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Numeral Systems

Binary, Decimal and Hexadecimal Numbers

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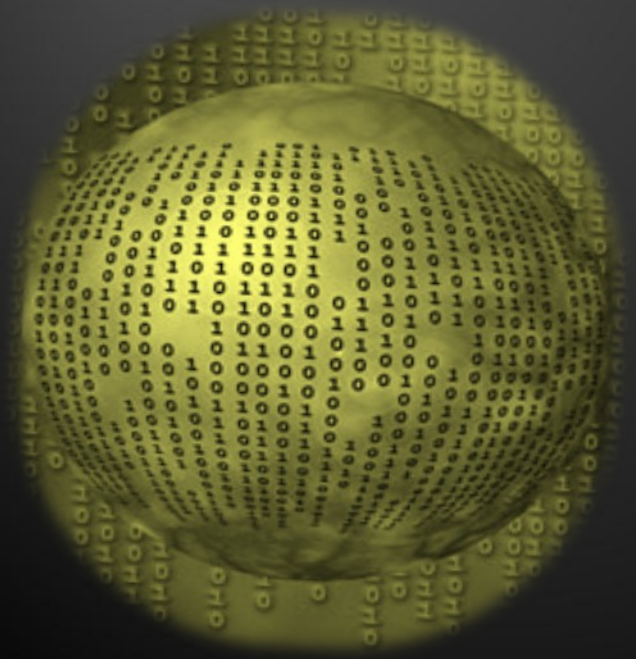
Numeral Systems

Conversion between Numeral Systems



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Decimal Numbers

- Decimal numbers (base 10)
  - Represented using 10 numerals: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
- Each position represents a power of 10:
  - $401 = 4*10^2 + 0*10^1 + 1*10^0 = 400 + 1$
  - $130 = 1*10^2 + 3*10^1 + 0*10^0 = 100 + 30$
  - $9786 = 9*10^3 + 7*10^2 + 8*10^1 + 6*10^0 = 9*1000 + 7*100 + 8*10 + 6*1$

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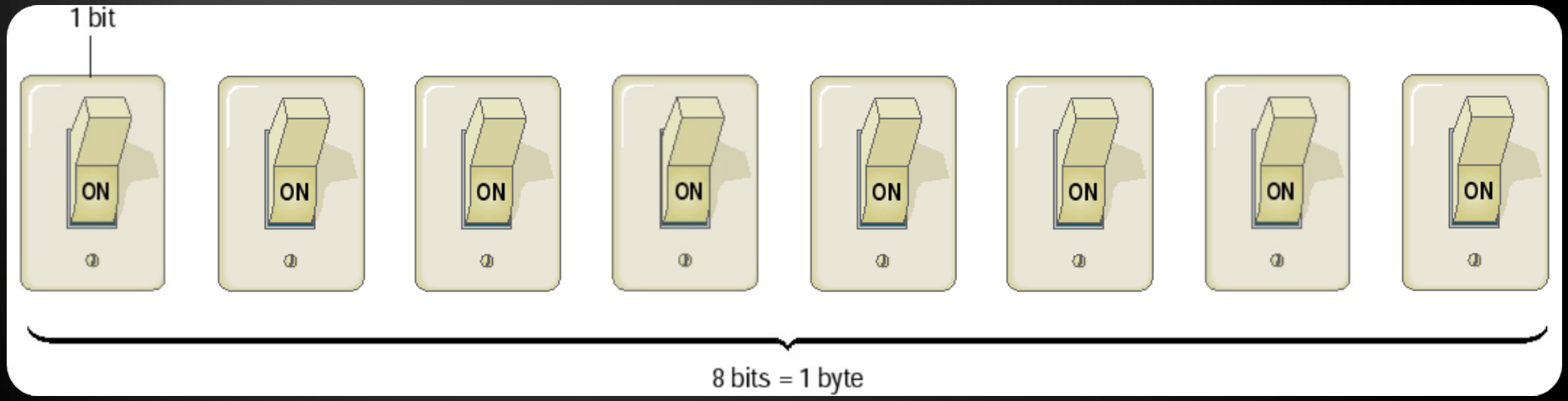
Binary Numeral System

- Binary numbers are represented by sequence of bits (smallest unit of information – 0 or 1)
  - Bits are easy to represent in electronics




```
1   1   1   1   1   1   1   1
```

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Binary Numbers

- Binary numbers (base 2)
  - Represented by 2 numerals: 0 and 1
- Each position represents a power of 2:
  - $101_b = 1*2^2 + 0*2^1 + 1*2^0 = 100_b + 1_b = 4 + 1 = 5$
  - $110_b = 1*2^2 + 1*2^1 + 0*2^0 = 100_b + 10_b = 4 + 2 = 6$
  - $110101_b = 1*2^5 + 1*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 1*2^0 = 32 + 16 + 4 + 1 = 53$

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Binary to Decimal Conversion

- Multiply each numeral by its exponent:
  - $1001_b = 1*2^3 + 1*2^0 = 1*8 + 1*1$
    $=$
    $= 9$
  - $0111_b = 0*2^3 + 1*2^2 + 1*2^1 + 1*2^0$
    $=$
    $= 100_b + 10_b + 1_b = 4 + 2 + 1 =$
    $= 7$
  - $110110_b = 1*2^5 + 1*2^4 + 0*2^3 + 1*2^2 + 1*2^1 =$ $= 100000_b + 10000_b + 100_b + 10_b =$

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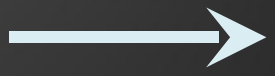
Decimal to Binary Conversion

- Divide by 2 and append the reminders in reversed order:

```
500/2   = 250 (0)
250/2   = 125 (0)
125/2   = 62  (1)
 62/2   = 31  (0)
 31/2   = 15  (1)
 15/2   = 7   (1)
  7/2   = 3   (1)
  3/2   = 1   (1)
  1/2   = 0   (1)
```



$500_d = 111110100_b$

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Hexadecimal Numbers

- Hexadecimal numbers (base 16)
  - Represented using 16 numerals:
    0, 1, 2, … 9, A, B, C, D, E and F
- Usually prefixed with 0x

```
0 → 0x0   8 → 0x8
1 → 0x1   9 → 0x9
2 → 0x2  10 → 0xA
3 → 0x3  11 → 0xB
4 → 0x4  12 → 0xC
5 → 0x5  13 → 0xD
6 → 0x6  14 → 0xE
```

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Hexadecimal Numbers (2)

- Each position represents a power of 16:
  - $9786_{hex} = 9*16^3 + 7*16^2 + 8*16^1 + 6*16^0$
    $=$
    $= 9*4096 + 7*256 + 8*16 + 6*1 =$
    $= 38790$
  - $0xABCDEF_{hex} = 10*16^5 + 11*16^4 + 12*16^3 +$
    $13*16^2 + 14*16^1 + 15*16^0 =$
    $= 11259375$

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Hexadecimal to Decimal Conversion

- Multiply each digit by its exponent
  - $1F4_{hex} = 1*16^2 + 15*16^1 + 4*16^0 =$
    $= 1*256 + 15*16 + 4*1 =$
    $= 500_d$
  - $FF_{hex} = 15*16^1 + 15*16^0 =$
    $= 240 + 15 =$
    $= 255_d$

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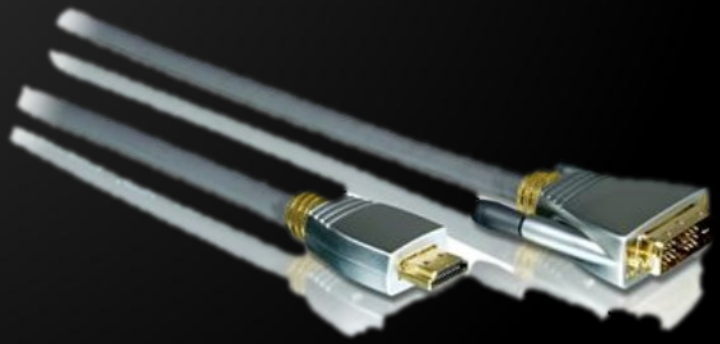
Decimal to Hexadecimal Conversion

- Divide by 16 and append the reminders in reversed order

```
500/16 = 31 (4)

31/16  = 1 (F)  ⟶  500d = 1F4hex

1/16   = 0 (1)
```



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Binary to Hexadecimal (and Back) Conversion

- The conversion from binary to hexadecimal (and back) is straightforward: each hex digit corresponds to a sequence of 4 binary digits:

```
0x0 = 00000x8 = 1000
0x1 = 00010x9 = 1001
0x2 = 00100xA = 1010
0x3 = 00110xB = 1011
0x4 = 01000xC = 1100
0x5 = 01010xD = 1101
0x6 = 01100xE = 1110
0x7 = 01110xF = 1111
```

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Numbers Representation

Positive and Negative Integers and Floating-Point Numbers

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Representation of Integers

- A `short` is represented by 16 bits
  - 100 = $2^6 + 2^5 + 2^2$ =
    = 00000000 01100100
- An `int` is represented by 32 bits
  - 65545 = $2^{16} + 2^3 + 2^0$ =
    = 00000000 00000001 00000000 00001001
- A `char` is represented by 16 bits
  - ‘0’ = 48 = $2^5 + 2^4$ =
    = 00000000 00110000

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Positive and Negative Numbers

- A number's sign is determined by the Most Significant Bit (MSB)
  - Only in signed integers: `sbyte`, `short`, `int`, `long`
  - Leading `0` means positive number
    - Leading `1` means negative number
  - Example: (8 bit numbers)

$0XXXXXXX_b$ > 0 e.g. 00010010_b = 18

00000000_b = 0

$1XXXXXXX_b$ < 0 e.g. 10010010_b = -110

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Positive and Negative Numbers (2)

- The largest positive 8-bit `sbyte` number is:
  $127\ (2^7 - 1) = 01111111_b$
- The smallest negative 8-bit number is:
  $-128\ (-2^7) = 10000000_b$
- The largest positive 32-bit `int` number is:
  $2\ 147\ 483\ 647\ (2^{31} - 1) = 01111\ldots11111_b$
- The smallest negative 32-bit number is:
  $-2\ 147\ 483\ 648\ (-2^{31}) = 10000\ldots00000_b$

Representation of 8-bit Numbers

- Positive 8-bit numbers have the format 0XXXXXXX
  - Their value is the decimal of their last 7 bits (XXXXXXX)
- Negative 8-bit numbers have the format 1YYYYYYY
  - Their value is 128 (2^7) minus (-) the decimal of YYYYYYY
  - $10010010_b = 2^7 - 10010_b =$
    $= 128 - 18 = -110$

```
+127 = 01111111
...
+3   = 00000011
+2   = 00000010
+1   = 00000001
+0   = 00000000
-1   = 11111111
-2   = 11111110
-3   = 11111101
...
-127 = 10000001
-128 = 10000000
```

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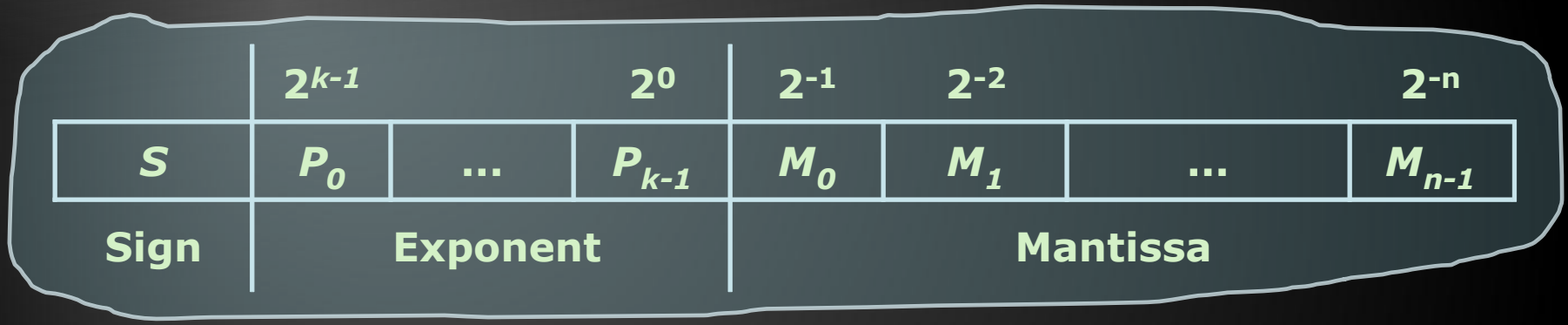
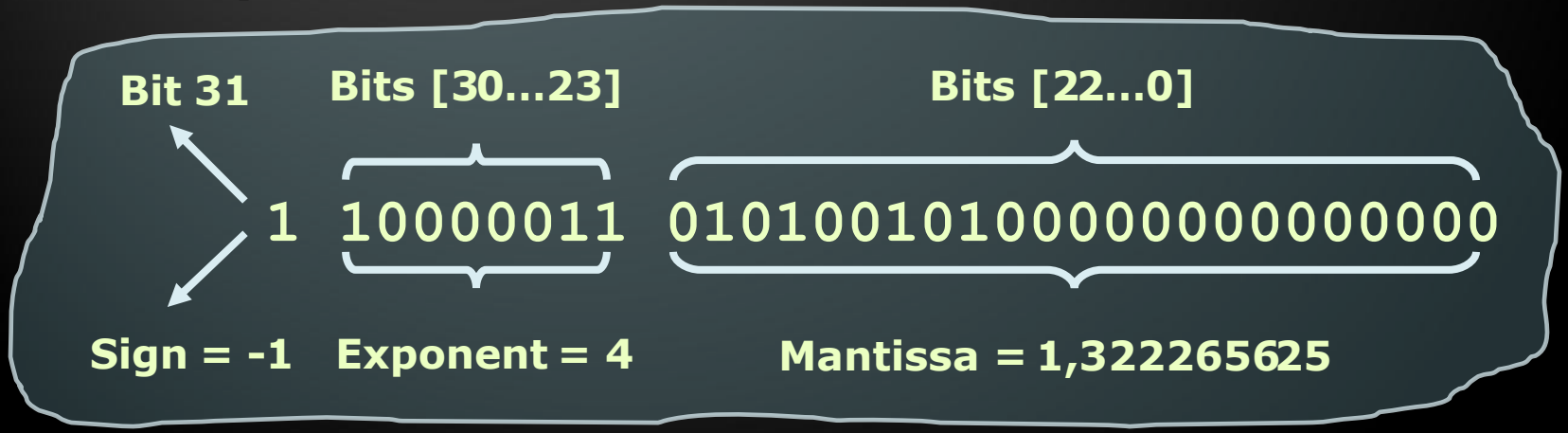
Floating-Point Numbers

- Floating-point numbers representation (according to the IEEE 754 standard*):



* See http://en.wikipedia.org/wiki/Floating_point

- Example:



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Text Representation in Computer Systems

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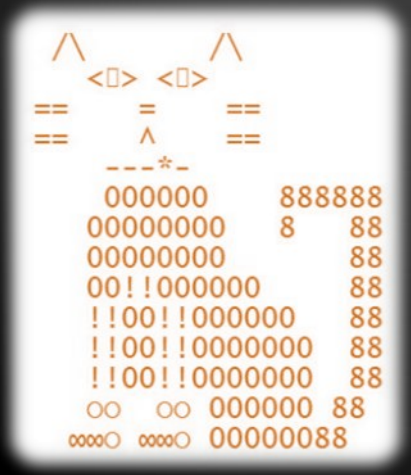
How Computers Represent Text Data?

- A text encoding is a system that uses binary numbers (1 and 0) to represent characters
  - Letters, numerals, etc.
- In the ASCII encoding each character consists of 8 bits (one byte) of data
  - ASCII is used in nearly all personal computers
- In the Unicode encoding each character consists of 16 bits (two bytes) of data
  - Can represent many alphabets

Character Codes – ASCII Table

Excerpt from the ASCII table



Binary Code	Decimal Code	Character
01000001	65	A
01000010	66	B
01000011	67	C
01000100	68	D
00100011	35	#
01100000	48	0
00110001	49	1
01111110	126	~

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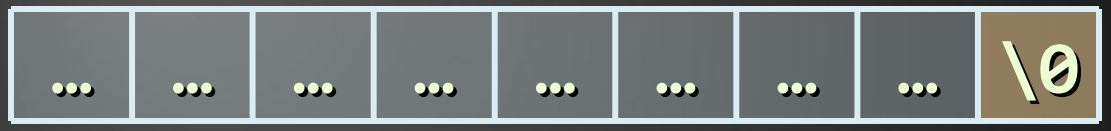
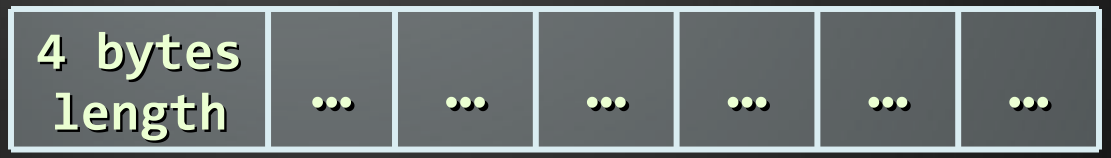
Strings of Characters

- Strings are sequences of characters
  - Null-terminated (like in C)



  - Represented by array



- Characters in the strings can be:
  - 8 bit (ASCII / windows-1251 / …)
  - 16 bit (UTF-16)

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Questions?

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Exercises

1. Write a program to convert decimal numbers to their binary representation.
2. Write a program to convert binary numbers to their decimal representation.
3. Write a program to convert decimal numbers to their hexadecimal representation.
4. Write a program to convert hexadecimal numbers to their decimal representation.
5. Write a program to convert hexadecimal numbers to binary numbers (directly).
6. Write a program to convert binary numbers to hexadecimal numbers (directly).

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Exercises (2)

1. Write a program to convert from any numeral system of given base `s` to any other numeral system of base `d` (2 ≤ s, d ≤ 16).
2. Write a program that shows the binary representation of given 16-bit signed integer number (the C# type `short`).
3. * Write a program that shows the internal binary representation of given 32-bit signed floating-point number in IEEE 754 format (the C# type `float`). Example: -27,25 → sign = 1, exponent = `10000011`, mantissa = `10110100000000000000000`.