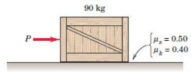
Chapter 6, Problem 1P

Problem

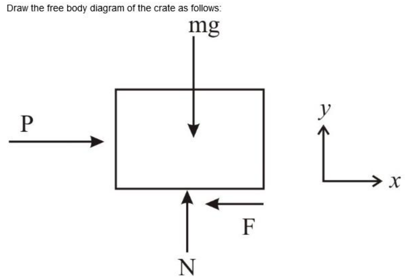
The force *P* is applied to the 90-kg crate, which is stationary before the force is applied. Determine the magnitude and direction of the friction force *F* exerted by the horizontal surface on the crate if (*a*) *P* = 300 N, (*b*) *P* = 400 N, and (*c*) *P* = 500 N.



Step-by-step solution

Step 1 of 7

Draw the free body diagram of the crate as follows:



In the above figure, the direction of the frictional force is towards left side since the frictional force always acts in the direction opposite to relative motion. Since the force P is acting towards the right side, the relative motion of the block with respect to ground will be towards the right side.

Step 2 of 7

The magnitude of the frictional force depends on the normal reaction on the block ($F = \mu N$).

So, calculate the normal reaction by considering the vertical equilibrium of the block as follows:

$$\sum F_y = 0$$
$$N - mg = 0$$
$$N = mg$$

Here, m is the mass of the block and g is the acceleration due to gravity.

Substitute $90\,\text{Kg}$ for m and $9.81\,\text{m/s}^2$ for g.

$$N = 90 \times 9.81$$
$$N = 882.9\text{ N}$$

Step 3 of 7

Calculate the maximum value static friction as follows:

$$F_{\max} = \mu_s N$$

Here, μ_s is the coefficient of static friction.

Substitute 0.5 for μ_s and 882.9 N for N.

$$F_{\max} = 0.5 \times 882.9$$
$$= 441.45\text{ N}$$

Step 4 of 7

Case (a): $P = 300\text{ N}$

Consider horizontal equilibrium.

$$\sum F_x = 0$$
$$P - F = 0$$
$$F = P$$

Here, F is the frictional force and P is the force applied.

Substitute 300 N for P.

$$F = 300\text{ N}$$

Since $F < F_{\max}$, block will not move, and the value of frictional force will be the which is obtained from the equilibrium equations.

Therefore, the magnitude of frictional force is $\boxed{300\text{ N}}$.

Step 5 of 7

Case (b): $P = 400\text{ N}$

Consider horizontal equilibrium:

$$\sum F_x = 0$$
$$P - F = 0$$
$$F = P$$

Substitute 400 N for P.

$$F = 400\text{ N}$$

Since $F < F_{\max}$, block will not move, and the value of frictional force will be the which is obtained from the equilibrium equations.

Therefore, the magnitude of frictional force is $\boxed{400\text{ N}}$.

Step 6 of 7

Case (c) $P = 500\text{ N}$

Consider horizontal equilibrium:

$$\sum F_x = 0$$
$$P - F = 0$$
$$F = P$$

Substitute 500 N for P.

$$F = 500\text{ N}$$

Step 7 of 7

Since $F > F_{\max}$, so the block will move, and equilibrium equations are invalid.

Hence the correct value of friction force can be obtained using the kinetic coefficient of friction

Calculate the value of kinetic friction as follows:

$$F = \mu_k N$$

Here, μ_k is the coefficient of kinetic friction.

Substitute 0.4 for μ_k.

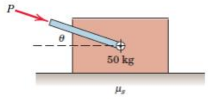
$$F = 0.4 \times 882.9$$
$$F = 353.16\text{ N}$$

Therefore, the magnitude of friction is $\boxed{353.16\text{ N}}$.

Chapter 6, Problem 2P

Problem

The 50-kg block rests on the horizontal surface, and a force $P = 200$ N, whose direction can be varied, is applied to the block. (*a*) If the block begins to slip when θ is reduced to 30°, calculate the coefficient of static friction μs between the block and the surface. (*b*) If P is applied with $\theta = 45°$, calculate the friction force F.



Step-by-step solution

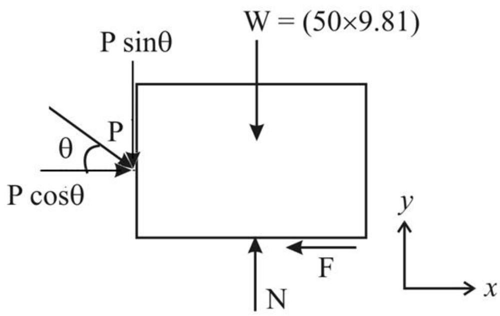
Step 1 of 6

Given that,

Mass of the block, $m = 50 \text{ kg}$

Force, $P = 200 \text{ N}$

Step 2 of 6



Comments (1)

Anonymous
excellent your solution

Step 3 of 6

(a) If $\theta = 30°$

From the free body diagram

$\sum F_x = 0$

$200\cos 30° - F = 0$

$F = 173.20 \text{ N}$

$\sum F_y = 0$

$N - W - P\sin\theta = 0$

$N - (50 \times 9.81) - 200\sin 30 = 0$

$N = 590.5 \text{ N}$

Step 4 of 6

Since, the body begins to slip

i.e., $F = F_{\max}$

$F = \mu_s N$

$173.20 = \mu_s \times 590.5$

$\boxed{\mu_s = 0.293}$

Step 5 of 6

(b) If $\theta = 45°$

$\sum F_x = 0$

$200\cos 45° - F = 0$

$F = 200\cos 45°$

$= 141.42 \text{ N}$

$\sum F_y = 0$

$N - W - P\sin 45° = 0$

$N = (50 \times 9.81) + 200\sin 45°$

$N = 631.92 \text{ N}$

Step 6 of 6

Maximum friction force, $F_{\max} = \mu_s N$

$F_{\max} = 0.293 \times 631.92$

$F_{\max} = 185.15 \text{ N}$

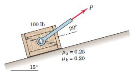
Since $F < F_{\max}$, so the block will not move

The friction force, $\boxed{F = 141.42 \text{ N}}$

Chapter 6, Problem 3P

Problem

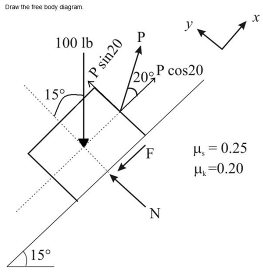
The force P is applied to the 100-lb block when it is at rest. Determine the magnitude and direction of the friction force exerted by the surface on the block if (*a*) $P = 0$, (*b*) $P = 40$ lb, and (*c*) $P = 60$ lb. (*d*) What value of P is required to initiate motion up the incline? The coefficients of static and kinetic friction between the block and the incline are $\mu s = 0.25$ and $\mu k = 0.20$, respectively.



Step-by-step solution

Step 1 of 8

Draw the free body diagram.



Step 2 of 8

Consider the equilibrium equations in horizontal direction.

$$\sum F_x = 0$$
$$P\cos 20° - F - 100\sin 15° = 0 \quad \text{...... (1)}$$
$$F = P\cos 20° - 100\sin 15°$$
$$= P\cos 20° - 25.88$$

Here, F is the frictional force and P is the applied load.

Consider the equilibrium equations in vertical direction.

$$\sum F_y = 0$$
$$N - 100\cos 15° + P\sin 20° = 0 \quad \text{...... (2)}$$
$$N = 100\cos 15° - P\sin 20°$$
$$= 96.598 - P\sin 20°$$

Here, N is the normal force.

Comments (1)

Anonymous
why isnt the normal force equal to the mass times 9.81 only the mass?

Step 3 of 8

a)

If $P = 0$ lb

Substitute 0 for P in equation (1).

$$F = P\cos 20° - 25.88$$
$$= 0\cos 20° - 25.88$$
$$= -25.88 \text{ lb}$$

Substitute 0 for P in equation (2).

$$N = 96.598 - P\sin 20°$$
$$= 96.598 - 0\sin 20°$$
$$= 96.598 \text{ lb}$$

Calculate the maximum frictional force.

$$F_{max} = \mu_s N$$

Here, μ_s is coefficient of static friction.

Substitute 0.25 for μ_s and 96.598 lb for N.

$$F_{max} = 0.25 \times 96.598$$
$$= 24.15 \text{ lb}$$

Since $|F| > F_{max}$, the block will be in motion.

Comments (1)

Anonymous
how come the F > Fmax since F = -25.88

Step 4 of 8

Calculate the frictional force.

$$F = \mu_k N$$

Here, μ_k is coefficient of kinetic friction.

Substitute 0.20 for μ_k and 96.598 lb for N.

$$F_{max} = 0.2 \times 96.598$$
$$= 19.32 \text{ lb}$$

Therefore, the frictional force is $\boxed{19.32 \text{ lb} (\text{up the incline})}$.

Step 5 of 8

b)

If $P = 40$ lb

Substitute 40 lb for P in equation (1).

$$F = P\cos 20° - 25.88$$
$$= 40\cos 20° - 25.88$$
$$= 11.71 \text{ lb}$$

Substitute 40 lb for P in equation (2).

$$N = 96.598 - P\sin 20°$$
$$= 96.598 - 40\sin 20°$$
$$= 82.92 \text{ lb}$$

Calculate the maximum frictional force.

$$F_{max} = \mu_s N$$

Here, μ_s is coefficient of static friction.

Substitute 0.25 for μ_s and 82.92 lb for N.

$$F_{max} = 0.25 \times 82.92$$
$$= 20.73 \text{ lb}$$

Since $|F| < F_{max}$, the block will be in stationary.

Therefore, the frictional force is $\boxed{11.70 \text{ lb} (\text{down the incline})}$.

Step 6 of 8

c)

If $P = 60$ lb

Substitute 60 lb for P in equation (1).

$$F = P\cos 20° - 25.88$$
$$= 60\cos 20° - 25.88$$
$$= 30.5 \text{ lb}$$

Substitute 60 lb for P in equation (2).

$$N = 96.598 - P\sin 20°$$
$$= 96.598 - 60\sin 20°$$
$$= 76.077 \text{ lb}$$

Calculate the maximum frictional force.

$$F_{max} = \mu_s N$$

Here, μ_s is coefficient of static friction.

Substitute 0.25 for μ_s and 76.077 lb for N.

$$F_{max} = 0.25 \times 76.077$$
$$= 19.02 \text{ lb}$$

Since $|F| > F_{max}$, the block will be in motion.

Step 7 of 8

Calculate the frictional force.

$$F = \mu_k N$$

Here, μ_k is coefficient of kinetic friction.

Substitute 0.20 for μ_k and 76.077 lb for N.

$$F_{max} = 0.2 \times 76.077$$
$$= 15.21 \text{ lb}$$

Therefore, the frictional force is $\boxed{15.21 \text{ lb} (\text{down the incline})}$.

Step 8 of 8

(d)

When the initiate motion up the incline.

Calculate the frictional force.

$$F = \mu_s N$$

Substitute 0.25 for μ_s.

$$F = 0.25N$$

Substitute 0.25N for P in equation (1).

$$0.25N = P\cos 20° - 25.88$$
$$P\cos 20° = 0.25N + 25.88 \quad \text{...... (3)}$$
$$P = 0.266N + 27.54$$

Substitute $96.598 - P\sin 20°$ for N in equation (3).

$$P = 0.266(96.598 - P\sin 20°) + 27.54$$
$$= 25.695 - 0.091P + 27.54$$
$$1.091P = 53.235$$
$$P = 48.8 \text{ lb}$$

Therefore, the load P is $\boxed{48.8 \text{ lb}}$.

Chapter 6, Problem 4P

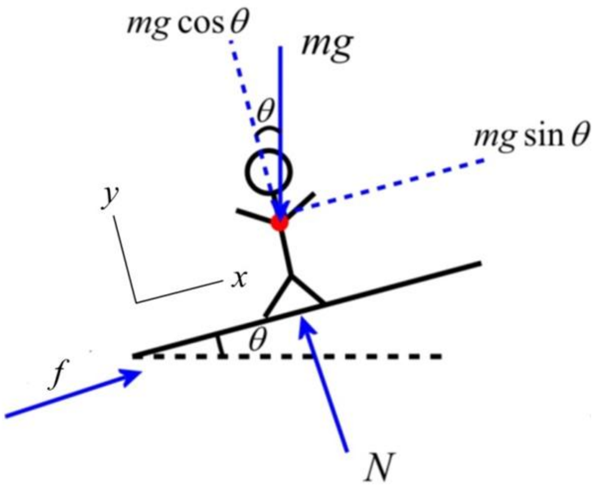
Problem

The designer of a ski resort wishes to have a portion of a beginner's slope on which a snowboarder's speed will remain fairly constant. Tests indicate the average coefficients of friction between a snowboard and snow to be μs = 0.11 and μk = 0.09. What should be the slope angle θ of the constant-speed section?

Step-by-step solution

Step 1 of 4

Draw the free body diagram of the skier as follows:



Step 2 of 4

Since, the snowboarder is in motion. Consider kinematic coefficient of friction.

Calculate the frictional force snowboard and inclined surface.

$$f = \mu_k N$$

Here, μ_k is the coefficient of kinetic friction between the snowboard and inclined surface and *N* is normal reaction acting on the snowboard.

Consider force equilibrium condition along the *x* direction.

$$\sum F_x = 0$$
$$f - mg\sin\theta = 0$$

Here, *m* is mass of the snowboarder and *g* is acceleration due to gravity.

Step 3 of 4

Substitute $\mu_k N$ for f.

$$\mu_k N - mg\sin\theta = 0$$
$$N = \frac{mg\sin\theta}{\mu_k}$$

Step 4 of 4

Consider force equilibrium condition along the *y* direction.

$$\sum F_y = 0$$
$$N - mg\cos\theta = 0$$

Substitute $\frac{mg\sin\theta}{\mu_k}$ for *N*.

$$\frac{mg\sin\theta}{\mu_k} - mg\cos\theta = 0$$
$$\tan\theta = \mu_k$$
$$\theta = \tan^{-1}(\mu_k)$$

Substitute 0.09 for μ_k.

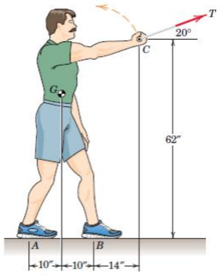
$$\theta = \tan^{-1}(0.09)$$
$$= 5.14°$$

Therefore, the snowboarder's slope angle is $\boxed{5.14°}$.

Chapter 6, Problem 5P

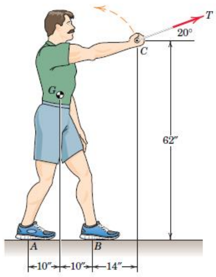
Problem

The 180-lb exerciser is repeated from Prob. 3/23. The tension T = 15 lb is developed against an exercise machine (not shown) as he is about to begin a biceps curl. Determine the minimum coefficient of static friction which must exist between his shoes and the floor if he is not to slip.



Problem. 3/23

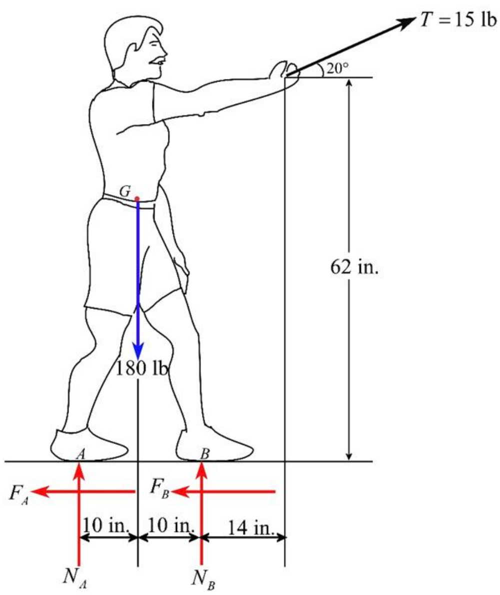
The 180-lb exerciser is beginning to execute some slow, steady bicep curls. As the tension T = 15 lb is developed against an exercise machine (not shown), determine the normal reaction forces at the feet *A* and *B*. Friction is sufficient to prevent slipping, and the exerciser maintains the position shown with center of gravity at *G*.



Step-by-step solution

Step 1 of 2

Draw the force diagram of the exerciser.



Step 2 of 2

Apply the equation of equilibrium along *x* direction.

$$\sum F_x = 0$$
$$-(F_A + F_B) + 15\cos 20° = 0 \quad \ldots\ldots (1)$$
$$-\mu_s (N_A + N_B) + 15\cos 20° = 0$$

Here, the coefficient of static friction between the exerciser shoes and the is floor μ_s, the normal reaction between the floor and his shoes *A* and *B* are N_A and N_B respectively, and the friction between his shoes *A* and *B* and the floor are F_A and F_B respectively.

Apply the equation of equilibrium along *y* direction.

$$\sum F_y = 0$$
$$N_A + N_B - 180 + 15\sin 20° = 0$$
$$N_A + N_B = (180 - 15\sin 20°)$$

Substitute $(180 - 15\sin 20°)$ for $N_A + N_B$ in equation (1).

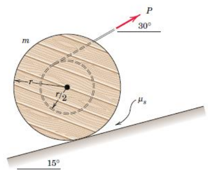
$$-\mu_s (180 - 15\sin 20°) + 15\cos 20° = 0$$
$$\mu_s = \frac{15\cos 20°}{(180 - 15\sin 20°)}$$
$$= 0.0806$$

Therefore, the coefficient of friction is $\boxed{0.0806}$.

Chapter 6, Problem 6P

Problem

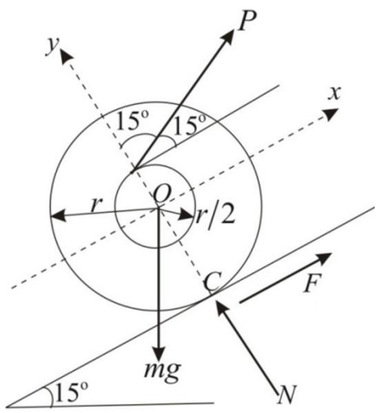
Determine the minimum coefficient of static friction μs which will allow the drum with fixed inner hub to be rolled up the 15° incline at a steady speed without slipping. What are the corresponding values of the force *P* and the friction force *F*?



Step-by-step solution

Step 1 of 5

Draw the free body diagram of the drum.



Step 2 of 5

Calculate the normal force acting on the drum using maximum frictional force exerted on the drum.

$$F = \mu_s \times N$$
$$N = \frac{F}{\mu_s}$$

Here, F is the frictional force exerted by the inclined surface on the drum, N is the normal force acting on the drum perpendicular to inclined surface, and μ_s is the coefficient of static friction between drum and inclined surface.

Apply the equations of equilibrium and calculate the forces along the *x*-direction.

$$\sum F_x = 0$$
$$P\cos 15° - mg\sin 15° + F = 0 \quad \text{...... (1)}$$

Here, P is the tensile force acting on the drum, m is the mass of the drum, and g is the acceleration due to gravity.

Apply the equations of equilibrium and calculate the forces along the *y*-direction.

$$\sum F_y = 0$$
$$P\sin 15° + N - mg\cos 15° = 0$$

Substitute $\frac{F}{\mu_s}$ for N.

$$P\sin 15° + \frac{F}{\mu_s} - mg\cos 15° = 0 \quad \text{...... (2)}$$

Step 3 of 5

Apply the equations of equilibrium and calculate the moments about the point *C*.

$$\sum M_C = 0$$
$$(mg\sin 15°)(r) - (P\cos 15°)\left(r + \frac{r}{2}\right) = 0 \quad \text{...... (3)}$$

Here, r is the radius of the drum, and $(r/2)$ is the radius of the inner hub.

Further Solve equation (3).

$$(mg\sin 15°)(r) = (P\cos 15°)\left(r + \frac{r}{2}\right)$$
$$(r)(mg\sin 15°) = (P\cos 15°)\left(\frac{3r}{2}\right)$$
$$P = \frac{2(mg\sin 15°)}{3\cos 15°}$$
$$P = 0.1786mg \quad \text{...... (4)}$$

Thus, the magnitude of force P which allow the drum to roll up at steady speed without slipping is $\boxed{0.1786mg}$.

Step 4 of 5

Calculate the frictional force by substituting equation (4) in equation (1).

$$P\cos 15° - mg\sin 15° + F = 0$$
$$(0.1786mg)\cos 15° - mg\sin 15° + F = 0$$
$$F = 0.0863mg \quad \text{...... (5)}$$

Thus, the magnitude of frictional force F which allow the drum to roll up at steady speed without slipping is $\boxed{0.0863mg}$.

Step 5 of 5

Calculate the coefficient of static friction by Substituting equation (4) and equation (5) in equation (2).

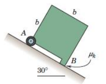
$$P\sin 15° + \frac{F}{\mu_s} - mg \times \cos 15° = 0$$
$$(0.1786mg)(\sin 15°) + \frac{(0.0863mg)}{\mu_s} - mg\cos 15° = 0$$
$$\mu_s = \frac{(0.0863)}{0.9197}$$
$$\mu_s = 0.0938$$

Thus, the minimum coefficient of static friction which allow the drum to roll up at steady speed without slipping is $\boxed{0.0938}$.

Chapter 6, Problem 8P

Problem

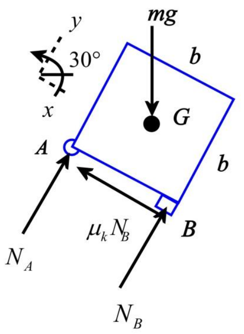
Determine the coefficient μk of kinetic friction which allows the homogeneous body to move down the 30° incline at constant speed. Show that this constant-speed motion is unlikely to occur if the ideal roller and small foot were reversed.



Step-by-step solution

Step 1 of 6

Draw the force diagram of the system.



Step 2 of 6

Equate the forces in the direction of *x* axis considering the fact that they are in equilibrium.

$$\sum F_x = 0$$
$$-\mu_k N_B + mg\sin 30° = 0 \quad \text{...... (1)}$$

Here, the coefficient of kinetic friction between the body and the floor is μ_s, the normal reaction between the floor and the foot *B* is N_B, the mass of the body is *m* and acceleration due to gravity is *g*.

and the friction between his shoes *A* and *B* and the floor are F_A and F_B respectively.

Now equate the forces of equilibrium in the direction of *y* axis.

$$\sum F_y = 0$$
$$N_A + N_B - mg\cos 30° = 0 \quad \text{...... (2)}$$

Here, the normal reaction between the floor and the foot *A* is N_A

Step 3 of 6

Equate the moments about the point *A*.

$$\sum M_A = 0$$
$$N_B(b) - mg\left(\frac{b}{2}\cos 30° + \frac{b}{2}\sin 30°\right) = 0$$
$$N_B - mg\left(\frac{1}{2}\cos 30° + \frac{1}{2}\sin 30°\right) = 0 \quad \text{...... (3)}$$
$$N_B = mg\left(\frac{1}{2}\cos 30° + \frac{1}{2}\sin 30°\right)$$

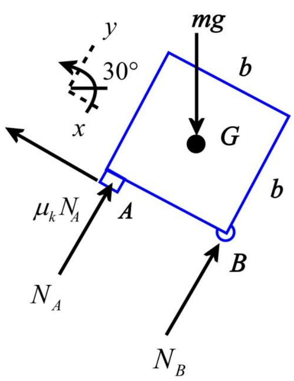
Substitute equation (3) in equation (1).

$$\sum F_x = 0$$
$$-\mu_k\left[mg\left(\frac{1}{2}\cos 30° + \frac{1}{2}\sin 30°\right)\right] + mg\sin 30° = 0$$
$$\mu_k = \frac{\sin 30°}{\left(\frac{\cos 30°}{2} + \frac{\sin 30°}{2}\right)}$$
$$= 0.732$$

Thus, the coefficient of friction is $\boxed{0.732}$

Step 4 of 6

Draw the free body diagram of the system if the roller and the foot are interchanged.



Step 5 of 6

Equate the forces in the direction of *x* axis considering the fact that they are in equilibrium.

$$\sum F_x = 0$$
$$-\mu_k N_A + mg\sin 30° = 0 \quad \text{...... (4)}$$

Equate the moments about the point *B*.

$$\sum M_B = 0$$
$$-N_A(b) + mg\left(\frac{b}{2}\cos 30° - \frac{b}{2}\sin 30°\right) = 0 \quad \text{...... (5)}$$
$$N_A = mg\left(\frac{1}{2}\cos 30° - \frac{1}{2}\sin 30°\right)$$

Step 6 of 6

Substitute equation (5) in equation (4).

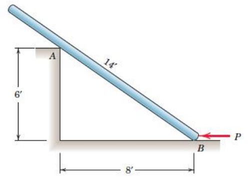
$$\sum F_x = 0$$
$$-\mu_k\left[mg\left(\frac{1}{2}\cos 30° - \frac{1}{2}\sin 30°\right)\right] + mg\sin 30° = 0$$
$$\mu_k = \frac{\sin 30°}{\left(\frac{1}{2}\cos 30° - \frac{1}{2}\sin 30°\right)}$$
$$= 2.732$$

The coefficient of friction is greater than one, which states that the frictional force is greater than the normal force and the body doesn't move. Thus, reversing the roller and foot indicates that the constant speed motion is unlikely to occur.

Chapter 6, Problem 9P

Problem

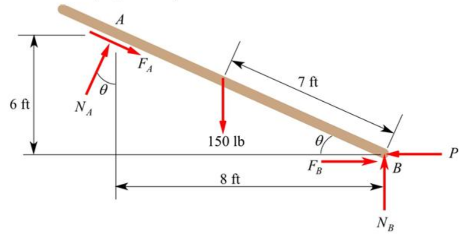
The uniform 14-ft pole weighs 150 lb and is supported as shown. Calculate the force *P* required to move the pole if the coefficient of static friction for each contact location is 0.40.



Step-by-step solution

Step 1 of 4

Draw the free body diagram of the pole.



Step 2 of 4

Calculate the inclination of the pole with the horizontal as follows:

$$\tan\theta = \frac{6}{8}$$

$$\theta = \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ$$

The weight of the pole acts vertically downwards from its centroid. This centroid is located at a distance 7 ft from either point *A* or *B* measured along the length of the pole.

Consider moment equilibrium condition about *B*.

$$\sum M_B = 0$$
$$150(7\cos\theta) - N_A\cos\theta(8) - N_A\sin\theta(6) = 0$$

Here, N_A is the normal reaction at *A*.

Substitute 36.87° for θ.

$$150(7\cos 36.87^\circ) - N_A\cos 36.87^\circ(8) - N_A\sin 36.87^\circ(6) = 0$$
$$150(5.6) - 6.4N_A - 3.6N_A = 0$$
$$10N_A = 840$$
$$N_A = 84 \text{ lb}$$

Comments (2)

Anonymous

Why do we not consider Friction at point A as a component of the moment about B. is it because it is in line with B?

Anonymous

Yes, the distance is 0

Step 3 of 4

Calculate the frictional force at *A* as follows:

$$F_A = \mu_s N_A$$

Here, μ_s is coefficient of static friction between the pole and surface.

Substitute 0.4 for μ_s and 84 lb for N_A.

$$F_A = 0.4\times 84 = 33.60 \text{ lb}$$

Consider force equilibrium condition along vertical direction.

$$\sum F_y = 0$$
$$N_B - 150 + N_A\cos\theta - F_A\sin\theta = 0$$

Here, N_B is the normal reaction at *B*.

Substitute 84 lb for N_A, 36.87° for θ, and 33.60 lb for F_A.

$$N_B - 150 + 84\cos 36.87^\circ - 33.60\sin 36.87^\circ = 0$$
$$N_B - 150 + 67.20 - 20.20 = 0$$
$$N_B = 103 \text{ lb}$$

Step 4 of 4

Calculate the frictional force at *B* as follows:

$$F_B = \mu_s N_B$$

Substitute 0.4 for μ_s and 103 lb for N_B.

$$F_B = 0.4\times 103 = 41.20 \text{ lb}$$

Consider force equilibrium condition along horizontal direction.

$$\sum F_x = 0$$
$$-P + F_B + N_A\sin\theta + F_A\cos\theta = 0$$

Substitute 41.20 lb for F_B, 84 lb for N_A, 36.87° for θ, and 33.60 lb for F_A.

$$-P + 41.20 + 84\sin 36.87^\circ + 33.60\cos 36.87^\circ = 0$$
$$P = 118.48 \text{ lb}$$

Therefore, the force, *P* required to move the pole is $\boxed{118.48 \text{ lb}}$.

Comments (1)

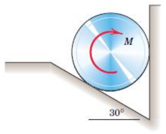
Anonymous

why is he not using weight with gravity (w= mg) ?

Chapter 6, Problem 11P

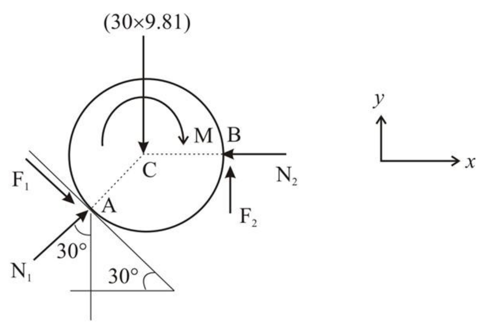
Problem

The 30-kg homogeneous cylinder of 400-mm diameter rests against the vertical and inclined surfaces as shown. If the coefficient of static friction between the cylinder and the surfaces is 0.30, calculate the applied clockwise couple *M* which would cause the cylinder to slip.



Step-by-step solution

Step 1 of 6



Step 2 of 6

Given that,

Mass of the cylinder = 30 kg

Diameter of the cylinder = 400 mm

Coefficient of static friction between cylinder and the surfaces is $\mu_s = 0.30$

Let $F_1 \,\&\, N_1$ are the friction and normal forces between cylinder and inclined surface respectively

$F_2 \,\&\, N_2$ are the friction and normal forces between cylinder and vertical surface respectively

For impending slippage, at both the surfaces

We have

$$F_1 = \mu_s N_1$$
$$= 0.3\ N_1$$
$$F_2 = \mu_s N_2$$
$$= 0.3\ N_2$$

Step 3 of 6

Writing equations of equilibrium

$$\sum F_x = 0$$
$$F_1 \cos 30 + N_1 \sin 30 - N_2 = 0 \qquad \rightarrow (1)$$
$$\sum F_y = 0$$
$$F_2 + N_1 \cos 30 - (30 \times 9.81) - F_1 \sin 30 = 0 \qquad \rightarrow (2)$$

Step 4 of 6

From equation (1)

$$0.3 N_1 \cos 30 + N_1 \sin 30 - N_2 = 0$$
$$0.7598 N_1 - N_2 = 0$$
$$N_2 = 0.7598 N_1 \qquad \rightarrow (3)$$

Step 5 of 6

From equation (2)

$$0.3 N_2 + N_1 \cos 30 - 294.3 - 0.30 N_1 \sin 30 = 0$$
$$0.3 N_2 + 0.716 N_1 - 294.3 = 0$$
$$0.3[0.7598 N_1] + 0.716 N_1 = 294.3$$
$$N_1 = 311.78\ \text{N}$$

From (3) $N_2 = 0.7598 N_1$

$$= 236.89\ \text{N}$$
$$F_1 = 0.3 N_1$$
$$= 0.3 \times 311.78\ \text{N}$$
$$= 93.534\ \text{N}$$
$$F_2 = 0.3 N_2$$
$$= 0.3 \times 236.89\ \text{N}$$
$$= 71.067\ \text{N}$$

Step 6 of 6

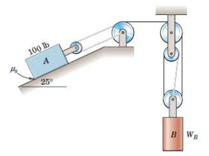
Writing moment – equilibrium equation about point 'C'

$$M - (F_2 \times 200) - (F_1 \times 200) = 0$$
$$M - (71.067 \times 200) - (93.534 \times 200) = 0$$
$$M = 32{,}920.2\ \text{N-mm}$$
$$\boxed{M = 32.92\ \text{N} \cdot \text{m}}$$

Chapter 6, Problem 12P

Problem

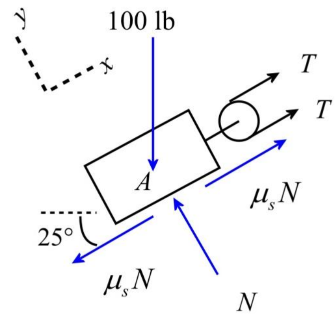
If the coefficient of static friction between block *A* and the incline is $\mu s = 0.30$, determine the range of cylinder weights *WB* for which the system will remain in equilibrium. Neglect all pulley friction.



Step-by-step solution

Step 1 of 5

Draw the force diagram of the block *A* as follows:



Step 2 of 5

Consider that the motion impends down the incline.

Obtain equilibrium equation as follows:

Calculate forces along y-direction.

$$\sum F_y = 0$$

$$N = 100\cos 25° = 90.6 \text{ lb}$$

Here, the normal reaction is *N*.

Calculate forces along x-direction.

$$\sum F_x = 0$$

$$2T - 100\sin 25° + \mu_s N = 0$$

Here, the coefficient of static friction is μ_s.

Substitute 0.3 for μ_s.

$$2T - 100\sin 25° + 0.3\times 90.6 = 0 \quad \text{...... (1)}$$

$$T = 7.54 \text{ lb}$$

Comments (1)

Anonymous

Why is friction not in the negative direction?

Step 3 of 5

Consider that the motion impends up the incline.

Obtain equilibrium equation as follows:

Calculate forces along x-direction.

$$\sum F_x = 0$$

$$2T - 100\sin 25° - \mu_s N = 0$$

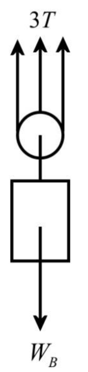
Substitute 0.3 for μ_s.

$$2T - 100\sin 25° - 0.3\times 90.6 = 0 \quad \text{...... (2)}$$

$$T = 34.7 \text{ lb}$$

Step 4 of 5

Draw the free body diagram of the pulley *B*.



Step 5 of 5

Obtain equilibrium equation as follows:

Calculate forces along y-direction.

$$\sum F_y = 0$$

$$3T - W_B = 0 \quad \text{...... (3)}$$

$$W_B = 3T$$

Substitute equation (1) in equation (3).

$$W_B = 3\times(7.54) = 22.6 \text{ lb}$$

Substitute equation (2) in equation (3).

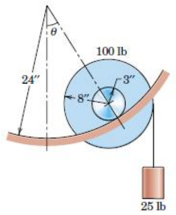
$$W_B = 3\times(34.7) = 104.2 \text{ lb}$$

Therefore, the range of cylinder weights for which the system remains in equilibrium is $\boxed{22.6 \le W_B \le 104.2}$.

Chapter 6, Problem 13P

Problem

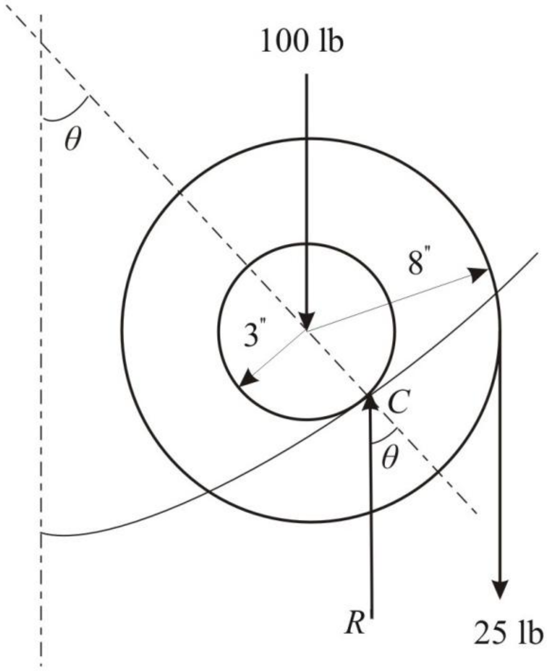
The 100-lb wheel rolls on its hub up the circular incline under the action of the 25-lb weight attached to a cord around the rim. Determine the angle θ at which the wheel comes to rest, assuming that friction is sufficient to prevent slippage. What is the minimum coefficient of static friction which will permit this position to be reached with no slipping?



Step-by-step solution

Step 1 of 3

Draw the free body diagram of the wheel.



Step 2 of 3

Apply the equations of equilibrium and calculate the moments about the point C.

$100\times(3\sin\theta)-25\times(8-3\times\sin\theta)=0$ ...... (1)

Here θ is the angle at which the wheel comes to the rest.

Calculate the angle θ for which comes to rest.

Solve equation (1).

$$100\times(3\sin\theta)-25\times(8-3\times\sin\theta)=0$$

$$300\sin\theta+75\sin\theta=200$$

$$\sin\theta=\frac{200}{375}$$

$$\theta=\sin^{-1}\left(\frac{8}{15}\right)$$

$$\theta=32.23°$$

Therefore, the value of angle θ at which the wheel comes to rest without slipping is $\boxed{32.23°}$.

Step 3 of 3

Calculate the minimum coefficient of friction required to reach the θ position.

$(\mu_s)_{\min}=\tan\theta$ ...... (2)

Here $(\mu_s)_{\min}$ is the minimum coefficient of friction.

Substitute $32.23°$ for θ in equation (2).

$$(\mu_s)_{\min}=\tan\theta$$

$$(\mu_s)_{\min}=\tan(32.23°)$$

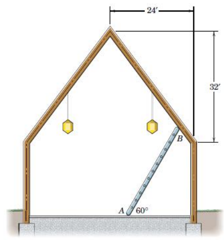
$$(\mu_s)_{\min}=0.630$$

Thus, the value of minimum coefficient of friction which allows the wheel to roll up to θ position without slipping is $\boxed{0.630}$.

Chapter 6, Problem 14P

Problem

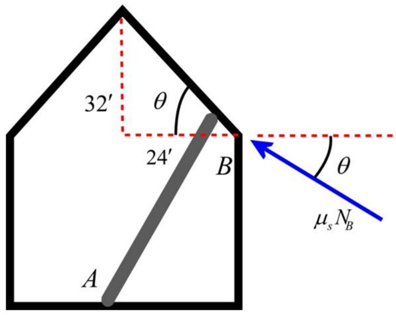
A uniform ladder is positioned as shown for the purpose of maintaining the light fixture suspended from the cathedral ceiling. Determine the minimum coefficient of static friction required at ends *A* and *B* to prevent slipping. Assume that the coefficient is the same at *A* and *B*.



Step-by-step solution

Step 1 of 5

Compute the angle between frictional force at *B* and the horizontal as follows:



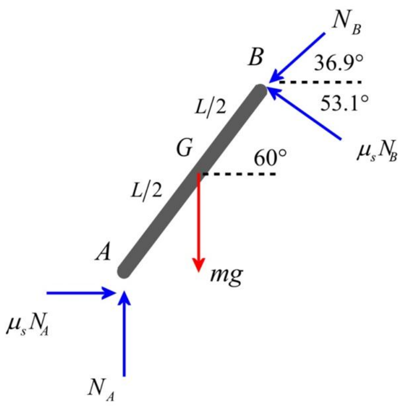
Step 2 of 5

Calculate the angle θ .

$$\tan\theta = \frac{32}{24}$$
$$\theta = \tan^{-1}\left(\frac{32}{24}\right)$$
$$= 53.1°$$

Step 3 of 5

Draw the free body diagram of the system:



Step 4 of 5

Obtain equilibrium equation as follows:

Calculate forces along x-direction.

$$\sum F_x = 0$$
$$\mu_s N_A - N_B \cos 36.9° - \mu_s N_B \cos 53.1° = 0 \quad \text{...... (1)}$$
$$\mu_s N_A - 0.8 N_B - 0.6 \mu_s N_B = 0$$

Here, the coefficient of static friction is μ_s and the normal reaction at *A* is N_A and the normal reaction at *B* is N_B.

Calculate forces along y-direction.

$$\sum F_y = 0$$
$$N_A - N_B \sin 36.9° + \mu_s N_B \sin 53.1° - mg = 0 \quad \text{...... (2)}$$
$$N_A - 0.6 N_B + 0.8 \mu_s N_B - mg = 0$$

Here, *m* is the mass of the ladder and *g* is the acceleration due to gravity.

Step 5 of 5

Calculate moment about point *B*.

$$\sum M_B = 0$$
$$mg\frac{L}{2}\cos 60° + \mu_s N_A L \sin 60° - N_A L \cos 60° = 0$$
$$\text{...... (3)}$$
$$\frac{mg}{2}\cos 60° + \mu_s N_A \sin 60° - N_A \cos 60° = 0$$
$$0.25mg + 0.866 \mu_s N_A - 0.5 N_A = 0$$

Here, L is the total length of the ladder.

Solve equations (1), (2) and (3) in any computational software and obtain the values N_A, N_B, and μ_s as follows:

$$N_A = 1.125mg$$
$$N_B = 0.364mg$$
$$\mu_s = 0.321$$

Therefore, the minimum coefficient of static friction is $\boxed{0.321}$.

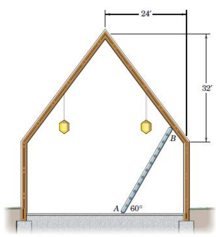
Chapter 6, Problem 15P

Problem

If there is a small frictionless roller on end *B* of the ladder of Prob. 6/14, determine the minimum coefficient of static friction required at end *A* in order to provide equilibrium. Compare with the results of the previous problem.

Problem. 6/14

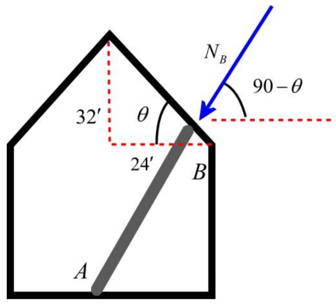
A uniform ladder is positioned as shown for the purpose of maintaining the light fixture suspended from the cathedral ceiling. Determine the minimum coefficient of static friction required at ends *A* and *B* to prevent slipping. Assume that the coefficient is the same at *A* and *B*.



Step-by-step solution

Step 1 of 5

Compute the angle between frictional force at *B* and the horizontal as follows:



Step 2 of 5

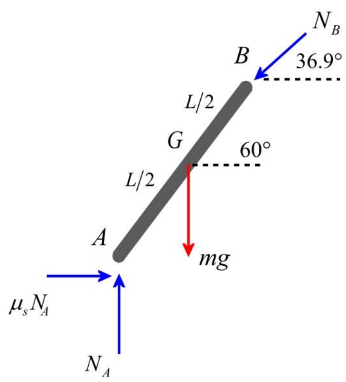
Calculate the angle θ.

$$\tan\theta = \frac{32}{24}$$

$$\theta = \tan^{-1}\left(\frac{32}{24}\right)$$
$$= 53.1^\circ$$

Step 3 of 5

Draw the free body diagram of the system considering the fact that there will be no frictional force at *B* due to the presence of roller.



Step 4 of 5

Obtain equilibrium equation as follows:

Calculate forces along x-direction.

$$\sum F_x = 0$$
$$\mu_s N_A - N_B \cos 36.9^\circ = 0 \quad \ldots\ldots (1)$$
$$\mu_s N_A - 0.8 N_B = 0$$

Here, the coefficient of static friction is μ_s and the normal reaction at *A* is N_A and the normal reaction at *B* is N_B.

Calculate forces along y-direction.

$$\sum F_y = 0$$
$$N_A - N_B \sin 36.9^\circ - mg = 0 \quad \ldots\ldots (2)$$
$$N_A - 0.6 N_B - mg = 0$$

Here, *m* is the mass of the ladder and *g* is the acceleration due to gravity.

Step 5 of 5

Calculate moment about point *B*.

$$\sum M_B = 0$$
$$mg\frac{L}{2}\cos 60^\circ + \mu_s N_A L \sin 60^\circ - N_A L \cos 60^\circ = 0$$
$$\frac{mg}{2}\cos 60^\circ + \mu_s N_A \sin 60^\circ - N_A \cos 60^\circ = 0 \quad \ldots\ldots (3)$$
$$0.25mg + 0.866\mu_s N_A - 0.5 N_A = 0$$

Solve equations (1), (2) and (3) in any computational software and obtain the values N_A, N_B, and μ_s as follows:

$$N_A = 1.382mg$$
$$N_B = 0.636mg$$
$$\mu_s = 0.368$$

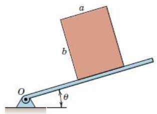
Therefore, the minimum coefficient of static friction is $\boxed{0.368}$.

Thus, the minimum coefficient of static friction is higher (0.368) in this case when compared to the previous problem value of (0.321).

Chapter 6, Problem 16P

Problem

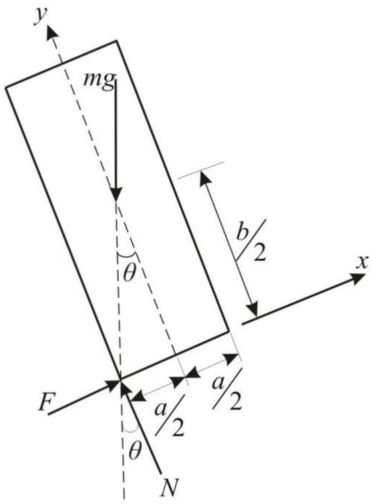
The homogeneous rectangular block of mass m rests on the inclined plane which is hinged about a horizontal axis through O. If the coefficient of static friction between the block and the plane is μ, specify the conditions which determine whether the block tips before it slips or slips before it tips as the angle θ is gradually increased.



Step-by-step solution

Step 1 of 6

Draw the free body diagram of the rectangular block.



Step 2 of 6

Apply the equations of equilibrium and calculate the normal reaction force along the y-direction.

$$\sum F_y = 0$$
$$N - (m \times g) \times \cos\theta = 0$$
$$N = (m \times g) \times \cos\theta$$

Here N is the normal force exerted by the inclined surface on the block, θ is the angle of inclination, m is the mass of the block, and g is the acceleration due to gravity.

Step 3 of 6

Calculate the maximum frictional force on the block assume that the block slips.

$F = \mu \times N$ ...... (1)

Here F is the maximum frictional force exerted by the inclined surface on the block, and μ is the coefficient of static friction.

Substitute $(m \times g) \times \cos\theta$ for N in equation (1).

$$F = \mu \times (m \times g) \times \cos\theta$$

Step 4 of 6

Apply the equations of equilibrium and calculate the forces along the x-direction.

$$\sum F_x = 0$$
$$F - (m \times g) \times \sin\theta = 0$$
$$F = (m \times g) \times \sin\theta$$

Calculate the angle of inclination. Substitute $\mu \times (m \times g) \times \cos\theta$ for F.

$$\mu \times (m \times g) \times \cos\theta = (m \times g) \times \sin\theta$$
$$\tan\theta = \mu$$
$$\theta = \tan^{-1}(\mu)$$

Step 5 of 6

Calculate the relation between the dimensions of the block from the geometry of the free body diagram. Write the tangent of the angle θ.

$\tan\theta = \dfrac{\left(a/2\right)}{\left(b/2\right)}$ ...... (2)

Here a is the width of the rectangular block and b is the height of the rectangular block.

Solve equation (2) for the relation between a, b, and μ.

$$\tan\theta = \frac{a}{b}$$
$$a = b \times \tan\theta$$

Substitute μ for $\tan\theta$.

$$\frac{a}{b} = \mu$$

Step 6 of 6

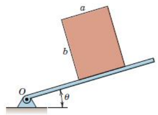
Thus, until $\frac{a}{b}$ equals to μ, the block does not slip. If $\frac{a}{b}$ becomes greater than μ, the block slips and if $\frac{a}{b}$ becomes less than μ, the block tips.

Therefore, the block slips first, if $\boxed{\frac{a}{b} > \mu}$ and the block tips if $\boxed{\frac{a}{b} < \mu}$.

Chapter 6, Problem 16P

Problem

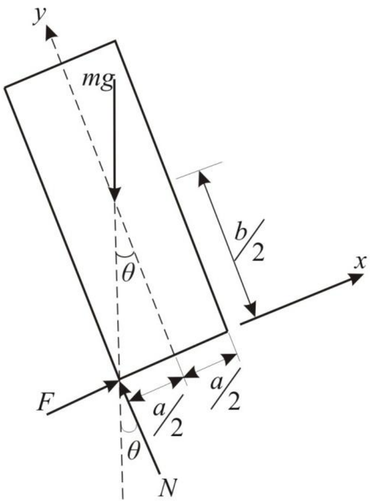
The homogeneous rectangular block of mass *m* rests on the inclined plane which is hinged about a horizontal axis through *O*. If the coefficient of static friction between the block and the plane is μ, specify the conditions which determine whether the block tips before it slips or slips before it tips as the angle θ is gradually increased.



Step-by-step solution

Step 1 of 6

Draw the free body diagram of the rectangular block.



Step 2 of 6

Apply the equations of equilibrium and calculate the normal reaction force along the *y*-direction.

$$\sum F_y = 0$$
$$N - (m \times g) \times \cos\theta = 0$$
$$N = (m \times g) \times \cos\theta$$

Here N is the normal force exerted by the inclined surface on the block, θ is the angle of inclination, m is the mass of the block, and g is the acceleration due to gravity.

Step 3 of 6

Calculate the maximum frictional force on the block assume that the block slips.

$F = \mu \times N$ ...... (1)

Here F is the maximum frictional force exerted by the inclined surface on the block, and μ is the coefficient of static friction.

Substitute $(m \times g) \times \cos\theta$ for N in equation (1).

$$F = \mu \times (m \times g) \times \cos\theta$$

Step 4 of 6

Apply the equations of equilibrium and calculate the forces along the *x*-direction.

$$\sum F_x = 0$$
$$F - (m \times g) \times \sin\theta = 0$$
$$F = (m \times g) \times \sin\theta$$

Calculate the angle of inclination. Substitute $\mu \times (m \times g) \times \cos\theta$ for F.

$$\mu \times (m \times g) \times \cos\theta = (m \times g) \times \sin\theta$$
$$\tan\theta = \mu$$
$$\theta = \tan^{-1}(\mu)$$

Step 5 of 6

Calculate the relation between the dimensions of the block from the geometry of the free body diagram. Write the tangent of the angle θ.

$\tan\theta = \dfrac{\left(a/2\right)}{\left(b/2\right)}$ ...... (2)

Here a is the width of the rectangular block and b is the height of the rectangular block.

Solve equation (2) for the relation between a, b, and μ.

$$\tan\theta = \frac{a}{b}$$
$$a = b \times \tan\theta$$

Substitute μ for $\tan\theta$.

$$\frac{a}{b} = \mu$$

Step 6 of 6

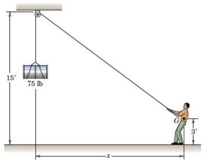
Thus, until $\frac{a}{b}$ equals to μ, the block does not slip. If $\frac{a}{b}$ becomes greater than μ, the block slips and if $\frac{a}{b}$ becomes less than μ, the block tips.

Therefore, the block slips first, if $\boxed{\frac{a}{b} > \mu}$ and the block tips if $\boxed{\frac{a}{b} < \mu}$.

Chapter 6, Problem 17P

Problem

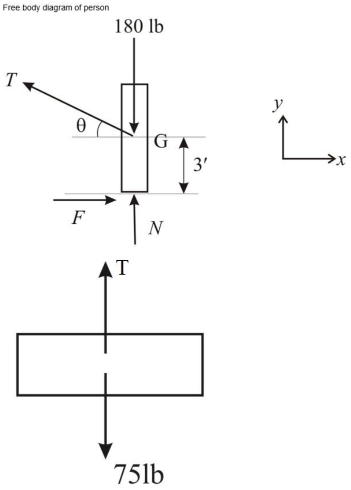
The 180-lb man with center of gravity *G* supports the 75-lb drum as shown. Find the greatest distance *x* at which the man can position himself without slipping if the coefficient of static friction between his shoes and the ground is 0.40.



Step-by-step solution

Step 1 of 8

Free body diagram of person



Step 2 of 8

Given that,

Weight of the person = 180 lb

Weight of the drum = 75 lb

Coefficient of static friction between person shoes and the ground is $\mu_s = 0.40$

Let θ be the angle that the cord makes with the horizontal as shown in the figure above

From the free body diagram of drum

$\sum F_y = 0$

$T = 75 \text{ lb}$

Step 3 of 8

From the free body diagram of person

$\sum F_y = 0$

$N + T\sin\theta - 180 = 0 \quad \rightarrow (1)$

$\sum F_x = 0$

$F - T\cos\theta = 0 \quad \rightarrow (2)$

Step 4 of 8

For the person to position himself without slipping or on the verge of slipping

$F = \mu_s N$

$F = 0.40N$

From equation (1)

$N + 75\sin\theta - 180 = 0$

$N = 180 - 75\sin\theta$

Step 5 of 8

From equation (2)

$F = T\cos\theta$

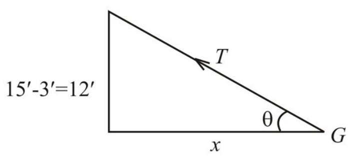
$0.40N = 75\cos\theta$

$0.40[180 - 75\sin\theta] = 75\cos\theta$

$72 - 30\sin\theta = 75\cos\theta$

$75\cos\theta + 30\sin\theta = 72 \quad \rightarrow (3)$

Step 6 of 8



Step 7 of 8

From the figure, we have

$\sin\theta = \dfrac{12}{\sqrt{12^2 + x^2}}$

$\cos\theta = \dfrac{x}{\sqrt{12^2 + x^2}}$

Step 8 of 8

From equation (3)

$75\left(\dfrac{x}{\sqrt{12^2 + x^2}}\right) + 30\left(\dfrac{12}{\sqrt{12^2 + x^2}}\right) = 72$

$75x + 360 = 72\sqrt{144 + x^2}$

$1.0417x + 5 = \sqrt{144 + x^2}$

$(1.0417x + 5)^2 = 144 + x^2$

$1.0851x^2 + 25 + 10.417x = 144 + x^2$

$0.0851x^2 + 10.417x - 119 = 0$

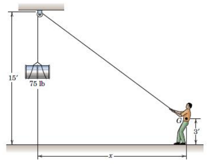
$x = 10.519'$

$\boxed{x = 10.52 \text{ ft}}$

Chapter 6, Problem 17P

Problem

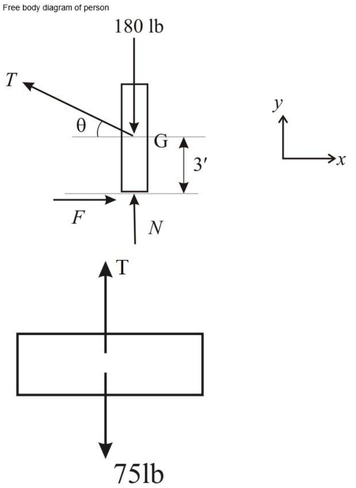
The 180-lb man with center of gravity G supports the 75-lb drum as shown. Find the greatest distance x at which the man can position himself without slipping if the coefficient of static friction between his shoes and the ground is 0.40.



Step-by-step solution

Step 1 of 8

Free body diagram of person



Step 2 of 8

Given that,

Weight of the person = 180 lb

Weight of the drum = 75 lb

Coefficient of static friction between person shoes and the ground is $\mu_s = 0.40$

Let θ be the angle that the cord makes with the horizontal as shown in the figure above

From the free body diagram of drum

$\sum F_y = 0$

$T = 75 \text{ lb}$

Step 3 of 8

From the free body diagram of person

$\sum F_y = 0$

$N + T\sin\theta - 180 = 0 \quad \rightarrow (1)$

$\sum F_x = 0$

$F - T\cos\theta = 0 \quad \rightarrow (2)$

Step 4 of 8

For the person to position himself without slipping or on the verge of slipping

$F = \mu_s N$

$F = 0.40N$

From equation (1)

$N + 75\sin\theta - 180 = 0$

$N = 180 - 75\sin\theta$

Step 5 of 8

From equation (2)

$F = T\cos\theta$

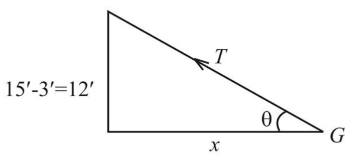
$0.40N = 75\cos\theta$

$0.40[180 - 75\sin\theta] = 75\cos\theta$

$72 - 30\sin\theta = 75\cos\theta$

$75\cos\theta + 30\sin\theta = 72 \quad \rightarrow (3)$

Step 6 of 8



Step 7 of 8

From the figure, we have

$$\sin\theta = \frac{12}{\sqrt{12^2 + x^2}}$$

$$\cos\theta = \frac{x}{\sqrt{12^2 + x^2}}$$

Step 8 of 8

From equation (3)

$$75\left(\frac{x}{\sqrt{12^2 + x^2}}\right) + 30\left(\frac{12}{\sqrt{12^2 + x^2}}\right) = 72$$

$75x + 360 = 72\sqrt{144 + x^2}$

$1.0417x + 5 = \sqrt{144 + x^2}$

$(1.0417x + 5)^2 = 144 + x^2$

$1.0851x^2 + 25 + 10.417x = 144 + x^2$

$0.0851x^2 + 10.417x - 119 = 0$

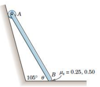
$x = 10.519'$

$\boxed{x = 10.52 \text{ ft}}$

Chapter 6, Problem 18P

Problem

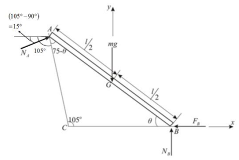
The uniform slender bar has an ideal roller at its upper end *A*. Determine the minimum value of the angle θ for which equilibrium is possible for $\mu s = 0.25$ and for $\mu s = 0.50$.



Step-by-step solution

Step 1 of 10

Draw the free body diagram of the slender bar:



Step 2 of 10

From the sine rule:

$$\frac{\sin(105^\circ)}{l} = \frac{\sin\theta}{\overline{AC}} = \frac{\sin(75-\theta)}{\overline{BC}}$$

Calculate the length between points *A* and *C*:

$$\overline{AC} = \frac{l\sin\theta}{\sin(105^\circ)}$$

Calculate the length between points *B* and *C*:

$$\overline{BC} = \frac{l\sin(75-\theta)}{\sin(105^\circ)}$$

Step 3 of 10

Calculate the maximum frictional force at *B*:

$$F_B = \mu_s \times N_B$$

Here, F_B is the frictional force acting at end *B*, N_B is the normal force at end *B*, and μ_s is the coefficient of static friction.

Apply the equations of equilibrium and calculate the forces along the *x*-direction.

$$N_A \times \cos 15^\circ = F_B \text{ ...... (1)}$$

Here, N_A is the normal force acting at end *A*.

The normal force at *A* acts perpendicular to the vertical wall.

Substitute $\mu_s \times N_B$ for F_B in equation (1).

$$N_A \times \cos 15^\circ = F_B$$

$$N_A \times \cos 15^\circ = \mu_s \times N_B$$

$$N_A = \frac{\mu_s \times N_B}{\cos 15^\circ} \text{ ...... (2)}$$

Step 4 of 10

Apply the equations of equilibrium and calculate the forces along the *y*-direction.

$$N_A \times \sin 15^\circ + N_B - mg = 0 \text{ ...... (3)}$$

Here, m is the mass of the slender bar and g is the acceleration due to gravity.

Calculate the normal force at *B*.

Substitute $\frac{\mu_s \times N_B}{\cos 15^\circ}$ for N_A in equation (3).

$$N_A \times \sin 15^\circ + N_B - mg = 0$$

$$\frac{\mu_s \times N_B}{\cos 15^\circ} \times \sin 15^\circ + N_B - mg = 0$$

$$N_B = \frac{mg}{\mu_s \times \tan 15^\circ + 1} \text{ ...... (4)}$$

Step 5 of 10

Calculate the normal force at *A*.

Substitute equation (4) in equation (2).

$$N_A = \frac{\mu_s \times N_B}{\cos 15^\circ}$$

$$N_A = \mu_s \times \frac{mg}{(\mu_s \times \tan 15^\circ + 1) \times \cos 15^\circ}$$

$$N_A = \frac{\mu_s \times mg}{(\mu_s \times \sin 15^\circ + \cos 15^\circ)}$$

Step 6 of 10

Apply the equations of equilibrium and calculate the moments about the point *C*.

$$-N_A \times \overline{AC} + N_B \times \overline{BC} - mg\left(\overline{BC} - \frac{l}{2}\cos\theta\right) = 0 \text{ ...... (5)}$$

Here, θ is the maximum angle of inclination of the bar at which the bar remains in equilibrium, and l is the length of the bar.

Substitute $\frac{\mu_s \times mg}{(\mu_s \times \sin 15^\circ + \cos 15^\circ)}$ for N_A, $\frac{mg}{\mu_s \times \tan 15^\circ + 1}$ for N_B, $\frac{l\sin\theta}{\sin(105^\circ)}$ for $\overline{AC}$, and $\frac{l\sin(75-\theta)}{\sin(105^\circ)}$ for $\overline{BC}$ in equation (5).

$$-N_A \times \overline{AC} + N_B \times \overline{BC} - mg\left(\overline{BC} - \frac{l}{2}\cos\theta\right) = 0$$

$$\left(\begin{array}{l} -\left(\frac{\mu_s \times mg}{(\mu_s \times \sin 15^\circ + \cos 15^\circ)}\right) \times \left(\frac{l\sin\theta}{\sin(105^\circ)}\right) + \left(\frac{mg}{\mu_s \times \tan 15^\circ + 1}\right) \times \left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) \\ -mg\left(\left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) - \frac{l}{2}\cos\theta\right) \end{array}\right) = 0 \text{ ......}$$

(6)

Step 7 of 10

Calculate the angle of inclination of the bar with coefficient of static friction value equal to 0.25.

Substitute 0.25 for μ_s in equation (6).

$$\left[\begin{array}{l} -\left(\frac{\mu_s \times mg}{(\mu_s \times \sin 15^\circ + \cos 15^\circ)}\right) \times \left(\frac{l\sin\theta}{\sin(105^\circ)}\right) + \left(\frac{mg}{\mu_s \times \tan 15^\circ + 1}\right) \times \left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) \\ -mg\left(\left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) - \frac{l}{2}\cos\theta\right) \end{array}\right] = 0$$

$$\left[\begin{array}{l} -\left(\frac{0.25 \times mg}{(0.25 \times \sin 15^\circ + \cos 15^\circ)}\right) \times \left(\frac{l\sin\theta}{\sin(105^\circ)}\right) + \left(\frac{mg}{0.25 \times \tan 15^\circ + 1}\right) \times \left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) \\ -mg\left(\left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) - \frac{l}{2}\cos\theta\right) \end{array}\right] = 0$$

$$\left[\begin{array}{l} -0.2426 \times mg \times (1.0353 \times \sin\theta) + 0.9372 \times mg \times (1.0353 \times \sin(75-\theta)) \\ -mg(1.0353 \times \sin(75-\theta)) + mg(0.5 \times \cos\theta) = 0 \end{array}\right] \text{ ...... (7)}$$

Step 8 of 10

Solve the equation (7).

$$\left[\begin{array}{l} -0.2426 \times mg \times (1.0353 \times \sin\theta) + 0.9372 \times mg \times (1.0353 \times \sin(75-\theta)) \\ -mg(1.0353 \times \sin(75-\theta)) + mg(0.5 \times \cos\theta) \end{array}\right] = 0$$

$$-0.2512 \times \sin\theta - 0.065 \times \sin(75-\theta) + 0.5 \times \cos\theta = 0$$

$$-0.2512 \times \sin\theta - 0.065 \times (\sin 75^\circ \times \cos\theta - \cos 75^\circ \times \sin\theta) + 0.5 \times \cos\theta = 0$$

$$-0.2512 \times \sin\theta - 0.06279 \times \cos\theta + 0.01682 \times \sin\theta + 0.5 \times \cos\theta = 0 \text{ ...... (8)}$$

$$0.23438 \times \sin\theta = 0.43721 \times \cos\theta$$

$$\tan\theta = 1.8654$$

$$\theta = \tan^{-1}(1.8654)$$

$$\theta = 61.8^\circ$$

Thus, the maximum angle of inclination of the bar, with coefficient of friction 0.25, at which the bar remains in equilibrium is $\boxed{61.8^\circ}$.

Step 9 of 10

Calculate the angle of inclination of the bar with coefficient of static friction value equal to 0.50.

Substitute 0.50 for μ_s in equation (6).

$$\left[\begin{array}{l} -\left(\frac{\mu_s \times mg}{(\mu_s \times \sin 15^\circ + \cos 15^\circ)}\right) \times \left(\frac{l\sin\theta}{\sin(105^\circ)}\right) + \left(\frac{mg}{\mu_s \times \tan 15^\circ + 1}\right) \times \left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) \\ -mg\left(\left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) - \frac{l}{2}\cos\theta\right) \end{array}\right] = 0$$

$$\left[\begin{array}{l} -\left(\frac{0.50 \times mg}{(0.50 \times \sin 15^\circ + \cos 15^\circ)}\right) \times \left(\frac{l\sin\theta}{\sin(105^\circ)}\right) + \left(\frac{mg}{0.50 \times \tan 15^\circ + 1}\right) \times \left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) \\ -mg\left(\left(\frac{l\sin(75-\theta)}{\sin(105^\circ)}\right) - \frac{l}{2}\cos\theta\right) \end{array}\right] = 0$$

$$\left[-0.4745 \times \sin\theta + 0.9167 \times \sin(75-\theta) - 1.0353 \times \sin(75-\theta) + 0.5 \times \cos\theta\right] = 0 \text{ ...... (9)}$$

Step 10 of 10

Further solve the equation (9).

$$-0.4745 \times \sin\theta + 0.9167 \times \sin(75-\theta) - 1.0353 \times \sin(75-\theta) + 0.5 \times \cos\theta = 0$$

$$-0.4745 \times \sin\theta - 0.11858 \times \sin(75-\theta) + 0.5 \times \cos\theta = 0$$

$$-0.4745 \times \sin\theta - 0.11858 \times (\sin 75^\circ \times \cos\theta - \cos 75^\circ \times \sin\theta) + 0.5 \times \cos\theta = 0$$

$$-0.4745 \times \sin\theta - 0.1145 \times \cos\theta + 0.0307 \times \sin\theta + 0.5 \times \cos\theta = 0$$

$$-0.4745 \times \sin\theta - 0.114536 \times \cos\theta + 0.0307 \times \sin\theta + 0.5 \times \cos\theta = 0$$

$$0.44381 \times \sin\theta = 0.385464 \times \cos\theta$$

$$\tan\theta = 0.8685$$

$$\theta = \tan^{-1}(0.8685)$$

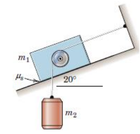
$$\theta = 40.9^\circ$$

Thus, the maximum angle of inclination of the bar, with coefficient of friction 0.50, at which the bar remains in equilibrium is $\boxed{40.9^\circ}$.

Chapter 6, Problem 19P

Problem

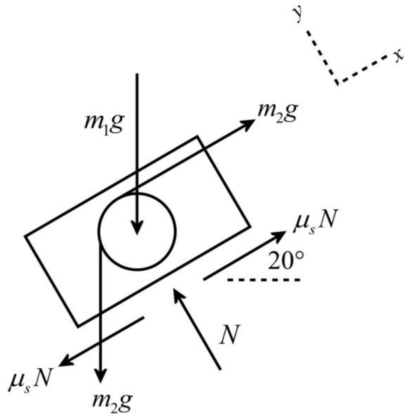
Determine the range of mass *m2* for which the system is in equilibrium. The coefficient of static friction between the block and the incline is $\mu s = 0.25$. Neglect friction associated with the pulley.



Step-by-step solution

Step 1 of 5

Draw the free body diagram of the system for motions up and down the inclined plane as follows:



Step 2 of 5

Consider that the motion impends up the inclined plane.

Obtain equilibrium equations as follows:

Calculate forces along the along the incline.

$$\sum F_x = 0$$
$$-\mu_s N - m_1 g\sin 20° - m_2 g\sin 20° + m_2 g = 0$$

Here, the coefficient of static friction is μ_s and the normal reaction is N, the mass of the pulley is m_2, the mass of the block on the inclined surface is m_1, and the acceleration due to gravity is *g*.

Substitute 0.25 for μ_s.

$$\sum F_x = 0$$
$$-0.25N - m_1 g\sin 20° - m_2 g\sin 20° + m_2 g = 0 \quad \text{...... (1)}$$

Step 3 of 5

Calculate forces along perpendicular to the incline.

$$\sum F_y = 0$$
$$N - m_1 g\cos 20° - m_2 g\cos 20° = 0 \quad \text{...... (2)}$$

Solve equations (1) and (2).

Multiply equation (2) with 0.25 and add equation (1) to it.

$$0.25\left(N - m_1 g\cos 20° - m_2 g\cos 20°\right)$$
$$+\left(-0.25N - m_1 g\sin 20° - m_2 g\sin 20° + m_2 g\right) = 0$$
$$\left(0.25N - 0.25m_1 g\cos 20° - 0.25m_2 g\cos 20°\right)$$
$$+\left(-0.25N - m_1 g\sin 20° - m_2 g\sin 20° + m_2 g\right) = 0$$
$$m_1 g\left(0.25\cos 20° + \sin 20°\right) = m_2 g\left(1 - \sin 20° - 0.25\cos 20°\right)$$
$$m_1\left(0.577\right) = m_2\left(0.423\right)$$
$$m_2 = 1.364m_1$$

Comments (1)

Anonymous

Why do you multiply eqn 2 by .25 and not to eqn 1

Step 4 of 5

Consider that the motion impends down the inclined plane.

Obtain equilibrium equations as follows:

Calculate forces along the incline.

$$\sum F_x = 0$$
$$\mu_s N - m_1 g\sin 20° - m_2 g\sin 20° + m_2 g = 0$$

Substitute 0.25 for μ_s.

$$\sum F_x = 0$$
$$0.25N - m_1 g\sin 20° - m_2 g\sin 20° + m_2 g = 0 \quad \text{...... (3)}$$

Calculate forces along perpendicular to the incline.

$$\sum F_y = 0$$
$$N - m_1 g\cos 20° - m_2 g\cos 20° = 0 \quad \text{...... (4)}$$

Step 5 of 5

Solve equations (3) and (4).

Multiply equation (4) with 0.25 and subtract equation (3) from it.

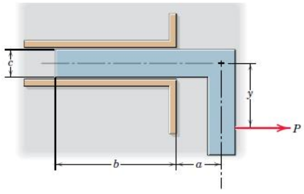
$$0.25\left(N - m_1 g\cos 20° - m_2 g\cos 20°\right)$$
$$-\left(0.25N - m_1 g\sin 20° - m_2 g\sin 20° + m_2 g\right) = 0$$
$$\left(0.25N - 0.25m_1 g\cos 20° - 0.25m_2 g\cos 20°\right)$$
$$-\left(0.25N - m_1 g\sin 20° - m_2 g\sin 20° + m_2 g\right) = 0$$
$$-m_1 g\left(0.25\cos 20° - \sin 20°\right) = m_2 g\left(1 - \sin 20° + 0.25\cos 20°\right)$$
$$-m_1\left(-0.1070\right) = m_2\left(0.8929\right)$$
$$m_2 = 0.1199m_1$$

Therefore, the range of mass for maintaining equilibrium is $\boxed{0.1199m_1 \le m_2 \le 1.364m_1}$.

Chapter 6, Problem 20P

Problem

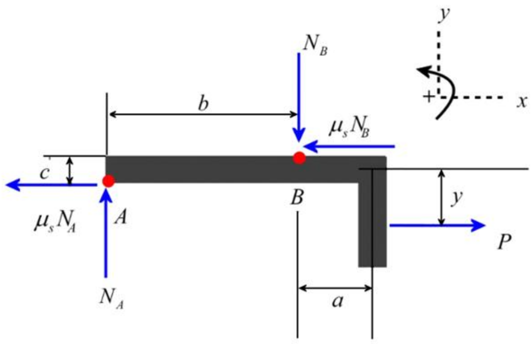
The right-angle body is to be withdrawn from the close-fitting slot by the force *P*. Find the maximum distance *y* from the horizontal centerline at which *P* may be applied without binding. The body lies in a horizontal plane, and friction underneath the body is to be neglected. Take the coefficient of static friction along both sides of the slot to be *μs*.



Step-by-step solution

Step 1 of 4

Draw the free body diagram of the system as follows:



Step 2 of 4

Consider that the motion impends up the inclined plane.

Obtain equilibrium equations as follows:

Calculate forces along x-direction.

$$\sum F_x = 0$$
$$-\mu_s (N_A + N_B) + P = 0 \quad \ldots\ldots (1)$$

Here, the coefficient of static friction is μ_s and the normal reaction at point *A* is N_A, the normal reaction at point *B* is N_B and the force to be applied is *P*.

Calculate forces along y-direction.

$$\sum F_y = 0$$
$$N_A - N_B = 0$$
$$N_A = N_B$$

Consider the following relation:

$$N_A = N_B = N \quad \ldots\ldots (2)$$

Step 3 of 4

Calculate the moment about the point *A*.

$$\sum M_A = 0$$
$$-N_B(b) + \mu_s N_B(c) + P\left(y - \frac{c}{2}\right) = 0 \quad \ldots\ldots (3)$$

Here, *a*, *b*, and *c* is the dimensional lengths.

Substitute equation (2) in equation (1).

$$-\mu_s (N_A + N_B) + P = 0$$
$$-\mu_s (N + N) + P = 0 \quad \ldots\ldots (4)$$
$$P = 2N\mu_s$$

Step 4 of 4

Substitute the values of equation (4) and (2) in equation (3).

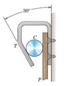
$$-N_B(b) + \mu_s N_B(c) + P\left(y - \frac{c}{2}\right) = 0$$
$$-N(b) + \mu_s N(c) + 2N\mu_s\left(y - \frac{c}{2}\right) = 0$$
$$2N\mu_s \frac{c}{2} - \mu_s N(c) + N(b) = 2N\mu_s \times y$$
$$N(b) = 2N\mu_s \times y$$
$$y = \frac{b}{2\mu_s}$$

Therefore, the maximum distance *y* from at which force many be applied without bending is $\boxed{\frac{b}{2\mu_s}}$.

Chapter 6, Problem 21P

Problem

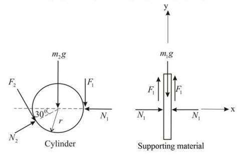
The inverted track T with freely floating cylinder C comprise a system which is designed to hold paper or other thin materials P in place. The coefficient of static friction is μ for all interfaces. What minimum value of μ ensures that the device will work no matter how heavy the supported material P is?



Step-by-step solution

Step 1 of 13

Draw the free body diagram of the cylinder and supporting material P separately.



Step 2 of 13

Apply the equations of equilibrium and calculate the forces on the supporting material along the *y*-direction.

$$\sum F_y = 0$$

$$F_1 + F_1 - m_1 \times g = 0$$

$$F_1 = \frac{m_1 \times g}{2}$$

Here F_1 is the frictional force acting on the supporting material on both sides, m_1 is the mass of the supporting material, and g is the acceleration due to gravity.

Step 3 of 13

Apply the equations of equilibrium and calculate the forces on the cylinder along the *x*-direction.

$$\sum F_x = 0$$

$$N_2 \times \cos 30° + F_2 \times \sin 30° - N_1 = 0 \text{ ...... (1)}$$

Here F_2 is the frictional force exerted by the inverted tract on the cylinder, N_1 is the normal force exerted by the supporting material on the cylinder, N_2 is the normal force exerted by the inverted tract on the cylinder.

Step 4 of 13

Apply the equations of equilibrium and calculate the forces on the cylinder along the *y*-direction.

$$\sum F_y = 0$$

$$N_2 \times \sin 30° - F_2 \times \cos 30° - m_2 \times g - F_1 = 0 \text{ ...... (2)}$$

Here m_2 is the mass of the cylinder which acts at center in downward direction, g is the acceleration due to gravity.

Substitute $\frac{m_1 \times g}{2}$ for F_1 in equation (2).

$$N_2 \times \sin 30° - F_2 \times \cos 30° - m_2 \times g - F_1 = 0$$

$$N_2 \times \sin 30° - F_2 \times \cos 30° - m_2 \times g - \frac{m_1 \times g}{2} = 0 \text{ ...... (3)}$$

Step 5 of 13

Apply the equations of equilibrium and calculate the moment of forces on cylinder about the center of the cylinder.

$$F_2 \times r - F_1 \times r = 0$$

Calculate the frictional force on the cylinder.

Substitute $\frac{m_1 \times g}{2}$ for F_1.

$$F_2 \times r - \frac{m_1 \times g}{2} \times r = 0$$

$$F_2 = \frac{m_1 \times g}{2} \text{ ...... (4)}$$

Step 6 of 13

Calculate the normal force exerted on the cylinder.

Substitute equation (4) in equation (3).

$$N_2 \times \sin 30° - F_2 \times \cos 30° - m_2 \times g - \frac{m_1 \times g}{2} = 0$$

$$N_2 \times \sin 30° - \frac{m_1 \times g}{2} \times \cos 30° - m_2 \times g - \frac{m_1 \times g}{2} = 0$$

$$N_2 \times 0.5 = (0.433 \times m_1 \times g) + (m_2 \times g) + (0.5 \times m_1 \times g)$$

$$N_2 = 1.866 \times m_1 \times g + (2 \times m_2 \times g) \text{ ...... (5)}$$

Step 7 of 13

Calculate the normal force exerted on the supporting material.

Substitute F_2 from equation (4) and N_2 from equation (5) in equation (1).

$$N_2 \times \cos 30° + F_2 \times \sin 30° - N_1 = 0$$

$$(1.866 \times m_1 \times g + 2 \times m_2 \times g) \times \cos 30° + \left(\frac{m_1 \times g}{2}\right) \times \sin 30° - N_1 = 0$$

$$N_1 = 1.616 \times m_1 \times g + (1.732 \times m_2 \times g) + 0.25 \times m_1 \times g$$

$$N_1 = 1.866 \times m_1 \times g + (1.732 \times m_2 \times g)$$

Substitute 9.81 N/kg for g.

$$N_1 = 1.866 \times m_1 \times 9.81 + (1.732 \times m_2 \times 9.81)$$

$$N_1 = 18.3 \times m_1 + 16.99 \times m_2$$

The frictional force F_1 is the maximum frictional friction force, when the slipping occurs on right side of the cylinder.

Step 8 of 13

Calculate the maximum frictional force acting on the supporting material.

$$F_1 = \mu \times N_1 \text{ ...... (6)}$$

Here μ is the coefficient of static friction for cylinder assembly.

Calculate the coefficient of static friction based on forces on supporting material.

Substitute $\frac{m_1 \times g}{2}$ for F_1 in equation (6).

$$F_1 = \mu \times N_1$$

$$\frac{m_1 \times g}{2} = \mu \times N_1$$

$$\mu = \left(\frac{m_1 \times g}{2 \times N_1}\right) \text{ ...... (7)}$$

Substitute $18.3 \times m_1 + 16.99 \times m_2$ for N_1 and 9.81 N/kg for g in equation (7).

$$\mu = \left(\frac{m_1 \times g}{2 \times N_1}\right)$$

$$\mu = \left(\frac{m_1 \times 9.81}{2 \times (18.3 \times m_1 + 16.99 \times m_2)}\right)$$

$$\mu = \left(\frac{m_1}{3.731 \times m_1 + 3.46 \times m_2}\right) \text{ ...... (8)}$$

Step 9 of 13

The frictional force F_2 is the maximum frictional friction force, when the slipping occurs on left side of the cylinder.

Calculate the maximum frictional force acting on the cylinder.

$$F_2 = \mu \times N_2$$

$$\mu = \frac{F_2}{N_2} \text{ ...... (9)}$$

Step 10 of 13

Calculate the coefficient of static friction based on forces on cylinder.

Substitute F_2 from equation (4) and N_2 from equation (5) in equation (9).

$$\mu = \frac{F_2}{N_2}$$

$$\mu = \frac{\left(\frac{m_1 \times g}{2}\right)}{(1.866 \times m_1 \times g + (2 \times m_2 \times g))}$$

Substitute 9.81 N/kg for g.

$$\mu = \frac{\left(\frac{m_1 \times 9.81}{2}\right)}{(1.866 \times m_1 \times 9.81 + (2 \times m_2 \times 9.81))}$$

$$\mu = \frac{m_1}{(3.73 \times m_1 + 4 \times m_2)} \text{ ...... (10)}$$

Step 11 of 13

Compare the coefficient of static frictions calculated using forces on cylinder and supporting material.

Compare equation (8) with equation (10).

$$\mu = \left(\frac{m_1}{3.731 \times m_1 + 3.46 \times m_2}\right) > \frac{m_1}{(3.73 \times m_1 + 4 \times m_2)}$$

$$\mu = \frac{F_1}{N_1} > \frac{F_2}{N_2}$$

Thus, the slipping first occurs on the right side of the cylinder.

Comments (1)

Anonymous

Shouldn't this be the left side since the max frictional force is lower on the left (F2, N2)?

Step 12 of 13

Calculate the value of coefficient of static friction for the cylinder assembly.

Neglect mass of the cylinder, assuming mass of the supporting material P is heavy. Here mass of the material P (m_1) is much higher than the mass of the cylinder m_2.

That is, when $m_1 >> m_2$, m_2 is equal to zero.

Neglect m_2 in equation (8).

$$\mu = \left(\frac{m_1}{3.731 \times m_1 + 3.46 \times m_2}\right)$$

$$\mu = \left(\frac{m_1}{3.731 \times m_1}\right)$$

$$\mu = 0.268$$

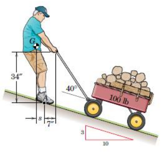
Step 13 of 13

Thus, the minimum value of coefficient of static friction μ for which the device will work for any heavy supported material P is $\boxed{0.268}$.

Chapter 6, Problem 23P

Problem

A 180-lb man pulls the 100-lb cart up the incline at steady speed. Determine the minimum coefficient μs of static friction for which the man's shoes will not slip. Also determine the distance s required for equilibrium of the man's body.



Step-by-step solution

Step 1 of 6

Given that,

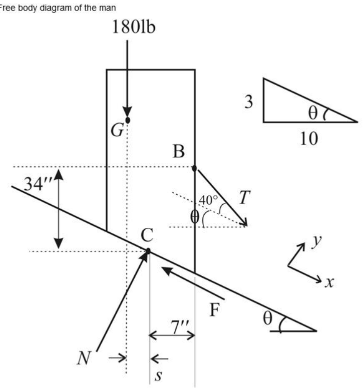
Weight of the man = 180 lb

Weight of the cart = 100 lb

Minimum coefficient of static friction for which the man's shoes will not slip is μ_s

Step 2 of 6

Free body diagram of the man



Step 3 of 6

From triangle,

$\tan\theta = \frac{3}{10}$

$\theta = 16.7°$

Writing equations of equilibrium

$\sum F_y = 0$

$N - 180\cos 16.7° - T\sin 40° = 0 \quad \rightarrow (1)$

$\sum F_x = 0$

$180\sin 16.7° + T\cos 40° - F = 0 \quad \rightarrow (2)$

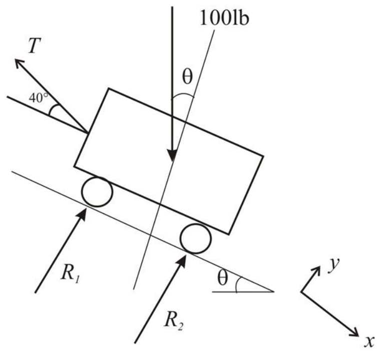
Taking moments about C

$\sum M_C = 0$

$180 \times s - (7 \times T\sin 56.7°) - (34 \times T\cos 56.7°) = 0 \quad \rightarrow (3)$

Step 4 of 6

Free body of the cart



Step 5 of 6

Assuming that there exists no friction between the wheels of the cart and the incline

$\sum F_x = 0$

$100\sin 16.7° - T\cos 40° = 0$

$T = \frac{100 \times \sin 16.7°}{\cos 40°}$

$= 37.51 \text{ lb}$

From (1)

$N = 180\cos 16.7° + 37.51\sin 40°$

$= 196.52 \text{ lb}$

From (2)

$F = 180\sin 16.7° + 37.51\cos 40°$

$= 80.46 \text{ lb}$

Step 6 of 6

On the verge of slip

$F = \mu_s N$

$\mu_s = \frac{80.46}{196.52}$

$= 0.4094$

From equation (3)

$180 \times s = [37.51 \times 7 \times \sin 56.7°] + [37.51 \times 34 \times \cos 56.7°]$

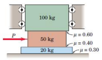
$s = 5.11''$

$\boxed{s = 5.11 \text{ in}}$

Chapter 6, Problem 24P

Problem

Determine the horizontal force *P* required to cause slippage to occur. The friction coefficients for the three pairs of mating surfaces are indicated. The top block is free to move vertically.



Step-by-step solution

Step 1 of 11

Given that,

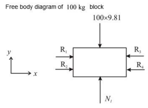
Coefficient of friction between 100 kg and 50 kg block is $\mu_1 = 0.60$

Coefficient of friction between 50 kg and 20 kg block is $\mu_2 = 0.40$

Coefficient of friction between 100 kg and 50 kg block is $\mu_3 = 0.30$

Step 2 of 11

Free body diagram of 100 kg block



R_1, R_2, R_3, R_4 are the normal forces

Step 3 of 11

As the 100 kg is free to move vertically, no friction acts on its guides.

$\sum F_y = 0$

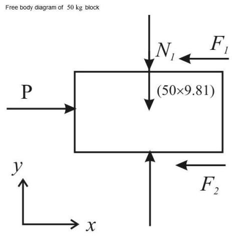
$N_1 - (100 \times 9.81) = 0$

$N_1 = 100 \times 9.81$

$= 981\text{ N}$

Step 4 of 11

Free body diagram of 50 kg block



Step 5 of 11

Writing equations of equilibrium

$\sum F_y = 0$

$N_2 - N_1 - (50 \times 9.81) = 0$

$N_2 = 981 + 490.5$

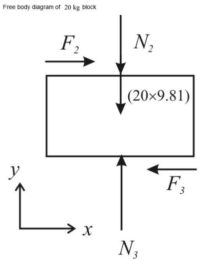
$= 1471.5\text{N}$

$\sum F_x = 0$

$P - F_1 - F_2 = 0 \rightarrow (1)$

Step 6 of 11

Free body diagram of 20 kg block



Step 7 of 11

Writhing equations of equilibrium

$\sum F_y = 0$

$N_3 - N_2 - (20 \times 9.81) = 0$

$N_3 = 1471.5 + 196.2$

$= 1667.7\text{ N}$

$\sum F_x = 0$

$F_2 - F_3 = 0 \qquad \rightarrow (2)$

Step 8 of 11

Here we can have two cases. In the first case, 50 kg block slips relative to its two blocks and 20 kg block remains in static equilibrium.

In the second case, the slipping occurs between 20 kg block and the ground and both $50\text{ kg}\ \&\ 20\text{ kg}$ blocks move together.

Step 9 of 11

First case

As slipping occurs on two surfaces of 50 kg block

$F_1 = F_{max}$

$= \mu_1 N_1$

$= 0.60 \times 981$

$= 588.6\text{ N}$

$F_2 = F_{max}$

$= \mu_2 N_2$

$= 0.40 \times 1471.5$

$= 588.6\text{ N}$

From equation (1)

$P - F_1 - F_2 = 0$

$P = F_1 + F_2$

$= 588.6 + 588.6$

$= 1177.2\text{ N}$

Step 10 of 11

From equation (2)

$F_2 = F_3$

$= 588.6\text{ N}$

Maximum possible static friction force on 20 kg block is

$F_{max} = \mu_3 N_3$

$= 0.3 \times 1667.7$

$= 500.3\text{ N}$

Here we can say that, $F_3 > F_{max}$ which makes our assumption incorrect that 20 kg block is in static equilibrium.

Comments (1)

Anonymous

So because F3>Fmax we cant use it? which is why we use F3=Fmax=N(mu)?

Step 11 of 11

Second case:

As slipping occurs between 20 kg block and the ground

$F_3 = F_{max}$

$= \mu_3 N_3$

$= 0.3 \times 1667.7$

$= 500.3\text{ N}$

From equation (2)

$F_2 - F_3 = 0$

$F_2 = 500.3\text{ N}$

From equation (1)

$P = F_1 + F_2$

$P = 588.6 + 500.3$

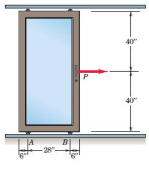
$P = 1088.9\text{ N}$

The required horizontal force to cause slippage is $\boxed{P = 1088.9\text{ N}}$

Chapter 6, Problem 25P

Problem

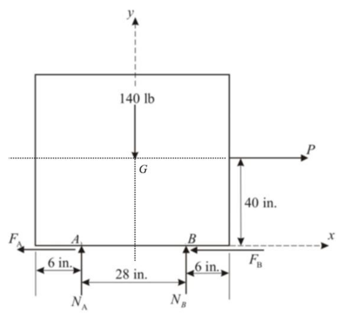
The sliding glass door rolls on the two small lower wheels *A* and *B*. Under normal conditions the upper wheels do not touch their horizontal guide. (*a*) Compute the force *P* required to slide the door at a steady speed if wheel *A* becomes "frozen" and does not turn in its bearing. (*b*) Rework the problem if wheel *B* becomes frozen instead of wheel *A*. The coefficient of kinetic friction between a frozen wheel and the supporting surface is 0.30, and the center of mass of the 140-lb door is at its geometric center. Neglect the small diameter of the wheels.



Step-by-step solution

Step 1 of 6

Draw the free body diagram of the glass door.



Step 2 of 6

(a)

There is no contact between wheel *B* and horizontal guide when wheel *A* is frozen. Thus F_B equals to zero, when wheel *A* is frozen.

$$F_B = 0$$

Calculate the maximum frictional force at wheel *A*.

$$F_A = \mu_K \times N_A$$

Here F_A is the frictional force at the contact of wheel *A*, N_A is the normal force at contact point *A*, and μ_K is the coefficient of kinetic friction.

Substitute 0.3 for μ_K.

$$F_A = \mu_K \times N_A$$

$$F_A = 0.3 \times N_A \quad \text{...... (1)}$$

Step 3 of 6

Consider the forces along the *x*-direction.

$$\sum F_x = 0$$
$$P - F_A = 0$$

Here P is the force required to slide the door when wheel *A* frozen.

From equation (1).

$$\begin{aligned} P - 0.3N_A &= 0 \\ P &= 0.3N_A \end{aligned} \quad \text{...... (2)}$$

Consider the moments about point *B*.

$$M_B = 0$$

$$140 \times \left(\frac{28}{2}\right) - N_A \times 28 - P \times 40 = 0$$

$$140 \times \left(\frac{28}{2}\right) - N_A \times 28 - (0.3 \times N_A) \times 40 = 0$$

$$N_A = 49 \text{ lb}$$

Step 4 of 6

Calculate the force required to slide the door when wheel *A* is frozen.

Substitute 49 lb for N_A in equation (2).

$$\begin{aligned} P &= 0.3N_A \\ &= 0.3 \times 49 \\ &= 14.7 \text{ lb} \end{aligned}$$

Therefore, the force P required to slide the door when wheel *A* becomes frozen is $\boxed{14.7 \text{ lb}}$.

Step 5 of 6

(b)

There is no contact between wheel *A* and horizontal guide when wheel *B* is frozen. Thus F_A equals to zero, when wheel *B* is frozen.

$$F_A = 0$$

Calculate the maximum frictional force at wheel *B*.

$$F_B = \mu_K \times N_B$$

Here F_B is the frictional force at the contact of wheel *B*, N_B is the normal force at contact point *B* and μ_K is the coefficient of kinetic friction.

Substitute 0.3 for μ_K.

$$F_B = 0.3 \times N_B \quad \text{...... (3)}$$

Consider the forces along the *x*-direction.

$$\sum F_x = 0$$
$$P - F_B = 0$$

Here, P is the force required to slide the door when wheel *B* is frozen.

From equation (3).

Substitute $0.3 \times N_B$ for F_B.

$$\begin{aligned} P - 0.3 \times N_B &= 0 \\ P &= 0.3N_B \end{aligned} \quad \text{...... (4)}$$

Step 6 of 6

Consider the moments about point *A*.

$$M_A = 0$$

$$N_B(28) - 140\left(\frac{28}{2}\right) - P(40) = 0$$

$$N_B(28) - 140\left(\frac{28}{2}\right) - (0.3N_B)(40) = 0$$

$$N_B = 122.5 \text{ lb}$$

Calculate the force required to slide the door when wheel *B* is frozen.

Substitute 122.5 lb for N_B in equation (4).

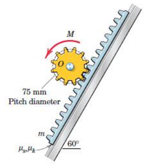
$$\begin{aligned} P &= 0.3 \times 122.5 \\ &= 36.8 \text{ lb} \end{aligned}$$

Therefore, the force P required to slide the door at a steady speed when wheel *B* becomes frozen is $\boxed{36.8 \text{ lb}}$.

Chapter 6, Problem 27P

Problem

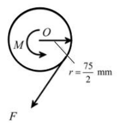
The rack has a mass m = 75 kg. What moment M must be exerted by the gear wheel in order to (a) lower and (b) raise the rack at a slow steady speed on the greased 60° rail? The coefficients of static and kinetic friction are μs = 0.10 and μk = 0.05. The fixed motor which drives the gear wheel is not shown.



Step-by-step solution

Step 1 of 7

Draw the free body diagram of the gear wheel as follows:



Step 2 of 7

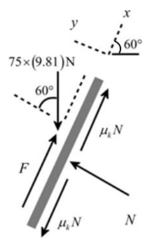
Calculate the moment about point O.

$$\sum M_O = 0$$
$$M - F \times r = 0 \quad \cdots\cdots (1)$$
$$M = F \times r$$

Here, the radius of gear wheel is r and the force exerted by the gear is F.

Step 3 of 7

Draw the free body diagram of the rack as follows:



Step 4 of 7

Consider that the rack is being lowered.

Obtain equilibrium equations as follows:

Calculate forces along y-direction.

$$\sum F_y = 0$$
$$N - 75 \times 9.81 \times \cos 60° = 0 \quad \cdots\cdots (2)$$
$$N = 368 \text{ N}$$

Here, N is the normal reaction force.

Equate the forces of equilibrium in the direction of x axis.

$$\sum F_x = 0$$
$$F + \mu_k N - 75 \times 9.81 \times \sin 60° = 0 \quad \cdots\cdots (3)$$

Here, the coefficient of kinetic friction is μ_k and the force exerted by the gear is F.

Step 5 of 7

Substitute equation (2) in equation (3).

$$\sum F_x = 0$$
$$F + 0.05 \times 368 - 75 \times 9.81 \times \sin 60° = 0$$
$$F = 619 \text{ N}$$

(a)

Calculate the moment required to lower the rack.

Substitute 619 N for F, and 37.5 mm for r in equation (1).

$$\begin{aligned} M &= F \times r \\ &= 619 \times 37.5 \\ &= 23.213 \text{ kN} \cdot \text{mm} \times \frac{10^{-3} \text{ m}}{\text{mm}} \\ &= 23.213 \text{ N} \cdot \text{m} \end{aligned}$$

Therefore, the moment required to lower the rack is $\boxed{23.213 \text{ N} \cdot \text{m}}$.

Step 6 of 7

Consider that the rack is being raised.

Obtain equilibrium equations as follows:

Calculate forces along x-direction.

$$\sum F_x = 0$$
$$F - \mu_k N - 75 \times 9.81 \times \sin 60° = 0 \quad \cdots\cdots (4)$$

Here, the coefficient of friction is μ_k and the force exerted by the gear is F.

Substitute equation (2) in equation (4).

$$\sum F_x = 0$$
$$F - 0.05 \times 368 - 75 \times 9.81 \times \sin 60° = 0$$
$$F = 656 \text{ N}$$

Step 7 of 7

(b)

Calculate the moment required to raise the rack.

Substitute 656 N for F, and 37.5 mm for r in equation (1).

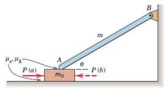
$$\begin{aligned} M &= F \times r \\ &= 656 \times 37.5 \\ &= 24.6 \text{ kN} \cdot \text{mm} \times \frac{10^{3} \text{m}}{\text{mm}} \\ &= 24.6 \text{ N} \cdot \text{m} \end{aligned}$$

Therefore, the moment required to raise the rack is $\boxed{24.6 \text{ N} \cdot \text{m}}$.

Chapter 6, Problem 28P

Problem

Determine the magnitude *P* of the horizontal force required to initiate motion of the block of mass *m*0 for the cases (*a*) *P* is applied to the right and (*b*) *P* is applied to the left. Complete a general solution in each case, and then evaluate your expression for the values θ = 30°, *m* = *m*0 = 3 kg, μ*s* = 0.60, and μ*k* = 0.50.



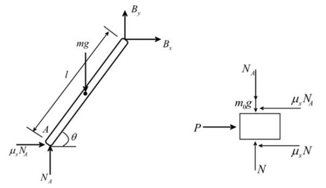
Step-by-step solution

Step 1 of 6

(a)

The force *P* is applied to right:

Draw the free body diagram for the rod and the block as follows:



Step 2 of 6

Consider the moment equilibrium condition about *B*:

$$\Sigma M_B = 0$$
$$mg\left(\frac{l}{2}\cos\theta\right) - N_A l\cos\theta + \mu_s N_A l\sin\theta = 0$$

Here, N_A is the normal reaction at *A*, *m* is the mass of the rod, *g* is acceleration due to gravity, μ_s is the coefficient of static friction, and θ is the inclination angle.

$$\frac{mg}{2}(\cos\theta) - N_A(\cos\theta - \mu_s\sin\theta) = 0$$
$$N_A = \frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$$

Consider the vertical force equilibrium condition:

$$\Sigma F_y = 0$$
$$N - N_A - m_0 g = 0$$
$$N = N_A + m_0 g$$

Substitute $\frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$ for N_A.

$$N = \frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} + m_0 g$$

Step 3 of 6

Consider the horizontal force equilibrium condition:

$$\Sigma F_x = 0$$
$$P - \mu_s N - \mu_s N_A = 0$$
$$P = \mu_s(N + N_A)$$

Substitute $\frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}$ for N_A and $\frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} + m_0 g$ for *N*.

$$P = \mu_s\left(\frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta - \mu_s\sin\theta)} + m_0 g + \frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}\right)$$
$$= \mu_s g\left(m_0 + \frac{m(\cos\theta)}{(\cos\theta - \mu_s\sin\theta)}\right)$$

Substitute 0.6 for μ_s, $30°$ for θ, 3 kg for m and m_0 and 9.81 m/s^2 for g.

$$P = 0.6(9.81)\left(3 + \frac{3(\cos 30°)}{(\cos 30° - 0.6\sin 30°)}\right)$$
$$= 44.675\text{ N}$$

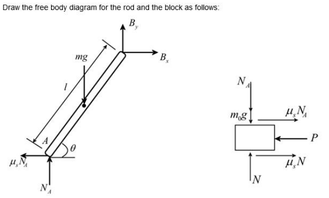
Therefore, the magnitude *P* of the required horizontal force is $\boxed{44.675\text{ N}}$.

Step 4 of 6

(b)

The force *P* is applied to left:

Draw the free body diagram for the rod and the block as follows:



Step 5 of 6

Consider the moment equilibrium condition about *B*:

$$\Sigma M_B = 0$$
$$mg\left(\frac{l}{2}\cos\theta\right) - N_A l\cos\theta - \mu_s N_A l\sin\theta = 0$$

Here, N_A is the normal reaction at *A*, *m* is the mass of the rod, *g* is acceleration due to gravity, μ_s is the coefficient of static friction, and θ is the inclination angle.

$$\frac{mg}{2}(\cos\theta) - N_A(\cos\theta + \mu_s\sin\theta) = 0$$
$$N_A = \frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}$$

Consider the vertical force equilibrium condition:

$$\Sigma F_y = 0$$
$$N - N_A - m_0 g = 0$$
$$N = N_A + m_0 g$$

Substitute $\frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}$ for N_A.

$$N = \frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta + \mu_s\sin\theta)} + m_0 g$$

Step 6 of 6

Consider the horizontal force equilibrium condition:

$$\Sigma F_x = 0$$
$$-P + \mu_s N + \mu_s N_A = 0$$
$$P = \mu_s(N + N_A)$$

Substitute $\frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}$ for N_A and $\frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta + \mu_s\sin\theta)} + m_0 g$ for *N*.

$$P = \mu_s\left(\frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta + \mu_s\sin\theta)} + m_0 g + \frac{\frac{mg}{2}(\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}\right)$$
$$= \mu_s g\left(m_0 + \frac{m(\cos\theta)}{(\cos\theta + \mu_s\sin\theta)}\right)$$

Substitute 0.6 for μ_s, $30°$ for θ, 3 kg for m and m_0 and 9.81 m/s^2 for g.

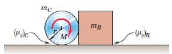
$$P = 0.6(9.81)\left(3 + \frac{3(\cos 30°)}{(\cos 30° + 0.6\sin 30°)}\right)$$
$$= 30.772\text{ N}$$

Therefore, the magnitude *P* of the required horizontal force is $\boxed{30.772\text{ N}}$.

Chapter 6, Problem 29P

Problem

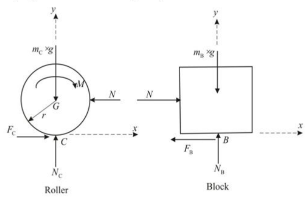
A clockwise couple *M* is applied to the circular cylinder as shown. Determine the value of *M* required to initiate motion for the conditions *mB* = 3 kg, *mC* = 6 kg, (*μs*)*B* = 0.50, (*μs*)*C* = 0.40, and *r* = 0.2 m. Friction between the cylinder *C* and the block *B* is negligible.



Step-by-step solution

Step 1 of 7

Draw the free body diagram of roller and block separately.



Step 2 of 7

Apply the equations of equilibrium and calculate the forces on the block along the *x*-direction.

$\sum F_x = 0$

$N - F_B = 0$

$N = F_B$ ...... (1)

Here F_B is the frictional force exerted by the horizontal surface on the block, N is the normal force exerted by the roller on the block.

Step 3 of 7

Calculate the maximum frictional force exerted on the block *B*.

$F_B = (\mu_S)_B \times N_B$ ...... (2)

Here N_B is the normal force exerted by the horizontal surface on block, and $(\mu_S)_B$ is the coefficient of static friction for block *B*.

Calculate the normal force exerted by the roller on the block.

Substitute equation (1) in equation (2).

$F_B = (\mu_S)_B \times N_B$

$N = (\mu_S)_B \times N_B$ ...... (3)

Step 4 of 7

Apply the equations of equilibrium and calculate the forces on the block along the *y*-direction.

$\sum F_y = 0$

$N_B - m_B \times g = 0$

$N_B = m_B \times g$ ...... (4)

Substitute equation (4) in equation (3).

$N = (\mu_S)_B \times N_B$

$N = (\mu_S)_B \times (m_B \times g)$ ...... (5)

Here m_B is the mass of the block and g is the acceleration due to gravity.

Step 5 of 7

Apply the equations of equilibrium and calculate the forces on the roller along the *x*-direction.

$\sum F_x = 0$

$F_C - N = 0$

$F_C = N$ ...... (6)

Here F_C is the frictional force exerted by the horizontal surface on the roller, N is the normal force exerted by the block on the roller.

Apply the equation of equilibrium along the *y*-direction and calculate the forces on the roller.

$\sum F_y = 0$

$N_C - (m_c \times g) = 0$

$N_C = (m_c \times g)$ ...... (7)

Here m_c is the mass of the roller, and N_C is the normal force exerted by the horizontal surface on the roller.

Step 6 of 7

Apply the equations of equilibrium and calculate the moments of forces on the roller about the point *G* of the roller.

$\sum M_G = 0$

$F_C \times r - M = 0$

$F_C \times r = M$ ...... (8)

Here M is the external clockwise moment applied to the roller.

Calculate the external clockwise moment acting on the roller.

Substitute equation (6) in equation (8).

$F_C \times r = M$

$M = N \times r$

Here, r is the radius of the roller.

$M = ((\mu_S)_B \times (m_B \times g)) \times r$

Substitute 0.5 for $(\mu_S)_B$, 3 kg for m_B, 9.81 N/kg for g, and 0.2 m for r.

$M = (0.5 \times (3 \times 9.81)) \times 0.2$
$M = 2.94\text{ N}\cdot\text{m}$

Calculate the maximum friction force exerted on the roller.

$(F_C)_{max} = (\mu_S)_C \times N_C$ ...... (10)

Here $(\mu_S)_C$ is the coefficient of static friction for roller *C*.

Substitute equation (7) in equation (10).

$(F_C)_{max} = (\mu_S)_C \times N_C$

$(F_C)_{max} = (\mu_S)_C \times (m_C \times g)$

Substitute 0.4 for $(\mu_S)_C$, 6 kg for m_C, 9.81 N/kg for g.

$(F_C)_{max} = 0.4 \times (6 \times 9.81)$
$(F_C)_{max} = 235\text{ N}$

Step 7 of 7

Check the validity of the assumption that the block only slips but roller does not.

Substitute equation (6) in equation (5).

$F_C = (\mu_S)_B \times (m_B \times g)$

Substitute 0.5 for $(\mu_S)_B$, 3 kg for m_B, and 9.81 N/kg for g.

$F_C = 0.5 \times (3 \times 9.81)$
$F_C = 14.72\text{ N}$

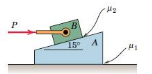
Since, F_C is less than $(F_C)_{max}$, the assumption that roller does not slip is valid.

Therefore, the value of moment that required for initiating motion under given conditions is $\boxed{2.94\text{ N}\cdot\text{m}}$.

Chapter 6, Problem 30P

Problem

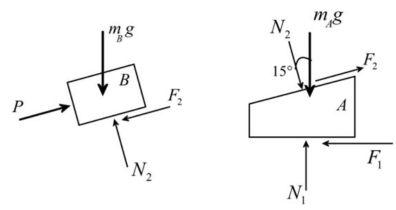
The horizontal force $P = 50$ N is applied to the upper block with the system initially stationary. The block masses are $m_A = 10$ kg and $m_B = 5$ kg. Determine if and where slippage occurs for the following conditions on the coefficients of static friction: (a) $\mu_1 = 0.40$, $\mu_2 = 0.50$ and (b) $\mu_1 = 0.30$, $\mu_2 = 0.60$. Assume that the coefficients of kinetic friction are 75 percent of the static values.



Step-by-step solution

Step 1 of 5

Draw the free body diagram for blocks *A* and *B* as follows:



Step 2 of 5

Consider the horizontal force equilibrium condition for block *A*:

$$\Sigma F_x = 0$$
$$N_2 \sin 15° + F_2 \cos 15° - F_1 = 0 \quad \text{...... (1)}$$
$$F_1 = 0.258 N_2 + 0.965 F_2$$

Here, F_1 and F_2 are the friction forces and N_2 is the normal force due to block *B*.

Consider the vertical force equilibrium condition for block *B*:

$$\Sigma F_y = 0$$
$$N_1 + F_2 \sin 15° - N_2 \cos 15° - m_A g = 0$$

Substitute 10 kg for m_A and $9.81\ \text{m/s}^2$ for g.

$$N_1 + 0.258 F_2 - 0.965 N_2 - (10)(9.81) = 0$$
$$N_1 + 0.258 F_2 - 0.965 N_2 - 98.1 = 0 \quad \text{....... (2)}$$

Consider the horizontal force equilibrium condition for block *B*:

$$\Sigma F_x = 0$$
$$P - F_2 \cos 15° - N_2 \sin 15° = 0$$

Substitute 50 N for *P*.

$$50 - 0.965 \cos 15° - 0.258 N_2 = 0 \quad \text{....... (3)}$$

Consider the vertical force equilibrium condition for block *B*:

$$\Sigma F_y = 0$$
$$-F_2 \sin 15° + N_2 \cos 15° - m_B g = 0$$

Substitute 5 kg for m_B and $9.81\ \text{m/s}^2$ for g.

$$-F_2 \sin 15° + N_2 \cos 15° - (5 \times 9.81) = 0$$
$$-0.258 F_2 + 0.965 N_2 - 49.01 = 0 \quad \text{....... (4)}$$

Step 3 of 5

Solve equations (3) and (4), obtain the following values:

$$N_2 = 60.37\ \text{N}$$
$$F_2 = 35.67\ \text{N}$$

Substitute 60.37 N for N_2 and 35.67 N for F_2 in equation (1).

$$\begin{aligned} F_1 &= 0.258 N_2 + 0.965 F_2 \\ &= 0.258(60.37) + 0.965(35.67) \\ &= 50\ \text{N} \end{aligned}$$

Substitute 60.37 N for N_2 and 35.67 N for F_2 in equation (2).

$$N_1 + 0.258 F_2 - 0.965 N_2 - 98.1 = 0$$
$$N_1 + 0.258(35.67) - 0.965(60.37) - 98.1 = 0$$
$$N_1 = 147.154\ \text{N}$$

Step 4 of 5

(a)

Calculate the maximum frictional force between the ground and block *A*.

$$F_{1,\max} = \mu_1 N_1$$

Substitute 0.4 for μ_1 and 147.154 N for N_1.

$$\begin{aligned} F_{1,\max} &= (0.4)(147.154) \\ &= 58.861\ \text{N} \end{aligned}$$

As $F_{1,\max} > F_1$, slippage would not occurs between the ground and block *A*.

Calculate the maximum frictional force between the block *B* and block *A*.

$$F_{2,\max} = \mu_2 N_2$$

Substitute 0.5 for μ_2 and 60.37 N for N_2.

$$\begin{aligned} F_{2,\max} &= (0.5)(60.37) \\ &= 30.185\ \text{N} \end{aligned}$$

As $F_{2,\max} < F_2$, slippage occurs between the blocks *A* and *B*.

Therefore, the slippage occurs between the blocks $\boxed{A \text{ and } B}$.

Step 5 of 5

(b)

Calculate the maximum frictional force between the ground and block *A*.

$$F_{1,\max} = \mu_1 N_1$$

Substitute 0.3 for μ_1 and 147.154 N for N_1.

$$\begin{aligned} F_{1,\max} &= (0.3)(147.154) \\ &= 44.146\ \text{N} \end{aligned}$$

As $F_{1,\max} < F_1$, slippage occurs between the ground and block *A*.

Calculate the maximum frictional force between the block *B* and block *A*.

$$F_{2,\max} = \mu_2 N_2$$

Substitute 0.6 for μ_2 and 60.37 N for N_2.

$$\begin{aligned} F_{2,\max} &= (0.6)(60.37) \\ &= 36.222\ \text{N} \end{aligned}$$

As $F_{2,\max} > F_2$, slippage would not occurs between the blocks *A* and *B*.

Therefore, the slippage occurs between the $\boxed{\text{ground surface and } A}$.

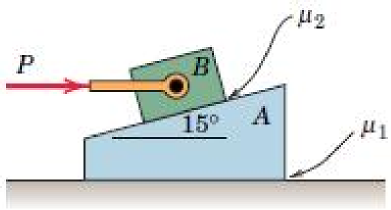
Chapter 6, Problem 31P

Problem

Reconsider the system of Prob. 6/30. If P = 40 N, $\mu 1$ = 0.30, and $\mu 2$ = 0.50, determine the force which block *B* exerts on block *A*. Assume that the coefficients of kinetic friction are 75 percent of the static values. The block masses remain mA = 10 kg and mB = 5 kg.

Problem. 6/30

The horizontal force P = 50 N is applied to the upper block with the system initially stationary. The block masses are mA = 10 kg and mB = 5 kg. Determine if and where slippage occurs for the following conditions on the coefficients of static friction: (*a*) $\mu 1$ = 0.40, $\mu 2$ = 0.50 and (*b*) $\mu 1$ = 0.30, $\mu 2$ = 0.60. Assume that the coefficients of kinetic friction are 75 percent of the static values.



Step-by-step solution

There is no solution to this problem yet.

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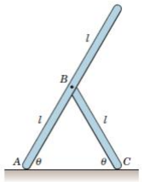
Chapter 6, Problem 32P

Problem

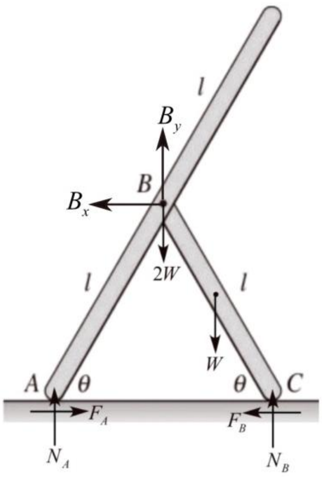
The two uniform slender bars constructed from the same stock material are freely pinned together at *B*. Determine the minimum angle θ at which slipping does not occur at either contact point *A* or *C*. The coefficient of static friction at both *A* and *C* is $\mu s = 0.50$. Consider only motion in the vertical plane shown.



Step-by-step solution

Step 1 of 3

Draw a free body diagram of the system.



Comments (1)

Anonymous

Wouldn't a subscript of "C" make more sense for the normal and friction forces at C? It would be less confusing for me.

Step 2 of 3

Apply the equation of equilibrium along *x* direction.

$$\sum F_x = 0$$
$$F_A - B_x = 0$$
$$F_A = B_x$$

Here, F_A is the reaction force at point *A* and B_x is the horizontal reaction force at Point *B*.

Apply the equation of equilibrium along *y* direction.

$$\sum F_y = 0$$
$$N_A + B_y = 2W$$
$$N_A + B_y = 2mg$$
$$B_y = 2mg - N_A$$

Here, N_A is the normal force at point *A*, *m* is the mass, *g* is the acceleration due to gravity, and B_y is the vertical reaction force at point *B*.

Apply moment about point *A*.

$$\sum M_A = 0$$
$$B_x \times l\sin\theta + B_y \times l\cos\theta = 2mg\left(l\cos\theta\right) \text{ ...... (1)}$$

Here, *l* is the length of member *AB* and θ is angle.

Step 3 of 3

Assume slippage at point *A*.

$$F_A = \mu_s N_A$$

Here, μ_s is the coefficient of static friction and N_A is the normal force at point *A*.

Substitute B_x for F_A and 0.5 for μ_s

$$B_x = 0.5N_A$$

Substitute $0.5N_A$ for B_x and $\left(2mg - N_A\right)$ for B_y in equation (1).

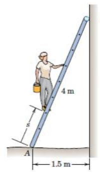
$$0.5N_A \times l\sin\theta + \left(2mg - N_A\right) \times l\cos\theta = 2mg\left(l\cos\theta\right)$$
$$0.5N_A \times l\sin\theta + \left(2mg - N_A\right) \times l\cos\theta - 2mg\left(l\cos\theta\right) = 0$$
$$0.5N_A\sin\theta - N_A\cos\theta = 0$$
$$0.5\sin\theta = \cos\theta$$
$$Tan\theta = 2$$
$$\theta = Tan^{-1}(2)$$
$$= 63.43°$$

Therefore, the minimum angle is $\boxed{63.43°}$.

Chapter 6, Problem 33P

Problem

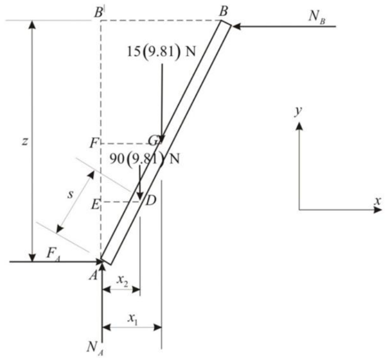
Determine the distance s to which the 90-kg painter can climb without causing the 4-m ladder to slip at its lower end A. The top of the 15-kg ladder has a small roller, and at the ground the coefficient of static friction is 0.25. The mass center of the painter is directly above her feet.



Step-by-step solution

Step 1 of 7

Draw the free body diagram of the ladder with painter at s m height on the ladder.



Step 2 of 7

The friction at the roller end *B* is neglected. Therefore only normal force acts at end *B*.

Calculate the vertical length of the wall from end *B* to end *A* using the geometry of the free body diagram.

$$z = \sqrt{(4^2 - 1.5^2)}$$
$$z = 3.71 \text{ m}$$

Here z is the vertical length of the wall from end *B* to end *A*.

Step 3 of 7

Calculate the horizontal distance from center of the ladder to the vertical axis passing through the end *A*.

Consider the similar triangles ABB' and AGF in the geometry of free body diagram.

$$\frac{3.71}{1.5} = \frac{\left(3.71/2\right)}{x_1}$$
$$x_1 = 0.75 \text{ m}$$

Here x_1 is the horizontal distance from center of the ladder to the vertical axis passing through the end *A*.

Step 4 of 7

Calculate the maximum frictional force at end *A* when the ladder slips.

$$F_A = (\mu_s)_A \times N_A \text{ ...... (1)}$$

Here F_A is the frictional force acting at end *A*, N_A is the normal force exerted at end *A*, and $(\mu_s)_A$ is the coefficient of static friction at end *A*.

Substitute 0.25 for $(\mu_s)_A$ in equation (1).

$$F_A = (\mu_s)_A \times N_A$$
$$F_A = 0.25 \times N_A$$

Step 5 of 7

Apply the equations of equilibrium and calculate the forces along the *y*-direction.

$$N_A - (15 \times 9.81) - (90 \times 9.81) = 0$$
$$N_A = 1030 \text{ N}$$

Consider an imaginary end B' on the vertical axis passing through end *A*.

Apply the equations of equilibrium and calculate the moments about the point B'.

$$(F_A \times 3.71) - (15 \times g \times x_1) - (90 \times g \times x_2) = 0 \text{ ...... (2)}$$

Here x_2 is the horizontal distance from painter position on the ladder to the vertical axis passing through the end *A*, g is the acceleration due to gravity.

Step 6 of 7

Calculate the horizontal distance from painter position on the ladder to the vertical axis.

Substitute $0.25 \times N_A$ for F_A, 1030 N for N_A, 9.81 N/kg for g and 0.75 m for x_1 in equation (2).

$$(F_A \times 3.71) - (15 \times g \times x_1) - (90 \times g \times x_2) = 0$$
$$(0.25 \times N_A \times 3.71) - (15 \times 9.81 \times 0.75) - (90 \times 9.81 \times x_2) = 0$$
$$(0.25 \times 1030 \times 3.71) - (15 \times 9.81 \times 0.75) - (90 \times 9.81 \times x_2) = 0$$
$$x_2 = 0.957 \text{ m.}$$

Step 7 of 7

Calculate the height that the painter climbs on the ladder.

Consider the similar triangles ADE and ABB' in the geometry of the free body diagram.

$$\frac{s}{x_2} = \frac{4}{1.5} \text{ ...... (3)}$$

Here s is the height that the painter climbs on the ladder.

Substitute 0.957 m for x_2 in equation (3).

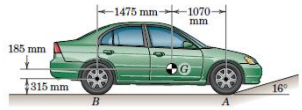
$$\frac{s}{x_2} = \frac{4}{1.5}$$
$$\frac{s}{0.957} = \frac{4}{1.5}$$
$$s = \frac{3.828}{1.5}$$
$$s = 2.552 \text{ m}$$

Therefore, the height of the ladder up to which the painter can climb without slipping of the ladder is $\boxed{2.552 \text{ m}}$.

Chapter 6, Problem 34P

Problem

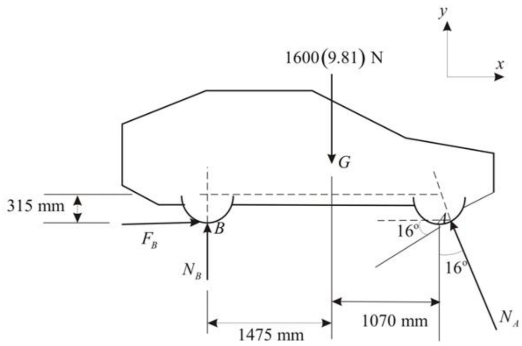
The 1600-kg car is just beginning to negotiate the 16° ramp. If the car has rear-wheel drive, determine the minimum coefficient of static friction required at *B*.



Step-by-step solution

Step 1 of 4

Draw the free body diagram of the car.



Step 2 of 4

Frictional force at front wheel *A* doesn't exist, as the car has rear wheel drive.

Apply the equations of equilibrium along the *x*-direction and calculate the forces.

$\sum F_x = 0$

$F_B - N_A \times \sin 16° = 0$ ...... (1)

Here F_B is the frictional force at rear wheel, and N_A is the normal force exerted on the front wheel by the ramp surface at an angle of 16 degrees.

Apply the equations of equilibrium along the *y*-direction and calculate the forces.

$\sum F_y = 0$

$N_B + N_A \times \cos 16° - (1600 \times 9.81) = 0$ ...... (2)

Here N_B is the normal force exerted by the ground on the rear wheel.

Step 3 of 4

Apply the equations of equilibrium and calculate the moments about the point *B*.

$\sum M_B = 0$

$N_A \times \cos 16° \times (1475 + 1070) + N_A \times \sin 16° \times 315 - (1600 \times 9.81) \times 1475 = 0$

Calculate the normal force exerted on the front wheel by the ramp surface.

$N_A \times 2533.23 = 23151600$

$N_A = 9139.16\text{ N}$ ...... (3)

Calculate the normal force exerted by the ground on the rear wheel.

Substitute N_A from equation (3) in equation (2).

$N_B + N_A \times \cos 16° - (1600 \times 9.81) = 0$

$N_B + (9139.16) \times \cos 16° - (1600 \times 9.81) = 0$

$N_B = 6910.9\text{ N}$ ...... (4)

Comments (1)

Anonymous

why is the moment Na sin(16) positive? isn't it supposed to be negative?

Step 4 of 4

Calculate the frictional force at rear wheel.

Substitute N_A from equation (3) in equation (1).

$F_B - N_A \times \sin 16° = 0$

$F_B - (9139.16) \times \sin 16° = 0$

$F_B = 2519.09\text{ N}$

Calculate the minimum coefficient of static friction required at rear wheel.

$\mu_s = \frac{F_B}{N_B}$

Substitute 2519.09 N for F_B and 6910.9 N for N_B.

$\mu_s = \frac{2519.09}{6910.9}$

$\mu_s = 0.3645$

Hence, the value of minimum coefficient of static friction required at rear wheel to move the car is $\boxed{0.3645}$.

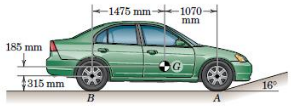
Chapter 6, Problem 35P

Problem

Repeat Prob. 6/34, but now the car has all-wheel drive. Assume that slipping occurs at *A* and *B* simultaneously.

Problem. 6/34

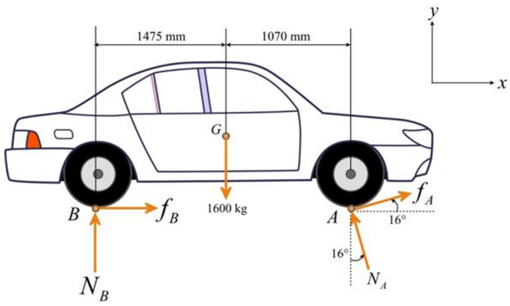
The 1600-kg car is just beginning to negotiate the 16° ramp. If the car has rear-wheel drive, determine the minimum coefficient of static friction required at *B*.



Step-by-step solution

Step 1 of 5

Draw the free body diagram of the car.



Step 2 of 5

Apply the moment equilibrium about the point *A*. to calculate the normal reaction force acting at point *B*.

$$-N_B(1475+1070)+(1600\times9.81)(1070)=0$$
$$2545N_B=16794720$$
$$N_B=6599.1\text{ N}$$

Here, N_B is the normal reaction force at point *B*.

Step 3 of 5

Apply the force equilibrium along *y*-direction to calculate the friction force acting at point *A*.

$$N_B+F_A\times\sin16°+N_A\times\cos16°-(1600\times9.81)=0$$

Here, f_A is the frictional force acting at point *A* and N_A is the normal reaction force at point *A*.

Substitute 6599.1N for N_B and μN_A for f_A, where μ is the coefficient of static friction.

$$6599.1+\mu N_A\times\sin16°+N_A\times\cos16°-(1600\times9.81)=0$$

$$\mu N_A\times\sin16°+N_A\times\cos16°=9096.9$$
$$N_A(\mu\sin16°+\cos16°)=9096.9$$
$$N_A=\frac{9096.9}{\mu\sin16°+\cos16°}$$

Step 4 of 5

Apply the force equilibrium along *x*-direction.

$$f_B+f_A\cos16°-N_A\times\sin16°=0$$

Substitute μN_B for f_B, and μN_A for f_A.

$$\mu N_B+\mu N_A\cos16°-N_A\times\sin16°=0$$

Substitute 6599.1N for N_B.

$$6599.1\mu+\mu N_A\cos16°-N_A\times\sin16°=0$$

Substitute $\frac{9096.9}{\mu\sin16°+\cos16°}$ for N_A.

$$6599.1\mu+\mu\left(\frac{9096.9}{\mu\sin16°+\cos16°}\right)\cos16°-\left(\frac{9096.9}{\mu\sin16°+\cos16°}\right)\times\sin16°=0$$

$$1818.96\mu^2+6343.46\mu+8744.5\mu-2507.44=0$$

$$1818.96\mu^2+15087.96\mu-2507.44=0 \;\ldots\ldots\; (1)$$

Step 5 of 5

Solve for the roots of the quadratic equation (1).

$$\mu=\frac{-15087.96\pm\sqrt{(15087.96)^2-4(1818.96)(-2507.44)}}{2(1818.96)}$$

$$\mu=\frac{-15087.96\pm15680.89}{3637.92}$$
$$\mu=-4.14\pm4.31$$
$$\mu=0.17,-8.45$$

Choose the positive root.

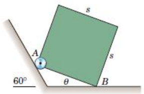
$$\mu=0.17$$

Therefore, the minimum coefficient of static friction required for all-wheel drive at *A* and *B* is $\boxed{0.17}$.

Chapter 6, Problem 36P

Problem

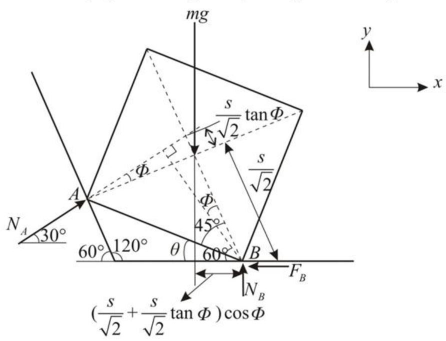
The homogeneous square body is positioned as shown. If the coefficient of static friction at *B* is 0.40, determine the critical value of the angle θ below which slipping will occur. Neglect friction at *A*.



Step-by-step solution

Step 1 of 5

Draw the free body diagram of the homogeneous square body positioned at an angle.



Step 2 of 5

Calculate the maximum frictional force at *B*.

$$F_B = \mu_s \times N_B$$

Here, F_B is the frictional force at contact *B*, N_B is the normal force exerted at contact point *B*, and μ_s is the coefficient of static friction at *B*.

Substitute 0.4 for μ_s.

$$F_B = 0.4 \times N_B$$

Apply the equations of equilibrium and calculate the forces along the *x*-direction.

$$F_B - N_A \times \cos 30^\circ = 0$$

Here, N_A is the normal force exerted at wheel contact *A*.

Substitute $0.4 \times N_B$ for F_B.

$$F_B - N_A \times \cos 30^\circ = 0$$
$$(0.4 \times N_B) - N_A \times \cos 30^\circ = 0$$
$$N_B = \frac{N_A \times \cos 30^\circ}{0.4}$$
$$N_B = 2.17 \times N_A$$

Step 3 of 5

Apply the equations of equilibrium and calculate the forces along the *y*-direction.

$$N_B + N_A \times \sin 30^\circ = mg$$

Here, m is the mass of the square body and g is the acceleration due to gravity.

Substitute $2.17 N_A$ for N_B.

$$(2.17 N_A) + N_A \times \sin 30^\circ = mg$$
$$2.67 \times N_A = mg$$
$$N_A = 0.375 \times mg$$

Step 4 of 5

Apply the equations of equilibrium and calculate the moments about the point *B*.

$$\sum M_B = 0$$
$$mg \times \frac{s}{\sqrt{2}}\cos(60^\circ + \phi) = N_A \times \left(\left(\frac{s}{\sqrt{2}} + \frac{s}{\sqrt{2}}\tan\phi\right)\cos\phi\right)$$

Here, ϕ is the angle between normal force at *A* and diagonal of square passing through *A*, and s is the length of the side of square.

Substitute $0.375 \times mg$ for N_A.

$$mg \times \frac{s}{\sqrt{2}}\cos(60^\circ + \phi) = N_A \times \left(\left(\frac{s}{\sqrt{2}} + \frac{s}{\sqrt{2}}\tan\phi\right)\cos\phi\right)$$
$$mg \times \frac{s}{\sqrt{2}}\cos(60^\circ + \phi) = (0.375 \times mg) \times \left(\left(\frac{s}{\sqrt{2}} + \frac{s}{\sqrt{2}}\tan\phi\right)\cos\phi\right)$$
$$\cos(60^\circ + \phi) = 0.375 \times ((1 + \tan\phi)\cos\phi)$$
$$\cos(60^\circ + \phi) = 0.375 \times \left(\left(1 + \frac{\sin\phi}{\cos\phi}\right)\cos\phi\right)$$
$$\cos(60^\circ + \phi) = 0.375 \times (\cos\phi + \sin\phi) \quad \cos(60^\circ + \phi) = 0.375 \times (\cos\phi + \sin\phi)$$
$$0.5\cos\phi - 0.866\sin\phi = 0.375\cos\phi + 0.375\sin\phi$$
$$0.125\cos\phi = 1.241\sin\phi$$
$$\tan\phi = 0.1$$
$$\phi = 5.7^\circ$$

Step 5 of 5

Calculate the critical angle for the square body below which the square would slip.

Consider the geometry of the square body and its angles in the free body diagram.

$$60^\circ + \phi = \theta + 45^\circ$$

Here, θ is the angle which the square body makes with the horizontal surface. Also here θ is the critical angle for the square body below which the square would slip.

Substitute 5.7° for ϕ.

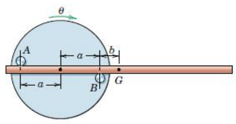
$$60^\circ + 5.7^\circ = \theta + 45^\circ$$
$$\theta = 20.7^\circ$$

Therefore, the critical value of angle θ below which the square body slips is $\boxed{20.7^\circ}$.

Chapter 6, Problem 37P

Problem

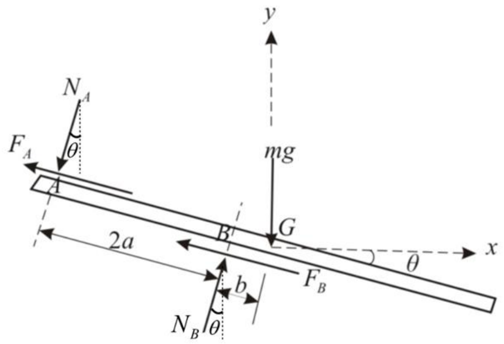
The uniform rod with center of mass at G is supported by the pegs A and B, which are fixed in the wheel. If the coefficient of friction between the rod and pegs is μ, determine the angle θ through which the wheel may be slowly turned about its horizontal axis through O, starting from the position shown, before the rod begins to slip. Neglect the diameter of the rod compared with the other dimensions.



Step-by-step solution

Step 1 of 5

Draw the free body diagram of the rod, when the wheel is rotated by angle θ as shown below.



Step 2 of 5

Calculate the algebraic sum of the moments of all the forces about point G and equate it zero.

$$\sum M_G = 0$$

Here, the algebraic sum of the moments of all the forces about point G is $\sum M_G$.

$$\left(N_A \times (2a+b)\right) - \left(N_B \times b\right) = 0$$

$$\frac{N_B}{N_A} = \frac{(2a+b)}{b}$$

Here, N_A and N_B are the normal forces exerted at pegs A and B respectively.

Step 3 of 5

Write the equations for the friction force at pegs A and B.

$$F_A = \mu N_A$$
$$F_B = \mu N_B$$

Here, the friction force at pegs A and B are F_A and F_B respectively and the coefficient of friction between the rod and the pegs A and B is μ.

Step 4 of 5

Apply force equilibrium equation for forces acting along the rod.

$$N_B \sin\theta - N_A \sin\theta - F_A \cos\theta - F_B \cos\theta = 0$$

Here, the angle of the wheel at which rod slips is θ.

Substitute μN_A for F_A and μN_B for F_B.

$$N_B \sin\theta - N_A \sin\theta - \mu N_A \cos\theta - \mu N_B \cos\theta = 0$$

$$(N_B - N_A)\sin\theta - \mu(N_A + N_B) \times \cos\theta = 0$$

$$(N_B - N_A)\sin\theta = \mu(N_A + N_B) \times \cos\theta$$

$$\tan\theta = \mu\left(\frac{N_A + N_B}{N_B - N_A}\right)$$

$$\tan\theta = \mu \frac{\left(1 + \frac{N_B}{N_A}\right)}{\left(\frac{N_B}{N_A} - 1\right)} \quad \text{...... (1)}$$

Step 5 of 5

Calculate the of θ by substituting the value $\frac{(2a+b)}{b}$ for $\frac{N_B}{N_A}$ in equation (1).

$$\tan\theta = \mu \times \frac{\left(1 + \frac{(2a+b)}{b}\right)}{\left(\frac{(2a+b)}{b} - 1\right)}$$

$$\tan\theta = \mu\left(\frac{a+b}{a}\right)$$

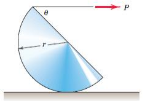
$$\theta = \tan^{-1}\left(\mu\left(\frac{a+b}{a}\right)\right)$$

Therefore, the value of angle θ for which the wheel can be rotated about its horizontal axis without slipping of the rod is $\boxed{\tan^{-1}\left(\mu\left(\frac{a+b}{a}\right)\right)}$.

Chapter 6, Problem 38P

Problem

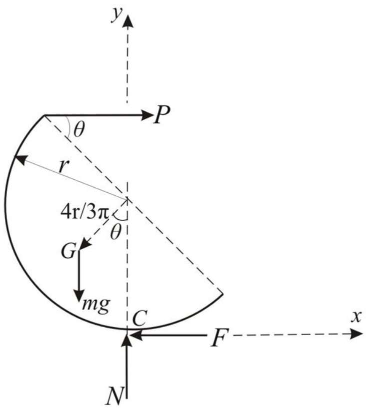
The solid semicylinder of mass *m* and radius *r* is rolled through an angle θ by the horizontal force *P*. If the coefficient of static friction is μs, determine the angle θ at which the cylinder begins to slip on the horizontal surface as *P* is gradually increased. What value of μs would permit θ to reach 90°?



Step-by-step solution

Step 1 of 6

Draw the free body diagram of the semi cylinder.



Step 2 of 6

Apply the equations of equilibrium and calculate the forces along the *y*-direction.

$$\sum F_y = 0$$

$$N = m \times g$$

Here N is the normal force acting at *C*, m is the mass of the semi cylinder, and g is the acceleration due to gravity.

Calculate the maximum frictional force.

$$F = \mu_s \times N$$

Here F is the friction force at contact *C*, and μ_s is the coefficient of static friction.

Step 3 of 6

Apply the equations of equilibrium and calculate the forces along the *x*-direction.

$$\sum F_x = 0$$

$$P = F$$

Substitute $\mu_s \times N$ for F.

$$P = \mu_s \times N$$

Here P is the external force acting on the semi cylinder.

Step 4 of 6

Apply the equations of equilibrium and calculate the moments about the point *C*.

$$P(r + r\sin\theta) - m \times g \times \left(\frac{4r}{3\pi}\sin\theta\right) = 0 \quad \text{...... (1)}$$

$$P(r + r\sin\theta) - m \times g \times \left(\frac{4r}{3\pi}\sin\theta\right) = 0$$

$$(\mu_s \times N)(r + r\sin\theta) = m \times g \times \left(\frac{4r}{3\pi}\sin\theta\right)$$

Substitute $m \times g$ for N.

$$(\mu_s \times (m \times g)) \times (r + r\sin\theta) = m \times g \times \left(\frac{4r}{3\pi}\sin\theta\right) \quad \text{...... (2)}$$

Step 5 of 6

Calculate the angle at which cylinder begins to slip on the horizontal surface.

Solve the equation (2).

$$(\mu_s \times (m \times g)) \times (r + r\sin\theta) = m \times g \times \left(\frac{4r}{3\pi}\sin\theta\right)$$

$$(\mu_s) \times r \times (1 + \sin\theta) = \left(\frac{4r}{3\pi}\sin\theta\right)$$

$$\frac{1 + \sin\theta}{\sin\theta} = \frac{4}{3\pi \times \mu_s}$$

$$\frac{1}{\sin\theta} + 1 = \frac{4}{3\pi \times \mu_s}$$

$$\frac{1}{\sin\theta} = \frac{(4 - 3\pi \times \mu_s)}{3\pi \times \mu_s}$$

$$\sin\theta = \frac{3\pi \times \mu_s}{(4 - 3\pi \times \mu_s)}$$

$$\theta = \sin^{-1}\left(\frac{3\pi \times \mu_s}{4 - 3\pi \times \mu_s}\right) \quad \text{...... (3)}$$

Hence, the value of angle θ for which cylinder begins to slip on the horizontal surface under load P is $\boxed{\sin^{-1}\left(\frac{3\pi \times \mu_s}{4 - 3\pi \times \mu_s}\right)}$.

Step 6 of 6

Calculate the coefficient of friction between semi cylinder and surface when angle θ equals 90 degrees.

Substitute $90°$ for θ in equation (3).

$$\theta = \sin^{-1}\left(\frac{3\pi \times \mu_s}{4 - 3\pi \times \mu_s}\right)$$

$$90° = \sin^{-1}\left(\frac{3\pi \times \mu_s}{4 - 3\pi \times \mu_s}\right)$$

$$\frac{3\pi \times \mu_s}{4 - 3\pi \times \mu_s} = 1$$

$$\mu_s = \frac{2}{3\pi}$$

$$\mu_s = 0.318$$

Thus, the coefficient of friction between semi cylinder and surface at an angle θ equals 90 degrees is $\boxed{0.318}$.

Comments (1)

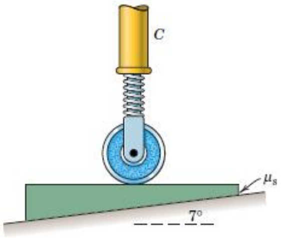
Anonymous

2/(3pi) = 0.2122

Chapter 6, Problem 53P

Problem

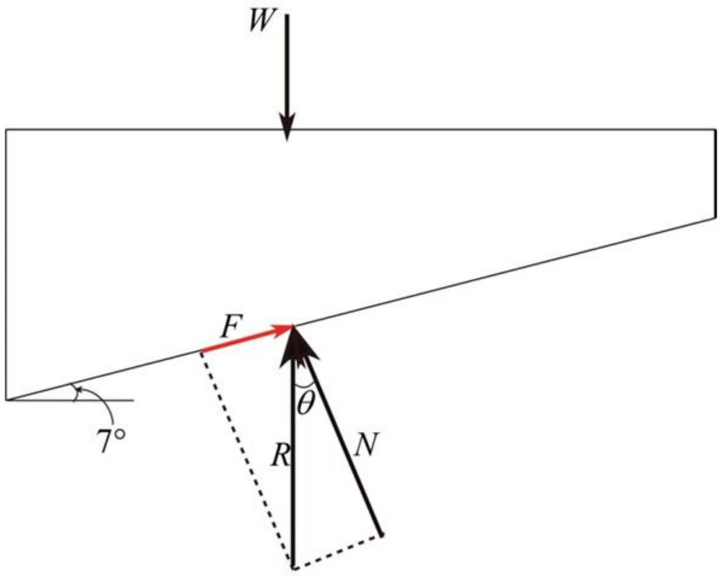
The 7° wedge is driven under the spring-loaded wheel whose supporting strut *C* is fixed. Determine the minimum coefficient of static friction μs for which the wedge will remain in place. Neglect all friction associated with the wheel.



Step-by-step solution

Step 1 of 3

Draw a free body diagram of the wedge..



Step 2 of 3

Here, *R* is the reaction force and *W* is the self-weight.

Step 3 of 3

Consider the relation for angle made by the force.

$$\tan\theta = \frac{F}{N}$$

Here, θ is the angle made by the normal force, *F* is the friction force, and *N* is the normal force.

Consider the relation for coefficient for static friction.

$$\mu_s = \frac{F}{N}$$

Substitute $\tan\theta$ for $\frac{F}{N}$.

$$\mu_s = \tan\theta$$

Substitute $7°$ for θ.

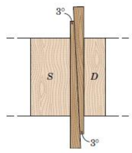
$$\mu_s = \tan(7°) \\ = 0.12278$$

Therefore, the minimum coefficient of static friction is $\boxed{0.12278}$.

Chapter 6, Problem 55P

Problem

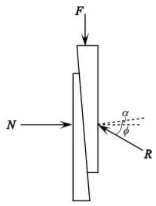
In wood-frame construction, two shims are frequently used to fill the gap between the framing *S* and the thinner window/door jamb *D*. The members *S* and *D* are shown in cross section in the figure. For the 3° shims shown, determine the minimum necessary coefficient of static friction so that the shims will remain in place.



Step-by-step solution

Step 1 of 5

Draw the schematic diagram.



Step 2 of 5

Calculate the forces acting on *x*-axis.

$\Sigma F_x = 0$

Consider the diagram.

$N - R\cos(\phi - \alpha) = 0$

$N = R\cos(\phi - \alpha)$ ...... (1)

Here, *R* is the reaction force, ϕ is the friction angle, α is the wedge angle, and *N* is the normal force.

Calculate the forces acting on *y*-axis.

$\Sigma F_y = 0$

Consider the diagram.

$R\sin(\phi - \alpha) - F = 0$

$F = R\sin(\phi - \alpha)$ ...... (2)

Step 3 of 5

Write the equation for the coefficient of static friction.

$\mu_s = \frac{F}{N}$

Substitute $R\cos(\phi - \alpha)$ for *N*, and $R\sin(\phi - \alpha)$ for *F*.

$\mu_s = \frac{R\sin(\phi - \alpha)}{R\cos(\phi - \alpha)}$

$\mu_s = \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)}$

Write the relation for the coefficient of static friction.

$\mu_s = \tan\phi$ ...... (3)

Substitute $\frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)}$ for μ_s.

$\frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} = \tan\phi$

$\frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} = \frac{\sin\phi}{\cos\phi}$

$\sin\phi\cos(\phi - \alpha) = \cos\phi\sin(\phi - \alpha)$ ...... (4)

Step 4 of 5

Substitute $3°$ for α in equation (4).

$\sin\phi\cos(\phi - \alpha) = \cos\phi\sin(\phi - \alpha)$

$\sin\phi\cos(\phi - 3°) = \cos\phi\sin(\phi - 3°)$ ...... (5)

Solve the equation by trial and error method.

Trial 1

Take $\phi = 1°$

Substitute $1°$ for ϕ in equation (5).

$\sin 1°\cos(1° - 3°) = \cos 1°\sin(1° - 3°)$

$0.0174 \neq -0.0348$

Trial 2

Take $\phi = 1.5°$

Substitute $1.5°$ for ϕ in equation (5).

$\sin 1.5°\cos(1.5° - 3°) = \cos 1°\sin(1.5° - 3°)$

$|0.0261| \neq |-0.0261|$

$0.0261 = 0.0261$

Hence, the friction angle ϕ is $1.5°$

Step 5 of 5

Substitute $1.5°$ for ϕ in equation (3).

$\mu_s = \tan\phi$

$\mu_s = \tan 1.5°$

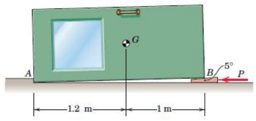
$= 0.0262$

Hence, the coefficient of static friction is $\boxed{0.0262}$.

Chapter 6, Problem 56P

Problem

The 100-kg industrial door with mass center at *G* is being positioned for repair by insertion of the 5° wedge under corner *B*. Horizontal movement is prevented by the small ledge at corner *A*. If the coefficients of static friction at both the top and bottom wedge surfaces are 0.60, determine the force *P* required to lift the door at *B*.



Step-by-step solution

Step 1 of 6

Given that,

Mass of the industrial door = 100 kg

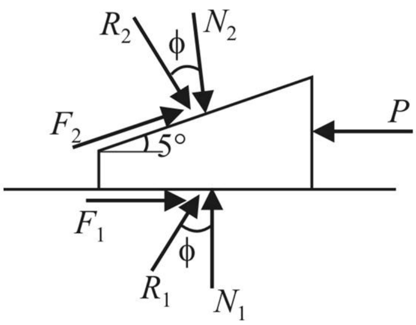
Wedge angle, $\alpha = 5°$

The coefficients of static friction at both the top and bottom wedge surfaces are $\mu = 0.6$

$\therefore$ The friction angle, $\phi = \tan^{-1}(\mu)$

$= 30.96°$

Step 2 of 6



Step 3 of 6

From the Free Body Diagram of wedge

Writing equations of equilibrium

$\sum F_x = 0$

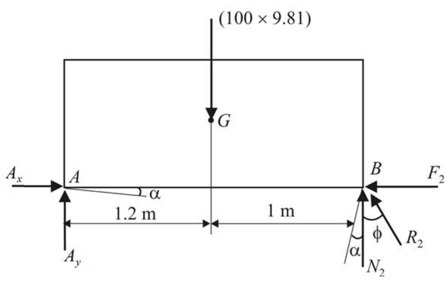
$R_1 \sin\phi + R_2 \sin(\phi+\alpha) = P$ ....(1)

$\sum F_y = 0$

$R_1 \cos\phi - R_2 \cos(\phi+\alpha) = 0$

$R_1 = \dfrac{R_2 \cos(\phi+\alpha)}{\cos\phi}$ ....(2)

Step 4 of 6



Comments (1)

Anonymous

hi , i dont understand the alpha place on the vertical

Step 5 of 6

From the free body diagram of door

Writing moment equilibrium equation about point 'A'

$-(100\times 9.81\times 1.2)+\left[R_2 \cos(\phi+\alpha)\times 2.2\right]=0$

$-(100\times 9.81\times 1.2)+\left[R_2 \cos(30.96°+5°)\times 2.2\right]=0$

$1177.2-1.78R_2=0$

$R_2 = 661.35\,\text{N}$

Step 6 of 6

From (2)

$R_1 = \dfrac{661.35\times\cos 35.96°}{\cos 30.96°}$

$R_1 = 624.25\,\text{N}$

From (1)

$624.25\times\sin 30.96° + 661.35\sin 35.96° = P$

$\boxed{P = 709.5\,\text{N}}$

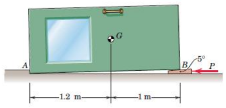
Chapter 6, Problem 57P

Problem

Calculate the rightward force P' which would remove the wedge from under the door of Prob. 6/56. Assume that corner A does not slip for your calculation of P', but then check this assumption; the coefficient of static friction at A is 0.60.

Problem. 6/56

The 100-kg industrial door with mass center at G is being positioned for repair by insertion of the 5° wedge under corner B. Horizontal movement is prevented by the small ledge at corner A. If the coefficients of static friction at both the top and bottom wedge surfaces are 0.60, determine the force P required to lift the door at B.



Step-by-step solution

Step 1 of 6

Given that,

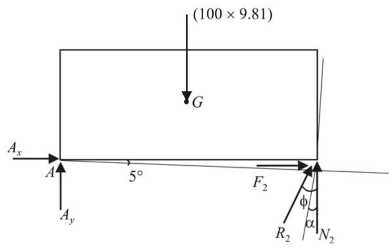
Mass of the industrial door = 100 kg

Wedge angle, $\alpha = 5°$

The coefficients of static friction at both the top and bottom wedge surfaces are $\mu = 0.6$

$\therefore$ The friction angle, $\phi = \tan^{-1}(\mu)$

$= 30.96°$



Step 2 of 6

From the free body diagram of door

Writing moment of equilibrium equation about point 'A'

$\sum M_A = 0$

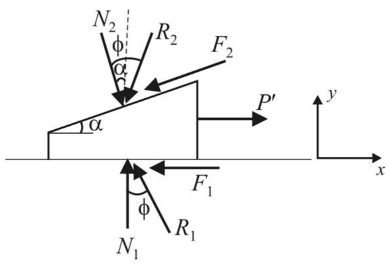
$-(100 \times 9.81 \times 1.2) + [R_2 \cos(\phi - \alpha) \times 2.2] = 0$

$-(100 \times 9.81 \times 1.2) + [R_2 \cos(30.96° - 5°) \times 2.2] = 0$

$1177.2 - 1.978 R_2 = 0$

$R_2 = 595.15\,\mathrm{N}$

Step 3 of 6



Step 4 of 6

For wedge

Writing equilibrium equations

$\sum F_y = 0$

$R_1 \cos\phi - R_2 \cos(\phi - \alpha) = 0$

$R_1 = \dfrac{R_2 \cos(\phi - \alpha)}{\cos\phi}$

$= \dfrac{595.15 \times \cos 25.96°}{\cos 30.96°}$

$R_1 = 624\,\mathrm{N}$

Step 5 of 6

$\sum F_x = 0$

$P' - R_1 \sin\phi - R_2 \sin(\phi - \alpha) = 0$

$P' = (624 \times \sin 30.96°) + (595.15 \times \sin 25.96°)$

$\boxed{P' = 581.53\,\mathrm{N}}$

Step 6 of 6

From the free body diagram of door

$\sum F_x = 0$

$A_x + R_2 \sin(\phi - \alpha) = 0$

$A_x = -595.15 \sin(30.96° - 5°)$

$A_x = -260.52\ \mathrm{N}$

$\sum F_y = 0$

$A_y + R_2 \cos(\phi - \alpha) - 981 = 0$

$A_y + 595.15 \cos(30.96° - 5°) - 981 = 0$

$A_y = 445.9\ \mathrm{N}$

Coefficient of static friction at A, $\mu_A = \dfrac{A_x}{A_y}$

$\mu_A = \dfrac{260.52}{445.9}$

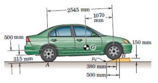
$\mu_A = 0.58$

Hence $\mu_A < 0.60$, so our assumption is correct i.e., corner A does not slip

Chapter 6, Problem 59P

Problem

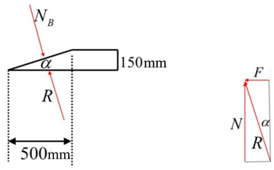
A 1600-kg rear-wheel-drive car is being driven up the ramp at a slow steady speed. Determine the minimum coefficient of static friction *μs* for which the portable ramp will not slip forward. Also determine the required friction force *FA* at each rear drive wheel.



Step-by-step solution

Step 1 of 10

Draw the free-body diagram:



Step 2 of 10

From the free body diagram calculate the angle made by the wedge.

$$\alpha = \tan^{-1}\left(\frac{150}{500}\right)$$
$$= 16.70^\circ$$

Calculate the coefficient of static friction:

Consider the relation for coefficient of static friction.

$$\mu_S = \frac{F}{N}$$

Here, F is the force applied and N is the normal force.

Consider the relation for helix angle.

$$\tan\alpha = \frac{F}{N}$$

Substitute $\frac{F}{N}$ for μ_S.

$$\tan\alpha = \mu_s$$

Here, α is the inclination made by the normal force.

Substitute 16.70° for α.

$$\mu_S = \tan 16.70^\circ$$
$$= 0.3$$

Step 3 of 10

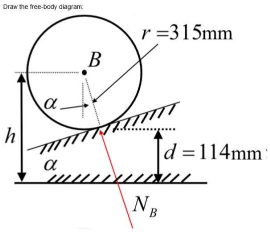
Therefore, the minimum coefficient of static friction is $\boxed{0.3}$.

Comments (1)

Anonymous
correct! thanks

Step 4 of 10

Draw the free-body diagram:



Step 5 of 10

Calculate the height attained by the front wheel:

$$d = \frac{l}{L} \times H$$

Here, *d* is the height attained by the wheel, *l* is the length traveled by the front wheel, *L* is the total length of the ramp and *H* is the height of the ramp.

Substitute 380 mm for *l*, 500 mm for *L*, and 150 mm for *H*.

$$d = \frac{380}{500} \times 150$$
$$= 114 \text{ mm}$$

Step 6 of 10

Calculate the total distance from the wheel center to the ramp base:

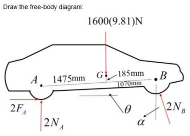
$$h = d + r\cos\alpha$$

Substitute 114 mm for *h*, 315 mm for *r*, and $\cos 16.70^\circ$ for α.

$$h = d + r\cos\alpha$$
$$= 114 + 315\cos 16.70^\circ$$
$$= 416 \text{ mm}$$

Step 7 of 10

Draw the free-body diagram:



Step 8 of 10

From the free-body diagram, calculate the angle θ.

$$\theta = \sin^{-1}\frac{416 - 315}{2545}$$
$$= \sin^{-1}\frac{101}{2545}$$
$$= 2.27^\circ$$

Comments (2)

Anonymous
where does the 416 comes from?

Anonymous
the height of the ramp

Step 9 of 10

Consider the moments about the point *A*.

Since, the entire system is in equilibrium, the sum of moments is equal to zero.

$$\sum M_A = 0$$
$$\left\{\begin{array}{l} -1600(9.81)(1475\cos\theta - 185\sin\theta) \\ +2N_B\cos(\alpha-\theta)(315\sin\theta + 2545) \\ \qquad +2N_B\sin(\alpha-\theta)(315\sin\theta) \end{array}\right\} = 0$$

Substitute 16.70° for α and 2.27° for θ.

$$\left\{\begin{array}{l} -1600(9.81)(1475\cos(2.27^\circ) - 185\sin(2.27^\circ)) + \\ 2N_B\cos(14.43^\circ)(315\sin(2.27^\circ) + 2545) + \\ 2N_B\sin(14.43^\circ)(315\sin(2.27^\circ)) \end{array}\right\} = 0$$
$$\left\{\begin{array}{l} -1600(9.81)(1475\cos(2.27^\circ) - 185\sin(2.27^\circ)) + \\ 2N_B\cos(16.70^\circ - 2.27^\circ)(315\sin(2.27^\circ) + 2545) + \\ 2N_B\sin(16.70^\circ - 2.27^\circ)(315\sin(2.27^\circ)) \end{array}\right\} = 0$$
$$N_B = 4500\text{N}$$

Step 10 of 10

Obtain the equilibrium of forces acting on the car.

Calculate the forces along *x*-direction:

Since, the entire system is in equilibrium, the net horizontal force is equal to zero.

$$\sum F_x = 0$$
$$2F_A - 2N_B\sin\alpha = 0$$

Substitute 4500 N for N_B, 16.70° for α.

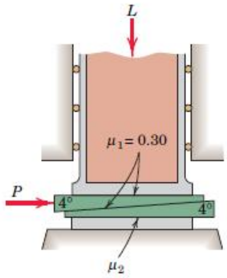
$$2F_A = 2(4500)\sin 16.70^\circ$$
$$F_A = (4500)\sin 16.70^\circ$$
$$= 1294\text{N}$$

Therefore, the required friction force at each rear wheel is $\boxed{1294\text{N}}$.

Chapter 6, Problem 63P

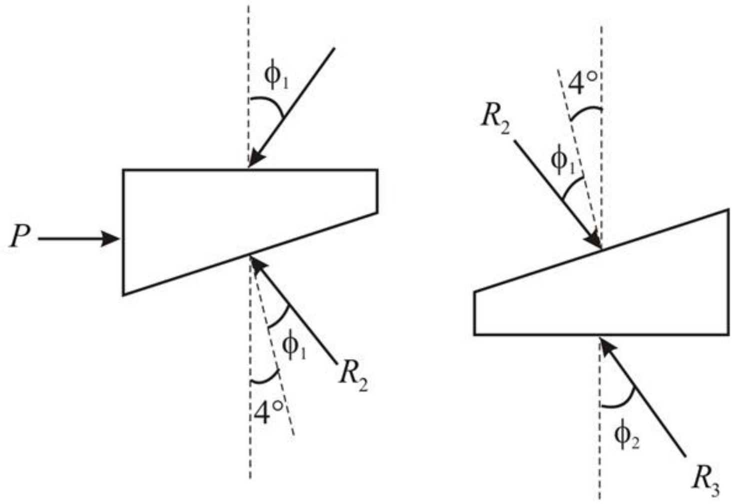
Problem

The two 4° wedges are used to position the vertical column under a load *L*. What is the least value of the coefficient of friction $\mu 2$ for the bottom pair of surfaces for which the column may be raised by applying a single horizontal force *P* to the upper wedge?



Step-by-step solution

Step 1 of 3



Step 2 of 3

Given that,

Wedge angle, $\alpha = 4°$

Coefficient of friction at the upper edge is $\mu_1 = 0.3$

Let R_1 be the resultant force at the top surface of the upper wedge

Let R_2 be the resultant force at the contact surfaces of the 2 wedge

Let R_3 be the resultant force at the bottom surface of the lower wedge

Let ϕ_1 be the friction angle of the upper wedge

Let ϕ_2 be the friction angle of the lower wedge

In order to raise the column the bottom wedge must remain stationary. Therefore μ_2 must be greater than $\tan\phi_2$

Friction angle for the upper wedge, $\phi_1 = \tan^{-1}(0.3)$

$= 16.7°$

Step 3 of 3

The minimum coefficient of friction for the bottom surface is $(\mu_2)_{\min} = \tan(\phi_1 + \alpha)$

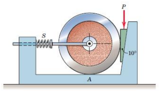
$= \tan(16.7° + 4°)$

$\boxed{(\mu_2)_{\min} = 0.378}$

Chapter 6, Problem 64P

Problem

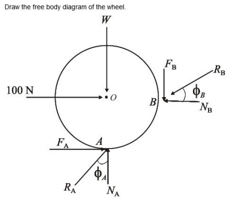
Compute the force P required to move the 20-kg wheel. The coefficient of friction at A is 0.25 and that for both pairs of wedge surfaces is 0.30. Also, the spring S is under a compression of 100 N, and the rod offers negligible support to the wheel.



Step-by-step solution

Step 1 of 11

Draw the free body diagram of the wheel.



Step 2 of 11

Calculate the algebraic sum of the moment of all the forces about point O and equate them to zero.

$$\Sigma M_O = 0$$
$$F_A r - F_B r = 0$$
$$F_A = F_B$$

Here, the algebraic sum of the moment of all the forces about point O is ΣM_O. F_A and F_B are the friction forces acting at points A and B respectively.

Step 3 of 11

Assume that the wheel slips at B first.

Calculate the friction force at B by using the equation:

$$F_B = \mu_B N_B$$

Here, the friction force at B is F_B, the coefficient of friction at B is μ_B and the normal reaction at B is N_B.

Substitute 0.3 for μ_B.

$$F_B = 0.3 N_B$$

Apply horizontal force equilibrium equation.

$$\Sigma F_x = 0$$
$$100 - N_B + F_A = 0$$

Here, the algebraic sum of all the horizontal forces is ΣF_x and the normal reaction at B is N_B.

Substitute $0.3N_B$ for F_A. $[\because F_A = F_B]$

$$100 - N_B + 0.3N_B = 0$$
$$N_B = \frac{100}{0.7} = 142.86\,\text{N}$$

Step 4 of 11

Calculate the friction force at B by using the equation:

$$F_B = \mu_B N_B$$

Substitute 0.3 for μ_B and $142.86\,\text{N}$ for N_B.

$$F_B = 0.3(142.86) = 42.86\,\text{N}$$

Hence,

$$F_A = F_B = 42.86\,\text{N}$$

Step 5 of 11

Calculate the weight of the ball by using the equation:

$$W = mg$$

Here, the weight of the ball is W, the mass of the ball is m and the acceleration due to gravity is g.

Substitute $20\,\text{kg}$ for m and $9.81\,\text{m/s}^2$ for g.

$$W = (20 \times 9.81) = 196.2\,\text{N}$$

Step 6 of 11

Apply vertical force equilibrium equation.

$$\Sigma F_y = 0$$
$$N_A - F_B - W = 0$$

Substitute $42.86\,\text{N}$ for F_B and $196.2\,\text{N}$ for W.

$$N_A - 42.86 - 20(9.81) = 0$$
$$N_A = 239.06\,\text{N}$$

Step 7 of 11

Calculate the coefficient of friction at A by using the equation:

$$\mu_A = \frac{F_A}{N_A}$$

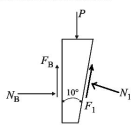
Substitute $42.86\,\text{N}$ for F_A and $239.06\,\text{N}$ for N_A.

$$\mu_A = \frac{42.86}{239.06} = 0.179 < 0.25$$

Hence, our assumption that the wheel slips at B first is correct.

Step 8 of 11

Draw the free Body diagram for wedge



Step 9 of 11

Calculate the friction force at the right side of the wedge by using the equation:

$$F_1 = \mu N_1$$

Here, the friction force at the right side of the wedge is F_1 and normal reaction at the right side of the wedge is N_1.

Substitute 0.3 for N_1.

$$F_1 = 0.3N_1$$

Step 10 of 11

Apply horizontal force equilibrium.

$$\Sigma F_x = 0$$
$$N_B + F_1 \sin 10° - N_1 \cos 10° = 0$$

Substitute $0.3N_1$ for F_1.

$$142.86 + 0.3\,N_1 \sin 10° - N_1 \cos 10° = 0$$
$$N_1 = \frac{142.86}{0.9327} = 153.168\,\text{N}$$

Step 11 of 11

Apply horizontal force equilibrium.

$$\Sigma F_y = 0$$
$$F_B - P + F_1 \cos 10° + N_1 \sin 10° = 0$$

Here, the force required to move the wheel is P.

Substitute $42.86\,\text{N}$ for F_B and $0.3N_1$ for F_1.

$$F_B - P + 0.3N_1 \cos 10° + N_1 \sin 10° = 0$$

Again substitute $153.168\,\text{N}$ for N_1.

$$42.86 - P + 0.3(153.168)\cos 10° + 153.168 \sin 10° = 0$$
$$P = 114.71\,\text{N}$$

Therefore, the force required to move the wheel is $\boxed{114.71\,\text{N}}$.

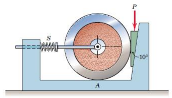
Chapter 6, Problem 65P

Problem

Work Prob. 6/64 if the compression in the spring is 200 N. All other conditions remain unchanged.

Problem. 6/64

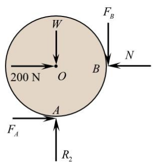
Compute the force *P* required to move the 20-kg wheel. The coefficient of friction at *A* is 0.25 and that for both pairs of wedge surfaces is 0.30. Also, the spring *S* is under a compression of 100 N, and the rod offers negligible support to the wheel.



Step-by-step solution

Step 1 of 8

Draw the free body of the wheel.



Step 2 of 8

Calculate the algebraic sum of the moment of all the forces about point *O* and equate them to zero.

$$\Sigma M_O = 0$$
$$F_A r - F_B r = 0 \quad \ldots\ldots (1)$$
$$F_A = F_B$$

Here, the algebraic sum of the moment of all the forces about point *O* is ΣM_O, F_A and F_B are the friction forces acting at points *A* and *B* respectively.

Step 3 of 8

Assume that the wheel slips at *A* but not *B*.

Calculate the friction force at *A* by using the equation:

$$F_A = \mu_A R_2$$

Here, the friction force at *A* is F_A, the coefficient of friction at *A* is μ_A and the normal reaction at *A* is R_2.

Substitute 0.25 for μ_A.

$$F_A = 0.25 R_2$$

Step 4 of 8

From equation (1).

$$F_A = F_B = 0.25 R_2$$

Apply the force equilibrium equation in *y* direction.

$$\sum F_y = 0$$
$$R_2 - W - F_B = 0$$
$$R_2 - mg - F_B = 0$$

Substitute $0.25R_2$ for F_B. $[\because F_A = F_B]$, 20 kg for *m*, and $9.81\ \text{m/s}^2$ for *g*.

$$R_2 - mg - F_B = 0$$
$$R_2 - 20\times 9.81 - 0.25R_2 = 0$$
$$(1-0.25)R_2 = 20\times 9.81$$
$$R_2 = \frac{20\times 9.81}{0.75} = 261.6\ \text{N}$$

Step 5 of 8

Apply the force equilibrium equation in *x* direction.

$$\sum F_x = 0$$
$$200 - N + F_A = 0$$

Substitute $0.25R_2$ for F_A. $[\because F_A = F_B]$

$$200 - N + 0.25R_2 = 0$$

Substitute 261.6 N for R_2.

$$200 - N + 0.25\times 261.6 = 0$$
$$N = 265.4\ \text{N}$$

Also, from equation (1).

$$F_A = F_B = 0.25\times 261.6 = 65.4\ \text{N}$$

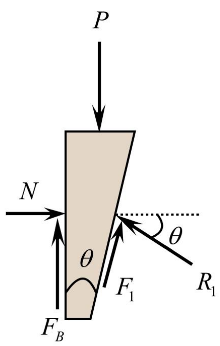
Since,

$$F_B < (\mu_B N)$$
$$65.4\ \text{N} < 0.3\times 265.4\ \text{N}$$
$$65.4\ \text{N} < 79.62\ \text{N}$$

Hence, our assumption is correct.

Step 6 of 8

Draw the free body diagram of the wedge.



Step 7 of 8

Apply the force equilibrium equation in *x* direction.

$$\sum F_x = 0$$
$$N - R_1\cos\theta + F_1\sin\theta = 0$$
$$N - R_1\cos\theta + (\mu_B R_1)\sin\theta = 0$$

Note: μ_B is the coefficient of friction on both the side of the wedge.

Here, R_1 is the reaction force at the wedge and θ is the angle of inclination.

Substitute 0.3 for μ_B, 265.4 N for *N* and $10°$ for θ.

$$265.4 - R_1\cos 10° + 0.3\times R_1\sin 10° = 0$$
$$R_1(\cos 10° - 0.3\sin 10°) = 265.4$$
$$R_1 = 284.54\ \text{N}$$

Step 8 of 8

Apply the force equilibrium equation in *y* direction.

$$\sum F_y = 0$$
$$F_B - P + F_1\cos 10° + R_1\sin 10° = 0$$
$$F_B - P + (\mu_B R_1)\cos 10° + R_1\sin 10° = 0$$

Substitute 65.4 N for F_B, 0.30 for μ_B, and 284.54 N for R_1.

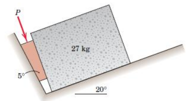
$$65.4 - P + (0.3\times 284.54)\cos 10° + 284.54\times\sin 10° = 0$$
$$P = 198.88\ \text{N}$$

Thus, the value of force *P* required to move the wheel is $\boxed{198.88}$.

Chapter 6, Problem 66P

Problem

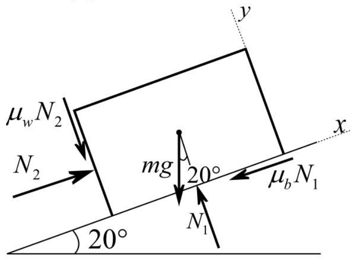
The coefficient of static friction for both wedge surfaces is 0.40 and that between the 27-kg concrete block and the 20° incline is 0.70. Determine the minimum value of the force P required to begin moving the block up the incline. Neglect the weight of the wedge.



Step-by-step solution

Step 1 of 7

Draw the free body diagram of the block.



Step 2 of 7

Apply the equation of equilibrium along the x-direction.

$$\sum F_x = 0$$
$$N_2 - mg\sin 20° - \mu_b N_1 = 09.81\ \text{m/s}^2$$

Here, the mass of the block is m, acceleration due to gravity is g, the coefficient of friction between the block and the plane is μ_b, the normal reaction on the block due to the wedge is N_2, and the normal reaction of the block on the plane is N_1.

Substitute $27\ \text{kg}$ for m, $9.81\ \text{m/s}^2$ for g, and 0.7 for μ_b.

$$N_2 - mg\sin 20° - \mu_b N_1 = 0$$
$$N_2 - (0.7 \times N_1) = 27 \times 9.81 \times \sin 20° \quad \text{....... (1)}$$
$$N_2 - (0.7 \times N_1) = 90.59\ \text{N}$$
$$N_2 = 90.59 + 0.7N_1$$

Comments (3)

Anonymous

Wouldn't the friction force for N2 be in the upwards direction to oppose the motion of the wedge moving down?

Anonymous

it would be if we were looking at just the system of the little block. Since we are looking at the system of the big block we have to look at the equal and opposite reaction from the little block.

Anonymous

You wrote inncorrectly when you wrote the first equation of equilibrium in the x axis. everything = 0 not g

Step 3 of 7

Apply the equation of equilibrium along the y-direction.

$$\sum F_y = 0$$
$$N_1 - mg\cos 20° - \mu_w N_2 = 0$$

Here, the coefficient of the friction of wedge surfaces is μ_w.

Substitute $27\ \text{kg}$ for m, $9.81\ \text{m/s}^2$ for g, and 0.4 for μ_w.

$$N_1 - mg\cos 20° - \mu_w N_2 = 0$$
$$N_1 - (0.4 \times N_2) = 27 \times 9.81 \times \cos 20° \quad \text{....... (2)}$$
$$N_1 - (0.4 \times N_2) = 248.89\ \text{N}$$

Step 4 of 7

Substitute equation (1) in equation (2).

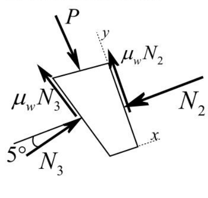
$$N_1 - \left[0.4 \times \left(90.59 + (0.7 \times N_1)\right)\right] = 248.89$$
$$N_1 - 0.28N_1 = 248.89 + 36.236$$
$$N_1 = 395.6\ \text{N}$$

Calculate the normal reaction on the block due to the wedge from the equation (1)

$$\begin{aligned} N_2 &= 90.59 + 0.7N_1 \\ &= 90.59 + (0.7 \times 395.6) \\ &= 367.51\ \text{N} \end{aligned}$$

Step 5 of 7

Draw the free body diagram of the wedge.



Step 6 of 7

Apply the equation of equilibrium along the x-direction.

$$\sum F_x = 0$$
$$N_3\cos 5° - N_2 - \mu_w N_3 \sin 5° = 0$$

Here, the normal reaction of the plane in the wedge is N_3.

Substitute 0.4 for μ_w, and $367.5\ \text{N}$ for N_2.

$$N_3\cos 5° - N_2 - \mu_w N_3 \sin 5° = 0$$
$$N_3\left(\cos 5° - (0.4 \times \sin 5°)\right) = 367.5$$
$$N_3 = 382.28\text{N}$$

Step 7 of 7

Apply the equation of equilibrium along the y-direction.

$$\sum F_y = 0$$
$$\mu_w N_2 + \mu_w N_3 \cos 5° + N_3 \sin 5° - P = 0$$

Here, the minimum applied force is P.

Substitute 0.4 for μ_w, $367.5\ \text{N}$ for N_2, and 382.28N for N_3.

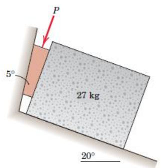
$$\mu_w N_2 + \mu_w N_3 \cos 5° + N_3 \sin 5° - P = 0$$
$$\begin{aligned} P &= \begin{Bmatrix} (0.4 \times 367.5) + (0.4 \times 382.28\cos 5°) \\ +(382.28\sin 5°) \end{Bmatrix} \\ &= 332.64\ \text{N} \end{aligned}$$

Therefore, the minimum force required to move up the block is $\boxed{332.64\ \text{N}}$.

Chapter 6, Problem 67P

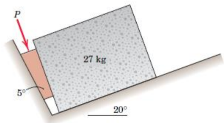
Problem

Repeat Prob. 6/66, only now the 27-kg concrete block begins to move down the 20° incline as shown. All other conditions remain as in Prob. 6/66.



Problem. 6/66

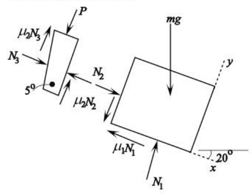
The coefficient of static friction for both wedge surfaces is 0.40 and that between the 27-kg concrete block and the 20° incline is 0.70. Determine the minimum value of the force *P* required to begin moving the block up the incline. Neglect the weight of the wedge.



Step-by-step solution

Step 1 of 5

Draw the schematic diagram.



Step 2 of 5

Consider the Block.

Calculate the forces acting on *x*-axis.

$$\Sigma F_x = 0$$

$$mg\sin 20° + N_2 - \mu_1 N_1 = 0$$

Here, μ_1 is the coefficient of static friction between the block and the surface, *N* is the normal force acting on the block, *m* is the mass of the block, and *g* is the acceleration due to gravity.

Substitute 27kg for *m*, 9.81 m/s^2 for *g*, and 0.7 for μ_1.

$$(27\times 9.81)\sin 20° + N_2 - 0.7N_1 = 0$$

$$90.59 + N_2 - 0.7N_1 = 0$$

$$N_2 = 0.7N_1 - 90.59 \text{ ...... (1)}$$

Step 3 of 5

Calculate the forces acting on *y*-axis.

$$\Sigma F_y = 0$$

$$-mg\cos 20° + N_1 - \mu_2 N_2 = 0$$

Here, μ_2 is the coefficient of static friction for both wedge surfaces.

Substitute 27kg for *m*, 9.81 m/s^2 for *g*, and 0.4 for μ_2.

$$-(27\times 9.81)\cos 20° + N_1 - 0.4N_2 = 0$$

$$-248.89 + N_1 - 0.4N_2 = 0$$

Substitute $0.7N_1 - 90.59$ for N_2.

$$-248.89 + N_1 - 0.4(0.7N_1 - 90.59) = 0$$

$$-248.89 + N_1 - 0.28N_1 + 36.236 = 0$$

$$0.72N_1 = 212.6$$

$$N_1 = 295\text{ N}$$

Step 4 of 5

Substitute 295 N for N_1 in equation (1).

$$N_2 = 0.7N_1 - 90.59$$

$$N_2 = 0.7(295\text{ N}) - 90.59$$

$$= 116.2\text{ N}$$

Consider the wedge.

Calculate the forces acting on *x*-axis.

$$\Sigma F_x = 0$$

$$N_3\cos 5° - N_2 - \mu_2 N_3 \sin 5° = 0$$

Substitute 116.2 N for N_2, and 0.4 for μ_2.

$$N_3\cos 5° - 116.2 - 0.4N_3\sin 5° = 0$$

$$0.996N_3 - 0.034N_3 = 116.2$$

$$0.961N_3 = 116.2$$

$$N_3 = 120.8\text{ N}$$

Step 5 of 5

Calculate the minimum value of force *P*.

Calculate the forces acting on *y*-axis.

$$\Sigma F_y = 0$$

$$N_3\sin 5° + \mu_2 N_3\cos 5° - P + \mu_2 N_2 = 0$$

Substitute 120.8 N for N_3, 116.2 N for N_2, and 0.4 for μ_2.

$$120.8\sin 5° + (0.4\times 120.8)\cos 5° - P + (0.4\times 116.2) = 0$$

$$10.52 + 48.14 - P + 46.48 = 0$$

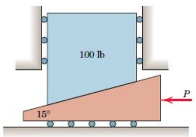
$$P = 105.1\text{ N}$$

Hence, the minimum value of force *P* is $\boxed{105.1\text{ N}}$.

Chapter 6, Problem 69P

Problem

The coefficient of static friction μs between the 100-lb body and the 15° wedge is 0.20. Determine the magnitude of the force P required to begin raising the 100-lb body if (a) rollers of negligible friction are present under the wedge, as illustrated, and (b) the rollers are removed and the coefficient of static friction μs = 0.20 applies at this surface as well.



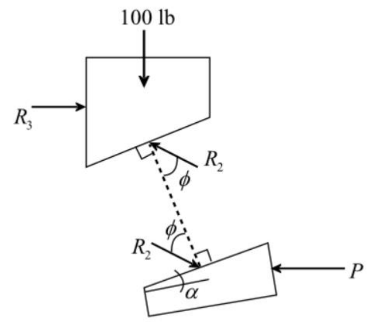
Step-by-step solution

Step 1 of 4

(a)

Consider the negligible friction between the rollers and the bottom wedge.

Draw the free body diagram for the wedges as follows:



Step 2 of 4

Calculate the friction angle as follows:

$\phi = \tan^{-1}\mu$

Here, μ is the coefficient of static friction.

Substitute 0.2 for μ.

$$\begin{aligned}\phi &= \tan^{-1}0.2\\ &= 11.31^\circ\end{aligned}$$

Consider the vertical force equilibrium condition for 100 lb wedge:

$$\Sigma F_y = 0$$
$$R_2\cos(\alpha+\phi) - 100 = 0$$

Substitute 15° for α and 11.31° for ϕ.

$$R_2\cos(15^\circ + 11.31^\circ) - 100 = 0$$
$$R_2 = 111.556 \text{ lb}$$

Consider the horizontal force equilibrium condition for the bottom wedge:

$$\Sigma F_x = 0$$
$$-P + R_2\sin(\alpha+\phi) = 0$$

Substitute 111.556 lb for R_2, 15° for α and 11.31° for ϕ.

$$-P + 111.556\sin(15^\circ + 11.31^\circ) = 0$$
$$P = 49.44 \text{ lb}$$

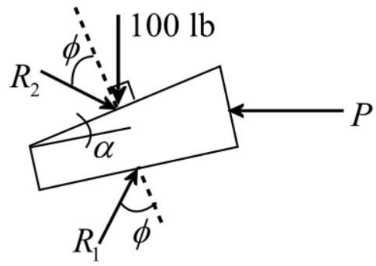
Therefore, the magnitude of force P is $\boxed{49.44 \text{ lb}}$.

Step 3 of 4

(b)

Consider the friction between the rollers and the bottom wedge.

Draw the free body diagram for the bottom wedges as follows:



Step 4 of 4

Consider the vertical force equilibrium condition:

$$\Sigma F_y = 0$$
$$R_1\cos\phi - 100 = 0$$
$$R_1\cos 11.31^\circ - 100 = 0$$
$$R_1 = 101.98 \text{ lb}$$

Consider the horizontal force equilibrium condition for the bottom wedge:

$$\Sigma F_x = 0$$
$$-P + R_2\sin(\alpha+\phi) + R_1\sin\phi = 0$$

Substitute 101.98 lb for R_1, 111.556 lb for R_2, 15° for α and 11.31° for ϕ.

$$-P + 111.556\sin(15^\circ + 11.31^\circ) + 101.98\sin 11.31^\circ = 0$$
$$P = 69.44 \text{ lb}$$

Therefore, the magnitude of force P is $\boxed{69.44 \text{ lb}}$.

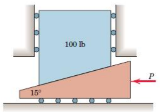
Chapter 6, Problem 70P

Problem

For both conditions (*a*) and (*b*) as stated in Prob. 6/69, determine the magnitude and direction of the force *P'* required to begin lowering the 100-lb body.

Problem. 6/69

The coefficient of static friction μs between the 100-lb body and the 15° wedge is 0.20. Determine the magnitude of the force *P* required to begin raising the 100-lb body if (*a*) rollers of negligible friction are present under the wedge, as illustrated, and (*b*) the rollers are removed and the coefficient of static friction μs = 0.20 applies at this surface as well.



Step-by-step solution

Step 1 of 6

Calculate the friction angle as follows:

$\phi = \tan^{-1}\mu$

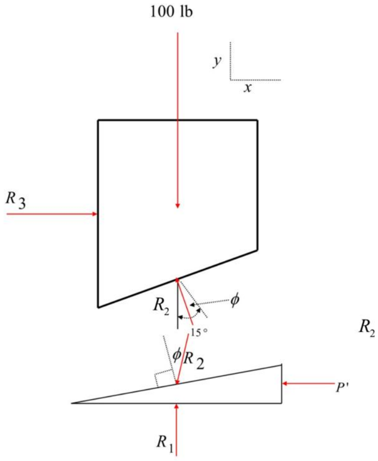
Here, μ is the coefficient of friction.

Substitute 0.2 for μ.

$$\begin{aligned}\phi &= \tan^{-1}0.2\\ &= 11.31°\end{aligned}$$

Step 2 of 6

Consider the free-body diagram as follows:



Step 3 of 6

(a)

Calculate the force required to begin lowering the 100-lb body if rollers of negligible friction are under the wedge.

Apply the equilibrium forces as follows:

Calculate the forces along *y*-axis.

$$\begin{aligned}&\sum F_y = 0:\\ &-100 + R_2\cos(15° - 11.31°) = 0\\ &R_2 = 100.2 \text{ lb}\end{aligned}$$

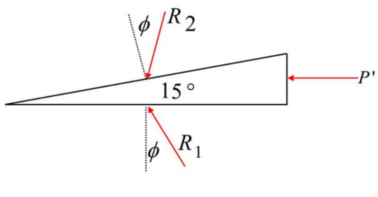
Calculate the forces along *x*-axis.

$$\begin{aligned}&\sum F_x = 0:\\ &R_2\sin(15° - 11.31°) - P' = 0\\ &P' = 6.45\,\text{lb}\,(\text{to the left})\end{aligned}$$

Therefore, the force required to begin lowering the 100-lb body if rollers of negligible friction are under the wedge is $\boxed{6.45 \text{ lb}}$.

Step 4 of 6

Consider the free-body diagram.



Step 5 of 6

(b)

The force required to begin lowering the 100-lb body if rollers are removed and the coefficient of static friction μ_s = 0.20 at the surface.

From the free-body diagram, obtain the value of R_1.

$$R_1 = \frac{W}{\cos\phi}$$

Here, *W* is the weight of the body.

Substitute 100 lb for *W* and $11.31°$ for ϕ.

$$\begin{aligned}R_1 &= \frac{100}{\cos 11.31°}\\ &= 102\,\text{lb}\end{aligned}$$

Step 6 of 6

Apply the equilibrium forces as follows:

Calculate the forces along *x*-axis.

$$\begin{aligned}&\sum F_x = 0:\\ &R_2\sin(15° - 11.31°) + P' - R_1\sin(11.31°) = 0\end{aligned}$$

Substitute 102 lb for R_1 and 100.2 lb for R_2 (remains unchanged), and $11.31°$ for ϕ.

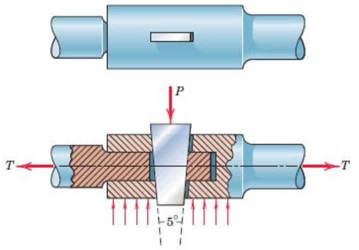
$$\begin{aligned}&(100.2 \text{ lb})\sin(15° - 11.31°) + P' - (102.0\,\text{lb})\sin(11.31°) = 0\\ &P' = 13.55\,\text{lb}\,(\text{to the right})\end{aligned}$$

Therefore, the force required to begin lowering the 100-lb body if rollers are removed and the coefficient of static friction μ_s = 0.20 at the surface is $\boxed{13.55\,\text{lb}}$.

Chapter 6, Problem 71P

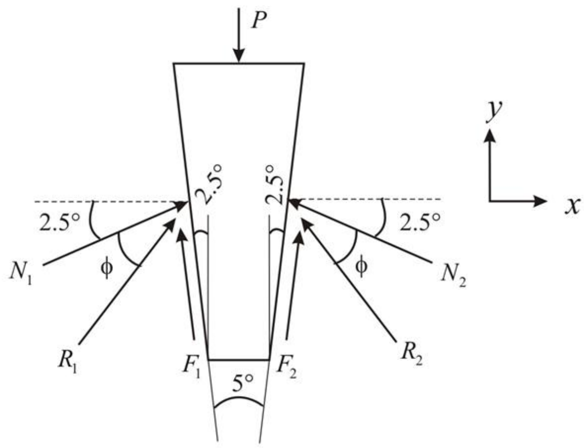
Problem

The design of a joint to connect two shafts by a flat 5° tapered cotter is shown by the two views in the figure. If the shafts are under a constant tension T of 200 lb, find the force P required to move the cotter and take up any slack in the joint. The coefficient of friction between the cotter and the sides of the slots is 0.20. Neglect any horizontal friction between the shafts.



Step-by-step solution

Step 1 of 4



Step 2 of 4

Given that,

Two shafts are connected by a flat $5°$ tapered cotter

The shafts are under tension $T = 200\text{ lb}$

The coefficient of friction between the cotter and the sides of the slots is $\mu = 0.20$

Friction angle, $\phi = \tan^{-1}(\mu)$

$\phi = \tan^{-1}(0.20)$

$\phi = 11.3°$

Step 3 of 4

Let $R_1 \,\&\, R_2$ are the resultant forces at both sides due the friction and normal forces on the sides of the cotter.

Step 4 of 4

From the shaft, the tension in the shaft $T = R_1 \cos(\phi + 2.5°)$

$$R_1 = \frac{T}{\cos(11.3° + 2.5°)}$$

Writing equations of equilibrium

$\Sigma F_x = 0$

$R_1 \cos(\phi + 2.5°) - R_2 \cos(\phi + 2.5°) = 0$

$R_1 = R_2$

$\Sigma F_y = 0$

$2R_1 \sin(\phi + 2.5°) - P = 0$

$P = 2R_1 \sin(11.3° + 2.5°)$

$$P = 2 \times \frac{T}{\cos(11.3° + 2.5°)} \sin(11.3° + 2.5°)$$

$= 2 \times 200 \times \tan 13.8°$

$\boxed{P = 98.25\text{ lb}}$