$$A\vec{x} = \vec{b}$$

$$\begin{pmatrix} 2 & 3 & -4 & 2 \\ -4 & -5 & 6 & -3 \\ 2 & 2 & 1 & 0 \\ -6 & -7 & 14 & -4 \end{pmatrix} \vec{x} = \begin{pmatrix} 4 \\ -8 \\ 9 \\ 6 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & 1 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} -6 & -7 & 14 & -4 \\ -4 & -5 & 6 & -3 \\ 2 & 2 & 1 & 0 \\ 2 & 3 & -4 & 2 \end{pmatrix}$$

pivot

$$\vec{p} = \begin{pmatrix} 3 \\ * \\ * \\ * \end{pmatrix}$$

$$R_1 - \left(\frac{-4}{-6}\right) R_0 \to R_1$$

$$R_2 - \left(\frac{2}{-6}\right) R_0 \to R_2$$

$$R_3 - \left(\frac{2}{-6}\right) R_0 \to R_3$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 & 0 \\ -\frac{1}{3} & * & 1 & 0 \\ \frac{2}{3} & * & * & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} -6 & -7 & 14 & -4 \\ 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & -\frac{1}{3} & \frac{17}{3} & -\frac{4}{3} \\ 0 & -\frac{1}{3} & -\frac{10}{3} & -\frac{1}{3} \end{pmatrix}$$

$$\vec{p} = \begin{pmatrix} 3 \\ 3 \\ * \\ * \end{pmatrix}$$

$$R_2 - \left(-\frac{1}{2}\right) R_1 \to R_2$$

$$R_3 - \left(-\frac{1}{2}\right) R_1 \to R_3$$

pivot

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{2} & 1 & 0 \\ \frac{2}{3} & -\frac{1}{2} & * & 1 \end{pmatrix} \quad U = \begin{pmatrix} -6 & -7 & 14 & -4 \\ 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 6 & -1 \\ 0 & 0 & -3 & 0 \end{pmatrix}$$

$$\vec{p} = \begin{pmatrix} 3 \\ 3 \\ 2 \\ * \end{pmatrix} \qquad R_3 - \left(-\frac{1}{2}\right) R_2 \to R_3$$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{2} & 1 & 0 \\ \frac{2}{3} & -\frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix} \quad U = \begin{pmatrix} -6 & -7 & 14 & -4 \\ 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 6 & -1 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

$$\vec{p} = \begin{pmatrix} 3 \\ 3 \\ 2 \\ 3 \end{pmatrix}$$

$$\vec{b} = \begin{pmatrix} 4 \\ -8 \\ 9 \\ 6 \end{pmatrix} \quad p_0 = 3 \begin{pmatrix} 6 \\ -8 \\ 9 \\ 4 \end{pmatrix} \quad p_1 = 3 \begin{pmatrix} 6 \\ 4 \\ 9 \\ -8 \end{pmatrix}$$

$p_2 = 2$ $\begin{pmatrix} 6 \\ 4 \\ 9 \\ -8 \end{pmatrix}$ $p_3 = 3$ $\quad \vec{d} = \begin{pmatrix} 6 \\ 4 \\ 9 \\ -8 \end{pmatrix}$

① $L\vec{y} = \vec{d}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{3} & 1 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{2} & 1 & 0 \\ \frac{2}{3} & -\frac{1}{2} & -\frac{1}{2} & 1 \end{pmatrix} \vec{y} = \begin{pmatrix} 6 \\ 4 \\ 9 \\ -8 \end{pmatrix}$$

R_0: $y_0 = 6$

R_1: $-2 + y_1 = 4 \rightarrow y_1 = 6$

R_2: $-2 - 3 + y_2 = 9 \rightarrow y_2 = 14$

R_3: $4 - 3 - 7 + y_3 = -8$

$y_3 = -8 + 7 + 3 - 4 = -2$

② $U\vec{x} = \vec{y}$

$$\begin{pmatrix} -6 & -7 & 14 & -4 \\ 0 & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 6 & -1 \\ 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \\ 14 \\ -2 \end{pmatrix}$$

R_3: $x_3 = 4$

R_2: $6x_2 - 4 = 14$

$x_2 = 3$

R_1: $\frac{2}{3}x_1 + \frac{6}{3} + \frac{8}{3} = \frac{18}{3}$

$\frac{2}{3}x_1 = \frac{4}{3}$

$x_1 = 2$

R_0: $-6x_0 - 14 + 42 - 16 = 6$

$-6x_0 = -6$

$x_0 = 1$

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$