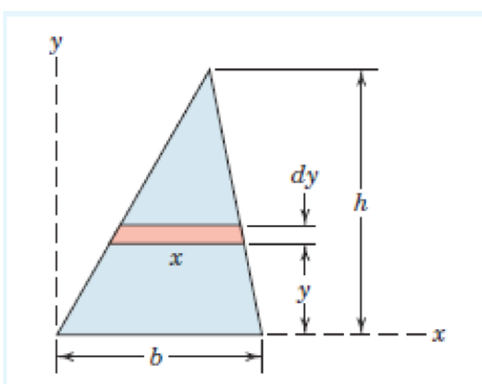
$$2\alpha r\bar{x} = 2r^2 \sin\alpha$$

$$\bar{x} = \frac{r\sin\alpha}{\alpha} \qquad \textit{Ans.}$$

For a semicircular arc $2\alpha = \pi$, which gives $\bar{x} = 2r/\pi$. By symmetry we see immediately that this result also applies to the quarter-circular arc when the measurement is made as shown.



21. Determine the Centroid distance of a triangle of base width b, and height h, from its base.



The x-axis is taken to coincide with the base. A differential strip of Area $dA = x\,dy$ is chosen.

By similar triangles $x/(h - y) = b/h$. Applying the Equation $\bar{y} = \dfrac{\int y_c\,dA}{A}$ gives

$$[A\bar{y} = \int y_c\,dA]$$

$$\frac{bh}{2}\bar{y} = \int_0^h y\frac{b(h-y)}{h}dy = \frac{bh^2}{6}$$

$$\bar{y} = \frac{h}{3}$$

VI MOMENT OF INERTIA

22. Explain with a neat sketch the moment of intertia of a plane lamina about X, Y and polar axis

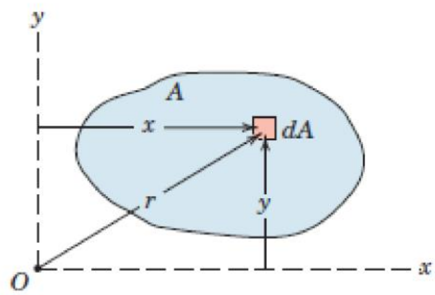


Fig.22 a)

Consider the area *A* in the *x*-*y* plane, Fig.22 a). The moments of inertia of the element *dA* about the *x*- and *y*-axes are, by definition, $dI_x = y^2\,dA$ and $dI_y = x^2\,dA$, respectively. The moments of inertia of *A* about the same axes are therefore

$$I_x = \int y^2\,dA$$

$$I_y = \int x^2\,dA$$

The moment of inertia of *dA* about the pole *O* (*z*-axis) is, by similar definition, $dI_z = r^2\,dA$. The moment of inertia of the entire area about *O* is

$$I_z = \int r^2\,dA$$

23. What is Radius of Gyration.

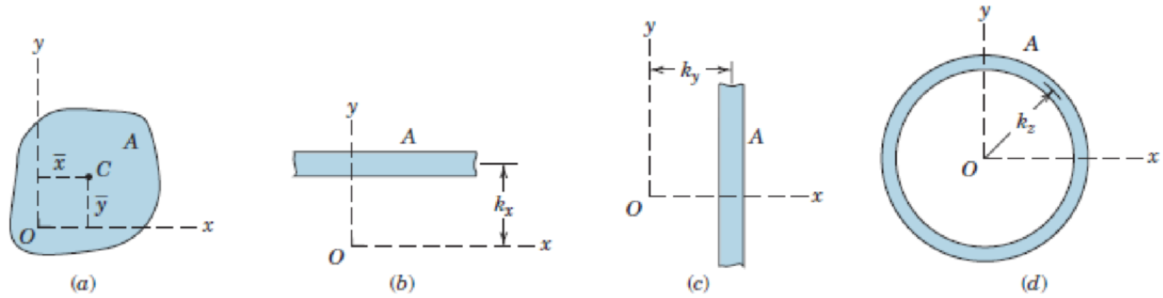


Fig.23

Consider an area *A*, Fig.23 a), which has rectangular moments of inertia I_x and I_y and a polar moment of inertia I_z about *O*. We now visualize this area as concentrated into a long narrow strip of area *A* a distance k_x from the *x*-axis, Fig.23 *b)*. By definition the moment of inertia of the strip about the *x*-axis will be the same as that of the original area if $k_x^2A = \bar{I}_x$. The distance k_x is called the *radius of gyration* of the area about the *x*-axis.

A similar relation for the *y*-axis is written by considering the area as concentrated into a narrow strip parallel to the *y*-axis as shown in Fig.23*c)*. Also, if we visualize the area as concentrated into a narrow ring of radius k_z as shown in Fig. 23 *d)*, we may express the polar moment of inertia as $k_z^2A = I_z$. In summary we write

$$\boxed{\begin{aligned} I_x &= k_x^2 A \\ I_y &= k_y^2 A \\ I_z &= k_z^2 A \end{aligned}} \quad \text{or} \quad \boxed{\begin{aligned} k_x &= \sqrt{I_x/A} \\ k_y &= \sqrt{I_y/A} \\ k_z &= \sqrt{I_z/A} \end{aligned}}$$

The radius of gyration, then, is a measure of the distribution of the area from the axis in question.

24. State and prove the parallel axis theorem.

Statement: The moment of inertia of any area about an axis in its plane is the sum of moment of inertia about a parallel axis passing through the centroid of the area (centroidal axis) and the product of area and square of the distance between the two parallel axes.

$Ix=I_{CG}+Ad^2$.

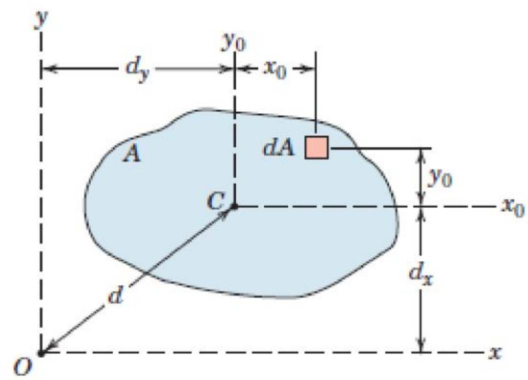


Fig.24 a)

The moment of inertia of an area about a non centroidal axis may be easily expressed in terms of the moment of inertia about a parallel centroidal axis. In Fig.24 a) the x_0-y_0 axes pass through the centroid *C* of the area. Let us now determine the moments of inertia of the area about the parallel *x*-*y* axes. By definition, the moment of inertia of the element *dA* about the *x*-axis is

$$dI_x = (y_0 + d_x)^2\, dA$$

Expanding and integrating give us

$$I_x = \int y_0^2\, dA + 2d_x \int y_0\, dA + d_x^2 \int dA$$

We see that the first integral is by definition the moment of inertia $\bar{I}_x$ about the centroidal x_0-axis. The second integral is zero, since $\int y_0\, dA = A\bar{y}_0$ and $\bar{y}_0$ is automatically zero with the centroid on the x_0-axis. The third term is simply Ad_x^2. Thus, the expression for I_x and the similar expression for I_y become

$$\begin{aligned} I_x &= \bar{I}_x + Ad_x^2 \\ I_y &= \bar{I}_y + Ad_y^2 \end{aligned} \qquad \text{Eq (1)}$$

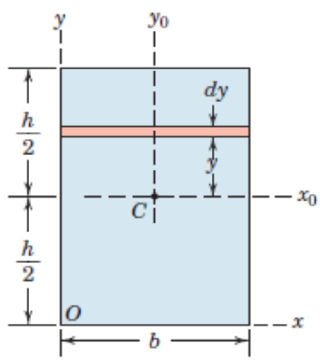
By equation $I_z = I_x + I_y$, the sum of above two equations gives

$$I_z = \bar{I}_z + Ad^2 \qquad \text{Eq (2)}$$

Equations (1) and (2) are called *parallel-axis theorems*. Two points in particular should be noted. First, the axes between which the transfer is made *must be parallel*, and second, one of the axes *must pass through the centroid of the area*.

If a transfer is desired between two parallel axes neither of which passes through the centroid, it is first necessary to transfer from one axis to the parallel centroidal axis and then to transfer from the centroidal axis to the second axis.

25. Determine the moment of inertia of a rectangular area about its centroidal x_o, y_o and polar z_o axis.



Solution. For the calculation of the moment of inertia $\bar{I}_x$ about the x_0-axis, a horizontal strip of area $b\,dy$ is chosen so that all elements of the strip have the same y-coordinate. Thus,

$$\left[I_x = \int y^2\,dA\right] \qquad \bar{I}_x = \int_{-h/2}^{h/2} y^2 b\,dy = \frac{1}{12}bh^3$$

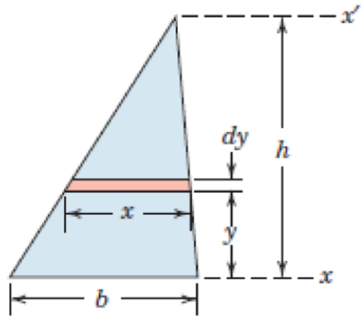
By interchange of symbols, the moment of inertia about the centroidal y_0-axis is

$$\bar{I}_y = \frac{1}{12}hb^3$$

The centroidal polar moment of inertia is

$$[\bar{I}_z = \bar{I}_x + \bar{I}_y] \qquad \bar{I}_z = \frac{1}{12}(bh^3 + hb^3) = \frac{1}{12}A(b^2 + h^2)$$

26. Determine the moment of inertia of a triangle about an axis passing through its base, centroid and its vertex.



Solution. A strip of area parallel to the base is selected as shown in the figure, and it has the area $dA = x\,dy = [(h - y)b/h]\,dy$. By definition

$$[I_x = \int y^2\,dA] \qquad I_x = \int_0^h y^2 \frac{h-y}{h} b\,dy = b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h = \frac{bh^3}{12}$$

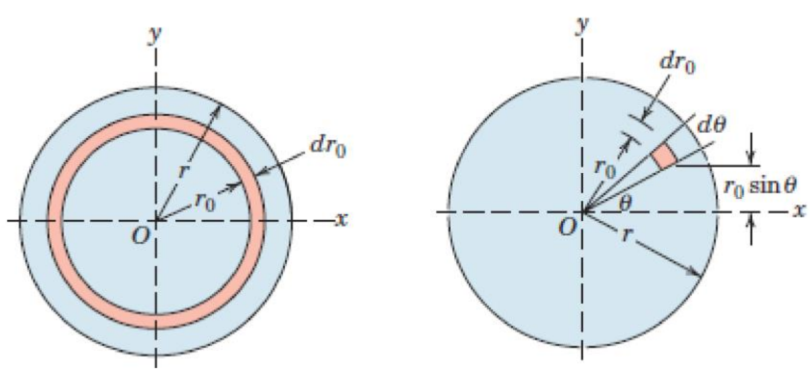
By the parallel-axis theorem the moment of inertia $\bar{I}$ about an axis through the centroid, a distance $h/3$ above the x-axis, is

$$[\bar{I} = I - Ad^2] \qquad \bar{I} = \frac{bh^3}{12} - \left(\frac{bh}{2}\right)\left(\frac{h}{3}\right)^2 = \frac{bh^3}{36}$$

A transfer from the centroidal axis to the x'-axis through the vertex gives

$$[I = \bar{I} + Ad^2] \qquad I_{x'} = \frac{bh^3}{36} + \left(\frac{bh}{2}\right)\left(\frac{2h}{3}\right)^2 = \frac{bh^3}{4}$$

27. Determine the moment of inertia of a circle about its centoidal x, y and polar z axis



Solution. A differential element of area in the form of a circular ring may be used for the calculation of the moment of inertia about the polar z-axis through O since all elements of the ring are equidistant from O. The elemental area is $dA = 2\pi r_0\, dr_0$, and thus,

$$\left[I_z = \int r^2\, dA\right] \qquad I_z = \int_0^r r_0^{\,2}(2\pi r_0\, dr_0) = \frac{\pi r^4}{2} = \frac{1}{2}Ar^2$$

The polar radius of gyration is

$$\left[k = \sqrt{\frac{I}{A}}\right] \qquad k_z = \frac{r}{\sqrt{2}}$$

By symmetry $I_x = I_y$, so that from Eq.

$$[I_z = I_x + I_y] \qquad I_x = \frac{1}{2}I_z = \frac{\pi r^4}{4} = \frac{1}{4}Ar^2$$

The radius of gyration about the diametral axis is

$$\left[k = \sqrt{\frac{I}{A}}\right] \qquad k_x = \frac{r}{2}$$

The foregoing determination of I_x is the simplest possible. The result may also be obtained by direct integration, using the element of area $dA = r_0\, dr_0\, d\theta$ shown in the lower figure. By definition

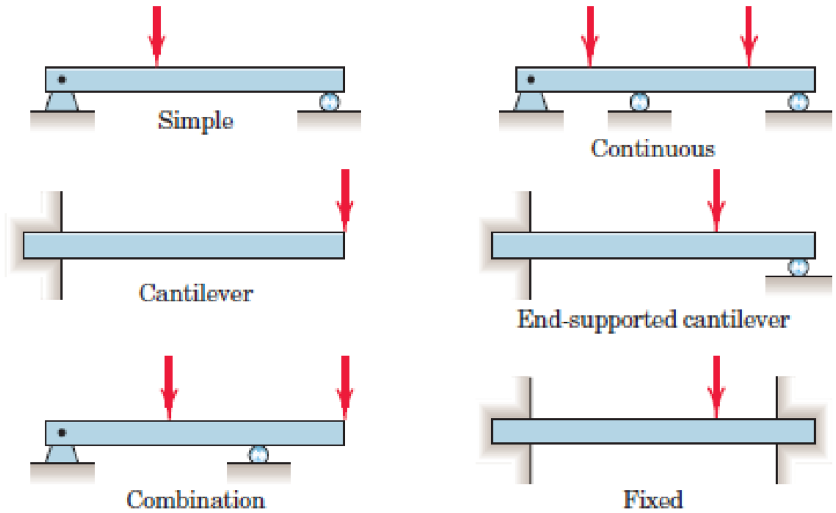
$$\left[I_x = \int y^2\, dA\right] \qquad I_x = \int_0^{2\pi}\int_0^r (r_0 \sin\theta)^2 r_0\, dr_0\, d\theta$$

$$= \int_0^{2\pi} \frac{r^4 \sin^2\theta}{4}\, d\theta$$

$$= \frac{r^4}{4}\frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} = \frac{\pi r^4}{4}$$

VII BEAMS

28. What are the different types of beams? Explain with sketches.



Simply Supported Beam:

It is a beam which consists of simple supports. Such a beam can resist forces normal to the axis of the beam

.

Continuous Beam:

It is a beam which consists of 3 or more supports.

Cantilever Beam:

It is a beam whose one end is fixed and the other end is free.

End supported Cantilever (or Propped Cantilever) Beam:

It is a beam whose one end is fixed and the other end is simply supported.

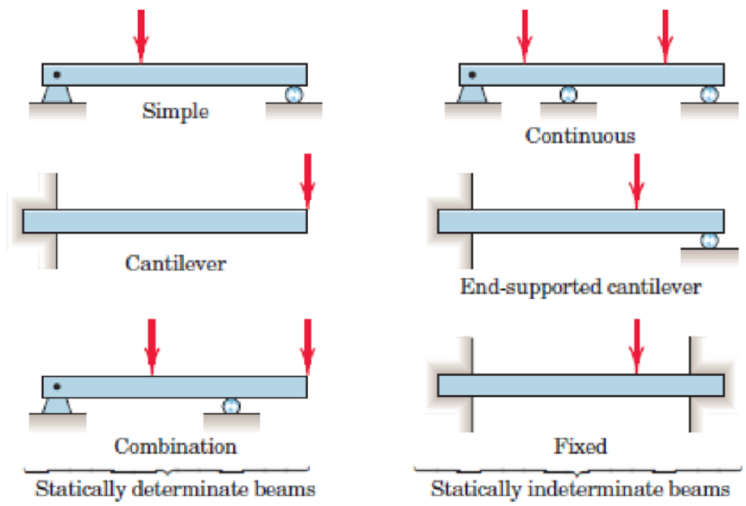
Combination or Over hanging Beam:

It is a beam which extends beyond supports

Fixed Beam:

It is a beam whose both the ends are fixed.

29. Differentiate between statically determinant and statically indeterminant beam.

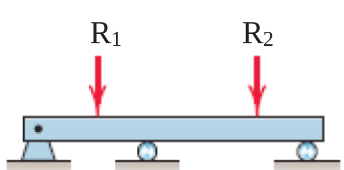


Beams supported so that their external support reactions can be calculated by the methods of statics alone are called *statically determinate beams*.

A beam which has more supports than needed to provide equilibrium is *statically indeterminate*. To determine the support reactions for such a beam we must consider its load-deformation properties in addition to the equations of static equilibrium.

30. Explain the different types of loadings on a beam.

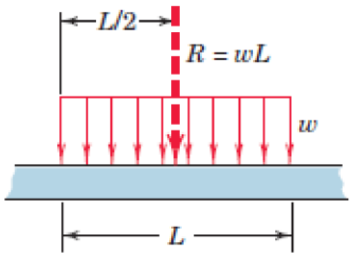
Concentrated or Point Load:



:

A load acting at a point on a beam is known as Concentrated or Point Load.

Uniformly Distributed Load (UDL):

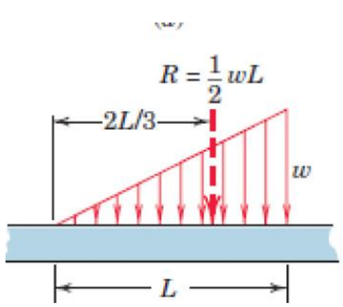


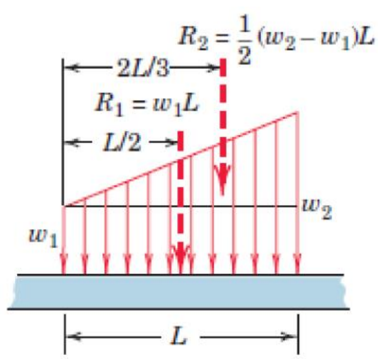
If a beam is loaded in such a way that, each unit length of the beam carries same intensity of the load, then that type of load is known as UDL.

The total load due to UDL is assumed to be acting at the centre of gravity of the UDL for all calculations.

In above Figure we see that the resultant load *R* is represented by the area formed by the intensity *w* (force per unit length of beam) and the length *L* over which the force is distributed. The resultant passes through the centroid of this area.

Uniformly Varying Load (UVL):

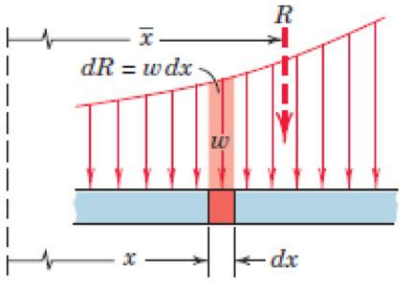




If a beam is loaded in such a way that, each unit length of the beam carries a uniformly varying intensity of loading, then this type of loading is known as Uniformly Varying Load.

The total load due to uniformly varying load is equal to the area of load diagram and is assumed to be acting at the Centre of Gravity(G) of the UVL for all calculations.

General Load Distribution:



For a more general load distribution, we must start with a differential increment of force $dR = w\,dx$. The total load *R* is then the sum of the differential forces, or

$$R = \int w\,dx$$

As before, the resultant *R* is located at the centroid of the area under consideration. The *x*-coordinate of this centroid is found by the principle of moments

$$R\bar{x} = \int xw\,dx,$$

or

$$\bar{x} = \frac{\int xw\,dx}{R}$$

VIII TRUSSES

31. Explain the terms internal redundancy and external redundancy as applied to trusses.

If a plane truss has more external supports than are necessary to ensure a stable equilibrium configuration, the truss as a whole is statically indeterminate, and the extra supports constitute *external* redundancy.

If a truss has more internal members than are necessary to prevent collapse when the truss is removed from its supports, then the extra members constitute *internal* redundancy and the truss is again statically indeterminate.

32. What do you understand by m+3=2j in case of a truss? What are implications if this equation is not satisfied?

For any plane truss, the equation m+3=2j will be satisfied if the truss is statically determinate internally. Where, m= No. of two force members and j= No of joints.

Truss becomes imperfect frame or statically indeterminate structure if it fails to satisfy above equation.

IX FRICTION

32. What are types of friction, briefly explain them.

(a) Dry Friction. Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide. A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place. The direction of this friction force always opposes the motion or impending motion. This type of friction is also called *Coulomb friction*.

(b) Fluid Friction. Fluid friction occurs when adjacent layers in a fluid (liquid or gas) are moving at different velocities. This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers. When there is no relative velocity, there is no fluid friction. Fluid friction depends not only on the velocity gradients within the fluid but also on the viscosity of the fluid, which is a measure of its resistance to shearing action between fluid layers.

(c) Internal Friction. Internal friction occurs in all solid materials which are subjected to cyclical loading. For highly elastic materials the recovery from deformation occurs with very little loss of energy due to internal friction. For materials which have low limits of elasticity and which undergo appreciable plastic deformation during loading, a considerable amount of internal friction may accompany this deformation.

33. Explain the theory of Dry (Coulomb) friction, with the help of sketches.

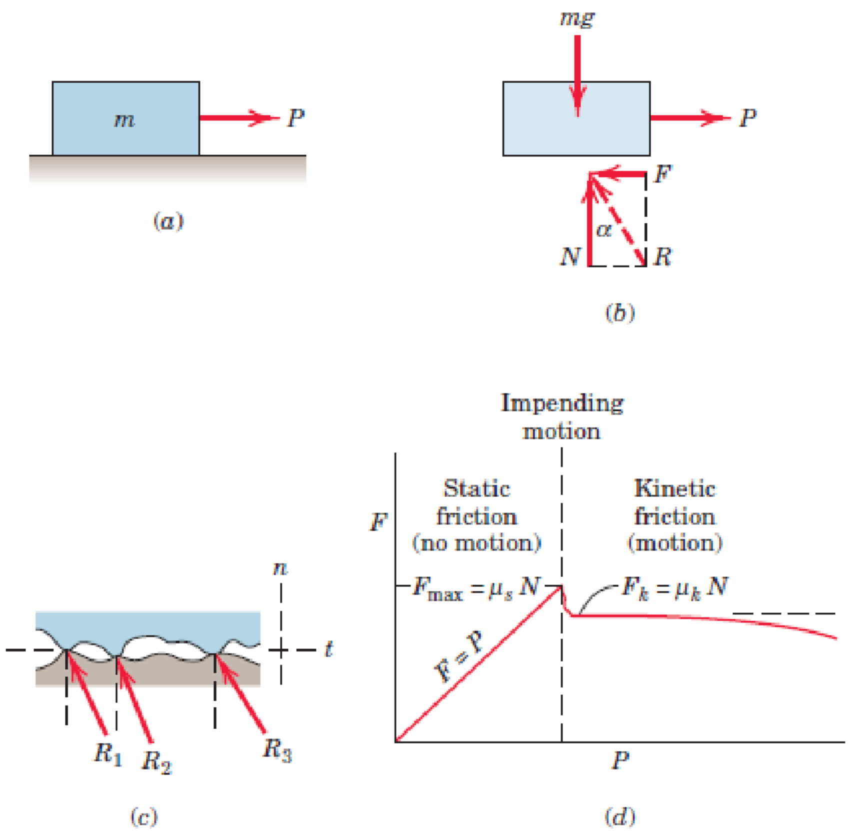


Fig.33 a)

Consider a solid block of mass *m* resting on a horizontal surface, as shown in Fig. 33 *a)*. We assume that the contacting surfaces have some roughness. The experiment involves the application of a horizontal force *P* which continuously increases from zero to a value sufficient to move the block and give it an appreciable velocity. The free-body diagram of the block for any value of *P* is shown in Fig. 33 *b)*, where the tangential friction force exerted by the plane on the block is labeled *F*. This friction force acting on the body will *always* be in a direction to oppose motion or the tendency toward motion of the body. There is also a normal force *N* which in this case equals *mg*, and the total force *R* exerted by the supporting surface on the block is the resultant of *N* and *F*.

A magnified view of the irregularities of the mating surfaces, Fig. 33 *c)*, helps us to visualize the mechanical action of friction. Support is necessarily intermittent and exists at the mating humps. The direction of each of the reactions on the block, *R*1, *R*2, *R*3, etc. depends not only on the geometric profile of the irregularities but also on the extent of local deformation at each contact point. The total normal force *N* is the sum of the *n*-components of the *R*'s, and the total frictional force F is the sum of the t-components of the R's. When the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the t-components of the R's are smaller than when the surfaces are at rest relative to one another. This observation helps to explain the well known fact that the force P necessary to maintain motion is generally less than that required to start the block when the irregularities are more nearly in mesh.

If we perform the experiment and record the friction force F as a function of P, we obtain the relation shown in Fig. 33 *d)*. When P is zero, equilibrium requires that there be no friction force. As P is increased, the friction force must be equal and opposite to P as long as the block does not slip. During this period the block is in equilibrium, and all forces acting on the block must satisfy the equilibrium equations. Finally, we reach a value of P which causes the block to slip and to move in the direction of the applied force. At this same time the friction force decreases slightly and abruptly. It then remains essentially constant for a time but then decreases still more as the velocity increases.

35. Derive an expression for Belt Friction

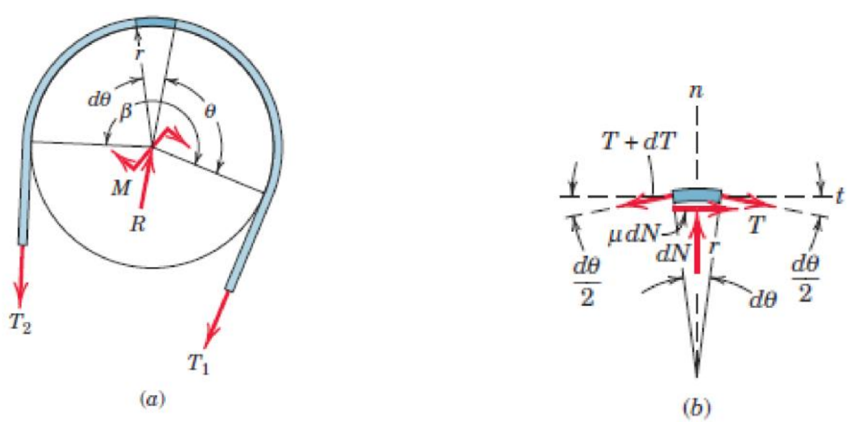


Fig. 35

The impending slippage of flexible cables, belts, and ropes over sheaves and drums is important in the design of belt drives of all types, band brakes, and hoisting rigs.

Figure 35 a) shows a drum subjected to the two belt tensions $T1$ and $T2$, the torque M necessary to prevent rotation, and a bearing reaction R. With M in the direction shown, $T2$ is greater than $T1$. The free body diagram of an element of the belt of length $r\ d\theta$ is shown in part b of the figure. We analyze the forces acting on this differential element by establishing the equilibrium of the element, in a manner similar to that used for other variable-force problems. The tension increases from T at the angle θ *to* $T + dT$ at the angle θ *to* $d\theta$. The normal force is a differential dN, since it acts on a differential element of area. Likewise the friction force, which must act on the belt in a direction to oppose slipping, is a differential and is $\mu\, dN$ for impending motion.

Equilibrium in the *t*-direction gives

$$T \cos \frac{d\theta}{2} + \mu\, dN = (T + dT) \cos \frac{d\theta}{2}$$

or $$\mu\, dN = dT$$

since the cosine of a differential quantity is unity in the limit. Equilibrium in the *n*-direction requires that

$$dN = (T + dT) \sin \frac{d\theta}{2} + T \sin \frac{d\theta}{2}$$

or $$dN = T\, d\theta$$

where we have used the facts that the sine of a differential angle in the limit equals the angle and that the product of two differentials must be neglected in the limit compared with the first-order differentials remaining.

Combining the two equilibrium relations gives

Integrating between $$\frac{dT}{T} = \mu\, d\theta$$ corresponding limits yields

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\beta} \mu\, d\theta$$

or $$\ln \frac{T_2}{T_1} = \mu\beta$$

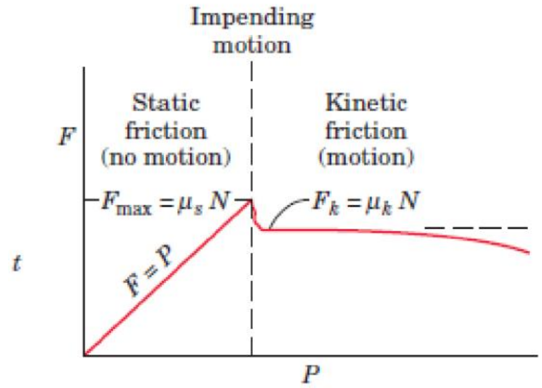
where the ln ($T2/T1$) is a natural logarithm (base e). Solving for $T2$ gives

$$T_2 = T_1 e^{\mu\beta}$$

Note that β is the total angle of belt contact and must be expressed in radians. If a rope were wrapped around a drum n times, the angle β would be $2\pi n$ radians.

36. Explain the terms

a. Coefficient of static friction **b. Coefficient of kinetic friction**



The region in above Figure up to the point of slippage or impending motion is called the range of *static friction*, and in this range the value of the friction force is determined by the *equations of equilibrium*. This friction force may have any value from zero up to and including the maximum value. For a given pair of mating surfaces the experiment shows that this maximum value of static friction F_{max} is proportional to the normal force N. Thus, we may write

$$F_{max} = \mu_s N$$

where μ_s is the proportionality constant, called the *coefficient of static friction*.

b. Coefficient of kinetic friction:

After slippage occurs, a condition of *kinetic friction* accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. The kinetic friction force F_k is also proportional to the normal force. Thus,

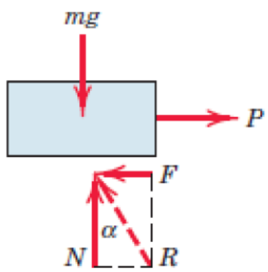
$$F_k = \mu_k N$$

where μ_k is the *coefficient of kinetic friction*.

37. Explain the terms

a. Angle of friction b. Cone of friction c. Angle of repose

a) Angle of friction



The direction of the resultant R in above figure measured from the direction of N is specified by $\tan\alpha = F/N$. When the friction force reaches its limiting static value Fmax, the angle α reaches a maximum value ϕ_s. Thus,

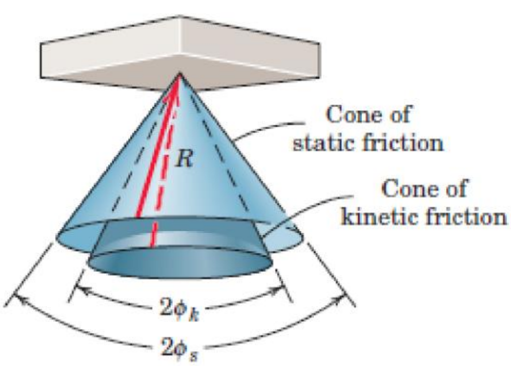
$$\tan \phi_s = \mu_s$$

When slippage is occurring, the angle α has a value ϕ_k corresponding to the kinetic friction force. In like manner,

$$\tan \phi_k = \mu_k$$

In practice we often see the expression $\tan \phi = \mu$, in which the coefficient of friction may refer to either the static or the kinetic case, depending on the particular problem. The angle ϕ_s is called the *angle of static friction*, and the angle ϕ_k is called the *angle of kinetic friction.*

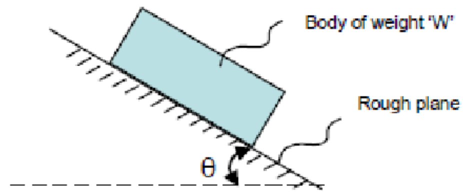
b. Cone of friction :



The friction angle for each case clearly defines the limiting direction of the total reaction R between two contacting surfaces. If motion is impending, R must be one element of a right-circular cone of vertex angle $2\phi_s$, as shown in above figure. If motion is not impending, R is within the cone.

This cone of vertex angle $2\phi_s$ is called the *cone of static friction* and represents the locus of possible directions for the reaction *R* at impending motion. If motion occurs, the angle of kinetic friction applies, and the reaction must lie on the surface of a slightly different cone of vertex angle $2\phi_k$. This cone is the *cone of kinetic friction*.

c. Angle of repose



Consider a body weighing 'w' placed on a rough inclined plane, which makes an angle 'θ' with the horizontal. When 'θ' value is small, the body is in equilibrium or rest without sliding. If 'θ' is gradually increased, a stage reaches when the body tends to slide down the plane

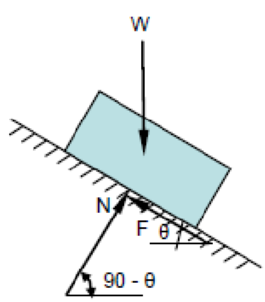
The maximum inclination of the plane with the horizontal, on which a body free from external forces can rest without sliding is called angle of repose.

Let $\theta_{max} = \Phi$

Where Φ = angle of repose

When = angle of repose.

Let us draw the free body diagram of the body before it slide.



Applying conditions of equilibrium.

$$\sum F_x = 0$$

$$N \cos(90 - \theta) - F \cos\theta = 0$$

$$N \sin\theta = F \cos\theta$$

$$\tan\theta = \frac{F}{N}$$

$$\tan\theta_{max} = \tan\Phi$$

$$\text{but } \frac{F}{N} = \mu$$

$$\mu = \tan\alpha$$

$$\tan\Phi = \tan\alpha$$

$$\Phi = \alpha$$

i.e. angle of repose is equal to angle of limiting friction