Kohonen Map Project Report

Unsupervised and Reinforcement Learning in Neural Networks

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1 Learning rate and convergence

In this part, we start with a Khohonen network of 6x6 neurons and use a Gaussian neighborhood function with standard deviation $\sigma = 3$.

We have tried several learning rate, the observations are show as below :

1. eta = 0.01 : numbre of iteration = 4
2. eta = 0.001 : numbre of iteration = 24
3. eta = 0.0001 : numbre of iteration = 48

The result is exactly the same to our intuition, small learning rates cause the algorithm to converge slower, while the larger ones help the algorithm to converge faster. However, we can not tell which one is better. As we know, the final solution of the small learning rate will be more stationary, but it might be stuck in local minima, while the large one might make the weights fail to converge.

For the three learning rate, the resulting figures have no big difference, as the one show above. What is important is that the three learning rates all leads to the convergence of the algorithm. So we can conclude that the bigger learning rate has no side-effect in this project. In addition, according to our observation, smaller learning rates doesn't improve our result. That means the quality of the result depends on other parameters which will be discussed in the later section. As a result, we choose eta = 0.01

In the origin matlab code downloaded from the Moodle, there is a simple convergence rule which just iterate the algorithm as many times as the number of the data points. In most of the case, it is not good enough, as some noticeable changes are observed. In order to improve this, we have defined a new convergence condtion which is descibed as following :

$$\frac{\|centers_{n+1} - centers_n\|}{\|centers_n\|} < 0.01 \tag{1}$$

where 0.01 is a converge criterion which can be chosen as needed and each row of the matrix centers is one of the prototypes.

In our convergence rule, every iteration uses all the data points to calculate the centers. After each iterartion, the convergence condition will be tested which will decide whether the algorithm will go on or not. If the change is small enough between the centers of two successive iterations, we can conclude that the algorithm converges. Mathmatically, that is to say the norm of the difference of the two centrers matrix is sufficent small (relative to the norm of the previous one).

2 Visualization and prototypes

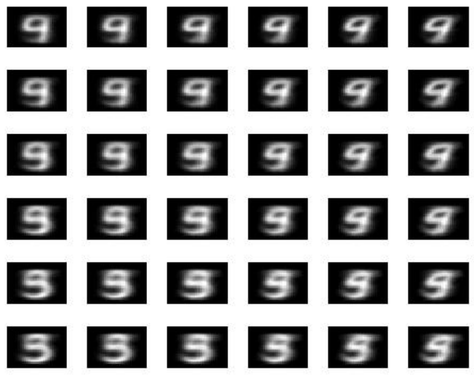


FIGURE 1 – Visulization of prototypes

Figure 1 is the Visulization of the prototypes in the map with standard parameters ($eta = 0.01$, $\sigma = 3$) for the digit set $3, 4, 5, 9$. From the figure we can't tell much difference between the prototypes. We can say that under this setting, the resulting map is not of a high quality.

3 Assignment method

In this step, we need a method to automatically assign the propre digit to each prototype that we found by executing the Kohonen map algorithm. An obvious idea is to look at the labels of the trainning data points that are close to the prototype, then decide the its label accroding to the majority in this surrounding. In fact, this task has a similarity with the k nearest neighbor classification algorithm, which assigns a label to a data sample based on the majority of its k nearest neighbors. So we decided to use the kNN method.

Before we apply this method directly to the problem, however, we need to determine the parameter, k, of this algorithm. Intuitively, this parameter depends on our trainning data, which covers the hand writing samples of the four chosen digits. So we do firstly a cross validation on kNN models with distinct k values (ranging from 3 to 300), which results in k equals 5.

Once the parameter is fixed, the application of kNN to the problem is fairly straightforward.

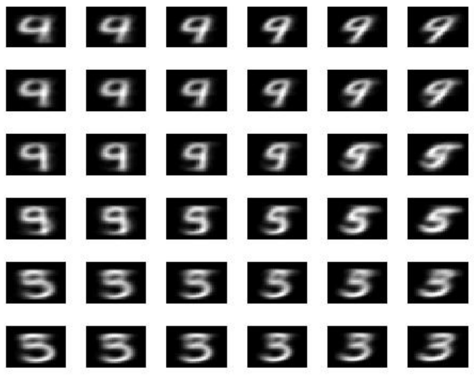
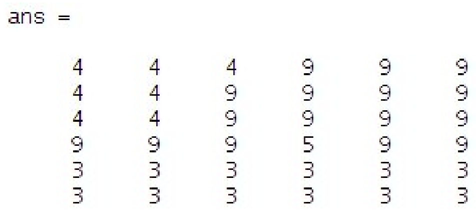
As a result, we have a matrix which contains the assigned digit to the corresponding prototype of the resulting figure matrix, as in figure 2.

4 σ and map size

In this section we will discuss the role of the width of the neighborhood function and the size of the Kohomen map.

4.1 Constant σ and map size

With different σ and size of map, we will get different resulting matrix. In order to descirbe and compare the qualty of the result map, we come up with a way to define the error rate :

```
ans =

     4     4     4     9     9     9
     4     4     9     9     9     9
     4     4     9     9     9     9
     9     9     9     5     9     9
     3     3     3     3     3     3
     3     3     3     3     3     3
```

Figure 2 – Prototypes and assigned digits

1. For each data point, we find the prototype which is closest to the data point, and assign the digit of the prototype to the data point
2. compare the digits of the data points to the labels, if the digit is not the same, we count this as an error.
3. finally, we get the error rate : $\frac{\#error}{\#dataPoints}$

For the test scheme, we firstly cut out dataset into training and test set with a $9:1$ ratio. Kohonen maps are constructed with the trainning data, then we apply them to classify data points in the test set to calculate the error rate mentioned earlier. In this way, the error rate can be regarded as a trustable measure of the map quality.

We have tried $\sigma = 1, 3, 5$ and $sizeK = 6, 7, 8$, the result is showed as table-1 :

	$\sigma = 1$	$\sigma = 3$	$\sigma = 5$
$sizeK = 6$	0.4665	0.4790	0.5360
$sizeK = 8$	0.3925	0.5270	0.5505
$sizeK = 10$	0.2635	0.5165	0.5765

Table 1 – error rate with different σ and $sizeK$

As a result, the optimal width does not depend on the size of the Kohomen map. For different sizes of the map, the smaller sigma is , the better quality the result has. That means the optimal sigma is always the smaller one, no matter how the size of the map changes. With the increase of the size of the Kohonen map, the quaity is improved. An intuitive interpretion of this is that each prototype covers fewer and fewer data points, which makes the prototype's position become more and more accurate during the convergence.

In fact, the sigma controls the strength and range of the neighborhood effect of the points of the map. With a small sigma, the algorithm is essentially the competitive learning algorithm, with possible dead units, because the initialization might not be good enough. With big sigmas, all prototypes learn the same thing, i.e. the center of the points.

For the map size, we've found that when width is small, it has a significant influence on the quality. However, when σ is bigger, the quality is dominated by this parameter, changing de map size do not alter much the error rate.

A propre size of the Kohonen map helps to prevent overifitting. Especially, in the unsupervised learning, it is better to fitting the data with a more or less regular grid that captures the essence of the underlying distribution than using a naive clustering algorithm with so many free parameters.

4.2 Variable σ

Finally, we adopt another approach that is to decrease the neighborhood width parameter with iterations.

We decrease this parameter with following scheme :

$$\sigma(n) = \sigma_0 \, exp(-\frac{n}{\tau_1}) \qquad n = 0, 1, 2... \tag{2}$$

moreover, we have

$$\tau_1 = \frac{1000}{log \, \sigma_0} \tag{3}$$

We fixed the initial width to a large value, say 10, and the result map is with very high quality even with a simple visual check. An interesting point is that if the initial value is not big enough, we'll have some badly-fitted prototype in the result map. This problem is probably due to the fact that the neighborhood of the winning neuron is decreasing with the runtime of the algorithm and if the initial value is not big enough, some unlucky prototype of the map may not be updated properly.

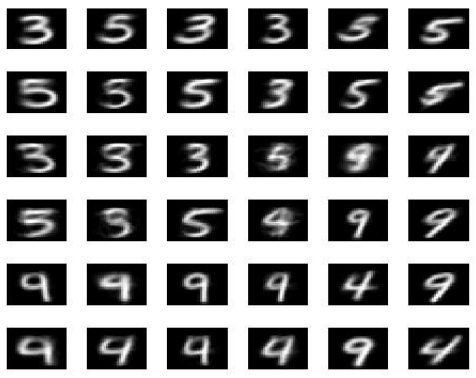


FIGURE 3 – Variable σ with large initial value

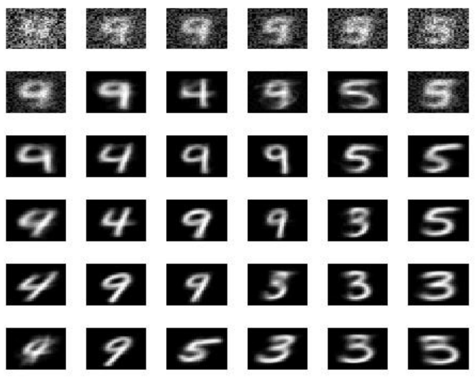


FIGURE 4 – Variable σ with small initial value

5 Conclusion

In this mini-project we've studied the self-organizing map by applying this technique to a concrete example. In the meanwhile, we analysed mainly the convergence condition of the map, as well as the influence of its different paratmeters to the final result. With this project, we fully understand the principle of self-organizing maps as a powerful tool in data analysis.