Volatility Filtering and Estimation - MTH9863 Final Project (Spring 2013)

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Contents

Summary

I have implemented the Extended Kalman Filter and estimated parameters using MLE, with a Powell Method optimizer constained by the following contraints:
$\omega \geq 0$
$\theta \geq 0$
$\xi \geq 0$
$-1 \leq \rho \leq 1$

The constrained optimization is achieved by running iterations of the Powell method until the parameters minimizing the MLE are outside of the contraints. The iteration is achieved by resetting the parameters outside of the permitted using a draw from a uniform distribution within a range of permissible values for the parameter in question.
For example: ρ is reset by drawing a random values from (-1.0, 0.0), this is delibarate as I want to keep the correlation negative and more in line with the real world. The results below are tabulated in order (below) for the Sqrt/Heston model $p = 0.5$, GARCH model $p = 1.0$ and 3-2 Model $p = 1.5$.
To test for the correctness of the model the following Diagnostics were applied and are detailed for each model choice below.

- MPE
- RMSE

- Box-Ljung Test for Autocorrelation of residuals
- χ-Squared Test for Normality of standardized residuals
- Histogram for visual inspection of bell curve
- Q-Q Plot for visual inspection of normality

NOTE: On running the code and implementation

The Filtering and MLE code was implemented using C++ on Visual Studio and compiled to run on a Win32 architecture.
VS project name is "Final Project"'. All the required code and configs are attached.
The main file of the C++ project FilterMain.cpp. This includes the code to run the EKF Filer and MLE using Powell Optimizer for $p = 05, 1.0$ and 1.5.
Please note that the input trade time series file needs to be saved as a csv file before it can be read by the program. Final-Project.csv.
$ifstreamdatafile = openForRead("../../Data/Final - Project.csv");$

The diagnostics were implemented in R. The C++ codes outputs the observation, estimates, means, variances and the residuals
for the the three types of models in a folder called "Data" at the same level as the Final Project folder for VS.
All file paths are relative. There are some pre-requisites to run the code successfully in R.
The workding directory should be set to the Data folder under the top level Final Project folder in the submission
(attached.). Filename is NormalityTests.R. The R code also relies on the packages "moments" and "randtoolbox".

Analysis of results

The only model to fit the data with the contraints on the parameters was the Square Root/Heston Model. However as the test
stats and graps below reveal that the residuals are only approximately normal.
The other models, GARCH and the 3-2 Model, both could not fit the data with the parameter constrainsts, however the residuals
appear to be much more normal than the square root model.
In conclusion, I would have to say that the Square Root model most accurate fits the data with the parameter set estimated.

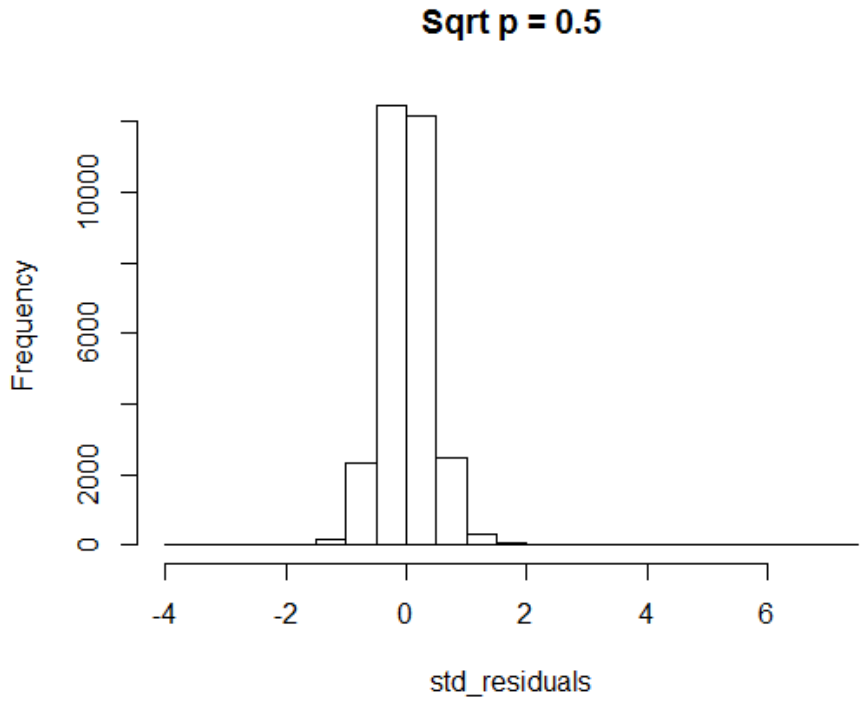
Filtering with p = 0.5 (Square Root Model)

Number of Powell iterations = 6

After Powell, minimum value = -174610 at:
$\kappa = 0.0350868, V = 10.843, \xi = 2.41281, \rho = 0.354961$
MPE: 0.00153469,RMSE: 0.0137258
Box-Ljung Test stat: 2651.57, which greater than the ChiSquared statistic for 20 df and 95 percent confidence interval
value 31.41

Chi-Squared Test stat: 253
P-Value: 0.7638247

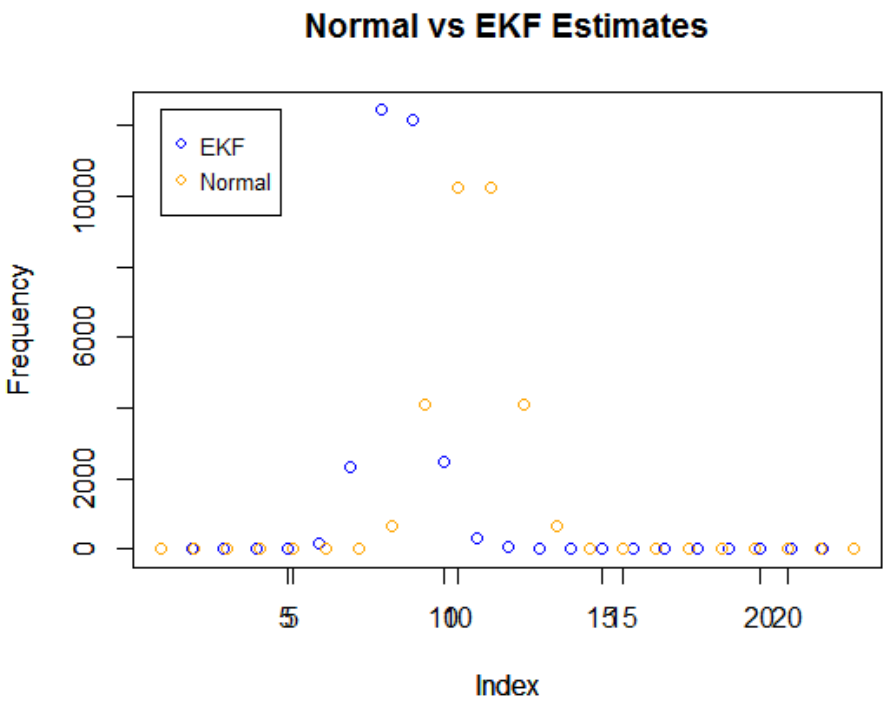
The histogram indicates approximate normality



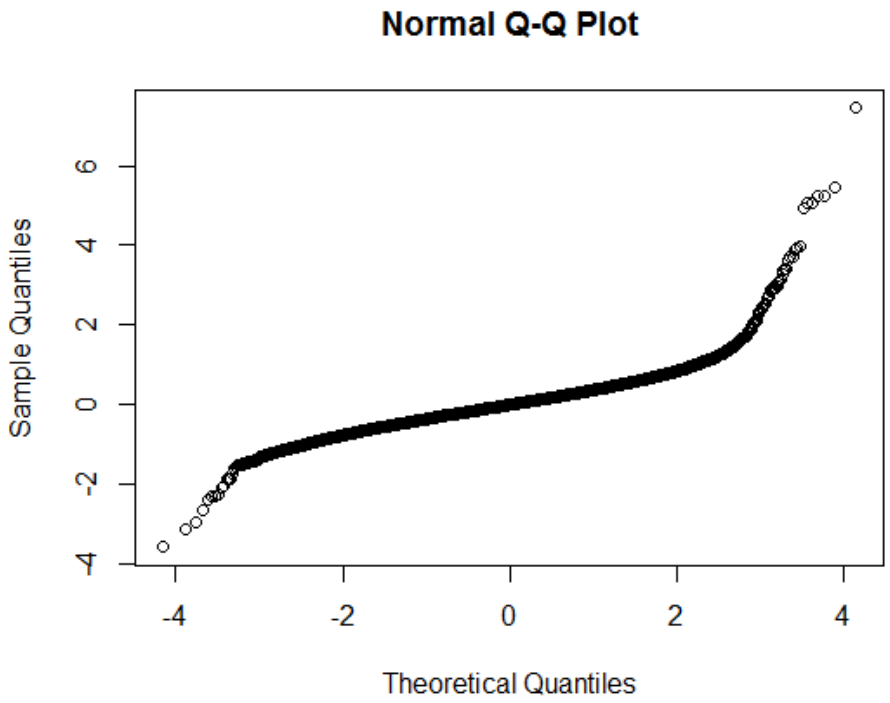
Model	MPE	RMSE	χ-Squared	P-Value	Box-Ljung
Sqrt	0.001534690	0.0137258	253	0.7638247	5529.66
GARCH	5.55174e-005	0.0136935	224	0.2416908	2654.5
3-2	5.55174e-005	0.0136935	224	0.2416908	2651.57

Table 1: Result Comparision

Plot of bins Normal vs EKF



Q-Q Plot is almost a straight line; approximately normal

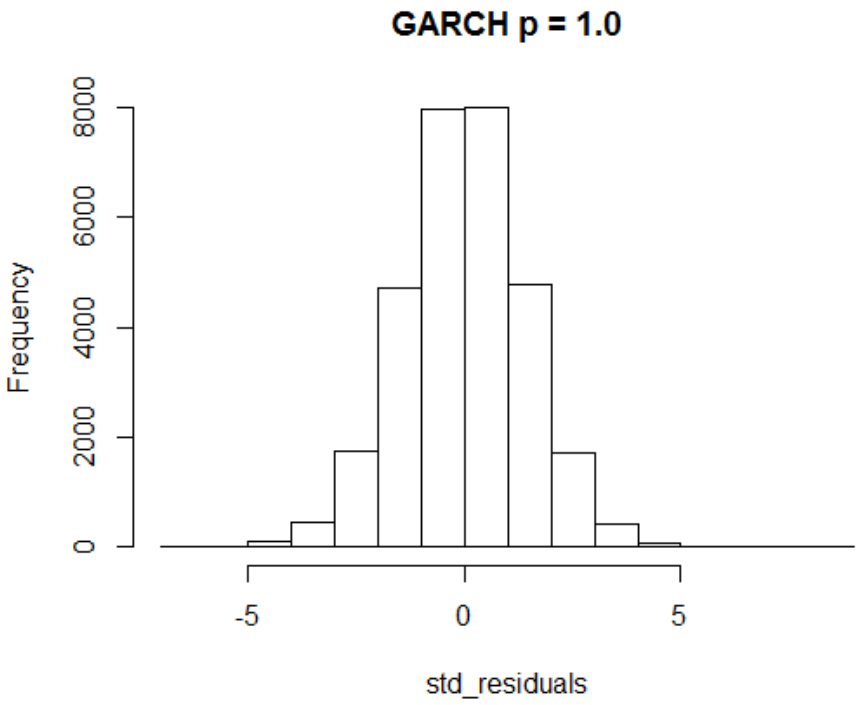


Filtering with p = 1.0 (GARCH Model)

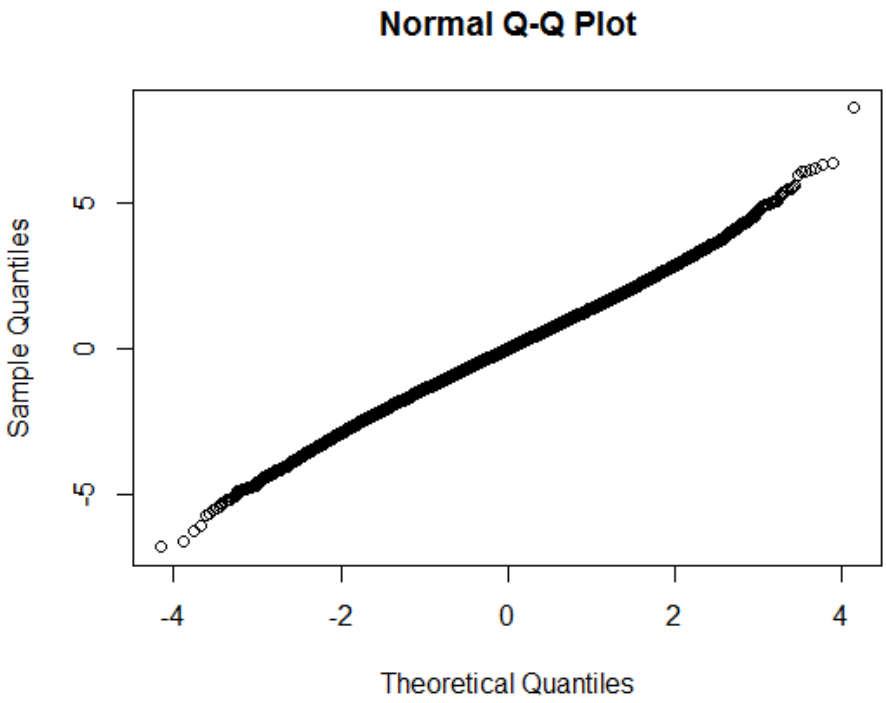
Number of Powell iterations = 4
After Powell, minimum value = -227436 at: $\kappa = 4.23291, V = -39.9416, \xi = -0.893428, \rho = 0.0453112$
$MPE : 5.55174e-005, RMSE : 0.0136935$
Box-Ljung Test stat: 2654.5, which greater than the ChiSquared statistic for 20 df and 95 percent confidence interval
value 31.41

Chi-Squared Test stat: 224
P-Value: 0.2416908

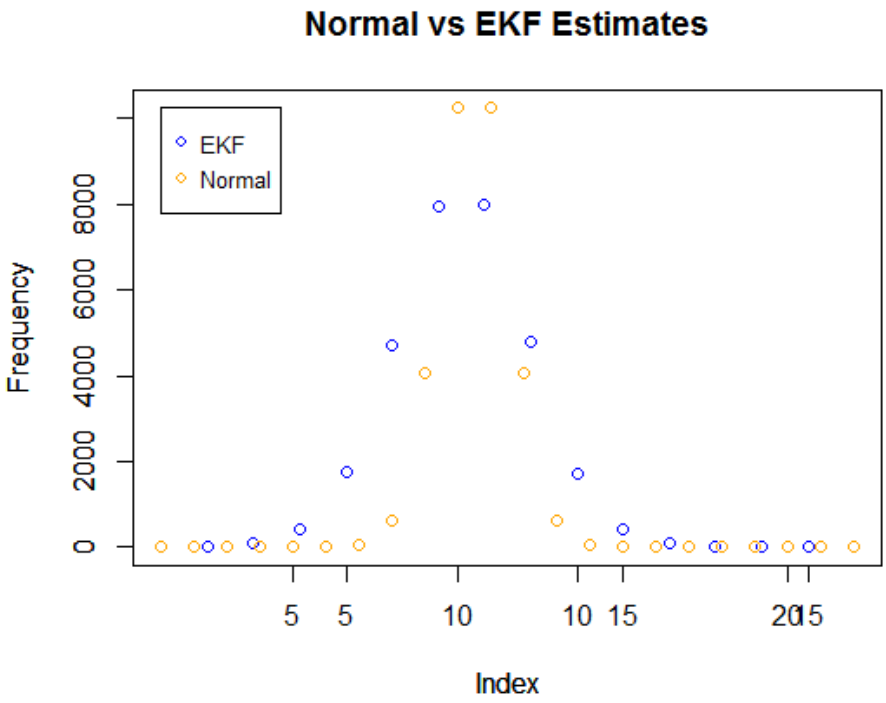
The histogram indicates approximate normality



Q-Q Plot is almost a straight line; approximately normal



Plot of bins Normal vs EKF



Filtering with p = 1.5 (3-2 Model)

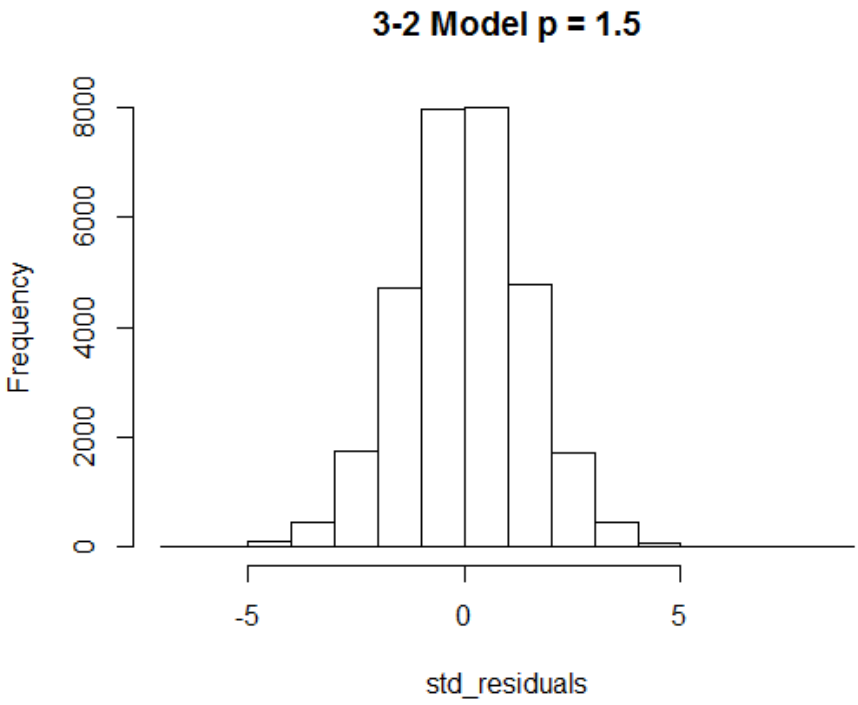
Number of Powell iterations = 3
After Powell, minimum value = -227436 at:
$\kappa = 4.23222, V = -0.466434, \xi = 0.758437, \rho = -0.0403623$

MPE: 5.55187e-005, RMSE: 0.0136935

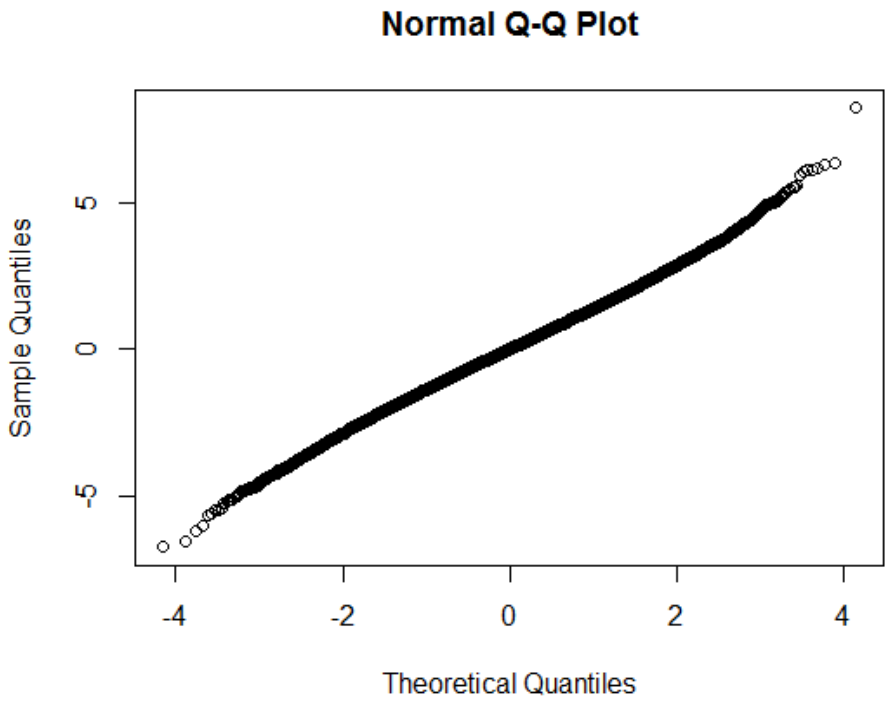
Box-Ljung Test stat: 2651.57, which greater than the ChiSquared statistic for 20 df and 95 percent confidence interval
value 31.41

Chi-Sqaured Test stat: 224
P-Value: 0.2416908

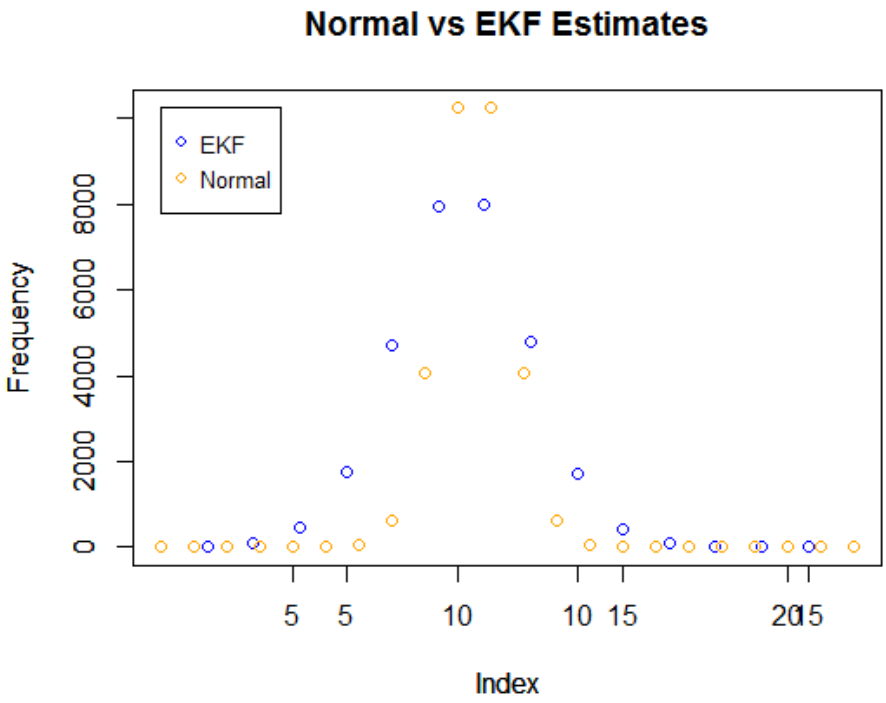
The histogram indicates approximate normality



Q-Q Plot is almost a straight line; approximately normal



Plot of bins Normal vs EKF


Normal vs EKF Estimates
EKF
Normal
Frequency
0
2000
4000
6000
8000
5
5
10
10
15
20
15
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