9 घातांक तथा करणी (SURDS AND INDICES)

सामान्य नियम

करणी : माना a एक परिमेय संख्या है तथा n एक धन-पूर्णांक है.

यदि a का nवाँ मूल एक अपरिमेय राशि हो तो $a^{1/n} = \sqrt[n]{a}$ को घात n की करणी कहा जाता है.

उदाहरण : (*i*) $\sqrt{3} = 3^{\frac{1}{2}}$, एक द्वितीय घात की करणी है.

(*ii*) $\sqrt[4]{5} = 5^{\frac{1}{4}}$, एक करणी है जिसकी घात 4 है.

घातांक के नियम (Laws of Indices) :

(*i*) $a^m \times a^n = a^{m+n}$ (*ii*) $(a^m) \div (a^n) = a^{m-n}$ (*iii*) $(a^m)^n = a^{mn}$

(*iv*) $(ab)^n = (a^n \times b^n)$ (*v*) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ (*vi*) $a^0 = 1$

करणी के नियम (Laws of Surds) :

(*i*) $\left(\sqrt[n]{a}\right)^n = \left(a^{1/n}\right)^n = a$ (*ii*) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ (*iii*) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

(*iv*) $\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ (*v*) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

साधित उदाहरण

प्रश्न 1. सरल कीजिए :

(*i*) $(81)^{\frac{3}{4}}$ (*ii*) $\left(\frac{1}{64}\right)^{\frac{-2}{3}}$ (*iii*) $(64)^{\frac{-1}{6}}$

हल : सरल करने पर :

(*i*) $(81)^{\frac{3}{4}} = \left(3^4\right)^{\frac{3}{4}} = 3^{\left(4\times\frac{3}{4}\right)} = 3^3 = 27.$

(*ii*) $\left(\frac{1}{64}\right)^{-\frac{2}{3}} = (64)^{\frac{2}{3}} = (4^3)^{\frac{2}{3}} = 4^{\left(3\times\frac{2}{3}\right)} = 4^2 = 16.$

(*iii*) $(64)^{-\frac{1}{6}} = \left(\frac{1}{64}\right)^{\frac{1}{6}} = \left(\frac{1}{2^6}\right)^{\frac{1}{6}} = \frac{1}{2^{\left(6\times\frac{1}{6}\right)}} = \frac{1}{2^1} = \frac{1}{2}.$

प्रश्न 2. $(\cdot 00032)^{-6} = ?$

हल : $(\cdot 00032)^{-6} = \left(\frac{32}{100000}\right)^{\frac{6}{10}} = \left(\frac{2^5}{10^5}\right)^{\frac{3}{5}} = \left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{3}{5}}$

$= \left(\frac{2}{10}\right)^{\left(5\times\frac{3}{5}\right)} = \left(\frac{1}{5}\right)^3 = (\cdot 2\times\cdot 2\times\cdot 2) = \cdot 008.$

प्रश्न 3. $\dfrac{(.625)^{.25}\times(25)^{2.6}}{(.625)^{.75}\times(5)^{1.2}}=?$

हल : दिया गया व्यंजक $=\dfrac{\left(5^4\right)^{.25}\times\left(5^2\right)^{2.6}}{\left(5^4\right)^{.75}\times(5)^{1.2}}=\dfrac{(5)^{(4\times.25)}\times5^{(2\times2.6)}}{(5)^{4\times.75}\times5^{(1.2)}}$

$=\dfrac{5^{25}\times5^{5.2}}{5^{27}\times5^{1.2}}=\dfrac{5^{(25+5.2)}}{5^{(27+1.2)}}=\dfrac{5^{30.2}}{5^{28.2}}=5^{(30.2-28.2)}$

$=5^2=25.$

प्रश्न 4. यदि $(\sqrt{3})^5\times9^2=3^n\times3\sqrt{3}$ हो, तो n का मान ज्ञात कीजिए.

हल : $(\sqrt{3})^5\times\left(3^2\right)^2=3^n\times3\times3^{\frac{1}{2}}\Rightarrow\left(3^{1.2}\right)^5\times3^4=3^n\times3\times3^{\frac{1}{2}}$

$\Rightarrow 3^{\left(\frac{5}{2}+4\right)}=3^{\left(n+1+\frac{1}{2}\right)}\Rightarrow3^{\frac{13}{2}}=3^{\left(n+\frac{3}{2}\right)}\Rightarrow n+\dfrac{3}{2}=\dfrac{13}{2}\Rightarrow n=\left(\dfrac{13}{2}-\dfrac{3}{2}\right)=\dfrac{10}{2}=5.$

अत: $n=5.$

प्रश्न 5. सरल कीजिए : $\left(\dfrac{x^a}{x^b}\right)^{(a^2+ab+b^2)}\cdot\left(\dfrac{x^b}{x^c}\right)^{(b^2+bc+c^2)}\cdot\left(\dfrac{x^c}{x^a}\right)^{(c^2+ca+a^2)}$

हल : दिया गया व्यंजक

$=x^{(a-b)(a^2+ab+b^2)}\cdot x^{(b-c)(b^2+bc+c^2)}\cdot x^{(c-a)(c^2+ca+a^2)}$

$=x^{(a^3-b^3)}\cdot x^{(b^3-c^3)}\cdot x^{(c^3-a^3)}=x^{(a^3-b^3+b^3-c^3+c^3-a^3)}=x^0=1.$

प्रश्न 6. $\sqrt{2}$ तथा $\sqrt[3]{3}$ में से कौन-सा बड़ा है?

हल : दी गई करणी क्रमश: घात 2 तथा 3 की हैं, जिनका ल०स० = 6. प्रत्येक को घात 6 की करणी में बदलने पर : $\sqrt{2}=2^{\frac{1}{2}}=\left(2^3\right)^{\frac{1}{6}}=(8)^{\frac{1}{6}},\qquad\sqrt[3]{3}=3^{\frac{1}{3}}=\left(3^2\right)^{\frac{1}{6}}=(9)^{\frac{1}{6}}.$

स्पष्ट है $(9)^{\frac{1}{6}}>(8)^{\frac{1}{6}}$. अत: $\sqrt[3]{3}>\sqrt{2}.$

प्रश्न 7. $\sqrt{2},\sqrt[3]{4},\sqrt[4]{6}$ को आरोही क्रम में लिखिए.

हल : दी गई करणियों की घात क्रमश: 2, 3, 4 हैं जिनका ल०स० = 12.

$\sqrt{2}=2^{\frac{1}{2}}=(2^6)^{\frac{1}{12}}=(64)^{\frac{1}{12}},\ \sqrt[3]{4}=4^{\frac{1}{3}}=(4^4)^{\frac{1}{12}}=(256)^{\frac{1}{12}}$

तथा $\sqrt[4]{6}=6^{\frac{1}{4}}=(6^3)^{\frac{1}{12}}=(216)^{\frac{1}{12}}.$

स्पष्ट है कि $(64)^{\frac{1}{12}}<(216)^{\frac{1}{12}}<(256)^{\frac{1}{12}}\Rightarrow\sqrt{2}<\sqrt[4]{6}<\sqrt[3]{4}.$

प्रश्न 8. यदि $3^{x-y}=27$ तथा $3^{x+y}=243$ हो, तो x का मान ज्ञात कीजिए.

हल : $3^{x-y}=27=3^3\Rightarrow x-y=3\quad\ldots(i)\qquad3^{x+y}=243=3^5\Rightarrow x+y=5\quad\ldots(ii)$

(i) तथा (ii) को जोड़ने पर, $2x=8\Rightarrow x=4.$

(ii) में से (i) घटाने पर, $2y=2\Rightarrow y=1.$

$\therefore x=4$ तथा $y=1.$

प्रश्नमाला 9A

निम्नलिखित प्रश्नों में से प्रत्येक में ठीक उत्तर को चिन्हांकित (✓) कीजिए :

1. यदि $\left(\frac{3}{5}\right)^{x} \cdot \left(\frac{3}{5}\right)^{-6} = \left(\frac{3}{5}\right)^{2x-1}$ हो, तो $x = ?$ **(एस०एस०सी० परीक्षा, 2010)**

(a) -2 (b) 2 (c) -1 (d) 1

2. $(2\times3)^{3} + (4\times9)^{2} \times (27\times8)^{2} = 6^{?}$ **(बैंक पी०ओ० परीक्षा, 2010)**

(a) 5 (b) 6 (c) 3 (d) 8 (e) इनमें से कोई नहीं

3. यदि $8^{x+1} = 64$ हो, तो 3^{2x+1} का मान क्या होगा ?

(a) 81 (b) 9 (c) 1 (d) 27

4. $(23)^{2\cdot8} \times (23)^{7\cdot2} \times (23)^{3\cdot6} = (23)^{?}$ **(बैंक पी०ओ० परीक्षा, 2009)**

(a) 13·6 (b) 12·6 (c) 12·8 (d) 13·8 (e) इनमें से कोई नहीं

5. $(21)^{?} \times (21)^{6\cdot5} = (21)^{12\cdot4}$ **(बैंक पी०ओ० परीक्षा, 2009)**

(a) 18·9 (b) 4·4 (c) 6·9 (d) 16·4 (e) इनमें से कोई नहीं

6. $(34)^{56} \times (34)^{-53} = ?$ **(बैंक पी०ओ० परीक्षा, 2009)**

(a) 39304 (b) 1156 (c) 170504 (d) 102 (e) इनमें से कोई नहीं

7. $(31)^{31} \times (31)^{-27} = ?$ **(बैंक पी०ओ० परीक्षा, 2008)**

(a) $(961)^2$ (b) $(31)^2$ (c) 29791 (d) 4 (e) इनमें से कोई नहीं

8. $(21)^{5\cdot5} \times (21)^{?} = (21)^{12}$

(a) 2·18 (b) $-17\cdot5$ (c) 6·5 (d) 5·5

9. $4^{2\cdot3} \times 2^{2\cdot3} \times 8^{1\cdot7} = ?$ **(बैंक पी०ओ० परीक्षा, 2008)**

(a) 64 (b) 512 (c) 16 (d) 4096 (e) इनमें से कोई नहीं

10. $8^{1\cdot3} \times 4^{0\cdot6} \times 16^{0\cdot2} = 2^{?}$ **(बैंक पी०ओ० परीक्षा, 2007)**

(a) 2·1 (b) 3·8 (c) 5·9 (d) 4·7 (e) इनमें से कोई नहीं

11. $2^{0\cdot2} \times 64 \times 8^{1\cdot3} \times 4^{0\cdot2} = 8^{?}$ **(बैंक पी०ओ० परीक्षा, 2007)**

(a) 2·7 (b) 2·5 (c) 3·7 (d) 3·2 (e) इनमें से कोई नहीं

12. $(1000)^{12} + (10)^{30} = ?$ **(बैंक पी०ओ० परीक्षा, 2008)**

(a) $(1000)^2$ (b) 10 (c) $(100)^2$ (d) 100 (e) इनमें से कोई नहीं

13. $\sqrt{(24)^4} + 224 = ? \times (20)^2$ **(बैंक पी०ओ० परीक्षा, 2008)**

(a) 20 (b) 4 (c) 2 (d) 16 (e) इनमें से कोई नहीं

14. $\left(3^{?}\right)^{?} = 19683$ **(बैंक पी०ओ० परीक्षा, 2007)**

(a) 6 (b) 9 (c) 4 (d) 8 (e) इनमें से कोई नहीं

15. यदि m तथा n ऐसे धनपूर्णांक हैं कि $m^n = 25$ हो, तो $n^m = ?$ **(बी०सी०ए० परीक्षा, 2008)**

(a) 4 (b) 10 (c) 25 (d) 32

16. यदि x तथा y ऐसी धनात्मक वास्तविक संख्यायें हों कि $x^{1/3} = y^{1/4}$ हो, तो :

(a) $x^3 = y^4$ (b) $x^3 = y$ (c) $x = y^4$ (d) $x^{20} = y^{15}$ **(एस०एस०सी० परीक्षा, 2006)**

17. यदि $\sqrt{3^n} = 81$ हो, तो $n = ?$
(a) 2 (b) 4 (c) 8 (d) 12

18. यदि $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$ हो, तो $x = ?$
(a) $\frac{3}{2}$ (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{3}{2}$

19. $8^{4.2} \times 64^{2.1} \times 7^{8.4} \times 56^{3.5} = (56)^?$
(a) 18·2 (b) 9·8 (c) 11·9 (d) 12·6 (e) इनमें से कोई नहीं
(बैंक पी०ओ० परीक्षा, 2005)

20. $(16)^{0.16} \times (16)^{0.04} \times (2)^{0.2} = ?$
(a) 1 (b) 2 (c) 4 (d) 16
(एस०एस०सी० परीक्षा, 2005)

21. $(8)^{11} \times (4)^{2.7} \times (2)^{3.3} = 2^?$
(a) 7·1 (b) 14 (c) ·5 (d) 9 (e) इनमें से कोई नहीं
(बैंक पी०ओ० परीक्षा, 2009)

22. $(9)^{8.6} \times (8)^{3.9} \times (72)^{4.4} \times (9)^{3.9} \times (8)^{8.6} = (72)^?$
(a) 15·1 (b) 29·4 (c) 20·9 (d) 17·9 (e) इनमें से कोई नहीं

23. $(1000)^9 \div (10)^{24} = ?$
(a) 100000 (b) 1000 (c) 10 (d) 100 (e) इनमें से कोई नहीं

24. $\left[3^{m^2} \div (3^m)^2\right]^{1/m} = 81$ हो, तो $m = ?$
(a) -3 (b) -6 (c) 3 (d) 6

25. यदि $\sqrt{4^n} = 1024$ हो, तो $n = ?$
(a) 5 (b) 8 (c) 12 (d) 10
(रेलवे परीक्षा, 2006)

26. $(27)^{2/3} \times (64)^{2/3} = ?$
(a) $\frac{1}{12}$ (b) $\frac{1}{144}$ (c) $\frac{1}{48}$ (d) 144

27. $(0{\cdot}6)^4 \times (0{\cdot}36)^2 \times (0{\cdot}216) = (0{\cdot}6)^?$
(a) 12 (b) 14 (c) 11 (d) 13 (e) इनमें से कोई नहीं

28. $(0{\cdot}00032)^{0.6} = ?$
(a) 0·08 (b) 0·008 (c) 0·8 (d) 8

29. $(64)^{3.1} \times 8^{4.3} = 8^?$
(a) 10·5 (b) 7·4 (c) 1·2 (d) 13·3 (e) इनमें से कोई नहीं

30. $\frac{3^{n+4} - 6 \times 3^{n+1}}{3^{n+2}} = ?$
(a) 7 (b) 5 (c) 4 (d) 1

31. $\frac{343 \times 49}{216 \times 16 \times 81} = ?$
(a) $\frac{7^5}{6^7}$ (b) $\frac{7^5}{6^8}$ (c) $\frac{7^6}{6^7}$ (d) $\frac{7^4}{6^8}$ (e) इनमें से कोई नहीं
(बैंक पी०ओ० परीक्षा, 2010)

32. $\frac{36 \times 18}{125 \times 75} = ?$
(a) $\frac{6^3}{5^4}$ (b) $\frac{6^2}{5^7}$ (c) $\frac{6^3}{5^5}$ (d) $\frac{6^2}{5^5}$ (e) इनमें से कोई नहीं
(बैंक पी०ओ० परीक्षा, 2010)

33. $\left\{\left(\sqrt[3]{x^{-3/5}}\right)^{-5/3}\right\}^5$ का सरलीकृत रूप है : (एस०एस०सी० परीक्षा, 2010)

(a) x^5 (b) x^{-5} (c) x (d) $\frac{1}{x}$ (e) इनमें से कोई नहीं

34. $(0{\cdot}1\times0{\cdot}01\times0{\cdot}001\times10^7)=?$ (एस०एस०सी० परीक्षा, 2010)

(a) 100 (b) $\frac{1}{10}$ (c) $\frac{1}{100}$ (d) 10

35. यदि $3^{x+y}=81$ तथा $(81)^{x-y}=3$ हो, तो $x=?$ (एस०एस०सी० परीक्षा, 2009)

(a) 42 (b) $\frac{15}{8}$ (c) $\frac{17}{8}$ (d) 39

36. यदि $2^{3\cdot6}\times4^{3\cdot6}\times4^{3\cdot6}\times32^{2\cdot3}=(32)^?$

(a) 5·9 (b) 9·5 (c) 7·7 (d) 13·1

37. यदि $x=2+2^{2/3}+2^{1/3}$ हो, तो $x^3-6x^2+6x=?$ (रेलवे परीक्षा, 2001)

(a) 1 (b) 3 (c) 2 (d) इनमें से कोई नहीं

38. यदि $x=5-5^{2/3}-5^{1/3}$ हो, तो $x^3-15x^2+60x-15=?$ (रेलवे परीक्षा, 2001)

(a) 20 (b) 15 (c) 5 (d) -5

39. यदि $2^x=3^y=6^{-z}$ हो, तो $\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=?$

(a) 0 (b) 1 (c) $-\frac{1}{2}$ (d) $\frac{3}{2}$

40. यदि $2^x=4^y=8^z$ हो, तथा $\left(\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}\right)=\frac{24}{7}$ हो, तो $z=?$

(a) $\frac{7}{16}$ (b) $\frac{7}{32}$ (c) $\frac{7}{48}$ (d) $\frac{7}{64}$

41. यदि $a^x=b^y=c^z$ तथा $b^2=ac$ हो, तो $y=?$

(a) $\frac{xz}{x+y}$ (b) $\frac{xz}{2(x-z)}$ (c) $\frac{xz}{2(z-x)}$ (d) $\frac{2xz}{(z+x)}$

42. यदि $2^{x-1}+2^{x+1}=320$ हो, तो x का मान है :

(a) 5 (b) 6 (c) 7 (d) 8

43. यदि $2^{x+4}-2^{x+2}=3$ हो, तो x का मान है :

(a) -2 (b) -1 (c) 0 (d) 2

44. यदि $2^a+3^b=17$ हो तथा $2^{a+2}-3^{b+1}=5$ हो, तो

(a) $a=2, b=3$ (b) $a=-2, b=3$ (c) $a=2, b=-3$ (d) $a=3, b=2$

45. यदि $\left(\frac{a}{b}\right)^{x-1}=\left(\frac{b}{a}\right)^{x-3}$ हो, तो $x=?$

(a) -1 (b) 1 (c) 2 (d) 3

46. यदि $x=y^a, y=z^b$ तथा $z=x^c$ हो, तो $abc=?$ (एम०बी०ए० परीक्षा, 2006)

(a) 1 (b) 2 (c) 3 (d) 4

47. यदि a, b, c वास्तविक संख्यायें हों, तो $\sqrt{a^{-1}b}\,.\,\sqrt{b^{-1}c}\,.\,\sqrt{c^{-1}a}=?$

(a) abc (b) $\frac{1}{abc}$ (c) $\sqrt{abc}$ (d) 1

48. $\frac{1}{1+a^{n-m}}+\frac{1}{1+a^{m-n}}=?$

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) a^{m+n}

49. यदि $abc=1$ हो, तो $\left(\frac{1}{1+a+b^{-1}}+\frac{1}{1+b+c^{-1}}+\frac{1}{1+c+a^{-1}}\right)$ का मान है :

(a) 0 (b) 1 (c) ab (d) $\frac{1}{ab}$

50. $\left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}\cdot\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}}\cdot\left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}}=?$

(a) 1 (b) $x^{\frac{1}{abc}}$ (c) $x^{\frac{1}{(ab+bc+ca)}}$ (d) इनमें से कोई नहीं

51. $\left(x^{b+c}\right)^{b-c}\cdot\left(x^{c+a}\right)^{c-a}\cdot\left(x^{a+b}\right)^{a-b}=?$

(a) 0 (b) 1 (c) x (d) इनमें से कोई नहीं

52. $\left(\frac{x^a}{x^b}\right)^{(a+b)}\cdot\left(\frac{x^b}{x^c}\right)^{(b+c)}\cdot\left(\frac{x^c}{x^a}\right)^{(c+a)}=?$

(a) 1 (b) 0 (c) x^{a+b+c} (d) x^{abc}

53. $\left(\frac{x^a}{x^b}\right)^{(a+b-c)}\cdot\left(\frac{x^b}{x^c}\right)^{(b+c-a)}\cdot\left(\frac{x^c}{x^a}\right)^{(c+a-b)}=?$

(a) 1 (b) x^{abc} (c) x^{a+b+c} (d) $x^{(ab+bc+ca)}$

54. $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{b-c}+x^{a-c}}=?$

(a) 0 (b) 1 (c) x^{a-b-c} (d) इनमें से कोई नहीं

55. यदि $a+\frac{1}{b}=1$ तथा $b+\frac{1}{c}=1$ हो, तो $c+\frac{1}{a}=?$

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

56. यदि $a=\frac{\sqrt{3}}{2}$ हो, तो $\sqrt{1+a}+\sqrt{1-a}$ का मान क्या होगा? **(एस०एस०सी० परीक्षा, 2007)**

(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $(2+\sqrt{3})$ (d) $(2-\sqrt{3})$

57. $\frac{(2^n+2^{n-1})}{(2^{n+1}-2^n)}$ का मान है : **(रेलवे परीक्षा, 2006)**

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $2^{\frac{(n-1)}{(n+1)}}$ (d) इनमें से कोई नहीं

58. यदि $2^{2x-1}=\frac{1}{8^{x-3}}$ हो, तो $x=?$ **(जीवन बीमा निगम परीक्षा, 2007)**

(a) -2 (b) -1 (c) 2 (d) 3

59. यदि $a^{2x+2}=1$, जहाँ a एक धनात्मक वास्तविक संख्या है तथा $a\neq 1$, तब $x=?$ **(एस०एस०सी० परीक्षा, 2007)**

(a) -2 (b) -1 (c) 0 (d) 1

60. यदि $\sqrt{24} = 4 \cdot 898$ हो, तो $\sqrt{\frac{8}{3}} = ?$ (एस०एस०सी० परीक्षा, 2007)

(a) 0·544 (b) 1·333 (c) 1·633 (d) 2·666

61. यदि $8^{x+1} = 64$ हो, तो 3^{2x+1} का मान क्या होगा? (रेलवे परीक्षा, 2006)

(a) 1 (b) 9 (c) 27 (d) 81

62. यदि $1 \cdot 5x = 0 \cdot 04y$ हो, तो $\frac{y-x}{y+x}$ का मान क्या होगा? (जीवन बीमा निगम परीक्षा, 2007)

(a) $\frac{730}{77}$ (b) $\frac{73}{77}$ (c) $\frac{73}{770}$ (d) $\frac{703}{77}$

63. यदि $4^x = \sqrt{2^{3y}}$ हो, तो : (रेलवे परीक्षा, 2006)

(a) $x = \frac{3}{4}y$ (b) $y = \frac{3}{4}x$ (c) $x = 3y$ (d) $x = \frac{1}{3}y$

64. यदि $x^{-\frac{4}{5}} = 81$ हो, तो $x = ?$

(a) $\frac{1}{243}$ (b) 243 (c) $\frac{1}{3}$ (d) 3

65. यदि $5^{(x+3)} = 25^{(3x-4)}$ हो, तो x का मान है :

(a) $\frac{5}{11}$ (b) $\frac{11}{5}$ (c) $\frac{11}{3}$ (d) $\frac{13}{5}$

66. यदि $(64)^{(2x-5)} = 4 \times 8^{(x-4)}$ हो, तो x का मान है :

(a) 2 (b) 11 (c) $\frac{10}{7}$ (d) $\frac{20}{9}$

67. यदि $\left(\frac{9}{4}\right)^x \cdot \left(\frac{8}{27}\right)^{(x-1)} = \frac{2}{3}$ हो, तो x का मान है :

(a) 1 (b) 2 (c) 3 (d) 4

68. $\left(64x^3 + 27a^{-3}\right)^{-\frac{2}{3}} = ?$ (रेलवे परीक्षा, 2006)

(a) $\frac{9ax}{16}$ (b) $\frac{9}{16ax}$ (c) $\frac{3}{4}x^{-2}a^{-2}$ (d) $\frac{9}{16x^2a^2}$

69. $\sqrt[3]{-8} \times \sqrt[4]{16} = ?$

(a) −4 (b) −2 (c) 2 (d) 4

70. $\left(\frac{1}{2}\right)^{-\frac{1}{2}} = ?$ (एस०एस०सी० परीक्षा, 2005)

(a) $\frac{1}{\sqrt{2}}$ (b) $2\sqrt{2}$ (c) $-\sqrt{2}$ (d) $\sqrt{2}$

71. $\left(\sqrt{8}\right)^{\frac{1}{3}} = ?$

(a) 2 (b) 4 (c) $\sqrt{2}$ (d) $2\sqrt{2}$

72. $\left(\frac{32}{243}\right)^{-\frac{4}{5}} = ?$

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{16}{81}$ (d) $\frac{81}{16}$

73. $\left(\frac{-1}{343}\right)^{-\frac{2}{3}} = ?$

(a) $\frac{-1}{49}$ (b) $\frac{1}{49}$ (c) -49 (d) 49

74. $\left(\frac{1}{216}\right)^{-\frac{2}{3}} + \left(\frac{1}{27}\right)^{-\frac{4}{3}} = ?$

(a) $\frac{3}{4}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{1}{8}$

75. यदि $(27)^{\frac{2}{3}} \times (81)^{-\frac{1}{2}} = 3^n$ हो, तो $n = ?$

(a) 0 (b) 1 (c) 27 (d) 81

76. $(16)^{1.75} = ?$

(a) 64 (b) $64\sqrt{2}$ (c) 128 (d) $128\sqrt{2}$

77. यदि $\sqrt{2^n} = 64$ हो, तो $n = ?$ (रेलवे परीक्षा, 2006)

(a) 2 (b) 4 (c) 6 (d) 12

78. $(256)^{0.16} \times (16)^{0.18} = ?$ (एस०एस०सी० परीक्षा, 2007)

(a) 4 (b) 16 (c) 64 (d) 256·25

79. $9^{8.6} \times 8^{3.9} \times (72)^{4.4} \times 9^{3.9} \times 8^{8.6} = (72)^?$ (जीवन बीमा निगम परीक्षा, 2005)

(a) 15·1 (b) 17·9 (c) 20·9 (d) 29·4 (e) इनमें से कोई नहीं

80. $6^{1.2} \times (36)^? \times (30)^{2.4} \times (25)^{1.3} = (30)^5$ (बैंक पी०ओ० परीक्षा, 2006)

(a) 0·1 (b) 0·7 (c) 1·4 (d) 2·6 (e) इनमें से कोई नहीं

81. $(0{\cdot}01024)^{\frac{1}{5}} = ?$

(a) 0·00004 (b) 0·04 (c) 0·4 (d) 4

82. $(3+\sqrt{5})$ का वर्गमूल है : (जीवन बीमा निगम परीक्षा, 2007)

(a) $\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}\right)$ (b) $\left(\frac{\sqrt{3}}{2} - \frac{1}{2}\right)$ (c) $\left(\frac{\sqrt{5}}{2} - \frac{1}{\sqrt{2}}\right)$ (d) $\left(\sqrt{\frac{5}{2}} + \sqrt{\frac{1}{2}}\right)$

83. यदि $x = (7 - 4\sqrt{3})$ हो, तो $\left(x + \frac{1}{x}\right)$ का मान क्या होगा? (रेलवे परीक्षा, 2005)

(a) $3\sqrt{3}$ (b) $8\sqrt{3}$ (c) $14 + 8\sqrt{3}$ (d) 14

84. यदि $x = (3 + \sqrt{8})$ हो, तो $\left(x^2 + \frac{1}{x^2}\right)$ का मान क्या होगा? (एस०एस०सी० परीक्षा, 2007)

(a) 38 (b) 36 (c) 34 (d) 30

85. $\sqrt{2}, \sqrt[3]{3}, \sqrt[4]{4}, \sqrt[6]{6}$ में से सबसे बड़ी संख्या कौन-सी है? (एस०एस०सी० परीक्षा, 2005)

(a) $\sqrt{2}$ (b) $\sqrt[3]{3}$ (c) $\sqrt[4]{4}$ (d) $\sqrt[6]{6}$

86. $\sqrt{3}, \sqrt[3]{4}, \sqrt[4]{6}, \sqrt[6]{8}$ में से सबसे बड़ी संख्या कौन-सी है? (एस०एस०सी० परीक्षा, 2006)

(a) $\sqrt{3}$ (b) $\sqrt[3]{4}$ (c) $\sqrt[4]{6}$ (d) $\sqrt[6]{8}$

87. $\sqrt[3]{2}, \sqrt{3}, \sqrt[3]{5}, 1{\cdot}5$ में से सबसे बड़ी संख्या कौन-सी है? (एस०एस०सी० परीक्षा, 2009)

(a) 1·5 (b) $\sqrt{3}$ (c) $\sqrt[3]{2}$ (d) $\sqrt[3]{5}$

88. $\sqrt{2}, \sqrt[3]{4}, \sqrt[4]{6}$ को बढ़ते क्रम में लिखने पर : (ए०ए०ओ० परीक्षा, 2006)

(a) $\sqrt{2} < \sqrt[3]{4} < \sqrt[4]{6}$ (b) $\sqrt[4]{6} < \sqrt{2} < \sqrt[3]{4}$

(c) $4\sqrt{6} < \sqrt[3]{4} < \sqrt{2}$ (d) $\sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}$

89. $2^{\frac{1}{3}}, 3^{\frac{1}{3}}, 8^{\frac{1}{8}}$ तथा $9^{\frac{1}{9}}$ में से कौन-सी संख्या सबसे बड़ी है ? (ए०ए०ओ० परीक्षा, 2010)

(a) $2^{\frac{1}{3}}$ (b) $3^{\frac{1}{3}}$ (c) $8^{\frac{1}{8}}$ (d) $9^{\frac{1}{9}}$

90. $\sqrt{8-2\sqrt{15}} = ?$ (एस०एस०सी० परीक्षा, 2007)

(a) $(\sqrt{5}+\sqrt{3})$ (b) $(5-\sqrt{3})$ (c) $(\sqrt{5}-\sqrt{3})$ (d) $(3-\sqrt{5})$

91. $\left\{(-2)^{(-2)}\right\}^{(-2)} = ?$ (एस०एस०सी० परीक्षा, 2005)

(a) – 16 (b) – 8 (c) 8 (d) 16

92. यदि $\sqrt{3} = 1 \cdot 732$ हो, तो $\dfrac{4+3\sqrt{3}}{\sqrt{7+4\sqrt{3}}} = ?$ (एस०एस०सी० परीक्षा, 2005)

(a) ·023 (b) ·464 (c) 2·464 (d) 3·023

93. यदि $\sqrt{3} = 1 \cdot 732$ हो, तो $\dfrac{(3+\sqrt{6})}{(5\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50})} = ?$ (एस०एस०सी० परीक्षा, 2007)

(a) 4·899 (b) 2·551 (c) 1·414 (d) 1·732

94. $\dfrac{12}{(3+\sqrt{5}+2\sqrt{2})} = ?$ (एस०एस०सी० परीक्षा, 2010)

(a) $1-\sqrt{5}+\sqrt{2}+\sqrt{10}$ (b) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$

(c) $1+\sqrt{5}-\sqrt{2}+\sqrt{10}$ (d) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

95. $\dfrac{\sqrt{7}}{\sqrt{16+6\sqrt{7}}-\sqrt{16-6\sqrt{7}}} = ?$ (एस०एस०सी० परीक्षा, 2010)

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$

96. यदि $a = \sqrt{7+2\sqrt{12}}$ तथा $b = \sqrt{7-2\sqrt{12}}$ हो, तो $(a^3+b^3) = ?$ (एस०एस०सी० परीक्षा, 2010)

(a) 40 (b) 44 (c) 48 (d) 52

97. यदि $x^{1/3} + y^{1/3} = z^{1/3}$ हो, तो $\left\{(x+y-z)^3 + 27xyz\right\} = ?$ (एस०एस०सी० परीक्षा, 2007)

(a) – 1 (b) 1 (c) 0 (d) 27

98. 2^{50} का दुगुना क्या होगा ?

(a) 2^{100} (b) 2^{55} (c) 2^{51} (d) इनमें से कोई नहीं

99. $\left(\dfrac{5^{-1}\times 7^2}{5^2\times 7^{-4}}\right)^{7/2} \times \left(\dfrac{5^{-2}\times 7^3}{5^3\times 7^{-5}}\right)^{\frac{-5}{2}} = ?$

(a) 150 (b) 125 (c) 175 (d) 90

100. $\left\{\frac{\frac{m}{x^{m-n}}}{\frac{m}{x^{m+n}}} \times \frac{\frac{n}{x^{n-m}}}{\frac{n}{x^{n+m}}}\right\}^{m+n} = ?$

(a) $\frac{x^m}{x^n}$ (b) $\frac{x^n}{x^m}$ (c) 1 (d) 2

101. यदि $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ तथा $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$ हो, तो $\frac{(a^2+ab+b^2)}{(a^2-ab+b^2)} = ?$ (एस०एस०सी० परीक्षा, 2005)

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{5}{3}$

102. यदि $x = (3+2\sqrt{2})$ हो, तो $\left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$ का मान कितना होगा?

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) $3\sqrt{3}$

103. $\frac{1}{(1+\sqrt{2})} + \frac{1}{(\sqrt{2}+\sqrt{3})} + \frac{1}{(\sqrt{3}+\sqrt{4})} + ... + \frac{1}{(\sqrt{99}+\sqrt{100})} = ?$ (एस०एस०सी० परीक्षा, 2006)

(a) 1 (b) 5 (c) 9 (d) 10

104. $\frac{1}{(\sqrt{3}+\sqrt{4})} + \frac{1}{(\sqrt{4}+\sqrt{5})} + \frac{1}{(\sqrt{5}+\sqrt{6})} + \frac{1}{(\sqrt{6}+\sqrt{7})} + \frac{1}{(\sqrt{7}+\sqrt{8})} + \frac{1}{(\sqrt{8}+\sqrt{9})} = ?$

(a) $\sqrt{3}$ (b) $3\sqrt{3}$ (c) $(3-\sqrt{3})$ (d) $5-\sqrt{3}$

(एस०एस०सी० परीक्षा, 2005)

105. $\frac{1}{(\sqrt{100}-\sqrt{99})} - \frac{1}{(\sqrt{99}-\sqrt{98})} + \frac{1}{(\sqrt{98}-\sqrt{97})} - ... - \frac{1}{(\sqrt{3}-\sqrt{2})} + \frac{1}{(\sqrt{2}-\sqrt{1})} = ?$

(a) 0 (b) 9 (c) 10 (d) 11

(एस०एस०सी० परीक्षा, 2005)

उत्तरमाला (प्रश्नमाला 9A)

1. (c)	2. (a)	3. (d)	4. (a)	5. (e)	6. (a)	7. (a)	8. (c)	9. (d)	10. (c)
11. (e)	12. (a)	13. (c)	14. (e)	15. (d)	16. (d)	17. (c)	18. (a)	19. (c)	20. (b)
21. (e)	22. (e)	23. (b)	24. (d)	25. (d)	26. (d)	27. (c)	28. (b)	29. (a)	30. (a)
31. (a)	32. (c)	33. (c)	34. (d)	35. (c)	36. (a)	37. (c)	38. (c)	39. (a)	40. (c)
41. (d)	42. (c)	43. (a)	44. (d)	45. (c)	46. (a)	47. (d)	48. (c)	49. (b)	50. (a)
51. (b)	52. (a)	53. (a)	54. (b)	55. (a)	56. (a)	57. (b)	58. (c)	59. (b)	60. (c)
61. (c)	62. (b)	63. (a)	64. (a)	65. (b)	66. (d)	67. (d)	68. (d)	69. (a)	70. (d)
71. (c)	72. (d)	73. (d)	74. (c)	75. (a)	76. (c)	77. (d)	78. (a)	79. (e)	80. (b)
81. (c)	82. (d)	83. (d)	84. (c)	85. (b)	86. (a)	87. (b)	88. (d)	89. (b)	90. (c)
91. (d)	92. (c)	93. (d)	94. (b)	95. (a)	96. (d)	97. (c)	98. (c)	99. (c)	100. (c)
101. (b)	102. (b)	103. (c)	104. (c)	105. (d)					

दिये गये प्रश्नों के हल : प्रश्नमाला 9A

1. $\left(\frac{3}{5}\right)^{3+(-6)} = \left(\frac{3}{5}\right)^{2x-1} \Rightarrow 2x-1=-3 \Rightarrow 2x=-2 \Rightarrow x=-1.$

2. $(6)^3+(36)^2\times(216)^2 = 6^x$

$\Rightarrow 6^3+(6^2)^2\times(6^3)^2 = 6^x \Rightarrow 6^3+6^4\times6^6 = 6^x$

$\Rightarrow 6^3\times\frac{1}{6^4}\times6^6 = 6^x \Rightarrow 6^{(3+6-4)} = 6^x \Rightarrow 6^5 = 6^x \Rightarrow x=5.$

3. $8^{x+1} = 64 = 8^2 \Rightarrow x+1=2 \Rightarrow x=1.$

$\therefore\ 3^{2x+1} = 3^{(2\times1)+1} = 3^3 = 27.$

4. $(23)^{2\cdot8+7\cdot2+3\cdot6} = (23)^x \Rightarrow (23)^{13\cdot6} = (23)^x \Rightarrow x = 13\cdot6.$

5. माना $(21)^x\times(21)^{6\cdot5} = (21)^{12\cdot4}$, तब $(21)^{x+6\cdot5} = (21)^{12\cdot4}$

$\therefore\ x+6\cdot5 = 12\cdot4 \Rightarrow x = (12\cdot4-6\cdot5) = 5\cdot9.$

6. $(34)^{56}\times(34)^{-53} = (34)^{56-53} = (34)^3 = 39304.$

7. $(31)^{31}\times(31)^{-27} = (31)^{31+(-27)} = (31)^4 = (961)^2.$

8. माना $(21)^{5\cdot5}\times(21)^x = (21)^{12}$, तब $(21)^{5\cdot5+x} = (21)^{12}$.

$\therefore\ 5\cdot5+x = 12 \Rightarrow x = (12-5\cdot5) = 6\cdot5.$

9. $4^{2\cdot3}\times2^{2\cdot3}\times8^{1\cdot7} = (2^2)^{2\cdot3}\times2^{2\cdot3}\times(2^3)^{1\cdot7}$

$= 2^{4\cdot6}\times2^{2\cdot3}\times2^{5\cdot1} = 2^{(4\cdot6+2\cdot3+5\cdot1)} = 2^{12} = 4096.$

10. $8^{1\cdot3}\times4^{0\cdot6}\times16^{0\cdot2} = (2^3)^{1\cdot3}\times(2^2)^{0\cdot6}\times(2^4)^{0\cdot2}$

$= 2^{3\cdot9}\times2^{1\cdot2}\times2^{0\cdot8} = 2^{(3\cdot9+1\cdot2+0\cdot8)} = 2^{5\cdot9}.$

11. $2^{0\cdot2}\times64\times8^{1\cdot3}\times4^{0\cdot2} = 8^x$

$\Rightarrow (2\times4)^{0\cdot2}\times8^2\times8^{1\cdot3} = 8^x \Rightarrow 8^{0\cdot2+2+1\cdot3} = 8^x \Rightarrow x = 3\cdot5.$

12. $\frac{(1000)^{12}}{(10)^{30}} = \frac{(10^3)^{12}}{(10)^{30}} = \frac{(10)^{36}}{(10)^{30}} = (10)^{(36-30)} = 10^6 = (10^3)^2 = (1000)^2.$

13. $\sqrt{(24)^4}+224 = x\times(20)^2$

$\Rightarrow (24)^2+224 = x\times400 \Rightarrow 576+224 = x\times400$

$\Rightarrow x\times400 = 800 \Rightarrow x = 2.$

14. माना $(3^x)^x = 19683 \Rightarrow 3^{x^2} = 3^3\times3^6 = 3^9$

$\Rightarrow x^2 = 9 \Rightarrow x = 3.$

3	19683
3	6561
3	2187
9	729
9	81
	9

15. $5^2 = 25 \Rightarrow m = 5$ तथा $n = 2.$

$\Rightarrow n^m = 2^5 = 32.$

16. $x^{1/3} = y^{1/4} \Rightarrow \left(x^{1/3}\right)^{12} = \left(y^{\frac{1}{4}}\right)^{12}$ [$\because$ 3 तथा 4 का ल०स० = 12]

$\Rightarrow x^4 = y^3 \Rightarrow (x^4)^5 = (y^3)^5 \Rightarrow x^{20} = y^{15}.$

17. $\sqrt{3^n} = 81 \Rightarrow \left(3^n\right)^{\frac{1}{2}} = 3^4 \Rightarrow 3^{\frac{n}{2}} = 3^4 \Rightarrow \frac{n}{2} = 4 \Rightarrow n = 8.$

18. $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1} \Rightarrow 2^{2x} + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}}$

$\Rightarrow 2^{(2x-1)}(2+1) = 3^{\left(x-\frac{1}{2}\right)}.(3+1)$

$\Rightarrow \frac{2^{(2x-1)}}{2^2} = \frac{3^{\left(x-\frac{1}{2}\right)}}{3}$

$\Rightarrow 2^{(2x-3)} = 3^{\left(x-\frac{3}{2}\right)}$

$\Rightarrow 2x-3=0$ तथा $x-\frac{3}{2}=0 \Rightarrow x=\frac{3}{2}$.

19. $8^{4\cdot2} \times (8^2)^{2\cdot1} \times 7^{8\cdot4} \times (56)^{3\cdot5} = (56)^x$

$\Rightarrow 8^{4\cdot2} \times 8^{4\cdot2} \times 7^{8\cdot4} \times (56)^{3\cdot5} = (56)^x$

$\Rightarrow 8^{8\cdot4} \times 7^{8\cdot4} \times (56)^{3\cdot5} = (56)^x \Rightarrow (8\times7)^{8\cdot4} \times (56)^{3\cdot5} = (56)^x$

$\Rightarrow (56)^{8\cdot4+3\cdot5} = (56)^x \Rightarrow x = 11\cdot9$.

20. $(16)^{0\cdot16} \times (16)^{0\cdot04} \times (2)^{0\cdot2} = (2^4)^{0\cdot16} \times (2^4)^{0\cdot04} \times (2)^{0\cdot2}$

$= 2^{0\cdot64} \times 2^{0\cdot16} \times 2^{0\cdot2} = 2^{(0\cdot8+0\cdot2)} = 2^1 = 2$.

21. $(8)^{1\cdot1} \times (4)^{2\cdot7} \times (2)^{3\cdot3} = (2^3)^{1\cdot1} \times (2^2)^{2\cdot7} \times (2)^{3\cdot3}$

$= 2^{3\cdot3} \times 2^{5\cdot4} \times 2^{3\cdot3} = 2^{(3\cdot3+5\cdot4+3\cdot3)} = 2^{12}$.

22. $(9)^{8\cdot6} \times (8)^{3\cdot9} \times (72)^{4\cdot4} \times (9)^{3\cdot9} \times (8)^{8\cdot6} = (72)^x$

$\Rightarrow (9\times8)^{8\cdot6} \times (8\times9)^{3\cdot9} \times (72)^{4\cdot4} = (72)^x$

$\Rightarrow (72)^{8\cdot6} \times (72)^{3\cdot9} \times (72)^{4\cdot4} = (72)^x \Rightarrow (72)^{8\cdot6+3\cdot9+4\cdot4} = (72)^x$

$\Rightarrow x = (8\cdot6+3\cdot9+4\cdot4) = 16\cdot9$.

23. $(1000)^9 + (10)^{24} = \frac{(1000)^9}{(10)^{24}} = \frac{\left\{(10)^3\right\}^9}{(10)^{24}} = \frac{10^{(3\times9)}}{(10)^{24}} = \frac{(10)^{27}}{(10)^{24}}$

$= (10)^{(27-24)} = (10)^3 = 1000$.

24. $\left\{\frac{3^{m^2}}{3^{2m}}\right\}^{\frac{1}{m}} = 81 \Rightarrow 3^{(m^2-2m)\times\frac{1}{m}} = 3^4$

$\Rightarrow 3^{(m-2)} = 3^4 \Rightarrow m-2=4 \Rightarrow m=6$.

25. $\sqrt{4^n} = 1024 = 4^5 \Rightarrow \left(4^n\right)^{\frac{1}{2}} = 4^5 \Rightarrow 4^{n/2} = 4^5$

$\Rightarrow \frac{n}{2} = 5 \Rightarrow n = 10$.

26. $(27)^{2/3} \times (64)^{2/3} = \left(3^3\right)^{\frac{2}{3}} \times \left(4^3\right)^{\frac{2}{3}}$

$= 3^{\left(3\times\frac{2}{3}\right)} \times 4^{\left(3\times\frac{2}{3}\right)} = (3^2 \times 4^2) = (9\times16) = 144$.

27. $(0\cdot6)^4 \times (0\cdot36)^2 \times (0\cdot216) = (0\cdot6)^x$

$\Rightarrow (0\cdot6)^4 \times \left\{(0\cdot6)^2\right\}^2 \times (0\cdot6)^3 = (0\cdot6)^x$

$\Rightarrow (0\cdot6)^4\times(0\cdot6)^4\times(0\cdot6)^3=(0\cdot6)^x \Rightarrow (0\cdot6)^{(4+4+3)}=(0\cdot6)^x$

$\Rightarrow x=(4+4+3)=11.$

28. $(0\cdot00032)^{0\cdot6}=\left(\frac{32}{10^5}\right)^{0\cdot6}=\left(\frac{2^5}{10^5}\right)^{0\cdot6}$

$=\frac{2^{(5\times0\cdot6)}}{10^{(5\times0\cdot6)}}=\frac{2^3}{10^3}=\frac{8}{1000}=0\cdot008.$

29. $(64)^{3\cdot1}\times8^{4\cdot3}=8^x \Rightarrow (8^2)^{3\cdot1}\times8^{4\cdot3}=8^x$

$\Rightarrow 8^{6\cdot2}\times8^{4\cdot3}=8^x \Rightarrow 8^{(6\cdot2+4\cdot3)}=8^x \Rightarrow x=10\cdot5.$

30. $\frac{3^{n+4}-6\times3^{n+1}}{3^{n+2}}=\frac{3^{n+4}-2\times3\times3^{n+1}}{3^{n+2}}=\frac{3^{n+4}-2\times3^{n+2}}{3^{n+2}}$

$=\frac{3^{n+2}\cdot[3^2-2]}{3^{n+2}}=(9-2)=7.$

31. $\frac{343\times49}{216\times16\times81}=\frac{7^3\times7^2}{6^3\times2^4\times3^4}=\frac{7^{(3+2)}}{6^3\times(2\times3)^4}=\frac{7^5}{6^3\times6^4}=\frac{7^5}{6^{(3+4)}}=\frac{7^5}{6^7}.$

32. $\frac{36\times18}{125\times75}=\frac{6\times6\times6\times3}{5^3\times5^2\times3}=\frac{6\times6\times6}{5^3\times5^2}=\frac{6^3}{5^5}.$

33. दिया गया व्यंजक $=\left[\left\{\left(x^{-3/5}\right)^{\frac{1}{5}}\right\}^{\frac{-5}{3}}\right]^5$

$=\left[\left\{x^{\left(\frac{-3}{5}\times\frac{1}{5}\right)}\right\}^{\frac{-5}{3}}\right]^5=\left[x^{\left\{\frac{-3}{25}\times\frac{(-5)}{3}\right\}}\right]^5$

$=x^{\left(\frac{1}{5}\times5\right)}=x^1=x.$

34. दिया गया व्यंजक $=\left(\frac{1}{10}\times\frac{1}{100}\times\frac{1}{1000}\times10^7\right)=\frac{1}{10\times10^2\times10^3}\times10^7$

$=\frac{1}{(10)^{(1+2+3)}}\times10^7=\frac{10^7}{10^6}=10^{(7-6)}=10.$

35. $3^{x+y}=81$ तथा $(81)^{x-y}=3$

$\Rightarrow 3^{x+y}=3^4$ तथा $(3^4)^{x-y}=3=3^1$

$\Rightarrow x+y=4$ तथा $4x-4y=1$

$\Rightarrow 4x+4y=16$ तथा $4x-4y=1 \Rightarrow x=\frac{17}{8}.$

36. $2^{3\cdot6}\times4^{3\cdot6}\times4^{3\cdot6}\times32^{2\cdot3}=(2\times4\times4)^{3\cdot6}\times(32)^{2\cdot3}$

$=(32)^{3\cdot6}\times(32)^{2\cdot3}=(32)^{(3\cdot6+2\cdot3)}=(32)^{5\cdot9}$

$\therefore x=5\cdot9.$

37. $(x-2)=2^{2/3}+2^{1/3} \Rightarrow (x-2)^3=(2^{2/3}+2^{1/3})^3$

$\Rightarrow x^3-8-6x(x-2)=2^2+2+3\times2^{2/3}\times2^{1/3}\left(2^{2/3}+2^{1/3}\right)$

$\Rightarrow x^3-6x^2+12x-8=6+3\times2\,(x-2)$

$\Rightarrow x^3-6x^2+12x-8=6+6x-12$

$\Rightarrow x^3-6x^2+6x=2.$

38. $(5-x)=(5^{2/3}+5^{1/3}) \Rightarrow (5-x)^3=(5^{2/3}+5^{1/3})^3$

$\Rightarrow 5^3-x^3-15x(5-x)=5^2+5+3\times5^{2/3}\times5^{1/3}\,(5^{2/3}+5^{1/3})$

$\Rightarrow 125-x^3-75x+15x^2=25+5+3\times5\times(5-x)$

$\Rightarrow 125-x^3-75x+15x^2=30+75-15x$

$\Rightarrow x^3-15x^2+60x-15=5$.

39. माना $2^x=3^y=6^{-z}=k$. तब

$2=(k)^{\frac{1}{x}}, 3=(k)^{\frac{1}{y}}$ तथा $6=(k)^{\frac{-1}{z}}$.

अब, $2\times3=6 \Rightarrow (k)^{\frac{1}{x}}\times(k)^{\frac{1}{y}}=(k)^{\frac{-1}{z}}$

$\Rightarrow (k)^{\left(\frac{1}{x}+\frac{1}{y}\right)}=(k)^{-\frac{1}{z}} \Rightarrow \frac{1}{x}+\frac{1}{y}=\frac{-1}{z} \Rightarrow \left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=0.$

40. $2^x=4^y=8^z \Rightarrow 2^x=2^{2y}=2^{3z} \Rightarrow x=2y=3z.$

$\frac{1}{2x}+\frac{1}{4y}+\frac{1}{6z}=\frac{24}{7} \Rightarrow \frac{1}{6z}+\frac{1}{6z}+\frac{1}{6z}=\frac{24}{7} \Rightarrow \frac{3}{6z}=\frac{24}{7} \Rightarrow \frac{1}{z}=\left(\frac{24}{7}\times2\right)=\frac{48}{7}.$

अत: $z=\frac{7}{48}$.

41. माना $a^x=b^y=c^z=k$. तब, $a=(k)^{\frac{1}{x}}, b=(k)^{\frac{1}{y}}$ तथा $c=(k)^{\frac{1}{z}}$.

$b^2=ac \Rightarrow (k)^{\frac{2}{y}}=k^{\frac{1}{x}}\times k^{\frac{1}{z}} \Rightarrow \frac{2}{y}=\frac{1}{x}+\frac{1}{z} \Rightarrow y=\frac{2xz}{(z+x)}.$

42. $2^{x-1}+2^{x+1}=320 \Rightarrow 2^{x-1}(1+2^2)=320$

$\Rightarrow 5\times2^{x-1}=320 \Rightarrow 2^{x-1}=64=2^6$

$\Rightarrow x-1=6 \Rightarrow x=7.$

43. $2^{x+4}-2^{x+2}=3 \Rightarrow 2^{x+2}\,(2^2-1)=3 \Rightarrow 2^{x+2}=1=2^0$

$\Rightarrow x+2=0 \Rightarrow x=-2.$

44. $2^a=x$ तथा $3^b=y$ रखने पर: $x+y=17$ ...(*i*)

$2^2\times2^a-3\times3^b=5 \Rightarrow 4x-3y=5$ ...(*ii*)

इन्हें हल करने पर $x=8$ तथा $y=9$.

$\therefore 2^a=8=2^3$ तथा $3^b=9=3^2 \Rightarrow a=3$ तथा $b=2$.

45. $\left(\frac{a}{b}\right)^{x-1}=\left(\frac{b}{a}\right)^{x-3}=\left(\frac{a}{b}\right)^{3-x} \Rightarrow x-1=3-x \Rightarrow 2x=4 \Rightarrow x=2.$

46. $x=y^a \Rightarrow x=(z^b)^a=(z)^{ba}=(x^c)^{ba}=x^{cba}$

$\Rightarrow cba=1 \Rightarrow abc=1.$

47. $\sqrt{a^{-1}b}\cdot\sqrt{b^{-1}c}\cdot\sqrt{c^{-1}a}=\sqrt{a^{-1}b\cdot b^{-1}c\cdot c^{-1}a}=1.$

48. दिया गया व्यंजक $=\frac{1}{1+\frac{a^n}{a^m}}+\frac{1}{1+\frac{a^m}{a^n}}=\frac{a^m}{(a^m+a^n)}+\frac{a^n}{(a^n+a^m)}$

$=\frac{(a^m+a^n)}{(a^m+a^n)}=1.$

49. दिया गया व्यंजक $= \dfrac{1}{(1+a+b^{-1})} + \dfrac{b^{-1}}{b^{-1}+1+b^{-1}c^{-1}} + \dfrac{a}{a+ac+1}$

$= \dfrac{1}{(1+a+b^{-1})} + \dfrac{a}{1+a+ac} + \dfrac{b^{-1}}{1+b^{-1}+b^{-1}c^{-1}}$

$= \dfrac{1}{(1+a+b^{-1})} + \dfrac{a}{(1+a+b^{-1})} + \dfrac{b^{-1}}{(1+a+b^{-1})}$ $\left[\because\ abc=1 \Rightarrow b^{-1}=ac,\ (cb)^{-1}=a \Rightarrow b^{-1}c^{-1}=a\right]$

$= \dfrac{(1+a+b^{-1})}{(1+a+b^{-1})} = 1.$

50. दिया गया व्यंजक $= \left(x^{a-b}\right)^{\frac{1}{ab}} \cdot \left(x^{b-c}\right)^{\frac{1}{bc}} \cdot \left(x^{c-a}\right)^{\frac{1}{ca}}$

$= x^{\left(\frac{a-b}{ab}\right)} \cdot x^{\left(\frac{b-c}{bc}\right)} \cdot x^{\left(\frac{c-a}{ca}\right)} \cdot x^{\left(\frac{1}{b}-\frac{1}{a}\right)} \cdot x^{\left(\frac{1}{c}-\frac{1}{b}\right)} \cdot x^{\left(\frac{1}{a}-\frac{1}{c}\right)}$

$= x^{\left(\frac{1}{b}-\frac{1}{a}+\frac{1}{c}-\frac{1}{b}+\frac{1}{a}-\frac{1}{c}\right)} = x^0 = 1.$

51. दिया गया व्यंजक $= x^{(b+c)(b-c)} \cdot x^{(c+a)(c-a)} \cdot x^{(a+b)(a-b)}$

$= x^{(b^2-c^2)} \cdot x^{(c^2-a^2)} \cdot x^{(a^2-b^2)}$

$= x^{(b^2-c^2+c^2-a^2+a^2-b^2)} = x^0 = 1.$

52. दिया गया व्यंजक $= x^{(a-b)(a+b)} \cdot x^{(b-c)(b+c)} \cdot x^{(c-a)(c+a)}$

$= x^{(a^2-b^2)} \cdot x^{(b^2-c^2)} \cdot x^{(c^2-a^2)}$

$= x^{(a^2-b^2+b^2-c^2+c^2-a^2)} = x^0 = 1.$

53. दिया गया व्यंजक $= x^{(a-b)(a+b-c)} \cdot x^{(b-c)(b+c-a)} \cdot x^{(c-a)(c+a-b)}$

$= x^{(a-b)(a+b)} \cdot x^{-c(a-b)} \cdot x^{(b-c)(b+c)} \cdot x^{-a(b-c)} \cdot x^{(c-a)(c+a)} \cdot x^{-b(c-a)}$

$= x^{(a^2-b^2+b^2-c^2+c^2-a^2)} \cdot x^{-ca+bc-ab+ca-bc+ab} = (x^0 \times x^0) = (1 \times 1) = 1.$

54. दिया गया व्यंजक $= \dfrac{1}{1+\dfrac{x^b}{x^a}+\dfrac{x^c}{x^a}} + \dfrac{1}{1+\dfrac{x^a}{x^b}+\dfrac{x^c}{x^b}} + \dfrac{1}{1+\dfrac{x^b}{x^c}+\dfrac{x^a}{x^c}}$

$= \dfrac{x^a}{(x^a+x^b+x^c)} + \dfrac{x^b}{(x^b+x^a+x^c)} + \dfrac{x^c}{(x^c+x^b+x^a)}$

$= \dfrac{(x^a+x^b+x^c)}{(x^a+x^b+x^c)} = 1.$

55. $\left\{\dfrac{1}{c} = (1-b) \Rightarrow c = \dfrac{1}{(1-b)}\right\}, \left\{a = \left(1-\dfrac{1}{b}\right) = \dfrac{(b-1)}{b} \Rightarrow \dfrac{1}{a} = \dfrac{b}{(b-1)}\right\}$

$\left\{\therefore \left(c+\dfrac{1}{a}\right) = \dfrac{1}{(1-b)} + \dfrac{b}{(b-1)} = \dfrac{1}{(1-b)} - \dfrac{b}{(1-b)} = \dfrac{(1-b)}{(1-b)} = 1\right\}$

56. माना $p = \sqrt{1+a} + \sqrt{1-a}$. तब, $p^2 = (1+a)+(1-a)+2\sqrt{1-a^2}$

$\Rightarrow p^2 = 2+2\sqrt{1-a^2} = 2+2\sqrt{1-\dfrac{3}{4}} = 2+2\times\dfrac{1}{2} = (2+1) = 3 \Rightarrow p = \sqrt{3}.$

57. $\frac{\left(2^n+2^{n-1}\right)}{\left(2^{n+1}-2^n\right)}=\frac{2^{n-1}(2+1)}{2^n(2-1)}=\frac{3}{2}$.

58. $2^{2x-1}=\frac{1}{(2^3)^{x-3}}=\frac{1}{2^{(3x-9)}}=2^{(9-3x)}$

$\therefore 2x-1=9-3x \Rightarrow 5x=10 \Rightarrow x=2$.

59. $a^{2x+2}=1=a^0 \Rightarrow 2x+2=0 \Rightarrow 2x=-2 \Rightarrow x=-1$.

60. $\sqrt{\frac{8}{3}}=\sqrt{\frac{8\times3}{3\times3}}=\frac{\sqrt{24}}{3}=\frac{1}{3}\times4\cdot898=1\cdot633$.

61. $8^{x+1}=64=8^2 \Rightarrow x+1=2 \Rightarrow x=1$.

$\therefore 3^{2x+1}=3^{(2\times1+1)}=3^3=27$.

62. $\frac{y}{x}=\frac{1\cdot50}{0\cdot04}=\frac{150}{4}=\frac{75}{2}$.

$\therefore \frac{y-x}{y+x}=\frac{\frac{y}{x}-1}{\frac{y}{x}+1}$ [अंश तथा हर में x से भाग देने पर]

$=\frac{\left(\frac{75}{2}-1\right)}{\left(\frac{75}{2}+1\right)}=\frac{73}{77}$.

63. $4^x=\left(2^{3y}\right)^{\frac{1}{2}} \Rightarrow 2^{2x}=2^{\frac{3y}{2}} \Rightarrow 2x=\frac{3y}{2} \Rightarrow x=\frac{3}{4}y$.

64. $x^{-\frac{4}{5}}=81 \Rightarrow \frac{1}{x^{4/5}}=81 \Rightarrow x^{4/5}=\frac{1}{81}$

$\Rightarrow \left(x^{1/5}\right)^4=\left(\frac{1}{3}\right)^4 \Rightarrow x^{1/5}=\frac{1}{3} \Rightarrow x=\left(\frac{1}{3}\right)^5=\frac{1^5}{3^5}=\frac{1}{243}$.

65. $5^{(x+3)}=(5^2)^{3x-4}=5^{6x-8} \Rightarrow x+3=6x-8 \Rightarrow 5x=11 \Rightarrow x=\frac{11}{5}$.

66. $(64)^{(2x-5)}=2^2\times2^{3(x-4)} \Rightarrow 2^{6(2x-5)}=2^{3x-12+2}$

$\therefore 6(2x-5)=3x-10 \Rightarrow 12x-30=3x-10 \Rightarrow 9x=20 \Rightarrow x=\frac{20}{9}$.

67. $\left\{\left(\frac{3}{2}\right)^2\right\}^x\cdot\left(\frac{2}{3}\right)^{3(x-1)}=\frac{2}{3}$

$\Rightarrow \left(\frac{3}{2}\right)^{2x}\cdot\left(\frac{2}{3}\right)^{(3x-3)}=\frac{2}{3} \Rightarrow \left(\frac{2}{3}\right)^{-2x}\cdot\left(\frac{2}{3}\right)^{(3x-3)}=\frac{2}{3}$

$\Rightarrow \left(\frac{2}{3}\right)^{(3x-3-2x)}=\left(\frac{2}{3}\right)^1 \Rightarrow x-3=1 \Rightarrow x=4$.

68. $\left(\frac{64x^3}{27a^{-3}}\right)^{-\frac{2}{3}}=\left(\frac{64x^3a^3}{27}\right)^{-\frac{2}{3}}=\left(\frac{27}{64x^3a^3}\right)^{\frac{2}{3}}=\left\{\left(\frac{3}{4xa}\right)^3\right\}^{\frac{2}{3}}$

$=\left(\frac{3}{4xa}\right)^{\left(3\times\frac{2}{3}\right)}=\left(\frac{3}{4xa}\right)^2=\frac{9}{16x^2a^2}.$

69. $\sqrt[3]{-8}\times\sqrt[4]{16}=(-8)^{\frac{1}{3}}\times(16)^{\frac{1}{4}}$

$=\left\{(-2)^3\right\}^{\frac{1}{3}}\times(2^4)^{\frac{1}{4}}=(-2)^{\left(3\times\frac{1}{3}\right)}\times2^{\left(4\times\frac{1}{4}\right)}=(-2)^1\times2^1=(-2)\times2=-4.$

70. $\left(\frac{1}{2}\right)^{\frac{-1}{2}}=2^{\frac{1}{2}}=\sqrt{2}.$

71. $(\sqrt{8})^{\frac{1}{3}}=\left(8^{\frac{1}{2}}\right)^{\frac{1}{3}}=8^{\left(\frac{1}{2}\times\frac{1}{3}\right)}=8^{\frac{1}{6}}=(2^3)^{\frac{1}{6}}=2^{\left(3\times\frac{1}{6}\right)}=2^{\frac{1}{2}}=\sqrt{2}.$

72. $\left(\frac{32}{243}\right)^{-\frac{4}{5}}=\left\{\left(\frac{2}{3}\right)^5\right\}^{-\frac{4}{5}}=\left(\frac{2}{3}\right)^{5\times\frac{(-4)}{5}}=\left(\frac{2}{3}\right)^{-4}=\left(\frac{3}{2}\right)^4=\frac{3^4}{2^4}=\frac{81}{16}.$

73. $\left(\frac{-1}{343}\right)^{\frac{-2}{3}}=(-343)^{\frac{2}{3}}=\left\{(-7)^3\right\}^{\frac{2}{3}}=(-7)^{\left(3\times\frac{2}{3}\right)}=(-7)^2=49.$

74. $\left(\frac{1}{216}\right)^{-\frac{2}{3}}=(216)^{\frac{2}{3}}=(6^3)^{\frac{2}{3}}=6^{\left(3\times\frac{2}{3}\right)}=6^2=36.$

$\left(\frac{1}{27}\right)^{-\frac{4}{3}}=(27)^{\frac{4}{3}}=(3^3)^{\frac{4}{3}}=3^{\left(3\times\frac{4}{3}\right)}=3^4=81.$

$\therefore$ दिया गया व्यंजक $=36+81=\frac{36}{81}=\frac{4}{9}.$

75. $3^n=\left(3^3\right)^{\frac{2}{3}}\times\left(\frac{1}{81}\right)^{\frac{1}{2}}\Rightarrow3^{\left(3\times\frac{2}{3}\right)}\times\frac{1}{9}=3^2\times\frac{1}{9}=9\times\frac{1}{9}=1\Rightarrow n=0.$

76. $(16)^{1\cdot75}=(16)^{\frac{175}{100}}=(16)^{\frac{7}{4}}=(2^4)^{\frac{7}{4}}=2^{\left(4\times\frac{7}{4}\right)}=2^7=128.$

77. $\sqrt{2^n}=2^6\Rightarrow2^{n/2}=2^6\Rightarrow\frac{n}{2}=6\Rightarrow n=12.$

78. $(256)^{0\cdot16}\times(16)^{0\cdot18}=(4^4)^{0\cdot16}\times(4^2)^{0\cdot18}$

$=(4)^{0\cdot64}\times4^{0\cdot36}=4^{(\cdot64+36)}=4^1=4.$

79. माना $(72)^x=(9\times8)^{8\cdot6}\times(8\times9)^{3\cdot9}\times(72)^{4\cdot4}$

$=(72)^{8\cdot6}\times(72)^{3\cdot9}\times(72)^{4\cdot4}=(72)^{(8\cdot6+3\cdot9+4\cdot4)}=(72)^{16\cdot9}$

$\Rightarrow x=16\cdot9.$

80. माना $6^{1\cdot2}\times(36)^x\times(30)^{2\cdot4}\times(25)^{1\cdot3}=(30)^5$

तब, $6^{1\cdot2}\times(6^2)^x\times(30)^{2\cdot4}\times(5^2)^{1\cdot3}=(30)^5$

तब, $6^{1\cdot2}\times(6^2)^x\times(6\times5)^{2\cdot4}\times(5^2)^{1\cdot3}=(30)^5$

$\Rightarrow6^{(1\cdot2+2x+2\cdot4)}\times5^{(2\cdot4+2\cdot6)}=(30)^5$

$\Rightarrow6^{(3\cdot6+2x)}\times5^5=(30)^5\Rightarrow3\cdot6+2x=5\Rightarrow2x=1\cdot4\Rightarrow x=0\cdot7.$

81. $(0{\cdot}01024)^{\frac{1}{5}} = \left(\frac{1024}{10^5}\right)^{\frac{1}{5}} = \left(\frac{4^5}{10^5}\right)^{\frac{1}{5}} = \left\{\left(\frac{4}{10}\right)^5\right\}^{\frac{1}{5}} = \left(\frac{4}{10}\right)^{\left(5\times\frac{1}{5}\right)} = \left(\frac{4}{10}\right) = 0{\cdot}4.$

82. माना $\sqrt{3+\sqrt{5}} = \sqrt{a}+\sqrt{b}$. तब $(3+\sqrt{5}) = a+b+2\sqrt{ab}$

$\therefore\ a+b=3$ तथा $2\sqrt{ab}=\sqrt{5} \Rightarrow 4ab=5 \Rightarrow ab=\frac{5}{4}$.

$(a-b)^2 = (a+b)^2-4ab = \left(3^2-4\times\frac{5}{4}\right) = (9-5) = 4 \Rightarrow (a-b)=2.$

$(a+b)=3$ तथा $(a-b)=2 \Rightarrow 2a=5$ तथा $2b=1 \Rightarrow a=\frac{5}{2}$ तथा $b=\frac{1}{2}$.

$\therefore$ अभीष्ट वर्गमूल $= \sqrt{\frac{5}{2}}+\sqrt{\frac{1}{2}}$.

83. $\frac{1}{x} = \frac{1}{(7-4\sqrt{3})}\times\frac{(7+4\sqrt{3})}{(7+4\sqrt{3})} = \frac{(7+4\sqrt{3})}{(49-48)} = (7+4\sqrt{3})$

$\therefore \left(x+\frac{1}{x}\right) = (7-4\sqrt{3})+(7+4\sqrt{3}) = 14.$

84. $\frac{1}{x} = \frac{1}{(3+\sqrt{8})}\times\frac{(3-\sqrt{8})}{(3-\sqrt{8})} = \frac{(3-\sqrt{8})}{(9-8)} = (3-\sqrt{8}).$

$\therefore \left(x+\frac{1}{x}\right) = (3+\sqrt{8})+(3-\sqrt{8}) = 6 \Rightarrow \left(x+\frac{1}{x}\right)^2 = 6^2 = 36.$

$\therefore\ x^2+\frac{1}{x^2}+2 = 36 \Rightarrow \left(x^2+\frac{1}{x^2}\right) = (36-2) = 34.$

85. 2, 3, 4, 6 का ल०स० = 12.

$\sqrt{2} = 2^{\frac{1}{2}} = (2^6)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$, $\sqrt[3]{3} = 3^{\frac{1}{3}} = (3^4)^{\frac{1}{12}} = (81)^{\frac{1}{12}}$,

$\sqrt[4]{4} = 4^{\frac{1}{4}} = (4^3)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$, $\sqrt[6]{6} = 6^{\frac{1}{6}} = (6^2)^{\frac{1}{12}} = (36)^{\frac{1}{12}}$.

$\therefore\ (81)^{\frac{1}{12}} > (64)^{\frac{1}{12}} > (36)^{\frac{1}{12}}$.

अत: सबसे बड़ी संख्या $= \sqrt[3]{3}$.

86. 2, 3, 4, 6 का ल०स० = 12.

$\sqrt{3} = 3^{\frac{1}{2}} = (3^6)^{\frac{1}{12}} = (729)^{\frac{1}{12}}$, $\sqrt[3]{4} = 4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$,

$\sqrt[4]{6} = 6^{\frac{1}{4}} = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}$, $\sqrt[6]{8} = 8^{\frac{1}{6}} = (8^2)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$.

$(729)^{\frac{1}{12}} > (256)^{\frac{1}{12}} > (216)^{\frac{1}{12}} > (64)^{\frac{1}{12}}$.

अत: $\sqrt{3}$ इनमें सबसे बड़ा है.

87. 3, 2 का ल०स० = 6.

$\sqrt[3]{2} = 2^{\frac{1}{3}} = (2^2)^{\frac{1}{6}} = (4)^{\frac{1}{6}}$, $\sqrt{3} = 3^{\frac{1}{2}} = (3^3)^{\frac{1}{6}} = (27)^{\frac{1}{6}}$.

$\sqrt[3]{5} = 5^{\frac{1}{3}} = (5^2)^{\frac{1}{6}} = (25)^{\frac{1}{6}}$, $\frac{3}{2} = \left\{\left(\frac{3}{2}\right)^6\right\}^{\frac{1}{6}} = \left(\frac{729}{64}\right)^{\frac{1}{6}} = \left(11\frac{25}{64}\right)^{\frac{1}{6}}$.

अत: इन सबमें बड़ा है $(27)^{\frac{1}{6}}$ अर्थात् $\sqrt{3}$.

88. 2, 3, 4 का ल०स० = 12.

$\sqrt{2} = \left(2^{\frac{1}{2}}\right) = \left(2^6\right)^{\frac{1}{12}} = (64)^{\frac{1}{12}}.$

$\sqrt[3]{4} = 4^{\frac{1}{3}} = (4^4)^{\frac{1}{12}} = (256)^{\frac{1}{12}}.$

$\sqrt[4]{6} = 6^{\frac{1}{4}} = (6^3)^{\frac{1}{12}} = (216)^{\frac{1}{12}}.$

स्पष्ट है कि $(64)^{\frac{1}{12}} < (216)^{\frac{1}{12}} < (256)^{\frac{1}{12}}$.

अर्थात् $\sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}.$

89. 3, 3, 8, 9 का ल०स० = 72.

$2^{\frac{1}{3}} = 2^{\left(\frac{1}{3}\times\frac{24}{24}\right)} = \left(2^{24}\right)^{\frac{1}{72}};\ 3^{\frac{1}{3}} = 3^{\left(\frac{1}{3}\times\frac{24}{24}\right)} = \left(3^{24}\right)^{\frac{1}{72}};$

$8^{\frac{1}{8}} = 2^{\left(\frac{3}{8}\times\frac{9}{9}\right)} = \left(2^{27}\right)^{\frac{1}{72}};\ 9^{\frac{1}{9}} = 3^{\left(\frac{2}{9}\times\frac{8}{8}\right)} = \left(3^{16}\right)^{\frac{1}{72}}.$

स्पष्ट है कि इन सबमें $\left(3^{24}\right)^{\frac{1}{72}}$ अर्थात् $\sqrt[3]{3}$ सबसे बड़ी संख्या है.

90. $(8-2\sqrt{15}) = (\sqrt{5})^2 + (\sqrt{3})^2 - 2\times\sqrt{5}\times\sqrt{3} = (\sqrt{5}-\sqrt{3})^2$

$\Rightarrow \sqrt{8-2\sqrt{15}} = (\sqrt{5}-\sqrt{3}).$

91. दिया गया व्यंजक $= (-2)^{(-2)(-2)} = (-2)^4 = 16.$

92. $(7+4\sqrt{3}) = 4+3+2\times2\times\sqrt{3}$

$= (2)^2 + (\sqrt{3})^2 + 2\times2\times\sqrt{3} = (2+\sqrt{3})^2$

$\Rightarrow \sqrt{7+4\sqrt{3}} = (2+\sqrt{3}).$

$\therefore$ दिया गया व्यंजक $= \dfrac{4+3\sqrt{3}}{2+\sqrt{3}} = \dfrac{2(2+\sqrt{3})+\sqrt{3}}{(2+\sqrt{3})} = 2+\dfrac{\sqrt{3}}{(2+\sqrt{3})}$

$= 2+\dfrac{1\cdot732}{3\cdot732} = \left(2+\dfrac{1732}{3732}\right) = \left(2+\dfrac{433}{933}\right) = 2+0\cdot464 = 2\cdot464.$

93. दिया गया व्यंजक $= \dfrac{(3+\sqrt{6})}{\left(5\sqrt{3}-2\sqrt{4\times3}-\sqrt{16\times2}+\sqrt{25\times2}\right)}$

$= \dfrac{(3+\sqrt{6})}{\left(5\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}\right)} = \dfrac{(3+\sqrt{6})}{\left(\sqrt{3}+\sqrt{2}\right)}\times\dfrac{(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})}$

$= \left(3\sqrt{3}-3\sqrt{2}+\sqrt{18}-\sqrt{12}\right) = \left(3\sqrt{3}-3\sqrt{2}+3\sqrt{2}-2\sqrt{3}\right)$

$= \sqrt{3} = 1\cdot732.$

94. दिया गया व्यंजक $= \dfrac{12}{\left(3+\sqrt{5}+2\sqrt{2}\right)}\times\dfrac{(3+\sqrt{5}-2\sqrt{2})}{\left(3+\sqrt{5}-2\sqrt{2}\right)} = \dfrac{12(3+\sqrt{5}-2\sqrt{2})}{\left(3+\sqrt{5}\right)^2-\left(2\sqrt{2}\right)^2} = \dfrac{12(3+\sqrt{5}-2\sqrt{2})}{9+5+6\sqrt{5}-8}$

$= \dfrac{12(3+\sqrt{5}-2\sqrt{2})}{6(\sqrt{5}+1)}\times\dfrac{(\sqrt{5}-1)}{(\sqrt{5}-1)}$

$= \dfrac{1}{2}\left(3\sqrt{5}-3+5-\sqrt{5}-2\sqrt{10}+2\sqrt{2}\right) = \dfrac{1}{2}\left(2+2\sqrt{5}+2\sqrt{2}-2\sqrt{10}\right)$

$= (1+\sqrt{5}+\sqrt{2}-\sqrt{10}).$

95. $(16+6\sqrt{7}) = 9+7+2\times3\times\sqrt{7} = 3^2+(\sqrt{7})^2+2\times3\times\sqrt{7} = (3+\sqrt{7})^2$.

$\Rightarrow \sqrt{16+6\sqrt{7}} = (3+\sqrt{7})$ तथा $\sqrt{16-6\sqrt{7}} = (3-\sqrt{7})$

$\Rightarrow \sqrt{16+6\sqrt{7}} - \sqrt{16-6\sqrt{7}} = (3+\sqrt{7})-(3-\sqrt{7}) = 2\sqrt{7}$.

$\therefore$ दिया गया व्यंजक $= \dfrac{\sqrt{7}}{2\sqrt{7}} = \dfrac{1}{2}$.

96. $(7+2\sqrt{12}) = 4+3+2\times2\times\sqrt{3} = 2^2+(\sqrt{3})^2+2\times2\times\sqrt{3} = (2+\sqrt{3})^2$

$\Rightarrow a = \sqrt{7+2\sqrt{12}} = (2+\sqrt{3})$ तथा $b = \sqrt{7-2\sqrt{12}} = (2-\sqrt{3})$

$\Rightarrow a+b = 4$ तथा $ab = (4-3) = 1$

$\Rightarrow (a^3+b^3) = (a+b)^3 - 3ab(a+b) = (4)^3 - 3\times1\times4 = (64-12) = 52.$

97. $(x^{1/3}+y^{1/3})^3 = \left(z^{1/3}\right)^3 \Rightarrow x+y+3x^{1/3}y^{1/3}\left(x^{1/3}+y^{1/3}\right) = z$

$\Rightarrow (x+y-z) = -3x^{1/3}y^{1/3}z^{1/3}$

$\Rightarrow (x+y-z)^3 = -27xyz$

$\Rightarrow (x+y-z)^3 + 27xyz = 0.$

98. 2^{50} का दुगुना $= (2\times2^{50}) = (2^{51})$.

99. दिया गया व्यंजक $= \left\{\dfrac{7^{(2+4)}}{5^{(2+1)}}\right\}^{\frac{7}{2}} \times \left\{\dfrac{7^{(3+5)}}{5^{(3+2)}}\right\}^{\frac{-5}{2}}$

$= \left(\dfrac{7^6}{5^3}\right)^{\frac{7}{2}} \times \left(\dfrac{7^8}{5^5}\right)^{\frac{-5}{2}} = \dfrac{7^{\left(6\times\frac{7}{2}\right)}\times7^{\left\{8\times\frac{(-5)}{2}\right\}}}{5^{\left(3\times\frac{7}{2}\right)}\times5^{\left\{5\times\frac{(-5)}{2}\right\}}}$

$= \dfrac{7^{21}\times7^{-20}}{5^{\left(\frac{21}{2}\right)}\times5^{\left(\frac{-25}{2}\right)}} = \dfrac{7^{(21-20)}}{5^{\left(\frac{21}{2}-\frac{25}{2}\right)}} = \dfrac{7^1}{5^{(-2)}} = (7\times5^2) = (7\times25) = 175.$

100. दिया गया व्यंजक $= \left\{\dfrac{x^{\frac{m}{(m-n)}+\frac{n}{(n-m)}}}{x^{\frac{m}{(m+n)}+\frac{n}{(n+m)}}}\right\}^{m+n} = \left\{\dfrac{x^{\left\{\frac{m}{(m-n)}-\frac{n}{(m-n)}\right\}}}{x^{\left\{\frac{m}{m+n}+\frac{n}{m+n}\right\}}}\right\}^{(m+n)}$

$= \left\{\dfrac{x^{\left(\frac{m-n}{m-n}\right)}}{x^{\frac{(m+n)}{(m+n)}}}\right\}^{m+n} = \left(\dfrac{x}{x}\right)^{m+n} = (1)^{m+n} = 1.$

101. $(a+b) = \dfrac{\sqrt{5}+1}{\sqrt{5}-1} + \dfrac{\sqrt{5}-1}{\sqrt{5}+1} = \dfrac{\left(\sqrt{5}+1\right)^2+\left(\sqrt{5}-1\right)^2}{(5-1)} = \dfrac{2(5+1)}{4} = 3.$

$(a-b) = \dfrac{\sqrt{5}+1}{\sqrt{5}-1} - \dfrac{\sqrt{5}-1}{\sqrt{5}+1} = \dfrac{\left(\sqrt{5}+1\right)^2-\left(\sqrt{5}-1\right)^2}{(5-1)} = \dfrac{4\times\sqrt{5}\times1}{4} = \sqrt{5},\ ab = 1$

$\dfrac{(a^2+ab+b^2)}{(a^2-ab+b^2)} = \dfrac{(a+b)^2-ab}{(a+b)^2-3ab} = \dfrac{(9-1)}{(9-3)} = \dfrac{8}{6} = \dfrac{4}{3}.$

102. $x=\left(3+2\sqrt{2}\right)\Rightarrow\frac{1}{x}=\frac{1}{\left(3+2\sqrt{2}\right)}\times\frac{(3-2\sqrt{2})}{(3-2\sqrt{2})}=\frac{(3-2\sqrt{2})}{(9-8)}=(3-2\sqrt{2}).$

$$\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2=\left(x+\frac{1}{x}-2\right)=\left(3+2\sqrt{2}+3-2\sqrt{2}-2\right)=4.$$

$$\therefore\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)=2.$$

103. $\frac{1}{\left(1+\sqrt{2}\right)}=\frac{1}{\left(\sqrt{2}+1\right)}\times\frac{\left(\sqrt{2}-1\right)}{\left(\sqrt{2}-1\right)}=\frac{\left(\sqrt{2}-1\right)}{(2-1)}=\left(\sqrt{2}-1\right).$

इसी प्रकार, $\frac{1}{\left(\sqrt{2}+\sqrt{3}\right)}=\frac{1}{\left(\sqrt{3}+\sqrt{2}\right)}\times\frac{\left(\sqrt{3}-\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)}=\frac{\left(\sqrt{3}-\sqrt{2}\right)}{(3-2)}=\left(\sqrt{3}-\sqrt{2}\right).$

$$\frac{1}{\left(\sqrt{3}+\sqrt{4}\right)}=\left(\sqrt{4}-\sqrt{3}\right),\ \frac{1}{\left(\sqrt{99}+\sqrt{100}\right)}=\left(\sqrt{100}-\sqrt{99}\right).$$

∴ दिया गया व्यंजक

$$=\left(\sqrt{2}-\sqrt{1}\right)+\left(\sqrt{3}-\sqrt{2}\right)+\left(\sqrt{4}-\sqrt{3}\right)+\left(\sqrt{5}-\sqrt{4}\right)+....+\left(\sqrt{99}-\sqrt{98}\right)+\left(\sqrt{100}-\sqrt{99}\right)$$
$$=\left(\sqrt{100}-\sqrt{1}\right)=(10-1)=9.$$

104. $\frac{1}{(\sqrt{3}+\sqrt{4})}=\frac{1}{\left(\sqrt{4}+\sqrt{3}\right)}\times\frac{\left(\sqrt{4}-\sqrt{3}\right)}{\left(\sqrt{4}-\sqrt{3}\right)}=\frac{\left(\sqrt{4}-\sqrt{3}\right)}{(4-3)}=\left(\sqrt{4}-\sqrt{3}\right)$ आदि आदि.

∴ दिया गया व्यंजक $=\left(\sqrt{4}-\sqrt{3}\right)+\left(\sqrt{5}-\sqrt{4}\right)+\left(\sqrt{6}-\sqrt{5}\right)+\left(\sqrt{7}-\sqrt{6}\right)+\left(\sqrt{8}-\sqrt{7}\right)+\left(\sqrt{9}-\sqrt{8}\right)$

$$=\left(\sqrt{9}-\sqrt{3}\right)=\left(3-\sqrt{3}\right).$$

105. $\frac{1}{\left(\sqrt{100}-\sqrt{99}\right)}=\frac{1}{\left(\sqrt{100}-\sqrt{99}\right)}\times\frac{\left(\sqrt{100}+\sqrt{99}\right)}{\left(\sqrt{100}+\sqrt{99}\right)}=\left(\sqrt{100}+\sqrt{99}\right)$ आदि आदि.

दिया गया व्यंजक $=\left(\sqrt{100}+\sqrt{99}\right)-\left(\sqrt{99}+\sqrt{98}\right)+\left(\sqrt{98}+\sqrt{97}\right)-...-\left(\sqrt{3}+\sqrt{2}\right)+\left(\sqrt{2}+\sqrt{1}\right)$

$$=\left(\sqrt{100}+\sqrt{1}\right)=(10+1)=11.$$

प्रश्नमाला 9B

1. यदि $x=\frac{1}{(2+\sqrt{3})}$ हो, तो $\frac{(3x-x^3)}{(1-3x^2)}$ का मान ज्ञात कीजिए. **(एस०एस०सी० परीक्षा, 2009)**

2. यदि $\sqrt{3}=1\cdot732$ हो, तो $(\sqrt{6}+\sqrt{2})+(\sqrt{6}-\sqrt{2})+\left\{(2+\sqrt{3})+2\sqrt{3}\right\}$ का मान ज्ञात कीजिए. **(एस०एस०सी० परीक्षा, 2008)**

3. यदि $\sqrt{3}=1\cdot732$ हो, तो $\frac{(2-\sqrt{3})}{(2+\sqrt{3})}$ का मान ज्ञात कीजिए. **(एस०एस०सी० परीक्षा, 2008)**

4. $\left(3+\frac{1}{\sqrt{3}}+\frac{1}{3+\sqrt{3}}-\frac{1}{3-\sqrt{3}}\right)$ का मान ज्ञात कीजिए. **(एस०एस०सी० परीक्षा, 2007)**

5. यदि $\sqrt{2}=1\cdot414$ हो, तो $\frac{\sqrt{3}}{\sqrt{6}}$ का मान ज्ञात कीजिए.

6. यदि $\left(\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}\right)=a+\sqrt{30}\,b$ हो, तो a तथा b के मान ज्ञात कीजिए.

7. यदि $\sqrt{5}=2\cdot236$ हो, तो $\left(\frac{3-\sqrt{5}}{3+2\sqrt{5}}\right)$ का मान दशमलव के तीन स्थानों तक ज्ञात कीजिए.

8. यदि $\sqrt{2}=1\cdot414, \sqrt{5}=2\cdot236$ तथा $\sqrt{10}=3\cdot162$ हो, तो $\frac{1}{(3+\sqrt{5}-2\sqrt{2})}$ का मान ज्ञात कीजिए.

9. यदि $\sqrt{2}=1\cdot414$ हो, तो $\left(\sqrt{63}+\sqrt{28}-\sqrt{175}+\sqrt{162}-\sqrt{32}\right)$ का मान ज्ञात कीजिए.

10. यदि $\sqrt{5}=2\cdot236$ तथा $\sqrt{10}=3\cdot162$ हो, तो $\frac{15}{\left(\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}\right)}$ का मान ज्ञात कीजिए.

11. सिद्ध कीजिए कि : $\frac{7\sqrt{3}}{\left(\sqrt{10}+\sqrt{3}\right)}-\frac{2\sqrt{5}}{\left(\sqrt{6}+\sqrt{5}\right)}-\frac{3\sqrt{2}}{\left(\sqrt{15}+3\sqrt{2}\right)}=1.$

12. सिद्ध कीजिए कि : $\frac{1}{\left(3-\sqrt{8}\right)}-\frac{1}{\left(\sqrt{8}-\sqrt{7}\right)}+\frac{1}{\left(\sqrt{7}-\sqrt{6}\right)}-\frac{1}{\left(\sqrt{6}-\sqrt{5}\right)}+\frac{1}{\left(\sqrt{5}-2\right)}=5.$

13. यदि $x=\frac{\sqrt{3}}{2}$ हो, तो $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}$ का मान ज्ञात कीजिए. **(एस०एस०सी० परीक्षा, 2002)**

14. यदि $x=\left(7+4\sqrt{3}\right)$ हो, तो $(i)\left(x^2+\frac{1}{x^2}\right)$ $(ii)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ के मान ज्ञात कीजिए.

15. यदि $x^{\frac{1}{a}}=y^{\frac{1}{b}}=z^{\frac{1}{c}}$ तथा $xyz=1$ हो, तो सिद्ध करो कि $a+b+c=0$.

16. यदि $3^x=5^y=(75)^z$ हो, तो सिद्ध करो कि $z=\frac{xy}{(2x+y)}$.

17. यदि $x=10^{0\cdot48}, y=10^{0\cdot7}$ तथा $x^z=y^2$ हो, तो z का मान ज्ञात करो.

18. यदि $a+b+c=0$ हो, तो सिद्ध करो कि $\frac{1}{\left(x^a+x^{-b}+1\right)}+\frac{1}{\left(x^b+x^{-c}+1\right)}+\frac{1}{\left(x^c+x^{-a}+1\right)}=1.$

19. $\left(\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\frac{1}{7\cdot10}+\frac{1}{10\cdot13}+\frac{1}{13\cdot16}\right)=?$ **(एस०एस०सी० परीक्षा, 2007)**

20. $\left(\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}\right)=?$

21. $\left(\frac{1}{35}+\frac{1}{63}+\frac{1}{99}+\frac{1}{143}+\frac{1}{195}+\frac{1}{255}\right)=?$

22. $\left(\frac{1}{54}+\frac{1}{108}+\frac{1}{180}+\frac{1}{270}+\frac{1}{378}\right)=?$

23. $\left(\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}+\frac{1}{132}\right)=?$ **(असिस्टैन्ट ग्रेड परीक्षा, 2000)**

24. $\left(\frac{1\cdot2\cdot4+2\cdot4\cdot8+3\cdot6\cdot12+.....}{1\cdot3\cdot9+2\cdot6\cdot18+3\cdot9\cdot27+.....}\right)^{\frac{1}{3}}$ का मान ज्ञात कीजिए.

उत्तरमाला (प्रश्नमाला 9B)

1. 1 **2.** 3·464 **3.** 0·072 **4.** 3 **5.** 0·707 **6.** $a=\frac{3}{7}, b=\frac{4}{21}$ **7.** 0·102 **8.** 0·415 **9.** 7·07

10. 5·398 **13.** $\sqrt{3}$ **14.** (*i*) 194 (*ii*) 4 **17.** $2\frac{11}{12}$ **19.** $\frac{5}{16}$ **20.** $\frac{3}{20}$ **21.** $\frac{6}{85}$ **22.** $\frac{5}{126}$ **23.** $\frac{11}{12}$ **24.** $\frac{2}{3}$

दिये गए प्रश्नों के हल प्रश्नमाला 9B

1. $x=\frac{1}{(2+\sqrt{3})}\times\frac{(2-\sqrt{3})}{(2-\sqrt{3})}=\frac{(2-\sqrt{3})}{(4-3)}=(2-\sqrt{3}).$

$\therefore x^2=(2-\sqrt{3})^2=(4+3-4\sqrt{3})=(7-4\sqrt{3})$

तथा $x^3=(7-4\sqrt{3})(2-\sqrt{3})=(14+12-15\sqrt{3})=(26-15\sqrt{3}).$

$\therefore \frac{(3x-x^3)}{(1-3x^2)}=\frac{(6-3\sqrt{3})-(26-15\sqrt{3})}{\{1-(21-12\sqrt{3})\}}=\frac{(-20+12\sqrt{3})}{(-20+12\sqrt{3})}=1.$

2. दिया गया व्यंजक $=\frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})}+\frac{(2+\sqrt{3})}{2\sqrt{3}}=\frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})}\times\frac{2\sqrt{3}}{(2+\sqrt{3})}$

$=\frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}-\sqrt{2})}\times\frac{(\sqrt{6}+\sqrt{2})}{(\sqrt{6}+\sqrt{2})}\times\frac{2\sqrt{3}}{(2+\sqrt{3})}\times\frac{(2-\sqrt{3})}{(2-\sqrt{3})}=\frac{(\sqrt{6}+\sqrt{2})^2\times(4\sqrt{3}-6)}{(6-2)\times(4-3)}$

$=\frac{(6+2+2\sqrt{12})\times(4\sqrt{3}-6)}{4\times1}=\frac{(8+4\sqrt{3})(4\sqrt{3}-6)}{4}$

$=\frac{4(2+\sqrt{3})(4\sqrt{3}-6)}{4}=(2+\sqrt{3})(4\sqrt{3}-6)=(8\sqrt{3}-12+12-6\sqrt{3})=2\sqrt{3}$

$=(2\times1\cdot732)=(3\cdot464).$

3. दिया गया व्यंजक $=\frac{(2-\sqrt{3})}{(2+\sqrt{3})}\times\frac{(2-\sqrt{3})}{(2-\sqrt{3})}=\frac{(2-\sqrt{3})^2}{(4-3)}=(4+3-4\sqrt{3})$

$=(7-4\sqrt{3})=(7-4\times1\cdot732)=(7-6\cdot928)=0\cdot072.$

4. दिया गया व्यंजक $=3+\frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}+\frac{1}{(3+\sqrt{3})}\times\frac{(3-\sqrt{3})}{(3-\sqrt{3})}-\frac{1}{(3-\sqrt{3})}\times\frac{(3+\sqrt{3})}{(3+\sqrt{3})}$

$=3+\frac{\sqrt{3}}{3}+\frac{(3-\sqrt{3})}{(9-3)}-\frac{(3+\sqrt{3})}{(9-3)}=3+\frac{\sqrt{3}}{3}+\frac{(3-\sqrt{3})}{6}-\frac{(3+\sqrt{3})}{6}$

$=3+\frac{\sqrt{3}}{3}+\frac{(3-\sqrt{3})-(3+\sqrt{3})}{6}=3+\frac{\sqrt{3}}{3}-\frac{\sqrt{3}}{3}=3.$

5. $\frac{\sqrt{3}}{\sqrt{6}}=\frac{\sqrt{3}}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{\sqrt{3\times6}}{6}=\frac{\sqrt{18}}{6}=\frac{\sqrt{2\times9}}{6}=\frac{3\sqrt{2}}{6}=\frac{\sqrt{2}}{2}=\frac{1\cdot414}{2}=0\cdot707.$

6. $\frac{(2\sqrt{6}-\sqrt{5})}{(3\sqrt{5}-2\sqrt{6})}=\frac{(2\sqrt{6}-\sqrt{5})}{(3\sqrt{5}-2\sqrt{6})}\times\frac{(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}+2\sqrt{6})}=\frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(45-24)}$

$=\frac{(6\sqrt{30}-2\sqrt{30}+24-15)}{21}=\frac{(4\sqrt{30}+9)}{21}=\left(\frac{9}{21}+\frac{4\sqrt{30}}{21}\right)=\left(\frac{3}{7}+\frac{4\sqrt{30}}{21}\right)$

$\therefore (a+\sqrt{30}\,b)=\left(\frac{3}{7}+\sqrt{30}\times\frac{4}{21}\right)\Rightarrow a=\frac{3}{7}$ तथा $b=\frac{4}{21}.$

7. $\frac{(3-\sqrt{5})}{(3+2\sqrt{5})}=\frac{(3-\sqrt{5})}{(3+2\sqrt{5})}\times\frac{(3-2\sqrt{5})}{(3-2\sqrt{5})}=\frac{(3-\sqrt{5})(3-2\sqrt{5})}{(9-20)}$

$=\frac{(9+10)-6\sqrt{5}-3\sqrt{5}}{-11}=\frac{19-9\sqrt{5}}{-11}=\left(\frac{9}{11}\times\sqrt{5}-\frac{19}{11}\right)$

$=\frac{(9\sqrt{5}-19)}{11}=\frac{(9\times2\cdot236-19)}{11}=\frac{(20\cdot124-19)}{11}=\frac{1\cdot124}{11}=0\cdot102.$

8. दिया गया व्यंजक $=\frac{1}{\left\{\left(3+\sqrt{5}\right)-2\sqrt{2}\right\}}\times\frac{\left(3+\sqrt{5}\right)+2\sqrt{2}}{\left(3+\sqrt{5}\right)+2\sqrt{2}}$

$=\frac{\left(3+\sqrt{5}\right)+2\sqrt{2}}{\left(3+\sqrt{5}\right)^2-8}=\frac{\left(3+\sqrt{5}\right)+2\sqrt{2}}{\left(9+5+6\sqrt{5}-8\right)}=\frac{3+\sqrt{5}+2\sqrt{2}}{6+6\sqrt{5}}$

$=\frac{\left(3+\sqrt{5}+2\sqrt{2}\right)}{6\left(\sqrt{5}+1\right)}\times\frac{\left(\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)}=\frac{3\sqrt{5}-3+5-\sqrt{5}+2\sqrt{10}-2\sqrt{2}}{6(5-1)}$

$=\frac{2+2\sqrt{5}+2\sqrt{10}-2\sqrt{2}}{24}=\frac{\left(1+\sqrt{5}+\sqrt{10}-\sqrt{2}\right)}{12}$

$=\frac{(1+2\cdot236+3\cdot162-1\cdot414)}{12}=\frac{4\cdot984}{12}=0\cdot415.$

9. दिया गया व्यंजक $=\left(\sqrt{9\times7}+\sqrt{4\times7}-\sqrt{25\times7}+\sqrt{81\times2}-\sqrt{16\times2}\right)$

$=\left(3\sqrt{7}+2\sqrt{7}-5\sqrt{7}+9\sqrt{2}-4\sqrt{2}\right)=5\sqrt{2}=(5\times1\cdot414)=7\cdot07.$

10. दिया गया व्यंजक $=\frac{15}{\sqrt{10}+\sqrt{4\times5}+\sqrt{4\times10}-\sqrt{5}-\sqrt{16\times5}}$

$=\frac{15}{\left(\sqrt{10}+2\sqrt{5}+2\sqrt{10}-\sqrt{5}-4\sqrt{5}\right)}=\frac{15}{\left(3\sqrt{10}-3\sqrt{5}\right)}$

$=\frac{15}{3\left(\sqrt{10}-\sqrt{5}\right)}\times\frac{\left(\sqrt{10}+\sqrt{5}\right)}{\left(\sqrt{10}+\sqrt{5}\right)}=\frac{5\times\left(\sqrt{10}+\sqrt{5}\right)}{(10-5)}=\left(\sqrt{10}+\sqrt{5}\right)$

$=(3\cdot162+2\cdot236)=5\cdot398.$

11. $\frac{7\sqrt{3}}{\left(\sqrt{10}+\sqrt{3}\right)}=\frac{7\sqrt{3}}{\left(\sqrt{10}+\sqrt{3}\right)}\times\frac{\left(\sqrt{10}-\sqrt{3}\right)}{\left(\sqrt{10}-\sqrt{3}\right)}=\frac{7\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)}{(10-3)}=\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)=\left(\sqrt{30}-3\right).$

$\frac{2\sqrt{5}}{\left(\sqrt{6}+\sqrt{5}\right)}=\frac{2\sqrt{5}}{\left(\sqrt{6}+\sqrt{5}\right)}\times\frac{\left(\sqrt{6}-\sqrt{5}\right)}{\left(\sqrt{6}-\sqrt{5}\right)}=\frac{2\sqrt{5}\left(\sqrt{6}-\sqrt{5}\right)}{(6-5)}=2\sqrt{5}\left(\sqrt{6}-\sqrt{5}\right)=2\sqrt{30}-10.$

$\frac{3\sqrt{2}}{\left(\sqrt{15}+3\sqrt{2}\right)}=\frac{3\sqrt{2}}{\left(\sqrt{15}+3\sqrt{2}\right)}\times\frac{\left(\sqrt{15}-3\sqrt{2}\right)}{\left(\sqrt{15}-3\sqrt{2}\right)}=\frac{3\sqrt{2}\left(\sqrt{15}-3\sqrt{2}\right)}{(15-18)}=-\sqrt{2}\left(\sqrt{15}-3\sqrt{2}\right)=\left(6-\sqrt{30}\right).$

$\therefore$ बायाँ पक्ष $=\left(\sqrt{30}-3\right)-\left(2\sqrt{30}-10\right)-\left(6-\sqrt{30}\right)$

$=\left(\sqrt{30}-2\sqrt{30}+\sqrt{30}\right)-3+10-6=1=$ दायाँ पक्ष.

12. $\frac{1}{(3-\sqrt{8})}=\frac{1}{(3-\sqrt{8})}\times\frac{(3+\sqrt{8})}{(3+\sqrt{8})}=\frac{(3+\sqrt{8})}{(9-8)}=(3+\sqrt{8})$.

$\frac{1}{(\sqrt{8}-\sqrt{7})}=\frac{1}{(\sqrt{8}-\sqrt{7})}\times\frac{(\sqrt{8}+\sqrt{7})}{(\sqrt{8}+\sqrt{7})}=\frac{(\sqrt{8}+\sqrt{7})}{(8-7)}=(\sqrt{8}+\sqrt{7})$.

इसी प्रकार, $\frac{1}{(\sqrt{7}-\sqrt{6})}=(\sqrt{7}+\sqrt{6}), \frac{1}{(\sqrt{6}-\sqrt{5})}=(\sqrt{6}+\sqrt{5}), \frac{1}{(\sqrt{5}-2)}=(\sqrt{5}+2)$.

$\therefore$ दिया गया व्यंजक $=(3+\sqrt{8})-(\sqrt{8}+\sqrt{7})+(\sqrt{7}+\sqrt{6})-(\sqrt{6}+\sqrt{5})+(\sqrt{5}+2)=(3+2)=5$.

13. $\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}=\frac{(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x}-\sqrt{1-x})}\times\frac{(\sqrt{1+x}+\sqrt{1-x})}{(\sqrt{1+x}+\sqrt{1-x})}=\frac{(\sqrt{1+x}+\sqrt{1-x})^2}{(1+x)-(1-x)}$

$=\frac{(1+x+1-x)+2\sqrt{1+x}.\sqrt{1-x}}{2x}=\frac{\left(1+\sqrt{1-x^2}\right)}{x}$

$=\frac{1+\sqrt{1-\frac{3}{4}}}{\left(\frac{\sqrt{3}}{2}\right)}=\frac{1+\sqrt{\frac{1}{4}}}{(\sqrt{3}/2)}=\frac{\left(1+\frac{1}{2}\right)}{(\sqrt{3}/2)}\quad\left[\because x=\frac{\sqrt{3}}{2}\right]$

$=\left(\frac{3}{2}\times\frac{2}{\sqrt{3}}\right)=\sqrt{3}$.

14. $\frac{1}{x}=\frac{1}{(7+4\sqrt{3})}\times\frac{(7-4\sqrt{3})}{(7-4\sqrt{3})}=\frac{(7-4\sqrt{3})}{(49-48)}=(7-4\sqrt{3})$.

$\therefore\quad\left(x+\frac{1}{x}\right)=(7+4\sqrt{3})+(7-4\sqrt{3})=14$.

$\Rightarrow\left(x+\frac{1}{x}\right)^2=(14)^2\Rightarrow x^2+\frac{1}{x^2}+2x.\frac{1}{x}=196\Rightarrow x^2+\frac{1}{x^2}+2=196\Rightarrow\left(x^2+\frac{1}{x^2}\right)=194$.

अब, $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2=x+\frac{1}{x}+2\sqrt{x}\times\frac{1}{\sqrt{x}}=\left(x+\frac{1}{x}+2\right)=(14+2)=16$.

अत: $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)=4$.

15. माना $x^{\frac{1}{a}}=y^{\frac{1}{b}}=z^{\frac{1}{c}}=k$. तब $x=k^a, y=k^b, z=k^c$.

$xyz=1\Rightarrow k^a\times k^b\times k^c=1=k^0$

$\Rightarrow k^{a+b+c}=k^0\Rightarrow a+b+c=0$.

16. माना $3^x=5^y=(75)^z=K$. तब $3=K^{\frac{1}{x}}, 5=K^{\frac{1}{y}}, 75=K^{\frac{1}{z}}$.

अब, $75=(3\times5^2)\Rightarrow K^{\frac{1}{z}}=K^{\frac{1}{x}}\times K^{\frac{2}{y}}=K^{\left(\frac{1}{x}+\frac{2}{y}\right)}\Rightarrow\frac{1}{z}=\left(\frac{1}{x}+\frac{2}{y}\right)=\frac{(y+2x)}{xy}\Rightarrow z=\frac{xy}{(y+2x)}$.

17. $x^z = y^2 \Rightarrow (10^{0.48})^z = (10^{0.7})^2 \Rightarrow 10^{(0.48)z} = 10^{(0.7\times2)} = 10^{(1.4)}$

$\Rightarrow 0.48\times z = 1.4 \Rightarrow z = \dfrac{1.40}{0.48} = \dfrac{140}{48} = \dfrac{35}{12} = 2\dfrac{11}{12}.$

अतः $z = 2\dfrac{11}{12}.$

18. दिया गया व्यंजक $= \dfrac{1}{\left(x^a + x^{-b} + 1\right)} + \dfrac{1}{(x^b + x^{-c} + 1)} + \dfrac{1}{(x^c + x^{-a} + 1)}$

$= \dfrac{x^{-a}}{\{1 + x^{(-a-b)} + x^{-a}\}} + \dfrac{x^c}{(x^{b+c} + 1 + x^c)} + \dfrac{1}{(x^c + x^{-a} + 1)}$

$= \dfrac{x^{-a}}{(1 + x^c + x^{-a})} + \dfrac{x^c}{(x^{-a} + 1 + x^c)} + \dfrac{1}{(x^c + x^{-a} + 1)}$ $[\because -(a+b) = c, (b+c) = -a]$

$= \dfrac{1}{(1 + x^{-a} + x^c)} + \dfrac{x^{-a}}{(1 + x^{-a} + x^c)} + \dfrac{x^c}{(1 + x^{-a} + x^c)} = \dfrac{(1 + x^{-a} + x^c)}{(1 + x^{-a} + x^c)} = 1.$

19. दिया गया व्यंजक $= \dfrac{1}{3}\cdot\left[\left(1 - \dfrac{1}{4}\right) + \left(\dfrac{1}{4} - \dfrac{1}{7}\right) + \left(\dfrac{1}{7} - \dfrac{1}{10}\right) + \left(\dfrac{1}{10} - \dfrac{1}{13}\right) + \left(\dfrac{1}{13} - \dfrac{1}{16}\right)\right]$

$= \dfrac{1}{3}\left(1 - \dfrac{1}{16}\right) = \left(\dfrac{1}{3}\times\dfrac{15}{16}\right) = \dfrac{5}{16}.$

20. दिया गया व्यंजक $= \left(\dfrac{1}{4} - \dfrac{1}{5}\right) + \left(\dfrac{1}{5} - \dfrac{1}{6}\right) + \left(\dfrac{1}{6} - \dfrac{1}{7}\right) + \left(\dfrac{1}{7} - \dfrac{1}{8}\right) + \left(\dfrac{1}{8} - \dfrac{1}{9}\right) + \left(\dfrac{1}{9} - \dfrac{1}{10}\right)$

$= \left(\dfrac{1}{4} - \dfrac{1}{10}\right) = \dfrac{3}{20}.$

21. दिया गया व्यंजक $= \dfrac{1}{2}\cdot\left[\left(\dfrac{1}{5} - \dfrac{1}{7}\right) + \left(\dfrac{1}{7} - \dfrac{1}{9}\right) + \left(\dfrac{1}{9} - \dfrac{1}{11}\right) + \left(\dfrac{1}{11} - \dfrac{1}{13}\right) + \left(\dfrac{1}{13} - \dfrac{1}{15}\right) + \left(\dfrac{1}{15} - \dfrac{1}{17}\right)\right]$

$= \dfrac{1}{2}\cdot\left(\dfrac{1}{5} - \dfrac{1}{17}\right) = \dfrac{1}{2}\times\dfrac{12}{85} = \dfrac{6}{85}.$

22. दिया गया व्यंजक $= \dfrac{1}{3}\cdot\left[\left(\dfrac{1}{6} - \dfrac{1}{9}\right) + \left(\dfrac{1}{9} - \dfrac{1}{12}\right) + \left(\dfrac{1}{12} - \dfrac{1}{15}\right) + \left(\dfrac{1}{15} - \dfrac{1}{18}\right) + \left(\dfrac{1}{18} - \dfrac{1}{21}\right)\right]$

$= \dfrac{1}{3}\left(\dfrac{1}{6} - \dfrac{1}{21}\right) = \left(\dfrac{1}{3}\times\dfrac{5}{42}\right) = \dfrac{5}{126}.$

23. दी गई श्रेणी $= \left(1 - \dfrac{1}{2}\right) + \left(\dfrac{1}{2} - \dfrac{1}{3}\right) + \left(\dfrac{1}{3} - \dfrac{1}{4}\right) + \ldots + \left(\dfrac{1}{10} - \dfrac{1}{11}\right) + \left(\dfrac{1}{11} - \dfrac{1}{12}\right) = \left(1 - \dfrac{1}{12}\right) = \dfrac{11}{12}.$

24. दिया गया व्यंजक $= \left(\dfrac{8 + 8\times8 + 8\times27 + \ldots..}{27 + 27\times8 + 27\times27 + \ldots..}\right)^{\frac{1}{3}} = \left\{\dfrac{8(1 + 8 + 27 + \ldots.)}{27(1 + 8 + 27 + \ldots..)}\right\}^{\frac{1}{3}}$

$= \left(\dfrac{8}{27}\right)^{\frac{1}{3}} = \left\{\left(\dfrac{2}{3}\right)^3\right\}^{\frac{1}{3}} = \left(\dfrac{2}{3}\right)^{\left(3\times\frac{1}{3}\right)} = \left(\dfrac{2}{3}\right)^1 = \left(\dfrac{2}{3}\right).$