

FEBRUARY 2021: IN SEMESTER ASSESSMENT B Tech 1 SEMESTER
TEST – 1
UE20MA101 : Engineering Mathematics - I

Time: 2 Hrs		Answer All Questions	Max Marks: 60
1	a)	Test the convergence of the series whose nth term is given by $\sqrt{n^4 + 1} - \sqrt{n^4 - 1}$.	4 M
	b)	Discuss the convergence of the series $1 + \frac{\alpha.\beta}{1.\gamma}x + \frac{\alpha.(1+\alpha).\beta.(1+\beta)}{1.2.\gamma(1+\gamma)}x^2 + \frac{\alpha.(1+\alpha).(2+\alpha).\beta.(1+\beta).(2+\beta)}{1.2.3.\gamma.(1+\gamma).(2+\gamma)}x^3 + \dots \infty$	6 M
2	a)	Discuss the convergence of the series $\left(\frac{3}{4}\right)x + \left(\frac{4}{5}\right)^2 x^2 + \left(\frac{5}{6}\right)^3 x^3 + \dots \infty, x > 0$	4 M
	b)	Apply Cauchy integral test to discuss the nature of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$	6 M
3	a)	If $\theta = t^n e^{\frac{-r^2}{4t}}$ what value of n will make $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \theta}{\partial r} \right) = \frac{\partial \theta}{\partial t}$?	4 M
	b)	If $u = x^3 \sin^{-1} \left(\frac{y}{x} \right) + x^4 \tan^{-1} \left(\frac{y}{x} \right)$ find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ at $x = 1, y = 1$.	6 M
4	a)	Find Taylor's expansion of $f(x, y) = \cot^{-1}(xy)$ in powers of $(x + 0.5)$ and $(y - 2)$ up to second degree terms. Hence compute $f(-0.4, 2.2)$ approximately.	6 M
	b)	A scope probe in the shape of ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth atmosphere and its surface begins to heat. After one hour the temperature at the point (x, y, z) on the surface is $T(x, y, z) = 8x^2 + 4yz - 16z + 600$. Find the hottest point on the probe surface.	4 M
5	a)	Solve $y - \cos x \frac{dy}{dx} = y^2(1 - \sin x) \cos x$.	6 M
	b)	Prove that the system of confocal and coaxial parabolas $y^2 = 4a(x + a)$ is self-orthogonal. Here 'a' is the parameter.	4 M
6	a)	Solve $2px + \tan^{-1}(xp^2) - y = 0$.	6 M
	b)	Water at temperature 10°C takes 5 minutes to warm up to 20°C in a room at temperature 40°C . i) Find the temperature after 20 minutes. ii) When will the temperature be 25°C ?	4 M

**MARCH 2021: IN SEMESTER ASSESSMENT B Tech 1 SEMESTER
(Chemistry Cycle)**

TEST – 2

UE20MA101 : Engineering Mathematics - I

Time: 90 Minutes		Answer All Questions	Max Marks: 40
1	a)	Solve $\frac{d^4x}{dt^4} = m^4x$.	4 M
	b)	Solve $(D^3 - D)y = 2x + 1 + 4\cos x + 2e^x$.	6 M
2	a)	Solve by the method of variation of parameters $y'' - 2y' + 2y = e^x \tan x$.	5 M
	b)	Solve $x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 3y = \frac{\log x}{x^2}$.	5 M
3	a)	Show that $\int_0^\infty \frac{x^4}{1+x^6} dx \cdot \int_0^2 (8-x^3)^{-\frac{1}{3}} dx = \frac{2\pi^2}{9\sqrt{3}}$.	6 M
	b)	Prove that $\int_0^\infty e^{-ax} x^{m-1} \sin bx dx = \frac{\Gamma(m)}{(a^2+b^2)^{\frac{m}{2}}} \sin m \left(\tan^{-1} \left(\frac{b}{a} \right) \right)$.	4 M
4	a)	Evaluate $\int x^3 J_0(x) dx$ in terms of J_0 and J_1 .	4 M
	b)	Show that $J_0^2 + 2J_1^2 + 2J_2^2 + 2J_3^2 + \dots = 1$.	6 M

APRIL 2021: END SEMESTER ASSESSMENT (ESA) B Tech I SEMESTER
(Chemistry Cycle)
UE20MA101 : Engineering Mathematics - I

Time: 3 Hrs	Answer All Questions	Max Marks: 100
-------------	----------------------	----------------

1	a)	Prove that the series $1 + \frac{1}{2} \frac{a}{b} + \frac{1.3}{2.4} \frac{a(a+1)}{b(b+1)} + \frac{1.3.5}{2.4.6} \frac{a(a+1)(a+2)}{b(b+1)(b+2)} + \dots \infty$ is convergent if $a > 0, b > 0$ and $b > a + \frac{1}{2}$.	7 M
	b)	Examine the convergence or divergence of the series $1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \dots + \frac{2^{n+1}-2}{2^{n+1}+1}x^n + \dots \infty$ ($x > 0$)	7 M
	c)	Test the convergence of the series $\sum \frac{[(n+1)x]^n}{n^{n+1}}$.	6 M
2	a)	If $(\sqrt{x} + \sqrt{y})\cot u - x - y = 0$, prove that $4x \frac{\partial u}{\partial x} + 4y \frac{\partial u}{\partial y} + \sin 2u = 0$.	4 M
	b)	If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, show that $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} + z^2 \frac{\partial u}{\partial z} = 0$.	4 M
	c)	Find the Taylor's expansion of $e^{ax} \sin by$ about the origin upto third degree terms.	6 M
	d)	A tent of a given volume has a square base of side $2a$, has its four-side vertical of length b and is surmounted by a regular pyramid of height h . Find the values of a and b in terms of h such that the canvas required for its construction is minimum.	6 M
3	a)	Solve: $x \sin x \frac{dy}{dx} + y(x \cos x - \sin x) = 2$	7 M
	b)	Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ is a parameter.	6 M
	c)	Solve: $y = 2px + p^n$.	7 M
4	a)	Solve: $(D^2 - 4D + 1)y = \sin^2 x + e^x + e^{3x}$	6 M
	b)	Solve the differential equation: $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = \frac{x^3}{1+x^2}$	7 M

	c)	A circuit consists of an inductance of 0.5 henrys, resistance of 6 ohms, capacitance of 0.02 farads and an e.m.f of voltage $E=24\sin 10t$. Find the charge and the current at time $t > 0$ given that the circuit carries no charge and no current at time $t=0$.	7 M
5	a)	Show that for $m, n > 0$ $\int_0^1 x^{m-1} \left(\log \frac{1}{x}\right)^{n-1} dx = \frac{\Gamma(n)}{m^n}$.	5 M
	b)	Show that for $m, n > 0$, $\int_{-a}^a (a+x)^{m-1} (a-x)^{n-1} dx = (2a)^{m+n-1} \beta(m, n)$.	5 M
	c)	Use Jacobi series to derive the Bessel's integral formula $J_n(x) = \frac{1}{\pi} \int_0^\pi \cos(n\theta - x \sin\theta) d\theta$ where n is a positive integer.	6 M
	d)	Evaluate $\int J_4(x) dx$.	4 M