Waterfall Optimization

Description

We are ordering a sequence of N unique events: $A_1 ... A_N$.
We are given a time budget T_{max}.
Each event may either succeed or fail.
Each event A_i may succeed with some probability $P(A_i)$.
It will fail with probability $1 - P(A_i)$.
Each event if successful will provide reward $R(A_i)$.
Each event if successful will cost time T_i^s.
Each event if unsuccessful will cost time T_i^f.
T_i^s, T_i^f are Gaussian random variables with known mean.
Assume $R(A_i)$ and $P(A_i)$ is known.
The ordering will be evaluated from start to finish until the first successful event. We must return an ordering of the events such that the expected value of the evaluation of the ordering is maximal.

Optimization

$Max_{(A_1 \ldots A_N)} \sum_i R_{A_i} * P_{A_i} * \prod_{j<i}(1 - P_{A_j}) * P(T_i^s + \sum_{j<i}(T_j^f) < T_{Max})$.
$A_i \in \mathbb{Z}$
$1 \leq A_i \leq N$
$AllDiff(A_i)$

Where $P(T_i^s + \sum_j(T_i^f) < T_{Max})$ is evaluated as a cumulative distribution function with mean $T_i^s + \sum_{j<i}(T_j^f)$ and known variance.

$CDF(x, \mu, \sigma) = \frac{1}{2}[1 + erf(\frac{x-\mu}{\sqrt{2}})]$
Where erf is the error function that can be estimated with:
$erf(w) = 1 - (a_1 t^1 + a_2 t^2 + a_3 * t^3 + a_4 * t^4 + a_5 * t^5)e^{-w^2}$

Where $t = \frac{1}{1+pw}$,
$p = 0.3275911$,
$a_1 = 0.254829592$,
$a_2 = 0.284496736$,
$a_3 = 1.421413741$,
$a_4 = 1.453152027$,
$a_5 = 1.061405429$