

Examiner: Xiangfeng Yang (Tel: 013 28 57 88).**Things allowed (Hjälpmedel):** a calculator.**Scores rating (Betygsgränser):** 8-11 points giving rate 3; 11.5-14.5 points giving rate 4; 15-18 points giving rate 5. Write down all necessary steps in solutions in order to receive as many points as possible.**1 (3 points)**

A box contains 10 balls, of which 4 are black and 6 are white. Now 3 balls are randomly taken out from the box at the same time. The following three events are defined:

$$A = \{\text{there is exactly 1 black ball in these 3 chosen balls}\},$$

$$B = \{\text{there is at least 1 black ball in these 3 chosen balls}\},$$

$$C = \{\text{there is at least 1 white ball in these 3 chosen balls}\}.$$
(1.1) (1p) Find the probability $P(A)$.(1.2) (1p) Find the probability $P(B)$.(1.3) (1p) Find the probability $P(A \cup B \cup C)$.*Solution.* (1.1) It is from combination that

$$P(A) = P(1 \text{ black and 2 white}) = \frac{\binom{4}{1} \cdot \binom{6}{2}}{\binom{10}{3}} = \frac{4 \cdot 15}{120} = \frac{1}{2} = 0.5.$$

(1.2)

$$\begin{aligned} P(B) &= P(1 \text{ black and 2 white}) + P(2 \text{ black and 1 white}) + P(3 \text{ black and 0 white}) \\ &= \frac{\binom{4}{1} \cdot \binom{6}{2}}{\binom{10}{3}} + \frac{\binom{4}{2} \cdot \binom{6}{1}}{\binom{10}{3}} + \frac{\binom{4}{3} \cdot \binom{6}{0}}{\binom{10}{3}} = \frac{60 + 36 + 4}{120} = \frac{5}{6} = 0.8333. \end{aligned}$$

(1.3) It is clear that A is contained in B , therefore $A \cup B = B$. It implies that

$$P(A \cup B \cup C) = P(B \cup C) = 1 - P(B' \cap C') = 1 - P(\text{no black} \cap \text{no white}) = 1 - 0 = 1.$$

□

2 (3 points)

Let (X, Y) be a two dimensional discrete random variable with the following joint probability mass function $p(x, y)$:

$X \setminus Y$	-1	1
-1	0.2	a
1	0.1	0.3

The table tells that both X and Y take values -1 and 1 .

(2.1) (0.5p) Find the value of a .(2.2) (1p) Find the conditional probability $P(X > 0 \mid Y < 0)$.(2.3) (0.5p) Find the mean $E(X)$ of X .(2.4) (1p) Are X and Y independent? Why?*Solution.* (2.1) Since the sum of $p(x, y)$ should be equal to 1, it holds that

$$1 = 0.2 + 0.1 + 0.3 + a \implies a = 0.4.$$

(2.2)

$$P(X > 0 \mid Y < 0) = \frac{P(X > 0 \cap Y < 0)}{P(Y < 0)} = \frac{P(X = 1 \cap Y = -1)}{P(Y = -1)} = \frac{0.1}{0.2 + 0.1} = \frac{1}{3}.$$

(2.3) The table for only X can be obtained as follows

X	-1	1
$p_X(x)$	0.6	0.4

The mean is

$$E(X) = -1 \cdot 0.6 + 1 \cdot 0.4 = -0.2.$$

(2.4) X and Y are NOT independent because $p(x, y) \neq p_X(x) \cdot p_Y(y)$. For example,

$$0.2 = p(-1, -1) \neq p_X(-1) \cdot p_Y(-1) = 0.6 \cdot 0.3 = 0.18.$$

□

3 (3 points)

Suppose that there is a program ⑤ at LiU which has 1000 students in total. Now a professor in this program ⑤ is planning to give a new AI course which is open to all these 1000 students. This AI course will be given only when there are at least 50 registered students. According to a survey, each student will register for this AI course with probability 4%. Assume that all these 1000 students are independent. What is the probability that the AI course will be given? (Namely, what is the probability that there are at least 50 registered students?).

Solution. Method-1 (CLT): Let $X_1, X_2, \dots, X_{1000}$ denote “register/non-register” for these 1000 students, then one can assume that $X_1, X_2, \dots, X_{1000}$ are independent and each has the same distribution as follows

X_i	0	1
$p(x)$	96%	4%

Then our aim is to find the probability $P(X_1 + X_2 + \dots + X_{1000} \geq 50)$. To this end, one needs to find the individual mean μ and individual variance σ^2 :

$$\mu = E(X_i) = 0 \cdot 96\% + 1 \cdot 4\% = 4\% = 0.04,$$

$$\sigma^2 = V(X_i) = E(X_i^2) - \mu^2 = (0^2 \cdot 96\% + 1^2 \cdot 4\%) - 4\%^2 = 0.04 - 0.0016 = 0.0384.$$

Therefore it is from CLT that

$$\begin{aligned} &P(X_1 + X_2 + \dots + X_{1000} \geq 50) \\ &= P\left(\frac{X_1 + X_2 + \dots + X_{1000}}{1000} \geq 50/1000\right) = P(\bar{X} \geq 0.05) \\ &= P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \geq \frac{0.05 - 0.04}{\sqrt{0.0384}/\sqrt{1000}}\right) = P(N(0, 1) \geq 1.61) = 1 - \Phi(1.61) = 1 - 0.9463 = 0.0537. \end{aligned}$$

Method-2 (Binomial): If we use X to denote the number of registered students, then X is a Binomial random variable: $X \sim \text{Bin}(1000, 4\%)$. Then we aim to find $P(X \geq 50)$. To this end, one uses normal approximation to Binomial (which is actually CLT) as follows:

$$\begin{aligned} &P(X \geq 50) = P(\text{Bin}(1000, 4\%) \geq 50) = P(N(1000 \cdot 4\%, 1000 \cdot 4\% \cdot (1 - 4\%)) \geq 50) = P(N(40, 38.4) \geq 50) \\ &= P(N(0, 1) \geq \frac{50 - 40}{\sqrt{38.4}}) = P(N(0, 1) \geq 1.61) = 1 - \Phi(1.61) = 1 - 0.9463 = 0.0537. \end{aligned}$$

□

4 (3 points)

Suppose that a population X has a probability density function as follows

$$f(x) = \begin{cases} \theta, & \text{if } 0 < x < 1, \\ 1 - \theta, & \text{if } 1 \leq x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

where $0 < \theta < 1$ is an unknown parameter. A sample $\{0.8, 1.6, 0.9\}$ is taken from this population.

(4.1) (1p) Find a point estimate $\hat{\theta}_{MM}$ of θ based on the method of moments.

(4.2) (2p) Find a point estimate $\hat{\theta}_{ML}$ of θ based on the maximum-likelihood method.

Solution. (4.1) We first compute the mean $E(X)$ of X as follows

$$E(X) = \int_{-\infty}^{+\infty} x \cdot f(x) dx = \int_0^1 x \cdot \theta dx + \int_1^2 x \cdot (1 - \theta) dx = \frac{1}{2}\theta + \frac{3}{2}(1 - \theta) = 1.5 - \theta.$$

Therefore, the equation $E(X) = \bar{x}$ gives (with $\bar{x} = \frac{0.8+1.6+0.9}{3} = 1.1$)

$$1.5 - \theta = \bar{x} \implies \hat{\theta}_{MM} = 1.5 - \bar{x} = 1.5 - 1.1 = 0.4.$$

(4.2) The likelihood function is

$$L(\theta) = f(x_1) \cdot f(x_2) \cdot \dots \cdot f(x_n) = f(0.8) \cdot f(1.6) \cdot f(0.9) = \theta \cdot (1 - \theta) \cdot \theta = \theta^2 - \theta^3.$$

The derivative $L'(\theta) = 0$ gives

$$0 = L'(\theta) = 2\theta - 3\theta^2 \implies 0 = 2 - 3\theta \implies \hat{\theta}_{ML} = \frac{2}{3} = 0.6667.$$

□

5 (3 points)

Assume that starting salaries X of all LiU year-2025 graduates follow a normal distribution $X \sim N(\mu, \sigma^2)$, where μ is the average starting salary of all LiU year-2025 graduates. To study μ , a sample of 16 LiU year-2025 graduates is chosen. The average salary of these 16 graduates is 51000 SEK/month and the salary standard deviation of these 16 graduates is 2000 SEK/month. Construct a 95% two-sided confidence interval I_μ of μ based on this sample.

Solution. This is Case 1.3:

$$I_\mu = \bar{x} \pm t_{\alpha/2}(n-1) \frac{s}{\sqrt{n}} = 51000 \pm t_{0.025}(15) \frac{2000}{\sqrt{16}} = 51000 \pm 2.13 \frac{2000}{4} = 51000 \pm 1065 = (49935, \quad 52065).$$

□

6 (3 points)

Recently, a 30-day training program is launched in a Fitness Center claiming that any adult attending this program can lose weight more than 5 kg. To study such a claim, 3 persons are randomly chosen from this program and their weights (before and after the program) are recorded as follows:

	Person 1	Person 2	Person 3
weight before	91.6	96.4	88.2
weight after	82.6	91.7	86.1
difference (weight before – weight after)	9.0	4.7	2.1

Assume that the differences $\{9.0, 4.7, 2.1\}$ are regarded as a sample from a normal population $X \sim N(\mu, \sigma^2)$. Therefore, the claim seems to be correct if $\mu > 5$. To check whether or not $\mu > 5$, hypothesis testing (HT) will be used with a significance level $\alpha = 5\%$.

(6.1) (1p) Write down H_0 and H_a .

(6.2) (2p) Is H_0 rejected? Why?

Solution. (6.1)

$$\begin{cases} H_0 : & \mu = 5 \\ H_a : & \mu > 5 \end{cases}$$

(6.2) First, it can be obtained that

$$\bar{x} = \frac{9.0 + 4.7 + 2.1}{3} = 5.2667, \quad s^2 = \frac{1}{3-1} ((9.0 - 5.2667)^2 + (4.7 - 5.2667)^2 + (2.1 - 5.2667)^2) = 12.1433.$$

The Test Statistic and the Rejection Region can be computed as follows:

$$TS = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{5.2667 - 5}{\sqrt{12.1433}/\sqrt{3}} = 0.1326,$$
$$C = (t_\alpha(n-1), +\infty) = (t_{0.05}(3-1), +\infty) = (2.92, +\infty).$$

Since $TS \notin C$, we do NOT reject H_0 (that is, the sample does not provide any evidence that $\mu > 5$, namely the claim is NOT supported by the sample). \square

1. Basic probability

$$(1.1) \text{ Conditional probability } P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

$$(1.2) \text{ Total probability } P(B) = \sum_{i=1}^k P(B|A_i)P(A_i) \text{ where } \{A_i\} \text{ are disjoint and } \bigcup_{i=1}^k A_i = S.$$

$$(1.3) \text{ Bayes' Theorem } P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^k P(B|A_i)P(A_i)} \text{ where } \{A_i\} \text{ are in (1.2).}$$

2. Random variables (r.v.s)

$$(2.1) \text{ Discrete r.v. } X \text{ has a pmf } p(x) = P(X = x) \text{ satisfying } p(x) \geq 0 \text{ and } \sum p(x_i) = 1,$$

$$\begin{array}{c|cccc} X & x_1 & x_2 & \cdots & x_n \\ \hline p(x) & p(x_1) & p(x_2) & \cdots & p(x_n) \end{array}$$

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \sum x_i p(x_i)$;

$$\text{Variance } \sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \sum x_i^2 p(x_i) - \left(\sum x_i p(x_i) \right)^2.$$

$$(2.2) \text{ Continuous r.v. } X \text{ has a pdf } f(x) \text{ satisfying } f(x) \geq 0 \text{ and } \int_{-\infty}^{\infty} f(x) dx = 1,$$

$$P(a < X < b) = \int_a^b f(x) dx.$$

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \int_{-\infty}^{\infty} x f(x) dx$;

$$\text{Variance } \sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \left(\int_{-\infty}^{\infty} x f(x) dx \right)^2.$$

$$(2.3) \text{ Cumulative distribution function (cdf) of a r.v. } X \text{ is } F(x) = P(X \leq x).$$

$$(2.4) X \text{ and } Y \text{ are r.v.s, } a, b \text{ and } c \text{ are scalars, then}$$

$$E(aX + bY + c) = aE(X) + bE(Y) + c,$$

$$V(aX + bY + c) = a^2 V(X) + b^2 V(Y) + 2ab \operatorname{cov}(X, Y),$$

$$E(g(X, Y)) = \begin{cases} \sum_{i,j} g(x_i, y_j) \cdot p(x_i, y_j), & \text{for discrete } (X, Y), \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \cdot f(x, y) dx dy, & \text{for continuous } (X, Y). \end{cases}$$

$$(2.5) \bullet \text{ Discrete r.v. } (X, Y) \text{ has a joint pmf } p(x, y) \text{ satisfying } p(x, y) \geq 0 \text{ and } \sum x_i \sum y_j p(x_i, y_j) = 1.$$

The *marginal pmf* of X is $p_X(x) = \sum_y p(x, y)$;

The *marginal pmf* of Y is $p_Y(y) = \sum_x p(x, y)$;

X and Y are *independent* if $p(x, y) = p_X(x) \cdot p_Y(y)$.

$$\bullet \text{ Continuous r.v. } (X, Y) \text{ has a joint pdf } p(x, y) \text{ satisfying } f(x, y) \geq 0 \text{ and } \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1.$$

The *marginal pdf* of X is $f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$;

The *marginal pdf* of Y is $f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$;

X and Y are *independent* if $f(x, y) = f_X(x) \cdot f_Y(y)$.

3. Several special r.v.s

$$(3.1) X \sim \operatorname{Bin}(n, p) \text{ has a pmf } p(x) = P(X = x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n.$$

$$E(X) = n \cdot p, \quad V(X) = n \cdot p \cdot (1-p).$$

$$(3.2) X \sim \operatorname{Po}(\lambda) \text{ has a pmf } p(x) = P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \dots$$

$$E(X) = \lambda, \quad V(X) = \lambda.$$

$$(3.3) X \sim \operatorname{Hypergeometric} \text{ has a pmf } p(x) = P(X = x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}.$$

$$(3.4) X \sim \operatorname{Exp}(\lambda) \text{ has a pdf}$$

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$E(X) = \frac{1}{\lambda}, \quad V(X) = \left(\frac{1}{\lambda}\right)^2.$$

$$(3.5) X \sim N(\mu, \sigma^2) \text{ has a pdf}$$

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, \quad -\infty < x < \infty.$$

$$E(X) = \mu, \quad V(X) = \sigma^2.$$

$$(3.6) X \sim U(a, b) \text{ has a pdf}$$

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

$$E(X) = \frac{a+b}{2}, \quad V(X) = \frac{(b-a)^2}{12}.$$

4. Central Limit Theorem (CLT)

Suppose that a population has mean $= \mu$ and variance $= \sigma^2$. A random sample $\{X_1, X_2, \dots, X_n\}$ from this population is given. Then for large $n \geq 30$,

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1). \quad (1)$$

$$\bullet \text{ If the population is normal, then (1) holds for any } n.$$

$$\bullet \text{ Note that } \mu = E(\bar{X}) \text{ and } (\sigma / \sqrt{n})^2 = V(\bar{X}).$$

5. Several notations in statistics

$$(5.1) \text{ Sample mean: } \bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} = \sum \frac{X_i}{n}; \quad \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \sum \frac{x_i}{n}.$$

$$(5.2) \text{ Sample variance:}$$

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{1}{n-1} \left(\sum X_i^2 - \frac{(\sum X_i)^2}{n} \right); \quad s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{1}{n-1} \left(\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right).$$

\bullet Capital letters \bar{X} and S^2 refer to the objects based on random sample (therefore they are in general r.v.s), while small letters \bar{x} and s^2 are the objects based on observations (so they are scalars).

$$(5.3) \text{ A point estimator of } \theta \text{ obtained by Methods of Moments is denoted as } \hat{\theta}_{MM}.$$

$$(5.4) \text{ A point estimator of } \theta \text{ obtained by Maximum Likelihood method is denoted as } \hat{\theta}_{ML}.$$

6. Confidence Interval (CI)

In this course, three types of confidence intervals are studied depending on the unknown population parameter(s): CI-1 (confidence intervals for population mean(s)), CI-2 (confidence intervals for population variance(s)), and CI-3 (confidence intervals for population proportion(s)).

CI-1: $(1 - \alpha)$ CI of a population mean μ

case 1.1 (any n) If population $X \sim N(\mu, \sigma^2)$ and σ^2 is known, then $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ and

$$I_\mu = \left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right) := \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.$$

case 1.2 ($n \geq 30$) For any population X , it holds that $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ and

$$I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \text{ or } I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}.$$

case 1.3 (any n) If population $X \sim N(\mu, \sigma^2)$ and σ^2 is unknown, then $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim T(n-1)$ and

$$I_\mu = \bar{x} \mp t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}.$$

CI-1': $(1 - \alpha)$ CI of the difference of two population means $\mu_X - \mu_Y$

case 1.1' (any n_1, n_2) If independent populations $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, and σ_X^2, σ_Y^2 are

known, then $\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1)$, and $I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}$.

case 1.2' ($n_1, n_2 \geq 30$) For any independent populations X and Y , it holds that

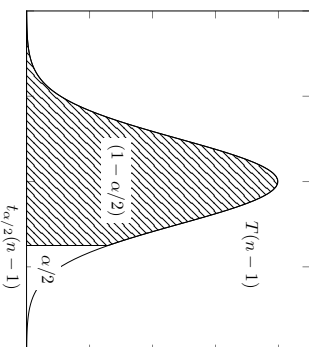
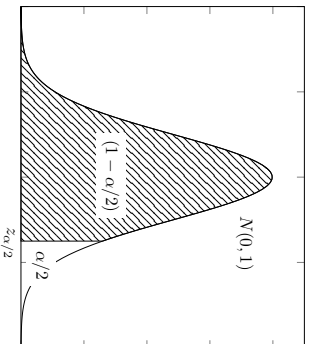
$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1) \text{ and}$$

$$I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}} \text{ or } I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{\sigma}_X^2}{n_1} + \frac{\hat{\sigma}_Y^2}{n_2}}.$$

case 1.3' (any n_1, n_2) If independent populations $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, where σ_X^2, σ_Y^2 are unknown but $\sigma_X^2 = \sigma_Y^2$, then

$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{S \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim T(n_1 + n_2 - 2), \text{ where } S^2 = \frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}, \text{ and}$$

$$I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp t_{\alpha/2}(n_1 + n_2 - 2) \cdot s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$



CI-2: $(1 - \alpha)$ CI of population variance(s) σ^2

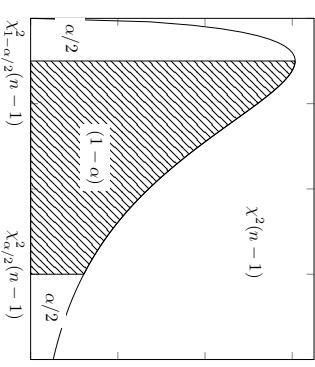
• If a population $X \sim N(\mu, \sigma^2)$ and σ^2 is unknown, then $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, and

$$I_{\sigma^2} = \left(\frac{(n-1)s^2}{\chi_{\alpha/2}^2(n-1)}, \frac{(n-1)s^2}{\chi_{1-\alpha/2}^2(n-1)} \right).$$

• If two independent populations $X \sim N(\mu_X, \sigma^2)$ and $Y \sim N(\mu_Y, \sigma^2)$, and σ^2 is unknown, then $\frac{(n_1+n_2-2)S^2}{\sigma^2} \sim \chi^2(n_1+n_2-2)$, and

$$I_{\sigma^2} = \left(\frac{(n_1+n_2-2)s^2}{\chi_{\alpha/2}^2(n_1+n_2-2)}, \frac{(n_1+n_2-2)s^2}{\chi_{1-\alpha/2}^2(n_1+n_2-2)} \right),$$

where $S^2 = \frac{(n_1-1)S_X^2 + (n_2-1)S_Y^2}{n_1+n_2-2}$.



CI-3: $(1 - \alpha)$ CI of population proportion(s)

• If a (large) population has an unknown proportion p , then $\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim N(0, 1)$ if $n\hat{p} \geq 10$ and

$n(1 - \hat{p}) \geq 10$ with $\hat{p} = x/n$, and $I_p = \hat{p} \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.

• If two independent (large) populations have unknown proportions p_1 and p_2 , then

$$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \sim N(0, 1)$$

if $n_i \hat{p}_i \geq 10$ and $n_i(1 - \hat{p}_i) \geq 10$ for $i = 1, 2$, and $I_{p_1 - p_2} = (\hat{p}_1 - \hat{p}_2) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$.

7. Hypothesis Test (HT)

	H_0 is true	H_0 is false and $\theta = \theta_1$
reject H_0	(type I error or significance level) α	(power) $h(\theta_1)$
don't reject H_0	$1 - \alpha$	(type II error) $\beta(\theta_1) = 1 - h(\theta_1)$

reject $H_0 \Leftrightarrow TS \in C \Leftrightarrow p\text{-value} < \alpha$

χ^2 tests for populations (non-parametric tests)

Suppose that for a random sample of a population X , the n elements of it are classified into k disjoint groups A_i , $1 \leq i \leq k$. For each group A_i , $1 \leq i \leq k$, suppose that there are N_i , $1 \leq i \leq k$ elements inside. Let $p_i = P(A_i)$ assuming a given distribution of X . Note that $p_1 + p_2 + \dots + p_k = 1$ and $N_1 + N_2 + \dots + N_k = n$. One wants to test the hypotheses

$$H_0 : P(A_i) = p_i, \quad 1 \leq i \leq k, \quad H_a : P(A_i) \neq p_i \text{ for some } 1 \leq i \leq k.$$

If n is large in the sense that $np_i \geq 5$ for all $1 \leq i \leq k$, then the test statistic is

$$\sum_{i=1}^k \frac{(N_i - np_i)^2}{np_i} \approx \chi^2(k-1).$$

Therefore the observation of the test statistic is

$$TS = \sum_{i=1}^k \frac{(n_i - np_i)^2}{np_i}, \text{ where } n_i \text{ is the observation of } N_i, 1 \leq i \leq k.$$

For the critical region C , one can take (note that if H_0 is true, then TS should be close to zero)

$$C = (\chi_{\alpha}^2(k-1), \infty).$$

The conclusion would be $TS \in C \iff H_0$ is rejected.

8. Linear and logistic regression

(Multiple) linear regression: $Y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon$, $\varepsilon \sim N(0, \sigma^2)$.

- Y : response variable (which is normal r.v.), $\{x_1, \dots, x_k\}$: predictors (which are scalars).
- sample: $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$.
- how to estimate $\beta_j \approx \hat{\beta}_j$: least square method, that is, to minimize $\sum_{i=1}^n (\hat{y}_i - y_i)^2$, where the estimated (multiple) linear regression line \hat{y} is

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k.$$

- $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \sim T(n-k-1)$, this helps determine whether or not the real $\beta_j = 0$?
- $\sigma^2 \approx \frac{SSE}{n-k-1}$, this gives an estimation of the size of the error.
- $R^2 = \frac{SSR}{SS_T}$, this gives how well the model is (if $R^2 \approx 1$, then the model fits the sample very well).
- How to test $\beta_1 = \dots = \beta_k = 0$? Use the random variable $\frac{SSR/k}{SSE/(n-k-1)} \sim F(k, n-k-1)$.

Logistic regression: Let Y can only take 0 or 1 with $P(Y=1) = p$ and $P(Y=0) = 1-p$.

$$E(Y) = p(x_1, \dots, x_k) = \frac{e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}{1 + e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}.$$

- Y : response variable (which is Bernoulli r.v. $P(Y=1) = p$ and $P(Y=0) = 1-p$, so $E(Y) = p$), $\{x_1, \dots, x_k\}$: predictors (which are scalars).
- sample: $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$.
- how to estimate $\beta_j \approx \hat{\beta}_j$: maximal likelihood method (maximize $\prod_{i=1}^n p(x_{i1}, \dots, x_{ik})^{y_i} (1 - p(x_{i1}, \dots, x_{ik}))^{1-y_i}$).
- $\frac{\hat{\beta}_j - \beta_j}{se(\hat{\beta}_j)} \approx N(0, 1)$ for large $n \geq 30$, this helps determine whether or not the real $\beta_j = 0$?
- Classification of a new object $Y(x_1, \dots, x_k)$ as 1 or 0 according

$$Y(x_1, \dots, x_k) = \begin{cases} 1, & \text{if } \hat{p}(x_1, \dots, x_k) \geq 0.5, \\ 0, & \text{if } \hat{p}(x_1, \dots, x_k) < 0.5, \end{cases}$$

where the estimated logit function $\hat{p}(x_1, \dots, x_k)$ is

$$\hat{p}(x_1, \dots, x_k) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}.$$

9. Tables

(9.1) Table for $N(0, 1)$ standard normal random variable $\Phi(x) = P(N(0, 1) \leq x)$, $x \geq 0$.
There is an important relation $\Phi(-x) = 1 - \Phi(x)$, $x \geq 0$.

x	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9935
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(9.2) Table for $T(f)$ random variable $F(x) = P(T(f) \leq x)$,
where f is a parameter called 'degrees of freedom'.

f	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.9995
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.96
7	0.71	1.41	1.89	2.36	3.00	3.50	4.03	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.44
12	0.70	1.36	1.78	2.18	2.68	3.05	3.43	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	4.22
14	0.69	1.35	1.76	2.14	2.62	2.98	3.33	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	4.01
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.97
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.88
20	0.69	1.33	1.72	2.09	2.53	2.85	3.15	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.79
23	0.69	1.32	1.71	2.07	2.50	2.81	3.10	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.75
25	0.68	1.32	1.71	2.06	2.49	2.79	3.08	3.73
26	0.68	1.31	1.71	2.06	2.48	2.78	3.07	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.67
29	0.68	1.31	1.70	2.05	2.46	2.76	3.04	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.55
50	0.68	1.30	1.68	2.01	2.40	2.68	2.94	3.50
60	0.68	1.30	1.67	2.00	2.39	2.66	2.91	3.46
100	0.68	1.29	1.66	1.98	2.36	2.63	2.87	3.39
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.29

(9.3) Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

f	$F(x)$														
	0.0005	0.001	0.005	0.01	0.025	0.05	0.10	0.20	0.30	0.40	0.50				
1	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.15	0.27	0.45				
2	0.00	0.00	0.01	0.02	0.05	0.10	0.21	0.45	0.71	1.02	1.39				
3	0.02	0.02	0.07	0.11	0.22	0.35	0.58	1.01	1.42	1.87	2.37				
4	0.06	0.09	0.21	0.30	0.48	0.71	1.06	1.65	2.19	2.75	3.36				
5	0.16	0.21	0.41	0.55	0.83	1.15	1.61	2.34	3.00	3.66	4.35				
6	0.30	0.38	0.68	0.87	1.24	1.64	2.20	3.07	3.83	4.57	5.35				
7	0.48	0.60	0.99	1.24	1.69	2.17	2.83	3.82	4.67	5.49	6.35				
8	0.71	0.86	1.34	1.65	2.18	2.73	3.49	4.59	5.53	6.42	7.34				
9	0.97	1.15	1.73	2.09	2.70	3.33	4.17	5.38	6.39	7.36	8.34				
10	1.26	1.48	2.16	2.56	3.25	3.94	4.87	6.18	7.27	8.30	9.34				
11	1.59	1.83	2.60	3.05	3.82	4.57	5.58	6.99	8.15	9.24	10.34				
12	1.93	2.21	3.07	3.57	4.40	5.23	6.30	7.81	9.03	10.18	11.34				
13	2.31	2.62	3.57	4.11	5.01	5.89	7.04	8.63	9.93	11.13	12.34				
14	2.70	3.04	4.07	4.66	5.63	6.57	7.79	9.47	10.82	12.08	13.34				
15	3.11	3.48	4.60	5.23	6.26	7.26	8.55	10.31	11.72	13.03	14.34				
16	3.54	3.94	5.14	5.81	6.91	7.96	9.31	11.15	12.62	13.98	15.34				
17	3.98	4.42	5.70	6.41	7.56	8.67	10.09	12.00	13.53	14.94	16.34				
18	4.44	4.90	6.26	7.01	8.23	9.39	10.86	12.86	14.44	15.89	17.34				
19	4.91	5.41	6.84	7.63	8.91	10.12	11.65	13.72	15.35	16.85	18.34				
20	5.40	5.92	7.43	8.26	9.59	10.85	12.44	14.58	16.27	17.81	19.34				
21	5.90	6.45	8.03	8.90	10.28	11.59	13.24	15.44	17.18	18.77	20.34				
22	6.40	6.98	8.64	9.54	10.98	12.34	14.04	16.31	18.10	19.73	21.34				
23	6.92	7.53	9.26	10.20	11.69	13.09	14.85	17.19	19.02	20.69	22.34				
24	7.45	8.08	9.89	10.86	12.40	13.85	15.66	18.06	19.94	21.65	23.34				
25	7.99	8.65	10.52	11.52	13.12	14.61	16.47	18.94	20.87	22.62	24.34				
26	8.54	9.22	11.16	12.20	13.84	15.38	17.29	19.82	21.79	23.58	25.34				
27	9.09	9.80	11.81	12.88	14.57	16.15	18.11	20.70	22.72	24.54	26.34				
28	9.66	10.39	12.46	13.56	15.31	16.93	18.94	21.59	23.65	25.51	27.34				
29	10.23	10.99	13.12	14.26	16.05	17.71	19.77	22.48	24.58	26.48	28.34				
30	10.80	11.59	13.79	14.95	16.79	18.49	20.60	23.36	25.51	27.44	29.34				
40	16.91	17.92	20.71	22.16	24.43	26.51	29.05	32.34	34.87	37.13	39.34				
50	23.46	24.67	27.99	29.71	32.36	34.76	37.69	41.45	44.31	46.86	49.33				
60	30.34	31.74	35.53	37.48	40.48	43.19	46.46	50.64	53.81	56.62	59.33				
100	59.90	61.92	67.33	70.06	74.22	77.93	82.36	87.95	92.13	95.81	99.33				

Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

f	$F(x)$														
	0.60	0.70	0.80	0.90	0.95	0.975	0.99	0.995	0.999	0.9995					
1	0.71	1.07	1.64	2.71	3.84	5.02	6.63	7.88	10.83	12.12					
2	1.83	2.41	3.22	4.61	5.99	7.38	9.21	10.60	13.82	15.20					
3	2.95	3.66	4.64	6.25	7.81	9.35	11.34	12.84	16.27	17.73					
4	4.04	4.88	5.99	7.78	9.49	11.14	13.28	14.86	18.47	20.00					
5	5.13	6.06	7.29	9.24	11.07	12.83	15.09	16.75	20.52	22.11					
6	6.21	7.23	8.56	10.64	12.59	14.45	16.81	18.55	22.46	24.10					
7	7.28	8.38	9.80	12.02	14.07	16.01	18.48	20.28	24.32	26.02					
8	8.35	9.52	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87					
9	9.41	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67					
10	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42					
11	11.53	12.90	14.63	17.28	19.68	21.92	24.72	26.76	31.26	33.14					
12	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82					
13	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48					
14	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11					
15	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.72					
16	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31					
17	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88					
18	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43					
19	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97					
20	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50					
21	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01					
22	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51					
23	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00					
24	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48					
25	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95					
26	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41					
27	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.64	55.48	57.86					
28	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30					
29	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73					
30	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16					
40	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.09					
50	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	86.66	89.56					
60	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.69					
100	102.95	106.91	111.67	118.50	124.34	129.56	135.81	140.17	149.45	153.17					

(9.4) Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1 - p) \geq n - k)$.

n	k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250
	1	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.7183	0.6480	0.5747	0.5000
4	0	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750
	1	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625
5	0	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125
	1	0.9974	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875
6	0	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000
	1	0.9999	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125
7	0	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9947	0.9898	0.9815	0.9688
	1	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156
8	0	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2233	0.1636	0.1094
	1	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438
9	0	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563
	1	1.0000	0.9999	0.9996	0.9984	0.9954	0.9891	0.9777	0.9659	0.9308	0.8906
10	0	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9982	0.9959	0.9917	0.9844
	1	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078
11	0	0.9356	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625
	1	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3164	0.2266
12	0	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000
	1	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734
13	0	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9575
	1	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9987	0.9964	0.9915	0.9848
14	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352
15	0	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445
	1	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633
16	0	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367
	1	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9747	0.9502	0.9115	0.8555
17	0	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9964	0.9915	0.9819	0.9648
	1	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961
18	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0207	0.0101	0.0046	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.1211	0.0705	0.0385	0.0195
19	0	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.3373	0.2218	0.1495	0.0898
	1	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.6089	0.4826	0.3614	0.2539
20	0	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.8283	0.7334	0.6214	0.5000
	1	1.0000	0.9999	0.9994	0.9990	0.9900	0.9747	0.9464	0.9006	0.8342	0.7461
21	0	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9888	0.9750	0.9502	0.9102
	1	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9986	0.9962	0.9909	0.9805
22	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9992	0.9980

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1 - p) \geq n - k)$.

n	k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0135	0.0060	0.0025	0.0010
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
11	0	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0860	0.0464	0.0233	0.0107
	1	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.2616	0.1673	0.0996	0.0547
12	0	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.5138	0.3823	0.2660	0.1719
	1	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.7515	0.6331	0.5044	0.3770
13	0	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.9051	0.8338	0.7384	0.6230
	1	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9740	0.9452	0.8980	0.8281
14	0	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9952	0.9877	0.9726	0.9453
	1	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983	0.9955	0.9893
15	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005
	1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0606	0.0302	0.0139	0.0059
16	0	0.9848	0.9104	0.7788	0.6174	0.4552	0.3127	0.2001	0.1189	0.0652	0.0327
	1	0.9984	0.9815	0.9306	0.8389	0.7133	0.5696	0.4256	0.2963	0.1911	0.1133
17	0	0.9999	0.9972	0.9841	0.9496	0.8854	0.7897	0.6683	0.5328	0.3971	0.2744
	1	1.0000	0.9997	0.9973	0.9883	0.9657	0.9218	0.8513	0.7535	0.6331	0.5000
18	0	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9499	0.9006	0.8262	0.7256
	1	1.0000	1.0000	1.0000	0.9998	0.9988	0.9957	0.9878	0.9707	0.9390	0.8867
19	0	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9980	0.9941	0.9852	0.9673
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9978	0.9941
20	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0003
	1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0424	0.0196	0.0083	0.0032
21	0	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.1513	0.0834	0.0421	0.0193
	1	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.3467	0.2253	0.1345	0.0730
22	0	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.5833	0.4382	0.3044	0.1938
	1	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.7873	0.6652	0.5269	0.3872
23	0	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.9154	0.8418	0.7393	0.6128
	1	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9745	0.9427	0.8883	0.8062
24	0	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9944	0.9847	0.9644	0.9270
	1	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9992	0.9972	0.9921	0.9807
25	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
26	0	0.5133	0.2542	0.1209	0.0550	0.0238	0.0097	0.0037	0.0013	0.0004	0.0001
	1	0.8646	0.6213	0.3983	0.2336	0.1267	0.0637	0.0296	0.0126	0.0049	0.0017
27	0	0.9755	0.8661	0.6920	0.5017	0.3326	0.2025	0.1132	0.0579	0.0269	0.0112
	1	0.9969	0.9658	0.8820	0.7473	0.5843	0.4206	0.2783	0.1686	0.0929	0.0461
28	0	0.9997	0.9935	0.9658	0.9009	0.7940	0.6543	0.5005	0.3530	0.2279	0.1334
	1	1.0000	0.9991	0.9925	0.9700	0.9198	0.8346	0.7159	0.5744	0.4268	0.2905
29	0	1.0000	0.9999	0.9987	0.9930	0.9757	0.9376	0.8705	0.7712	0.6437	0.5000
	1	1.0000	1.0000	0.9998	0.9988	0.9944	0.9818	0.9538	0.9023	0.8212	0.7095
30	0	1.0000	1.0000	1.0000	0.9998	0.9990	0.9960	0.9874	0.9679	0.9302	0.8666
	1	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9993	0.9975	0.9797	0.9539
31	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9888
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9983
32	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1 - p) \geq n - k)$.

n	k	p														
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50					
14	0	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001					
	1	0.8470	0.5846	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0029	0.0009					
	2	0.9699	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065					
	3	0.9958	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287					
	4	0.9996	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898					
	5	1.0000	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120					
	6	1.0000	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953					
	7	1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047					
	8	1.0000	1.0000	1.0000	0.9996	0.9978	0.9967	0.9957	0.9947	0.9811	0.7880					
	9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9970	0.9954	0.9825	0.9102					
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9961	0.9886	0.9713					
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9978	0.9935					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999					
15	0	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000					
	1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005					
	2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037					
	3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176					
	4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592					
	5	0.9999	0.9832	0.9389	0.8389	0.6816	0.5216	0.3608	0.2168	0.1509	0.0836					
	6	1.0000	0.9997	0.9664	0.8819	0.7434	0.5869	0.4252	0.2682	0.1509	0.0836					
	7	1.0000	1.0000	0.9994	0.9819	0.8627	0.7216	0.5548	0.3868	0.2168	0.1509					
	8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964					
	9	1.0000	1.0000	1.0000	0.9999	0.9999	0.9993	0.9972	0.9931	0.9745	0.9408					
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9974	0.9948					
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9995					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000					
16	0	0.4401	0.1853	0.0743	0.0281	0.0100	0.0033	0.0010	0.0003	0.0001	0.0000					
	1	0.8108	0.5147	0.2839	0.1407	0.0635	0.0261	0.0098	0.0033	0.0010	0.0003					
	2	0.9571	0.7892	0.5614	0.3518	0.1971	0.0994	0.0451	0.0183	0.0066	0.0021					
	3	0.9930	0.9316	0.7899	0.5981	0.4050	0.2459	0.1339	0.0651	0.0281	0.0106					
	4	0.9991	0.9830	0.9209	0.7982	0.6302	0.4499	0.2892	0.1666	0.0853	0.0384					
	5	0.9999	0.9967	0.9765	0.9183	0.8103	0.6598	0.4900	0.3288	0.1976	0.1051					
	6	1.0000	0.9995	0.9949	0.9733	0.9204	0.8247	0.6881	0.5272	0.3660	0.2272					
	7	1.0000	0.9999	0.9989	0.9930	0.9729	0.9256	0.8406	0.7161	0.5629	0.4018					
	8	1.0000	1.0000	0.9998	0.9985	0.9925	0.9743	0.9771	0.9417	0.8759	0.7728					
	9	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9971	0.9947	0.9514	0.8949					
	10	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9983	0.9809	0.9514	0.9166					
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9998	0.9951	0.9851	0.9616					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9965	0.9844					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9994	0.9979					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997					
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000					

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1 - p) \geq n - k)$.

n	k	p														
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50					
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000	0.0000					
	1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0067	0.0021	0.0006	0.0001					
	2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0327	0.0123	0.0041	0.0012					
	3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.1028	0.0464	0.0184	0.0064					
	4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.2348	0.1260	0.0596	0.0245					
	5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5968	0.4197	0.2639	0.1471	0.0717					
	6	1.0000	0.9992	0.9917	0.9623	0.8929	0.7752	0.6188	0.4478	0.2902	0.1662					
	7	1.0000	0.9999	0.9983	0.9891	0.9588	0.8954	0.7872	0.6405	0.4743	0.3145					
	8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.9006	0.8011	0.6626	0.5000					
	9	1.0000	1.0000	1.0000	0.9995	0.9969	0.9944	0.9873	0.9617	0.9081	0.8166					
	10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9998	0.9880	0.9652	0.9174	0.8338					
	11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9970	0.9894	0.9699	0.9283					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9914	0.9755					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9981	0.9936					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9988					
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999					
18	0	0.3972	0.1501	0.0536	0.0180	0.0056	0.0016	0.0004	0.0001	0.0000	0.0000					
	1	0.7735	0.4503	0.2241	0.0991	0.0395	0.0142	0.0046	0.0013	0.0003	0.0001					
	2	0.9419	0.7338	0.4797	0.2713	0.1353	0.0600	0.0236	0.0082	0.0025	0.0007					
	3	0.9891	0.9018	0.7202	0.5010	0.3057	0.1646	0.0783	0.0328	0.0120	0.0038					
	4	0.9855	0.9718	0.8794	0.7164	0.5187	0.3327	0.1866	0.0942	0.0411	0.0154					
	5	0.9998	0.9936	0.9581	0.8671	0.7175	0.5344	0.3550	0.2088	0.1077	0.0481					
	6	1.0000	0.9988	0.9882	0.9487	0.8610	0.7217	0.5491	0.3723	0.2258	0.1189					
	7	1.0000	0.9998	0.9973	0.9837	0.9431	0.8593	0.7283	0.5634	0.3915	0.2403					
	8	1.0000	1.0000	0.9995	0.9957	0.9807	0.9404	0.8609	0.7368	0.5778	0.4073					
	9	1.0000	1.0000	0.9999	0.9991	0.9946	0.9790	0.9403	0.8653	0.7473	0.5927					
	10	1.0000	1.0000	1.0000	0.9998	0.9988	0.9988	0.9939	0.9788	0.9424	0.8720					
	11	1.0000	1.0000	1.0000	1.0000	0.9998	0.9996	0.9938	0.9797	0.9463	0.8811					
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9986	0.9792	0.9463	0.8811					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9846					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9962	0.9846					
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9983					
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999						
19	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000	0.0000					
	1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002	0.0000					
	2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0170	0.0055	0.0015	0.0004					
	3	0.9868	0.8550	0.6841	0.4551	0.2631	0.1332	0.0591	0.0230	0.0077	0.0022					
	4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.1500	0.0696	0.0280	0.0096					
	5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.2968	0.1629	0.0777	0.0318					
	6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.4812	0.3081	0.1727	0.0835					
	7	1.0000	0.9997	0.9959	0.9767	0.9225	0.7713	0.6161	0.4145	0.2665	0.1399					
	8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.8145	0.6675	0.4940	0.3238					
	9	1.0000	1.0000	0.9999	0.9984	0.9914	0.9674	0.9125	0.8139	0.6710	0.5000					
	10	1.0000	1.0000	1.0000	0.9997	0.9977	0.9895	0.9772	0.9686	0.9648	0.9129					
	11	1.0000	1.0000	1.0000	1.0000	0.9995	0.9972	0.9895	0.9833	0.9699	0.9582					
	12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9969	0.9884	0.9658	0.9165					
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9969	0.9891	0.9682					
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9974	0.9904					
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9978					
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996						

(9.5) Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	μ									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9536	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
<hr/>										
k	μ									
	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
2	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
3	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
4	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
5	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868	0.9834
6	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
7	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
<hr/>										
k	μ									
	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146	0.1991
2	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460	0.4232
3	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696	0.6472
4	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318	0.8153
5	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258	0.9161
6	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713	0.9665
7	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901	0.9881
8	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969	0.9962
9	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991	0.9989
10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	μ									
	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
0	0.0408	0.0334	0.0273	0.0224	0.0183	0.0150	0.0123	0.0101	0.0082	0.0067
1	0.1712	0.1468	0.1257	0.1074	0.0916	0.0780	0.0663	0.0563	0.0477	0.0404
2	0.3799	0.3397	0.3027	0.2689	0.2381	0.2102	0.1851	0.1626	0.1425	0.1247
3	0.6025	0.5584	0.5152	0.4735	0.4335	0.3954	0.3594	0.3257	0.2942	0.2650
4	0.7806	0.7442	0.7064	0.6678	0.6288	0.5898	0.5512	0.5132	0.4763	0.4405
5	0.8946	0.8705	0.8441	0.8156	0.7851	0.7531	0.7199	0.6858	0.6510	0.6160
6	0.9654	0.9421	0.9267	0.9091	0.8893	0.8675	0.8436	0.8180	0.7908	0.7622
7	0.9832	0.9769	0.9682	0.9599	0.9509	0.9419	0.9321	0.9214	0.9049	0.8867
8	0.9943	0.9917	0.9883	0.9840	0.9786	0.9721	0.9642	0.9549	0.9442	0.9319
9	0.9982	0.9973	0.9960	0.9942	0.9919	0.9889	0.9851	0.9805	0.9749	0.9682
10	0.9995	0.9992	0.9987	0.9981	0.9972	0.9959	0.9943	0.9922	0.9896	0.9863
11	0.9999	0.9998	0.9996	0.9994	0.9991	0.9986	0.9980	0.9971	0.9960	0.9945
12	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9993	0.9990	0.9986
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9998	0.9997	0.9995	0.9993
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
<hr/>										
k	μ									
	5.2	5.4	5.6	5.8	6.0	6.5	7.0	7.5	8.0	8.5
0	0.0055	0.0045	0.0037	0.0030	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002
1	0.0342	0.0289	0.0244	0.0206	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019
2	0.1088	0.0948	0.0824	0.0715	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093
3	0.2381	0.2133	0.1906	0.1700	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301
4	0.4061	0.3733	0.3422	0.3127	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744
5	0.5589	0.5461	0.5119	0.4783	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496
6	0.7324	0.7017	0.6703	0.6384	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562
7	0.8449	0.8217	0.7970	0.7710	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856
8	0.9181	0.9027	0.8857	0.8672	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231
9	0.9603	0.9512	0.9409	0.9292	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530
10	0.9823	0.9775	0.9718	0.9651	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634
11	0.9927	0.9904	0.9875	0.9841	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487
12	0.9972	0.9962	0.9949	0.9932	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091
13	0.9990	0.9986	0.9980	0.9973	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486
14	0.9997	0.9995	0.9993	0.9990	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726
15	0.9999	0.9998	0.9998	0.9996	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862
16	1.0000	0.9999	0.9999	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	μ										
	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	
0	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
1	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	
2	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	
3	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	
4	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	
5	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	
6	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	
7	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	
8	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	
9	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	
10	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	
11	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	
12	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	
13	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	
14	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	
15	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	
16	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	
17	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	
18	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	
19	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	
20	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	
21	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	
22	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	
23	1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	
24	1.0000	1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	
25	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	
26	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	
27	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	
28	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9986	
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9996	
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	