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Write down all necessary steps in solutions in order to receive as many points as possible.

1 (3 points)

Suppose that a box contains 6 red balls and 8 blue balls. One now randomly takes 2 balls at once from the box. Define the following three events:

$$\begin{aligned} A &= \{\text{2 balls have same color}\}, \\ B &= \{\text{1 red ball and 1 blue ball}\}, \\ C &= \{\text{at least 1 red ball}\}. \end{aligned}$$

- (1.1) (1p) Find the probability $P(A)$.
- (1.2) (1p) Find the probability $P(A \cap C)$.
- (1.3) (1p) Find the conditional probability $P(B | C)$.

Solution. (1.1)

$$P(A) = P(\text{2 balls are red}) + P(\text{2 balls are blue}) = \frac{\binom{6}{2}}{\binom{14}{2}} + \frac{\binom{8}{2}}{\binom{14}{2}} = \frac{15}{91} + \frac{28}{91} = \frac{43}{91} = 0.4725.$$

(1.2)

$$P(A \cap C) = P(\text{2 balls are red}) = \frac{\binom{6}{2}}{\binom{14}{2}} = \frac{15}{91} = 0.1648.$$

(1.3)

$$\begin{aligned} P(B | C) &= \frac{P(B \cap C)}{P(C)} = \frac{P(\text{1 red ball and 1 blue ball})}{1 - P(\text{no red ball})} = \frac{\frac{\binom{6}{1} \cdot \binom{8}{1}}{\binom{14}{2}}}{1 - \frac{\binom{8}{2}}{\binom{14}{2}}} \\ &= \frac{\frac{48}{91}}{1 - \frac{28}{91}} = \frac{48}{63} = \frac{16}{21} = 0.7619. \end{aligned}$$

□

2 (3 points)

During the period 2024.1.1-2024.5.27, suppose that 25% of all LiU students have visited IKEA Linköping.

(2.1) (1p) Among 10 randomly selected LiU students, what is the probability that there are at most 5 who have visited IKEA Linköping during the period 2024.1.1-2024.5.27?

(2.2) (2p) Among 100 randomly selected LiU students, what is the probability that there are at most 35 who have visited IKEA Linköping during the period 2024.1.1-2024.5.27?

Solution. (2.1) Let X be the number of LiU students who have visited IKEA Linköping during the period 2024.1.1-2024.5.27 among these 10 randomly selected persons, then $X \sim \text{Bin}(10, 25\%)$. Therefore

$$P(\text{at most } 5) = P(X \leq 5) = (\text{Binomial table}) = 0.9803.$$

(2.2) Let Y be the number of LiU students who have visited IKEA Linköping during the period 2024.1.1-2024.5.27 among these 100 randomly selected persons, then $Y \sim \text{Bin}(100, 25\%)$. Normal approximation to Binomial shall be used

$$\begin{aligned} P(\text{at most } 35) &= P(Y \leq 35) = P(N(100 \cdot 25\%, 100 \cdot 25\% \cdot (1 - 25\%)) \leq 35) \\ &= P(N(25, 18.75) \leq 35) = P(N(0, 1) \leq \frac{35 - 25}{\sqrt{18.75}}) \\ &= P(N(0, 1) \leq 2.31) = \Phi(2.31) = 0.9896. \end{aligned}$$

□

3 (3 points)

Lars is table tennis player, and he spends some time everyday for training. Let X denote the time (in hours) Lars spends in one day for training. Assume that X is a continuous random variable with a probability density function:

$$f(x) = \begin{cases} \frac{x}{8}, & \text{if } 0 \leq x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

(3.1) (1p) Find the mean $\mu = E(X)$ and variance $\sigma^2 = V(X)$ of X .

(3.2) (2p) Let X_1, X_2, \dots, X_{100} denote the times Lars spends for training in 100 days, and assume that X_1, X_2, \dots, X_{100} are independent. Find the probability that the total training time in these 100 days is less than 280 hours, i.e. $P(X_1 + X_2 + \dots + X_{100} < 280)$.

Solution. (3.1)

$$\begin{aligned} \mu = E(X) &= \int_0^4 xf(x)dx = \frac{1}{8} \int_0^4 x^2 dx = \frac{1}{8 \cdot 3} [x^3]_0^4 = \frac{8}{3} = 2.6667, \\ \sigma^2 = V(X) &= E(X^2) - \mu^2 = \int_0^4 x^2 f(x)dx - \left(\frac{8}{3}\right)^2 = \frac{1}{8 \cdot 4} [x^4]_0^4 - \left(\frac{8}{3}\right)^2 = 8 - \frac{64}{9} = \frac{8}{9} = 0.8889. \end{aligned}$$

(3.2)

$$\begin{aligned} P(X_1 + X_2 + \dots + X_{100} < 280) &= P(\bar{X} < 2.8) = P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} < \frac{2.8 - \mu}{\sigma/\sqrt{n}}\right) \\ &= P\left(N(0, 1) < \frac{2.8 - \frac{8}{3}}{\sqrt{\frac{8}{9}/\sqrt{100}}}\right) = P\left(N(0, 1) < \frac{28 - \frac{80}{3}}{\sqrt{\frac{8}{9}}}\right) = P\left(N(0, 1) < \frac{28 \cdot 3 - 80}{\sqrt{8}}\right) \\ &= P(N(0, 1) < 1.41) = \Phi(1.41) = 0.9207. \end{aligned}$$

□

4 (3 points)

(4.1) (1p) A discrete population X has the following distribution

X	1	5	8
$p(x)$	$(1 - \theta)/3$	$1/3$	$(1 + \theta)/3$

where θ is unknown. To estimate θ , a sample $\{x_1, x_2, \dots, x_n\}$ is taken from this population. Use the method of moments to find a point estimate $\hat{\theta}_{MM}$ of θ .

(4.2) (2p) Another population Y is a continuous random variable with a probability density function

$$f(y) = \frac{a}{y^{a+1}}, \quad \text{for } y \geq 1,$$

where $a > 1$ is unknown. A sample $\{y_1, y_2, \dots, y_n\}$ is taken from this population. Use the maximum-likelihood method to find a point estimate \hat{a}_{ML} of a .

Solution. (4.1) The mean $E(X)$ of the population can be computed as

$$E(X) = 1 \cdot \frac{1-\theta}{3} + 5 \cdot \frac{1}{3} + 8 \cdot \frac{1+\theta}{3} = \frac{1}{3}(14+7\theta).$$

It is then from the equation $E(X) = \bar{x}$ that

$$\frac{1}{3}(14+7\theta) = \bar{x} \Rightarrow \hat{\theta}_{MM} = \frac{3\bar{x}-14}{7}.$$

(4.2) The likelihood function is

$$L(a) = f(y_1) \cdot f(y_2) \cdot \dots \cdot f(y_n) = \frac{a}{y_1^{a+1}} \cdot \frac{a}{y_2^{a+1}} \cdot \dots \cdot \frac{a}{y_n^{a+1}} = \frac{a^n}{(y_1 y_2 \dots y_n)^{a+1}}.$$

The log likelihood function is then

$$\ln L(a) = n \ln(a) - (a+1) \ln(y_1 y_2 \dots y_n).$$

The equation $0 = \ln' L(a)$ gives that

$$0 = \frac{n}{a} - \ln(y_1 y_2 \dots y_n),$$

which implies that

$$\hat{a}_{ML} = \frac{n}{\ln(y_1 y_2 \dots y_n)}.$$

□

5 (3 points)

One wants to study waiting times for meals in 2 fast food restaurants: McDonald's Tornby and Burger King Tornby.

(5.1) (1p) For McDonald's Tornby, suppose that the distribution X of waiting times is $X \sim N(\mu_x, \sigma^2)$ where μ_x is the average waiting time. A sample is taken from X with $n = 16$, $\bar{x} = 4.2$ and $s_x = 2.2$. Construct a 95% (two-sided) confidence interval I_{μ_x} of μ_x .

(5.2) (2p) For Burger King Tornby, suppose that the distribution Y of waiting times is $Y \sim Exp(\mu_y)$ where μ_y (the average waiting time) is the mean of Y . A sample is taken from Y with $m = 36$ and $\bar{y} = 5.5$. Construct a 95% (two-sided) confidence interval I_{μ_y} of μ_y .

Solution. (5.1) This is Case 1.3, and the confidence interval is

$$I_{\mu_x} = \bar{x} \mp t_{\alpha/2}(n-1) \frac{s_x}{\sqrt{n}} = 4.2 \mp t_{0.025}(15) \frac{2.2}{\sqrt{16}} = 4.2 \mp 2.13 \frac{2.2}{\sqrt{16}} = 4.2 \mp 1.1715 = (3.0285, 5.3715).$$

(5.2). This is Case 1.2, and the confidence interval is (because σ is unknown)

$$I_{\mu_y} = \bar{y} \mp z_{\alpha/2} \frac{\hat{\sigma}}{\sqrt{m}} = 5.5 \mp 1.96 \frac{5.5}{\sqrt{36}} = 5.5 \mp 1.7967 = (3.7033, 7.2967).$$

where we have used the fact that for exponential random variable $\sigma = \mu$ implying that $\hat{\sigma} = \hat{\mu} = \bar{x}$.

□

6 (3 points)

Suppose that a population $X \sim N(\mu, 3^2)$, and a sample is taken from this population with $n = 9$, $\bar{x} = 6.6$ and $s_x = 3.3$. We want to test the following hypotheses with a significance level $\alpha = 5\%$,

$$H_0 : \mu = 6 \quad \text{against} \quad H_a : \mu > 6,$$

(6.1) (1p) Is H_0 rejected based on TS (observed test statistic under H_0) and C (rejection region)?

(6.2) (1p) Is H_0 rejected based on p -value?

(6.3) (1p) Find the power when the true value is $\mu = 7$.

Solution. (6.1) Since here σ^2 is known, one has HT-1 (case 1.1) and

$$TS = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{6.6 - 6}{3/\sqrt{9}} = 0.6, \quad C = (z_\alpha, \infty) = (1.64, \infty),$$

and $TS \notin C$, which implies that H_0 is not rejected.

(6.2) In this case, one can calculate the p -value as

$$p\text{-value} = P(UL \geq TS) = P(N(0, 1) \geq 0.6) = 1 - P(N(0, 1) < 0.6) = 1 - \Phi(0.6) = 1 - 0.7257 = 0.2743.$$

Since p -value $\not< \alpha$, we do not reject H_0 .

(6.3) The power is computed as follows

$$\begin{aligned} h(7) &= P(H_0 \text{ is rejected when } H_0 \text{ is false and } \mu = 7) = P(TS \in C, \text{ when } \mu = 7) \\ &= P\left(\frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} > 1.64, \text{ when } \mu = 7\right) \\ &= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > 1.64 + \frac{(\mu_0 - \mu)}{\sigma/\sqrt{n}}, \text{ when } \mu = 7\right) \\ &= P(N(0, 1) > 0.64) = 1 - P(N(0, 1) \leq 0.64) = 1 - \Phi(0.64) = 1 - 0.7389 = 0.2611. \end{aligned}$$

□

1. Basic probability

(1.1) Conditional probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

(1.2) Total probability $P(B) = \sum_{i=1}^k P(B|A_i)P(A_i)$ where $\{A_i\}$ are disjoint and $\cup_{i=1}^k A_i = S$.

(1.3) Bayes' Theorem $P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^k P(B|A_i)P(A_i)}$ where $\{A_i\}$ are in (1.2).

2. Random variables (r.v.s)

(2.1) Discrete r.v. X has a pmf $p(x) = P(X = x)$ satisfying $p(x) \geq 0$ and $\sum p(x_i) = 1$,

X	x_1	x_2	\cdots	x_n	\cdots
$p(x)$	$p(x_1)$	$p(x_2)$	\cdots	$p(x_n)$	\cdots

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \sum x_i p(x_i)$;
 Variance $\sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \sum x_i^2 p(x_i) - (\sum x_i p(x_i))^2$.

(2.2) Continuous r.v. X has a pdf $f(x)$ satisfying $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$,

$$P(a < X < b) = \int_a^b f(x)dx.$$

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \int_{-\infty}^{\infty} xf(x)dx$;

Variance $\sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - (\int_{-\infty}^{\infty} xf(x)dx)^2$.

(2.3) Cumulative distribution function (cdf) of a r.v. X is $F(x) = P(X \leq x)$.

(2.4) X and Y are r.v.s, a , b and c are scalars, then

$$E(aX + bY + c) = aE(X) + bE(Y) + c,$$

$$V(aX + bY + c) = a^2 V(X) + b^2 V(Y) + 2ab \text{cov}(X, Y),$$

$$E(g(X, Y)) = \begin{cases} \sum_{i,j} g(x_i, y_j) \cdot p(x_i, y_j), & \text{for discrete } (X, Y), \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \cdot f(x, y)dxdy, & \text{for continuous } (X, Y). \end{cases}$$

(2.5) • Discrete r.v. (X, Y) has a joint pmf $p(x, y)$ satisfying $p(x, y) \geq 0$ and $\sum_{x_i} \sum_{y_i} p(x_i, y_i) = 1$.

The marginal pmf of X is $p_X(x) = \sum_y p(x, y)$;

The marginal pmf of Y is $p_Y(y) = \sum_x p(x, y)$;

X and Y are *independent* if $p(x, y) = p_X(x) \cdot p_Y(y)$.

• Continuous r.v. (X, Y) has a joint pdf $f(x, y)$ satisfying $f(x, y) \geq 0$ and $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)dxdy = 1$.

The marginal pdf of X is $f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy$;

The marginal pdf of Y is $f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx$;

X and Y are *independent* if $f(x, y) = f_X(x) \cdot f_Y(y)$.

3. Several special r.v.s

(3.1) $X \sim Bin(n, p)$ has a pmf $p(x) = P(X = x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$, $x = 0, 1, 2, \dots, n$.

$E(X) = n \cdot p$, $V(X) = n \cdot p \cdot (1-p)$.

(3.2) $X \sim Po(\lambda)$ has a pmf $p(x) = P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$, $x = 0, 1, 2, \dots$.

$E(X) = \lambda$, $V(X) = \lambda$.

(3.3) $X \sim Hypergeometric$ has a pmf $p(x) = P(X = x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$.

(3.4) $X \sim Exp(\lambda)$ has a pdf

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(3.5) $X \sim N(\mu, \sigma^2)$ has a pdf

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty.$$

$E(X) = \mu$, $V(X) = \sigma^2$

(3.6) $X \sim U(a, b)$ has a pdf

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

$$E(X) = \frac{a+b}{2}, \quad V(X) = \frac{(b-a)^2}{12}.$$

4. Central Limit Theorem (CLT)

Suppose that a population has mean = μ and variance = σ^2 . A random sample $\{X_1, X_2, \dots, X_n\}$ from this population is given. Then for large $n \geq 30$,

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1).$$

- If the population is normal, then (1) holds for any n .
- Note that $\mu = E(\bar{X})$ and $(\sigma/\sqrt{n})^2 = V(\bar{X})$.

5. Several notations in statistics

(5.1) Sample mean: $\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} = \frac{\sum X_i}{n}$; $\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum x_i}{n}$.

(5.2) Sample variance:

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{1}{n-1} \left(\sum X_i^2 - \frac{(\sum X_i)^2}{n} \right); \quad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{1}{n-1} \left(\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right).$$

- Capital letters X and S^2 refer to the objects based on random sample (therefore they are in general r.v.s), while small letters \bar{x} and s^2 are the objects based on observations (so they are scalars).

(5.3) A point estimator of θ obtained by Maximum Likelihood method is denoted as $\hat{\theta}_{ML}$.

6. Confidence Interval (CI)

In this course, three types of confidence intervals are studied depending on the unknown population parameter(s): CI-1 (confidence intervals for population mean(s)), CI-2 (confidence intervals for population variance(s)), and CI-3 (confidence intervals for population proportion(s)).

CI-1: $(1 - \alpha)$ CI of a population mean μ

case 1.1 (any n) If population $X \sim N(\mu, \sigma^2)$ and σ^2 is known, then $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ and

$$I_\mu = \left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right) := \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.$$

case 1.2 ($n \geq 30$) For any population X , it holds that $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ and

$$I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \text{ or } I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}.$$

case 1.3 (any n) If population $X \sim N(\mu, \sigma^2)$ and σ^2 is unknown, then $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim T(n-1)$ and

$$I_\mu = \bar{x} \mp t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}.$$

CI-1': $(1 - \alpha)$ CI of the difference of two population means $\mu_X - \mu_Y$

case 1.1' (any n_1, n_2) If independent populations $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, and σ_X^2, σ_Y^2 are known, then $\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1)$, and $I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}$.

case 1.2' ($n_1, n_2 \geq 30$) For any independent populations X and Y , it holds that

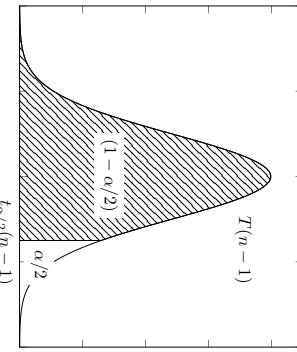
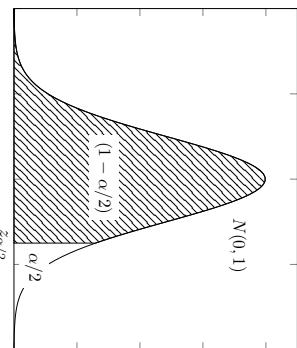
$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1) \text{ and}$$

$$I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}} \text{ or } I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{\sigma}_X^2}{n_1} + \frac{\hat{\sigma}_Y^2}{n_2}}.$$

case 1.3' (any n_1, n_2) If independent populations $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, where σ_X^2, σ_Y^2 are unknown but $\sigma_X^2 = \sigma_Y^2$, then

$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim T(n_1 + n_2 - 2), \text{ where } S^2 = \frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}, \text{ and}$$

$$I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp t_{\alpha/2}(n_1 + n_2 - 2) \cdot s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$



CI-2: $(1 - \alpha)$ CI of population variance(s) σ^2

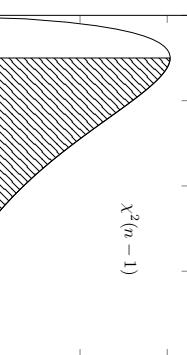
- If a population $X \sim N(\mu, \sigma^2)$ and σ^2 is unknown, then $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, and

$$I_{\sigma^2} = \left(\frac{(n-1)s^2}{\chi_{\alpha/2}^2(n-1)}, \frac{(n-1)s^2}{\chi_{1-\alpha/2}^2(n-1)} \right).$$

- If two independent populations $X \sim N(\mu_X, \sigma^2)$ and $Y \sim N(\mu_Y, \sigma^2)$, and σ^2 is unknown, then $\frac{(n_1+n_2-2)S^2}{\sigma^2} \sim \chi^2(n_1 + n_2 - 2)$, and

$$I_{\sigma^2} = \left(\frac{(n_1 + n_2 - 2)s^2}{\chi_{\alpha/2}^2(n_1 + n_2 - 2)}, \frac{(n_1 + n_2 - 2)s^2}{\chi_{1-\alpha/2}^2(n_1 + n_2 - 2)} \right),$$

where $S^2 = \frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}$.



CI-3: $(1 - \alpha)$ CI of population proportion(s)

- If a (large) population has an unknown proportion p , then $\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim N(0, 1)$ if $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ with $\hat{p} = x/n$, and $I_p = \hat{p} \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- If two independent (large) populations have unknown proportions p_1 and p_2 , then

$$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \sim N(0, 1)$$

if $n_i\hat{p}_i \geq 10$ and $n_i(1 - \hat{p}_i) \geq 10$ for $i = 1, 2$, and $I_{p_1 - p_2} = (\hat{p}_1 - \hat{p}_2) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$.

7. Hypothesis Test (HT)

	H_0 is true	H_0 is false and $\theta = \theta_1$
reject H_0	(type I error or significance level) α	(power) $h(\theta_1) = 1 - h(\theta_1)$
don't reject H_0	1 - α	(type II error) $\beta(\theta_1) = 1 - h(\theta_1)$

reject $H_0 \Leftrightarrow TS \in C \Leftrightarrow p\text{-value} < \alpha$

χ^2 tests for populations (non-parametric tests)

Suppose that for a random sample of a population X the n elements of it are classified into k disjoint groups $A_i, 1 \leq i \leq k$. For each group $A_i, 1 \leq i \leq k$, suppose that there are $N_i, 1 \leq i \leq k$ elements inside. Let $p_i = P(A_i)$ assuming a given distribution of X . Note that $p_1 + p_2 + \dots + p_k = 1$ and $N_1 + N_2 + \dots + N_k = n$. One wants to test the hypotheses

$$H_0 : P(A_i) = p_i, \quad 1 \leq i \leq k, \quad H_a : P(A_i) \neq p_i \text{ for some } 1 \leq i \leq k.$$

If n is large in the sense that $np_i \geq 5$ for all $1 \leq i \leq k$, then the test statistic is

$$\sum_{i=1}^k \frac{(N_i - np_i)^2}{np_i} \approx \chi^2(k-1).$$

Therefore the observation of the test statistic is

$$TS = \sum_{i=1}^k \frac{(n_i - np_i)^2}{np_i}, \text{ where } n_i \text{ is the observation of } N_i, 1 \leq i \leq k.$$

For the critical region C , one can take (note that if H_0 is true, then TS should be close to zero)

$$C = (\chi_\alpha^2(k-1), \infty).$$

The conclusion would be $TS \in C \iff H_0$ is rejected.

8. Linear and logistic regression

(Multiple) linear regression: $Y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon, \varepsilon \sim N(0, \sigma^2)$.

\bullet Y : response variable (which is normal r.v.), $\{x_1, \dots, x_k\}$: predictors (which are scalars).

\bullet sample: $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$.
 \bullet how to estimate $\beta_j \approx \hat{\beta}_j$: least square method, that is, to minimize $\sum_{i=1}^n (\hat{y}_i - y_i)^2$, where the estimated (multiple) linear regression line \hat{y} is

$$\hat{y} = \beta_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k.$$

\bullet $\frac{\hat{\beta}_j - \beta_j}{s.e(\hat{\beta}_j)} \sim T(n-k-1)$, this helps determine whether or not the real $\beta_j = 0$?

\bullet $\sigma^2 \approx \frac{SSE}{n-k-1}$, this gives an estimation of the size of the error.

$\bullet R^2 = \frac{SSE}{SS_T}$ this gives how well the model is (if $R^2 \approx 1$, then the model fits the sample very well).

\bullet How to test $\beta_1 = \dots = \beta_k = 0$? Use the random variable $\frac{SS_R/k}{SSE/(n-k-1)} \sim F(k, n-k-1)$.

Logistic regression: Let Y can only take 0 or 1 with $P(Y=1) = p$ and $P(Y=0) = 1-p$,

$$E(Y) = p(x_1, \dots, x_k) = \frac{e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}{1 + e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}.$$

$\bullet Y$: response variable (which is Bernoulli r.v. $P(Y=1) = p$ and $P(Y=0) = 1-p$, so $E(Y) = p$), $\{x_1, \dots, x_k\}$: predictors (which are scalars).

\bullet sample: $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$.

\bullet how to estimate $\beta_j \approx \hat{\beta}_j$: maximal likelihood method (maximize $\prod_{i=1}^n p(x_{i1}, \dots, x_{ik})^{y_i} (1 - p(x_{i1}, \dots, x_{ik}))^{1-y_i}$).

\bullet $\frac{\hat{\beta}_j - \beta_j}{s.e(\hat{\beta}_j)} \approx N(0, 1)$ for large $n \geq 30$, this helps determine whether or not the real $\beta_j = 0$?

\bullet Classification of a new object $Y(x_1, \dots, x_k)$ as 1 or 0 according

$$Y(x_1, \dots, x_k) = \begin{cases} 1, & \text{if } \hat{p}(x_1, \dots, x_k) \geq 0.5, \\ 0, & \text{if } \hat{p}(x_1, \dots, x_k) < 0.5, \end{cases}$$

where the estimated logit function $\hat{p}(x_1, \dots, x_k)$ is

$$\hat{p}(x_1, \dots, x_k) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}.$$

9. Tables

(9.1) Table for $N(0,1)$ standard normal random variable $\Phi(x) = P(N(0,1) \leq x)$, $x \geq 0$.

There is an important relation $\Phi(-x) = 1 - \Phi(x)$, $x \geq 0$.

x	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7957	0.8023	0.8051	0.8078	0.8106	0.8133	0.8160
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9222	0.9256	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.9330
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0.9965
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9977	0.9978	0.9978	0.9979	0.9979	0.9980	0.9981	0.9981
2.9	0.9981	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9990	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9993	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(9.2) Table for $T(f)$ random variable $F(x) = P(T(f) \leq x)$, where f is a parameter called 'degrees of freedom'.

f	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.9995	$F(x)$
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	636.62	
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	31.60	
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	12.92	
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	8.61	
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	6.87	
6	0.72	1.44	2.45	3.14	3.71	4.32	5.96	10.47	
7	0.71	1.41	1.89	2.36	3.00	3.50	4.03	5.41	
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	5.04	
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.78	
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.59	
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.44	
12	0.70	1.36	1.78	2.18	2.68	3.05	3.43	4.32	
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	4.22	
14	0.69	1.35	1.76	2.14	2.62	2.98	3.33	4.14	
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	4.07	
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	4.01	
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.97	
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.92	
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.88	
20	0.69	1.33	1.72	2.09	2.53	2.85	3.15	3.85	
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.82	
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.79	
23	0.69	1.32	1.71	2.07	2.50	2.81	3.10	3.77	
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.75	
25	0.68	1.32	1.71	2.06	2.49	2.79	3.08	3.73	
26	0.68	1.31	1.71	2.06	2.48	2.78	3.07	3.71	
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.69	
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.67	
29	0.68	1.31	1.70	2.05	2.46	2.76	3.04	3.66	
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.65	
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.55	
50	0.68	1.30	1.68	2.01	2.40	2.68	2.94	3.50	
60	0.68	1.30	1.67	2.00	2.39	2.66	2.91	3.46	
100	0.68	1.29	1.66	2.00	2.36	2.63	2.87	3.39	
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.29	

(9.3) Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

f	$F(x)$										
	0.005	0.001	0.005	0.01	0.025	0.05	0.10	0.20	0.30	0.40	0.50
1	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.15	0.27	0.45	1
2	0.00	0.00	0.01	0.02	0.05	0.10	0.21	0.45	0.71	1.39	2
3	0.02	0.02	0.07	0.11	0.22	0.35	0.58	1.01	1.42	2.37	3
4	0.06	0.09	0.21	0.30	0.48	0.71	1.06	1.65	2.19	2.75	4
5	0.16	0.21	0.41	0.55	0.83	1.15	1.61	2.34	3.00	3.66	5
6	0.30	0.38	0.68	0.87	1.24	1.64	2.20	3.07	3.83	4.57	6
7	0.48	0.60	0.99	1.24	1.69	2.17	2.83	3.82	4.67	5.49	7
8	0.71	0.86	1.34	1.65	2.18	2.73	3.49	4.59	5.53	6.42	8
9	0.97	1.15	1.73	2.09	2.70	3.33	4.17	5.38	6.39	7.36	9
10	1.26	1.48	2.16	2.56	3.25	3.94	4.87	6.18	7.27	8.30	10
11	1.59	1.83	2.60	3.05	3.82	4.57	5.58	6.99	8.15	9.24	11
12	1.93	2.21	3.07	3.57	4.40	5.23	6.30	7.81	9.03	10.18	12
13	2.31	2.62	3.57	4.11	5.01	5.89	7.04	8.63	9.93	11.13	13
14	2.70	3.04	4.07	4.66	5.63	6.57	7.79	9.47	10.82	12.08	14
15	3.11	3.48	4.60	5.23	6.26	7.26	8.55	10.31	11.72	13.03	15
16	3.54	3.94	5.14	5.81	6.91	7.96	9.31	11.15	12.62	13.98	16
17	3.98	4.42	5.70	6.41	7.56	8.67	10.09	12.00	13.53	14.94	17
18	4.44	4.90	6.26	7.01	8.23	9.39	10.86	12.86	14.44	15.89	18
19	4.91	5.41	6.84	7.63	8.91	10.12	11.65	13.72	15.35	16.85	19
20	5.40	5.92	7.43	8.26	9.59	10.85	12.44	14.58	16.27	17.81	20
21	5.90	6.45	8.03	8.90	10.28	11.59	13.24	15.44	17.18	18.77	21
22	6.40	6.98	8.64	9.54	10.98	12.34	14.04	16.31	18.10	19.73	22
23	6.92	7.53	9.26	10.20	11.69	13.09	14.85	17.19	19.02	20.69	23
24	7.45	8.08	9.89	10.86	12.40	13.85	15.66	18.06	19.94	21.65	24
25	7.99	8.65	10.52	11.52	13.12	14.61	16.47	18.94	20.87	22.62	25
26	8.54	9.22	11.16	12.20	13.84	15.38	17.29	19.82	21.79	23.58	26
27	9.09	9.80	11.81	12.88	14.57	16.15	18.11	20.70	22.72	24.54	27
28	9.66	10.39	12.46	13.56	15.31	16.93	18.94	21.59	23.65	25.51	28
29	10.23	10.99	13.12	14.26	16.05	17.71	19.77	22.48	24.58	26.48	29
30	10.80	11.59	13.79	14.95	16.79	18.49	20.60	23.36	25.51	27.44	30
40	16.91	17.92	20.71	22.16	24.43	26.51	29.05	32.34	34.87	37.13	40
50	23.46	24.67	27.99	29.71	32.36	34.76	37.69	41.45	44.31	46.86	50
60	30.34	31.74	35.53	37.48	40.48	43.19	46.46	50.64	53.81	56.62	60
100	59.90	61.92	67.33	70.06	74.22	77.93	82.36	87.95	92.13	95.81	100

Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

f	$F(x)$									
	0.60	0.70	0.80	0.90	0.95	0.975	0.99	0.995	0.999	0.9995
1	0.71	1.07	1.64	2.71	3.84	5.02	6.63	7.88	10.83	12.12
2	1.83	2.41	3.22	4.61	5.99	7.38	9.21	10.60	13.82	15.20
3	2.95	3.66	4.64	6.25	7.81	9.35	11.34	12.84	16.27	17.73
4	4.04	4.88	5.99	7.78	9.49	11.14	13.28	14.86	18.47	20.00
5	5.13	6.06	7.29	9.24	11.07	12.83	15.09	16.75	20.52	22.11
6	6.21	7.23	8.56	10.64	12.59	14.45	16.81	18.55	22.46	24.10
7	7.28	8.38	9.80	12.02	14.07	16.01	18.48	20.28	24.32	26.02
8	8.35	9.52	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87
9	9.41	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67
10	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42
11	11.53	12.90	14.63	17.28	19.68	21.92	24.72	26.76	31.14	33.14
12	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82
13	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48
14	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11
15	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.75
16	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31
17	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88
18	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43
19	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97
20	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50
21	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01
22	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51
23	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00
24	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48
25	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95
26	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41
27	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.64	55.48	57.86
28	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30
29	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73
30	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16
40	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.09
50	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	86.66	89.56
60	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.69
100	102.95	106.91	111.67	118.50	124.34	129.56	135.81	140.17	149.45	153.17

(9.4) Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
 If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1-p) \geq n-k)$.

n	k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	
3	0	0.9575	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500	
4	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	
5	0	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.7183	0.6480	0.5747	0.5000	
6	0	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750	
7	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	
8	0	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125	
9	0	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875	
10	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313	
11	0	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875	
12	0	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000	
13	0	0.9999	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125	
14	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
15	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	
16	0	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094	
17	0	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438	
18	0	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563	
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
20	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	
21	0	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625	
22	0	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3124	0.2266	
23	0	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000	
24	0	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734	
25	0	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375	
26	0	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922	0.9900	
27	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	
28	0	1.0000	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352	
29	0	2.0000	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445	
30	0	3.0000	0.9950	0.9786	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633	0.2700	
31	0	4.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367	
32	0	5.0000	1.0000	0.9998	0.9988	0.9887	0.9747	0.9502	0.9115	0.8555	0.7700	
33	0	6.0000	1.0000	1.0000	0.9999	0.9987	0.9964	0.9915	0.9819	0.9643	0.9300	
34	0	7.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961	0.9900	
35	0	8.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
36	0	9.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
37	0	10.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
38	0	11.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
39	0	12.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
 If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1-p) \geq n-k)$.

n	k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
11	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005	
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002	
13	0	0.5040	0.2242	0.1087	0.0544	0.0274	0.0121	0.0055	0.0023	0.0009	0.0003	
14	0	0.4816	0.1659	0.0843	0.0443	0.0274	0.0158	0.0085	0.0042	0.0016	0.0003	
15	0	0.4552	0.1174	0.0574	0.0317	0.0197	0.0102	0.0053	0.0026	0.0011	0.0003	
16	0	0.4256	0.0982	0.0496	0.0285	0.0158	0.0085	0.0043	0.0021	0.0009	0.0003	
17	0	0.3971	0.0778	0.0389	0.0197	0.0102	0.0055	0.0030	0.0015	0.0006	0.0002	
18	0	0.3744	0.0681	0.0348	0.0177	0.0088	0.0044	0.0022	0.0010	0.0004	0.0002	
19	0	0.3532	0.0583	0.0297	0.0145	0.0065	0.0033	0.0016	0.0007	0.0003	0.0001	
20	0	0.3345	0.0488	0.0246	0.0115	0.0053	0.0026	0.0011	0.0005	0.0002	0.0001	
21	0	0.3125	0.0392	0.0197	0.0088	0.0044	0.0021	0.0009	0.0004	0.0002	0.0001	
22	0	0.2960	0.0303	0.0158	0.0065	0.0033	0.0016	0.0007	0.0003	0.0001	0.0001	
23	0	0.2823	0.0267	0.0121	0.0051	0.0025	0.0011	0.0005	0.0002	0.0001	0.0001	
24	0	0.2660	0.0236	0.0102	0.0046	0.0021	0.0009	0.0004	0.0002	0.0001	0.0001	
25	0	0.2500	0.0205	0.0087	0.0040	0.0018	0.0007	0.0003	0.0001	0.0001	0.0001	
26	0	0.2333	0.0174	0.0065	0.0031	0.0013	0.0005	0.0002	0.0001	0.0001	0.0001	
27	0	0.2134	0.0143	0.0050	0.0023	0.0009	0.0003	0.0001	0.0001	0.0001	0.0001	
28	0	0.1935	0.0112	0.0038	0.0016	0.0006	0.0002	0.0001	0.0001	0.0001	0.0001	
29	0	0.1734	0.0081	0.0027	0.0010	0.0004	0.0001	0.0001	0.0001	0.0001	0.0001	
30	0	0.1534	0.0050	0.0017	0.0006	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	
31	0	0.1334	0.0029	0.0009	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
32	0	0.1134	0.0008	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
33	0	0.0933	0.0007	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
34	0	0.0734	0.0006	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
35	0	0.0533	0.0005	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
36	0	0.0334	0.0004	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
37	0	0.0134	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
38	0	0.0033	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	
39	0	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1-p) \geq n-k)$.

n	k	0.05	0.10	0.15	0.20	0.25	p	0.30	0.35	0.40	0.45	0.50
14	0	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001	
1	0.8470	0.5284	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0009			
2	0.9699	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065		
3	0.9558	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287	0.0123	
4	0.9996	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898	0.0464	
5	1.0000	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120	0.1260	0.0596
6	1.0000	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953	0.2348	0.0245
7	1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047	0.4478	0.2902
8	1.0000	1.0000	1.0000	0.9996	0.9978	0.9917	0.9757	0.8811	0.7830	0.6405	0.4743	0.3145
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9940	0.9825	0.9574	0.9102	0.8011	0.6626
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9961	0.9886	0.9713	0.9177	0.6855
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9978	0.9935	0.9777	0.8338
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9894	0.9283
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9755
14	0	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000	
1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005		
2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037		
3	0.9445	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176		
4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592		
5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509		
6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036		
7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000		
8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964		
9	1.0000	1.0000	1.0000	0.9990	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491		
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408		
11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824	0.9524		
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963	0.9790		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9999		
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
15	0	0.4401	0.1853	0.0743	0.0281	0.0100	0.0033	0.0010	0.0003	0.0001	0.0000	
1	0.8108	0.5147	0.2839	0.1407	0.0635	0.0261	0.0098	0.0033	0.0010	0.0003		
2	0.9571	0.7892	0.5614	0.3518	0.1971	0.0994	0.0451	0.0183	0.0066	0.0021		
3	0.9360	0.9316	0.7899	0.5981	0.4050	0.2459	0.1339	0.0651	0.0281	0.0106		
4	0.9991	0.9830	0.9209	0.7982	0.6302	0.4499	0.2892	0.1666	0.0853	0.0384		
5	0.9999	0.9967	0.9765	0.9183	0.8103	0.6598	0.4900	0.3288	0.1976	0.1051		
6	1.0000	0.9995	0.9944	0.9733	0.9204	0.8247	0.6881	0.5272	0.3660	0.2272		
7	1.0000	0.9999	0.9930	0.9729	0.9256	0.8406	0.7161	0.5629	0.4018	0.2409		
8	1.0000	1.0000	0.9998	0.9985	0.9795	0.9743	0.9329	0.8579	0.7441	0.5982		
9	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9771	0.9417	0.8759	0.7728		
10	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9809	0.9514	0.8949	0.8287		
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9951	0.9851	0.9616	0.9214		
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9965	0.9894	0.9529		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9999		
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1-p) \geq n-k)$.

n	k	0.05	0.10	0.15	0.20	0.25	p	0.30	0.35	0.40	0.45	0.50
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000	0.0000	
1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0067	0.0021	0.0006	0.0001		
2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0327	0.0123	0.0041	0.0012		
3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.1028	0.0464	0.0184	0.0064		
4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.2348	0.1260	0.0596	0.0245		
5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5968	0.4197	0.2639	0.1471	0.0717		
6	1.0000	0.9992	0.9623	0.8929	0.7752	0.6188	0.4478	0.2902	0.1662			
7	1.0000	0.9999	0.9883	0.9598	0.8787	0.7872	0.6405	0.4743	0.3145			
8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.9006	0.8011	0.6626	0.5000		
9	1.0000	1.0000	1.0000	0.9999	0.9999	0.9970	0.9894	0.9699	0.9283	0.7597		
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
16	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000		
1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002	0.0000		
2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0170	0.0055	0.0015	0.0004		
3	0.9836	0.8850	0.6841	0.4551	0.2631	0.1332	0.0591	0.0230	0.0077	0.0022		
4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.1500	0.0696	0.0280	0.0096		
5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.2968	0.1629	0.0777	0.0318		
6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.4812	0.3081	0.1727	0.0835		
7	1.0000	0.9997	0.9959	0.9767	0.9225	0.8180	0.6656	0.4878	0.3169	0.1796		
8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.8145	0.6675	0.4940	0.3238		
9	1.0000	1.0000	0.9999	0.9974	0.9674	0.9125	0.8139	0.6710	0.5000	0.3238		
10	1.0000	1.0000	1.0000	0.9997	0.9797	0.9563	0.9115	0.8159	0.6762	0.4940		
11	1.0000	1.0000	1.0000	0.9995	0.9972	0.9886	0.9648	0.9129	0.8204	0.6405		
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9969	0.9884	0.9658	0.9165	0.7597		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		

(9.5) Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	μ									
k	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9956
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
k	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.9329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.6890	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
2	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
3	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
4	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
5	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9834	0.9775
6	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
7	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
k	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
0	0.0408	0.0334	0.0273	0.0224	0.0183	0.0150	0.0123	0.0101	0.0082	0.0067
1	0.1712	0.1468	0.1257	0.1074	0.0916	0.0780	0.0663	0.0563	0.0477	0.0404
2	0.3799	0.3397	0.3027	0.2689	0.2381	0.2102	0.1851	0.1626	0.1425	0.1247
3	0.6025	0.5584	0.5152	0.4735	0.4335	0.3954	0.3594	0.3257	0.2942	0.2650
4	0.7806	0.7442	0.7064	0.6678	0.6288	0.5898	0.5512	0.5132	0.4763	0.4405
5	0.8946	0.8705	0.8441	0.8156	0.7851	0.7531	0.7199	0.6858	0.6510	0.6160
6	0.9554	0.9421	0.9267	0.9091	0.8893	0.8675	0.8436	0.8180	0.7908	0.7622
7	0.9832	0.9769	0.9692	0.9559	0.9489	0.9361	0.9214	0.9049	0.8867	0.8666
8	0.9943	0.9917	0.9883	0.9840	0.9786	0.9721	0.9642	0.9549	0.9442	0.9319
9	0.9995	0.9992	0.9987	0.9981	0.9972	0.9959	0.9943	0.9922	0.9896	0.9863
k	5.2	5.4	5.6	5.8	6.0	6.5	7.0	7.5	8.0	8.5
0	0.0055	0.0045	0.0037	0.0030	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002
1	0.0342	0.0289	0.0244	0.0206	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019
2	0.1088	0.0948	0.0824	0.0715	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093
3	0.2381	0.2133	0.1906	0.1700	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301
4	0.4061	0.3733	0.3422	0.3127	0.2851	0.2237	0.1730	0.1321	0.0966	0.0744
5	0.5809	0.5461	0.5119	0.4783	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496
6	0.7324	0.7017	0.6703	0.6384	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562
7	0.8449	0.8149	0.7970	0.7710	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856
8	0.9181	0.9027	0.8857	0.8672	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231
9	0.9603	0.9512	0.9409	0.9292	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530
10	0.9823	0.9775	0.9718	0.9651	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634
11	0.9904	0.9875	0.9841	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8047
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
k	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146	0.1991
2	0.6496	0.6227	0.5960	0.5597	0.5348	0.5184	0.4936	0.4695	0.4460	0.4232
3	0.8386	0.8194	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696	0.6474	0.6142
4	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8477	0.8318	0.8153	0.7915
5	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258	0.9161
6	0.9941	0.9925	0.9884	0.9828	0.9794	0.9756	0.9713	0.9665	0.9623	0.9573
7	0.9985	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901	0.9881	0.9850
8	0.9997	0.9995	0.9994	0.9989	0.9985	0.9981	0.9976	0.9969	0.9962	0.9953
9	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	0.9992
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Poisson random variable $P(Po(\mu) \leq k)$.

Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	μ
0	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000
4	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0000
5	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0000
6	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0000
7	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0000
8	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0000
9	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0000
10	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0000
11	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0000
12	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0000
13	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.0000
14	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.0000
15	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.0000
16	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.0000
17	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.0000
18	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.0000
19	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.0000
20	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.0000
21	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.0000
22	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.0000
23	1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.0000
24	1.0000	1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.0000
25	1.0000	1.0000	1.0000	0.9999	0.9997	0.9974	0.9938	0.9869	0.9748	0.9548	0.0000
26	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.0000
27	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.0000
28	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9900	0.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9973	0.9947	0.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9976	0.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9980	0.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000