Generalized Predicate Completion

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Abstract

Circumscription, proposed by McCarthy, is a formalism of non-monotonic reasoning. Predicate completion is an approach, proposed by Clark, to closed world reasoning which assumes that given sufficient conditions on a predicate are also necessary. Reiter has shown that for clausal sentences which are Horn in a predicate p, the circumscription of p logically subsumes the predicate completion of p. In this paper, we present a *generalized completion* of a predicate p, which is appropriate for clausal sentences which are not Horn in p. The main results of this paper are: (1) for *non-overlapping* clausal sentences (which may not necessarily be Horn in the predicate p), the circumscription of p logically subsumes the generalized completion of p; (2) for *non-overlapping* clausal sentences which are *collapsible wrt* (with respect to) the predicate p, the generalized completion of p is even logically equivalent to the circumscription of p.

1. Introduction

Non-monotonic reasoning is an area of growing significance to artificial intelligence [Clark, 1978; McCarthy, 1980; McCarthy, 1986; Lifschitz, 1985; Reiter, 1978; Reiter, 1980]. Especially, McCarthy's *circumscription* [McCarthy, 1980; McCarthy, 1986; Lifschitz, 1985] turns out to be an influential formalism which attempts to characterize a rule of conjecture - the objects that can be shown to have a certain property from certain facts are only those that satisfy this property. Prior to McCarthy's circumscription, Reiter has proposed the *closed world assumption* (CWA) [Reiter, 1978], which says that the implicit representation of negative facts presumes total knowledge. Circumscription is very similar to CWA in terms of "minimal entailment" and "minimal inference" which captures some characters of human plausible reasoning. As we consider clausal sentences, CWA can efficiently be implemented via Clark's *negation as failure* [Clark, 1978]. Furthermore, that can be proved with negation as failure inference rule from a clausal sentence is a logical consequence of the predicate completion of this sentence [Clark, 1978]. Predicate

completion simply states that the given sufficient conditions on a predicate are also necessary.

Since both predicate completion and circumscription attempt to capture some similar phenomena in the aspect of non-monotonic character, it is therefore important to achieve better understanding of the relationship between them. Recently, Reiter has shown that for clausal sentences which are Horn in a predicate p, Clark's predicate completion is implied by McCarthy's circumscription [Reiter, 1982]. Clearly, the completion is a non-trivial logical consequence of circumscription. That predicate completion is subsumed by circumscription for a wide class of clausal sentences is of some theoretical and computational interests. From these points of view, we shall enlarge the class of first-order clausal sentences for which predicate completion can be subsumed by circumscription. In this paper, we shall present a *generalized completion* of a predicate p, which refines on the definition of Clark's predicate completion. The generalized predicate completion is appropriate for clausal sentences which are not Horn in p. Clark's predicate completion and Reiter's result mentioned above are covered by our generalized predicate completion and results. Our main results of this paper are: (1) for *non-overlapping* clausal sentences (which may not necessarily be Horn in the predicate p), the circumscription of p subsumes the generalized completion of p; (2) for non-overlapping clausal sentences which are *collapsible wrt* (with respect to) the predicate p, the generalized completion of p is even logically equivalent to the circumscription of p.

This paper is organized as follows. In *Section* 2 we recall the definition of McCarthy's circumscription and some of its useful characterizations; In *Section* 3 we recall the definition of Clark's predicate completion and Reiter's result; In *Section* 4 we make some investigation on Clark's predicate completion, propose a generalized predicate completion and show our results. This paper is concluded in *Section* 5.

2. Circumscription and Minimal Entailment

Circumscription [McCarthy, 1980; McCarthy, 1986; Lifschitz, 1985] is an approach to the problem of non-monotonic reasoning, which augments formulas with a refinement of minimal inference. In this paper, we are particularly interested in clausal sentences. A *clausal sentence* is a conjunction of clauses (equivalently, a finite set of clauses). A *clause* is a universally quantified disjunction of literals, written as $l_1 \vee \cdots \vee l_n$, which is logically identified with $\forall x.\ (l_1 \vee \cdots \vee l_n)$, where x is the tuple of variables appearing in the clause and l_i is a *literal* (an *atom* or the negation of an atom) for i, $1 \leq i \leq n$.

Definition 1. Let T be a clausal sentence, p and z distinct predicate symbols. The *circumscription* of p in T with parameter z, denoted by Circum(T; p; z), is defined as the second-order formula:

$$T \wedge \forall p', z'.\ [\ T(p', z') \wedge \forall x.\ (\ p'(x) \supset p(x)\ ) \supset \forall x.\ (\ p(x) \supset p'(x)\ )\ ],$$