$A\vec{x} = \vec{b} \rightarrow LU\vec{x} = \vec{b}$

$$A = \begin{pmatrix} 2 & 3 & -4 & 2 \\ -4 & -5 & 6 & -3 \\ 2 & 2 & 1 & 0 \\ -6 & -7 & 14 & -4 \end{pmatrix} \qquad \vec{b} = \begin{pmatrix} 4 \\ -8 \\ 9 \\ 6 \end{pmatrix}$$

$A = LU$

① $L\vec{y} = \vec{b}$ (forward)

② $U\vec{x} = \vec{y}$ (backward)

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ * & 1 & 0 & 0 \\ * & * & 1 & 0 \\ * & * & * & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 2 & 3 & -4 & 2 \\ -4 & -5 & 6 & -3 \\ 2 & 2 & 1 & 0 \\ -6 & -7 & 14 & -4 \end{pmatrix}$$

pivot

R_0 R_1 R_2 R_3

C_0 C_1 C_2 C_3

$R_1 - \left(\frac{-4}{2}\right) R_0 \rightarrow R_1$

$R_2 - \left(\frac{2}{2}\right) R_0 \rightarrow R_2$

$R_3 - \left(\frac{-6}{2}\right) R_0 \rightarrow R_3$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & * & 1 & 0 \\ -3 & * & * & 1 \end{pmatrix} \qquad U = \begin{pmatrix} 2 & 3 & -4 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 5 & -2 \\ 0 & 2 & 2 & 2 \end{pmatrix}$$

pivot

$R_2 - \left(\frac{-1}{1}\right) R_1 \to R_2$

$R_3 - \left(\frac{2}{1}\right) R_1 \to R_3$

pivo

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -3 & 2 & * & 1 \end{pmatrix} \quad U = \begin{pmatrix} 2 & 3 & -4 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 6 & 0 \end{pmatrix}$$

$R_3 - \left(\frac{6}{3}\right) R_2 \to R_3$

$$L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -3 & 2 & 2 & 1 \end{pmatrix} \quad U = \begin{pmatrix} 2 & 3 & -4 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix}$$

$A = LU$

① $L\vec{y} = \vec{b}$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -3 & 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \\ 9 \\ 6 \end{pmatrix}$$

$R_0 : \quad y_0 = 4$

R_1 : $-8 + y_1 = -8$

$y_1 = 0$

R_2 : $4 - 0 + y_2 = 9$

$y_2 = 5$

R_3 : $-3y_0 + 2y_1 + 2y_2 + y_3 = 6$

$-12 + 10 + y_3 = 6$

$y_3 = 8$

② $U\vec{x} = \vec{y}$

$$\begin{pmatrix} 2 & 3 & -4 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 3 & -1 \\ 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 5 \\ 8 \end{pmatrix}$$

R_3 : $2x_3 = 8$

$x_3 = 4$

R_2 : $3x_2 - 4 = 5$

$x_2 = 3$

R_1 : $x_1 - 6 + 4 = 0$

$x_1 = 2$

R_0 : $2x_0 + 6 - 12 + 8 = 4$

$$x_0 = 1$$

$$\vec{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \qquad A\vec{x} = \vec{b}$$