Standard error of mean where S is the sample size and N is the number of observations:

$$\frac{S}{\sqrt{N}} \tag{1}$$

Variance Also knows as the 2^{nd} moment:

$$Variance = \Sigma_{i=i}^{n} \frac{(x_i - \overline{x})^2}{n-1} \tag{2}$$

Standard error of Variance

$$\frac{\sigma}{\sqrt{N}} \tag{3}$$

Skewness (3^{rd} Moment): Data is said to be skewed if its leaning of the mean (i.e. it is assymetric).

$$\frac{n}{(n-1)(n-2)} \Sigma \left(\frac{x_j - \overline{x}}{\sigma_{sample}} \right)^3 \tag{4}$$

Error of Skewness:

$$\sqrt{\frac{6}{n}} \tag{5}$$

Kurtosis (4^{th} Moment): Does the Data have fat tails.

$$\frac{n(n+1)}{(n-1)(n-2)(n-3)} \Sigma \left(\frac{x_j - \overline{x}}{\sigma_{sample}} \right)^4 - \frac{3(n-1)^2}{(n-2)(n-3)} \tag{6}$$

Error of Kurtosis: where n is number of observations.

$$\sqrt{\frac{24}{n}} \tag{7}$$

Standard Deviation:

$$\sqrt{\Sigma_{i=1}^{n} \frac{(x_i - \overline{x})^2}{N-1}} \tag{8}$$

Q. Test the null hypothesis at the 95% confidence level that the 3rd and 4th moments = 0 and that the serial correlation =0:
A. The methadology is as follows:
1. Figure out the statistic, for 99% it equals 2.33∗ Standard Error and for 95% it equals 1.64∗ Standard Error

2. To see if the data likes within the confidence interval, check to see if it lies beween + or - the above data.

Ways to calculate VAR:
1. Monte-Carlo.
2. Variace/Covariace.
3. Historical Simulation.
4. Simple One Asset.

Simple One asset:
For Normal Securities:

$$2.33 * \sigma * DollarPosition. \tag{9}$$

For Yeild Based Assets VAR:

$$2.33 * PV01 * yield * yield\_close * \sigma_{yield} * DollarPosition * 100 \tag{10}$$

Duration: First derivative of the bond pricing forumla with respect to yield. It measures the average time of a securities cash flows, where the weighting is the cash flow. Duration also shows the percentage change in price per change in yield. Traders think in terms of dollar duration because it reflects their pnl while a portfolio manager thinks in terms of duration because it shows them the percentage return.

$$D = -\frac{\delta P}{\delta y P} \tag{11}$$

$$\$D = D * P = -\frac{\delta P}{\delta y} \tag{12}$$

Covariance: Gives us an indication of how far one variable is from its mean when we observe one variable a certain distance from its mean. i.e. It tells us how much x moves when y moves. It provides a measure for every variable with respect to the other variable.
Importance of Covariance:
1. If we know the variances and covariances of all the securites in a portfolio, we can assess the risk of the entire portfolio.
2. We can also assess the risk of any sub-portfolio.
3. This is the basis of much of modern potfolio theory.

$$\Sigma_{i=1}^{n} \frac{(x_i - \overline{x})(y_i - \overline{y})}{(n-1)} \tag{13}$$

Basic Statistical Principals of Covariance

$$\sigma_{a+B}^2 = \sigma_a^2 + \sigma_b^2 + 2\sigma_{ab} \tag{14}$$

where $2\sigma_{ab}$ is the covariance.

$$var(xa) = x^2 var(a) \tag{15}$$

$$var(xa + yb) = x^2 var(a) + y^2 var(b) + 2xycov(a, b) \tag{16}$$

Portfolio Variance of n stocks (vector of) in vector Y and Covariance vector M

$$Y^t * M * Y \tag{17}$$

Corelation: is obtained by scaling covariance to get correlation. It is unit free unlike covariance. Correlation is an index of linearity.

$$Cor(a, b) = \frac{Cov(a, b)}{\sigma_a \sigma_b} \tag{18}$$

Time period coverage
Exponential Weigthing: Weights more recent observations more havily and declines exponentially. The equation for co-variance is of the form:

$$\Sigma_{i=1}^{n} \frac{\omega^i (x_i - \overline{x})(y_i - \overline{y})}{\Sigma \omega^i} \tag{19}$$

Bucket Weigthing: Example bucket uncorelated assets with a similar vol. It is done to make things simple.

e.g. of Scaling of volatility:

$$\sigma_{annual} = \sigma_{daily} * \sqrt{252} \tag{20}$$

or

$$\sigma_{annual} = \sigma_{weekly} * \sqrt{52} \tag{21}$$

Special Fixed Income Considerations
1. For fixed income we measure yields and not prices.
2. We must multiply the position with $DV01$ to reflect the *price* sensitivity to the change in *yeild*.
3. The adjustment is made in the position vector.

Bull Stepener is phenomenon which sees the short term yield fall more than the yeild of the long term rates thus steeping the yield curve, but causing a rise in bond prices.

Bear Stepener Short term rates are going up and the long term rates are going up faster than the short term rates thus steeping the yield.

Bull Flattener Long term rates are falling faster than the short term rates.

Bear Flattener Short term rate is increasing faster than the long term rate.

Regression Formula (least squares)

$$A^t A x = A^t b \tag{22}$$

Absolute VaR It is simple the VaR of a single postion (bucket). It calcualted in the following ways:
1. Same as single asset approach

$$2.33 * \sigma * p$$

2. Use the covariance Matrix approach, just set the postion for all the other buckets to 0

$$D^t \sigma_{cov} D$$

Marginal VaR Reasons for using this:
1.Primarily to see how an individual position contributes to portfolio VaR
2.Illuminates natural hedges
3.Shows efficacy of hedge portfolios

Strengths of Historical Simulation:
1. Simple.
2. Intuitively Applealing.
3. Non parametric.
4. Aggregation Easy.

Weaknesses of Historical Simulation:
1. Need lots of data for good sample.
2. May need historical data for things for which there is little history.
3. How do you assume new products would behave?
4. Assumes stationarity (i.e. no drift).

Bucketing Methods:

1. equal VaR.
2. duration weighting.
3. hedge positions.
Sensitivity Calculation Methods:
1. For equities, FX and Comodities it is very easy.
2. For fixed income: $pos * DV01 * close * 100$
3. Convexity not a big problem.
3. For options, Δ or Δ & Γ. Using the greeks will not work for volatile markets or ATM options.

Variance And Covariance of PnL vectors: where y is the overall portfolio historical return's vector and $x_1, x_2 ... x_n$ are the returns of assets/desks within the portfolio
Varaince of a portfolio:

$$\sigma_y^2 = E\left[(y-\overline{y})^2\right] = \Sigma_{j=1}^{n} E[(y-\overline{y})(x_j-\overline{x_j})]$$

Standard Deviation of a portfolio:

$$\sigma_y = \Sigma_{j=1}^{n} MVaR_j$$

where

$$MVaR_j = \frac{E[(y-\overline{y})(x_j-\overline{x_j})]}{\sigma_y}$$

Marginal VaR using Variance/Co-Variance Ratio:

$$MVaR_{child} = \frac{Cov(Parent, Child)}{\sigma_{parent}^2} * VaR_{parent}$$

Fund (Buy) side risk:
1 Concentration Risk.
2 Asset Allocatin
3 Performance Attribution
4 Systemic vs. Idiosyncratic Risk
5 Margin Optimization
6 Liquidity
7 Regulatory/Reputation Risk.

Bank (Sell) side risk:
1 Market Risk
2 Credit Risk
3 Counterparty Risk
4 Operational Risk
5 Liquidity Risk.

6 Reputational Risk.

Information Ratio:

$$\frac{E[R(P) - R(BL)]}{\sqrt{var(R(P) - R(BL))}} \tag{23}$$

Sharpe Ratio:

$$\frac{E[R(P) - R(Rf)]}{\sqrt{var(R(P) - R(Rf))}} \tag{24}$$

Leverage and NAV

$$L = \frac{LMV + Cash}{Capital} \tag{25}$$

To get leverage
-Unsecured
-Secured : Brokerage and RegT(borrow 50 % of market price of security), Bank & Collateral -Others: Repos, Reverses, Securities Lending
Give Ups:
Executing bank assigns the trade to another bank(PB) to settle. Transfer of credit risk. PB has not market risk. Also called Intermediation.
OTC Cleaing Impact:
Fun ends up clearing with only 1 or 2 Clearing Houses, if a Clring Member defaults, positionare netted and offset to another member.

Liquidity
-Periodicity, size of trade, fund liability posn. when due
Buy Side Liquidity
Mutual funds
Liquid assets,100% of capital required on Day1,100% invested Day 2,Daily redemption
Hedge Funds
100% of capital required Day1 for existing fund; new funds may ramp up,Fully invested Day 2, Possibility of side-pockets; liquidating trusts,Quarterly or annual redemption with 30-90 day advance notice
Private Equity structure
May include a ramp-up period; could last 3 year or more, Tail period of unwind could be much longer than original projections,Quarterly or annual redemption with 30-90 day

Liquidity Measure

Inability to exec. trade, relutance to move price (VWAP vs Hi/Lo), breadth(no of 2 way participants) and depth(size of trade w/o moving mkt.)

Definition of Credit Risk Credit risk represents the volatitility/uncertainity of credit events such as default or rating downgrades.

Types of Credit Risk Lending Risk, Issuer Risk and Counterparty risk.

Components of Credit Risk Exposure, Credit Worthiness, Loss Severity (Recovery Rate)

$$L_p(t) = \Sigma_{i=1}^{n} max[X_t(t), 0] d_i(0,t)[1 - \delta_i(t)]B(0,t) \tag{26}$$

Credit Exposure Current Exposure, Potential Future Exposure, Settlement Exposure

Market Risk vs Credit Risk
Time Horizon MR: Short (1-10) days. CR: Over a longer period of time.
Nature of Loss MR: Adverse Mrket Risk. CR: Counterparty Defaulting when you make money.
Distribution MR: Approximately Normal. CR: Large Negative Skew (low probability of a huge loss).
Hedging: MR: Positions can generally be synthetically replicated trivially. CR: Netting Agreements (mitigates the risk of you not getting but paying), Mutual Termiantions before CP goes bankrupt, Diversification, Collateral.

Note on risky swaps Add some spread for the risk.

Differnece between Credit/Lending (LR) and Counterparty risk (CR)
CR: Exposure could be +- depending on the state of the market. LR is always additive. CR can be hedged by working with multiple dealers.
Time based factors that effect risk *Diffusion of Interest rates* increase risk and *Ammortization decreases risk.* When graphing it, we get an inverted parabola.
Assymetry of diffusion Under a lognormal model, the upper half of the diffusion area is larger than the lower half. Formula is

$$R(t) = e^{-\frac{1}{2}\sigma^2 + 1.65\sigma\sqrt{t}} \tag{27}$$

Because of this effect, the person reciving LIBOR tends to have a higher upside consequently a higher counterparty risk.

Ways to model interest rate curves [Risk Factor Evolution]

Single Factor Interest Rate evolution. When z is positive the entire curve shift up, when z is negative the curve shifts down.

$$R_1 = r_i e^{-0.5\sigma^2 + z\sigma_1} \tag{28}$$

CVA Stands for Credit Valuation Adjustment. It equals the Net Present Value of the risk due to counterparty default. CVA = PV (Expected Positive Exposure) x Credit Spread

CVA Formula's where EPE = Expected Positive Exposure, PD = Cumulative Probability of Default, $\Delta PD = PT(t_{i+1}) - PD(t_i), LGD = LossGivenDefault, DF = DiscountFactor$

$CVA = EL_1 + EL_2 + \cdots + EL_N (29)$

$$EL_k = EPE_k \cdot \Delta PD_k \cdot LGD_k \cdot DF_k \tag{30}$$

Simple CVA

$$CVA = EL * Spread * yrs \tag{31}$$

Regulatory Credit Capital under Basel II is equal to 8 of the Risk-Weighted Asset (RWA)

- RWA Banks asset weighted according to its credit risk
- Risk-Weighted Asset (RWA) = Credit Risk Weight * Exposure at Default = CRW * EAD
- CRW is the weight assigned to Banks particular asset
- EAD (Exposure at Default) is the time-weighted average of the Expected Positive Exposure (EPE) Profile times a multiplier equal to 1.4

Moving average time series

$$Y_t = \mu + \epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \cdots + \theta_q \epsilon_{t-q} \tag{32}$$

Auto regressive time series

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \alpha_2 Y_{t-1} + \cdots + \alpha_p Y_{t-p} + \epsilon_t \tag{33}$$

ϵ is a standard normal variable with variance σ^2

Variance of AR1 $V(Y_t) = \frac{\sigma^2}{1-\alpha_1^2}$

Box-Jenkings tests help us identify lags.

Auto Corelation Function of order k is the corelation between the current value and the kth lag.

Partial Auto Corelation Function of order k is the corelation between the current value and the kth lag controlling for the corelation for everything in between.

Stationary Process: A stationary process is a stochastic process whose joint probability distribution does not change when shifted in time or space.

Strictly Stationary Process: A series is strictly stationary if its probability distribution does not change when shifted in time or space.

Covariance stationary: A series is covariance stationary if neither its mean nor its autocovariances depend on the date. $E(Y_t) = \mu \forall t$ and $E(Y_t - \mu)(Y_{t-j} - \mu) = \gamma_j \forall t$

Unit root timeseries: is a time series where the $(\alpha) is set to 1. For eg. y_t = \alpha y_{t-1} + \epsilon_t$. When $\alpha = 1$, it becomes $y_t = y_{t-1} + \epsilon_t$. Then $y_t = y_0 + \Sigma_{j=1}^{t} \epsilon_j$ with variance $\sigma^2 = t\sigma^2$

Non-Stationarity: Reasons for non-stationarity:

- Trends e.g. the level of the SP over the past 100 years
- Seasonality
- Heteroscedasticity well known problem in financial time-series, e.g. volatility of markets before and after the Lehmann collapse.

Integration: Non statioary series can be made stationary by converting the numbers. For example consering log returns instead of regular returns or considering change in volatility instead of the change in prices.

Co-integration: Two timeseries might be non stationary by themselves, but when looked at togther might be stationary for example GDP and employment. equation $\Delta E_t = \alpha_0 + \alpha_1(\beta_1 E_{t-1} + \beta_2 GDP_{t-1}) + \alpha_2 \Delta GDP_t + \epsilon_t (33)$

Whites test for heteroscedasticity: White test is a statistical test that establishes whether the residual variance of a variable in a regression model is constant (homoscedasticity).

- Regress the squared regression residuals on the model regressors (coefficents)
- The test statistic is $LM = nR^2$
- Distributed chi square, (n-1) degrees of freedom where n is the number of independent variables

Durbin-Watson test for first order serial correlation The sum of the differences of all consecutive approximations divided by the sum of the squares of the obervations should be around 2. equation d = $\Sigma_{t=2}^{T}(e_t - e_{t-1})^2 \frac{}{\Sigma_{t=1}^{T} e_t^2 (33)}$

Best way to deal with panel data: There are two ways:

- PCA
- Regression

Bassel Summary:

- Basel I VaR + standardized charges
- Basel II VaR + IDRC + standardized charges
- Basel 2.5 VaR + Stressed VaR + IRC + CRM + standardized charges + securitization framework (Starts 1/1/2012)
- Basel III new CVA (Credit Value Adjustment) treatment, higher capital ratios (starts 2013)

Bassel I:

- In principle, a 10-day 99measure
- Assumed liquid, reasonably functioning markets

Bassel II:

- Incremental risk default charge

Bassel 2.5:

- Stressed VaR: Regular VaR over a 12-month period of stress
- Incremental Risk Charge: IDRC including credit migration
- Introduce CRM
- Comprehensive Risk Measure: all substantial risks in the correlation trading portfolio
- Traditional approach to capital was non-model based, not very risk-sensitive, standardized charges

- The crisis showed the underlying premise that VaR could adequately capture trading book risks was flawed

CRM: Comprehensive Risk Measure

- Spread risk
- Implied correlation risk
- Default risk (including ordering of defaults)
- Recovery rate risk
- Index-single name basis risk
- No Greek short-cuts in calculating P and L
- At 99.9, one-year capital horizon

Gaussian Copulla: Workhorse of default risk.

Liquidity Horizon: Assume a given liquidity horizon for a given product, take risk to that horizon, and then assume the product is refreshed at its original level of risk.

Augmented Dickey Fuller Test: Used to test for a unit root process for time series models.

equation $\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \epsilon_t (33)$

If $\alpha = 0 \& \beta = 0 then Random Walk$

$If \beta = 0 then Random Walk with drift$

$If \gamma = 0 then Unit Root$

Panel Data:$Estimate using$

- Fixed Effect Reg. or Random Effect Reg.

- PCA

$$\Delta Y_i t = \beta X_{ijt} + \alpha_i + \delta_t + \epsilon_{it} \tag{34}$$

Move any time invariant fixed effect parameter to the Random effect part. Almost always do a fixed effect regression. If you take differences the fixed effect on levels cancels out.

Hausman Test: Run Fixed and Random, takes the params and comapres, if they ar every different from each other then null hypothesis is true.

OCC 2000-16 Concentrates on Model Validation
Elements of sound validation policy

- Independent Review - internal independent validation unit supplemented by internal/external auditors

- Defined Resp. - validation unit needs to validate, audit group needs to verify that validation performed before production.
- Model Documentation - Create memory, guide for users, validators etc.
- Ongoing Validation - Changes revalidated for evading risk limits, reapproved by decision makers, subjected to change ctrl.
- Audit Oversight - Internal audit to ensure model validation and model-validation units adhere to formal policy.

Validating the Model Inputs Comp.

- Data - Internal, verify agains GL and Contracts. External, against sevral srcs. Apprise audit, decision makers of data prbs.
- Assumptions - clear rationales for choice b/w private and public assumptions. Mthly reports i.e comparisons with observed to senior mgmt.

Validating the Model Process'g Comp.

- Code and Mathematics - 1. Assign model validation personnel. 2.Compare model results to a benchmark model. Apply same process for in-house and vendor models
- Theory - qualified model developer, independent review, internal review. Comparison to True Model

Model Reports (MIS) Clear and exlpain context

- Validating Model Results - Back-Testing, Out of sample tesing.
- Validating Model Context - Exec. Summary, stmt of model purpose and limitations.

OCC 2011-12 Concentrates on Model Risk Mgmt as a whole An effective validation framework should include three core elements:

- Evaluation of conceptual soundness, including developmental evidence
- Ongoing monitoring, including process verification and benchmarking
- Outcomes analysis, including back-testing

Three Parts
1. Model Development, Implementation & Use
2. Model Validation
3. Governance, Policies & Controls

Governance, Policies & Controls:

- Board of Directors and Senior Management
- Policies and Procedures
- Roles and Responsibilities
- Internal Audit
- External Resources
- Model Inventory
- Documentation

Canabarro & Duffie
Liquidity Puts - Liquidity puts give the parties the right to settle and terminate trades on pre-specified future dates.

PFE models:

- databases (trades, agreements, legal entities, legal opinions, collateral holdings, risk limits)
- Monte Carlo simulation engines
- trade pricing calculators
- exposure calculators
- reporting tools

Exposure Calculation 1. calculating the exposure in each netting node; 2. adding all netting node exposures; 3. calculating the collateral posted/received for each margin node; 4. adding collateral posted/received; and 5. calculating the net exposure to the counterparty as (2) minus (4).
Extreme Value Therorem: Statistical technique to model extreme events i.e observations deviate extremely from the median of a prob distribution. Maxima RVs in a period or all obs. greater than a threshold(Peak Over Threshold). Limiting distribution are GPD(Generalized PAreto Distribution) GEV (Generalized Extreme Value).

Operational Risk Risk Governance & Culture
1. Risk Culture
2. Risk Gov.

3. Risk Leadership
4. The Use Test - be demonstrable by involvement of staff at all levels in the decision-making process, from the board down as appropriate. Document and reports leading to action by decision makers
5. Incentivising risk mgmt
Risk Gov Framework: Three lines of defense. 1. Board - Ultimate stakeholders and biz owners
2. Risk Oversight - Risk Mgmt, OR Risk Mgmt, Legal, Compliance, IT Security, HR
3. Independent Assurance - Internal and Ext. Audit

Mortgage Risk
1. Agency: Pass thru prepayment risk, credit risk
2. Non-Agency: Keeps those risks

Players

1. Originators (Countrywide, Washington Mutual etc.)
2. Servicers (Countrywide, Saxon etc.)
3. Warehouse Lenders (Bear, Lehman, RBS Greenwich Capital etc.)
4. Broker/ Dealers (Morgan Stanley, RBS Greenwich Capital etc.)
5. Rating Agencies
6. Investors (Institutional investors, Insurance companies etc.)
7. Trustee (Bank of New York etc.)
8. Monolines

Z-test Used for testing the mean of a population versus a standard, or comparing the means of 2 populations, with large (n $\geq 30) samples whether you know the population sta$
$\rho \approx N(\rho_0, \frac{1-{\rho_0}^2}{n})$

Monte-Carlo

1. Negative Eigen Values
2. PRecision
3. Computation Time
4. Data Storage
5. Audit Trail

6. Assumptions of Normality
7. Must adjuts to simulate mean reversion
8. Must adjuts to simulate jump diffusion