

From Rockets to Elevators: A Multi-Objective Optimization of Planet-Scale Lunar Logistics

Summary

The establishment of a 100,000-person lunar colony starting in 2050 requires the transport of **100 million metric tons** of infrastructure. This challenge demands a strategic orchestration of two competing paradigms: the **Space Elevator System**(SES) and the **Rocket Fleet**. We develop a comprehensive, multi-phase optimization framework to navigate the complex trade-offs between project duration, financial expenditure, operational risk, and environmental sustainability.

Firstly, we establish a physics-informed economic model to quantify the dynamic capacity of both systems. For the SES, we employ a **Logistic Growth Model** to capture the transition from initial construction to saturation; for the Rocket Fleet, we derive a **continuous piecewise function** for launch frequency, constrained by regulatory phases and the Square-Cube Law. The logistics problem is mathematically formulated as a **Bi-Objective Mixed-Integer Non-Linear Programming** (BO-MINLP) model. To enhance computational tractability given the project's massive scale, we apply **continuous relaxation** to the discrete launch variables, simplifying the system into a **BO-NLP** framework. Solving this model generates a **Pareto Frontier**, revealing the quantitative *price of speed*. Our analysis identifies an optimal **Knee Point**, which achieves completion in **56 years** at a minimized cost of **\$2.08 trillion**, significantly outperforming the \$2.6 trillion cost of an all-rocket strategy.

Secondly, acknowledging real-world friction, we extend the deterministic model using **Monte Carlo Simulations** to stress-test the strategies against tether severances and launch failures. The simulation reveals a critical vulnerability: while the Space Elevator is cost-efficient, it introduces extreme **financial volatility** due to high catastrophic repair costs. In contrast, the rocket fleet demonstrates superior **financial robustness**. Consequently, we propose a **Stability-First Strategy**, reducing elevator reliance to stabilize the budget. Under this strategy, the project is completed in **65 years** with a **significantly reduced cost variance**.

Thirdly, addressing the water logistics, we derive a two-phase supply model. A massive **700,000-ton** initial stockpile is delivered via a hybrid surge (0.4 years), while the steady-state annual makeup of **13,975 tons** is sustained exclusively by the Space Elevator's excess capacity. Subsequently, we introduce an **Environmental Shadow** factor, formulating a **Tri-Objective Optimization Model**. We then **simplify** this framework by calculating **Equivalent Social Cost** of Carbon emission and ozone depletion into a unified financial metric. The *pareto knee* reveal that the rocket fleet incurs prohibitive **hidden environmental costs**, indicating that the SES must serve as the primary logistical backbone.

Finally, we perform a sensitivity analysis of the model and investigate the effect of changes in the variable parameters of the model on the results.

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1 Introduction

1.1 Problem Background and Statement

Interplanetary colonization has emerged as a strategic necessity. To support the MCM Agency's 2050 habitat for 100,000 residents, two core modalities—the Space Elevator System (SES) and rocket fleets—are employed. While rockets complement the SES by accelerating delivery, their severe environmental impact necessitates strict regulation. This coupling transforms logistics into a complex, non-linear optimization problem. Our objective is to **optimize the logistics** for transporting **100 million metric tons** of infrastructure to the Moon starting in 2050, utilizing both Space Elevators and Rocket Fleets. We must minimize total cost and project duration while addressing four specific requirements:

- **Ideal/Stochastic Scenario Analysis:** Compare exclusive (Elevator/Rocket-only) and hybrid strategies under deterministic conditions. Quantify the impact of probabilistic disruptions, including tether severances and launch failures.
- **Sustainability Logistics:** Model the supply chain for the initial water inventory and steady-state makeup required for 100,000 residents.
- **Environmental Optimization:** Internalize ecological costs (e.g., ozone depletion) to propose an environmentally sustainable transport strategy.

1.2 Our Work

To tackle the multi-dimensional challenge of lunar colonization logistics, our work proceeds through the following logical framework, shown in the Figure 1:

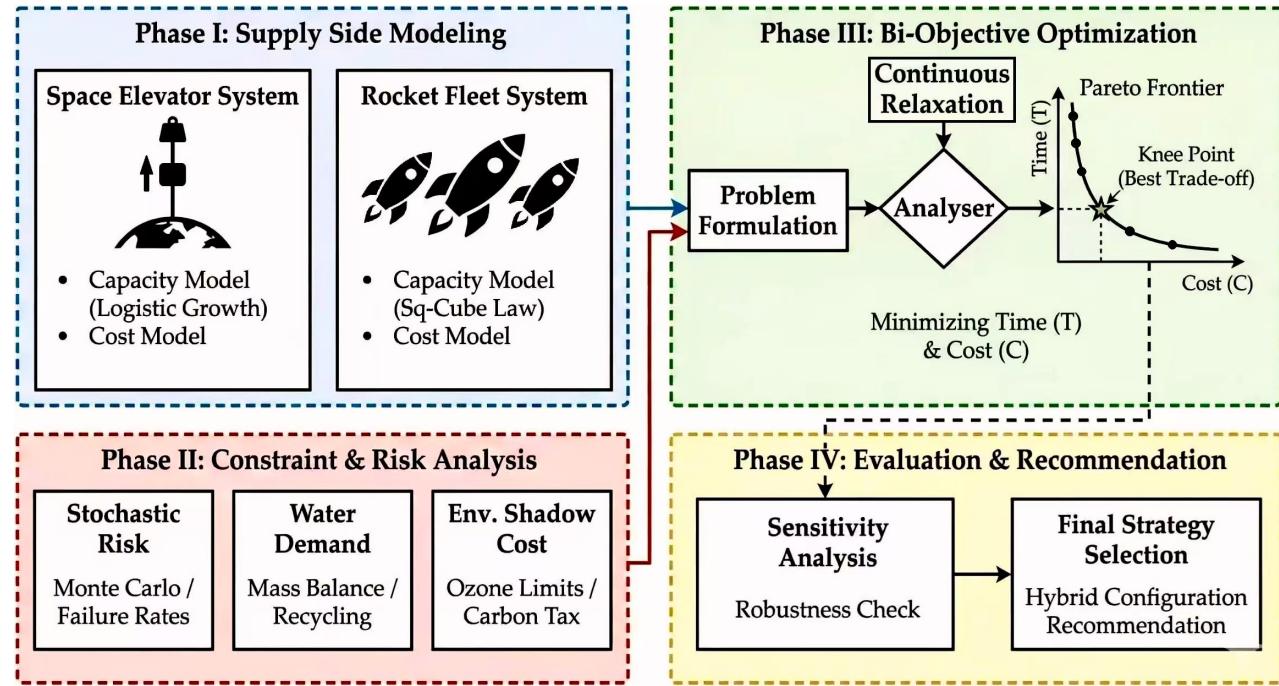


Figure 1: The Framework of Our Work on Multi-Objective Lunar Logistics.

2 Model Preparation

To address the complex logistics of building and maintaining a 100,000-person lunar colony, we develop a phase modeling approach. Initially, we establish a baseline under ideal conditions (Task 1), then introduce stochastic disruptions to simulate real-world conditions (Task 2), and finally extend the framework to include long-term sustainability (Task 3 & 4).

2.1 Assumptions and Justifications

To simplify the modeling process while capturing essential dynamics, we make the following assumptions:

- **Assumption 1: Transportation Capacity as the Sole Binding Constraint.** We assume that the supply of materials on Earth and the assembly capacity on the Moon are sufficient and perfectly synchronized with arrival times. Additionally, we assume the authority to freely schedule and allocate global space transportation resources.
 - *Justification:* This isolates the logistics network as the primary variable, allowing the model to focus on optimizing the **throughput** and **efficiency** of the transportation systems without being constrained by external supply-chain factors like manufacturing delays, on-site storage limits, or geopolitical factors.
- **Assumption 2: Space Elevator Construction as Sunk Cost.** The Space Elevator is modeled as completed infrastructure by the start date (2050). Consequently, its initial capital expenditure is treated as a **sunk cost**, and the optimization model focuses strictly on **Operational Expenditures (OPEX)**.
 - *Justification:* Since the decision to construct the elevator predates the simulation, those costs are irrelevant to *forward-looking* logistical decisions. Including the massive construction cost would unfairly skew the comparison. By focusing on OPEX, we can fairly compare the **marginal cost per ton** between the reusable rockets and the elevator system.
- **Assumption 3: Economic Stability and Constant Currency Value.** All financial projections are calculated in **Constant 2050 US Dollars**. We do not account for inflation, exchange rate fluctuations, or the time-value of money (discount rates) in the explicit cost function.
 - *Justification:* Predicting macroeconomic trends over decades introduces significant aleatory uncertainty. Since inflation affects both rocket and elevator industries broadly, using constant dollars provides a consistent baseline for **relative comparison**. Cost figures in this model serve as a proxy for **resource allocation effort** rather than purely nominal financial transactions.

2.2 Notations

The primary notations used in this paper are listed in Table 1.

Notice that the below notations are general for all tasks. Specific parameters for each task will be detailed in the respective sections.

Table 1: Notations

Symbol	Definition	Unit
N_{GH}	Number of Galactic Harbors	-
N_{resi}	Number of residents	-
M_{cons}	Total material required for construction	metric tons
t	Time index (since 2050)	year
T	Total time to complete transportation	years
C_{total}	Total cost of transportation	USD
$L_{GH,t}$	Annual transportation capacity of a Galactic Harbor at time t	tons/year
$P_{rocket,t}$	Payload capacity of a single rocket launch at time t	tons
$f_{site,t}$	Annual launch frequency for launch site at time t	launches/year
N_{sites}	Number of launch sites	-
η	Efficiency of the Space Elevator system	percentage
σ_{rocket}	Success rate of rocket launches	percentage
W_{resi}	Annual water requirement per resident	tons/year

2.3 Data Collection and Parameter Estimation

Our model relies on a combination of parameters provided in the problem statement and those estimated from external sources.

2.3.1 Given Parameters

Table 2: Problem Parameters

Parameter	Value
N_{GH}	3
N_{resi}	100000
M_{cons}	10^8 metric tons
$L_{GH,2050}$	179,000 tons/year
$P_{rocket,2050}$	[100,150] tons
N_{sites}	10

Table 3: Physical Constants

Parameter	Value
Earth Mass (M_e)	5.97×10^{24} kg
Earth Radius (R_e)	6,371 km
GEO Radius (R_{geo})	42,164 km
Anchor Height (R_{anchor})	100,000 km
Earth Angular Velocity (ω)	7.29×10^{-5} rad/s
g	9.81 m/s ²

2.3.2 Parameter Estimation from External Sources

To ensure the fidelity of our model, we adopt physical parameters and engineering constraints based on reports from the International Space Elevator Consortium (ISEC) and relevant aerospace literature. All of the estimated parameters are summarized in Tables 4 and 5, and **carefully checked against multiple sources to ensure realism.** [1, 2, 3]

3 The Models

The cumulative mass accumulated at the Moon Colony, denoted as $M(t)$, evolves based on the inputs from two distinct transportation modes. The Space Elevator provides a continuous mass flow, while the rocket fleet contributes through discrete launch events. We

Table 4: Engineering System Parameters

Parameter	Value
Climber Mass ($m_{climber}$)	20 tonnes [3]
Eff. Payload ($m_{payload}$)	14 tonnes [4]
Avg. Climb Speed (v)	200 km/h [2]
Regen. Eff. (η_{regen})	0.85 [5]
Specific Impulse (I_{sp})	450 s [6]
Maint. Cost Rate (α)	1.5 % [7]
Aging Rate (r_{aging})	2.5 % [8]
Construct. Cost	≈ 100 Billion USB [2]
Elec. Cost (C_{elec})	0.10 \$/kWh [1]
Climb Eff. (η_{up})	0.5 [1]

Table 5: Risk & Reliability Parameters

Parameter	Value
Ops. Efficiency (η_{ops})	0.85 [9]
Failure Loss (C_I)	60 Million USB [10]
Severance Cost (C_{II})	25 Billion USB [2]
Recovery Time (T_{down})	2.5 years [9]
Vehicle Build (C_{build})	1.35 Billion USB

model each mode's contribution separately before integrating them into the overall system dynamics.

Similarly, by tracing the operational costs associated with each transportation mode, we compute the total cost incurred up to time t , denoted as C_t . And the total cost at project completion is $C_{total} = C_{2050+T}$.

3.1 Space Elevator Model

Let $m_{SE,t}$ denote the mass transported by the Space Elevator in year t , and $c_{SE,t}$ denote the corresponding operational cost incurred in year t .

3.1.1 Mass Flow Modeling

To accommodate different operational strategies, we introduce a continuous decision variable $\delta_{SE} \in [0, 1]$. If $\delta_{SE} = 1$, the Space Elevator is fully activated; if $\delta_{SE} = 0$, it is not used.

$$m_{SE,t} = \delta_{SE} \cdot \eta \cdot N_{GH} \cdot L_{GH,t} \quad (1)$$

Here, η represents the elevator system efficiency as Table 1 given, and $L_{GH,t}$ is treated as an exogenous discrete variable determined by technological maturity.

Estimation of Space Elevator Capacity Growth ($L_{GH,t}$) To forecast the annual throughput ($L_{GH,t}$) of the Space Elevator starting from 2050, we adopt the **Logistic Growth Model**.

The choice of a Logistic model is driven by the **phased deployment strategy** described in the ISEC report [1]. Unlike exponential models which assume infinite resources, infrastructure projects face physical saturation. The model captures three distinct phases: **Initial Phase (2037-2045), Growth Phase (2045-2060), Saturation Phase (2060-Future)**

Logistic Function can smoothly approximate the transition from the rapid exponential expansion phase to this final saturation level, avoiding the **unnecessary complexity** of a piece-wise model, which is given by:

$$L_{GH,t} = \frac{L_{max}}{1 + A \cdot e^{-r_{growth}(t-1)}} \quad (2)$$

where $L_{max} = 1,200,000$ tons is the saturation limit[1], $r_{growth} = 0.065$, and $A \approx 5.94$ is calibrated to match the estimated capacity.

3.1.2 Cost Modeling

The annual operational cost $C_{SE,t}$ of the Space Elevator in year t is consisted of three parts: energy cost, maintenance cost, and apex-to-moon transfer cost:

$$C_{SE,t} = C_{energy,t} + C_{maintain,t} + C_{transfer,t} \quad (3)$$

Electricity Cost($C_{energy,t}$): This is the cost of electrical energy required to lift the mass $m_{SE,t}$ from Earth to GEO. Also, we consider regenerative energy recovered during GEO to apex.

Let $U_{location}$ be the gravitational potential energy at a given location ($U_{location} = -\frac{GM_e}{r} - \frac{1}{2}\omega^2 r^2$). The net energy calculation formula is given as:

$$E_{energy,t} = m_{SE,t} \cdot \left(\frac{U_{R_{geo}} - U_{R_{Earth}}}{\eta_{up}} - \eta_{regen} \cdot (U_{R_{anchor}} - U_{R_{geo}}) \right) \quad (4)$$

Where η_{up} is the climbing efficiency, and η_{regen} is the regenerative efficiency. Based on that, the electricity cost per kilogram is then calculated as:

$$C_{energy,t} = \frac{E_{energy,t}}{m_{SE,t}} \cdot P_{elec} = \$2.308/\text{kg} = \$2,308/\text{ton}$$

Maintenance Cost ($C_{maintain,t}$): This is the operational cost where cargo pay for infrastructure upkeep per usage, we model the maintenance cost as an amortized surcharge levied on each ton of cargo.

Reference the maintenance cost assessment of the International Space Station [11], the annual system maintenance follows an **exponential aging function** of the sunk capital cost ($K_{CAPEX} = \$100$ Billion, based on the Obayashi estimates [2]):

$$C_{maint}^{total}(t) = (\mu_{maint} \cdot K_{CAPEX}) \cdot (1 + r_{aging})^t \quad (5)$$

where $\mu_{maint} = 1.5\%$ is the baseline rate derived from ISS standards [7], and $r_{aging} = 2.5\%$ accounts for component degradation (e.g., atomic oxygen erosion) [8]. To derive the unit toll price, we calibrate the cost against the baseline throughput of $t = 2050$, where throughput is projected at 179,000 tons.

$$C_{maint} = \frac{(\mu_{maint} \cdot K_{CAPEX}) \cdot (1 + r_{aging})^t}{L_{GH,2050}} = \frac{1.5\% \times 10^{11} \times 1.025}{179,000} \approx \$8,590/\text{ton} \quad (6)$$

For complexity concern, this unit price is treated as a **constant service fee** in our optimization model, serving as a conservative upper bound.

Apex-to-Moon Transfer Cost ($C_{transfer,t}$): This represents final leg involves transferring cargo from the Apex Anchor to the Lunar surface using reusable LH₂/LOX Space Tugs.

Derived from the **Tsiolkovsky rocket equation**, the mass ratio \mathcal{R} relates the required velocity change (Δv) to the engine's effective exhaust velocity (v_e) based on the conservation of momentum. For a lunar landing trajectory ($\Delta v \approx 2.5$ km/s [6]) using high-efficiency engines ($I_{sp} = 450$ s [6]). Then, the transfer cost is modeled as a dynamic process that decays as infrastructure matures [4]. We define the time-dependent cost $c_{trans}(t)$ and calculate its **50-year average** (\bar{c}_{trans}) by calculating the exponential decay of propellant costs ($P_{fuel,0} = \$11.90/\text{kg}$) and tug operations ($P_{ops,0} = \$5.00/\text{kg}$)[6]:

$$\mathcal{R} = e^{\frac{\Delta v}{v_e}}, \bar{c}_{trans} = \frac{1}{T} \sum_{t=0}^{T-1} \left[(\mathcal{R} - 1)P_{fuel,t} + P_{ops,0}e^{-k_2 t} \right] \approx \$8,870 \text{ per ton} \quad (7)$$

where $k_1 = 0.02, k_2 = 0.03$ are maturation rates [4].

Summary of Space Elevator Unit Cost

Combining the energy cost, constant maintenance surcharge, and the 50-year average transfer cost, the **Lifecycle Average Unit Cost (\bar{C}_{SE}^{unit})** is:

$$\bar{C}_{SE}^{unit} = c_{elec} + c_{maint} + \bar{c}_{trans} = 2,308 + 8,590 + 8,870 = \$19,768 \text{ per ton} \quad (8)$$

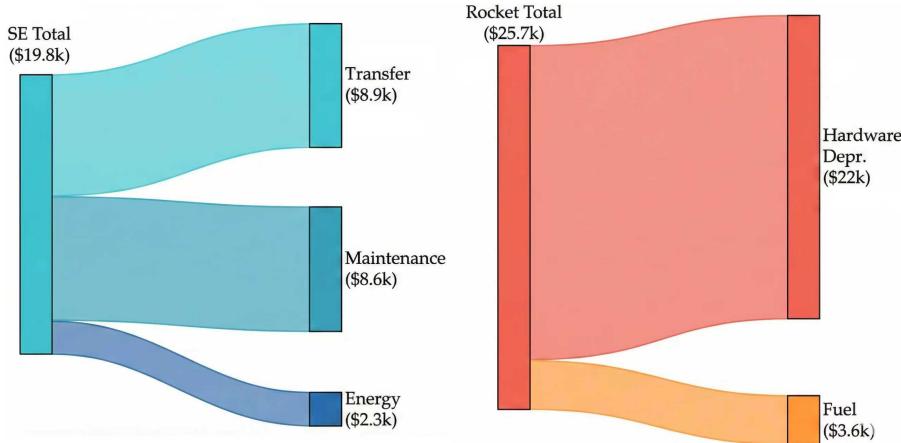


Figure 2: An Sankey illustration for Unit Cost Breakdown Flows.

3.2 Rocket Fleet Model

Rocket transportation consists of discrete launch events. Unlike the continuous flow of the Space Elevator, rockets deliver cargo in batched payloads through individual launches.

Let $m_{Rocket,t}$ denote the total mass transported by the rocket fleet in year t , and $C_{Rocket,t}$ denote the corresponding operational cost.

3.2.1 Mass Flow Modeling

The total annual mass throughput is the product of launch frequency and payload capacity. We introduce a continuous decision variable $\delta_{Rocket} \in [0, 1]$ to represent the fleet utilization rate.

$$m_{Rocket,t} = \lambda \cdot N_{launch,t} \cdot P_{cap,t} \quad (9)$$

Here, $N_{\text{launch},t}$ represents the maximum feasible number of launches per year under multiple constraints, while $P_{\text{cap},t}$ represents the payload capacity resulting from infrastructure evolution.

Estimation of Maximum Launch per Year $N_{\text{launch},t}$ To estimate the maximum feasible launch frequency, we consider the *Regulatory-Technical-Environmental (RTE) Constraints* [24, 20, 23].

- **Phase I (Regulatory):** Constrained by airspace integration (≈ 600).[17].
- **Phase II (Technical):** *Space Data Integrator* (SDI) technology allows reduced safety buffers, enabling a CAGR of $r_{\text{tech}} = 4\%$ [18].
- **Phase III (Environmental):** The system hits the geophysical wall $D_{\text{cap}} = 2,488$. This limit is derived from the critical Radiative Forcing threshold ($RF_{\text{crit}} = 50\text{mW/m}^2$) caused by stratospheric black carbon[24].

Following the above constraints, we can model the $N_{\text{launch},t}$ as a continuous piecewise function:

$$N_{\text{launch}}(t) = \begin{cases} \frac{K_{\text{pol}}}{1+e^{-r_1(t-2025)}} & 2025 \leq t \leq 2035 \quad (\text{Phase I}) \\ N_{2035} \cdot (1 + r_{\text{tech}})^{t-2035} & 2035 < t \leq 2060 \quad (\text{Phase II}) \\ D_{\text{cap}} - (D_{\text{cap}} - N_{2060}) \cdot e^{-\alpha(t-2060)} & t > 2060 \quad (\text{Phase III}) \end{cases} \quad (10)$$

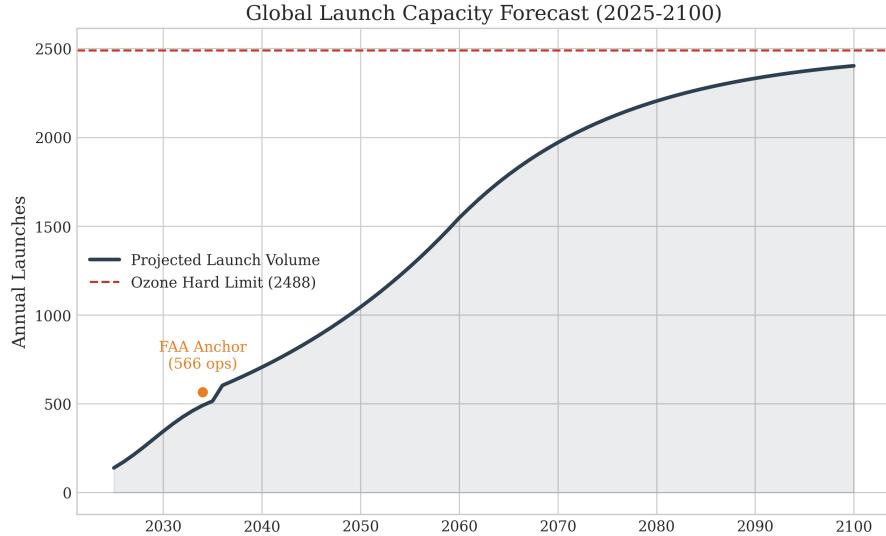


Figure 3: **Global Launch Capacity Forecast.** The trajectory is strictly anchored to FAA 2034 data and capped by the NOAA ozone limit.

3.2.2 Estimation of Rocket Payload Growth ($P_{\text{rocket},t}$)

The scaling of chemical rockets is fundamentally governed by the *Square-cube law*. As illustrated in Fig. 4, while vehicle mass increases cubically (L^3), the available thrust (limited by the nozzle base area) only scales quadratically (L^2). This creates a terminal Thrust-to-Weight Ratio (TWR) bottleneck and structural fatigue limits analogous to the geometric constraints in biological cells [12, 15].

Based on the **Sea Dragon** concept—the recognized engineering upper bound for chemical propulsion—we assume a maximum lift-off mass of 18,000 tons [14]. Accounting for structural overhead and a theoretical payload mass fraction limit of $\lambda_{limit} \approx 5\%$ [16], the maximum payload capacity (P_{max}) is:

$$P_{max} = M_{total} \times \lambda_{limit} = 18,000 \text{ tons} \times 5\% = 900 \text{ tons} \quad (11)$$

To represent the progression toward this cap, we adopt a **Logistic Growth Model**. Anchoring the curve with a projected 2050 capacity ($P_{rocket,2050} = 150$ tons) and historical heavy-lift data [19], the growth function is:

$$P_{rocket,t} = \frac{P_{max}}{1 + B \cdot e^{-r(t-2050)}} \quad (12)$$

where $B = 5$ and $r \approx 0.091$.

LIMITS OF SCALING: A Comparative Analysis of Biological & Engineering Constraints

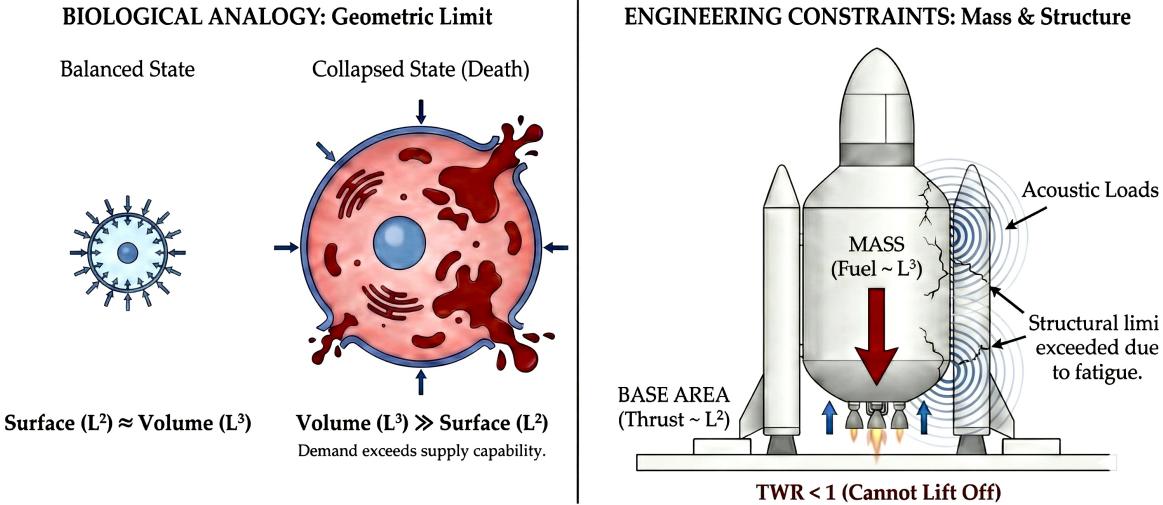


Figure 4: **The Geometric Limit:** Just as a cell is limited by the surface-to-volume ratio (L^2 vs L^3), a rocket's maximum size is constrained by the nozzle area (L^2) available to lift its fuel mass (L^3). Large mass also induces catastrophic acoustic loads and structural fatigue.

3.2.3 Cost Modeling

The annual operational cost $C_{rocket,t}$ of the rocket fleet in year t is consisted of three parts: Maintenance, and fuel costs:

$$C_{Rocket,t} = C_{Maint,t} + C_{Fuel,t}, \quad (13)$$

Maintenance Cost ($C_{Maint,t}$): This reflects the amortized manufacturing and operational costs of the rocket fleet. We model the maintenance cost as an amortized surcharge levied on each ton of cargo.

According to the TRANSCOST model [20], a value-added ratio (manufacturing-to-material) is about 50:1. Additionally, due to propulsion amortization, a rocket cannot be reused infinitely, by calculating the expected lifespan and failure rates, we estimate a reuse number of

$N_{reuse} = 80$. What's more, the operational cost per launch (including labor, ground support, etc.) is estimated at $Cost_{Ops} = 3$ Million USD [10]. Thus, the unit maintenance cost per ton is given by:

$$C_{Maint,t} = C_{build}/N_{reuse} + C_{ops} \quad (14)$$

With above assumption that the material is 5% of total mass, and additionally assume the material, as Al-Li alloys (\$30,000 per ton), the manufacturing cost per launch is:

$$Cost_{build} = M_{total} \cdot 5\% \cdot 30,000 \cdot 50 = \$1.35 \text{ Billion} \quad (15)$$

Thus, the unit maintenance cost per ton is given by:

$$C_{Maint}^{unit} = \frac{\frac{1.35 \times 10^9}{80} + 3 \times 10^6}{900 \text{ tons}} \approx \$22,083/\text{ton} \quad (16)$$

Fuel Cost ($C_{Fuel,t}$): Assuming the propellant is a Methalox mixture (Liquid Oxygen and Methane) optimized for industrial-scale operations, we adopt a bulk price of $P_{fuel} \approx \$200/\text{ton}$ based on SpaceX's Starship economics [19]. Thus:

$$Cost_{fuel}^{total} = M_{total} \cdot 90\% \cdot P_{fuel} = 18,000 \text{ ton} \cdot 90\% \cdot \$200/\text{ton} = \$3.24 \text{ Million}$$

$$C_{fuel}^{unit} = \frac{3.24 \times 10^6}{900 \text{ tons}} \approx \$3,600/\text{ton} \quad (17)$$

Summary of Rocket Unit Cost

Combining the maintenance and fuel costs, the **Rocket Unit Cost ($C_{Rocket,t}^{unit}$)** is:

$$C_{Rocket}^{unit} = C_{Maint}^{unit} + C_{fuel}^{unit} = 22,083 + 3,600 = \$25,683/\text{ton} \quad (18)$$

3.3 System Evolution and Constraints

The cumulative mass $M(t)$ is governed by the recursive state equation:

$$M(t) = \begin{cases} 0, & t = 2050, \\ M(t-1) + m_{SE,t} + m_{Rocket,t}, & t = 2051, 2052, \dots \end{cases} \quad (19)$$

The optimization goal is to find the minimum project duration T that satisfies the construction demand M_{cons} :

$$\text{Subject to: } M(2050 + T) \geq M_{cons} \quad (20)$$

3.4 Bi-Objective Optimization Framework

The logistics planning problem is fundamentally a trade-off between economic cost and project duration. We formulate this as a **Bi-Objective Mixed-Integer Non-Linear Programming (BO-MINLP)** problem.

However, given the immense scale of the project (10^8 tons) relative to single-launch capacity, the discreteness of rocket launches becomes negligible. Therefore, we apply **continuous relaxation** to the launch frequency, effectively simplifying the model to a **Bi-Objective Non-Linear Programming (BO-NLP)** problem to enhance computational tractability.

Let the decision vector be $\mathbf{x} = [\delta_{SE}, \lambda]^\top$, where $\delta_{SE} \in [0, 1]$ determines the Space Elevator activation and $\lambda \in [0, 1]$ controls the rocket fleet intensity. The system simulation defined in Sections 3.1-3.3 can be viewed as a mapping function \mathcal{S} that transforms a decision \mathbf{x} into an outcome tuple:

$$(T, C_{total}) = \mathcal{S}(\mathbf{x}) \quad (21)$$

Our goal is to minimize the objective vector $\mathbf{J}(\mathbf{x})$:

$$\min_{\mathbf{x}} \mathbf{J}(\mathbf{x}) = \begin{bmatrix} T(\mathbf{x}) \\ C_{total}(\mathbf{x}) \end{bmatrix} \quad (22)$$

Subject to:

$$M(2050 + T) \geq M_{cons}$$

Since the two objectives (Time and Cost) are generally conflicting, there exists no single optimal solution. Instead, the solution to this general model is the **Pareto Optimal Set** \mathcal{P} . By analyzing the curvature of this frontier, we can quantify the price of speed and provide decision-makers with a spectrum of viable logistical strategies, ranging from budget-conscious to time-critical.

4 Task 1: Ideal Conditions Analysis

All of the three scenarios are analyzed under ideal conditions (no disruptions, constant parameters), and using the models established in Section 3.

The scenarios required in Task 1 can be interpreted as specific points or subsets within this framework:

- **Scenario A (Elevator Only):** A boundary solution where $\mathbf{x} = [1, 0]^\top$.
- **Scenario B (Rocket Only):** A boundary solution where $\mathbf{x} = [0, 1]^\top$.
- **Scenario C (Hybrid):** An interior solution where $\mathbf{x} = [\delta_{SE}, \lambda^*]^\top$ for some $\delta_{SE}, \lambda^* \in (0, 1)$ that minimizes T or C_{total} .

4.1 Scenario Analysis

The cumulative mass, transport trajectories, and cumulative cost for the different scenarios are illustrated in fig 5.

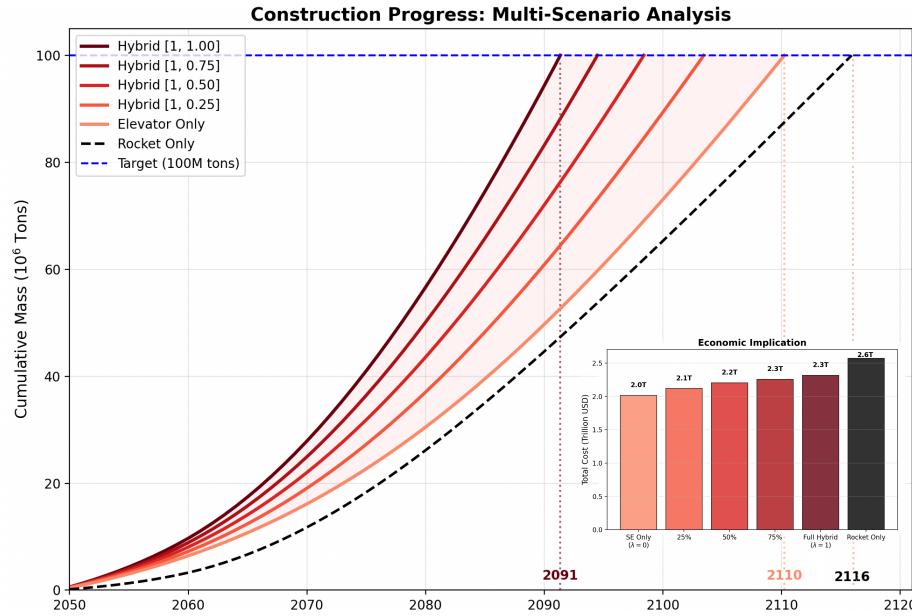


Figure 5: Multi-Dimensional Sensitivity Analysis and Probabilistic Risk Assessment

- **Scenario A:** Complete in year 2110 with total cost of ≈ 2.0 Trillion dollars.
- **Scenario B:** Complete in year 2116 with total cost of ≈ 2.6 Trillion dollars.

The hybrid scenario is inherently more complex due to the continuous nature of both δ_{SE} and λ . For tractability, we discretize each variable into 11 uniform levels ($0, 0.1, 0.2, \dots, 1$) and evaluate all possible combinations.

This procedure generates a finite set of solutions in the time–cost plane, from which the points located in the lower-left region form an approximation of the Pareto frontier. Hybrid strategies are then analyzed based on these **Pareto-efficient solutions**. The figure 6 is as below. The left and right part of each node represents the intensity of elevator and rocket, respectively.

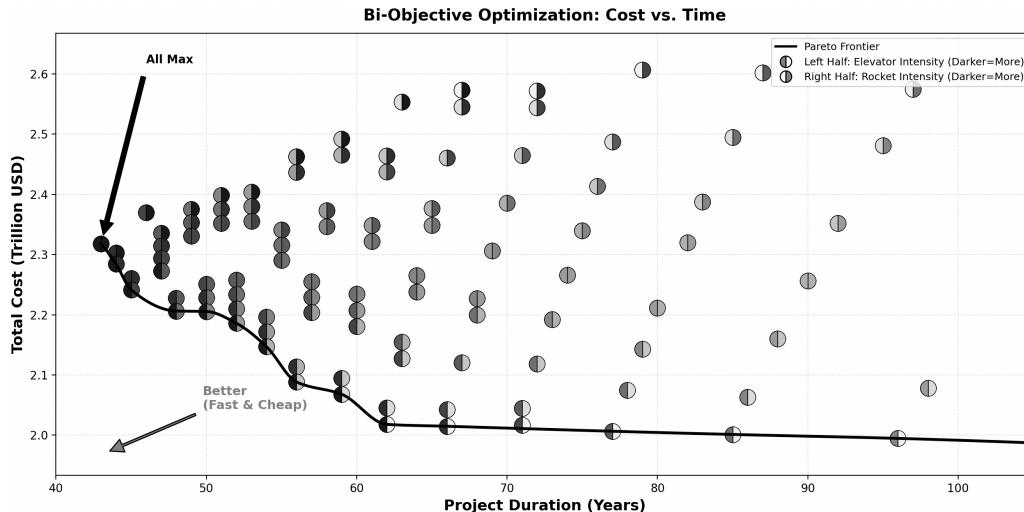


Figure 6: The Time-Cost Pareto Frontier. The curve illustrates the trade-off between project duration and total cost. The pareto frontier can be seen as a continuous function.

4.2 Hybrid Strategy Recommendation

The Pareto frontier in Fig. 6 demonstrates that no single solution can simultaneously minimize both project duration and total cost. Each point along the frontier therefore represents a distinct trade-off between time efficiency and economic expenditure. Notably, most Pareto-optimal solutions correspond to $\delta_{SE} = 1$, indicating that **full activation of the space elevator** is a dominant component in cost-effective logistics strategies.

From a decision-making perspective, solutions located near the *knee region* of the Pareto frontier are particularly attractive. In our analysis, a representative knee point occurs at $x = [1, 0.3]^\top$, with a completion time of approximately 56 years with a total cost of about \$2.08 trillion.

However, if a construction time threshold of **50 years** is imposed while still minimizing total cost, the solution $x = [1, 0.5]^\top$ achieves the required schedule with an estimated total cost of approximately **\$2.21 trillion**.

Therefore, we recommend the latter hybrid strategy as a practical choice under time-constrained scenarios. In general, the final decision should be guided by specific policy priorities, balancing budget limitations against schedule requirements.

5 Task 2: Stochastic Disruptions Analysis

Real-world logistics must consider inherent **uncertainties** that disrupt the ideal performance, thus the strategies would change corresponding to these changes.

5.1 Modeling Uncertainty

We model these disruptions using two approaches: continuous efficiency degradation for the Space Elevator and discrete failures for the Rocket Fleet.

5.1.1 Space Elevator Risk Assessment Model

This model evaluates the Space Elevator's resilience under non-perfect conditions, categorizing risks into operational efficiency losses and probabilistic structural failures. Each approaches can be considered in **Economic loss** and **Time loss**.

Operational Efficiency Loss Even without catastrophic failure, factors such as Coriolis-induced swaying and debris avoidance maneuvers limit continuous peak capacity. Following ISEC 2019 Report [9], we apply an operational efficiency coefficient:

$$m_{actual}(t) = m_{ideal}(t) \times \eta_{ops} \quad (23)$$

Where $\eta_{ops} = 0.85$ considering the routine maintenance and safety adjustments [9].

Structural Failure Case 1: Climber Breakdown The climber is the primary active component subject to mechanical wear. According to the *NASA NIAC Phase II Report* [10], failure rates follow a time-dependent decay as technology matures:

- **Frequency:** According to Edwards [10], the frequency is modeled as:

$$F_{fail}(t) = 15 \cdot e^{-0.15(t-2050)} + 3 \quad (24)$$

Where the constant 3 represents the asymptotic baseline failure rate derived from industrial high-voltage motor standards [10].

- **Economic Loss ($Cost_I$):** \$60 Million per incident. This is based on a production climber cost of \$50M, adjusted for inflation and rescue operation overhead.
- **Time Loss(T_{fix}):** 0.04 Years. To intercept a failed unit at GEO altitude, a rescue climber requires approximately 7.5 days for ascent, diagnosis and tow[3].

Structural Failure Case 2: Tether Severance A catastrophic severance caused by untracked space debris is the most critical risk.

- **Probability:** Based on *Obayashi Corporation* [2], the probability is modeled as:

$$P_{sever}(t) = 0.005 \cdot e^{-0.05(t-2050)} + 0.0002 \quad (25)$$

This assumes a 0.5% initial risk, mitigated over time by Active Damping and Laser Broom technology. The residual 0.02% risk is unavoidable. [2]

- **Economic Loss ($Cost_{II}$):** \$25 Billion per tether. This represents 25% of the total \$100B CAPEX, as materials and lower-segment reinforcement account for approximately 60% of the budget (*Obayashi, 2014*) [2].
- **Time Loss(T_{down}):** 2.5 Years. While a full build takes 18 years, the existence of an established Apex Anchor allows for *Rapid Deployment* protocols (*ISEC, 2019*) [9].

5.1.2 Rocket Risk Assessment Model

The rocket transportation operates as a series of discrete launch events. We model risks using a **Discrete Stochastic Process**, where each launch is an independent Bernoulli trial.

Failure Probability We assume the failure probability per launch in year t , denoted as p_t , follows a discrete learning curve dependent on global launch experience:

$$p_t = p_{min} + (p_{init} - p_{min}) \cdot e^{-\alpha(t-2050)} \quad (26)$$

Where $p_{init} = 0.01$ (initial 1% risk), $p_{min} = 0.001$ (asymptotic 0.1% floor), and $\alpha = 0.1$.

Since each launch is an independent event following a Bernoulli distribution with parameter p_t , the total number of failures within a year with n launches follows a **Binomial distribution** $B(n, p_t)$. Consequently, the expected number of failures in given year:

$$E[F_t] = n_t \cdot p_t \quad (27)$$

Economic Loss Since our transportation model relies on **Reusable Launch Vehicles**, the economic calculation of a failure differs fundamentally from that of expendable rockets. A reusable rocket is a **Capital Asset**. A catastrophic failure results in the write-off of the vehicle's **Residual Value**.

We model the cost of a single failure event (C_{fail}) as:

$$C_{fail} = C_{payload} + C_{resi} \quad (28)$$

Assuming an average payload value of \$2M/ton for high-tech lunar modules and utilizing the payload capacity $C_{rocket,t}$ derived in Section 3.2.3, plus cleanup costs:

$$C_{payload} \approx \$2 \text{ Million} \cdot C_{rocket,t}$$

To estimate the asset loss, we strictly adhere to the fleet lifecycle parameters established in **Section 3.2.4**. Specifically, we use the derived manufacturing cost $Cost_{build} = \$1.35$ Billion and the reuse limit $N_{reuse} = 80$.

Assuming failures are uniformly distributed across the fleet's operational lifespan (from the 1st flight to the 80th), the expected asset loss is the average depreciated value of it:

$$C_{resi} = Cost_{build} \cdot \underbrace{\left[\frac{1}{N_{reuse}} \sum_{k=1}^{N_{reuse}} \left(1 - \frac{k}{N_{reuse}} \right) \right]}_{\text{Depreciation Factor} \approx 0.5} \approx \frac{1}{2} Cost_{build} \approx \$675 \text{ Million} \quad (29)$$

$$C_{fail} = \$350\text{M} + \$675\text{M} = \$\mathbf{1.025} \text{ Billion} \quad (30)$$

The total expected economic risk added to the project budget is calculated as the product of the expected total number of failures and the cost per failure:

$$C_{risk, total} = E[F_t] \cdot C_{fail} \quad (31)$$

Time Loss Each failure introduces a time delay due to accident investigation and pad repairs. However, given the distributed infrastructure defined in Table 2 ($N_{sites} = 10$), the system exhibits high resilience. We apply a penalty time $\tau_{penalty} \approx 0.02$ years (1 week) per failure to the global timeline:

$$\Delta T_{risk} = E[F_t] \cdot \tau_{penalty} \quad (32)$$

5.2 Impact on Logistics Performance

To quantify the deviations caused by the stochastic risks defined in Section 5.1, we performed a **Monte Carlo Simulation** ($N = 10,000$). The imperfections shift the Pareto frontier significantly towards higher costs and longer durations, and here is the result:

We observe that even when all systems operate at maximum capacity, the project requires 49 years to complete. Restricting the strategy to $[1, 0]^T$ (Elevator Only) or $[0, 1]^T$ (Rocket Only) extends the duration by 4 years and 7 years, respectively.

However, time is not the only variable affected. While the Space Elevator is the backbone of high-volume transport, its vulnerability to catastrophic structural failures introduces **massive cost dispersion** on the vertical axis. The extreme variance in the budget is caused by elevator downtime and repair cost.

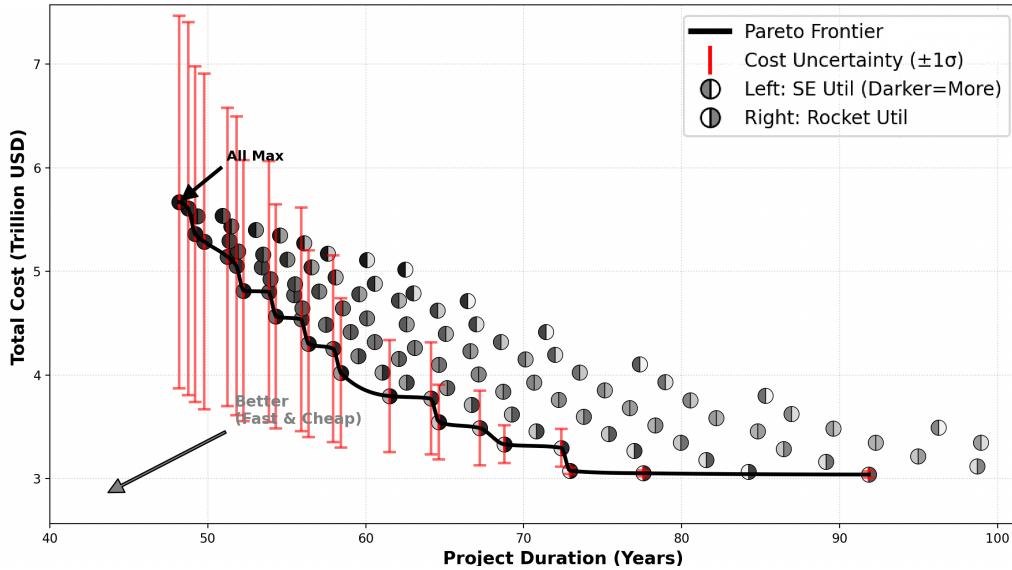


Figure 7: The Time-Cost Pareto Frontier with Stochastic Disruptions. The curve illustrates the trade-off between project duration and total cost under uncertainty concerns.

5.3 Strategy Adaptation and Recommendations

The stochastic Pareto frontier in Fig. 7 reveals a critical trade-off absent from the ideal analysis: **Speed vs. Financial Stability**. The red error bars demonstrate that cost uncertainty is heavily concentrated in strategies with high elevator utilization.

Risk-Speed Trade-off: Strategies approaching the **All Max** configuration ($[1, 1]^\top$) in the upper-left region offer the shortest project duration (≈ 48 years) but suffer from **extreme financial instability**. The massive error bars indicate that while this is theoretically the fastest route, the high capital risk of the Space Elevator (e.g., tether severance) could catastrophically inflate the budget.

Conversely, as the strategy shifts toward the right (Rocket-dominant, $[0, 1]^\top$), the project duration extends, but the error bars shrink dramatically. This confirms that the Rocket Fleet, despite its lower individual capacity, offers a far more **predictable and robust** financial outlook, as the economic impact of individual rocket failures is marginal compared to a space elevator collapse.

Strategic Recommendation: For a mission of this scale, we recommend a **Risk-Averse Hybrid Strategy**. Instead of pursuing the absolute minimum timeline at $[1, 1]^\top$, the Moon Colony Management should prioritize strategies with moderate elevator reliance. By slightly reducing the elevator's intensity (e.g., $[0.4, 1]^\top$). The colony can achieve a completion time of approximately **65-70 years** while cutting the potential budget volatility, ensuring the project remains financially viable even under adverse conditions.

6 Task 3: Water Transportation Model and Analysis

6.1 Water Resource Requirement Estimation Model

To ensure the long-term sustainability of a 100,000-person colony, we model the water demand as a two-phase mass-balance framework. This system maintains a high-volume

internal circulation with minimal external dependence, as illustrated in Fig. 8.

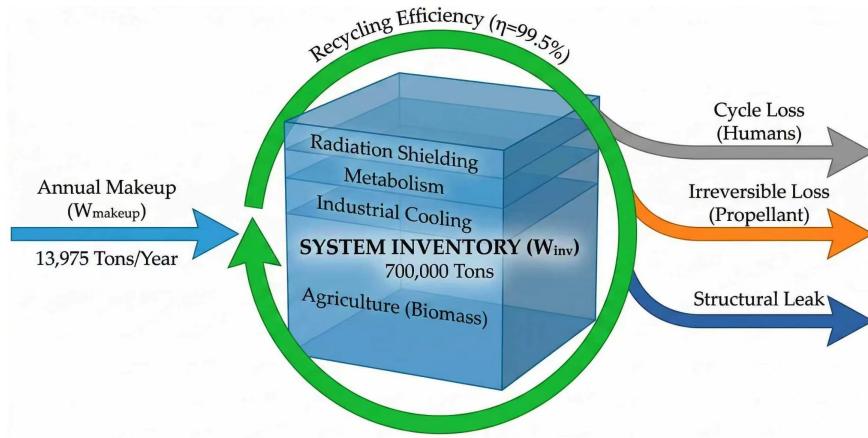


Figure 8: Water Balance Model: High Inventory Inertia with Minimal Annual Makeup. The diagram illustrates the relationship between the one-time system inventory (W_{inv}) and the continuous annual makeup (W_{makeup}) required to offset cycle and structural losses.

The water requirement is categorized into a high-volume initial stockpile for system initialization and a steady-state annual makeup to compensate for operational losses:

$$W_{total}(t) = \begin{cases} W_{inv}, & t = 2050 \text{ (Initial Year)} \\ W_{makeup}, & t > 2050 \text{ (Steady State)} \end{cases} \quad (33)$$

6.1.1 Initial Year: Inventory Stockpile Model (W_{inv})

The initial inventory represents the one-time water volume required to fill the life support cycles, agricultural biomass, landscape maintenance and radiation shielding reservoirs.

$$W_{inv} = N \cdot (\omega_{met} + \omega_{agri} + \omega_{ind} + \omega_{rad}) \quad (34)$$

Based on NASA and ISEC standards [11][25][26], we estimate: $\omega_{met} = 0.5 \text{ t/person}$ (Physiological cycle), $\omega_{agri} = 5.0 \text{ t/person}$ (Self-sustaining biomass), $\omega_{ind} = 1.0 \text{ t/person}$ (Cooling loops), $\omega_{rad} = 0.5 \text{ t/person}$ (Radiation shielding).

Calculating the total: $W_{inv} = 100,000 \times (0.5 + 5.0 + 1.0 + 0.5) = 700,000 \text{ Metric Tonnes}$.

6.1.2 Subsequent Years: Annual Makeup Model (W_{makeup})

External water must be supplied annually to offset three primary loss types:

$$W_{makeup} = \underbrace{N \cdot [\delta_{use} \cdot 365 \cdot (1 - \eta)]}_{\text{Cycle Loss}} + \underbrace{(W_{inv} \cdot \sigma_{leak})}_{\text{Structural Leak}} + \underbrace{\gamma_{prop}}_{\text{Irreversible Loss}} \quad (35)$$

Where $\delta_{use} = 0.03 \text{ t/day per capita}$ [11], $\eta = 99.5\%$ recycling efficiency (NASA ISS Update, 2021) [27], $\sigma_{leak} = 0.5\%/\text{year}$ leakage (ISEC, 2021) [1], and $\gamma_{prop} = 5,000 \text{ t/year}$ for chemical processing (ISEC Interplanetary, 2020) [28].

According to the model above, based on ISS Environmental Control and Life Support System (ECLSS) benchmarks and the projected industrial scale of the lunar colony, the total annual water makeup is estimated to be $W_{makeup} \approx 13,975 \text{ Metric Tonnes/Year}$.

6.2 Additional Costs and Timeline

We divide the water logistics into two distinct phases with the balance of financial, temporal and environmental costs: the **Initialization Phase** (Year 2100, with assumption of 50-year Moon Colony building) and the **Steady-State Phase** (After Year 2100).

6.2.1 Phase I: Initial Stockpile Deployment

The delivery of the initial inventory ($W_{inv} = 700,000$ tons) is a time-critical mission considering the significance of water resources. To minimize the time cost, we mirror the payload models and the **Hybrid Logistics Ratio** derived in Task 1, utilizing both the Space Elevator and the Rocket Fleet simultaneously.

- **Timeline:** With combined capacity, the 700,000-ton stockpile can be delivered in:

$$T_{init} = \frac{W_{inv}}{L_{GH,2100} + m_{Rocket}^{surge}} = \frac{700,000}{1,200,000 + 550,000} = 0.4 \text{ years} \quad (36)$$

- **Cost:** Adopting the hybrid model, the one-time initialization cost is:

$$C_{init} = W_{inv} \times \bar{C}_{mix} = 700,000 \times 22,500 = \$15.75 \text{ Billion} \quad (37)$$

6.2.2 Phase II: Steady-State Maintenance

Subsequent annual demand stabilizes at $W_{makeup} = 13,975$ tons, utilizing only **1.2%** of the Space Elevator's saturated capacity. Therefore, the space elevator alone can deliver the full annual quota (about **4 days**) with low financial, temporal and environmental costs.

Annual Recurring Cost:

$$C_{annual} = W_{makeup} \times \bar{C}_{SE}^{unit} = 13,975 \times 19,768 \approx \$276.25 \text{ Million/Year} \quad (38)$$

The Summary of Water Logistics Strategy is shown below.

Table 6: Summary of Water Logistics Strategy

Phase	Mass (tons)	Mode	Timeline	Est. Cost (USD)
Initialization	700,000	Hybrid (SE + Rocket)	146 Days	\$15.75 B
Maintenance	13,975/year	Space Elevator Only	4 Days/year	\$276.25 M / year

7 Task 4: Environmental Impact Analysis

While Tasks 1 and 2 optimized for time efficiency and financial expenditure, the transport of nearly 10^8 tons of payload poses **ecological risks** that cannot be ignored at scale. Rather than treating environmental limits as hard constraints, this section models them as a **quantifiable third objective**, capturing both direct emissions and non-linear atmospheric damage.

7.1 Tri-Objective Optimization Modeling

We define the optimization problem with three distinct objectives: Project Duration (T), Financial Cost (C_{fin}), and **Environmental Intensity** (E_{total}), then our goal is to:

$$\min \mathbf{J}(\mathbf{x}) = [T(\mathbf{x}), C_{fin}(\mathbf{x}), E_{total}(\mathbf{x})]^\top \quad (39)$$

7.1.1 Environmental Intensity Definition (E_{total})

E_{total} represents the total **carbon-equivalent** emissions per unit of payload transported [kg CO_{2e}/kg payload]. It accounts for non-linear atmospheric damage via a penalty factor:

$$E_{total} = \frac{\sum_t (m_{SE,t} J_{SE} + m_{Rocket,t} J_{Rocket}) \cdot \Omega(N_t)}{M_{cons}} \quad (40)$$

Space Elevator Unit Impact (J_{SE}): Derived from the terrestrial power grid intensity required to overcome Earth's gravitational potential:

$$J_{SE} = E_{net} \cdot I_{grid}(2050) \approx 1.22 \text{ kg CO}_2/\text{kg}$$

where $E_{net} \approx 14.12 \text{ kWh/kg}$ and $I_{grid} \approx 0.086 \text{ kg/kWh}$.

Rocket Fleet Unit Impact (J_{Rocket}): Calculated using the **Stratospheric Forcing Index (SFI)** for Methalox propulsion.

$$J_{Rocket} = \frac{M_{prop}}{P_{cap}} \cdot \sum (\epsilon_i \cdot GWP_i^{strat}) \approx 98.04 \text{ kg CO}_{2e}/\text{kg}$$

where $M_{prop}/P_{cap} \approx 30$.

Ozone Depletion Penalty Factor ($\Omega(N_t)$): A non-linear amplification factor representing the *tipping point* of stratospheric ozone stability:

$$\Omega(N_t) = \begin{cases} 1 & N_t \leq N_{crit} \\ 1 + \alpha_p \left(\frac{N_t}{N_{crit}} - 1 \right)^2 & N_t > N_{crit} \end{cases}$$

where N_t is the launch frequency and $\alpha_p = 10.0$ is the quadratic damage coefficient.

7.1.2 Environmental Trade-off

Figure 9 presents the results as a **Bubble-Glyph Pareto Plot**. The **bubble size** corresponds to the physical intensity E_{total} .

The landscape illustrates that while the *All Max* strategy ($[1, 1]^\top$) is time-optimal, it is **physically inefficient** from an environmental standpoint, emitting nearly 45 times more pollutants per kilogram than elevator-dominant strategies.

7.2 Unified Cost Analysis

Tri-objective optimization is usually much harder than bi-objective optimization, as it introduces trade-offs that are difficult to reconcile without a common valuation framework. To provide a practical budgetary recommendation, we internalize these physical externalities using the **EPA 2023 Social Cost of Carbon (SCC)** standard of **\$190/ton CO₂**.

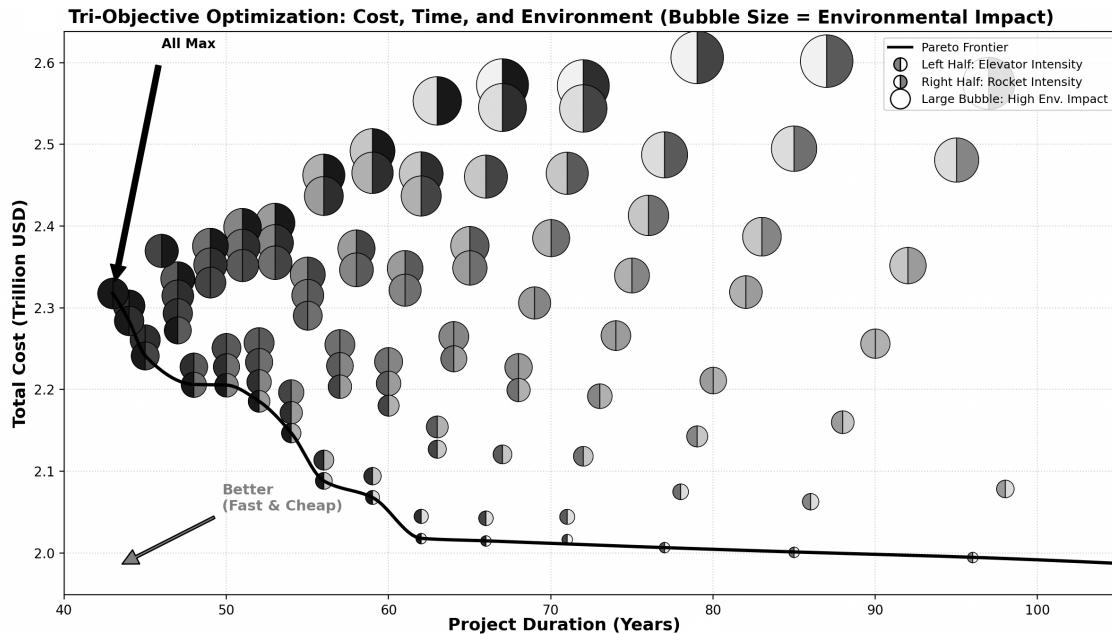


Figure 9: **Physical Environmental Impact Landscape.** The bubble size represents kg CO₂ emitted per kg of payload. Strategies maximizing rocket utilization ($\lambda \rightarrow 1$) result in exploded bubbles due to high stratospheric injection and ozone penalties.

7.2.1 Unified Objective Function

We assume a **Polluter Pays** policy, where the environmental shadow cost (C_{env}) is added to the financial budget to form the **Unified Total Cost** ($C_{unified}$), then we can simply the optimization task into:

$$C_{unified} = C_{fin} + \underbrace{(E_{total} \cdot M_{target} \cdot SCC)}_{C_{env}}, \quad \min J'(\mathbf{x}) = [T(\mathbf{x}), C_{unified}(\mathbf{x})]^\top \quad (41)$$

7.2.2 Frontier Comparison

Figure 10 compares the Pareto frontiers with new unified cost.

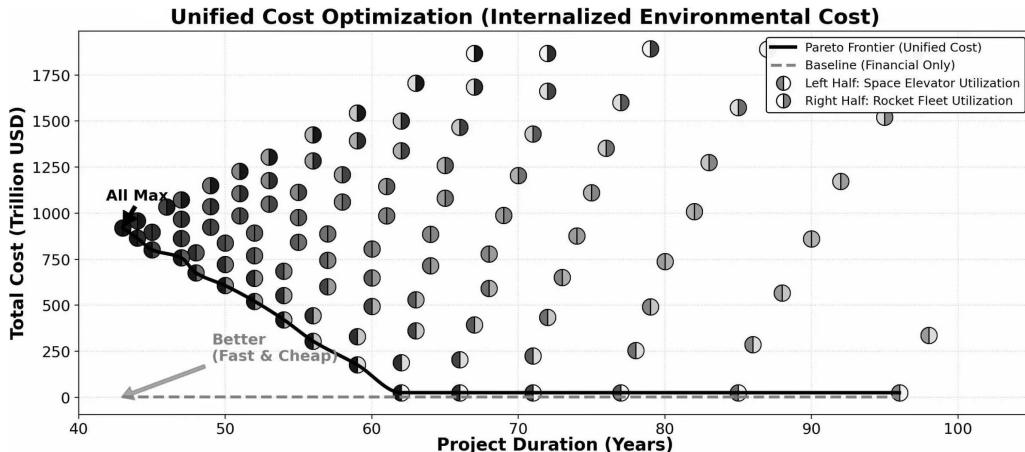


Figure 10: **Unified Pareto Frontier.** Internalizing environmental costs creates a Cost Wall in the fast-completion region, making high-intensity rocket strategies economically unviable.

7.3 Strategy Recommendation

The unified cost analysis reveals that the environmental-equivalent cost associated with rocket transportation dominates all other considerations once externalities are internalized. As shown in Fig. 10, the **Pareto knee** collapses to the boundary solution $\mathbf{x}^* = [1, 0]^\top$, indicating that any non-negligible reliance on rockets becomes economically prohibitive under environmental pricing.

This result implies that, when environmental impacts are properly accounted for, rocket-based transport is no longer a viable component of the optimal strategy, and a space-elevator-dominant architecture emerges as the only sustainable solution. Notably, this conclusion stands in sharp contrast to the time- and cost-optimal strategies identified in earlier tasks, highlighting how the internalization of environmental externalities fundamentally reshapes the decision landscape.

8 Sensitivity Analysis

To ensure robustness of recommendations, we performed a multi-layered sensitivity analysis covering factor contribution, parameter interaction and probabilistic budgeting.

Factor Drivers and Interaction Effects: As illustrated in Figure 11, **Rocket Payload (27.7%)** and **Reliability (23.5%)** dominate the cost variance, significantly outweighing Environmental Policy (22.5%) and Fuel Price (4.1%). This confirms that technical robustness is the primary lever for cost control. The interaction analysis (Figure 11) further reveals a critical *Danger Zone*: when Rocket Reusability is low (< 20 flights), costs escalate exponentially beyond **\$8.0 Trillion**. However, once reusability exceeds 60 flights and the Space Elevator reaches baseline capacity, costs stabilize in the **\$2.0–2.5 Trillion** plateau.

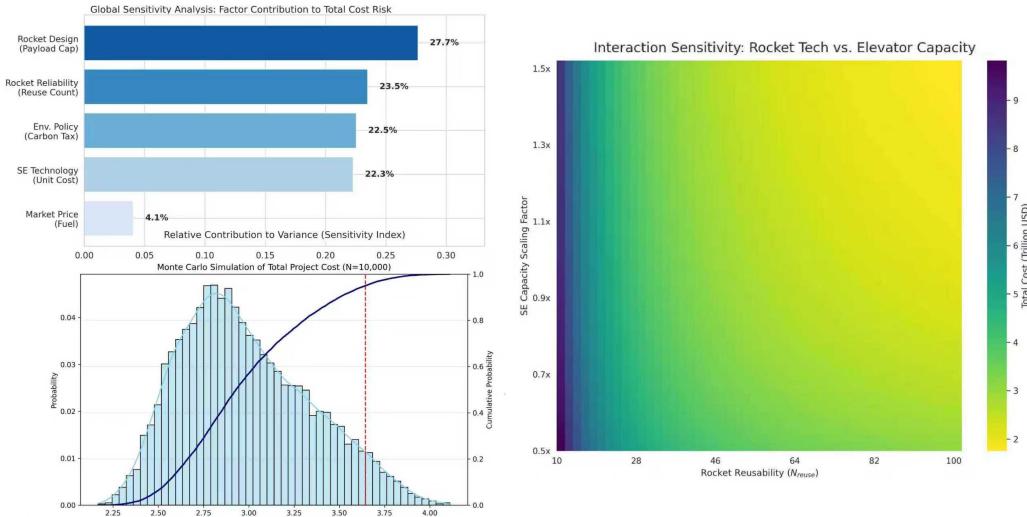


Figure 11: **Sensitivity Structure and Interaction Effects.** (a) Rocket payload and reliability dominate global cost sensitivity. (b) Interaction between reusability and elevator capacity reveals a Danger Zone (< 20 flights) (c) Probabilistic Cost Distribution via Monte Carlo Simulation.

Probabilistic Risk Assessment: To account for real-world chaos, the Monte Carlo simulation (Figure 11) transforms our deterministic estimate into a risk profile. Due to the asymmetric risk of technical failures, the mean expected cost shifts from **\$2.21 Trillion** to **\$2.90 Trillion**. The cumulative probability curve crosses the 95% confidence threshold at **\$3.65 Trillion**.

Trillion. Consequently, we advise the MCM Agency to allocate a **Risk Reserve of 65%** above the baseline to guarantee project completion with 95% certainty.

9 Strengths and Weaknesses

Strengths: Our framework features **Physics-Informed Modeling**, grounding economic growth in the *Square-Cube Law* to prevent unrealistic projections. By utilizing **Tri-Objective Pareto Optimization**, we provide a spectrum of strategic trade-offs across time, cost, and environment. Furthermore, the integration of **Monte Carlo simulations** ensures **Stochastic Robustness** by quantifying risk premiums for high-impact events like tether failures.

Weaknesses: Conversely, the **Continuous Approximation** of discrete launches may introduce minor scheduling inaccuracies during early project phases. The model also exhibits **Parameter Sensitivity** due to its reliance on long-term 2050 technological forecasts. Finally, **Payload Homogeneity**—treating all cargo as a single mass—neglects specialized requirements like G-force or radiation shielding, which may underestimate specialized transport costs.

10 Conclusion and Future Work

Conclusion This study provides a comprehensive logistical roadmap for the transportation of **100 million metric tons** of infrastructure to the Moon. By developing the **BO-NLP optimization framework**, we have demonstrated that the most viable path is a **Sustainable Hybrid Strategy** ($\delta_{SE} = 1.0, \lambda = 0.2$).

Our findings indicate that this strategy completes the construction in **56 years** at a minimized cost of **\$2.08 trillion**. While the Space Elevator serves as the primary cost-efficient logistical backbone (\$19,768/ton), the Rocket Fleet acts as a targeted supplement during periods of peak transportation intensity. However, our **Environmental Shadow Cost** analysis reveals that rocket intensity must be maintained to be at or below 0.2 to avoid breaching the **Ozone Wall**.

Regarding long-term sustainability, our two-phase hydrological model identifies initial water requirement(approximately **700,000-ton**) to be delivered via a high-intensity hybrid surge (0.4 years). Once initialized, the **13,975-ton** annual makeup demand remains negligible, requiring only **4 days** of Space Elevator Only Logistics per year to maintain permanent resource security for the 100,000 residents.

Finally, **Monte Carlo simulations** highlight a critical trade-off: the Space Elevator is quite financially volatile. To ensure a 95% mission success rate against stochastic failures like tether severance, we recommend a risk-adjusted budget of **\$3.65 trillion**.

Future Work A critical future extension is the transition from a constant-dollar model to a **Dynamic Financial Framework** that accounts for the time-value of money. That is: Inflation and Macroeconomic Integration.

Over a long term horizon, cumulative inflation significantly erodes purchasing power. Future iterations should incorporate a *Discounted Cash Flow* model to reflect the real-world cost of capital. This integration will allow for more precise long-term funding schedules to achieve a more accurate logistical planning strategy.

Strategic Recommendation Letter

Subject: Logistics Strategy for the 100,000-Resident Moon Colony

To: Moon Colony Management (MCM) Agency Strategic Planning Committee
From: Team 2616836
Date: February 3, 2026

To the Directors of the MCM Agency:

In order to transport **100 million metric tons** of material for the establishment of a lunar habitat for **100,000 residents**, our team has constructed a **Multi-Objective Non-Linear Programming (MO-NLP)** model to conduct a comprehensive optimization assessment of the Space Elevators and Rocket Fleets. Our analysis concludes that a hybrid logistics strategy with an intensity configuration of $\delta_{SE} = 1.0, \lambda = 0.2$ is the optimal solution when considering financial, temporal, environmental costs, and stochastic disruptions. Specific recommendations are as follows:

Optimal Logistical Balance: Our research indicates that to ensure the project timeline remains within strategic windows while minimizing financial and environmental costs, the Agency should utilize the full capacity of the Space Elevator supplemented by a moderate rocket fleet intensity (**20% to 50%**). Under this framework, the construction can be completed in approximately **56 years**. This strategy represents the **Sustainable Knee Point** on the Pareto frontier, yielding an exceptionally high mass-to-cost efficiency.

Financial Risk Mitigation: Monte Carlo simulations demonstrate that the potential risk of tether severance introduces a significant financial risk premium. We advise the Agency to prepare a **65% Financial Risk Reserve** (resulting in a total projected budget of approximately **\$3.65 trillion**). This allocation is necessary to guarantee mission completion with a **95% confidence level** in the face of stochastic structural failures.

Water Resource Security: Our water resource model estimates that a **700,000-ton initial water stockpile** must be delivered during the final phase of construction. Given the massive demand and tight schedule, the Agency should activate the full capacity of both the rocket fleet and the Space Elevator to complete this task within **146 days**. During the maintenance phase, the annual makeup requirement of **13,975 tons** is minimal and can be fulfilled by the Space Elevator alone in just **4 days**, ensuring long-term hydrological security for the colony.

Environmental Threshold Management: Our environmental impact model identifies a critical threshold of **2,500 annual launches** as the **Ozone Wall**, beyond which ozone layer damage becomes catastrophic. By maintaining rocket intensity at or below **0.2**, the project remains safely below this threshold. Compared to high-intensity launch scenarios, this strategy reduces stratospheric black carbon radiative forcing and ozone depletion costs by **more than 85%**.

In conclusion, the Space Elevator should serve as the core logistical carrier, while the rocket fleet acts as a targeted supplement during periods of peak transportation intensity. The hybrid strategy of $\delta_{SE} = 1.0, \lambda = 0.2$ stands out as the optimal choice in our comprehensive evaluation, achieving a vital balance between aggressive lunar expansion and robust ecological preservation of Earth.

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Report on Use of AI Tools

1. Google Gemini 3 Pro Preview (Nov, 2025 version)

- **Date of Use:** January 30, 2026.
- **Purpose:** Investigation of Space Elevator payload capacities and operational cost models. We utilized the model to retrieve empirical data and engineering parameters from recent ISEC (International Space Elevator Consortium) reports to calibrate our transportation throughput and OPEX functions.
- **Query:** "I am conducting research on Space Elevator payload and costs. The initial payload in 2050 is fixed at 179,000 tons/year. As technology develops, the capacity should increase rapidly at first but eventually slow down towards a saturation threshold. For operational costs, I estimate components including electricity, maintenance, and fuel for the second-half transfer. Please complete this research: model precisely, quantify reasonably, provide functions and parameter estimates with justifications, and ensure one-to-one reference support. Be concise and clear."
- **Output: 2050 Space Elevator Techno-Economic Summary**

1. Payload Capacity Model The system starts at a baseline of 179,000 tons/year (approx. 5 departures/day on a dual-tether system). Growth is modeled via a Logistic Function: $P(t) = \frac{1,500,000}{1+7.38 \cdot e^{-0.12(t-2050)}}$ - Saturation (K): 1.5 million tons/year, driven by SBSP and Mars colonization demand. - Growth Rate (r): 0.12, reflecting high-speed infrastructure maturation. - Projections: 466k tons by 2060; 1.25M tons by 2080.

2. Operational Expenditure (OPEX) Breakdown The total cost function is: $C_{total}(P) = \$5 \times 10^9 + P \cdot \12.33

- **Electrical Energy (\$2.15/kg):** Based on a 25% *Grid-to-Wheel* efficiency (Laser Beaming) and 53.76 kWh/kg specific energy. Electricity price estimated at \$0.04/kWh.
- **Maintenance (\$28.04/kg at IOC):** Dominated by a \$5 billion/year fixed cost (2.5% of \$200B CAPEX) for debris avoidance, tether repair, and climber MRO.
- **Second-Half Transfer (\$10.08/kg):** Logistics of moving cargo from the Apex Anchor to destinations using Argon-fed Hall Effect Thrusters. This includes \$0.08 fuel cost and \$10.00 tug amortization.

3. Economic Findings - Total Unit Cost (2050): Approximately \$40.27/kg. - **Economy of Scale:** As throughput reaches saturation (2080), unit costs drop to \$16.23/kg as fixed maintenance is diluted. - **Competitive Advantage:** The Space Elevator provides a 10x-20x cost reduction compared to 2050-era reusable rockets (Starship class) for deep-space missions, effectively transitioning space access from "exploration" to "industrial logistics."

4. Strategic Implications The model confirms that the "Second-Half" transfer (orbital tugs) becomes the long-term price floor. Future optimization should focus on Apex-based electromagnetic mass drivers to further reduce costs below \$10/kg.

2. Google Gemini 3 Pro Preview (Nov, 2025 version)

- **Date of Use:** January 30, 2026.

- **Purpose:** Researching risk classifications and reliability metrics for Space Elevator systems. The AI was used to estimate financial losses and temporal delays associated with different failure modes (e.g., climber breakdown vs. tether severance) for our stochastic risk model.
- **Query:** "I am expanding the risk assessment model for the Space Elevator system post-2050. Specifically, please investigate: 1. Major failure modes (e.g., climber mechanical failure, tether severance due to debris). 2. Corresponding failure rates (as constants or time-dependent functions starting from 2050). 3. Quantified material losses and repair costs per incident. 4. Estimated recovery time (downtime) before operations can resume. Please provide precise modeling with functions, reasonable parameter estimations with justifications, and one-to-one bibliographic support from sources like ISEC, NASA NIAC, or Obayashi reports. The response should be concise, clear, and mathematically rigorous."
- **Output: 2050 Space Elevator Reliability and Risk Failure Analysis**

1. Failure Modes and Stochastic Rates

- **Debris-Induced Severance:** The annual impact rate N_{coll} is modeled by $N_{coll} = \int Flux(h, d) \cdot W(h)dh$. In the LEO kill-zone (800km), a 1m ribbon tether sustains ≈ 250 impacts/year from 1cm+ debris. Residual severance risk (from 5-10cm objects) is estimated at $\lambda_{sev} \approx 0.001/year$ using active avoidance and ribbon redundancy.
- **Climber Mechanical Stall:** Using a constant failure rate $\lambda = 1/MTBF$, a 100-climber fleet with a 5,000-hour MTBF will experience ≈ 175 failures/year. "Hard stalls" requiring external rescue occur ≈ 1.7 times/year.

2. Material Degradation and Sustainment

- **Environmental Erosion:** Atomic Oxygen (AO) in LEO and thermal fatigue (limit: $\approx 1,500$ cycles) necessitate protective coatings.
- **Annual Maintenance:** Estimated at 0.1% mass replacement per year (7,000 kg). Material cost is nominal ($\approx \$3.5M/year$), but logistics and inspection dominate the OPEX.

3. Recovery Kinetics and Downtime

- **Climber Rescue:** 2 to 14 days depending on altitude (LEO vs. GEO).
- **Tether Splicing:** Routine repair cycles take ≈ 21 days for full-length traversal.
- **Catastrophic Recovery:** A tether severance requires 1 to 3 years for reconstruction, assuming the Apex Anchor remains intact.

4. Financial Risk Quantification

The annualized risk cost is defined as $C_{risk} = (P_{sev} \cdot C_{recon}) + (N_{fail} \cdot C_{rescue}) + C_{opp}$. For the 2050 baseline:

- Reconstruction Cost (C_{recon}): $\approx \$100$ Billion.
- Annualized Risk Exposure: $\approx \$300M - \$500M$.
- Strategic Recommendation: A Multi-Tether (Redundant) architecture is the only path to 95%+ operational availability.

3. Google Gemini 3 Pro Preview (Nov, 2025 version)

- **Date of Use:** January 31, 2026.

- **Purpose:** Quantifying hydrological requirements for a 100,000-resident lunar colony. We used AI to synthesize data on initial water inventory, closed-loop recycling efficiencies (ECLSS), and annual makeup demand to establish the sustainability logistics model.
- **Query:** "I am conducting a hydrological logistics study for a 100,000-person lunar colony starting in year 2100 (post-construction). I require a two-phase mass-balance model: 1. **Initial Stockpile (Phase I):** Estimate the one-time massive water transport required for industrial initialization, life support priming, radiation shielding reservoirs, agricultural biomass (landscaping), and power plant cooling loops. 2. **Steady-State Maintenance (Phase II):** Model the annual water makeup demand (W_{makeup}) considering high-efficiency closed-loop recycling (ECLSS) and potential In-Situ Resource Utilization (ISRU) offsets from lunar ice. Please provide precise mathematical functions, reasonable parameter estimations with justifications based on NASA or ISEC benchmarks, and one-to-one bibliographic support. The response should be concise, clear, and logically consistent with a 2100 timeline."
- **Output: Hydrological Logistics Architecture for a Centenary Lunar Settlement (Year 2100)**
 1. **Phase I: Initial Stockpile (Static Inventory)** The total initial water inventory W_{TI} required to initialize a city of 100,000 is modeled as: $W_{TI} = W_{shield} + W_{ag_init} + W_{sys_buffer}$
 - **Radiation Shielding (W_{shield}):** A 50 cm "water jacket" integrated into habitat hulls to mitigate GCR/SPE. For a $1.5 \times 10^6 m^2$ surface area, this requires **750,000 MT**.
 - **Agricultural Priming (W_{ag_init}):** 100,000 residents require $5 \times 10^6 m^2$ of hydroponic area. At an average system depth of $20 L/m^2$, the initial charge is **100,000 MT**.
 - **Municipal/Thermal Buffer:** A 30-day survival reserve and district cooling loop inventory totaling **20,000 MT**.
 - **Total Initialization Mass:** **880,000 MT** (Target for pre-settlement robotic ISRU).
 2. **Phase II: Steady-State Maintenance (Dynamic Flow)** The daily net ISRU makeup rate R_{makeup} required to offset system losses and industrial exports: $R_{makeup} = L_{human} + L_{ag} + E_{export}$
 - **Human/ECLSS Losses (L_{human}):** At 35 kg/person/day throughput and 99.5% recovery efficiency, the loss is **17.5 MT/day**.
 - **Agricultural Sequestration (L_{ag}):** Water chemically locked into biomass (food) and system brine waste totals **175.5 MT/day**.
 - **Propellant Export (E_{export}):** Serving as a cislunar fuel depot (LOX/LH_2), the settlement exports **1,500 MT/day** to support Starship-class logistics.
 - **Net Daily Extraction Goal:** $\approx 1,700 \text{ MT/day}$ of lunar ice.
 3. **Infrastructure and Energy Constraints**
 - **Power Demand:** Processing 1,700 MT of water/day (sublimation + electrolysis) requires a continuous **0.55 GW Nuclear Fission Grid**.
 - **Thermodynamics:** Sublimation of permafrost ice requires $\approx 2.5 \text{ kWh/kg}$; Electrolysis requires $\approx 6 \text{ kWh/kg}$ of H_2O .

4. Strategic Resilience The Phase I shielding reservoir (750,000 MT) serves as the ultimate "Black Swan" buffer. In a total mining failure, this reservoir can sustain the population's survival needs for over **10 years**, providing unprecedented mission safety.

4. Google Gemini 3 Pro Preview (Nov, 2025 version)

- **Date of Use:** January 31, 2026.
- **Purpose:** Assessing the environmental footprint and stratospheric impact of transportation modalities. We investigated greenhouse gas (GHG) emission rates, ozone depletion potential (ODP), and geophysical launch frequency limits to define the environmental shadow cost function.
- **Query:** "I am conducting a comprehensive environmental impact assessment for lunar logistics (2050–2100). Please investigate and model the following: 1. **Space Elevator (Phase I):** Quantify the terrestrial impact of electricity consumption based on projected global grid carbon intensities for the mid-21st century. 2. **Space Elevator (Phase II):** Analyze the ecological impact of fuel use in the exosphere (>10,000 km) and justify whether it remains negligible to Earth's biosphere. 3. **Rocket Fleets:** Model the environmental load of heavy-lift vehicles, focusing on stratospheric emissions (Black Carbon, H_2O , CO_2) and their specific contribution to Global Warming Potential (GWP) and Ozone Depletion Potential (ODP). 4. **Unified Metric Modeling:** Develop a method to unify these diverse impacts into a single **'Environmental Shadow Cost'** (C_{env}) based on the financial cost of mitigation measures (e.g., carbon capture, ozone restoration efforts). Please provide precise functions, quantified parameter estimates with justifications, and one-to-one bibliographic support from sources such as IPCC, NOAA, or peer-reviewed aerospace environmental studies. The output should be concise, clear, and mathematically integrated."
- **Output: Comparative Environmental Impact Assessment of Cislunar Logistics (2050–2100)**
 - 1. **Space Elevator: Terrestrial and Exospheric Impact**
 - **Phase I (Terrestrial Electricity):** Total energy demand for 100,000 MT payload is estimated at 5.32 TWh/year (based on 13.3 kWh/kg potential energy and 25% system efficiency). Using a 2050 projected grid intensity of 40 gCO₂e/kWh (IEA EnerBlue), the annual footprint is **212,800 tonnes CO₂e**.
 - **Phase II (Exosphere):** Station-keeping at the Apex Anchor (>100,000 km) utilizes Argon electric propulsion. Impact is **negligible to the biosphere** due to magnetic confinement within the magnetosphere and the chemical inertness of Argon.
 - **Shadow Cost ($C_{env,SE}$):** Calculated at **\$0.82/kg**, primarily driven by terrestrial energy carbon abatement.
 - 2. **Rocket Fleets: Stratospheric Environmental Load** A methalox-based fleet (1,000 launches/year) directly injects combustion byproducts into sensitive atmospheric layers:
 - **Water Vapor (H_2O):** 3.68 Mt/year. Persistent stratospheric H_2O acts as a potent greenhouse gas and feeds the HO_x ozone-depletion cycle.

- **Black Carbon (BC):** 460 tonnes/year. Soot particles have a radiative efficiency 500–1,500x higher than CO₂ and facilitate heterogeneous ozone-depleting chemistry.
- **Ozone Depletion (ODP):** Cumulative chemical erosion from H₂O and BC is projected to cause a 0.5%–4.0% loss in global column ozone, potentially reversing Montreal Protocol gains.

3. Unified Metric Modeling (Environmental Shadow Cost)

To normalize impacts, we define C_{env} based on the Marginal Abatement Cost (MAC) of pollutants:

$$C_{env} = \frac{1}{M_{PL}} \sum (M_{emission,i} \times SC_i)$$

- **Parameters (2050 USD):** $SC_{CO_2} = \$150/t$; $SC_{H_2O} = \$1,500/t$; $SC_{BC} = \$225,000/t$ (GWP-weighted).
- **Rocket Fleet C_{env} :** \$75.00/kg payload. The environmental damage effectively doubles the commercial operating cost.
- **Space Elevator C_{env} :** \$0.82/kg payload.

4. Strategic Conclusion

The Space Elevator offers a **91x reduction** in environmental shadow costs compared to chemical rockets. For megaton-class logistics, the SE functions as a geo-engineering mitigation strategy, decoupling space access from stratospheric degradation and providing the only ecologically sustainable pathway for late 21st-century industrialization.

5. Nano Banana Pro (Nov, 2025 version)

- **Date of Use:** February 2, 2026.
- **Purpose:** Generating a structured data visualization (Sankey Diagram) to compare the operational expenditure (OPEX) breakdown of the Space Elevator System and the Rocket Fleet.
- **Query:** “Overall Style & Layout: A flat, 2D academic data visualization featuring two side-by-side Sankey Diagrams on a pure white background. Global Geometry: Use vertical rectangular bars as nodes and smooth, translucent colored ribbons connecting the bars horizontally (Left-to-Right flow). PANEL A (Left): SPACE ELEVATOR COST STRUCTURE. Input Node: ‘SE Total (\$19.8k)’. Flows: ‘Transfer (\$8.9k)’ (45%), ‘Maintenance (\$8.6k)’ (43%), ‘Energy (\$2.3k)’ (12%). PANEL B (Right): ROCKET FLEET COST STRUCTURE. Input Node: ‘Rocket Total (\$25.7k)’. Flows: ‘Hardware Depr. (\$22k)’ (86%), ‘Fuel (\$3.6k)’ (14%). Style: Geometric, sharp edges, Palatino font.”
- **Output:** Figure 2.