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## UNIT - 03: Digital Electronics

- \* Boolean Algebra :- Boolean Algebra & logic gates
- \* Basic theorems and Properties of Boolean algebra
- \* Boolean functions, Canonical & Standard form
- \* Digital logic gates
  - Combinational logic Circuits
  - Sequential logic Circuits : Half adder  
full adder
  - Multiplexer & Demultiplexer
  - Flip Flops : RS, D, T, JK
  - Registers : SISO
  - Counters : 3 bit Synchronous and Asynchronous counter.

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## Boolean Algebra & logic gates:

- \* Digital Electronics :- The branch of electronics where the circuits process the data which contains binary values 1s and 0s (logical levels)
- \* Logic 1 is referred as HIGH Voltage / TRUE / ON state
- \* Logic 0 is referred as LOW Voltage / FALSE / OFF state.
- \* A Binary digit either '0' or '1' is called a "bit"
- \* Digital information is stored using a series of binary values of 1s & 0s.
- \* All digital systems store & process the data in terms of 0s & 1s.  
Ex:- Digital telephones, computers, digital camera etc
- \* Boolean algebra is a branch of mathematics which is used to analyse and simplify logic circuits / digital circuits.
- \* Digital circuits are constructed by using logic gates.
- \* Logic gates are building blocks of digital systems performs logical functions based on boolean algebra.



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\* Logic gates have one / more inputs but only one output.

\* If any logic gate has "n" number of inputs then it can have  $2^n$  possible input states.

\* Logic gates are implemented using diodes or transistors which acts as electronic switches.

\* There are two categories of logic gates

i) Basic gates: AND, OR, NOT

ii) Derived gates: NAND, NOR, EXOR & EXNOR gates.

\* AND Gate:-

> It is an electronic circuit which gives output as logic 1 (HIGH) iff all inputs are at logic "1". Otherwise output is logic "0" (LOW).

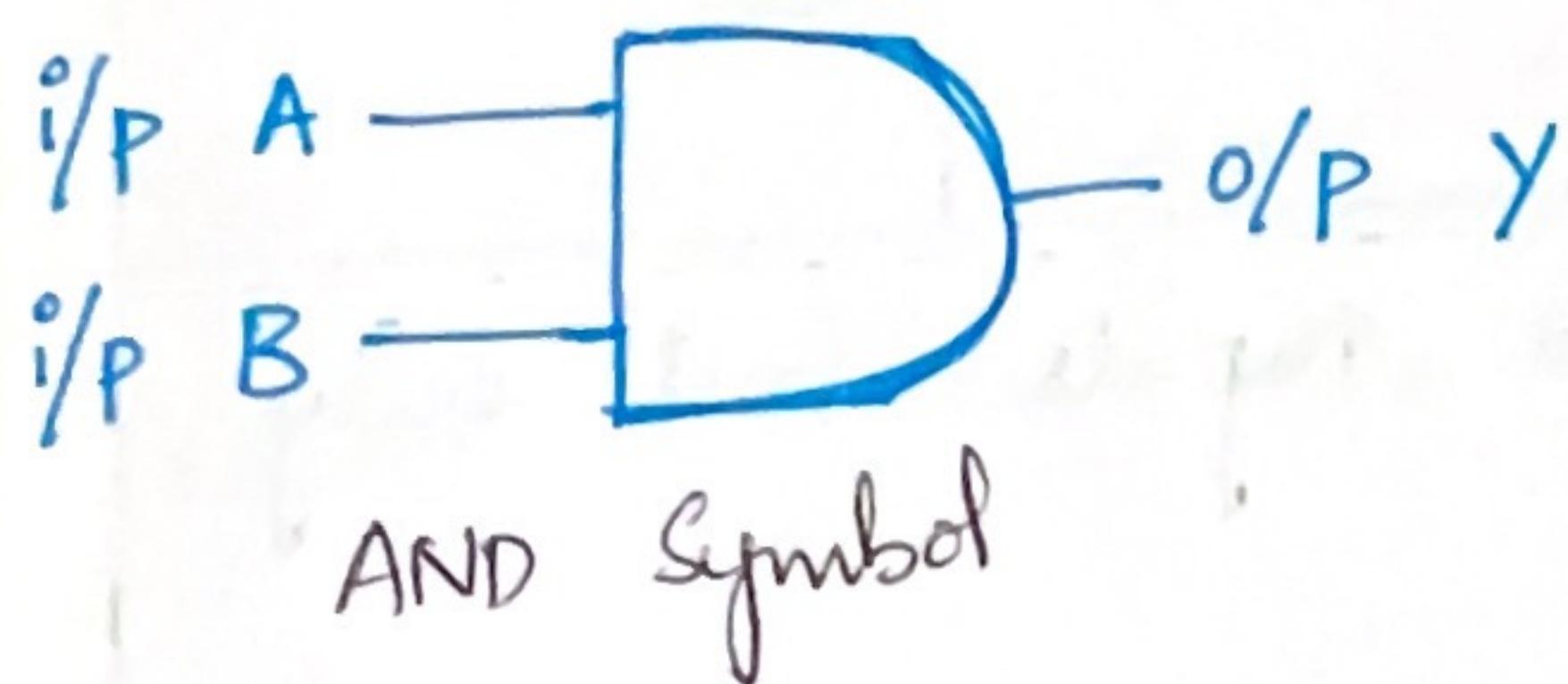
> Boolean algebra representation for AND gate is

$$Y = A \cdot B$$

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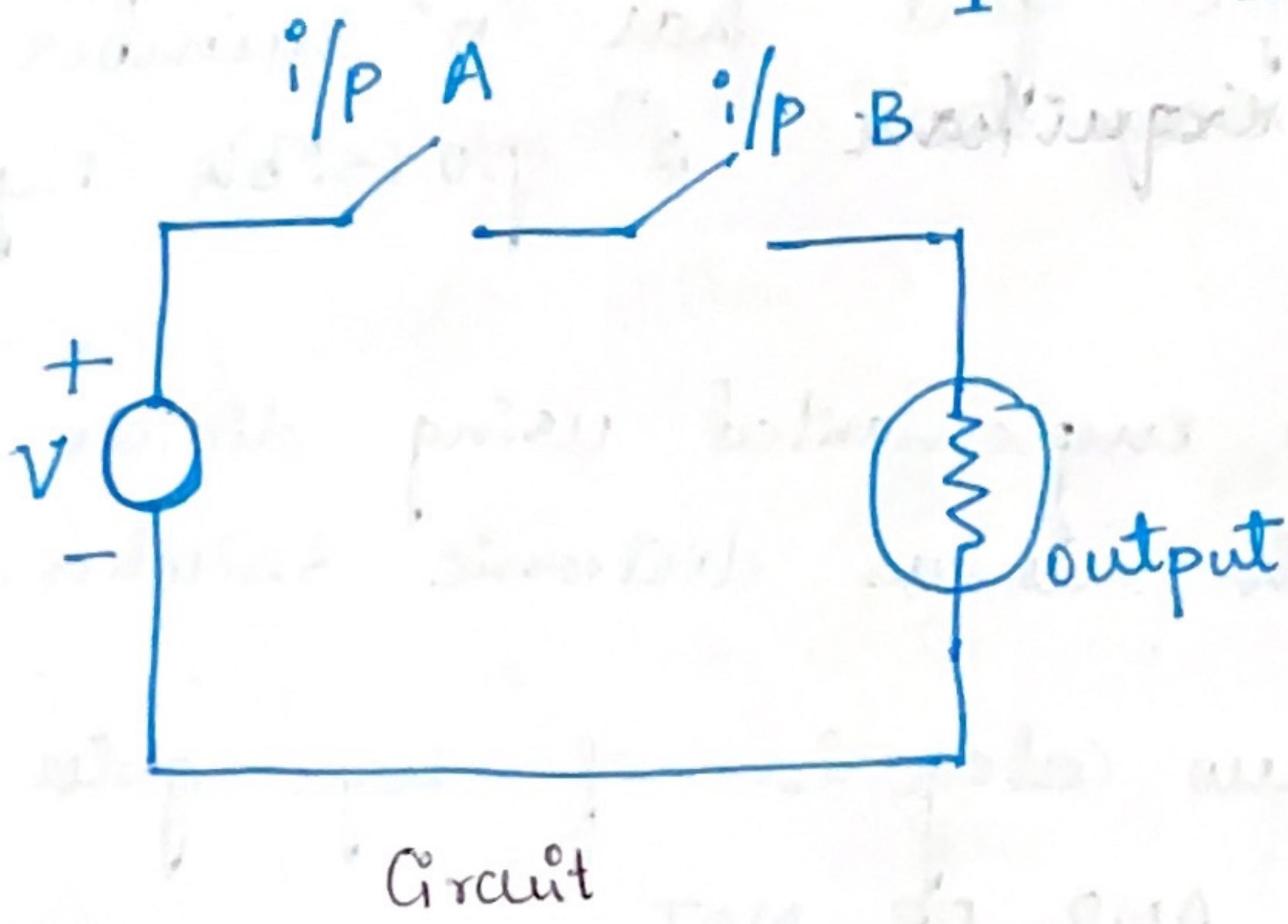
\* AND gate is represented as "two switches connected in series".

### TRUTH TABLE



Inputs

A	B	Output Y
0	0	0
0	1	0
1	0	0
1	1	1



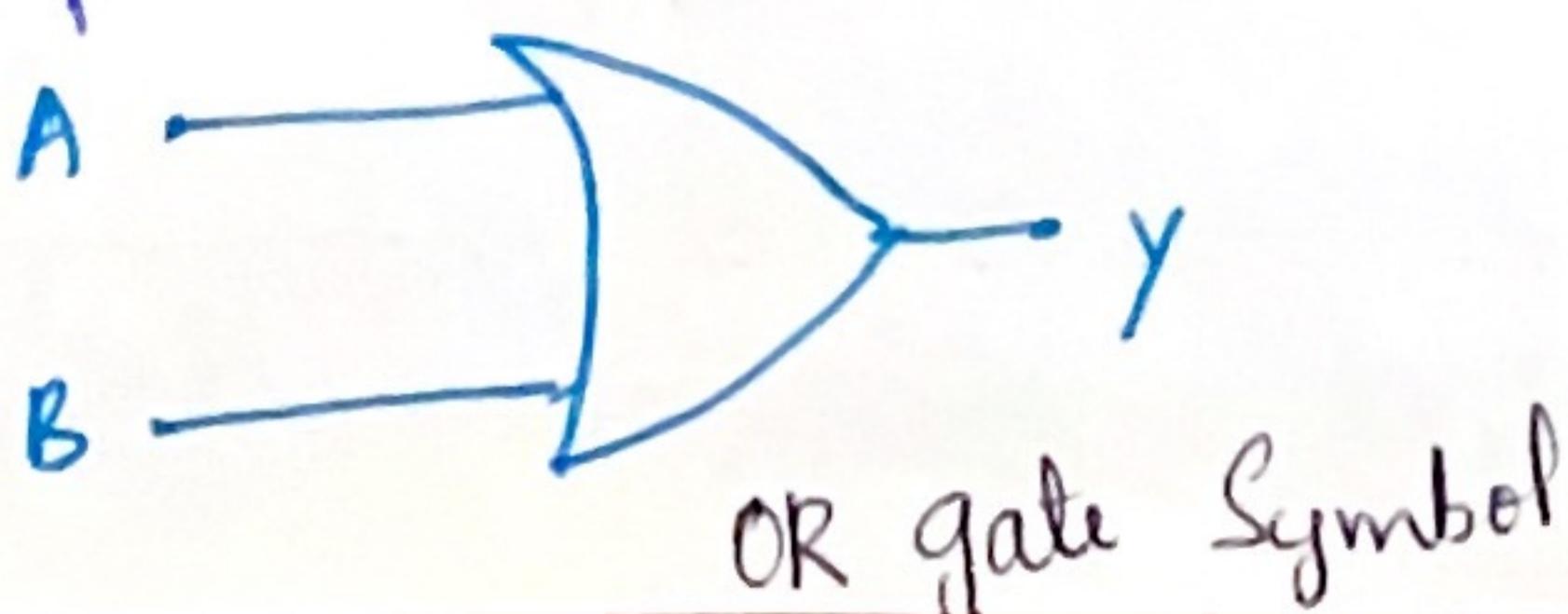
Circuit

### \* OR Gate:-

- > It is an electronic digital circuit which gives output as 1 when either of the input is logic 1 and output as logic 0 iff all inputs are logic "0".
- > Boolean algebra representation for OR gate is

$$y = A + B$$

- > OR gate is represented as two switches connected in parallel



OR gate Symbol

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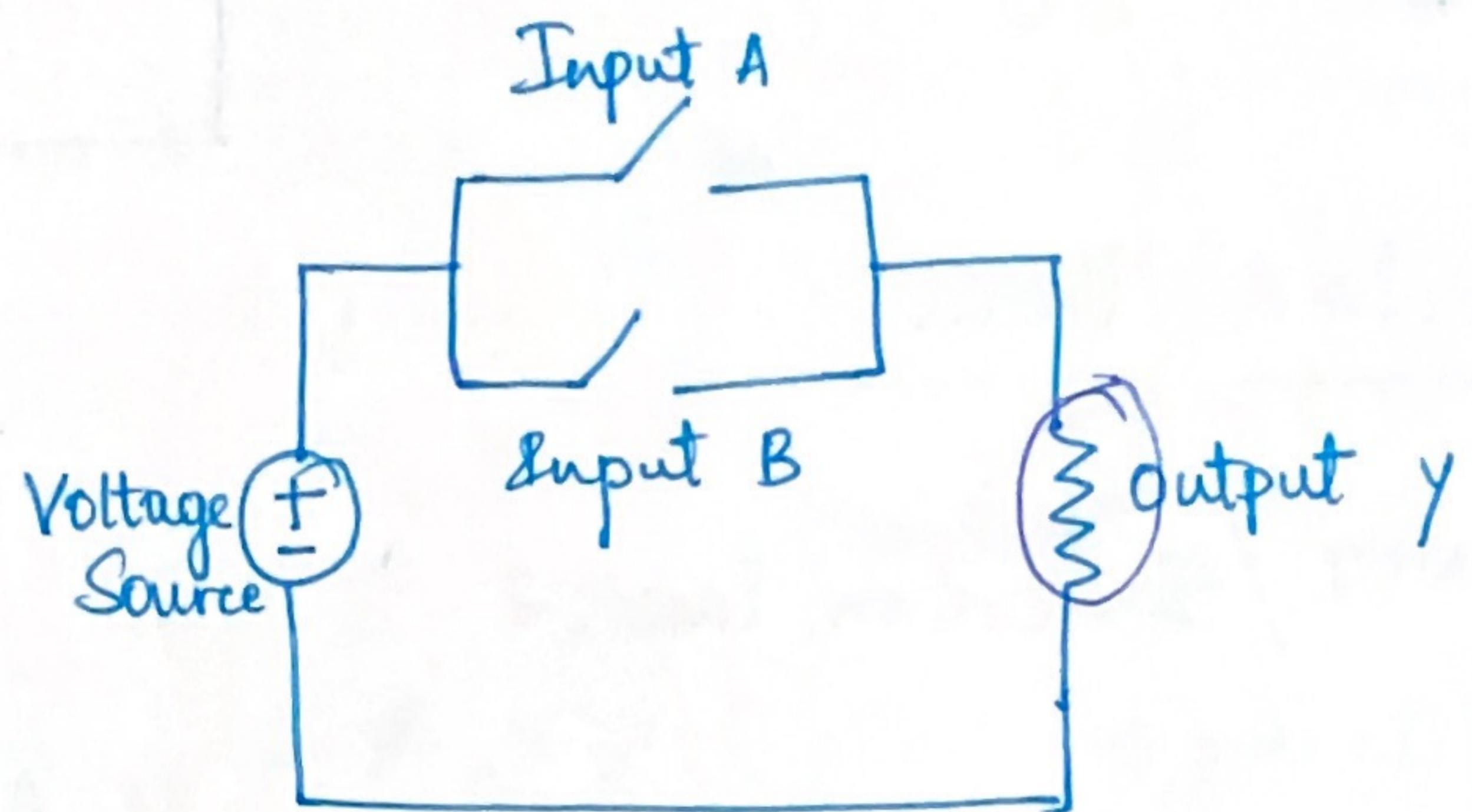
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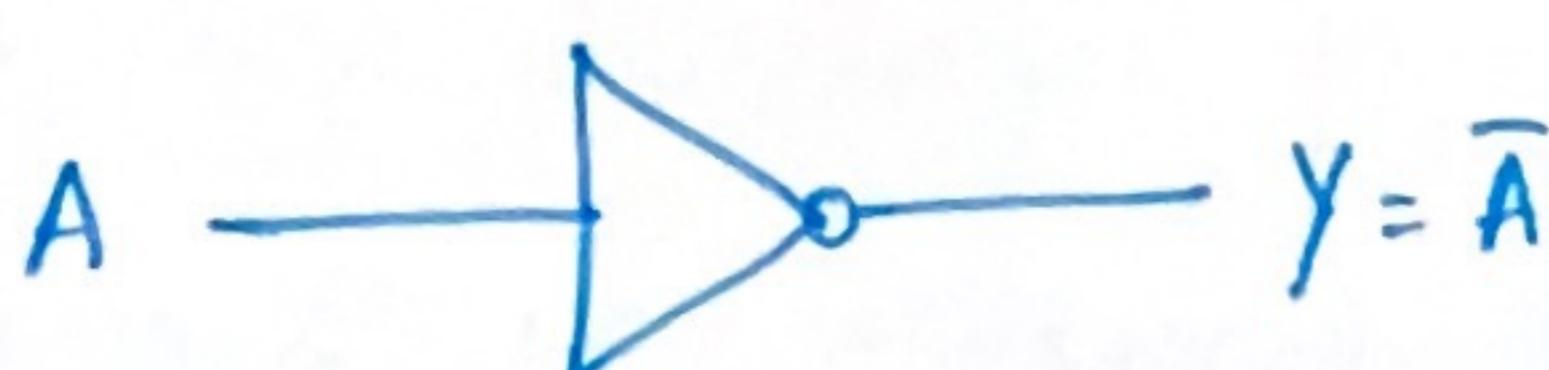
### TRUTH TABLE

Inputs		output
A	B	y
0	0	0
0	1	1
1	0	1
1	1	1



### \* NOT Gate

- > It is a Single input - Single output gate which performs inversion of the input signal. Hence it is also called as Inverter gate.
- > Boolean expression for NOT gate is
$$y = \bar{A}$$
- > NOT Gate is represented by connecting a switch parallel to load.



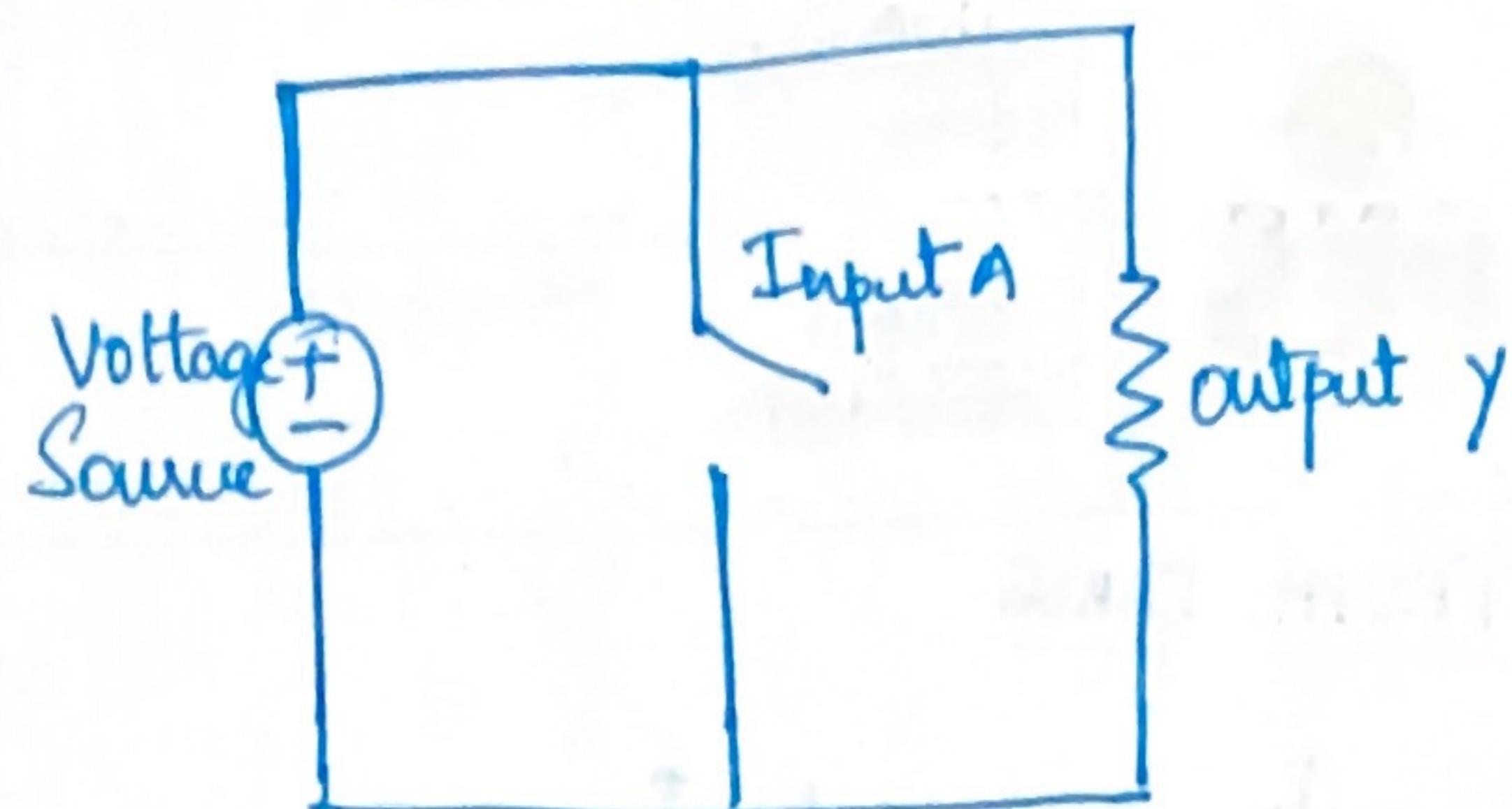
NOT gate symbol

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## TRUTH TABLE

Input A	Output y
0	1
1	0

## CIRCUIT



## Boolean laws

1] NOT / Inversion law :-

$$\begin{aligned} \bar{1} &= 0 \quad \therefore \text{if } A = 1 \Rightarrow \bar{A} = 0 \\ \bar{0} &= 1 \quad \quad \quad A = 0 \Rightarrow \bar{A} = 1 \quad \text{and} \\ &\quad \therefore (A')' = A \end{aligned}$$

2] AND law :-

$$\begin{aligned} A \cdot 0 &= 0 \\ A \cdot 1 &= A \\ A \cdot A &= A \\ A \cdot A' &= 0 \end{aligned}$$

3] OR law

$$\begin{aligned} A + 0 &= A \\ A + 1 &= 1 \\ A + A &= A \\ A + A' &= 1 \end{aligned}$$

4] Principle of duality

In Boolean Algebra, one type of expression can be converted into another type of expression by replacing '0' with '1' and '1' with '0' and also by replacing (+) with (.) and (.) with (+).

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AND ( $\cdot$ )	OR (+)	Expression	Dual Expression
$0 \cdot 0 = 0$	$0+0=1$	$\bar{0} = 1$	$\bar{1}=0$
$0 \cdot 1 = 0$	$1+0=1$	$0 \cdot 1 = 0$	$1+0 = 1$
$1 \cdot 0 = 0$	$0+1=1$	$A \cdot 0 = 0$	$A + 1 = 1$
$1 \cdot 1 = 1$	$0+0 = 0$	$A \cdot B = B \cdot A$	$A+B = B+A$
		$A \cdot \bar{A} = 0$	$A + \bar{A} = 1$

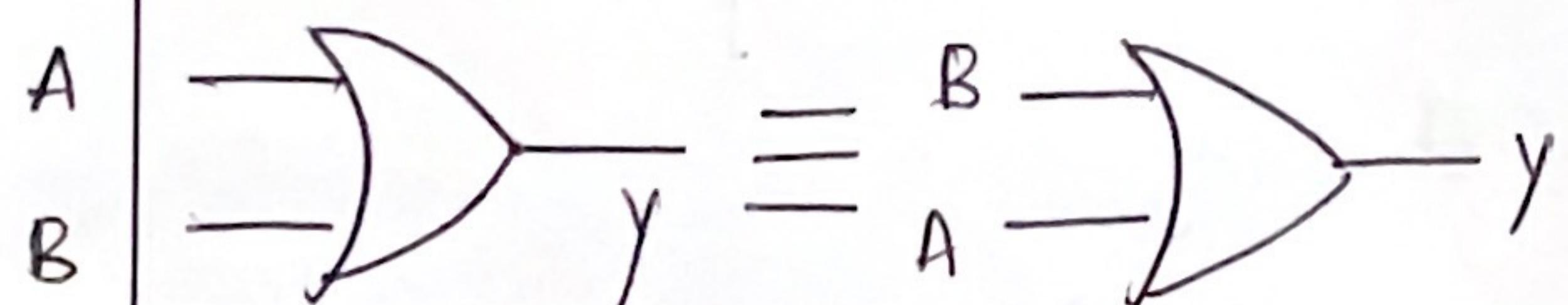
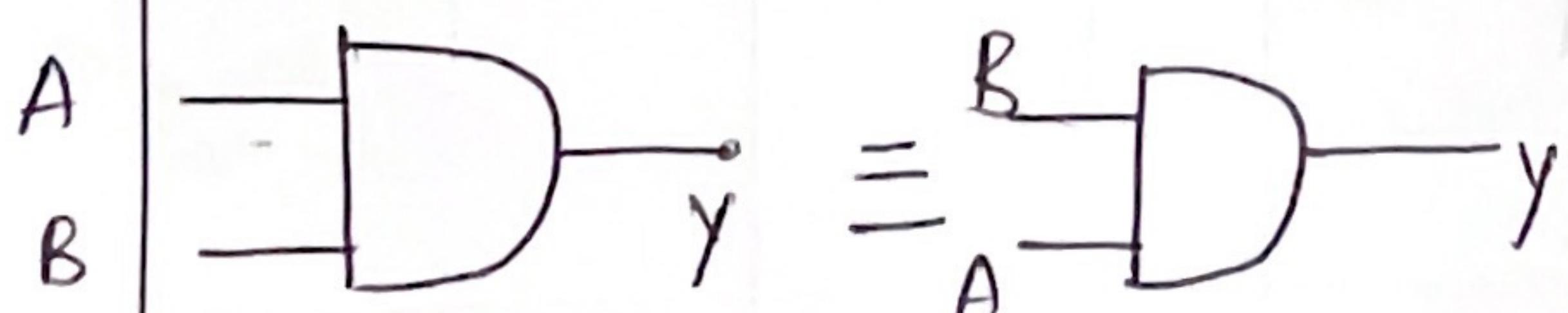
## 5] Basic Theorem and Properties of Boolean algebra.

Boolean laws and theorems are used to simplify Boolean expression, and reduce the number of logic gates.

### i] Commutative Law:

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$



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Proof:

<u>C<sub>1</sub></u>	<u>C<sub>2</sub></u>	<u>C<sub>3</sub></u>	<u>C<sub>4</sub></u>	<u>C<sub>5</sub></u>	<u>C<sub>6</sub></u>
A	B	A·B	B·A	A+B	B+A
0	0	0	0	0	0
0	1	0	0	1	1
1	0	0	0	1	1
1	1	1	1	1	1

$$C_3 = C_4 \quad \because A \cdot B = B \cdot A$$

$$C_5 = C_6 \quad \therefore A+B = B+A$$

ii] Associative law:

$$(A+B)+C = A+(B+C)$$

Proof:

A	B	C	A+B	(A+B)+C	B+C	A+(B+C)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

From the Principle of duality.

$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

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iii] Distributive Law:

$$A + B \cdot C = (A+B) \cdot (A+C)$$

Proof:

A	B	C	$B \cdot C$	$A+B$	$A+C$	$\bar{A}+BC$	$(A+B) \cdot (A+C)$
0	0	0	0	0	0	0	0
0	0	1	0	0	1	0	0
0	1	0	0	1	0	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

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Equality of distributive law:

$$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$$

A	B	C	$B+C$	$A \cdot (B+C)$	$A \cdot B$	$A \cdot C$	$(A \cdot B) + (A \cdot C)$
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1
1	1	1	1	1	1	1	1

### 6] De-Morgan's theorem

(i) The complement of sum of two variables is equal to Product of complement of individual variables.

$$\overline{A+B} = A' \cdot B'$$

(ii) The complement of Product of two variables is equal to Sum of complement of individual variables.

$$\overline{A \cdot B} = A' + B'$$

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Proof

A	B	A+B	$\overline{A+B}$	$A'$	$B'$	$A \cdot B$	$\overline{A \cdot B}$	$A' + B'$	$A' \cdot B'$
0	0	0	1	1	1	0	1	1	1
0	1	1	0	1	0	0	1	1	0
1	0	1	0	0	1	0	1	1	0
1	1	1	0	0	0	1	0	0	0

i] Absorption theorem :

$$\boxed{i) A + AB = A}$$

$$\begin{aligned}
 & A + AB \\
 &= A(1+B) \\
 &= A(1) \quad \because \text{OR law} \\
 &= A
 \end{aligned}$$

$$\boxed{ii) A + \overline{A}B = A + B}$$

$$\begin{aligned}
 & A + \overline{A}B \\
 &= (A + \overline{A}) \cdot (A + B) \quad (\text{distributive law}) \\
 &= 1 \cdot (A + B) \quad (\text{OR law}) \\
 &= A + B \quad (\text{AND law})
 \end{aligned}$$

$$\boxed{iii) A(A+B) = A}$$

$$\begin{aligned}
 & A(A+B) \\
 &= A \cdot A + A \cdot B \\
 &= A + AB \quad (\because \text{AND law}) \\
 &= A(1+B) \quad (\because \text{OR law}) \\
 &= A //
 \end{aligned}$$

$$\boxed{iv) A \cdot (\overline{A}+B) = AB}$$

$$\begin{aligned}
 & A \cdot (\overline{A}+B) \\
 &= A\overline{A} + AB = 0 + AB
 \end{aligned}$$

$$= AB$$

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## 8] Consensus theorem

Ex:

$$AB + \bar{A}C + BC = AB + \bar{A}C$$

$$\text{LHS} = \underline{AB} + \underline{\bar{A}C} + \cancel{BC}$$

$$= AB + \bar{A}C \quad (\because BC \text{ is redundant term})$$

Boolean functions, Canonical form & Standard Canonical form

Boolean function:

- \* It is an algebraic expression consist of binary Variables, Constants (0 or 1) and logical operation symbol.
- \* Boolean function is evaluated to logic 1 or logic 0 for the given value of binary variables.
- \* Boolean function can be represented in a truth table and it can be implemented as digital Circuite constructed using logic gates.

Ex:-  $f = A + B'C$

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If  $A=0, B=0$  and  $C=1$

$$F = 0 + 1 \cdot 1 = 1$$

If  $A=0, B=1$  and  $C=0$

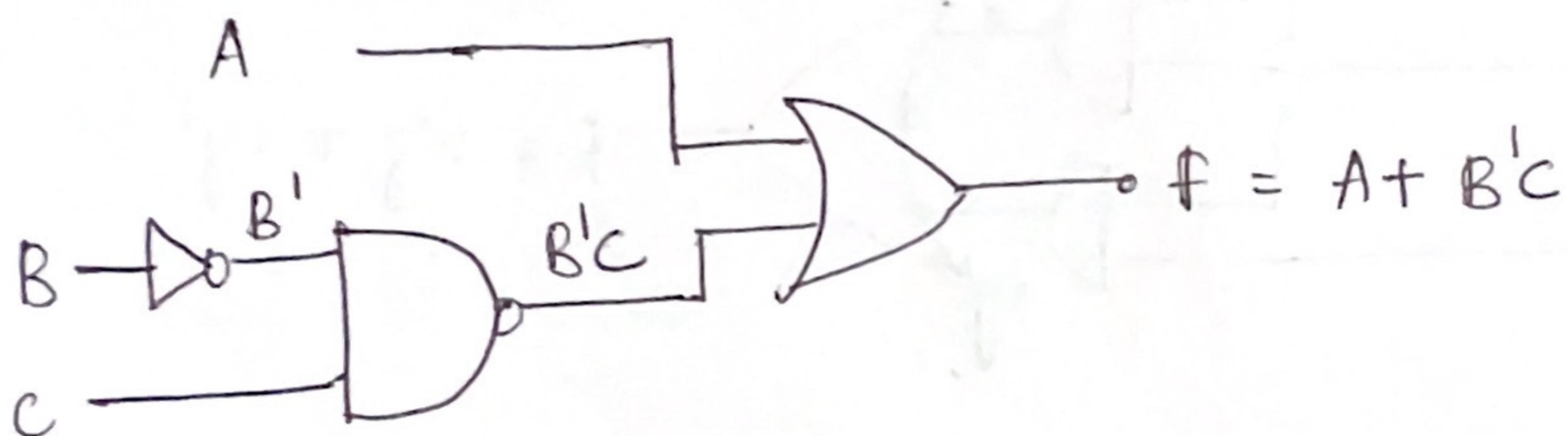
$$F = 0 + 0 \cdot 0 = 0$$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$\therefore F$  can be  
either "0" or  
"1"

Based on the value  
of A, B & C.

The given boolean expression can be constructed using  
1 NOT gate, 1 OR gate and 1 AND gate.



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Simplify and realize the given Boolean function using basic gates.

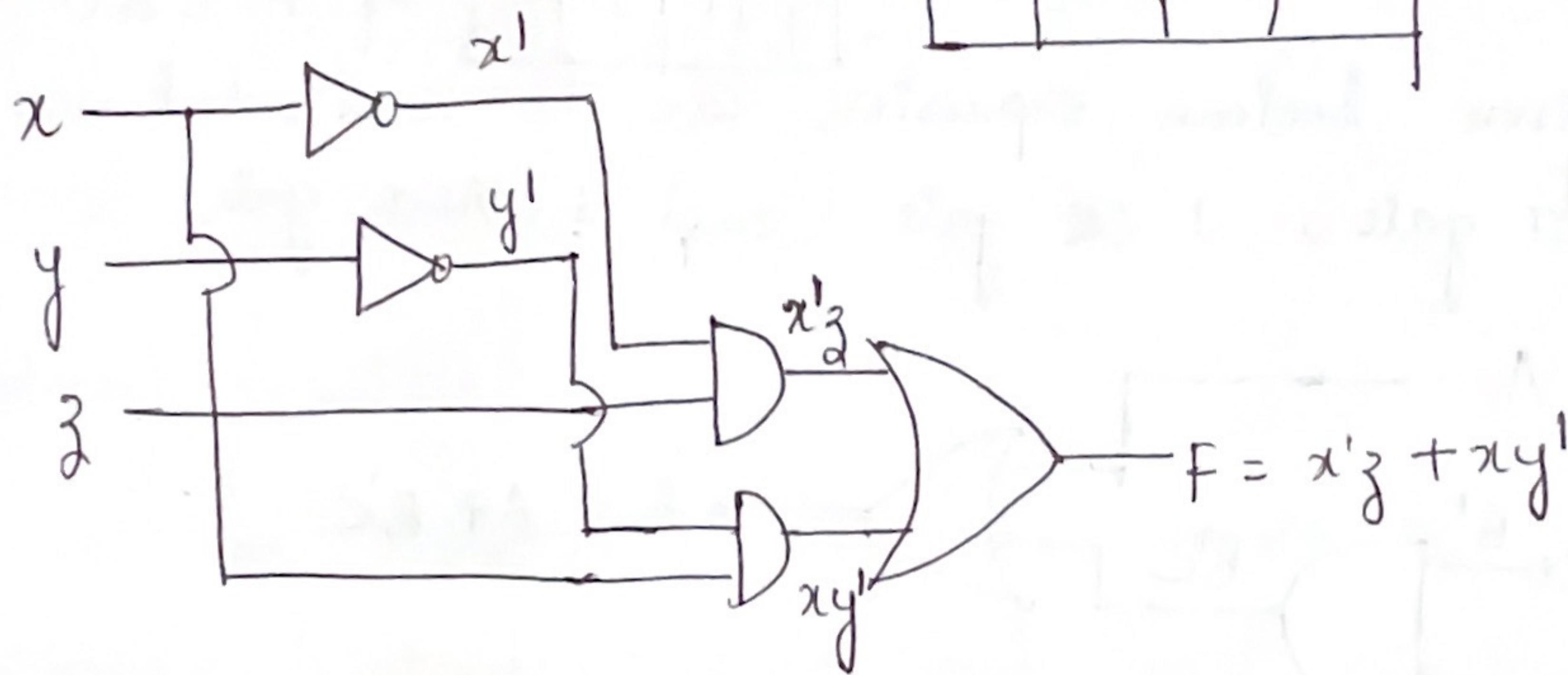
$$F = x'y'z + x'yz + xy'$$

$$x'z(y' + y) + xy'$$

$$x'z(1) + xy'$$

$$x'z + xy'$$

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0



### Sum of Products (SOP) :-

\* SOP is a boolean expression which contains sum of (ORing of) Product terms (ANDing of one or more literals)

Example  $F = \underbrace{y'}_{\text{ORing}} + \underbrace{xy}_{\text{Product term}} + \underbrace{x'y'z}_{\text{Product term}}$

AND terms  $\oplus$  Product terms



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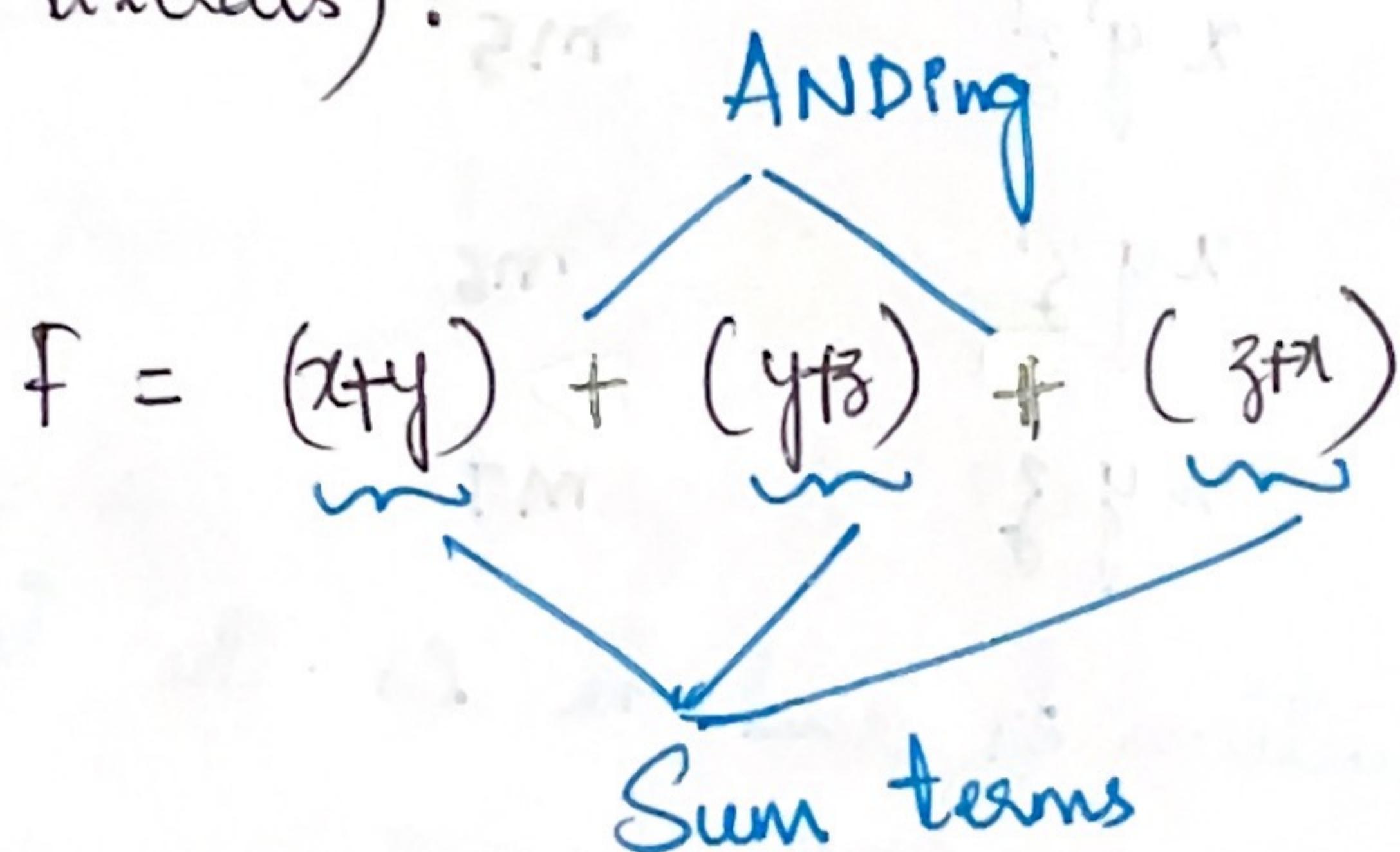
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Product of Sum: POS

\* POS is a boolean expression which contains Product of (Anding) of Sum terms (ORing of one or more literals).

Example:



Canonical SOP form:-

\* It is a SOP boolean expression where each Product term should contain all the literals either in true or complemented form.

\* Here each product term is called "Minterm".

\* Ex:  $f = xy + x'y + yz$  (NOT in Canonical form)

$F = xyz + x'y'z + x'y'z$  (in Canonical form because all terms contains all literals in true / Complemented form)

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## Truth table (three Variable function) :

### Minterm

x	y	z	Term	Designation
0	0	0	$x'y'z'$	$m_0$
0	0	1	$x'y'z$	$m_1$
0	1	0	$x'yz'$	$m_2$
0	1	1	$x'yz$	$m_3$
1	0	0	$xy'z'$	$m_4$
1	0	1	$xy'z$	$m_5$
1	1	0	$xyz'$	$m_6$
1	1	1	$xyz$	$m_7$

If  $x$  is a variable  
 Variables in true state  
 is written as "x" and  
 Variable in complimented  
 state is written as  
 "x'".

Consider the function  $f_1$  and  $f_2$  in the truth table and  
 write Canonical form SOP.

x	y	z	$f_1$	$f_2$
0	0	0	0 $m_0$	0 $m_0$
0	0	1	1 $m_1$	0 $m_1$
0	1	0	0 $m_2$	0 $m_2$
0	1	1	0 $m_3$	1 $m_3$
1	0	0	0 $m_4$	0 $m_4$
1	0	1	0 $m_5$	1 $m_5$
1	1	0	0 $m_6$	1 $m_6$
1	1	1	1 $m_7$	1 $m_7$

Canonical SOP form for  $f_1$

$$f_1 = m_1 + m_4 + m_7 \\ = x'y'z + xy'z' + xyz.$$

$$f_2 = m_3 + m_5 + m_6 + m_7 \\ = x'yz + xy'z + xyz' + xyz$$

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## Canonical POS form:-

- \* It is a POS boolean expression where all sum terms should contains all the literals in its true or complemented form.
- \* Here each sum term is called "Minterm".
- \* Ex:  $F = (x+y) \cdot (x'+y+z) \cdot (x+z)$  → Not in Canonical POS form.

$F = (x+y+z') \cdot (x'+y'+z) \cdot (x+y+z)$  → In Canonical POS form. because

### Truth table :

Minterm

x	y	z	Term	Designation
0	0	0	$x+y+z$	$M_0$
0	0	1	$x+y+z'$	$M_1$
0	1	0	$x+y'+z$	$M_2$
0	1	1	$x+y'+z'$	$M_3$
1	0	0	$x'+y+z$	$M_4$
1	0	1	$x'+y+z'$	$M_5$
1	1	0	$x'+y'+z$	$M_6$
1	1	1	$x'+y'+z'$	$M_7$

all sum terms contain all the literals in either true/complemented form.

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Consider the function  $f_1$  and  $f_2$  in the truth table.  
find Canonical POS form of  $f_1$  and  $f_2$ .

x	y	z	$f_1$	$f_2$	Canonical POS form for $f_1$
0	0	0	0	0	$M_0$
0	0	1	1	0	$M_1$
0	1	0	0	0	$M_2$
0	1	1	0	1	$M_3$
1	0	0	1	0	$M_4$
1	0	1	0	1	$M_5$
1	1	0	0	1	$M_6$
1	1	1	1	1	$M_7$

=  $(x+y+z) \cdot (x+y'+z) \cdot (x+y'+z')$  ·  
 $(x'+y+z') \cdot (x'+y'+z)$ .

(Canonical POS form of  $f_2$ )

$f_2 = M_0 \cdot M_1 \cdot M_2 \cdot M_4$   
 $= (x+y+z) \cdot (x+y'+z) \cdot (x'+y+z)$  ·  
 $(x'+y+z')$

Find the Minterms for the given expression  $F = A + BC$

Note: Convert the given expression into Canonical SOP form

$$F = A + BC$$

$$= A \cdot 1 + 1 \cdot BC$$

$$= A(B+B')(C+C') + (A+A')BC$$

$$= (AB+AB')(C+C') + ABC + A'BC$$

$$= \underline{ABC} + ABC' + AB'C + AB'C' + \underline{ABC} + A'BC$$

$$= ABC + ABC' + AB'C + AB'C' + A'BC$$

$$= m_7 + m_6 + m_5 + m_4 + m_3$$

$$F = \sum (3, 4, 5, 6, 7) //$$

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Find the minterm for the given expression:  $F = (x'y)(x+z)(y+z)$

Convert the given expression into Canonical POS form.

$$F = (x'y)(x+z)(y+z)$$

$$= (x'y + zz') (x + yy' + z) (x' + y + z)$$

$$= \underbrace{(x'y + zz')}_{A} \underbrace{(x + yy' + z)}_{BC} \underbrace{(y + z + x')}_{A BC}$$

$$= \underline{(x'y + z)} \cdot \underline{(x' + y + z)} \cdot \underline{(x + yy' + z)} \cdot \underline{(x + y + z)} \cdot \underline{(x' + y + z)}$$

$\therefore$  distributive law.

$$= (x'y + z) \cdot (x' + y + z) \cdot (x + y + z) \cdot (x + y' + z)$$

$$= M_4 \quad M_5 \quad M_0 \quad M_2$$

$$\therefore F = \prod M(0, 2, 4, 5)$$

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## Other logic gates      (Derived gate)

### 1] XOR Gate :

\* It performs modulo sum operation without including carry.

$$\begin{aligned}0+0 &= 0 \\0+1 &= 1 \\1+0 &= 1 \\1+1 &= 0\end{aligned}$$

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A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

\* The output of XOR gate is logic 1 if odd number of inputs are at logic 1. and output is logic 0 iff Even no of inputs are for 2 input XOR gate output is logic 1 when Two inputs are unequal. and output is logic 0 if both the inputs are same.

\* Boolean Expression for XOR gate.

$$Y = A'B + AB' \quad (\text{using Canonical SOP})$$

$$Y = A \oplus B.$$

Prove that  $A'B + AB' = A \oplus B$

$$AB' + A'B$$

$$\text{if } A=0 \text{ & } B=0$$

$$0 \cdot 1 + 1 \cdot 0 = 0 + 0 = \boxed{0}$$

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ii]  $A=0, B=1$

$$0 \cdot 0 + 1 \cdot 1 = 0 + 1 = 1$$

iii]  $A=1, B=0$

$$1 \cdot 1 + 0 \cdot 0 = 1 + 0 = 1$$

Hence

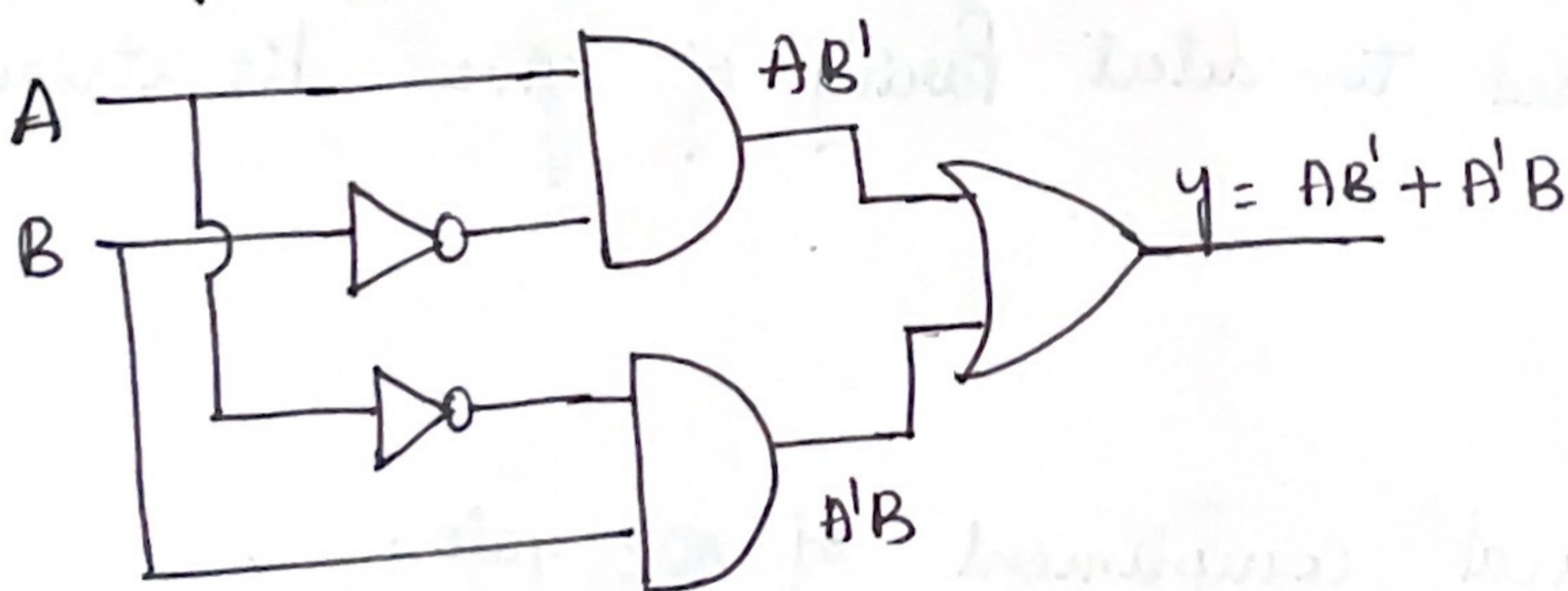
$$AB' + A'B = \underline{A \oplus B}$$

iv]  $A=1, B=1$

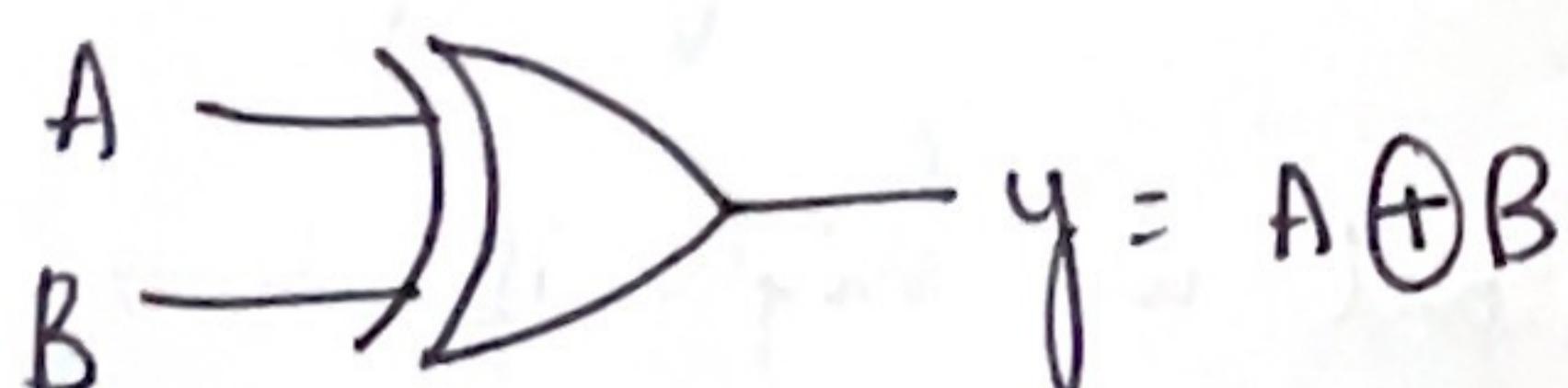
$$1 \cdot 0 + 0 \cdot 1 = 0 + 0 = 0$$

Implementation of XOR gate using basic gates.

$$y = AB' + A'B$$



Symbol for XOR gate



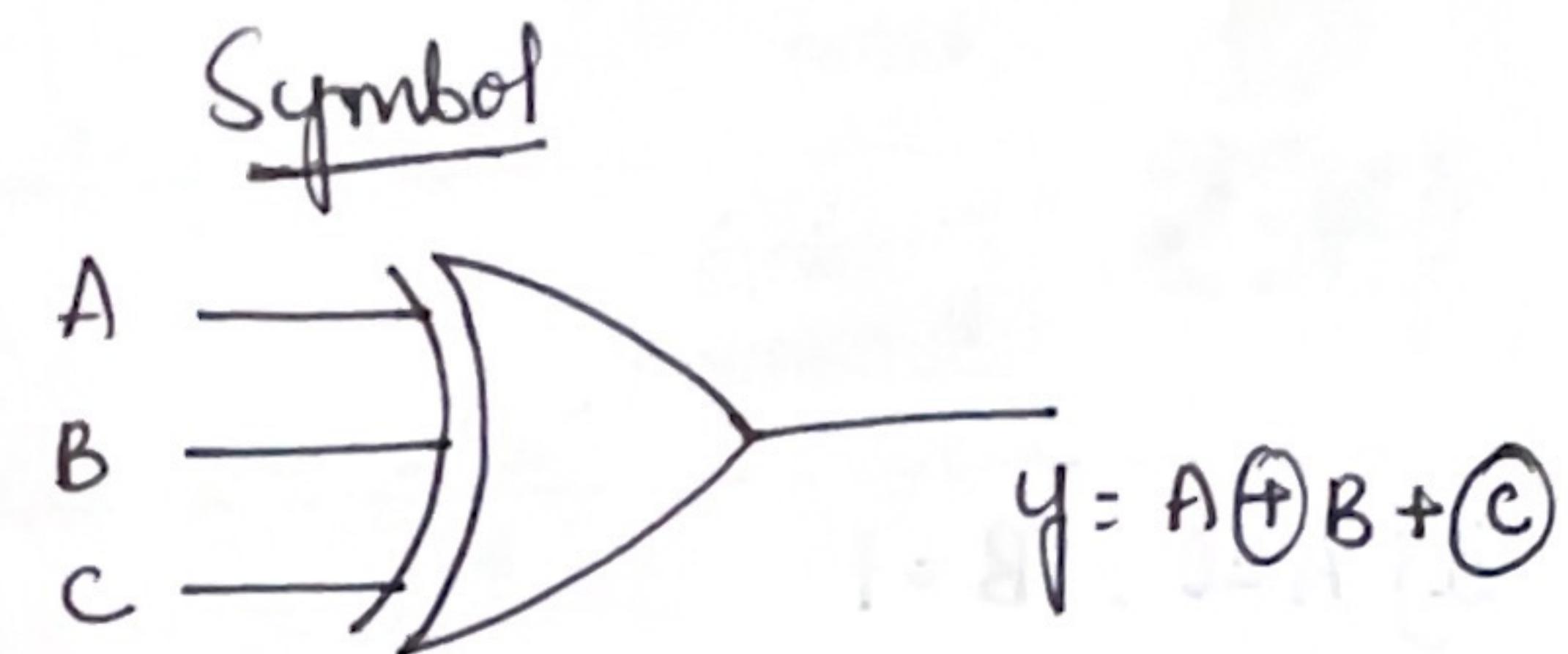
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### 3 input XOR gate

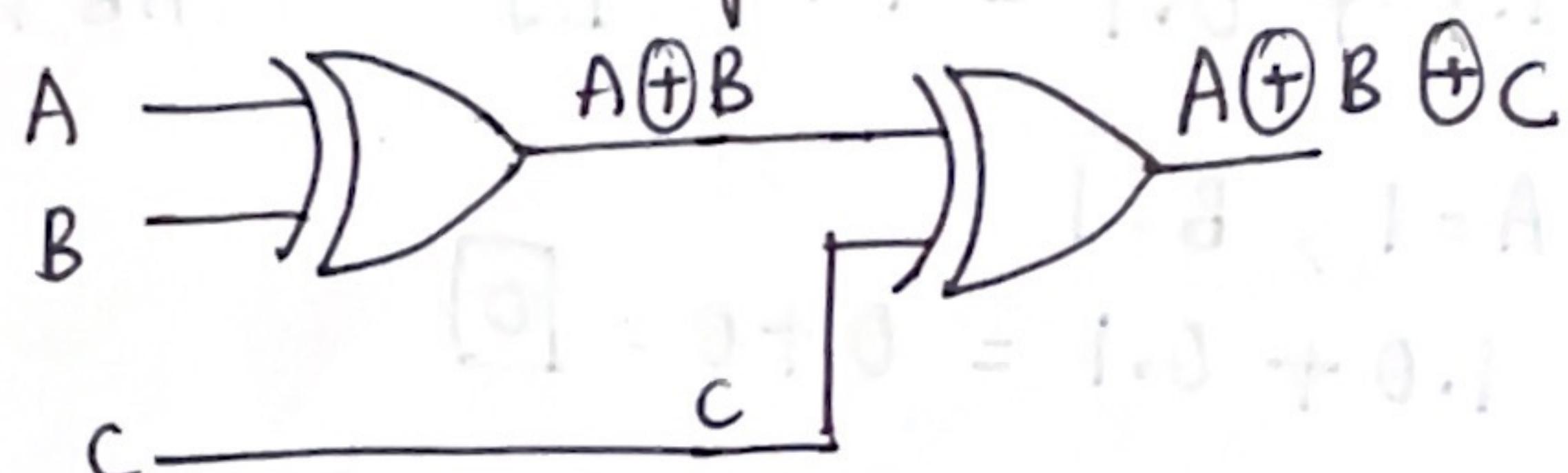
$y = A \oplus B \oplus C \rightarrow$  Boolean Expression.

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A	B	C	y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1



3 input XOR gate using 2 input XOR gate:



XOR gate is used to detect Parity of given bit stream & Error detection.

### 2] XNOR gate:

- \* It is a logical complement of XOR gate.
- \* Output of XNOR gate is logic 1 if Even no of inputs are high and is logic 0 if odd no of inputs are high.
- \* For 2 input XNOR gate output is high if two inputs are same and is low if two inputs are unequal.



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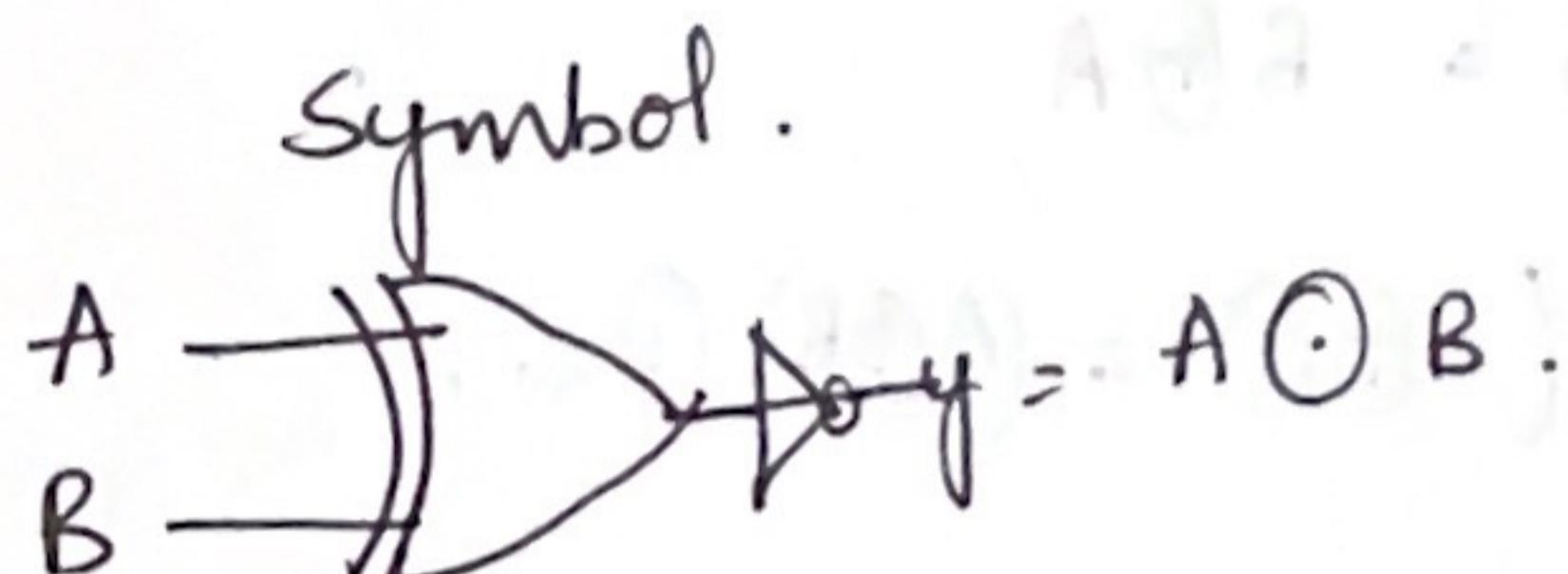
$$y = \overline{A \oplus B}$$

$$= \overline{AB' + A'B}$$

; Boolean Equation :  $y = A \odot B$

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A	B	y
0	0	1
0	1	0
1	0	0
1	1	1



Boolean Equation for XNOR gate

$$y = AB' + A'B$$

$$= (AB')' \cdot (A'B)'$$

$$= A' + (B')' \cdot (A')' + B'$$

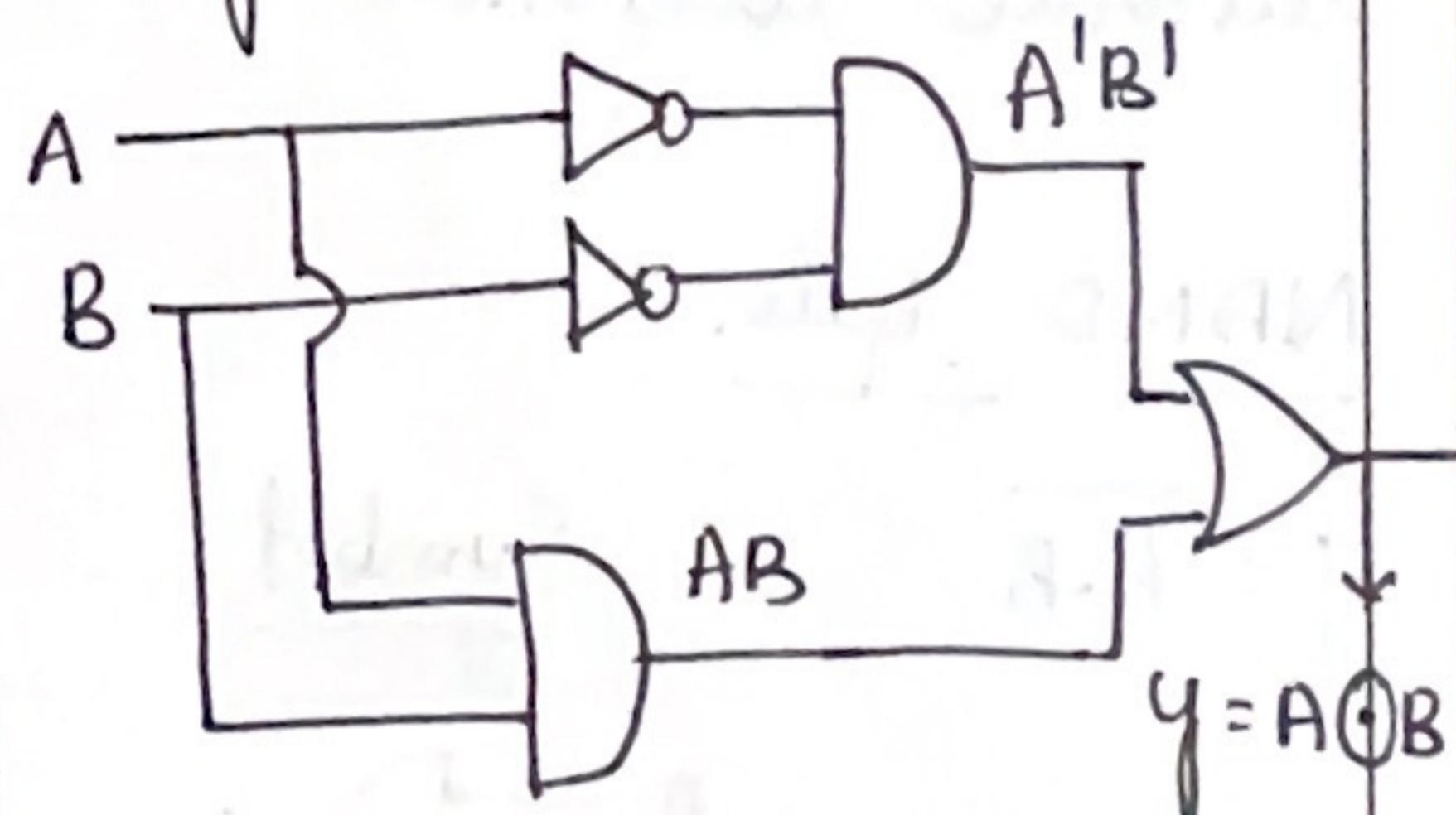
$$= (A' + B) \cdot (A + B')$$

$$= \cancel{AA'} + \cancel{A'B'} + AB + \cancel{BB'} \cancel{B'B}$$

$$= A'B' + AB$$

Implementation of XNOR gate using

$$y = A'B' + AB$$



\* XNOR gate is used in Equality detector

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## Properties of XOR gate :-

$$\textcircled{1} \ A \oplus 0 = A$$

$$\textcircled{2} \ A \oplus 1 = A'$$

$$\textcircled{3} \ A \oplus A = 0$$

$$\textcircled{4} \ A \oplus A' = 1$$

$$\textcircled{5} \ A \oplus B = B \oplus A$$

$$\textcircled{6} \ A \oplus (B \oplus C) = (A \oplus B) \oplus C.$$

## Universal Gate :-

\* Any gate can be constructed using either NAND or NOR gate are called universal gate.

\* NAND and NOR gate are easier to fabricate with electronic components. Hence they are used in all ICs.

### NAND Gate:

$$y = \overline{A \cdot B}$$

Symbol

T.T

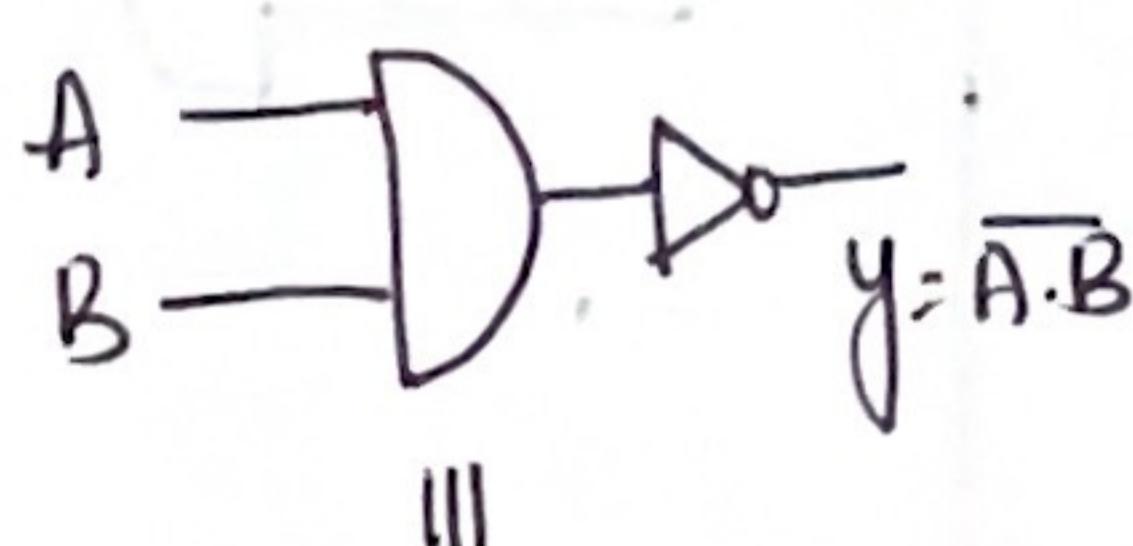
$$A \cdot B$$

$$0 \ 0 \quad |$$

$$0 \ 1 \quad |$$

$$1 \ 0 \quad |$$

$$1 \ 1 \quad 0$$



### NOR Gate :

$$y = \overline{A + B}$$

Symbol

T.T

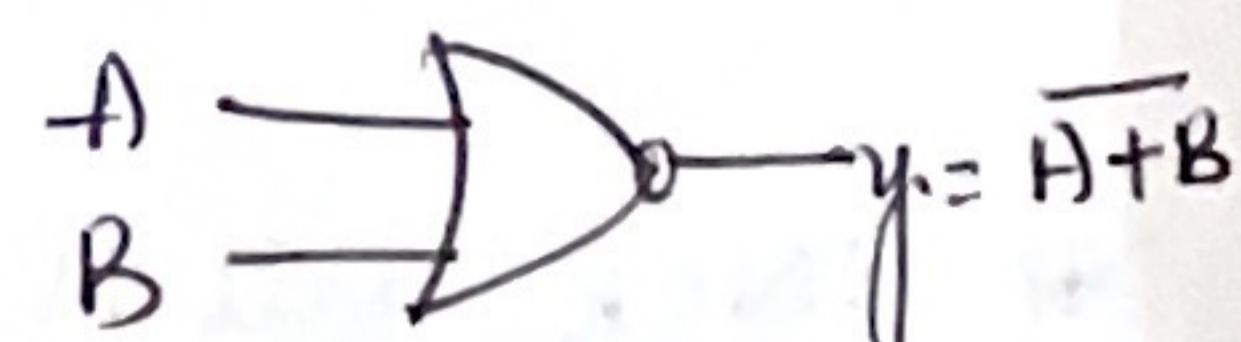
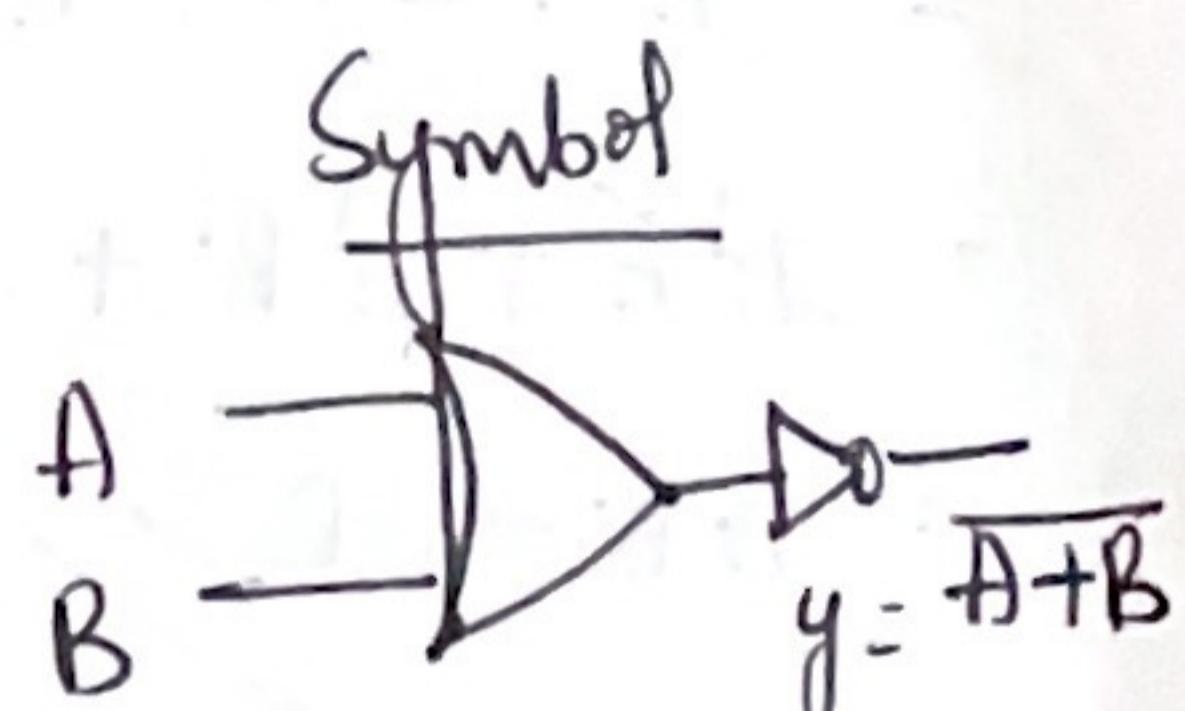
$$A \cdot B \cdot y$$

$$0 \ 0 \quad 1$$

$$0 \ 1 \quad 0$$

$$1 \ 0 \quad 0$$

$$1 \ 1 \quad 0$$



Experiment  
 Name:

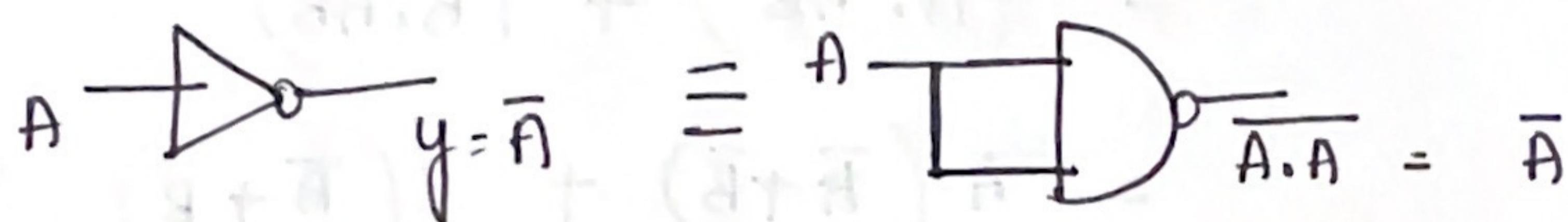
Expt. No.

 Problem  
 Statement:

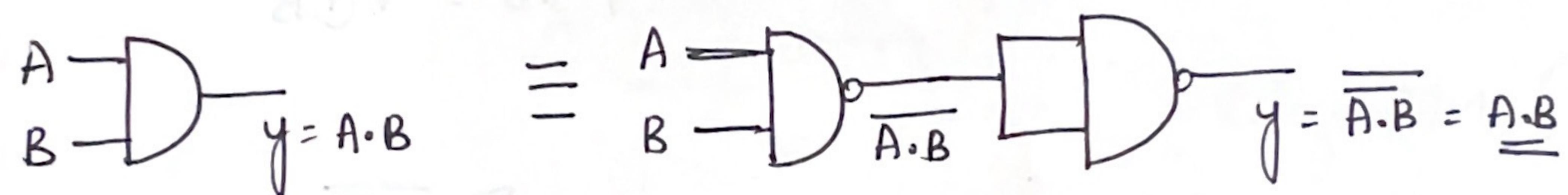
Date

Realization of logic gates using NAND gate;

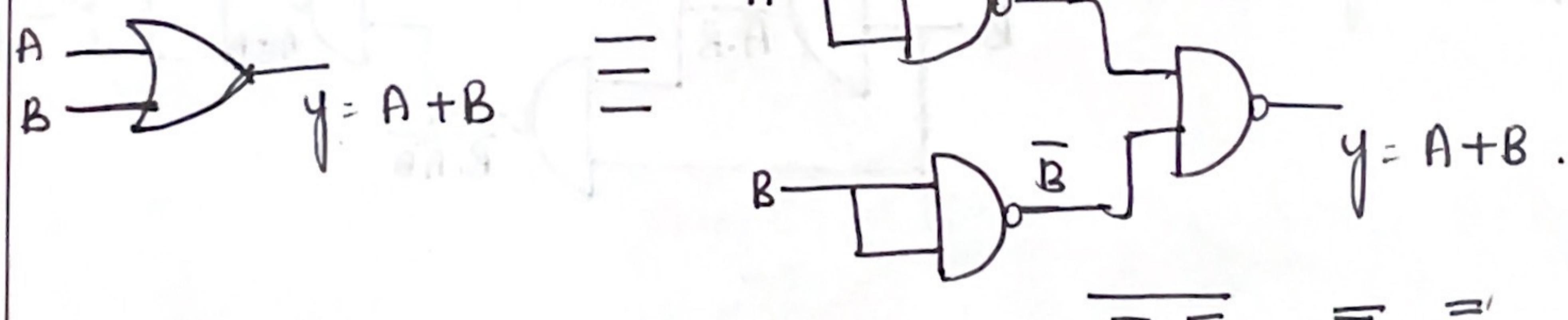
i) NOT gate:



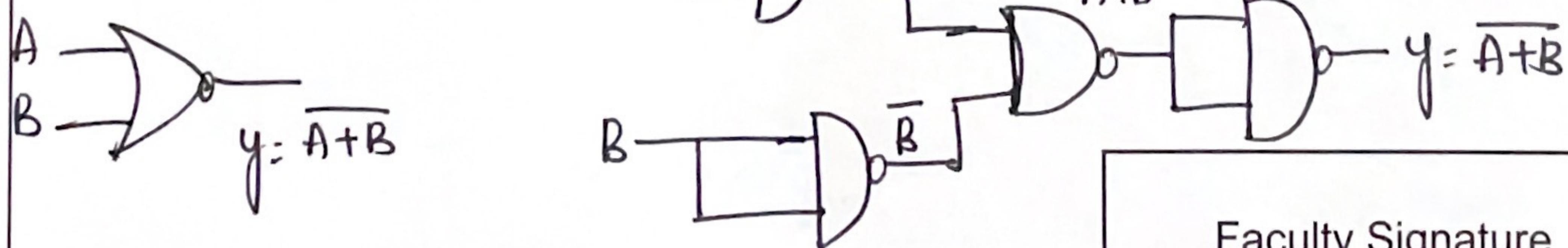
ii) AND gate:



iii) OR gate:

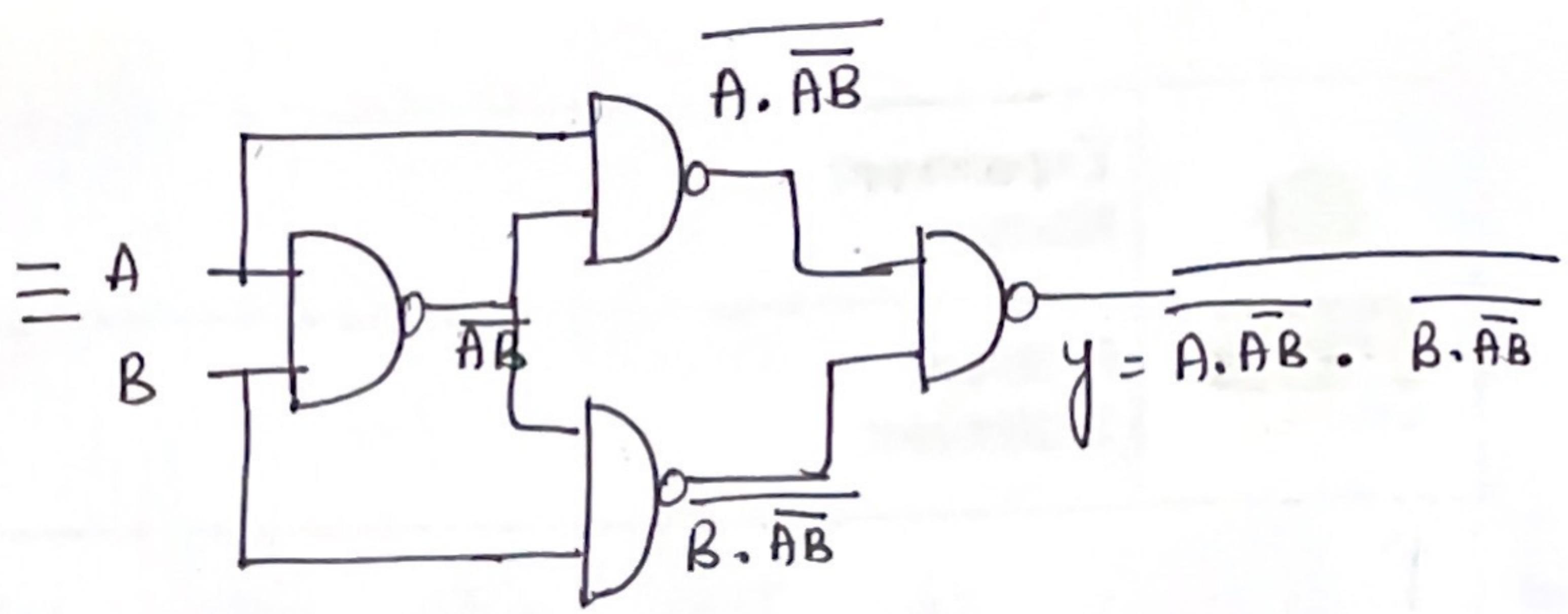
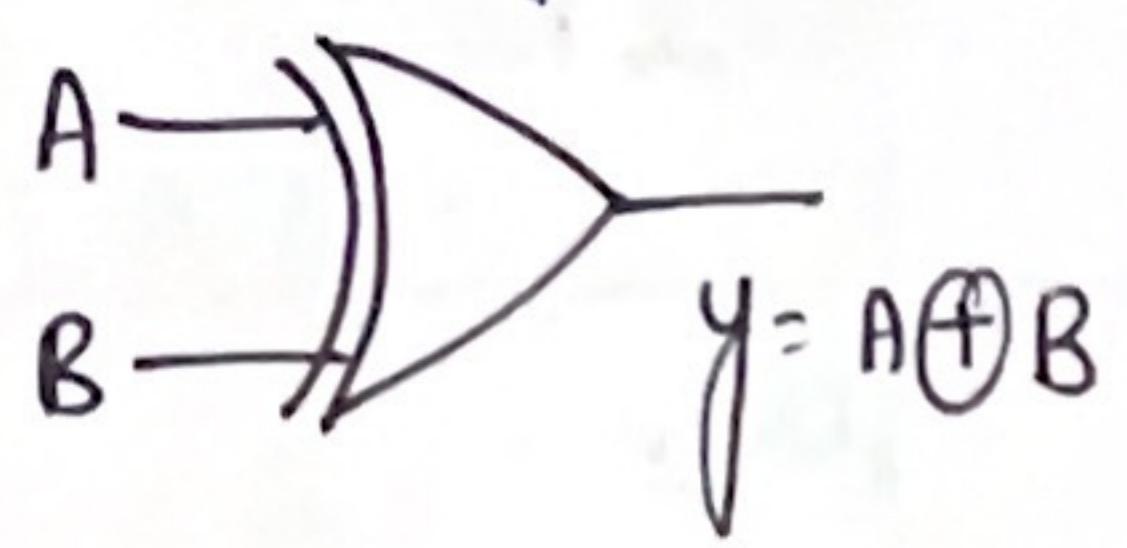


iv) NOR gate:



Faculty Signature

v) XOR gate:



$$y = \overline{A \cdot \overline{B}} \cdot \overline{B \cdot \overline{A}} = \overline{A \cdot \overline{B}} + \overline{B \cdot \overline{A}}$$

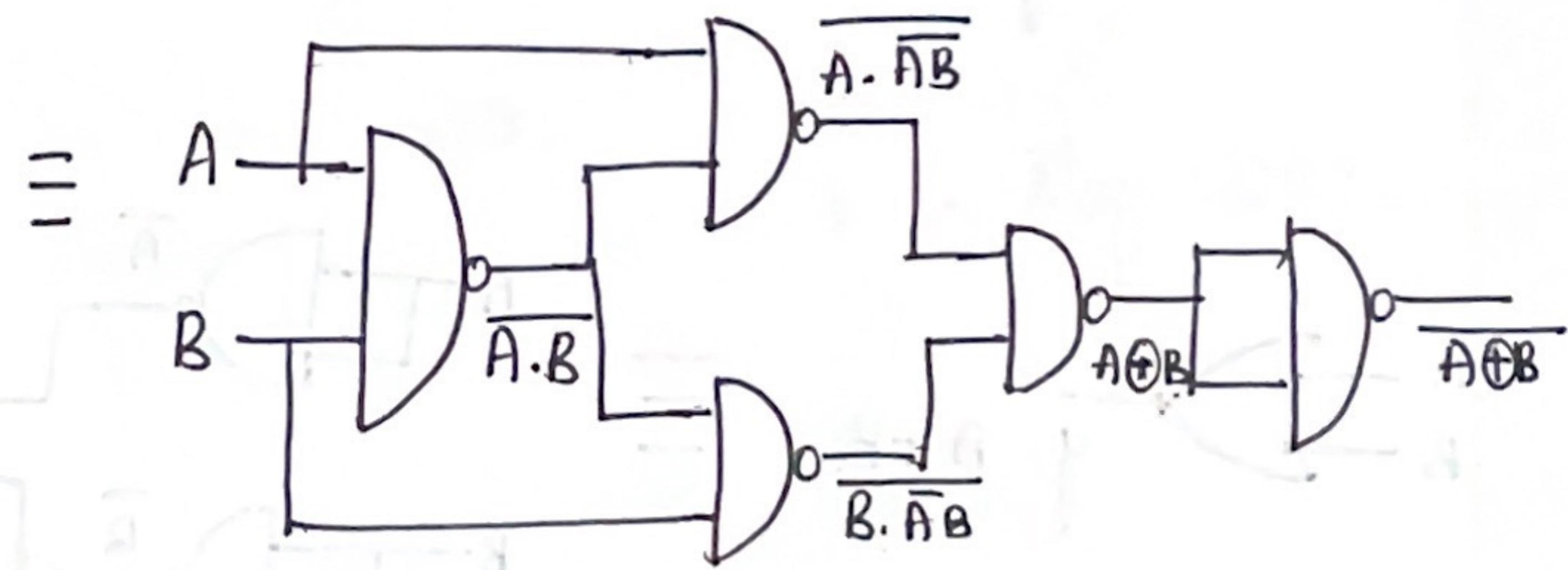
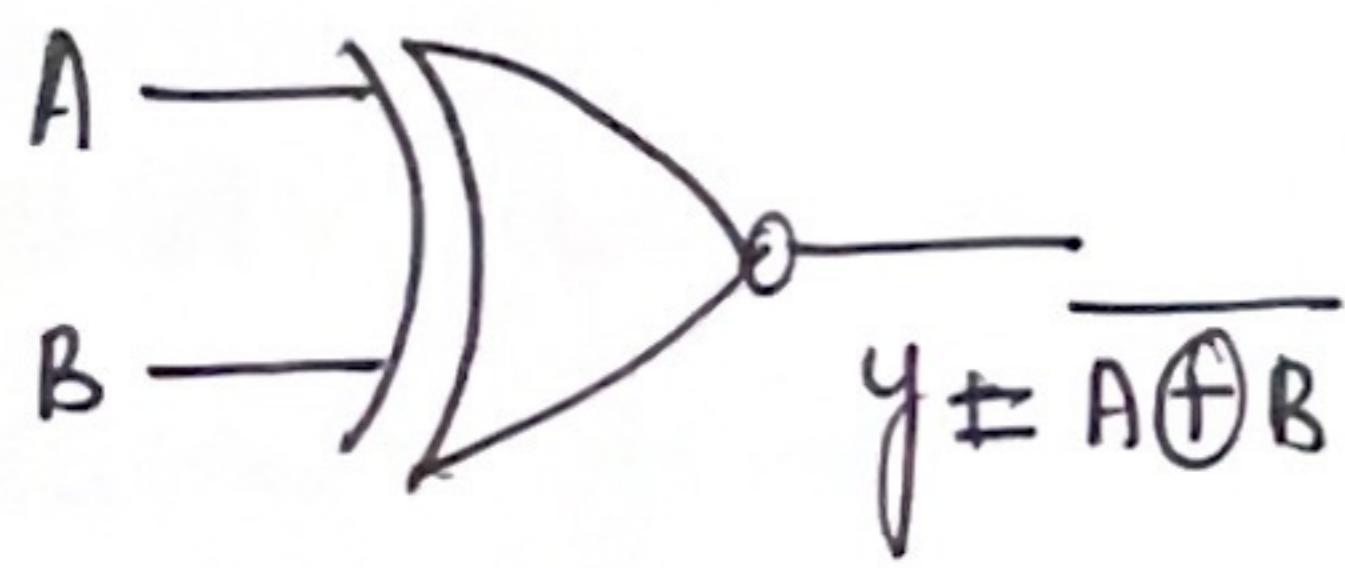
$$= (A \cdot \overline{B}) + (B \cdot \overline{A})$$

$$= A(\overline{A} + \overline{B}) + B(\overline{A} + \overline{B})$$

$$= \cancel{A\overline{A}} + \overline{AB} + B\overline{A} + \cancel{B\overline{B}}$$

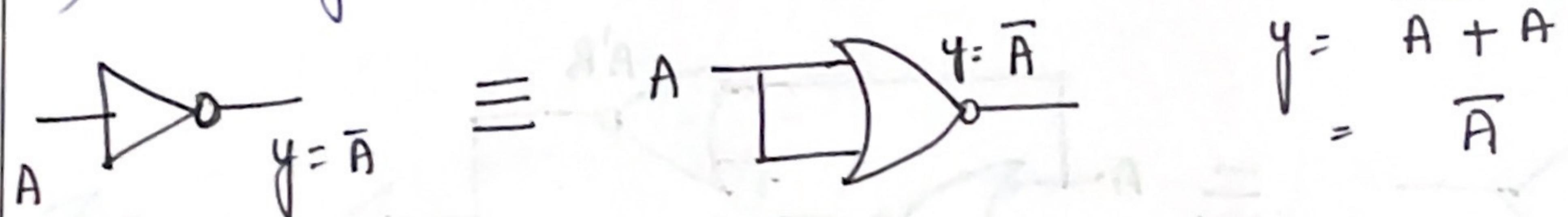
$$\therefore \overline{AB} + \overline{BA} = A \oplus B$$

vi) XNOR gate:

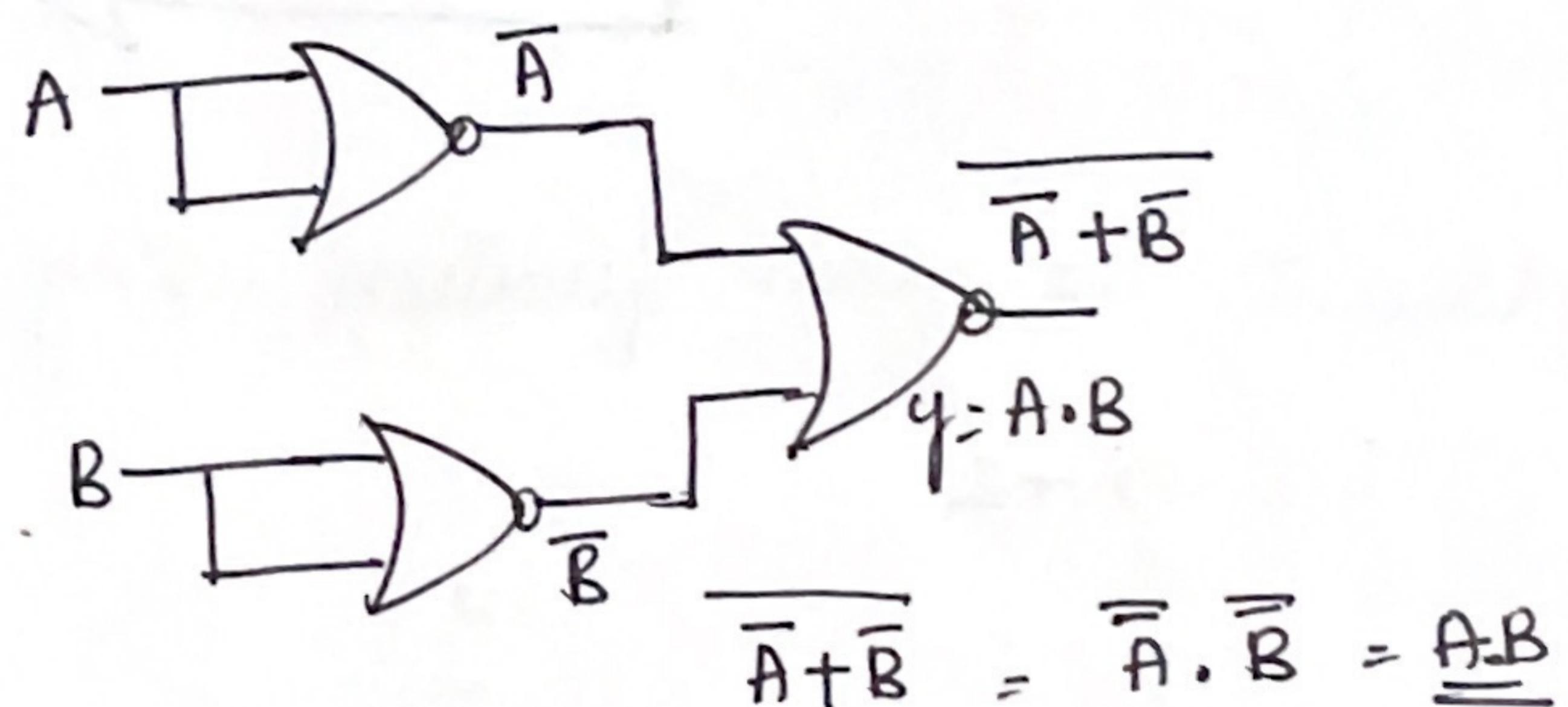
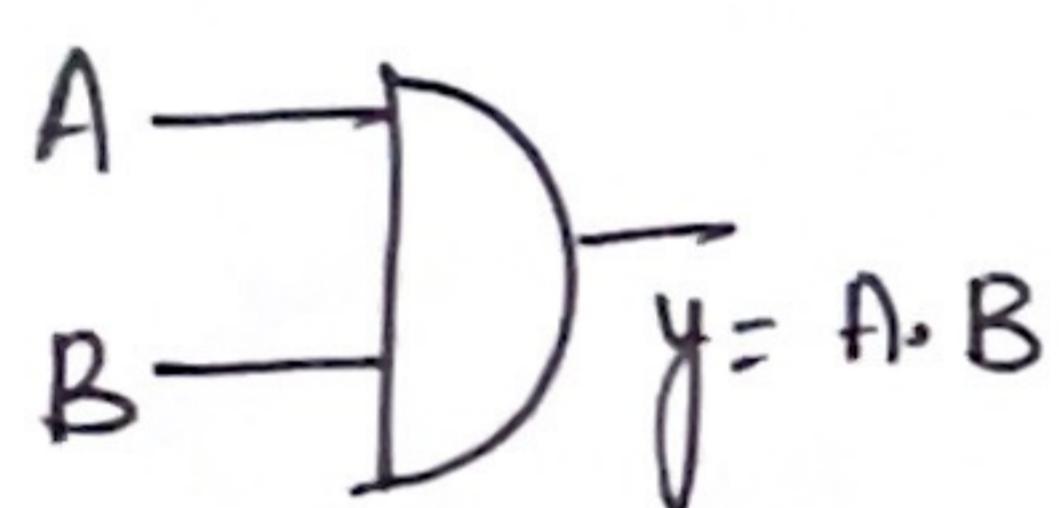


Realization of logic gate using NOR gate:

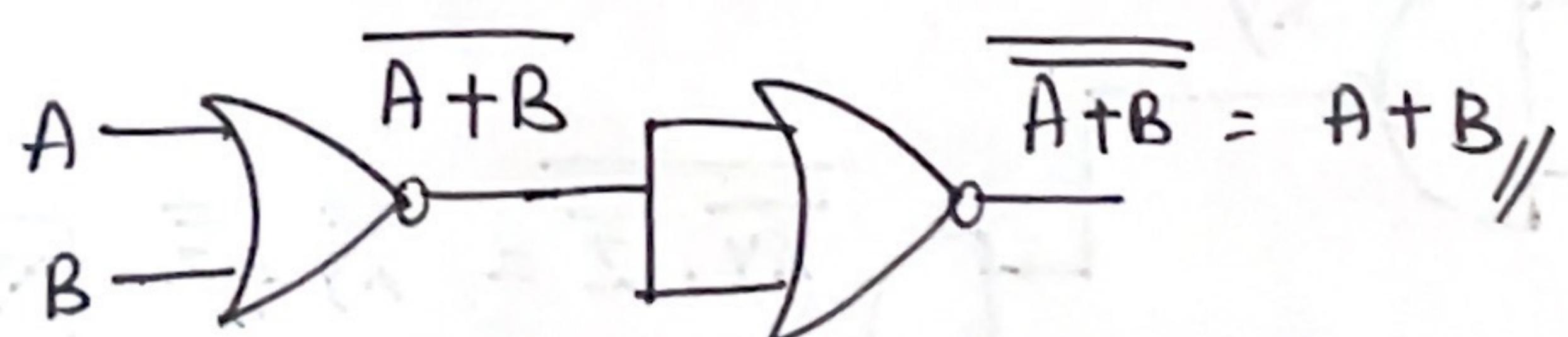
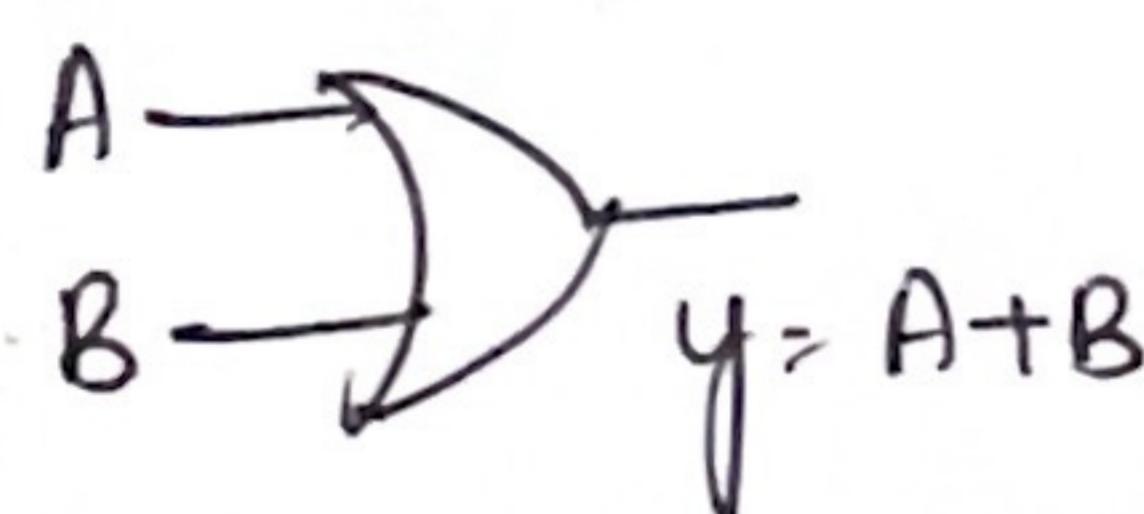
i) NOT gate:



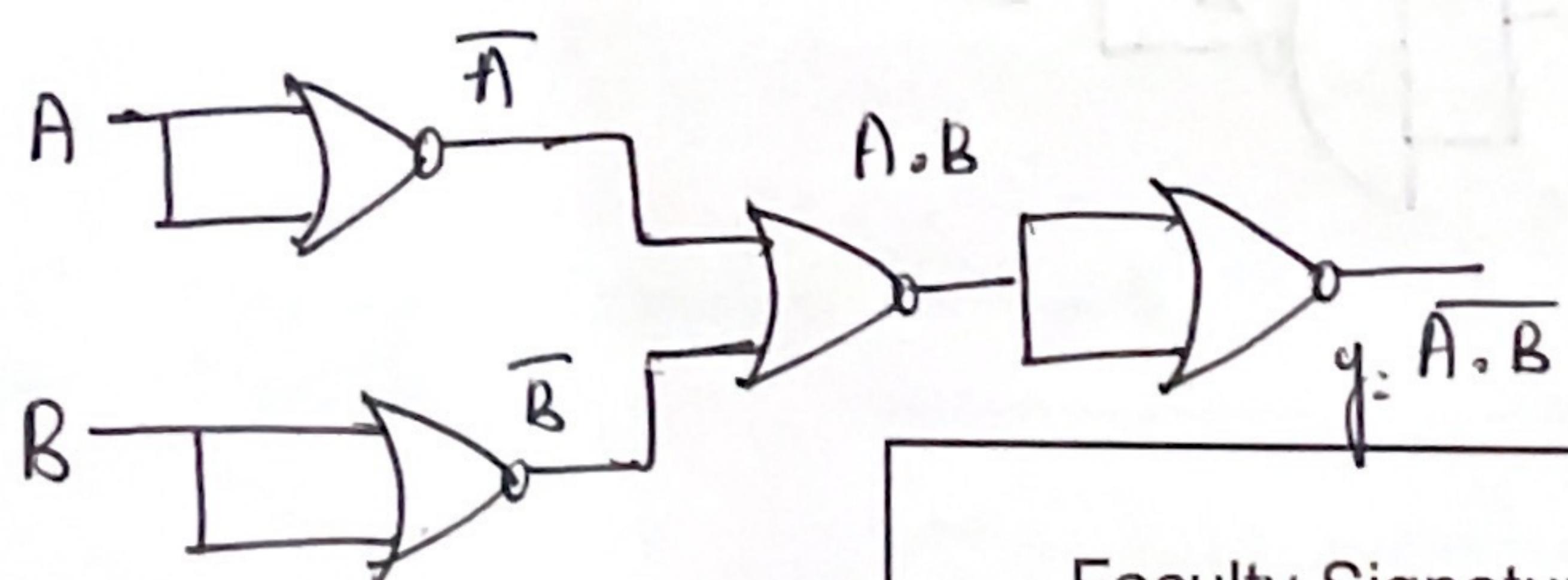
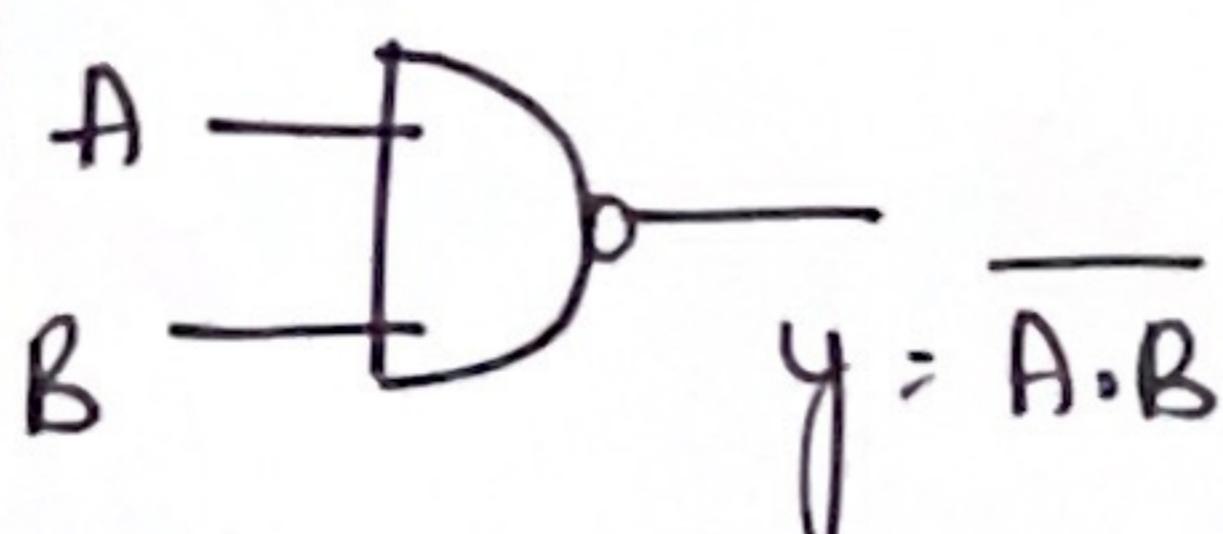
ii) AND gate:



iii) OR gate:

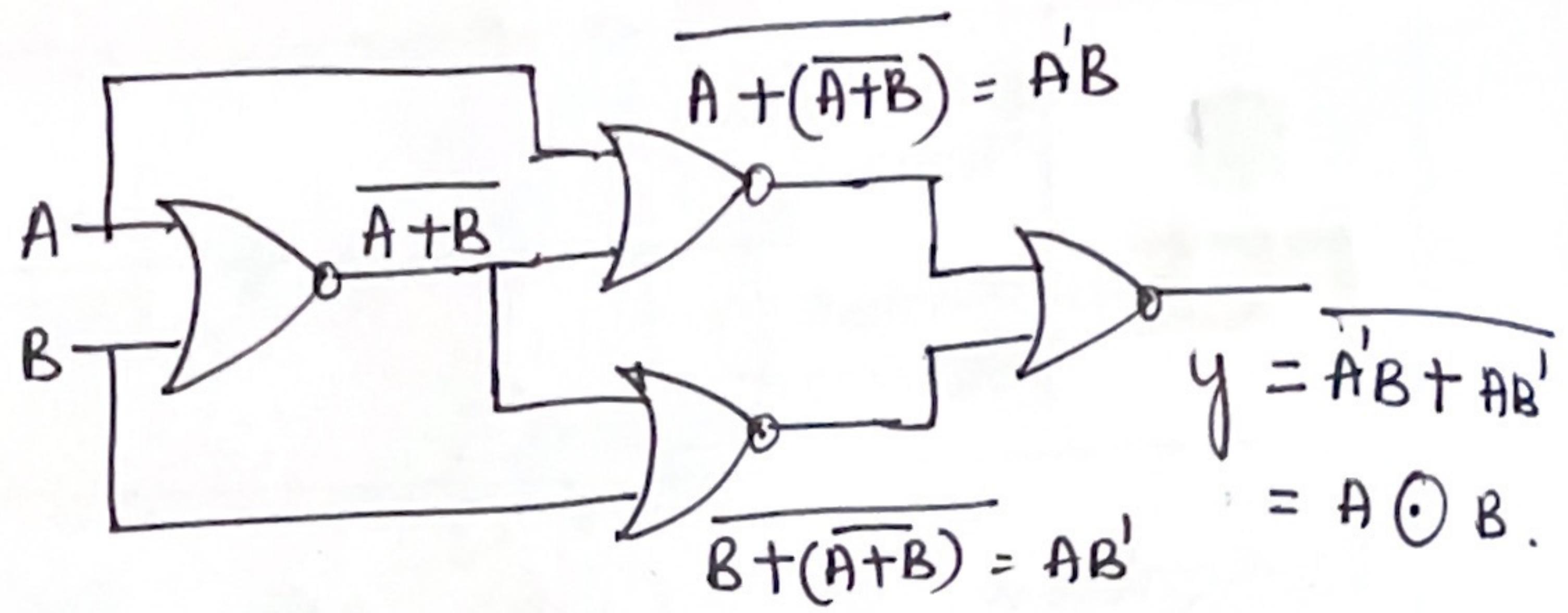
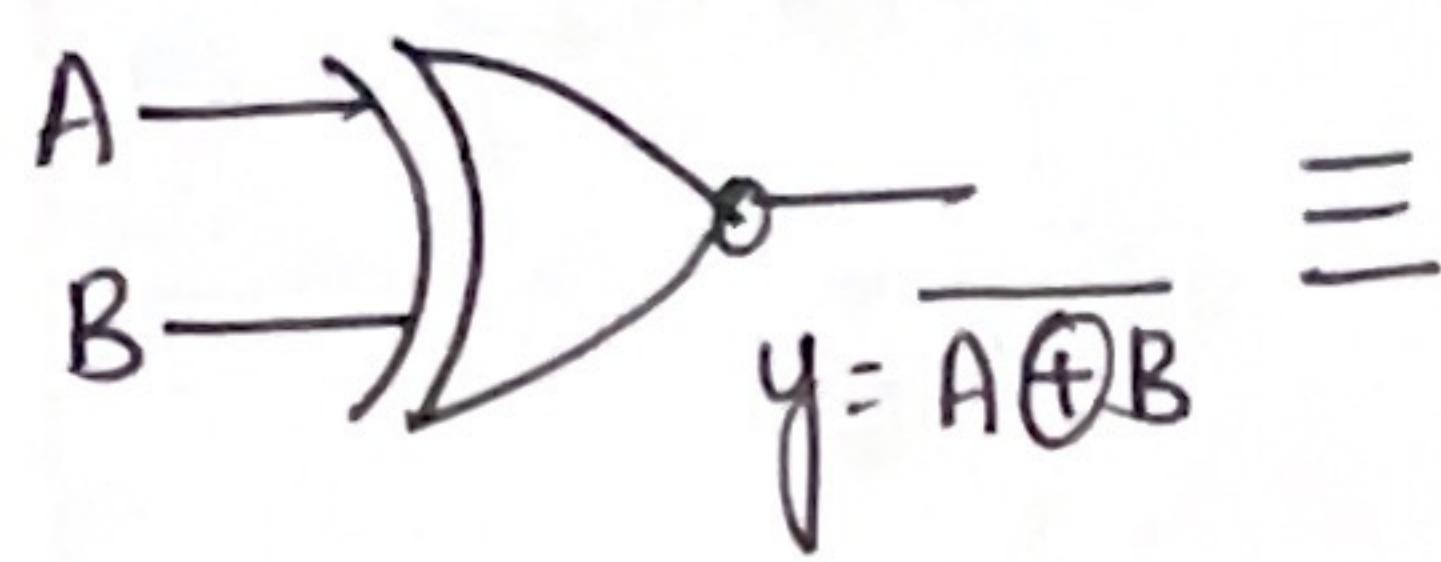


iv) NAND gate:

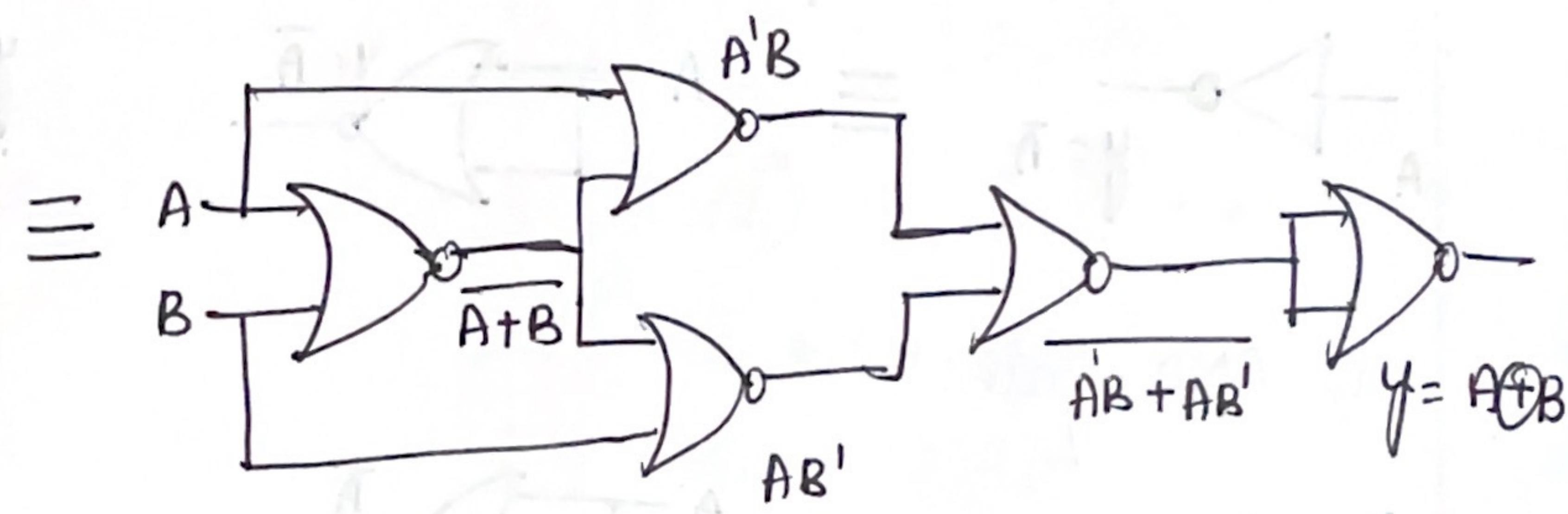
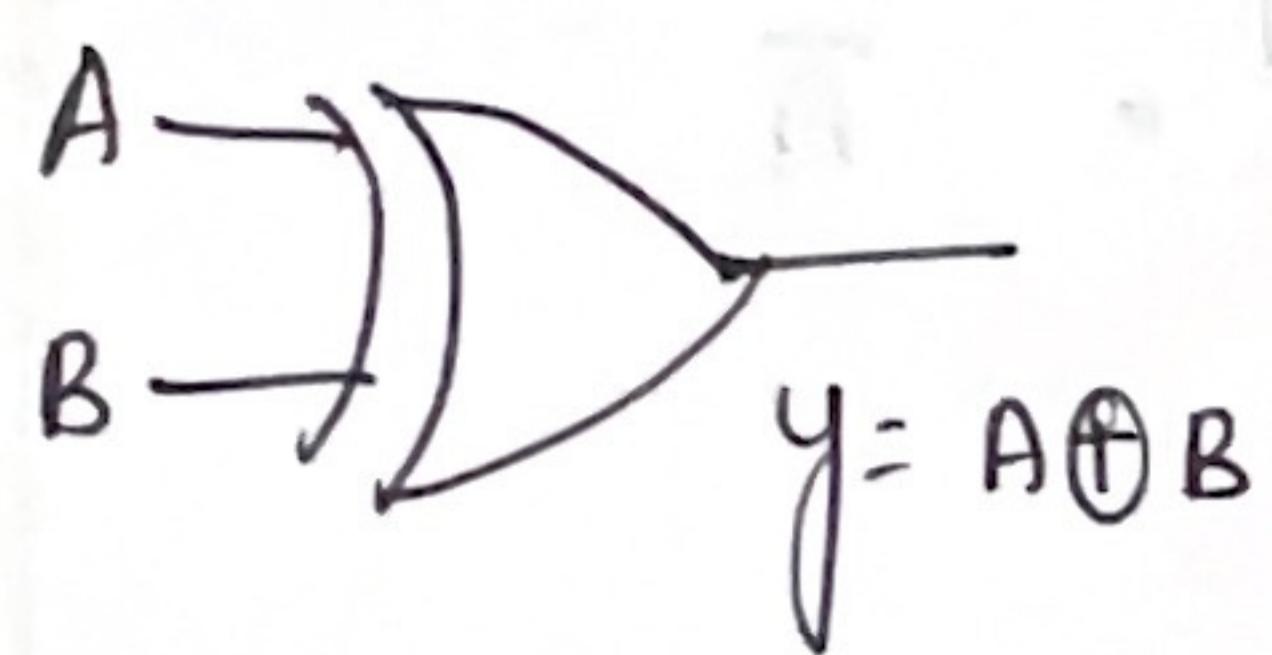


Faculty Signature

v) XNOR gate

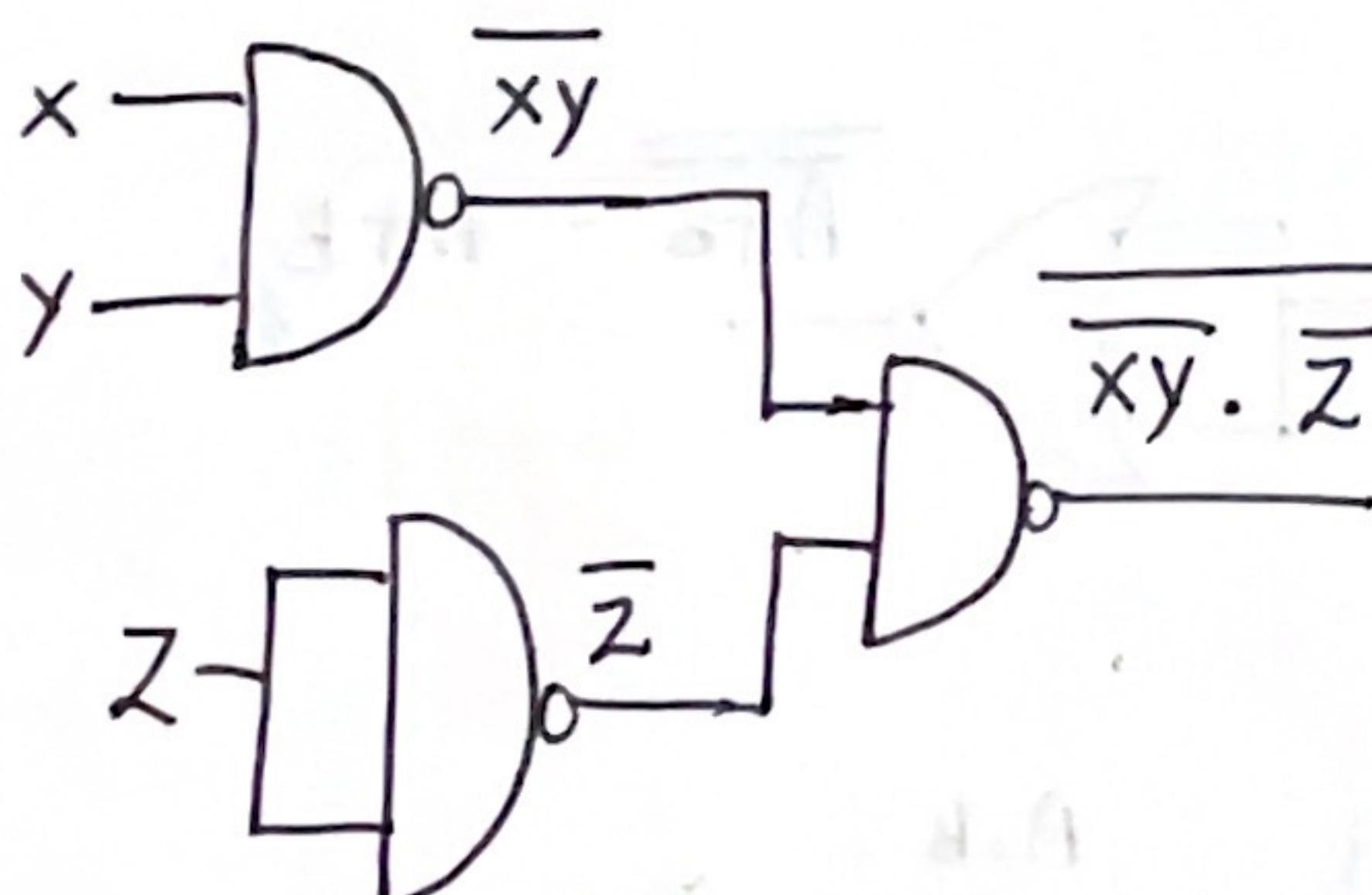
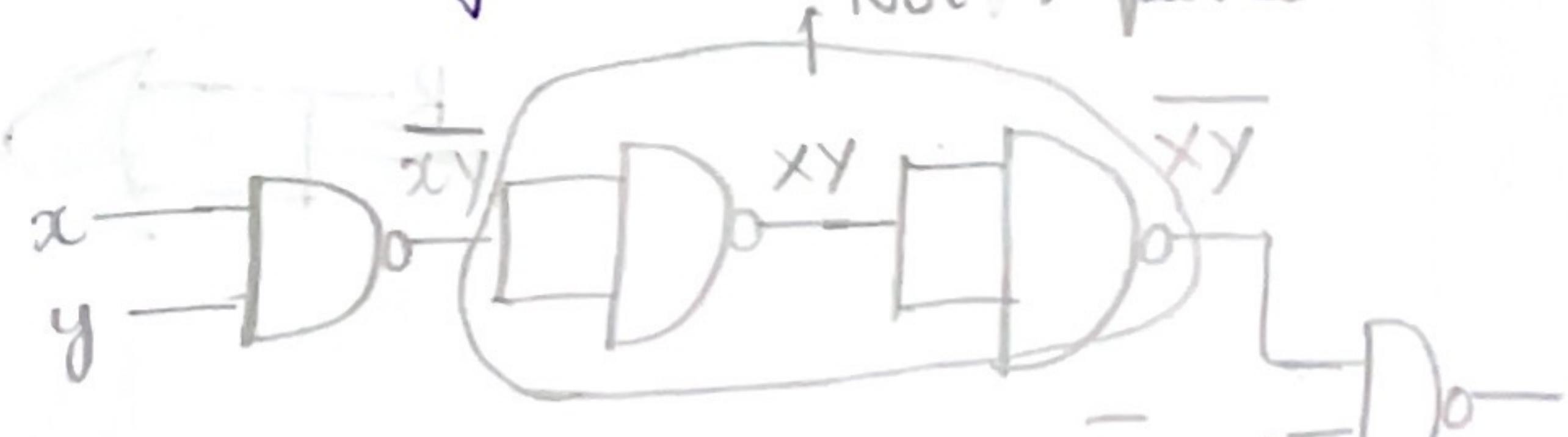


v) XOR gate



Implement the given function using NAND Gate Only.  
NOT required

$$F = XY + Z$$



$$\overline{xy} \cdot \overline{z} = \overline{\overline{xy} + \overline{z}} = \boxed{XY + Z = F}$$

$$\overline{\overline{xy} + \overline{z}} = \overline{\overline{xy}} + \overline{\overline{z}} = \overline{xy} + z$$

$$\overline{xy} + z$$

Experiment  
 Name:

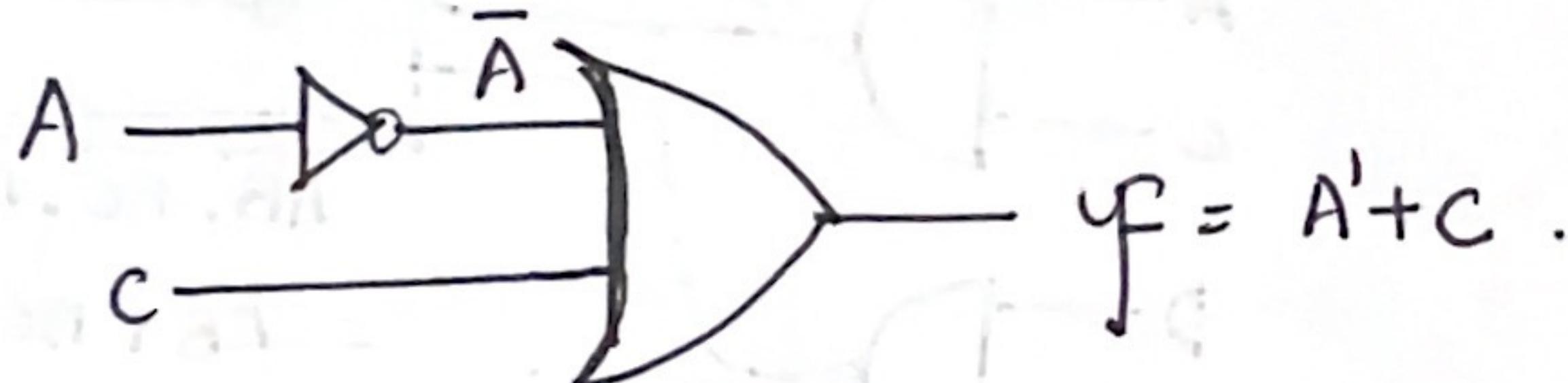
Expt. No.

 Problem  
 Statement:

Date

Simplify the given boolean expression and implement using basic gates.

$$\begin{aligned}
 F &= A \cdot B \cdot C + A' + A \cdot B' \cdot C \\
 &= A \cdot B \cdot C + A \cdot B' \cdot C + A' \\
 &= AC [B + B']^1 + A' \\
 &= AC [1] + A' \\
 &= A' + AC \quad \because \text{Distributive law} \\
 &= (\cancel{A'} + \cancel{A})^1 (A' + C) \\
 &= A' + C
 \end{aligned}$$

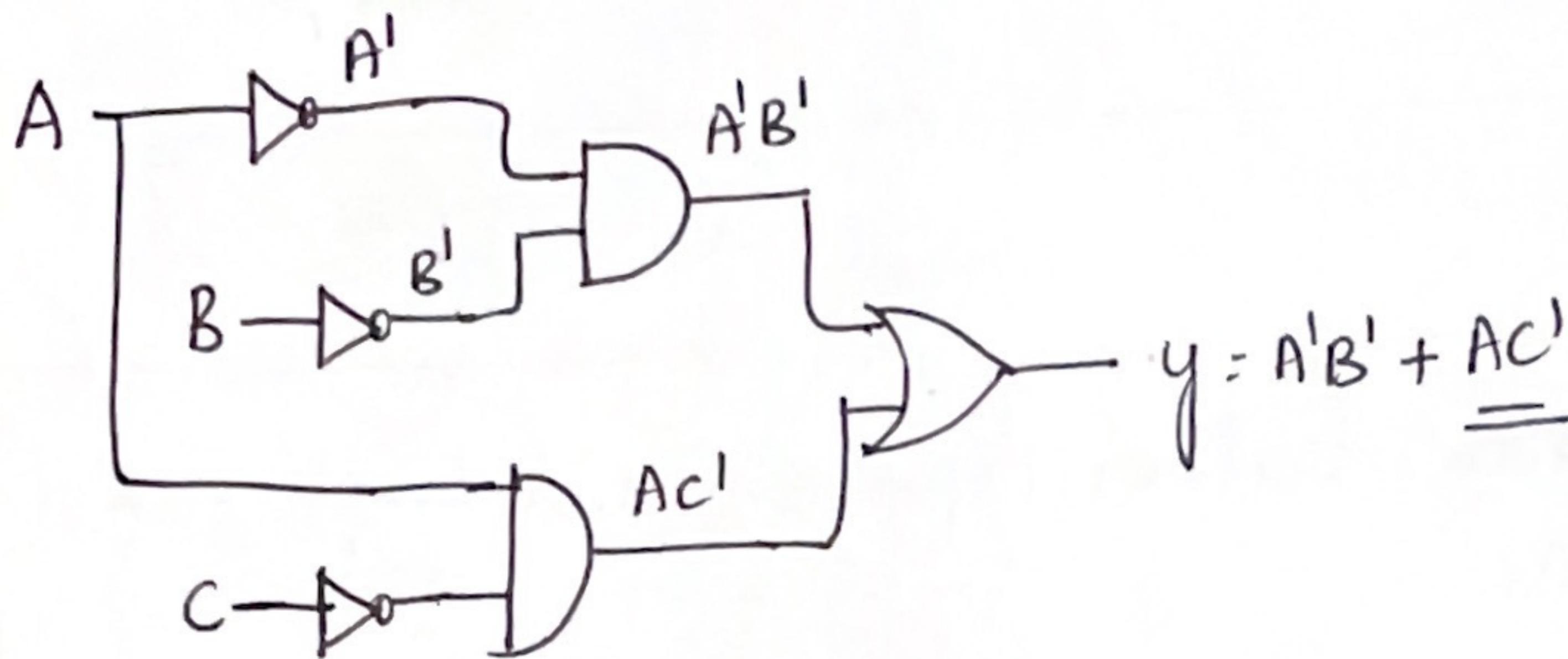


Simplify and realize the given function using basic gate.

$$\begin{aligned}
 Y &= A'B'C' + AB'C' + A'B' + AC' \\
 &= B'C' (\cancel{A'} + \cancel{A})^1 + A'B' + AC' \\
 &= B'C' + \underline{A'} \underline{B'} + \underline{A} \underline{C'} \\
 &= A'B' + AC' \quad (\text{using consensus theorem})
 \end{aligned}$$

Faculty Signature

$$Y = A'B' + AC'$$



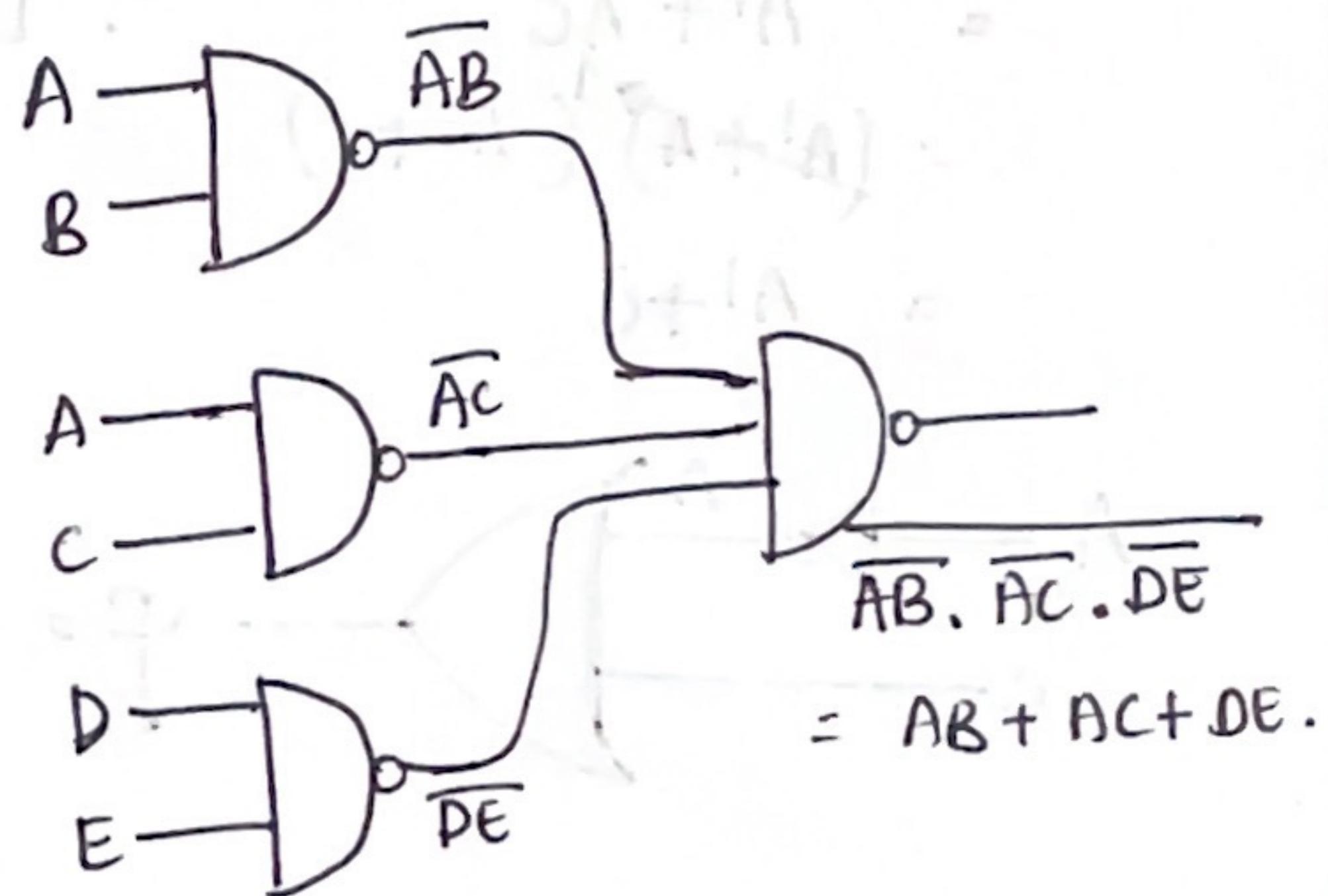
Simplify and implement the given function using NAND gate only.

$$F = AC(B+C) + DE$$

$$F = AB + AC + DE$$

Apply De-Morgan's law

$$\begin{aligned} F &= \overline{AB + AC + DE} \\ &= \overline{\overline{AB} \cdot \overline{AC} \cdot \overline{DE}} \end{aligned}$$



Simplify the given Boolean expression.

$$Y = A + A'B + A'B'C + A'B'C'D + A'B'C'D'E$$

$$= A + A'(B + B'C + B'C'D + B'C'D'E)$$

$$= A + B + B'C + B'C'D + B'C'D'E \quad \because A + A'B = A + B$$

$$= A + B + B'(C + C'D + C'D'E) \quad \text{absorption law.}$$

$$= A + B + C + C'D + C'D'E$$

$$= A + B + C + C'(D + D'E)$$

$$= A + B + C + D + D'E = A + B + C + D + E //$$

Experiment  
Name:

Expt. No.

Problem  
Statement:

Date

Simplify the given Boolean expression and implement the circuit using basic gates and using NAND gate

$$F = AB + A(B+C) + B(B+C)$$

$$F = AB + AB + AC + B \cdot B + BC$$

$$= AB + AC + B + BC$$

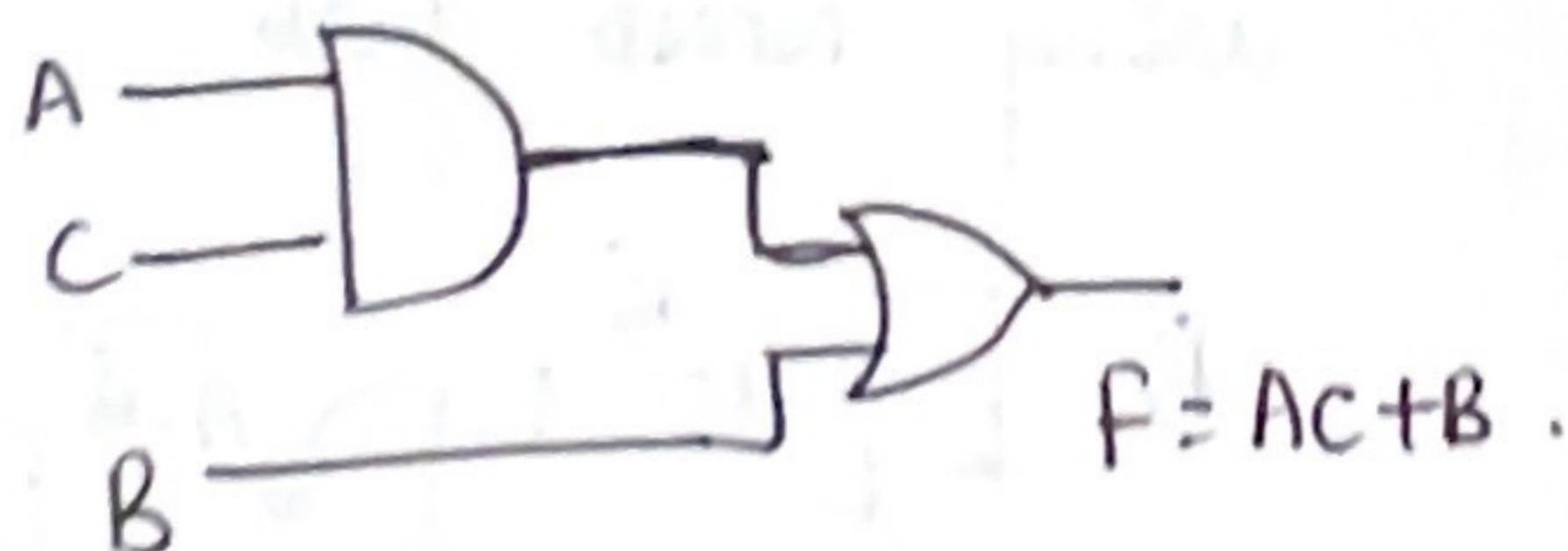
$$= AB + AC + B(1+C)$$

$$= AB + AC + B$$

$$= B(1+A) + AC$$

$$\boxed{F = AC + B}$$

logic circuit using basic gates

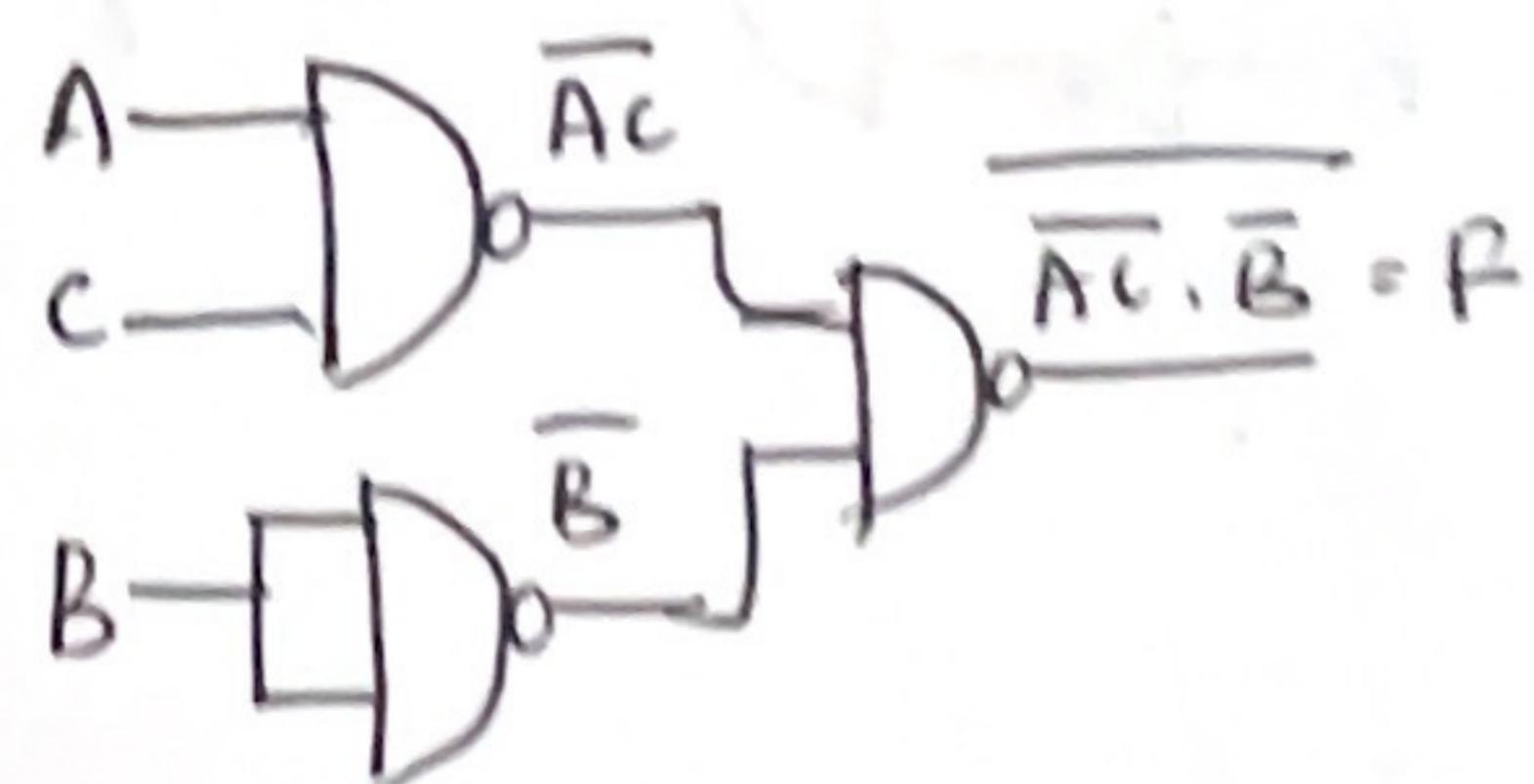


using NAND gate

$$F = AC + B$$

$$= \overline{\overline{AC} + B}$$

$$= \overline{AC} \cdot \overline{B}$$



Faculty Signature

Simplify and realize the given function using

- i) Basic gates
- ii) NAND gate

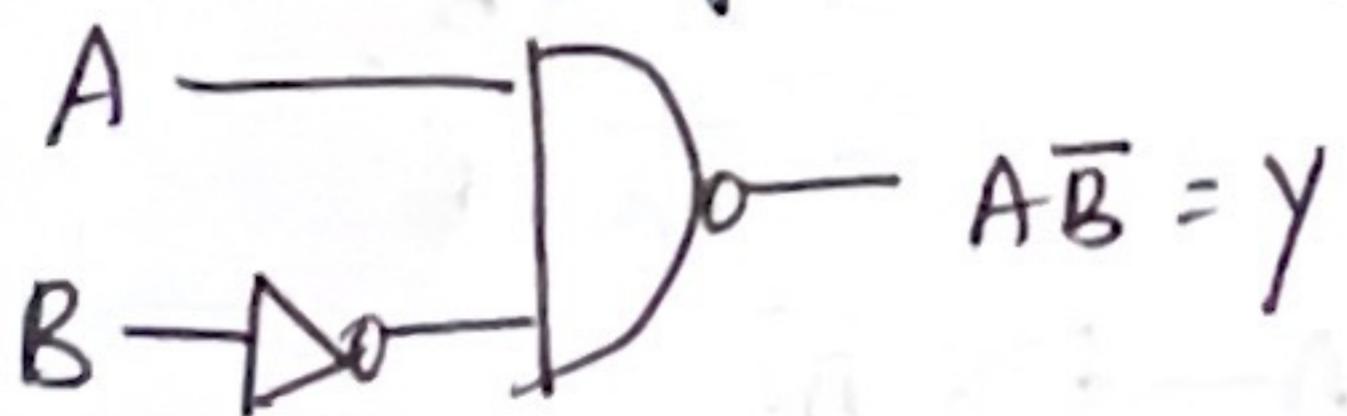
$$Y = (A \cdot B + A' + A \cdot B)^t$$

Apply De-Morgan's Law.

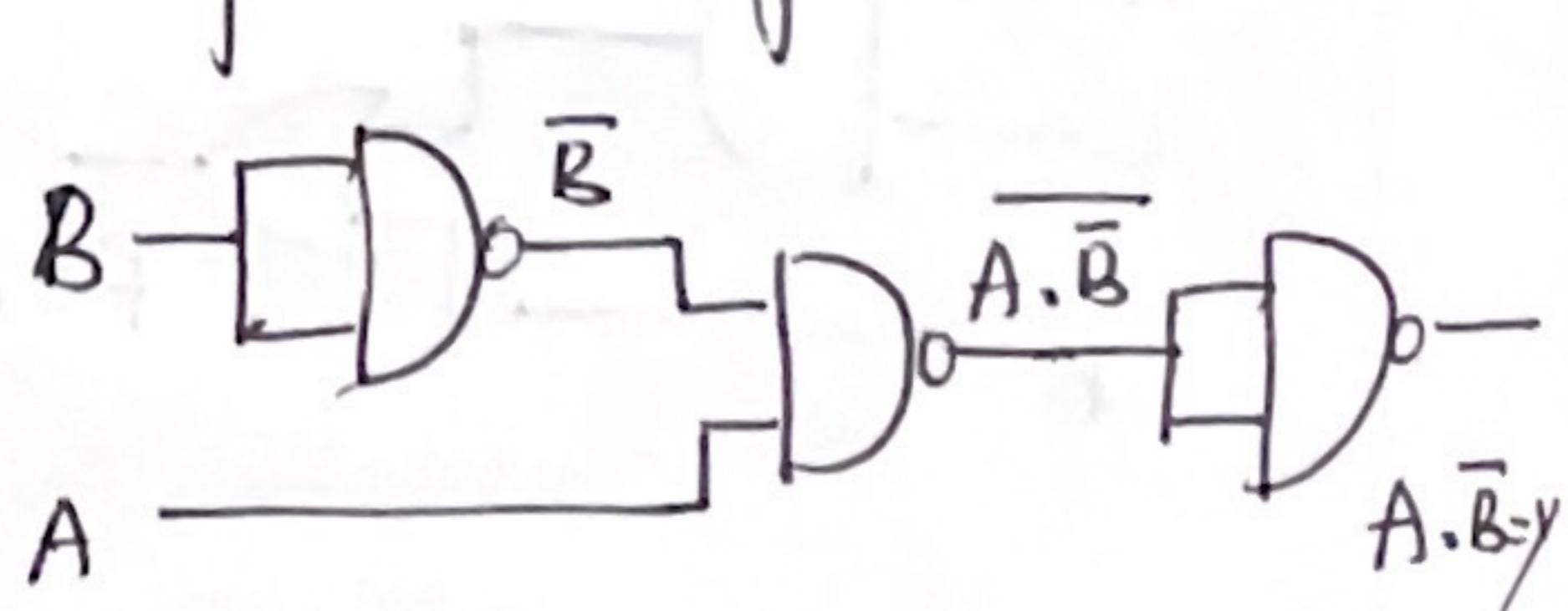
$$\begin{aligned} Y &= \overline{A'B} \cdot \overline{\overline{A}} \cdot \overline{A \cdot B} \\ &= (\overline{\overline{A}} + \overline{B}) \cdot A \cdot (\overline{A} + \overline{B}) \\ &= (A + \overline{B}) \cdot A \cdot (\overline{A} + \overline{B}) \\ &= (A \cdot A + A \cdot \overline{B}) (\overline{A} + \overline{B}) \\ &= A \cdot \overline{A} + A \cdot A \cdot \overline{B} + A \cdot \overline{B} \cdot \overline{A} + A \cdot \overline{B} \cdot \overline{B} \\ &= A \overline{B} + A \overline{B} \end{aligned}$$

$$\boxed{Y = A \overline{B}}$$

Circuit using basic gates.

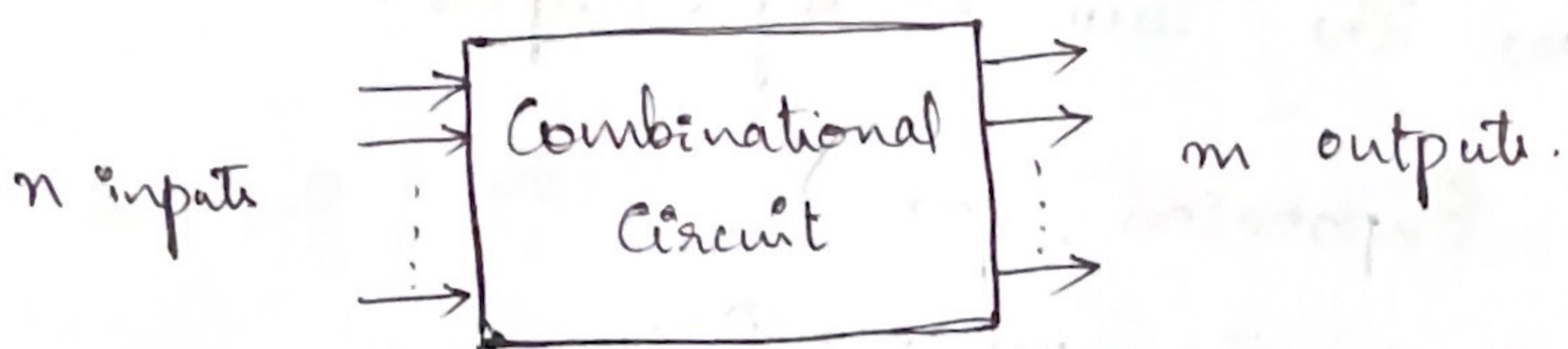


using NAND gate



## Combinational logic circuits:

- \* Combinational logic circuits are digital circuits which are interconnection logic gates whose output depends on current combination of inputs.



- \* Operation performed by a combinational circuit is specified by a "boolean function".
- \* Example: Binary adders & Multiplexers etc.

## Binary adders

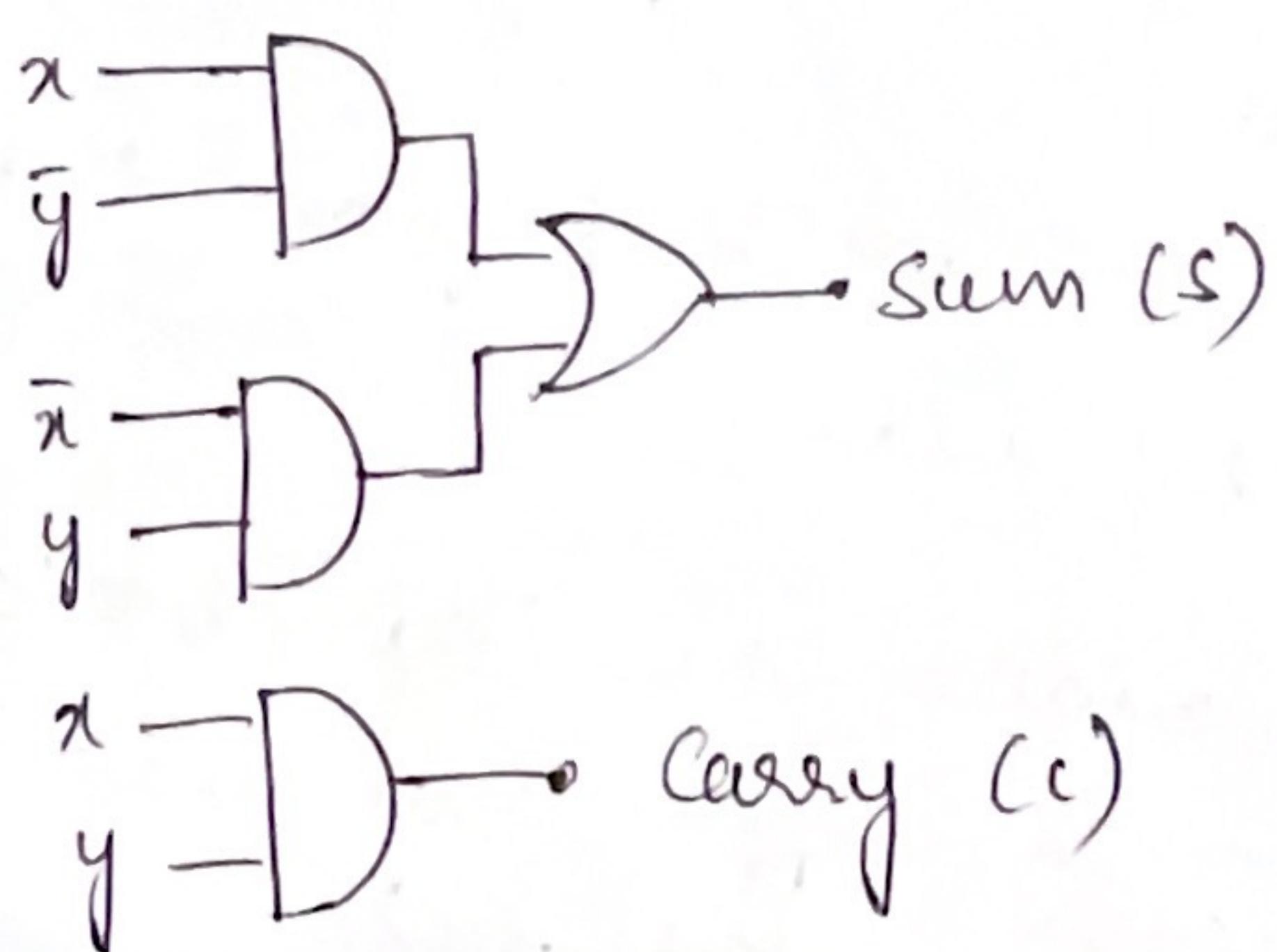
- \* There are two binary adders
  - 1] Half adder
  - 2] Full adder
- \* A combinational circuit which performs 2 bit addition is called Half adder.
- \* A combinational circuit which performs 3 bit addition is called full adder.
- \* Half adders/full adders are the basic components of adders such as: Ripple carry adder, Carry look ahead adder & other fast adders.

## Half adder:-

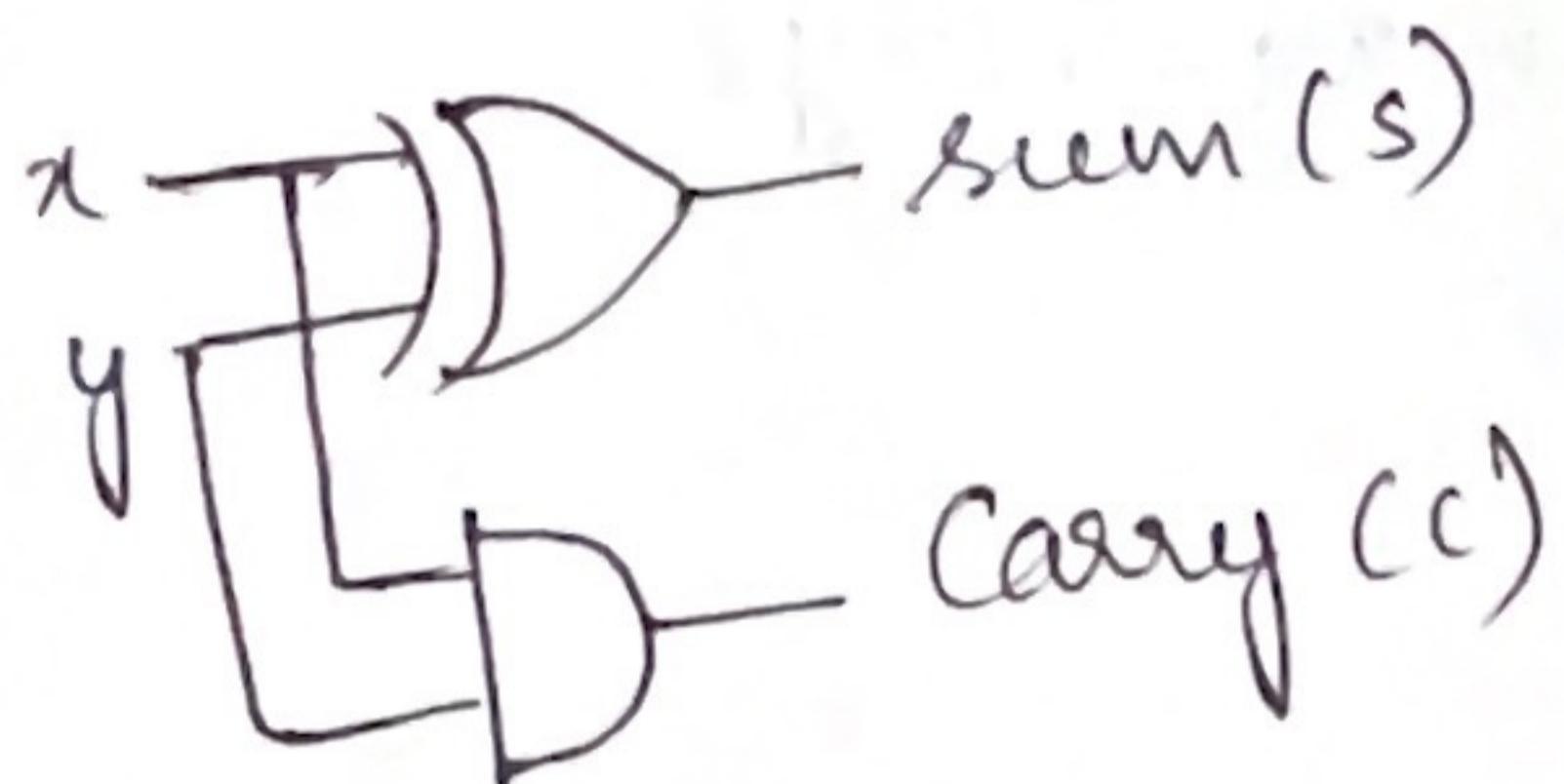
- \* It performs 2 bit binary addition
- \*  $x$  &  $y$  are two binary inputs, Sum ( $s$ ) carry ( $c$ ) are the two binary outputs.
- \* Boolean Expression for  
Sum,  $s = \bar{x}y + x\bar{y} = x \oplus y$   
Carry,  $c = xy$
- \* Truth table .

$x$	$y$	$s$	$c$
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

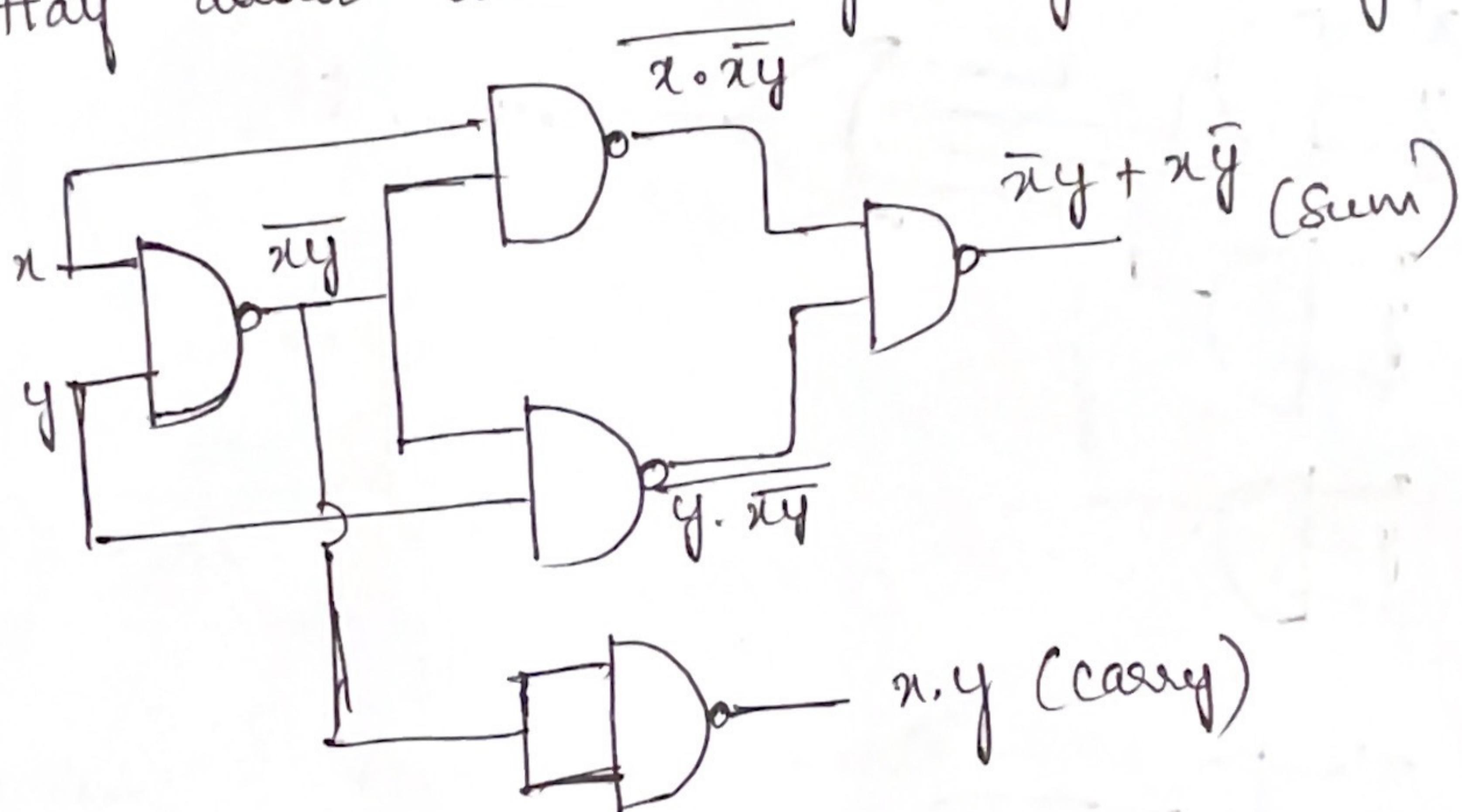
- \* Circuit diagram .



\* Half adder circuit can also be written as follows:-



\* Half adder circuit using only NAND gates.



### Full adder:-

- \* It performs 3 bit binary addition
- \*  $x, y$  &  $z$  are its binary inputs and  $s \& c$  are its output.

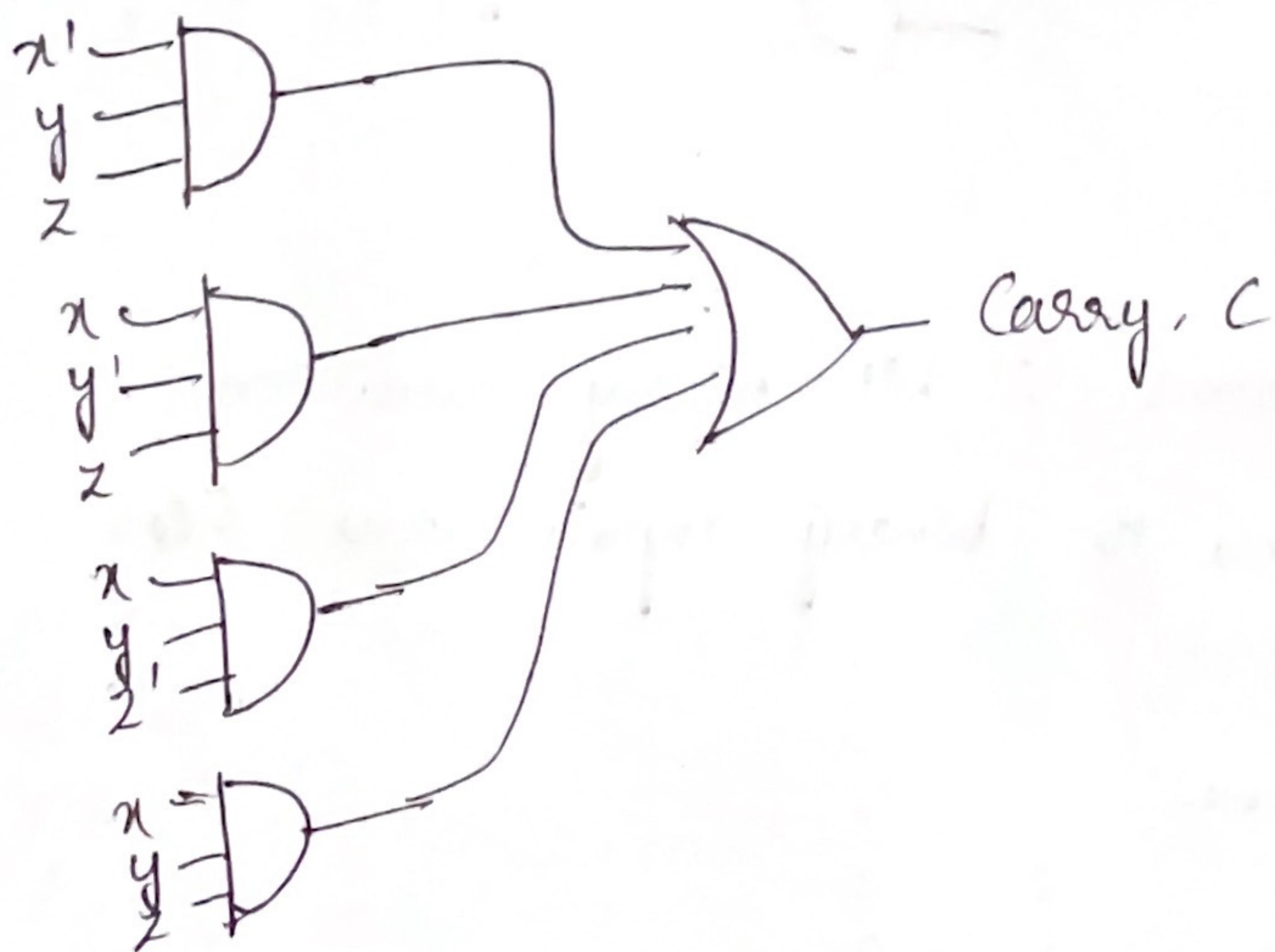
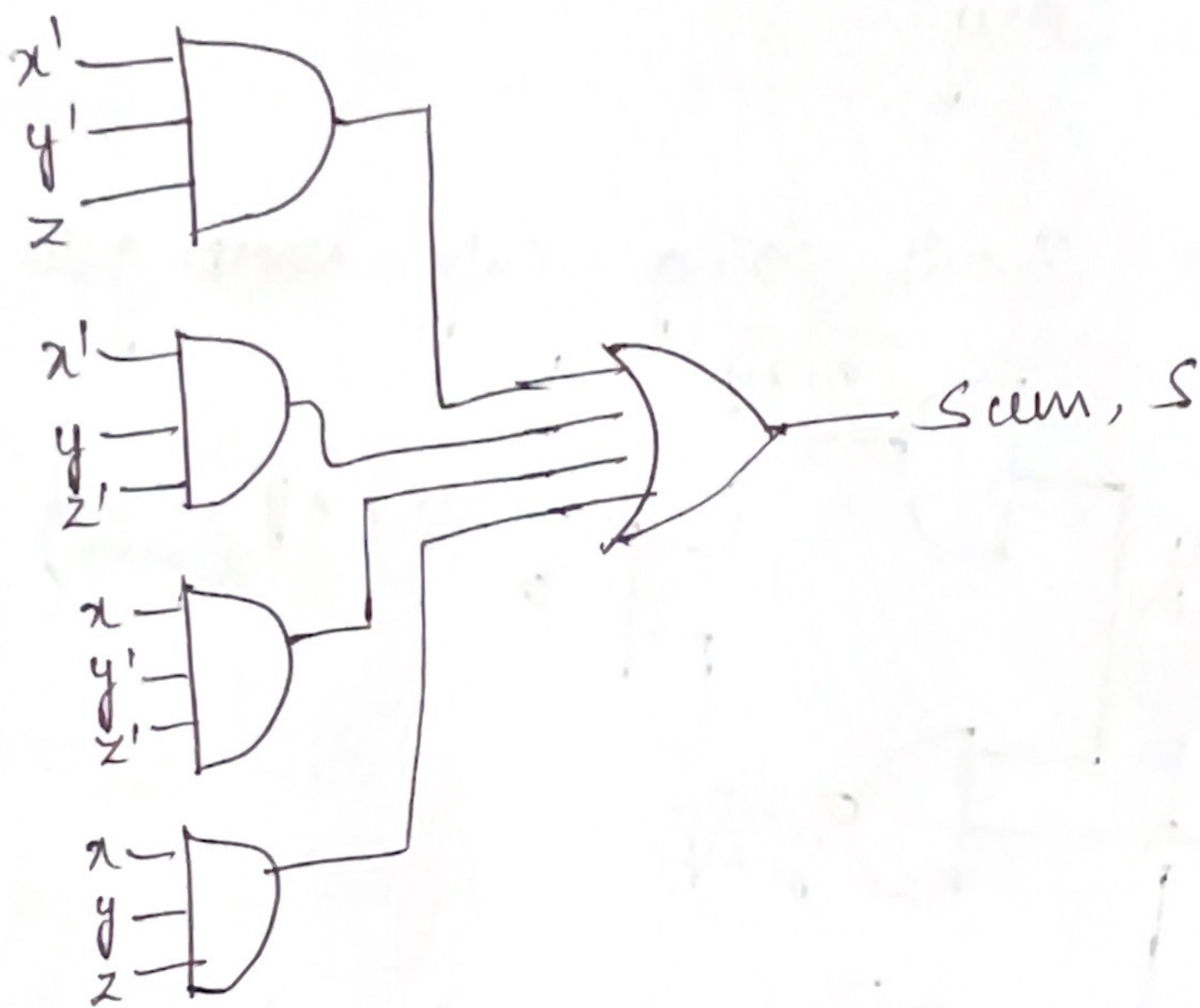
\* Truth Table

$x$	$y$	$z$	$s$	$c$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	1
1	1	0	1	1

\* Boolean Expression.

$$\text{Sum } S = x'y'z + x'yz' + xy'z' + xyz$$

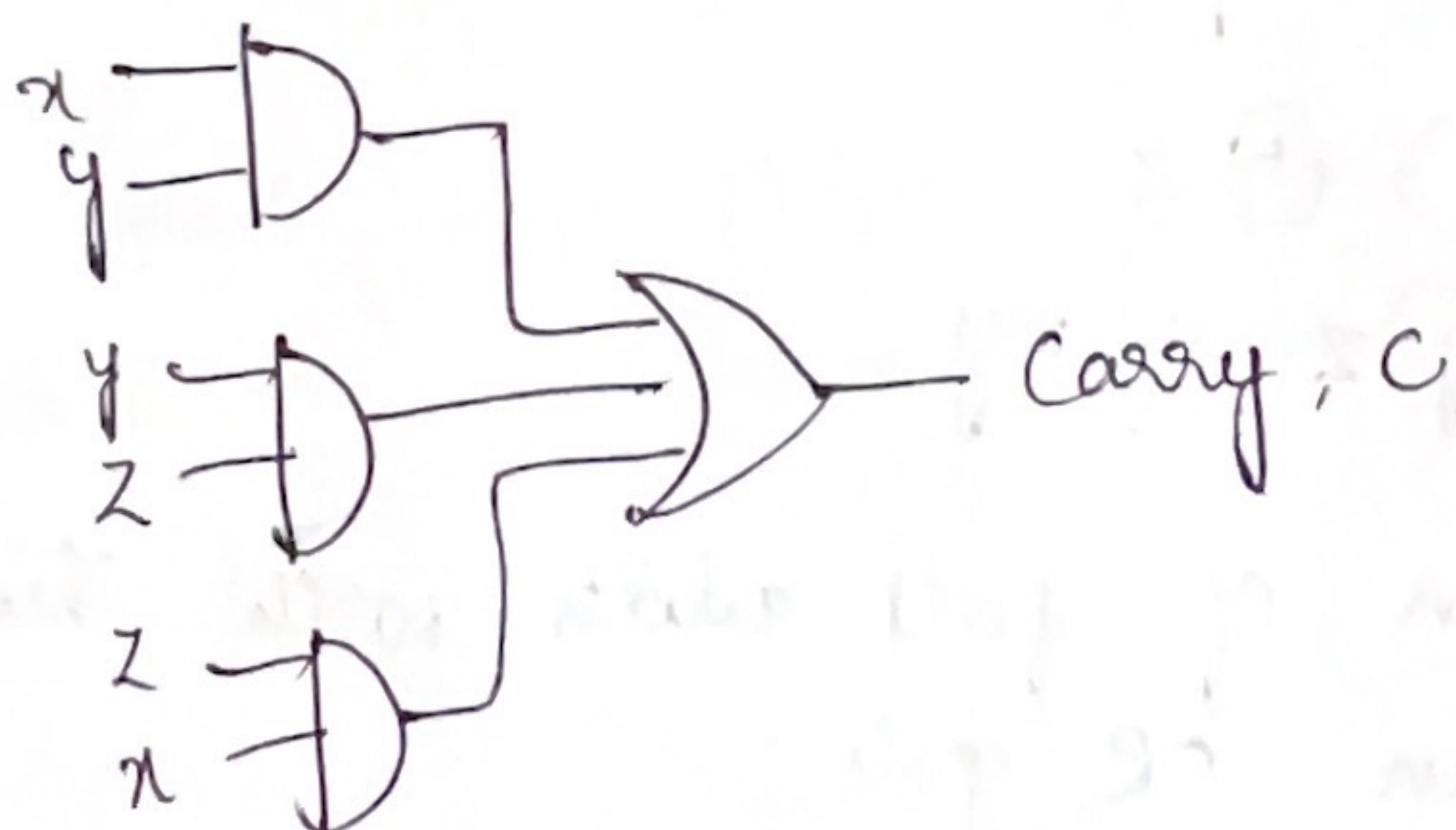
$$\text{Carry } C = x'yz + xy'z + xyz' + xyz$$



\* Carry expression:

$$\begin{aligned}C &= x'y'z + xy'z + xyz' + xyz \\&= x'y'z + xy'z + xy(x+x') \\&= x'y'z + \bar{x}y'z + xy \\&= x'y'z + x(y+y'z) \\&= x'y'z + x(y+z) \\&= x'y'z + xy + xz \\&= z(x+x'y) + xy \\&= z(x+y) + xy \\&= zx + xy + xy \\&\boxed{C = xy + yz + zx}\end{aligned}$$

: Simplified circuit to generate carry.



\* Boolean expression for sum:

$$\begin{aligned}
 S &= x'y'z + x'y'z' + xy'z' + xyz \\
 &= x'(y'z + yz') + x(y'z' + yz) \\
 &= x'(y \oplus z) + x(y \oplus z)'
 \end{aligned}$$

$S = x \oplus (y \oplus z)$

\* Boolean expression for Carry:

$$\begin{aligned}
 C &= x'y'z + xy'z + x'y'z' + xyz \\
 &= z(x'y + xy') + xy(2 + z')^1
 \end{aligned}$$

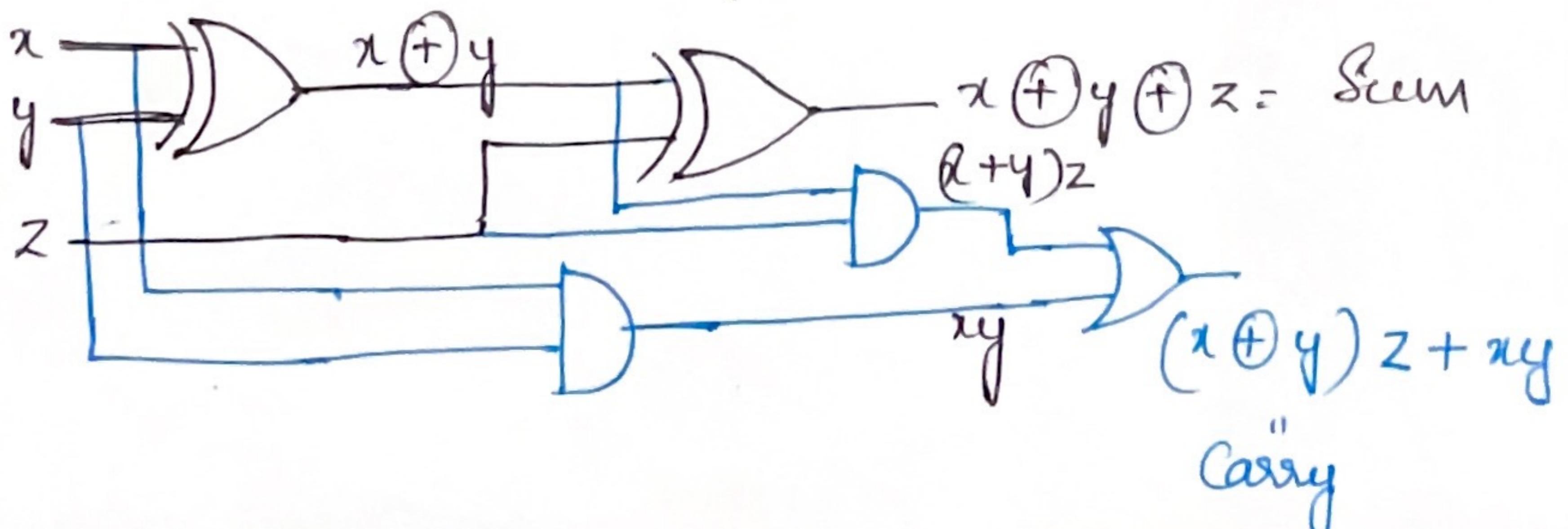
$C = z(x \oplus y) + xy$

\* ∴ Full adder expressions

$$S = (x \oplus y) \oplus z$$

$$C = (x \oplus y)z + xy$$

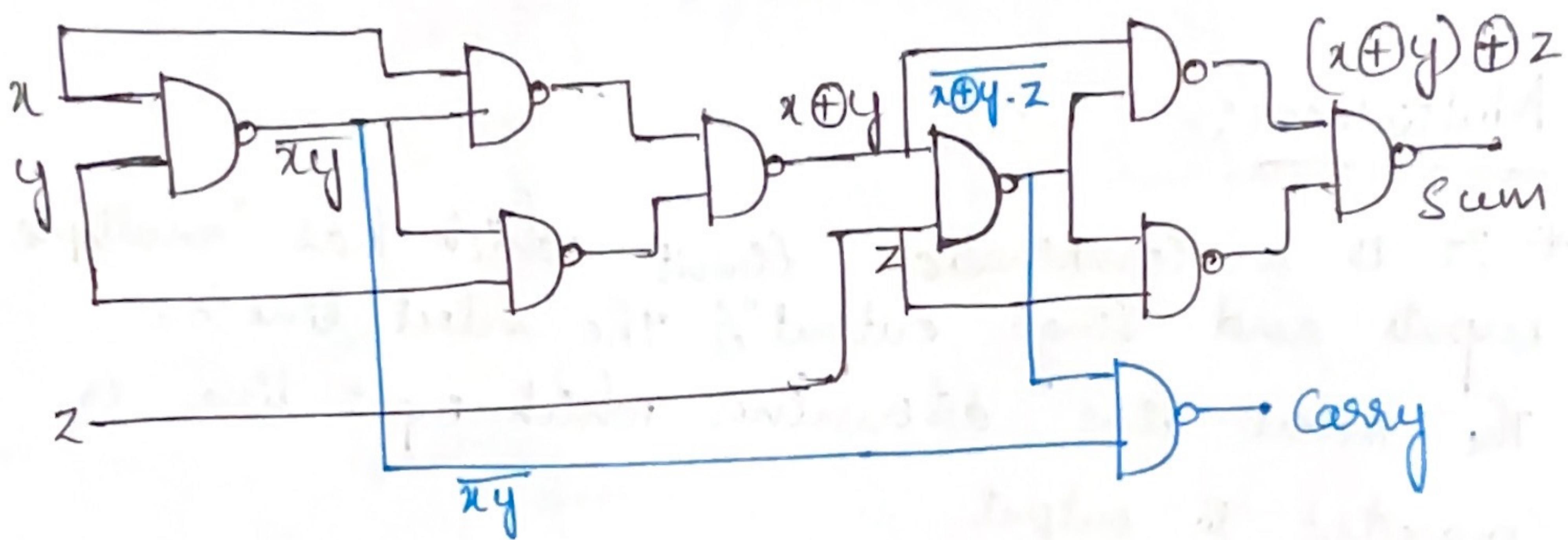
\* Implementation of full adder with two half adders and an OR gate.



\* Full adder circuit using ·NAND gates

WKT

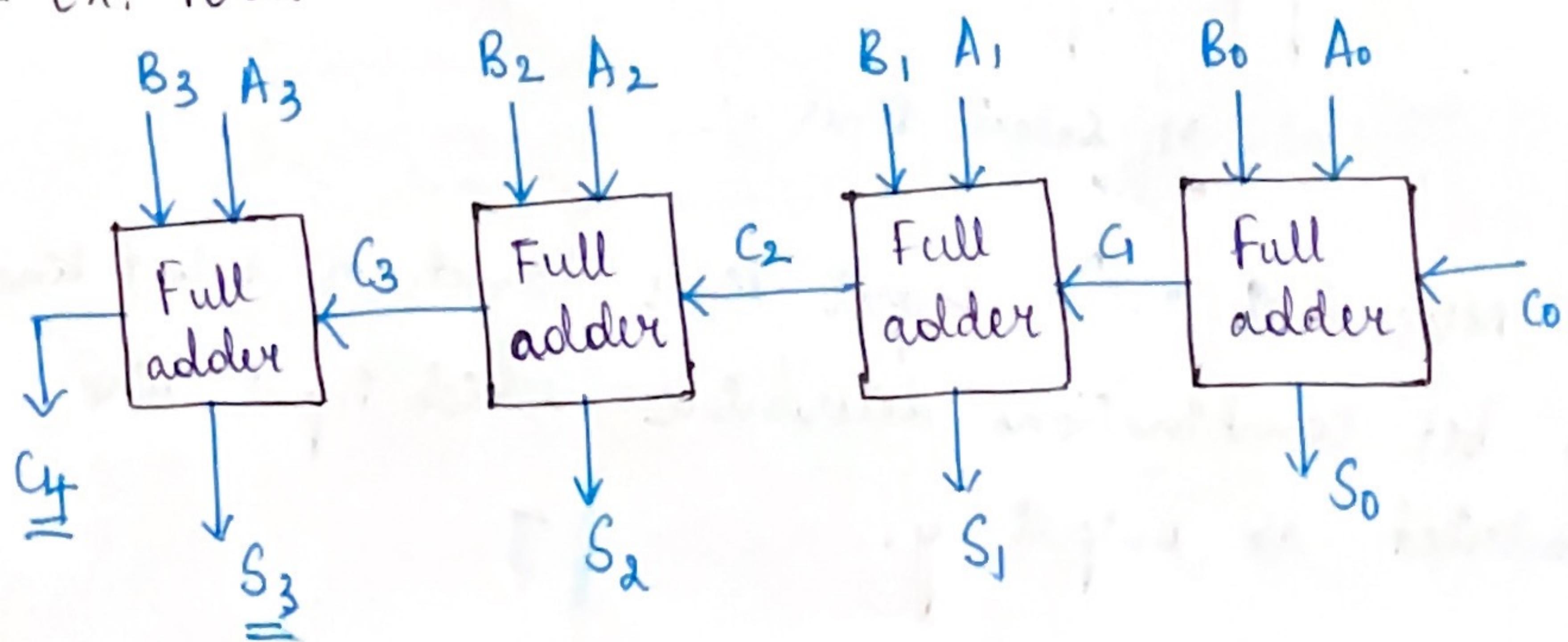
$$\begin{aligned} \text{Sum} &= (x \oplus y) \oplus z \\ \text{Carry} &= \overline{(x \oplus y)z + xy} \\ &= \overline{(x \oplus y)z} \cdot \overline{xy} \end{aligned}$$



### \* Ripple Carry Adder \*

\* Adder circuit can perform upto 3 bit addition. For n bit addition, we have to cascade adders.

\* Ex: four bit adder



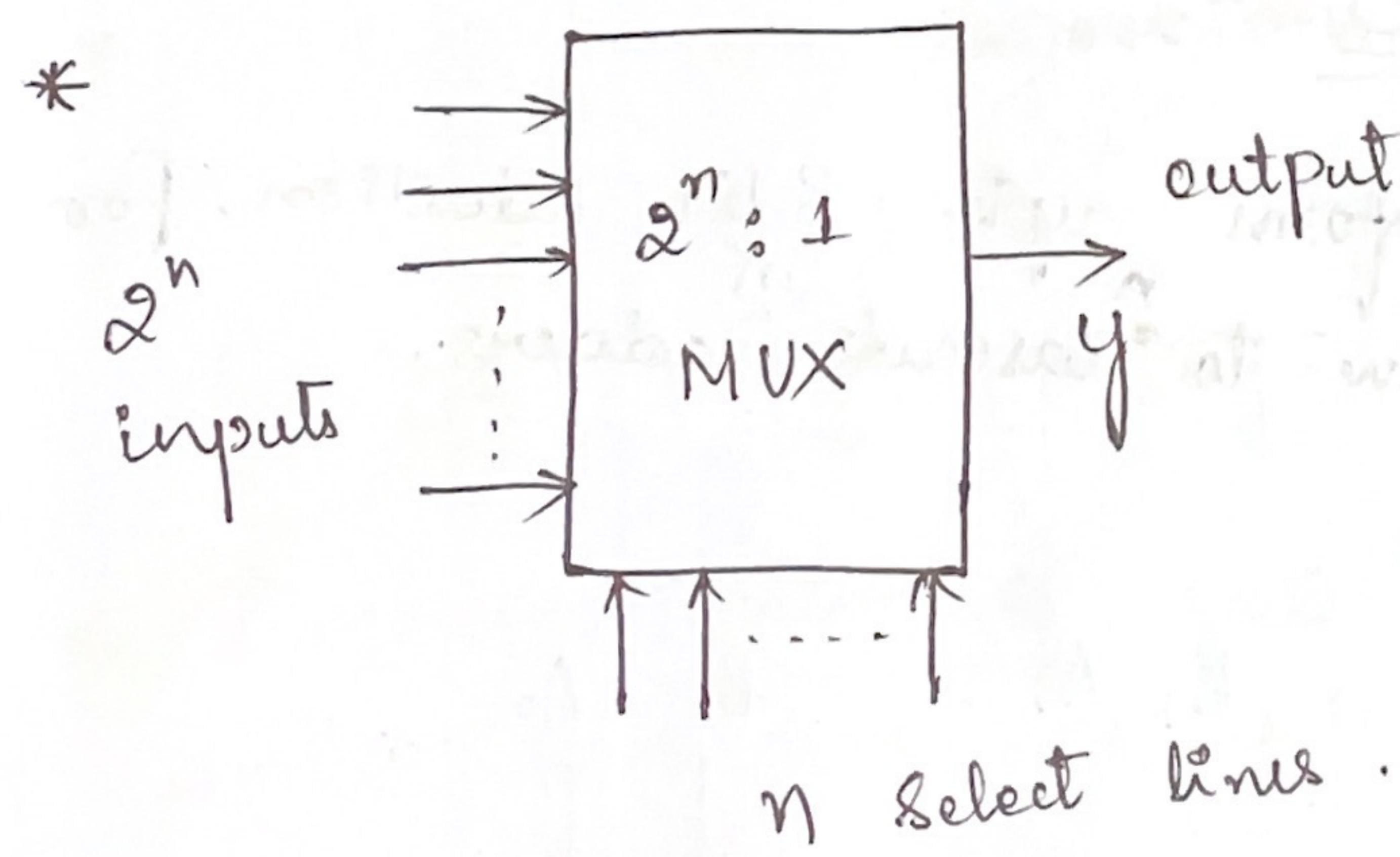
\* Example:-

$$\begin{array}{r} \text{C}_3 \text{ C}_2 \text{ C}_1 \text{ C}_0 \\ \text{1} \text{ 1} \text{ 1} \text{ 0} \\ \hline A = \text{1} \text{ 1} \text{ 0} \text{ 1} \\ \text{A}_3 \text{ A}_2 \text{ A}_1 \text{ A}_0 \\ \hline B = \text{0} \text{ 1} \text{ 1} \text{ 1} \\ \text{B}_3 \text{ B}_2 \text{ B}_1 \text{ B}_0 \\ \hline \text{1} \text{ 0} \text{ 1} \text{ 0} \text{ 0} \\ / \quad S_3 \text{ } S_2 \text{ } S_1 \text{ } S_0 \\ \text{c}_4 \end{array}$$

## Multiplexer and Demultiplexer :-

### Multiplexer:-

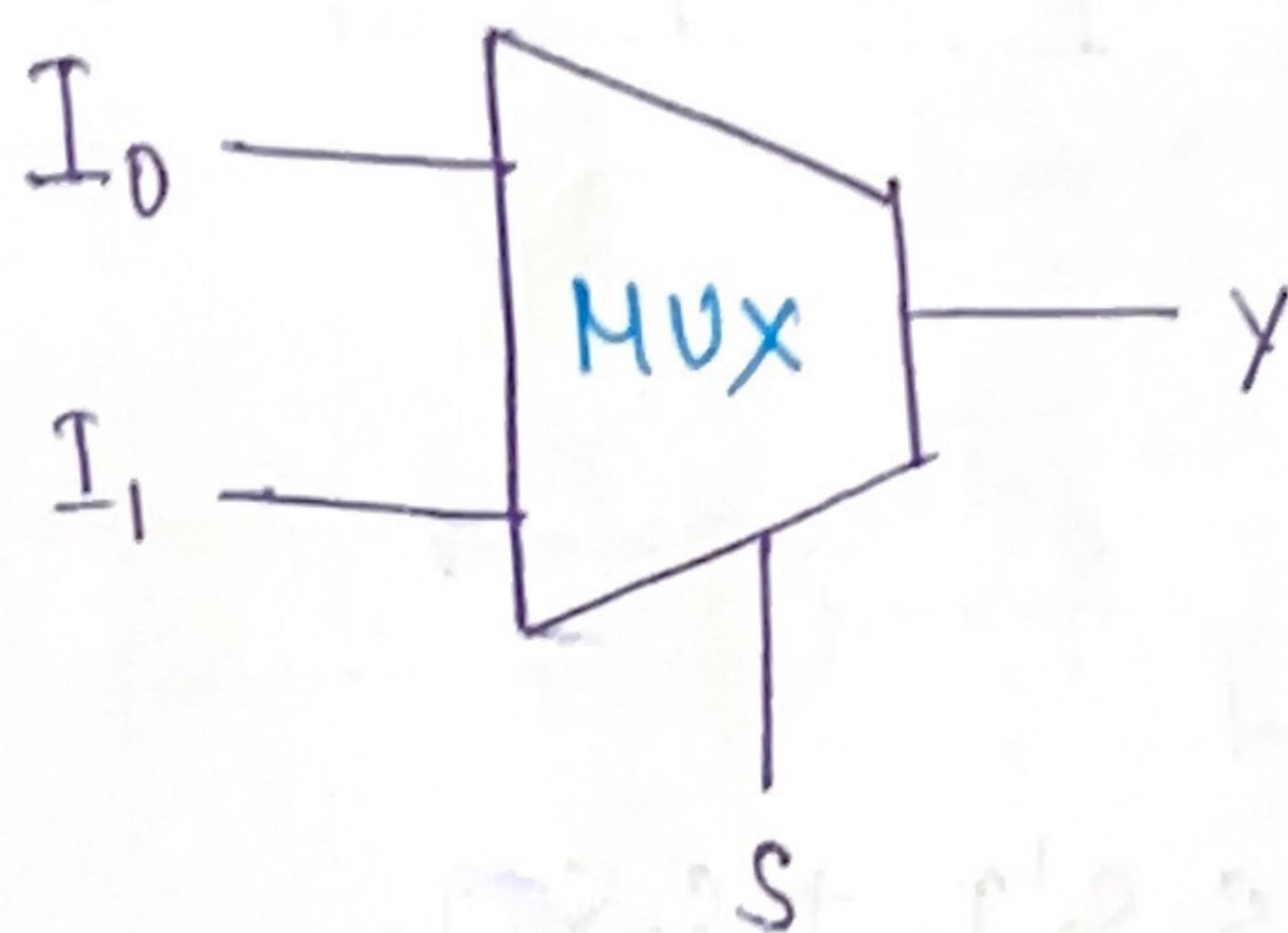
\* It is a combination circuit which has "multiple inputs and single output" & the select line/s. The select line determines which input line is connected to output.



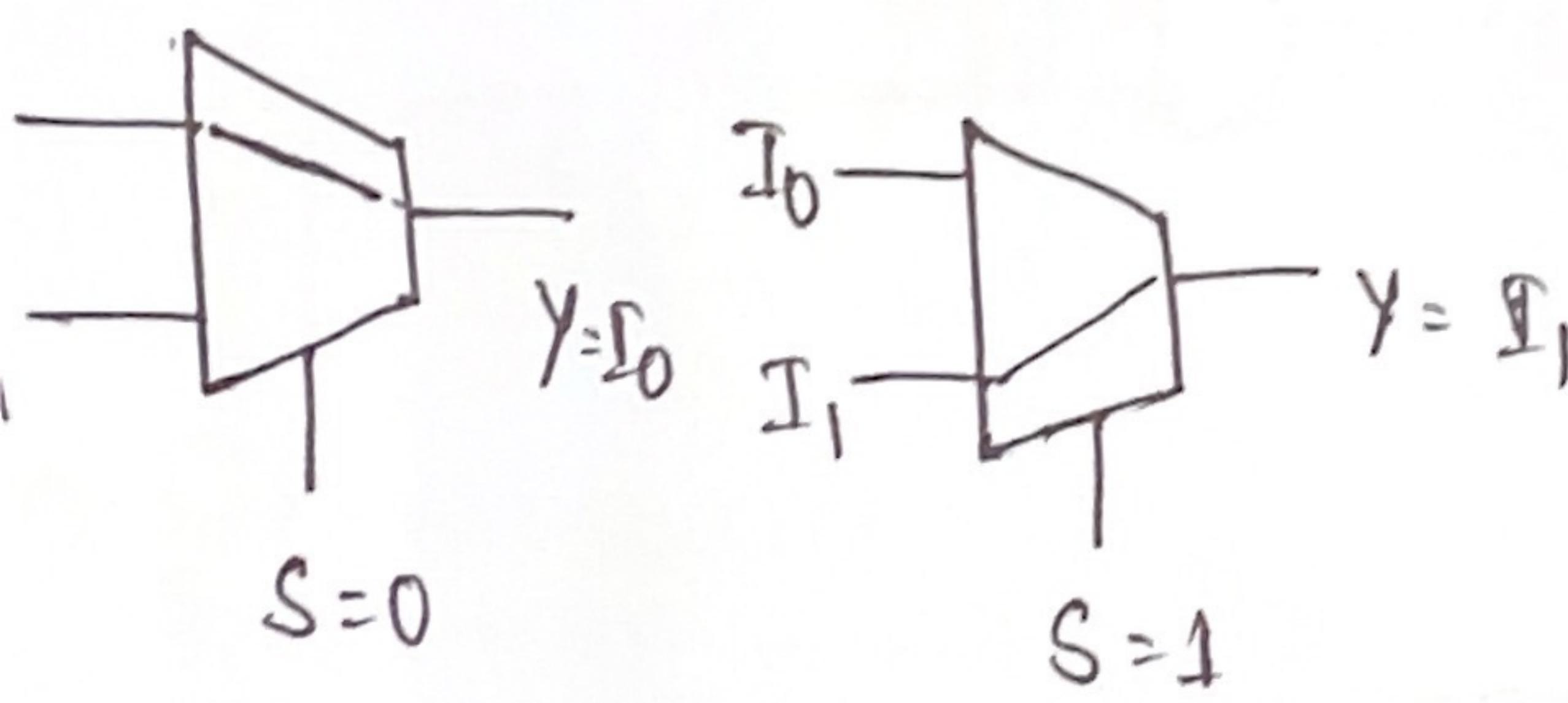
\* A MUX has  $2^n$  input lines and n select lines whose bit combinations determine which input line is selected as output y.

- \* Application of Mux is mainly in data composition.
- \* The function of demultiplexer is to inverse the function of multiplexer.

### \* 2:1 MUX (Two to one line multiplexer)

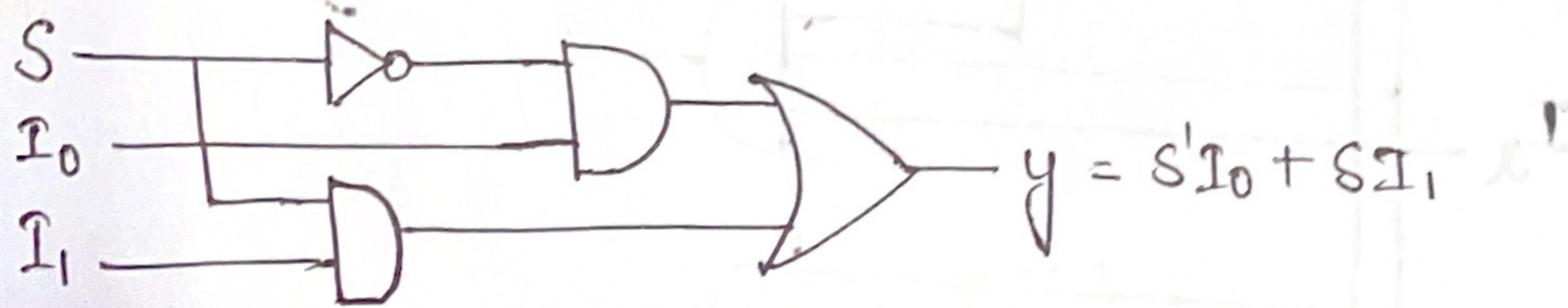


S	y
0	I0
1	I1

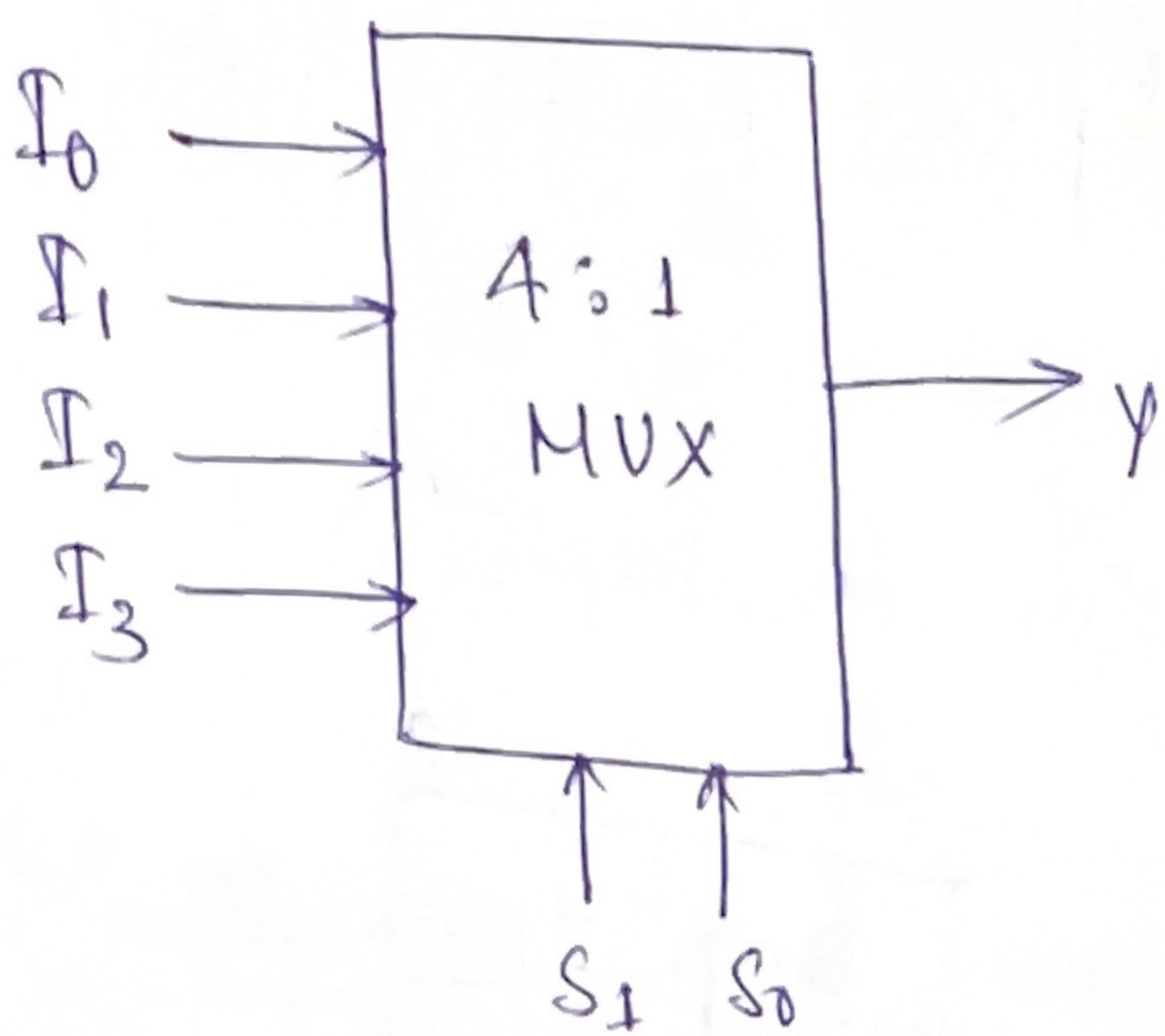


\* Boolean Expression:  $S'I_0 + SI_1$

\* Circuit diagram:



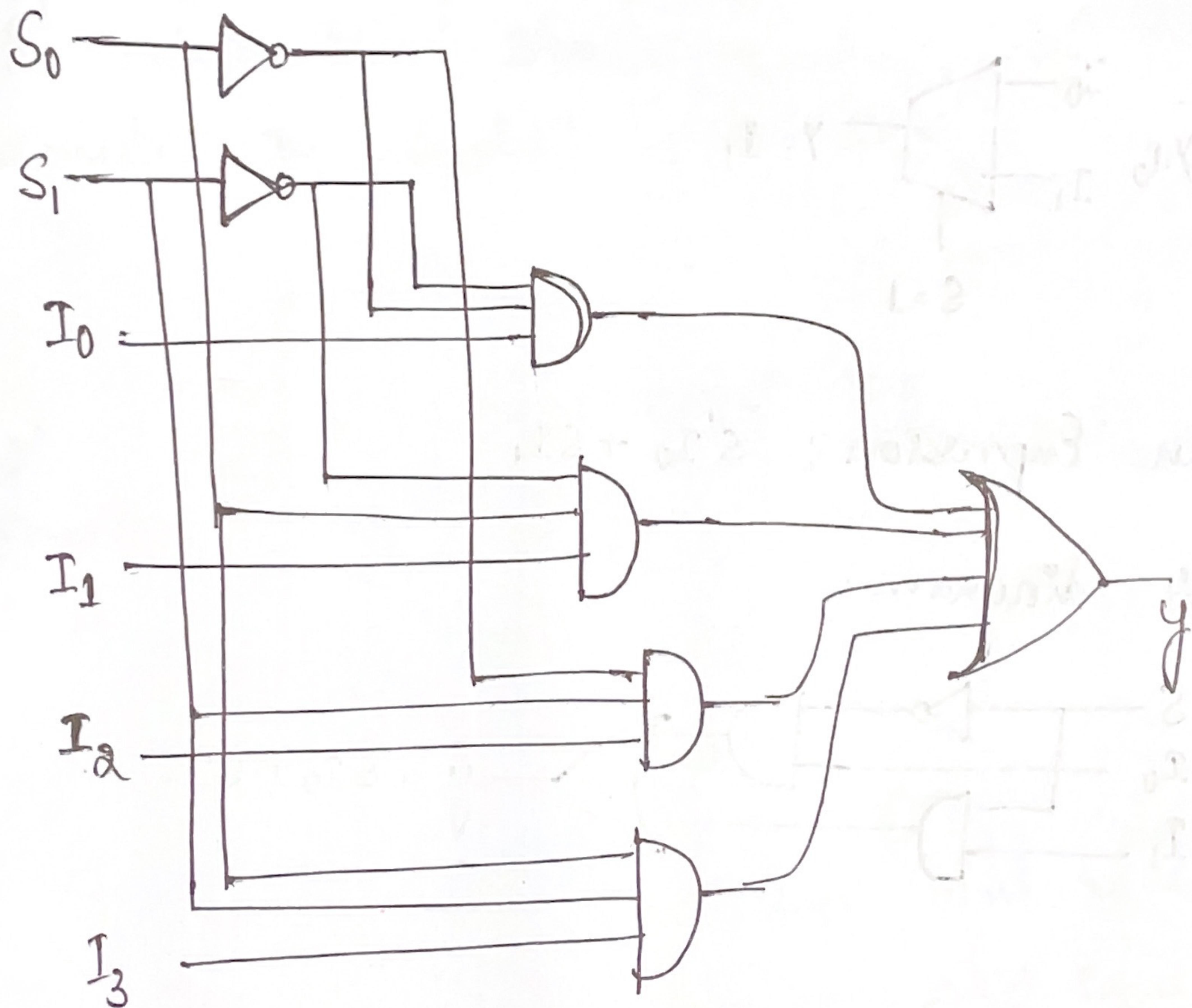
\* 4:1 MUX (four to one line multiplexer)



$S_1$	$S_0$	$y$
0	0	$I_0$
0	1	$I_1$
1	0	$I_2$
1	1	$I_3$

\* Boolean expression

$$y = S_0' S_1' I_0 + S_1' S_0 I_1 + S_1 S_0' I_2 + S_1 S_0 I_3$$



\* 2:1 MUX using NAND gates only.

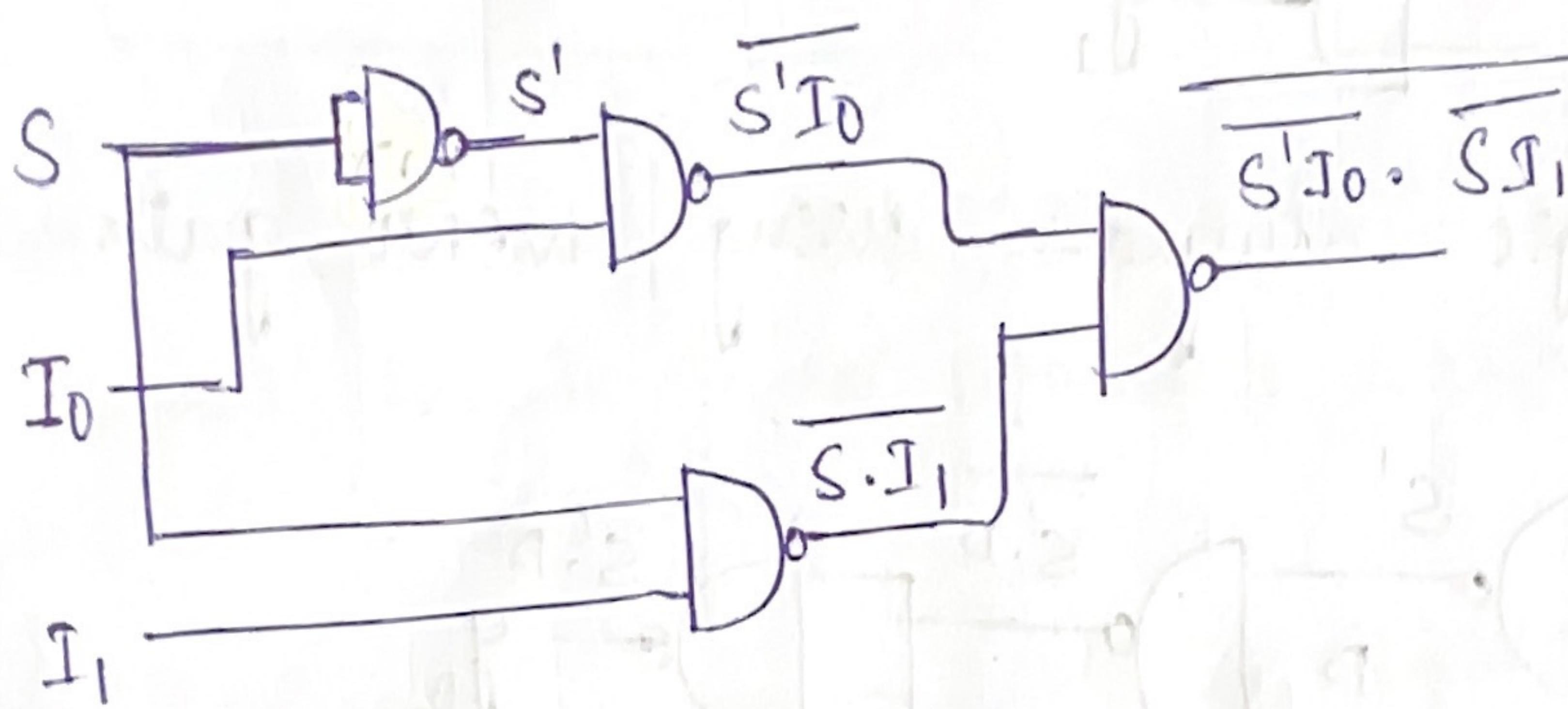
WRT

Boolean expression for 2:1 MUX is

$$y = S' I_0 + S I_1$$

$$y = \overline{S} I_0 + S \overline{I}_1$$

$$y = \overline{S' I_0} \cdot \overline{S \cdot I_1}$$

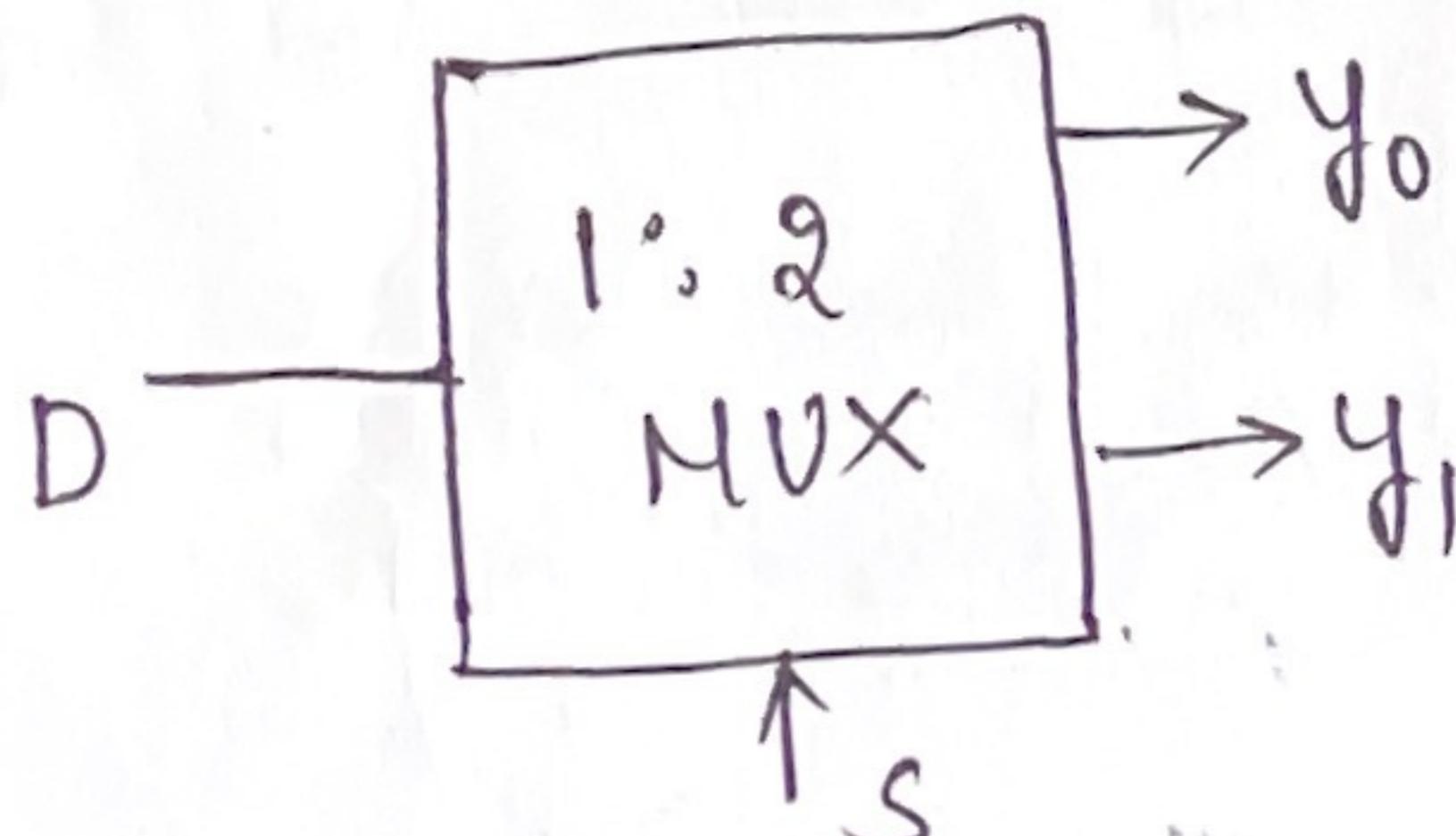


### \* De-multiplexer \*

- \* It is a combinational circuit which has one input line and multiple output lines.
- \* The select line decides which output line is connected to the input line.

### \* 1:2 Demux

Block diagram



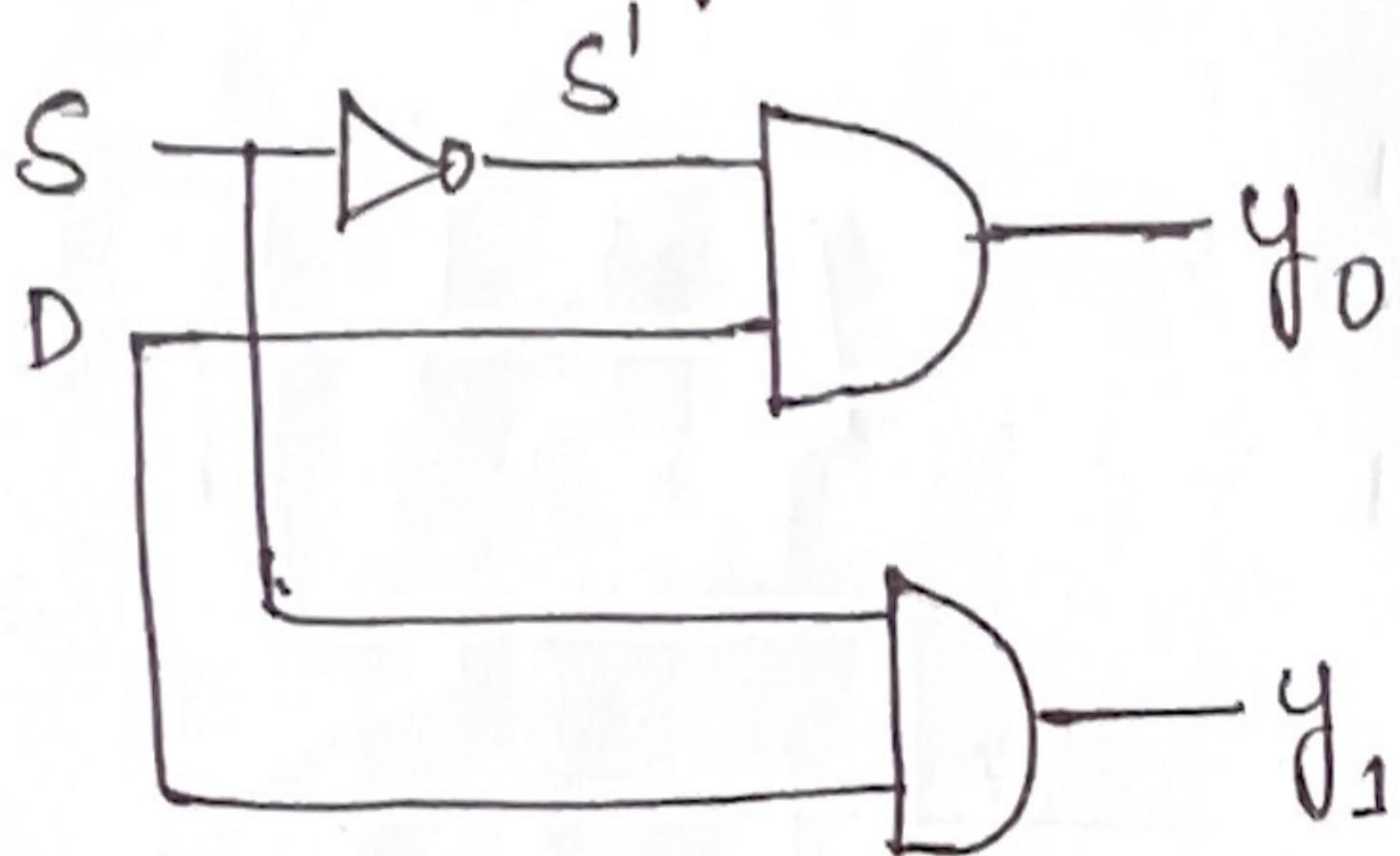
S	y <sub>0</sub>	y <sub>1</sub>
0	D	-
1	-	D

\* Boolean expression is.

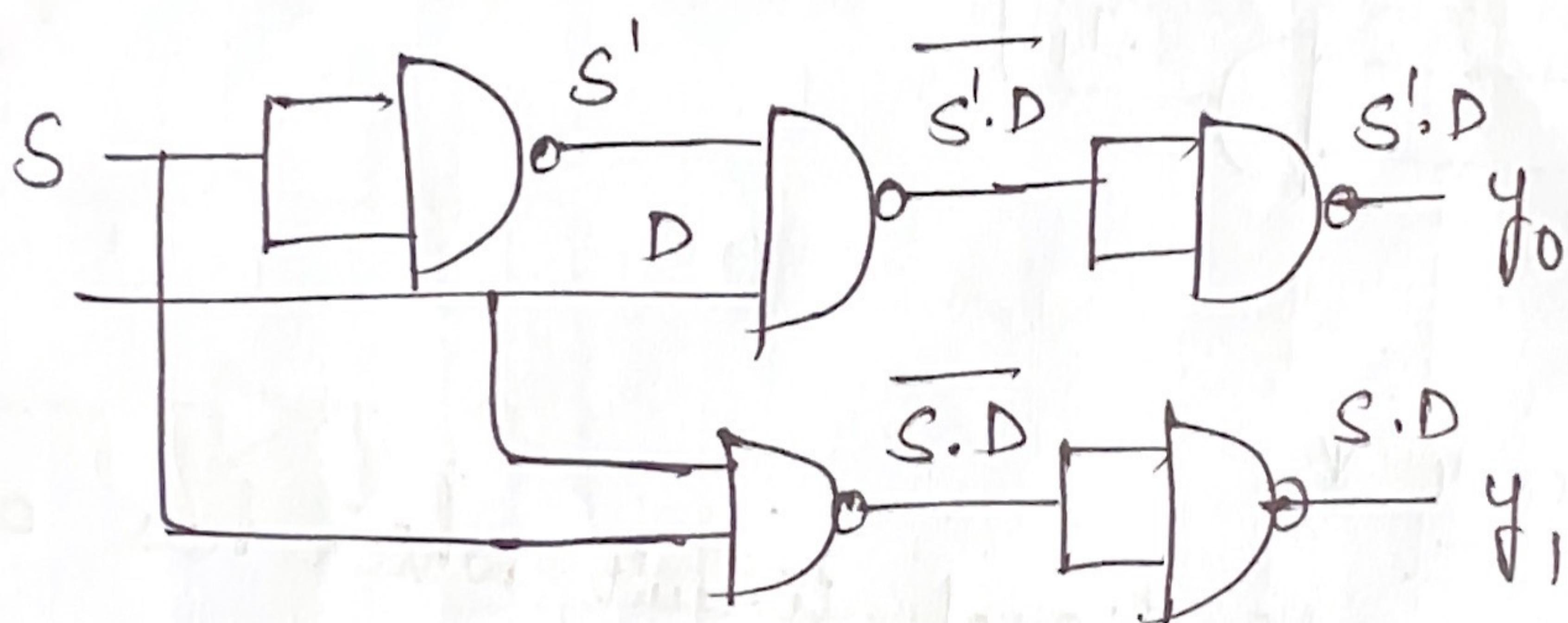
$$y_0 = S'D$$

$$y_1 = SD$$

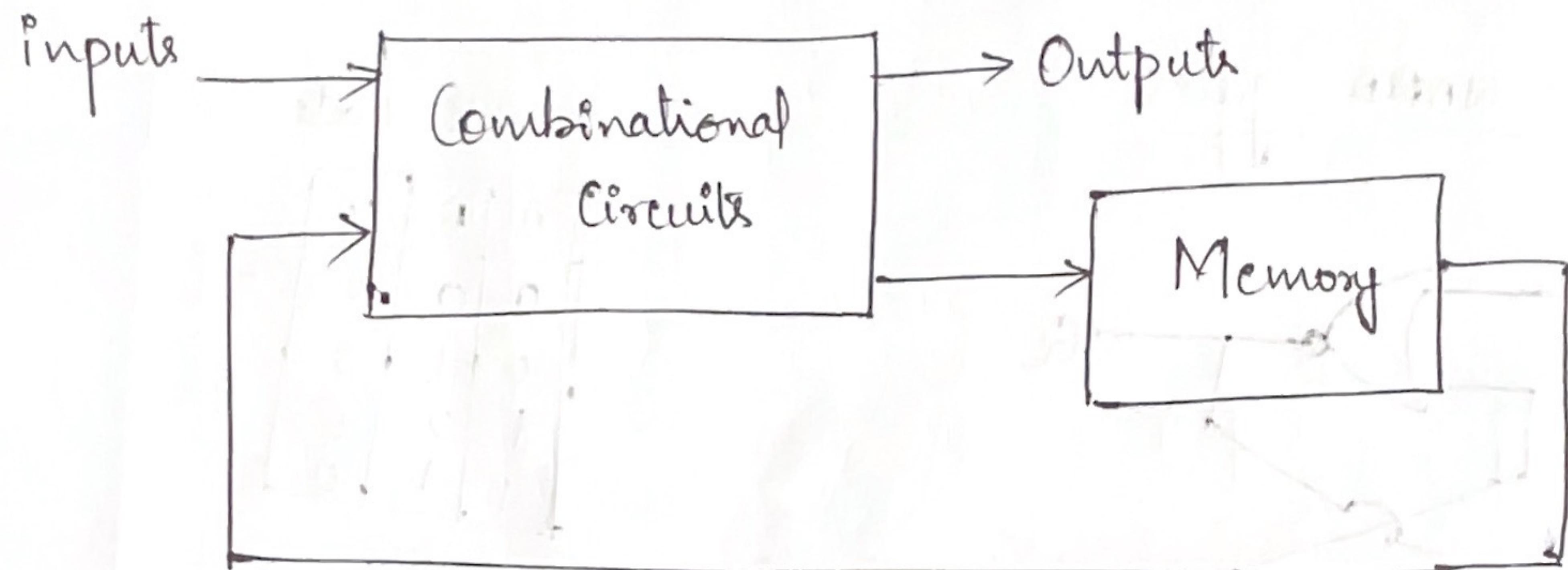
\* Logic diagram



\* Demux logic diagram using NAND gates.



## \* Sequential Circuits \*



\* The circuit in which the output depends on the current inputs and previous outputs is called sequential circuit.

Ex:- Flip-flops, Counters, Shift registers

\* The storage elements are the devices capable of storing binary information. Ex: Latch, Flip-flops.

### \* FLIP-FLOPS :-

\* Flip flops are digital circuit which are capable of storing data.

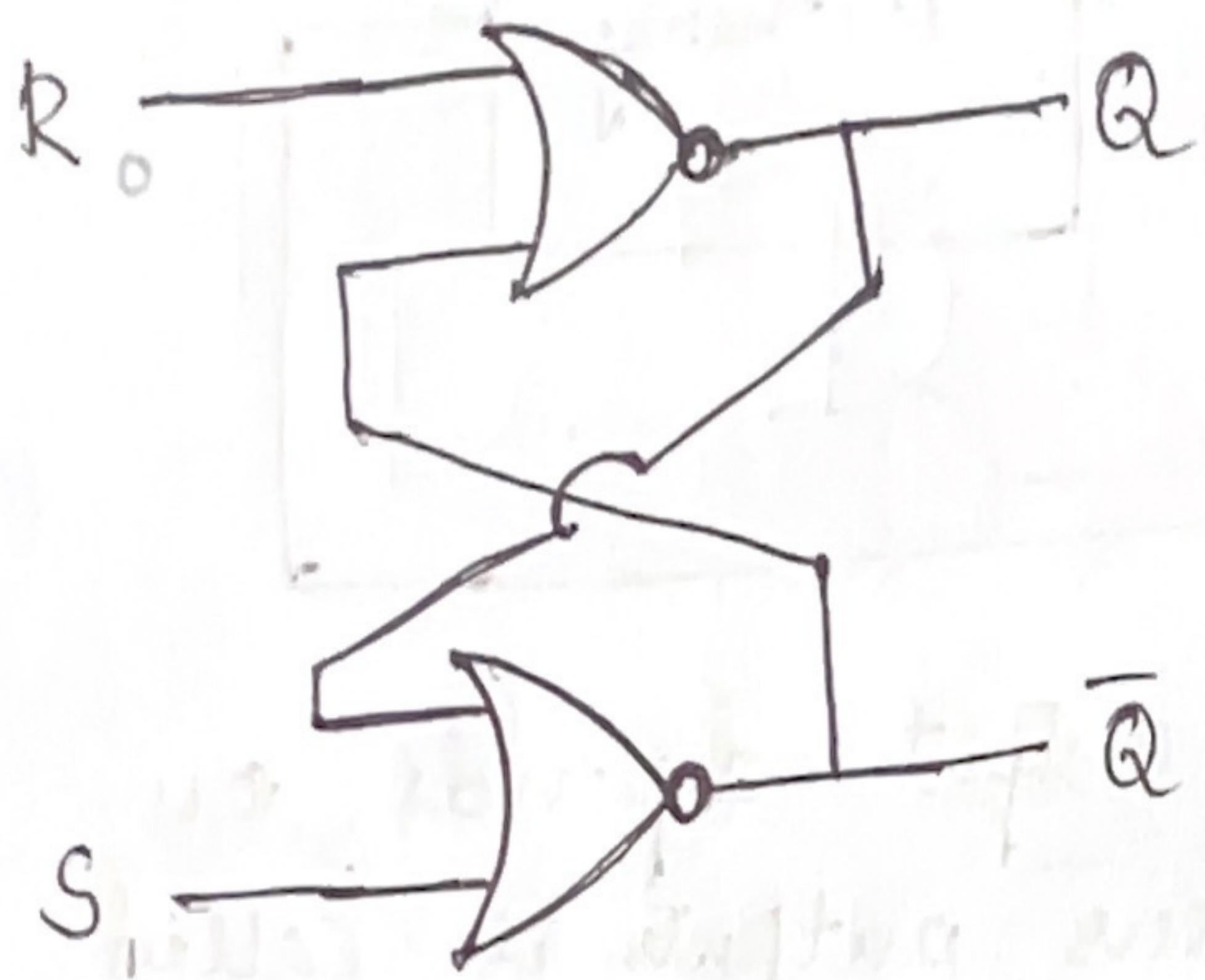
\* Each flip-flop will store one bit data (either 0 or 1).

\* The stored data remains for indefinite time unless and until it is changed by input or clock signal.

-: SR Latch (Set-Reset) using NAND gate / NOR

\* Circuit diagram

using NOR gate:



NOR Gate

A	B	y
0	0	1
0	1	0
1	0	0
1	1	0

Case 1 :-  $R=0$  and  $S=1$

$$Q=1 \text{ and } \bar{Q}=0$$

$R=0$  and  $S=0$

$$Q=1 \text{ and } \bar{Q}=0 \rightarrow \text{Memory state}$$

Case 2 :-  $R=1$  and  $S=0$

$$Q=0 \text{ and } \bar{Q}=1$$

$R=0$  and  $S=0$

$$Q=0 \text{ and } \bar{Q}=1 \rightarrow \text{Memory state}$$

Case 3 :-  $R=1$  and  $S=1$

$$Q=0 \text{ and } \bar{Q}=0 \text{ (Forbidden state / Not used state)}$$

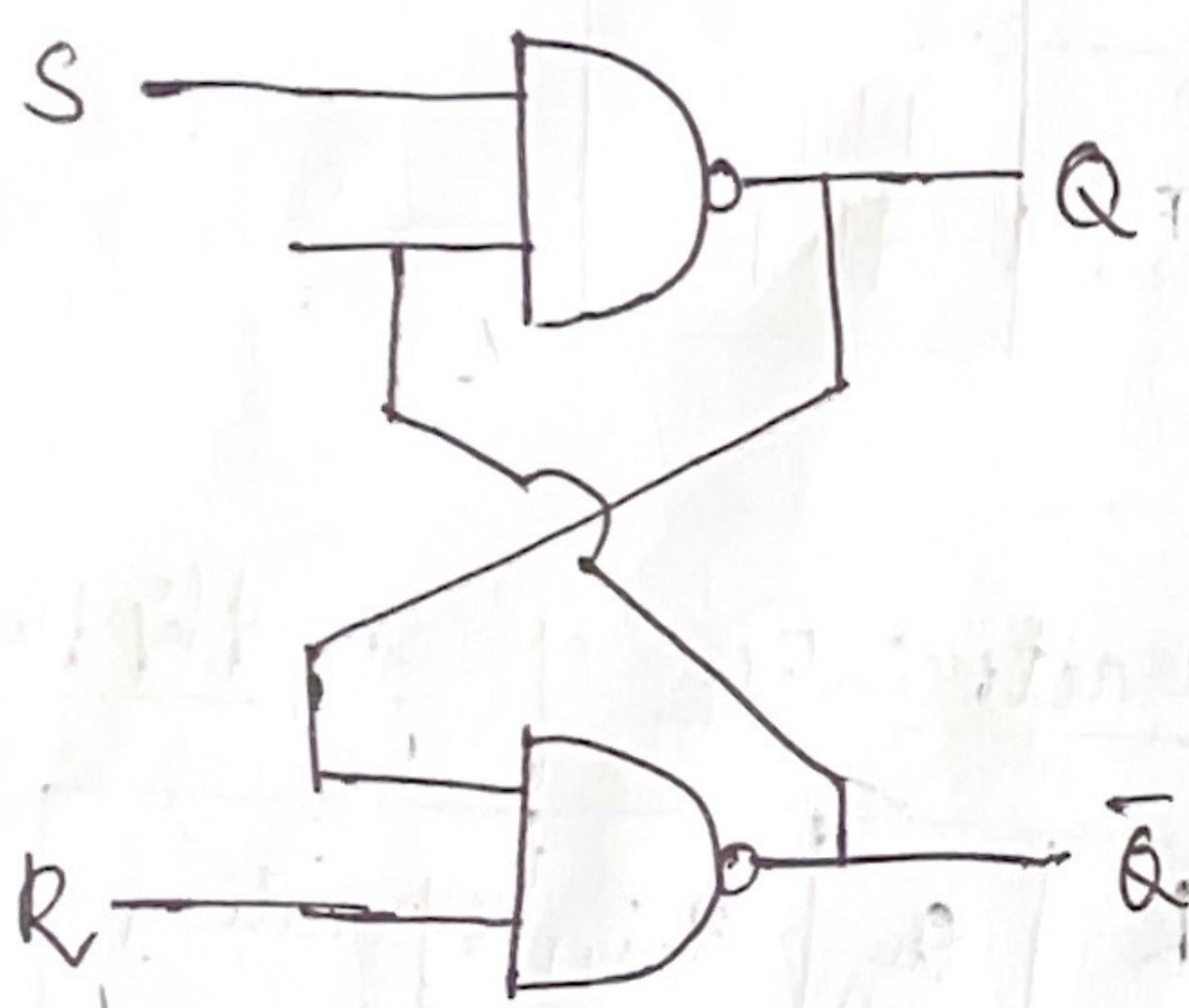
$R=0 \& S=0$

$$Q=0 \& \bar{Q}=1 \text{ (No memory)}$$

∴ Truth table is as follows

S	R	$Q$	$\bar{Q}$
0	0	Memory	
0	1	0	1
1	0	1	0
1	1	Not used	

\* SR flip-flop can also be constructed using NAND gate as follows.



Truth table

S	R	$Q$	$\bar{Q}$
0	0	Not used	
0	1	1	0
1	0	0	1
1	1	Memory	

NAND Gate

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

i)  $S=0$  and  $R=1$

$Q=1$  and  $\bar{Q}=0$

$S=1$  &  $R=1$

$Q=1$  and  $\bar{Q}=0$  (Memory)

ii)  $S=1$  and  $R=0$

$Q=0$  and  $\bar{Q}=1$

$S=1$  &  $R=1$

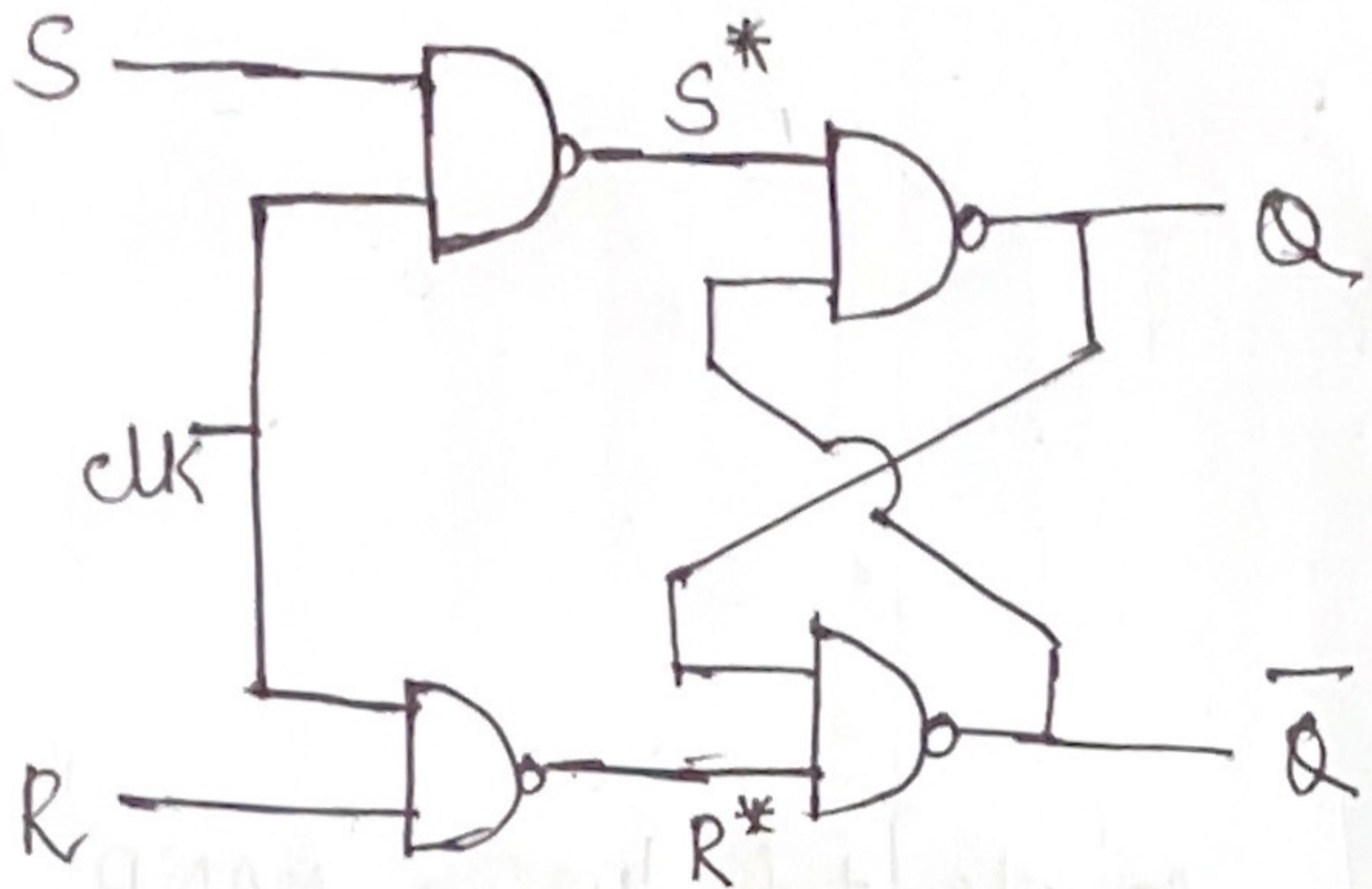
$Q=0$  and  $\bar{Q}=1$  (Memory)

iii)  $S=0$  and  $R=0$

$Q=1$  and  $\bar{Q}=1$  (Forbidden state / NOT USED state)

\* SR Flip flop Using NAND gate.

Circuit diagram



SR Latch T.T

S	R	Q	$\bar{Q}$
0	0	Not used	
0	1	1	0
1	0	0	1
1	1	Memory	

$$S^* = \overline{(S \cdot \text{clk})} = \overline{S} + \overline{\text{clk}}$$

$$R^* = \overline{(R \cdot \text{clk})} = \overline{R} + \overline{\text{clk}}$$



-; Truth table of SR flip flop:-

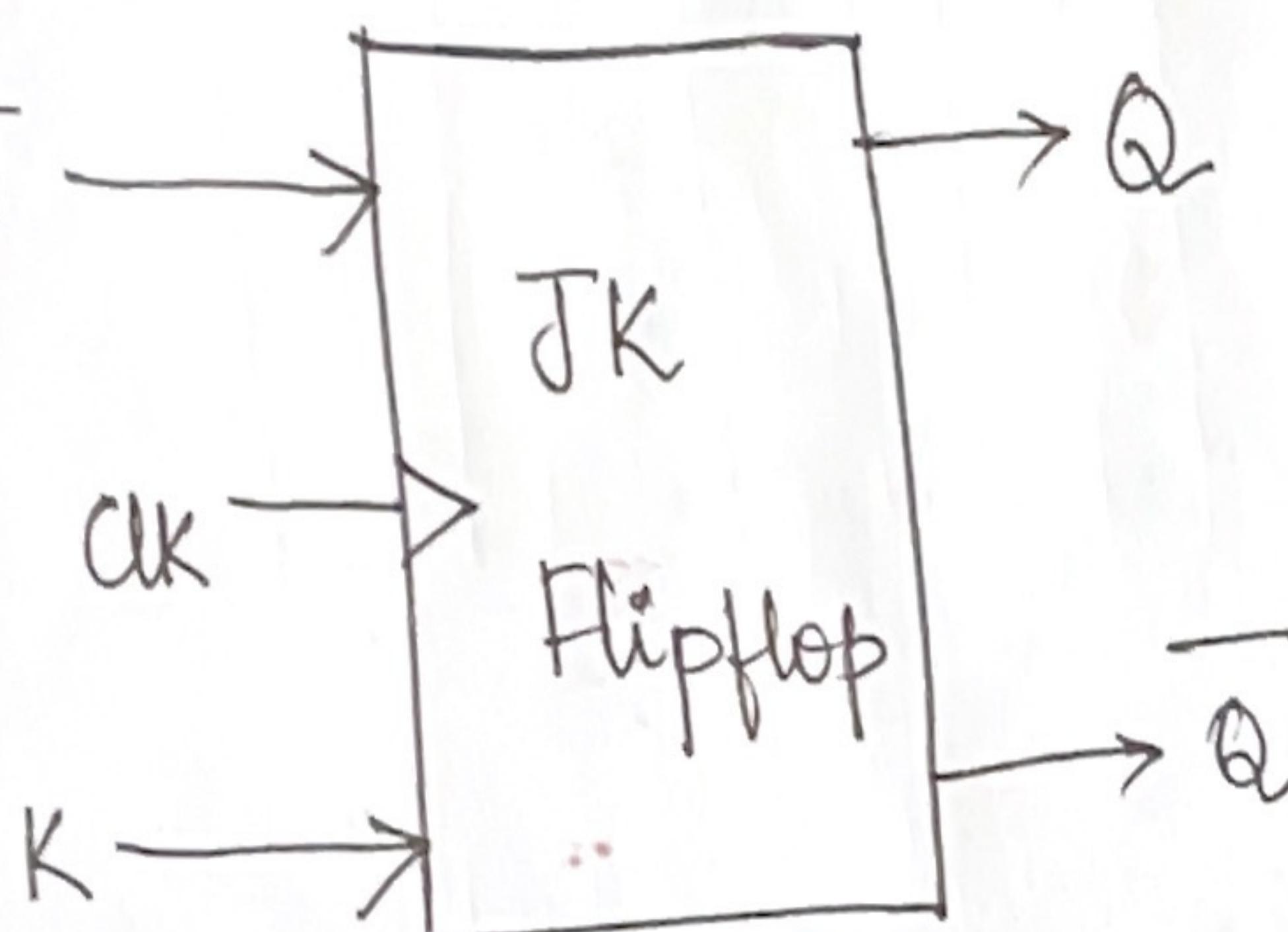
CLK	S	R	Q	$\bar{Q}$
0	X	X	Memory	
1	0	0	Memory	
1	0	1	0	1
1	1	0	1	0
1	1	1	Not used	

characteristic of SR flip flop

CLK	S	R	$Q_n$	$Q_{n+1}$	Operation
1	0	0	0	0	Memory(0)
1	0	0	1	1	Memory(1)
1	0	1	0	0	Reset
1	0	1	1	0	Reset
1	1	0	0	1	Set
1	1	0	1	1	Set
1	1	1	0	Not used	
1	1	1	1	Not used	
0	X	X	X	$Q_n$	Memory

# JK Flip flop :- (Jack - Kilby) flip flop

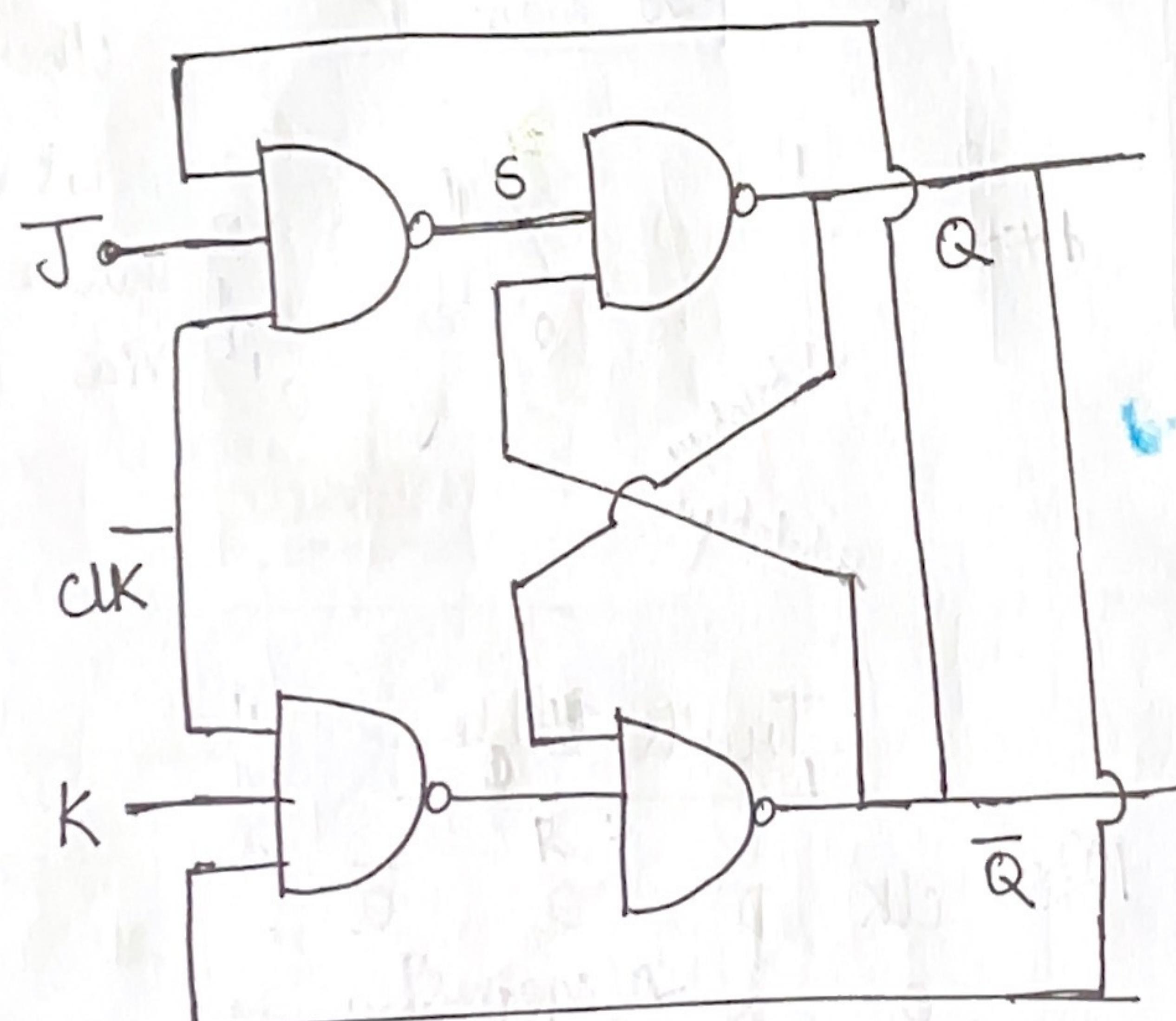
Symbol



Truth table

clk	J	K	Q	̄Q
0	x	x	Memory	
1	0	0	Memory	
1	0	1	0	1
1	1	0	1	0
1	1	1	Toggle	

Circuit diagram



$clk = 0 \rightarrow \text{Memory}$

$clk = 1, J = 0, K = 1$

$Q = 0 \& \bar{Q} = 1$

$clk = 1, J = 1, K = 0$

$Q = 1 \& \bar{Q} = 0$

$clk = 1, J = 1, K = 1$

assume  $Q = 0 \& \bar{Q} = 1$

$Q = 0, 1, 0, 1, \dots$

$\bar{Q} = 1, 0, 1, 0, \dots$

2I NAND

A	B	C	y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

A	B	y
0	0	1
0	1	0
1	0	0
1	1	1

i)  $J=0, K=1, clk=1$

$$Q = \overline{1 \cdot \bar{Q}} = \overline{1} + \bar{Q} = \bar{Q}$$

$$\bar{Q} = \overline{\bar{Q} \cdot Q} = \overline{0} = 1 \Rightarrow \underline{Q=0}$$

$\therefore J=0, K=1 \Rightarrow Q=0 \& \bar{Q}=1$

ii)  $J=0, K=0, clk=0$

$S=1 \& R=1$

? Memory

iv)  $J=1, K=1, clk=1$

Let  $Q=0 \quad \bar{Q}=1$

$Q=1, 0, \dots$

$\bar{Q}=0,$

ii)  $J=1, K=0, clk=1$

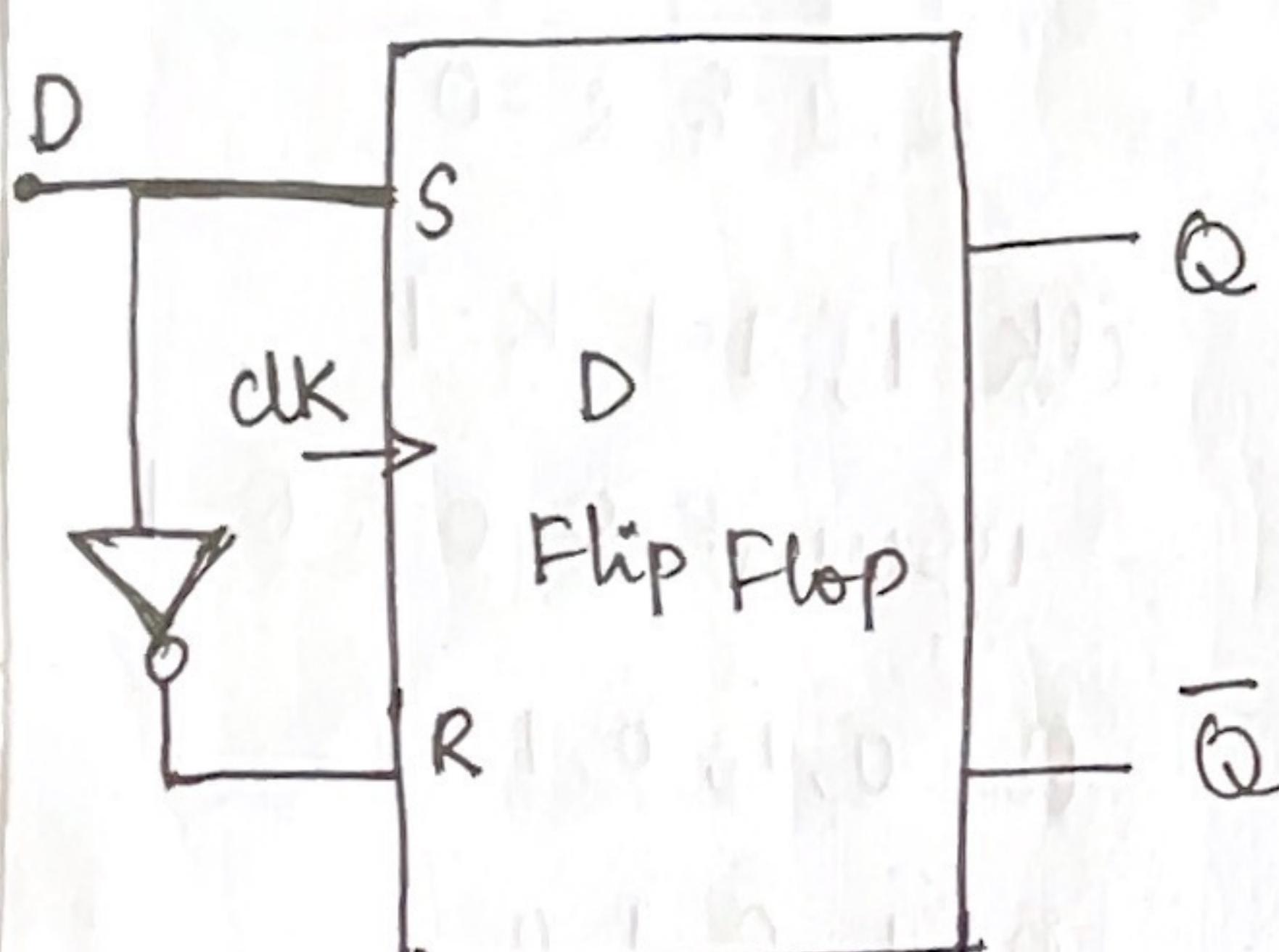
$$Q = \overline{\bar{Q} \cdot Q} = \overline{0} = 1 \Rightarrow \underline{Q=1 \therefore Q=0}$$

## \* Characteristics Table

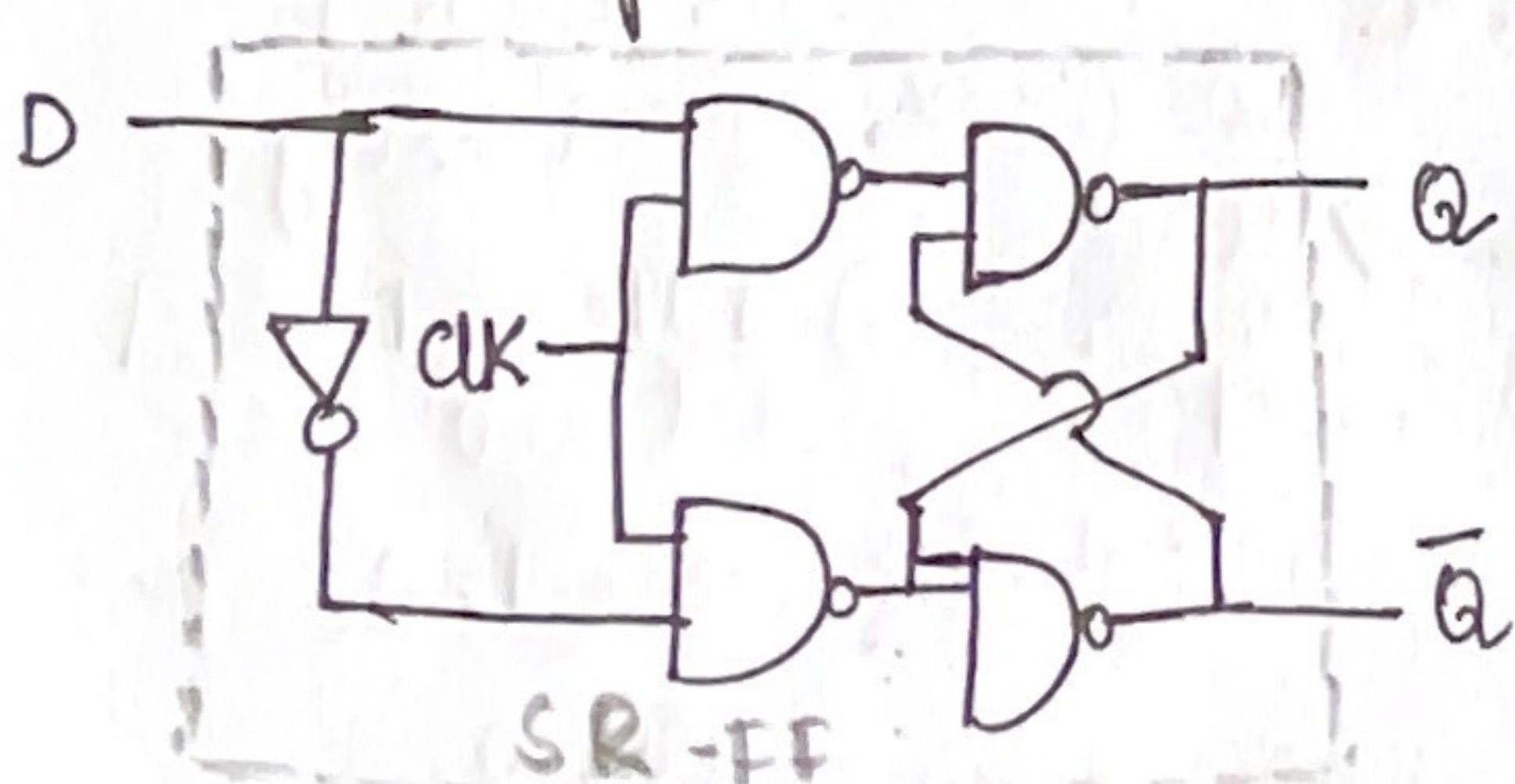
clk	$Q_n$	J	K	$Q_{n+1}$
↑	0	0	0	0
↑	0	0	1	0
↑	0	1	0	1
↑	0	1	1	1
↑	1	0	0	1
↑	1	0	1	0
↑	1	1	0	1
↑	1	1	1	0
0	x	x	x	Memory

\* D - Flip Flop (Data flip flop) :-

Symbol



Circuit diagram:-



S & R are complement to each other.

∴ Let us short then 2 terminals via NOT gate

S	R	Q	$\bar{Q}$
0	0	MEMORY	
0	1	0	1
1	0	1	0
1	1	NOT USED	

Truth Table

clk	D	Q	$\bar{Q}$
0	x	Memory	
1	0	0	1
1	1	1	0

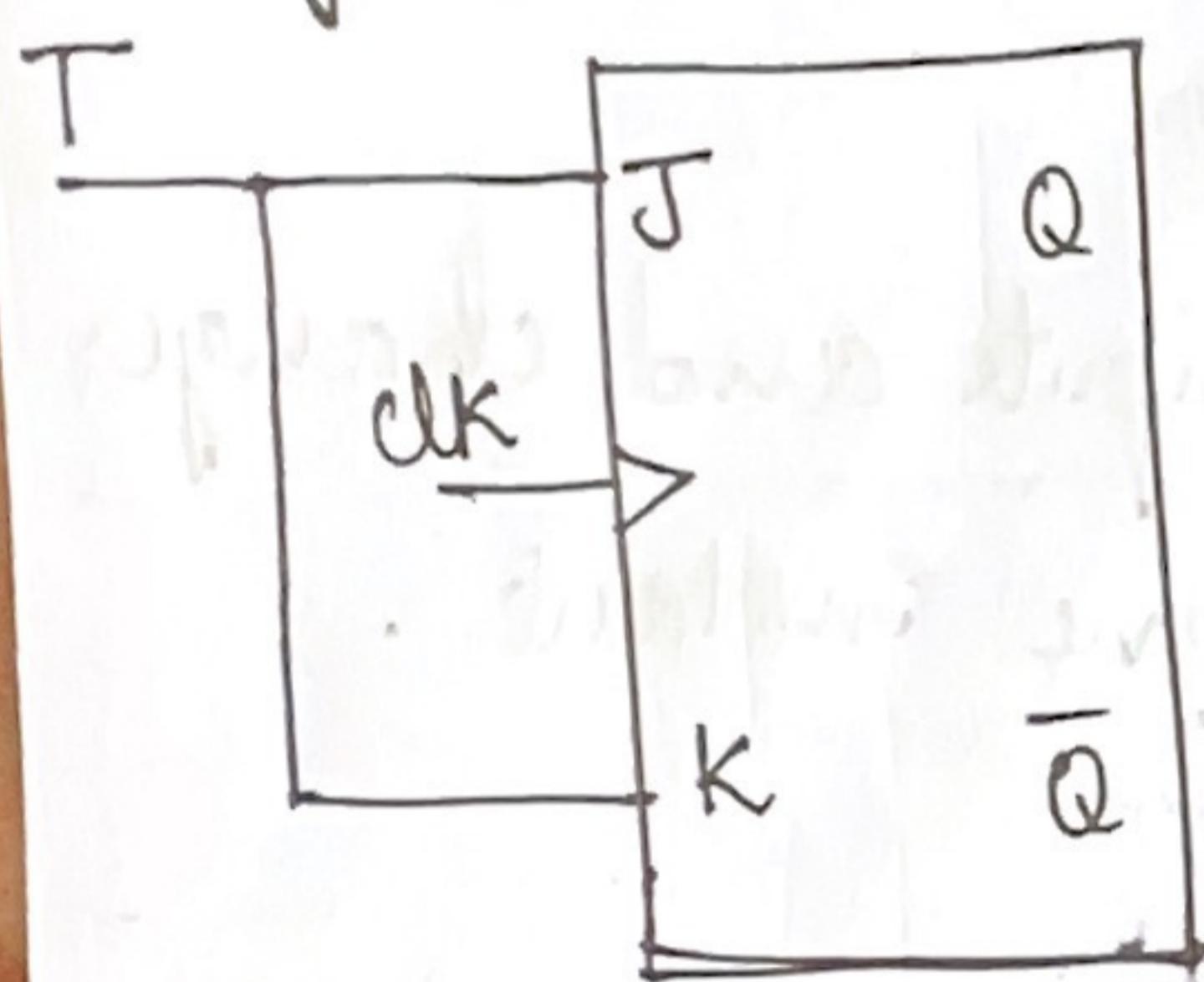
## \* Characteristic table

clk	$Q_n$	D	$Q_{n+1}$	$\bar{Q}_{n+1}$
0	x	x	Memory	
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

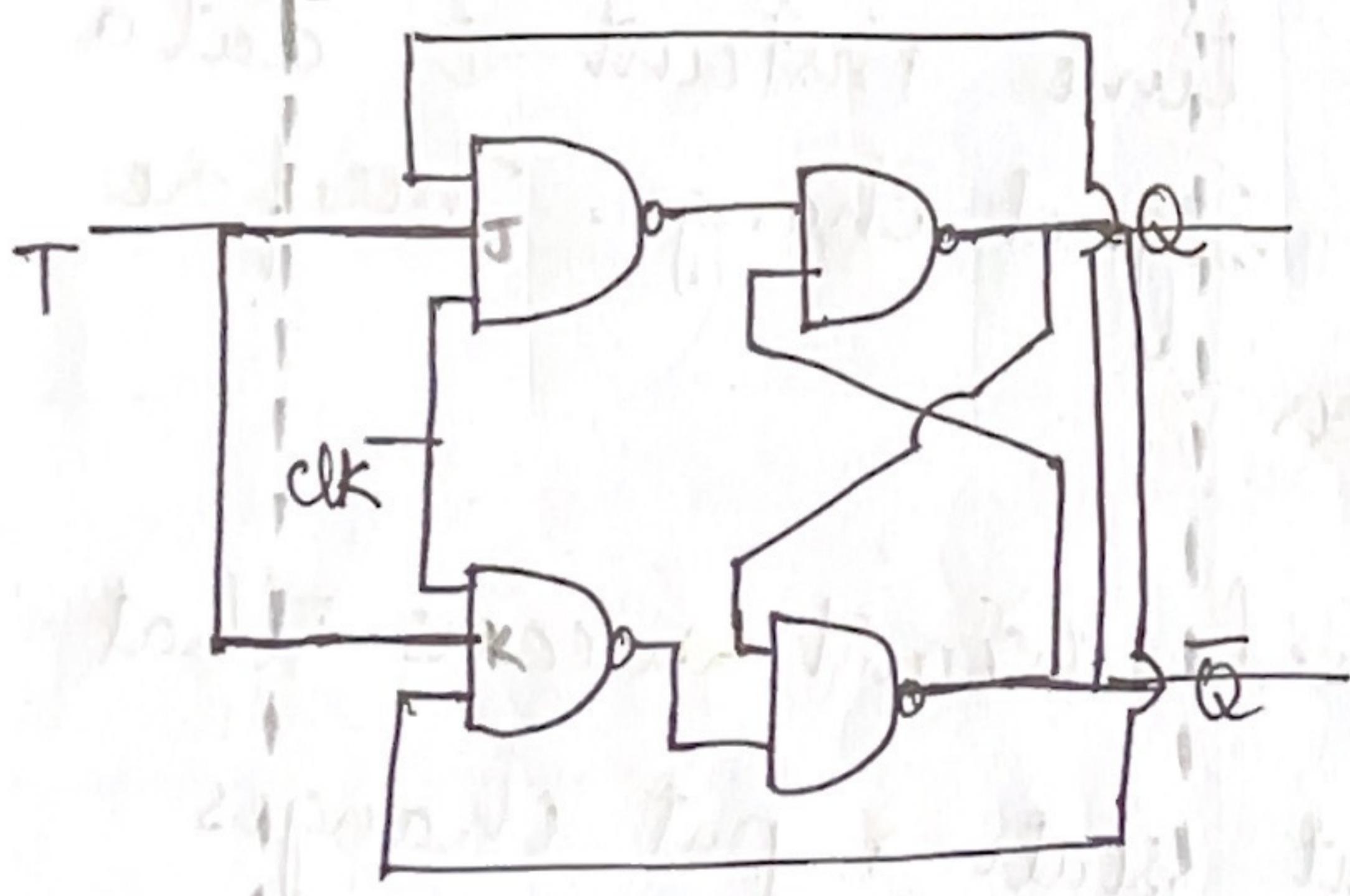
∴ Present output is equals to present input and independent of Previous output.

## \* T flip flop (Toggle flip flop)

### Symbol



### Circuit diagram:



J	K	$Q_n$	$\bar{Q}_n$
0	0	Memory	
0	1	0	1
1	0	1	0
1	1	Toggle state	

### Truth Table

clk	T	$Q$	$\bar{Q}$
0	x	Memory	
1	0	Memory	
1	1	Toggle	

∴ To get Toggling output T should be 1 otherwise the flip flop is in memory state.

## \*Characteristic Table

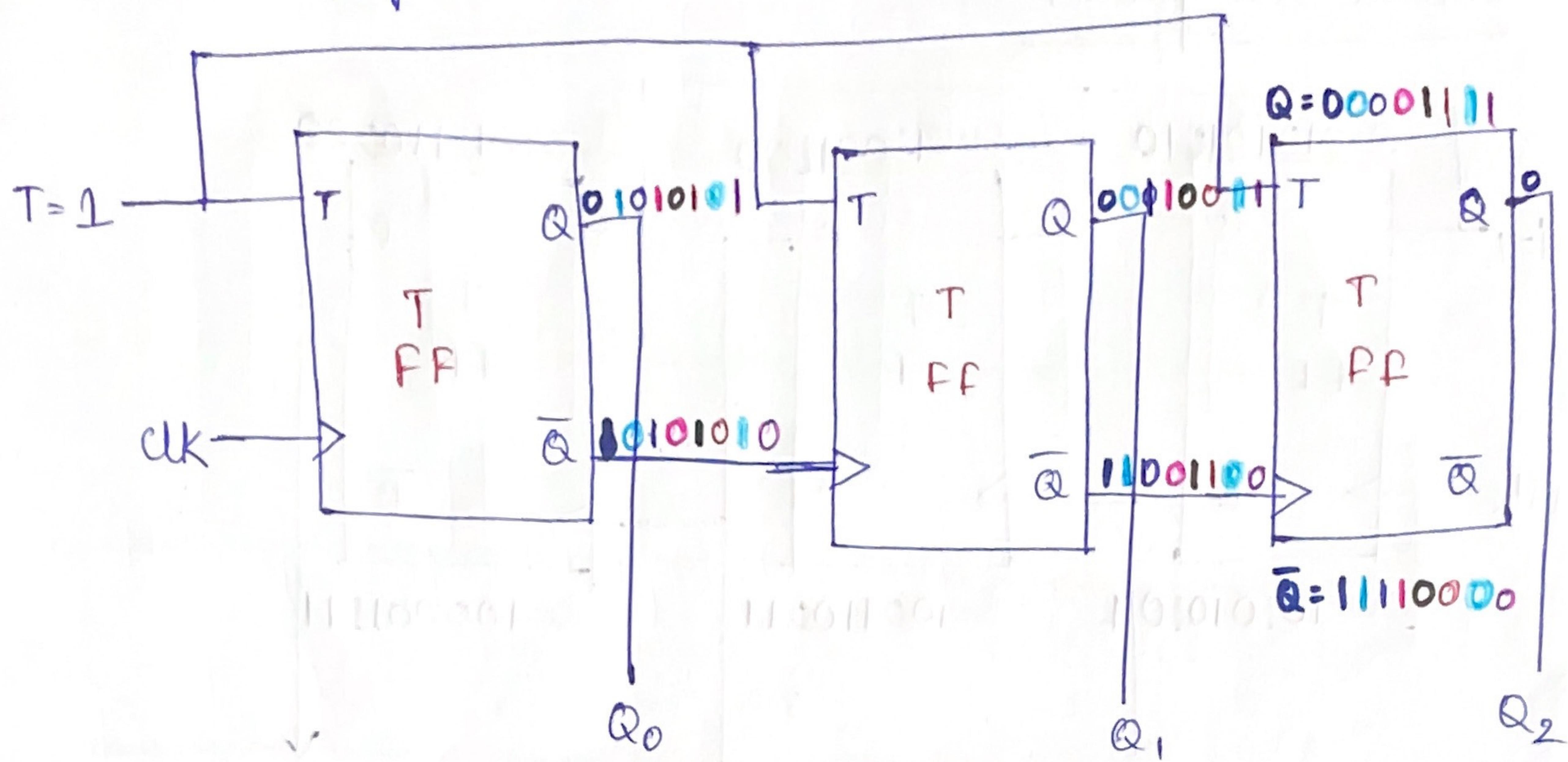
clk	Q <sub>n</sub>	T	Q <sub>n+1</sub>	$\bar{Q}_{n+1}$
0	x	x	Memory	
1	0	0	Memory	
1	0	1	1	0
1	1	0	Memory	
1	1	1	0	1

## ;Conclusion on flip flops:-

- \* flip flops are the building blocks of sequential circuits. these are built from latches.
- \* flip-flops continuously checks its inputs and changes its outputs correspondingly only at time instant determined by the clock signal.
- \* flip-flops are sensitive to signal change. they transfer data only at single time instant & data can't be changed till next signal change. therefore they are used as registers.
- \* It is an edge triggered circuit means that the output and the next state input changes when there is a change in the clock pulse whether it can be +ve / Negative.

# \* 3 bit Asynchronous up - counter \*

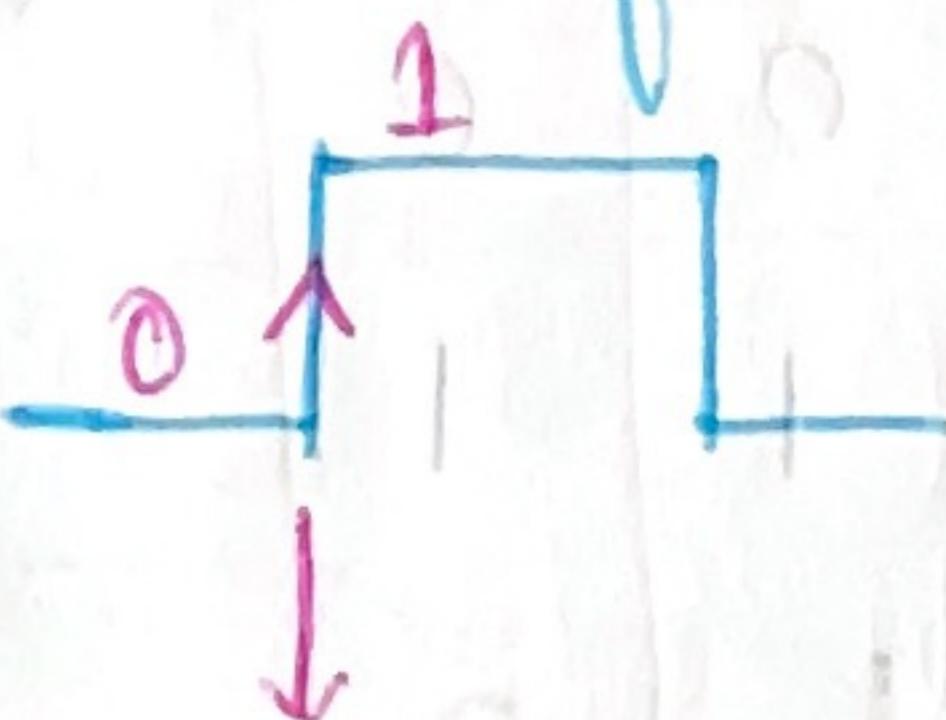
Circuit diagram:-



Q <sub>2</sub>	Q <sub>1</sub>	Q <sub>0</sub>	Counter state
0	0	0	initially 0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Notes:-

clock=1  $\Rightarrow$  Positive edge of clock which is nothing but transition from 0 to 1



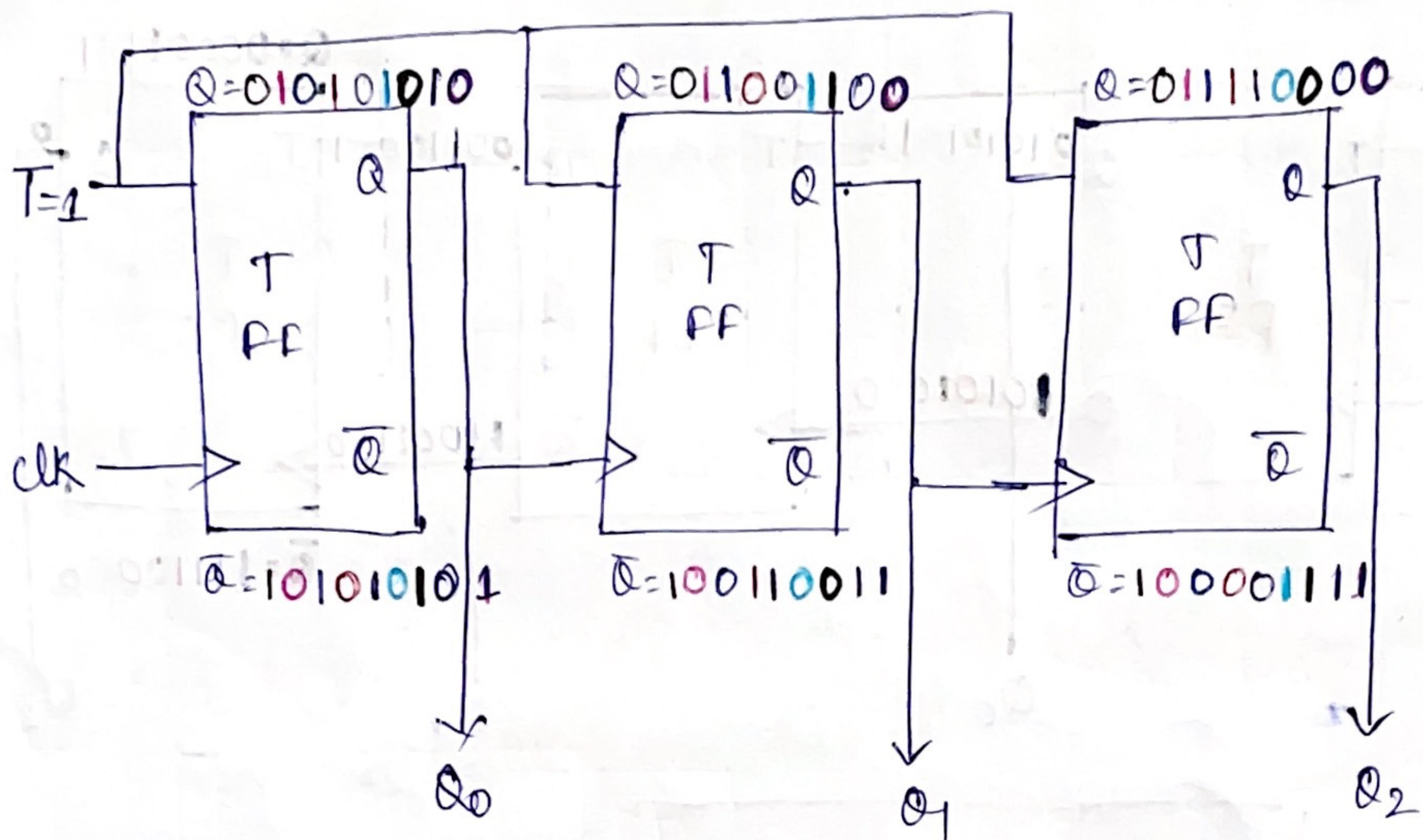
0 to 1 transition is positive edge.

J-Flip flop TT

clk	T	Q	$\bar{Q}$
0	X	Memory	
1	0	Memory	
1	1	Toggle	

# \* 3 bit Asynchronous down Counter

Circuit diagram:-

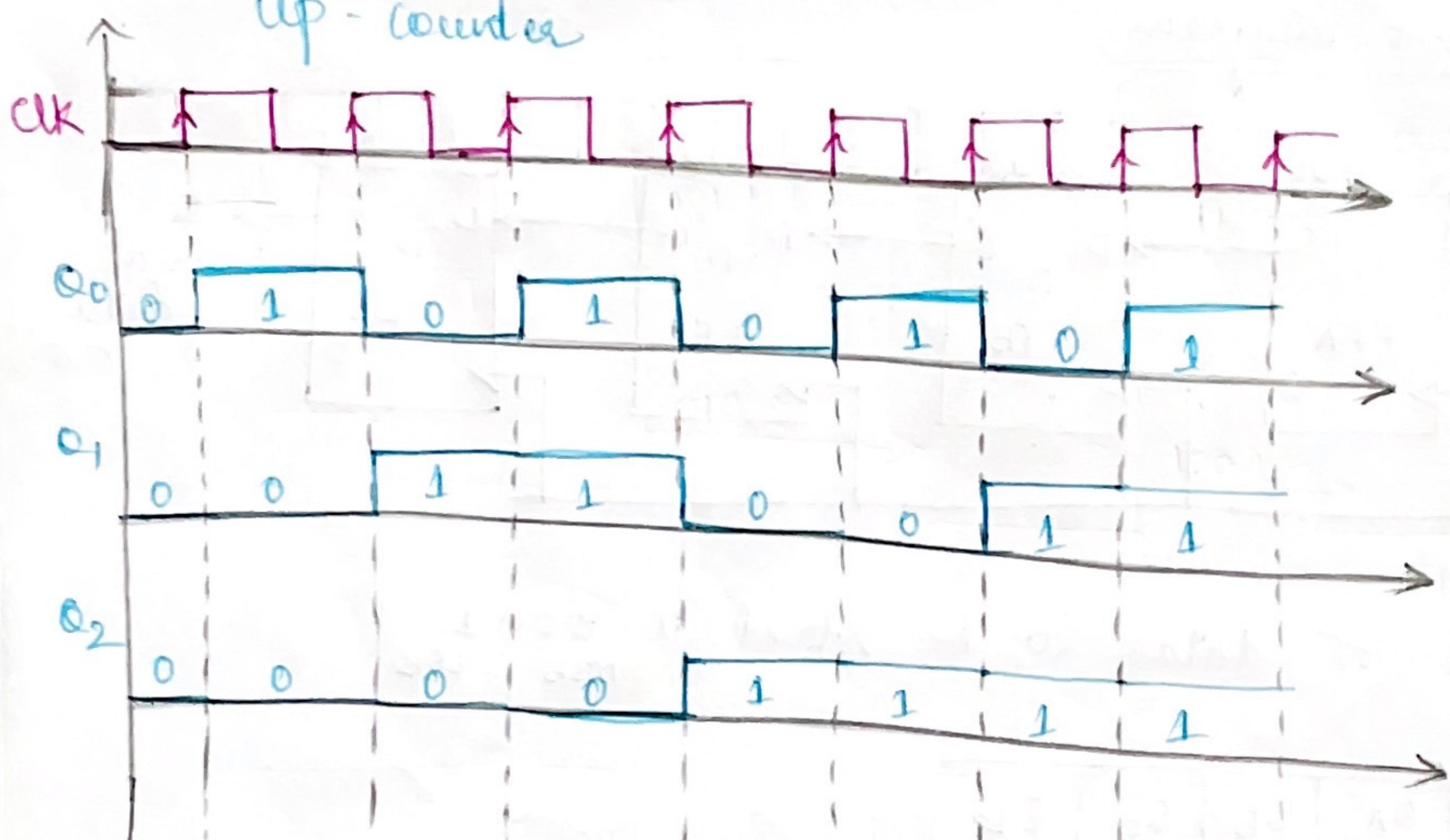


clk	$Q_2$	$Q_1$	$Q_0$	Counter state
*	0	0	0	initially (0)
↓	1	1	1	7
↓	1	1	0	6
↓	1	0	1	5
↓	1	0	0	4
↓	0	1	1	3
↓	0	1	0	2
↓	0	0	1	1
↓	0	0	0	0

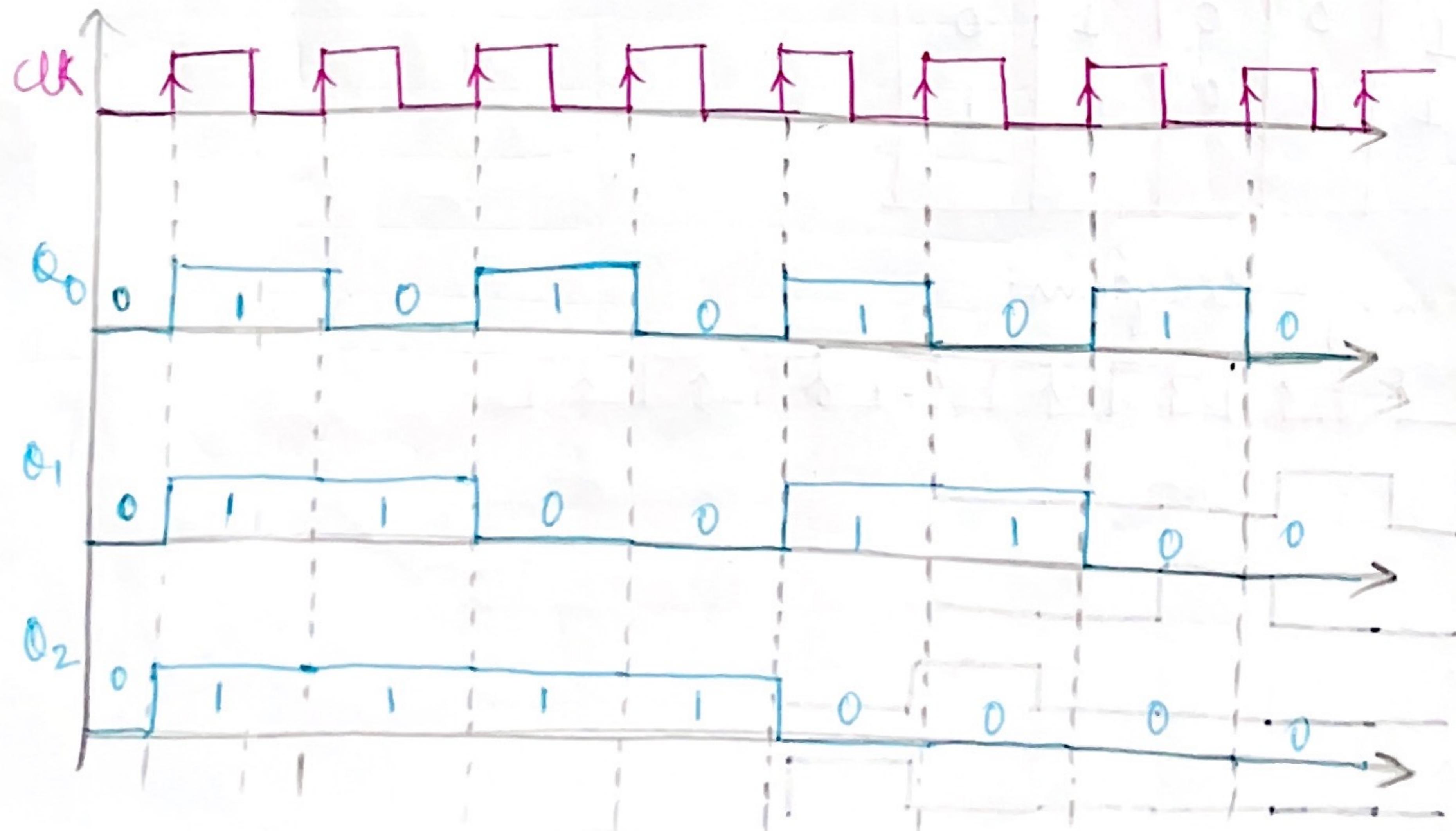
extra part

Timing diagram:-

Up - counter

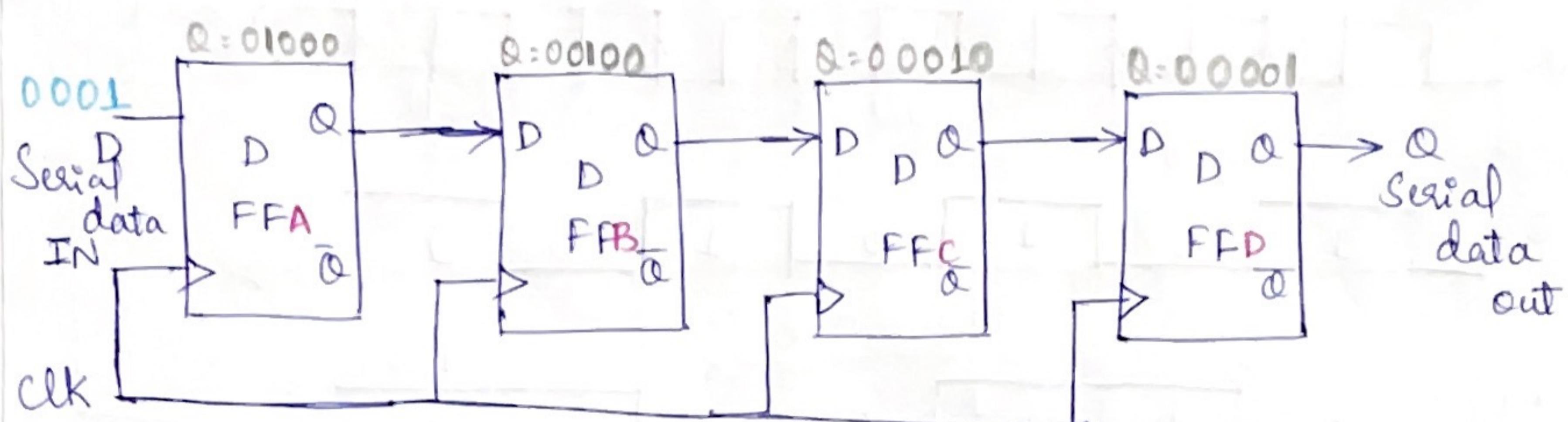


Down counter



# 4 Bit Serial i/p - Serial o/p Shift register

Circuit diagram:-



\* Let the data to be stored is 0001  
MSB      LSB

clk	QA	QB	QC	QD
0	0	0	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	1	0
1	0	0	0	1

Note:- The Previous data of Q is passed to next flip-flop's input.

Timing diagram:-

