A comparison theorem, Sobolev imbeddings and Konrachov theorem for Riemannian manifolds

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Contents

This post is the first part of my reading note for [**?**]. The second part is here.

In this part, we will first establish the Sobolev imbeddings theorem and the Kondrachov theorem for Riemannian manifolds from the Euclidean version of these theorems. This includes 2 tasks:

1. Compare the volume form of the Riemannian metric g near a point and that of the Euclidean metric on the tangent space at that point. Theorem 1 gives an equivalent between the integral under g and the integral under Euclidean metric via the exponential map.

2. Reasonably use partition of unity to establish global results from local results (the Euclidean case). We will need a covering lemma (Calabi's lemma), which essentially reduces to a combinatorial result (Vitali's covering lemma).

Finally, we will apply imbedding theorems to solve the equation $-\Delta u = f$ on a Riemannian manifold when f is square-integrable.

1 Local comparison with Euclidean space

Theorem 1 (comparison of volume forms).

2 Some covering lemmas

3 Sobolev imbeddings for Riemannian manifolds

4 Kondrachov theorem for compact Riemannian manifolds

5 Solving $\Delta u = f$ on a Riemannian manifold.

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