

Examiner: Xiangfeng Yang (Tel: 013 28 57 88).**Things allowed (Hjälpmmedel):** a calculator.**Scores rating (Betygsgränser):** 8-11 points giving rate 3; 11.5-14.5 points giving rate 4; 15-18 points giving rate 5.
Write down all necessary steps in solutions in order to receive as many points as possible.

1 (3 points)

A factory produces plenty of small calculators. Among all defective calculators, 20% have only hardware errors, 70% have only software errors, and 10% have both hardware and software errors.

(1.1) (1p) Find the probability that a randomly selected defective calculator has hardware errors.

(1.2) (1p) Find the probability that a randomly selected defective calculator has at least one of hardware and software errors.

(1.3) (1p) Given that a randomly selected defective calculator has hardware errors, find the probability that the defective calculator has also software errors.

2 (3 points)

Let (X, Y) be a two dimensional discrete random variable with the following joint probability mass function

$X \setminus Y$	5	10	15
1	0.10	0.20	0.18
2	0.15	0.05	0.32

The table tells that X takes values 1 and 2, and Y takes values 5, 10 and 15.

(2.1) (1p) Find the probability $P(X + Y \geq 8)$.

(2.2) (1p) Find the mean $E(X)$ of X , and the variance $V(Y)$ of Y .

(2.3) (1p) Are X and Y independent? Why?

3 (3 points)

A battery manufacturer claims that the distribution X of lifetimes of a certain type of batteries has a mean 40 hours and a standard deviation 5 hours. Find the probability that the average lifetime of 100 randomly selected such batteries is less than 39.5 hours.

4 (3 points)

(4.1) (1p) A coin has two sides (“Head” denoted as 1 and “Tail” denoted as 0). Let X be the distribution of the two sides, and assume that $P(X = 1) = p$ and $P(X = 0) = 1 - p$. To determine whether or not the coin is fair, one tosses the coin 10 times with the following results:

$$\{1, 1, 0, 1, 0, 0, 0, 1, 1, 0\}$$

Find a point estimate \hat{p}_{MM} of p based on the method of moments.

(4.2) (2p) Let Y denote the distribution of weights of a certain type of apples from Sweden and assume that Y is normal $Y \sim N(100, \sigma^2)$. To estimate σ , a sample $\{y_1, y_2, \dots, y_n\}$ is taken from Y . Find a point estimate $\hat{\sigma}_{ML}$ of σ based on the maximum-likelihood method.

5 (3 points)

A study in Sweden is conducted to investigate if there is a difference between girls and boys at age ten in terms of heights. To this end, 46 girls at age ten are randomly chosen with an average height 145cm and a standard deviation 4.2cm; 56 boys at age ten are randomly chosen as well with an average height 141cm and a standard deviation 3.8cm. Assume that girls and boys are independent. Let μ_g and μ_b denote the average heights of all girls and all boys at age ten in Sweden respectively.

(5.1) (1.5p) Do the above samples provide any evidence that $\mu_g > \mu_b$? Answer this using an appropriate one-sided 97.5% confidence interval.

(5.2) (1.5p) Do the above samples provide any evidence that $\mu_g > \mu_b$? Answer this using appropriate hypotheses testing with a significance level $\alpha = 2.5\%$.

6 (3 points)

Let X denote the distribution of numbers of daily car accidents on the high way E4 between Linköping and Norrköping. One suspects that X is a Binomial random variable $X \sim \text{Bin}(2, 0.25)$. To check this, a sample with 100 observations is taken from X and they are classified into three groups as follows

values	0	1	2
frequencies	55	25	20

Based on this sample, is there any evidence showing that the population $X \sim \text{Bin}(2, 0.25)$ with a significance level $\alpha = 1\%$? (Hint: χ^2 -test).

1. Basic probability

(1.1) Conditional probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

(1.2) Total probability $P(B) = \sum_{i=1}^k P(B|A_i)P(A_i)$ where $\{A_i\}$ are disjoint and $\cup_{i=1}^k A_i = S$.

(1.3) Bayes' Theorem $P(A_j|B) = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^k P(B|A_i)P(A_i)}$ where $\{A_i\}$ are in (1.2).

2. Random variables (r.v.s)

(2.1) Discrete r.v. X has a pmf $p(x) = P(X = x)$ satisfying $p(x) \geq 0$ and $\sum p(x_i) = 1$,

X	x_1	x_2	\cdots	x_n	\cdots
$p(x)$	$p(x_1)$	$p(x_2)$	\cdots	$p(x_n)$	\cdots

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \sum x_i p(x_i)$;
 Variance $\sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \sum x_i^2 p(x_i) - (\sum x_i p(x_i))^2$.

(2.2) Continuous r.v. X has a pdf $f(x)$ satisfying $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$,

$$P(a < X < b) = \int_a^b f(x)dx.$$

Expectation (or *Expected value* or *mean*) $\mu_X = E(X) = \int_{-\infty}^{\infty} xf(x)dx$;

Variance $\sigma_X^2 = V(X) = E(X - \mu_X)^2 = E(X^2) - \mu_X^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - (\int_{-\infty}^{\infty} xf(x)dx)^2$.

(2.3) Cumulative distribution function (cdf) of a r.v. X is $F(x) = P(X \leq x)$.

(2.4) X and Y are r.v.s, a , b and c are scalars, then

$$E(aX + bY + c) = aE(X) + bE(Y) + c,$$

$$V(aX + bY + c) = a^2V(X) + b^2V(Y) + 2ab \text{cov}(X, Y),$$

$$E(g(X, Y)) = \begin{cases} \sum_{i,j} g(x_i, y_j) \cdot p(x_i, y_j), & \text{for discrete } (X, Y), \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \cdot f(x, y)dxdy, & \text{for continuous } (X, Y). \end{cases}$$

(2.5) • Discrete r.v. (X, Y) has a joint pmf $p(x, y)$ satisfying $p(x, y) \geq 0$ and $\sum_{x_i} \sum_{y_i} p(x_i, y_i) = 1$.

The marginal pmf of X is $p_X(x) = \sum_y p(x, y)$;
 The marginal pmf of Y is $p_Y(y) = \sum_x p(x, y)$;

X and Y are *independent* if $p(x, y) = p_X(x) \cdot p_Y(y)$.

• Continuous r.v. (X, Y) has a joint pdf $f(x, y)$ satisfying $f(x, y) \geq 0$ and $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)dxdy = 1$.

The marginal pdf of X is $f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy$;
 The marginal pdf of Y is $f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx$;

X and Y are *independent* if $f(x, y) = f_X(x) \cdot f_Y(y)$.

3. Several special r.v.s

(3.1) $X \sim Bin(n, p)$ has a pmf $p(x) = P(X = x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x}$, $x = 0, 1, 2, \dots, n$.

$$E(X) = n \cdot p, \quad V(X) = n \cdot p \cdot (1-p).$$

(3.2) $X \sim Po(\lambda)$ has a pmf $p(x) = P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$, $x = 0, 1, 2, \dots$.

$$E(X) = \lambda, \quad V(X) = \lambda.$$

(3.3) $X \sim Hypergeometric$ has a pmf $p(x) = P(X = x) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$.

(3.4) $X \sim Exp(\lambda)$ has a pdf

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(3.5) $X \sim N(\mu, \sigma^2)$ has a pdf

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty < x < \infty.$$

$E(X) = \mu, \quad V(X) = \sigma^2$

(3.6) $X \sim U(a, b)$ has a pdf

$$f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

$$E(X) = \frac{a+b}{2}, \quad V(X) = \frac{(b-a)^2}{12}.$$

4. Central Limit Theorem (CLT)

Suppose that a population has mean = μ and variance = σ^2 . A random sample $\{X_1, X_2, \dots, X_n\}$ from this population is given. Then for large $n \geq 30$,

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1).$$

- If the population is normal, then (1) holds for any n .
- Note that $\mu = E(\bar{X})$ and $(\sigma/\sqrt{n})^2 = V(\bar{X})$.

5. Several notations in statistics

(5.1) Sample mean: $\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n} = \frac{\sum X_i}{n}; \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum x_i}{n}$.

(5.2) Sample variance:

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{1}{n-1} \left(\sum X_i^2 - \frac{(\sum X_i)^2}{n} \right); s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{1}{n-1} \left(\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right).$$

• Capital letters \bar{X} and S^2 refer to the objects based on random sample (therefore they are in general r.v.s), while small letters \bar{x} and s^2 are the objects based on observations (so they are scalars).

(5.3) A point estimator of θ obtained by Maximum Likelihood method is denoted as $\hat{\theta}_{ML}$.

6. Confidence Interval (CI)

In this course, three types of confidence intervals are studied depending on the unknown population parameter(s): CI-1 (confidence intervals for population mean(s)), CI-2 (confidence intervals for population variance(s)), and CI-3 (confidence intervals for population proportion(s)).

CI-1: $(1 - \alpha)$ CI of a population mean μ

case 1.1 (any n) If population $X \sim N(\mu, \sigma^2)$ and σ^2 is known, then $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ and

$$I_\mu = \left(\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \right) := \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}.$$

case 1.2 ($n \geq 30$) For any population X , it holds that $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$ and

$$I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \text{ or } I_\mu = \bar{x} \mp z_{\alpha/2} \cdot \frac{\hat{\sigma}}{\sqrt{n}}.$$

case 1.3 (any n) If population $X \sim N(\mu, \sigma^2)$ and σ^2 is unknown, then $\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim T(n-1)$ and

$$I_\mu = \bar{x} \mp t_{\alpha/2}(n-1) \cdot \frac{s}{\sqrt{n}}.$$

CI-1': $(1 - \alpha)$ CI of the difference of two population means $\mu_X - \mu_Y$

case 1.1' (any n_1, n_2) If independent populations $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, and σ_X^2, σ_Y^2 are known, then $\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1)$, and $I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}$.

case 1.2' ($n_1, n_2 \geq 30$) For any independent populations X and Y , it holds that

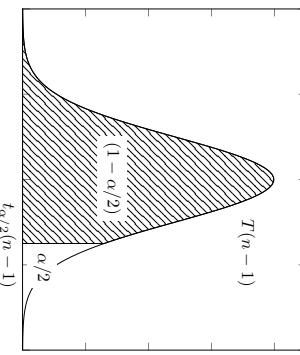
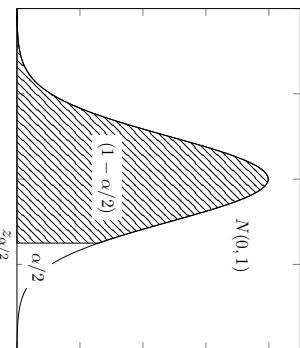
$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}}} \sim N(0, 1) \text{ and}$$

$$I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\sigma_X^2}{n_1} + \frac{\sigma_Y^2}{n_2}} \text{ or } I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{\sigma}_X^2}{n_1} + \frac{\hat{\sigma}_Y^2}{n_2}}.$$

case 1.3' (any n_1, n_2) If independent populations $X \sim N(\mu_X, \sigma_X^2)$, $Y \sim N(\mu_Y, \sigma_Y^2)$, where σ_X^2, σ_Y^2 are unknown but $\sigma_X^2 = \sigma_Y^2$, then

$$\frac{(\bar{X} - \bar{Y}) - (\mu_X - \mu_Y)}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim T(n_1 + n_2 - 2), \text{ where } S^2 = \frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}, \text{ and}$$

$$I_{\mu_X - \mu_Y} = (\bar{x} - \bar{y}) \mp t_{\alpha/2}(n_1 + n_2 - 2) \cdot s \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$



CI-2: $(1 - \alpha)$ CI of population variance(s) σ^2

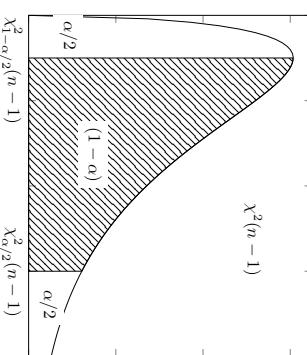
- If a population $X \sim N(\mu, \sigma^2)$ and σ^2 is unknown, then $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$, and

$$I_{\sigma^2} = \left(\frac{(n-1)s^2}{\chi_{\alpha/2}^2(n-1)}, \frac{(n-1)s^2}{\chi_{1-\alpha/2}^2(n-1)} \right).$$

- If two independent populations $X \sim N(\mu_X, \sigma^2)$ and $Y \sim N(\mu_Y, \sigma^2)$, and σ^2 is unknown, then $\frac{(n_1+n_2-2)S^2}{\sigma^2} \sim \chi^2(n_1 + n_2 - 2)$, and

$$I_{\sigma^2} = \left(\frac{(n_1 + n_2 - 2)s^2}{\chi_{\alpha/2}^2(n_1 + n_2 - 2)}, \frac{(n_1 + n_2 - 2)s^2}{\chi_{1-\alpha/2}^2(n_1 + n_2 - 2)} \right),$$

where $S^2 = \frac{(n_1 - 1)S_X^2 + (n_2 - 1)S_Y^2}{n_1 + n_2 - 2}$.



CI-3: $(1 - \alpha)$ CI of population proportion(s)

- If a (large) population has an unknown proportion p , then $\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \sim N(0, 1)$ if $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$ with $\hat{p} = x/n$, and $I_p = \hat{p} \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$.
- If two independent (large) populations have unknown proportions p_1 and p_2 , then

$$\frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}} \sim N(0, 1)$$

if $n_i\hat{p}_i \geq 10$ and $n_i(1 - \hat{p}_i) \geq 10$ for $i = 1, 2$, and $I_{p_1 - p_2} = (\hat{p}_1 - \hat{p}_2) \mp z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$.

7. Hypothesis Test (HT)

	H_0 is true	H_0 is false and $\theta = \theta_1$
reject H_0	(type I error or significance level) α	(power) $h(\theta_1) = 1 - h(\theta_1)$
don't reject H_0	1 - α	(type II error) $\beta(\theta_1) = 1 - h(\theta_1)$

reject $H_0 \Leftrightarrow TS \in C \Leftrightarrow p\text{-value} < \alpha$

χ^2 tests for populations (non-parametric tests)

Suppose that for a random sample of a population X , the n elements of it are classified into k disjoint groups $A_i, 1 \leq i \leq k$. For each group $A_i, 1 \leq i \leq k$, suppose that there are $N_i, 1 \leq i \leq k$ elements inside. Let $p_i = P(A_i)$ assuming a given distribution of X . Note that $p_1 + p_2 + \dots + p_k = 1$ and $N_1 + N_2 + \dots + N_k = n$. One wants to test the hypotheses

$$H_0 : P(A_i) = p_i, \quad 1 \leq i \leq k, \quad H_a : P(A_i) \neq p_i \text{ for some } 1 \leq i \leq k.$$

If n is large in the sense that $np_i \geq 5$ for all $1 \leq i \leq k$, then the test statistic is

$$\sum_{i=1}^k \frac{(N_i - np_i)^2}{np_i} \approx \chi^2(k-1).$$

Therefore the observation of the test statistic is

$$TS = \sum_{i=1}^k \frac{(n_i - np_i)^2}{np_i}, \text{ where } n_i \text{ is the observation of } N_i, 1 \leq i \leq k.$$

For the critical region C , one can take (note that if H_0 is true, then TS should be close to zero)

$$C = (\chi_\alpha^2(k-1), \infty).$$

The conclusion would be $TS \in C \iff H_0$ is rejected.

8. Linear and logistic regression

(Multiple) linear regression: $Y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon, \varepsilon \sim N(0, \sigma^2)$.

\bullet Y : response variable (which is normal r.v.), $\{x_1, \dots, x_k\}$: predictors (which are scalars).

\bullet sample: $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$.
 \bullet how to estimate $\beta_j \approx \hat{\beta}_j$: least square method, that is, to minimize $\sum_{i=1}^n (\hat{y}_i - y_i)^2$, where the estimated (multiple) linear regression line \hat{y} is

$$\hat{y} = \beta_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k.$$

$\bullet \frac{\hat{\beta}_j - \beta_j}{s.e(\hat{\beta}_j)} \sim T(n-k-1)$, this helps determine whether or not the real $\beta_j = 0$?

$\bullet \sigma^2 \approx \frac{SSE}{n-k-1}$, this gives an estimation of the size of the error.

$\bullet R^2 = \frac{SSE}{SS_T}$ this gives how well the model is (if $R^2 \approx 1$, then the model fits the sample very well).

\bullet How to test $\beta_1 = \dots = \beta_k = 0$? Use the random variable $\frac{SS_R/k}{SSE/(n-k-1)} \sim F(k, n-k-1)$.

Logistic regression: Let Y can only take 0 or 1 with $P(Y=1) = p$ and $P(Y=0) = 1-p$,

$$E(Y) = p(x_1, \dots, x_k) = \frac{e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}{1 + e^{\beta_0 + \beta_1 x_1 + \dots + \beta_k x_k}}.$$

$\bullet Y$: response variable (which is Bernoulli r.v. $P(Y=1) = p$ and $P(Y=0) = 1-p$, so $E(Y) = p$), $\{x_1, \dots, x_k\}$: predictors (which are scalars).

\bullet sample: $\{(x_{11}, \dots, x_{1k}; y_1), (x_{21}, \dots, x_{2k}; y_2), \dots, (x_{n1}, \dots, x_{nk}; y_n)\}$.

\bullet how to estimate $\beta_j \approx \hat{\beta}_j$: maximal likelihood method (maximize $\prod_{i=1}^n p(x_{i1}, \dots, x_{ik})^{y_i} (1 - p(x_{i1}, \dots, x_{ik}))^{1-y_i}$).

$\bullet \frac{\hat{\beta}_j - \beta_j}{s.e(\hat{\beta}_j)} \approx N(0, 1)$ for large $n \geq 30$, this helps determine whether or not the real $\beta_j = 0$?

\bullet Classification of a new object $Y(x_1, \dots, x_k)$ as 1 or 0 according

$$Y(x_1, \dots, x_k) = \begin{cases} 1, & \text{if } \hat{p}(x_1, \dots, x_k) \geq 0.5, \\ 0, & \text{if } \hat{p}(x_1, \dots, x_k) < 0.5, \end{cases}$$

where the estimated logit function $\hat{p}(x_1, \dots, x_k)$ is

$$\hat{p}(x_1, \dots, x_k) = \frac{e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}{1 + e^{\hat{\beta}_0 + \hat{\beta}_1 x_1 + \dots + \hat{\beta}_k x_k}}.$$

9. Tables

(9.1) Table for $N(0,1)$ standard normal random variable $\Phi(x) = P(N(0,1) \leq x)$, $x \geq 0$.

There is an important relation $\Phi(-x) = 1 - \Phi(x)$, $x \geq 0$.

x	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7957	0.8023	0.8051	0.8078	0.8106	0.8133	0.8160
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9222	0.9256	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	0.9330
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0.9965
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0.9981	0.9981
2.9	0.9981	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9990	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9993	0.9993	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

(9.2) Table for $T(f)$ random variable $F(x) = P(T(f) \leq x)$, where f is a parameter called 'degrees of freedom'.

f	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.9995	$F(x)$
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	636.62	
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	31.60	
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	12.92	
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	8.61	
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	6.87	
6	0.72	1.44	2.45	3.14	3.71	4.32	5.96	10.47	
7	0.71	1.41	1.89	2.36	3.00	3.50	4.03	5.41	
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	5.04	
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.78	
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.59	
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.44	
12	0.70	1.36	1.78	2.18	2.68	3.05	3.43	4.32	
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	4.22	
14	0.69	1.35	1.76	2.14	2.62	2.98	3.33	4.14	
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	4.07	
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	4.01	
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.97	
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.92	
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.88	
20	0.69	1.33	1.72	2.09	2.53	2.85	3.15	3.85	
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.82	
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.79	
23	0.69	1.32	1.71	2.07	2.50	2.81	3.10	3.77	
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.75	
25	0.68	1.32	1.71	2.06	2.49	2.79	3.08	3.73	
26	0.68	1.31	1.71	2.06	2.48	2.78	3.07	3.71	
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.69	
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.67	
29	0.68	1.31	1.70	2.05	2.46	2.76	3.04	3.66	
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.65	
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.55	
50	0.68	1.30	1.68	2.01	2.40	2.68	2.94	3.50	
60	0.68	1.30	1.67	2.00	2.39	2.66	2.91	3.46	
100	0.68	1.29	1.66	2.00	2.36	2.63	2.87	3.39	
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.29	

(9.3) Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

f	$F(x)$										
	0.005	0.001	0.005	0.01	0.025	0.05	0.10	0.20	0.30	0.40	0.50
1	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.15	0.27	0.45	1
2	0.00	0.00	0.01	0.02	0.05	0.10	0.21	0.45	0.71	1.39	2
3	0.02	0.02	0.07	0.11	0.22	0.35	0.58	1.01	1.42	2.37	3
4	0.06	0.09	0.21	0.30	0.48	0.71	1.06	1.65	2.19	2.75	4
5	0.16	0.21	0.41	0.55	0.83	1.15	1.61	2.34	3.00	3.66	5
6	0.30	0.38	0.68	0.87	1.24	1.64	2.20	3.07	3.83	4.57	6
7	0.48	0.60	0.99	1.24	1.69	2.17	2.83	3.82	4.67	5.49	7
8	0.71	0.86	1.34	1.65	2.18	2.73	3.49	4.59	5.53	6.42	8
9	0.97	1.15	1.73	2.09	2.70	3.33	4.17	5.38	6.39	7.36	9
10	1.26	1.48	2.16	2.56	3.25	3.94	4.87	6.18	7.27	8.30	10
11	1.59	1.83	2.60	3.05	3.82	4.57	5.58	6.99	8.15	9.24	11
12	1.93	2.21	3.07	3.57	4.40	5.23	6.30	7.81	9.03	10.18	12
13	2.31	2.62	3.57	4.11	5.01	5.89	7.04	8.63	9.93	11.13	13
14	2.70	3.04	4.07	4.66	5.63	6.57	7.79	9.47	10.82	12.08	14
15	3.11	3.48	4.60	5.23	6.26	7.26	8.55	10.31	11.72	13.03	15
16	3.54	3.94	5.14	5.81	6.91	7.96	9.31	11.15	12.62	13.98	16
17	3.98	4.42	5.70	6.41	7.56	8.67	10.09	12.00	13.53	14.94	17
18	4.44	4.90	6.26	7.01	8.23	9.39	10.86	12.86	14.44	15.89	18
19	4.91	5.41	6.84	7.63	8.91	10.12	11.65	13.72	15.35	16.85	19
20	5.40	5.92	7.43	8.26	9.59	10.85	12.44	14.58	16.27	17.81	20
21	5.90	6.45	8.03	8.90	10.28	11.59	13.24	15.44	17.18	18.77	21
22	6.40	6.98	8.64	9.54	10.98	12.34	14.04	16.31	18.10	19.73	22
23	6.92	7.53	9.26	10.20	11.69	13.09	14.85	17.19	19.02	20.69	23
24	7.45	8.08	9.89	10.86	12.40	13.85	15.66	18.06	19.94	21.65	24
25	7.99	8.65	10.52	11.52	13.12	14.61	16.47	18.94	20.87	22.62	25
26	8.54	9.22	11.16	12.20	13.84	15.38	17.29	19.82	21.79	23.58	26
27	9.09	9.80	11.81	12.88	14.57	16.15	18.11	20.70	22.72	24.54	27
28	9.66	10.39	12.46	13.56	15.31	16.93	18.94	21.59	23.65	25.51	28
29	10.23	10.99	13.12	14.26	16.05	17.71	19.77	22.48	24.58	26.48	29
30	10.80	11.59	13.79	14.95	16.79	18.49	20.60	23.36	25.51	27.44	30
40	16.91	17.92	20.71	22.16	24.43	26.51	29.05	32.34	34.87	37.13	40
50	23.46	24.67	27.99	29.71	32.36	34.76	37.69	41.45	44.31	46.86	50
60	30.34	31.74	35.53	37.48	40.48	43.19	46.46	50.64	53.81	56.62	60
100	59.90	61.92	67.33	70.06	74.22	77.93	82.36	87.95	92.13	95.81	100

Table for $\chi^2(f)$ random variable $F(x) = P(\chi^2(f) \leq x)$, where f is a parameter.

f	$F(x)$									
	0.60	0.70	0.80	0.90	0.95	0.975	0.99	0.995	0.999	0.9995
1	0.71	1.07	1.64	2.71	3.84	5.02	6.63	7.88	10.83	12.12
2	1.83	2.41	3.22	4.61	5.99	7.38	9.21	10.60	13.82	15.20
3	2.95	3.66	4.64	6.25	7.81	9.35	11.34	12.84	16.27	17.73
4	4.04	4.88	5.99	7.78	9.49	11.14	13.28	14.86	18.47	20.00
5	5.13	6.06	7.29	9.24	11.07	12.83	15.09	16.75	20.52	22.11
6	6.21	7.23	8.56	10.64	12.59	14.45	16.81	18.55	22.46	24.10
7	7.28	8.38	9.80	12.02	14.07	16.01	18.48	20.28	24.32	26.02
8	8.35	9.52	11.03	13.36	15.51	17.53	20.09	21.95	26.12	27.87
9	9.41	10.66	12.24	14.68	16.92	19.02	21.67	23.59	27.88	29.67
10	10.47	11.78	13.44	15.99	18.31	20.48	23.21	25.19	29.59	31.42
11	11.53	12.90	14.63	17.28	19.68	21.92	24.72	26.76	31.14	33.14
12	12.58	14.01	15.81	18.55	21.03	23.34	26.22	28.30	32.91	34.82
13	13.64	15.12	16.98	19.81	22.36	24.74	27.69	29.82	34.53	36.48
14	14.69	16.22	18.15	21.06	23.68	26.12	29.14	31.32	36.12	38.11
15	15.73	17.32	19.31	22.31	25.00	27.49	30.58	32.80	37.70	39.79
16	16.78	18.42	20.47	23.54	26.30	28.85	32.00	34.27	39.25	41.31
17	17.82	19.51	21.61	24.77	27.59	30.19	33.41	35.72	40.79	42.88
18	18.87	20.60	22.76	25.99	28.87	31.53	34.81	37.16	42.31	44.43
19	19.91	21.69	23.90	27.20	30.14	32.85	36.19	38.58	43.82	45.97
20	20.95	22.77	25.04	28.41	31.41	34.17	37.57	40.00	45.31	47.50
21	21.99	23.86	26.17	29.62	32.67	35.48	38.93	41.40	46.80	49.01
22	23.03	24.94	27.30	30.81	33.92	36.78	40.29	42.80	48.27	50.51
23	24.07	26.02	28.43	32.01	35.17	38.08	41.64	44.18	49.73	52.00
24	25.11	27.10	29.55	33.20	36.42	39.36	42.98	45.56	51.18	53.48
25	26.14	28.17	30.68	34.38	37.65	40.65	44.31	46.93	52.62	54.95
26	27.18	29.25	31.79	35.56	38.89	41.92	45.64	48.29	54.05	56.41
27	28.21	30.32	32.91	36.74	40.11	43.19	46.96	49.64	55.48	57.86
28	29.25	31.39	34.03	37.92	41.34	44.46	48.28	50.99	56.89	59.30
29	30.28	32.46	35.14	39.09	42.56	45.72	49.59	52.34	58.30	60.73
30	31.32	33.53	36.25	40.26	43.77	46.98	50.89	53.67	59.70	62.16
40	41.62	44.16	47.27	51.81	55.76	59.34	63.69	66.77	73.40	76.09
50	51.89	54.72	58.16	63.17	67.50	71.42	76.15	79.49	86.66	89.56
60	62.13	65.23	68.97	74.40	79.08	83.30	88.38	91.95	99.61	102.69
100	102.95	106.91	111.67	118.50	124.34	129.56	135.81	140.17	149.45	153.17

(9.4) Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
 If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1-p) \geq n-k)$.

n	k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.4225	0.3600	0.3025	0.2500	
3	0	0.9575	0.9900	0.9775	0.9600	0.9375	0.9100	0.8775	0.8400	0.7975	0.7500	
4	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2746	0.2160	0.1664	0.1250	
5	0	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.7183	0.6480	0.5747	0.5000	
6	0	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9571	0.9360	0.9089	0.8750	
7	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1785	0.1296	0.0915	0.0625	
8	0	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.5630	0.4752	0.3910	0.3125	
9	0	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8735	0.8208	0.7585	0.6875	
10	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.1160	0.0778	0.0503	0.0313	
11	0	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.4284	0.3370	0.2562	0.1875	
12	0	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.7648	0.6826	0.5931	0.5000	
13	0	0.9999	0.9995	0.9978	0.9933	0.9844	0.9692	0.9460	0.9130	0.8688	0.8125	
14	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
15	0	0.7351	0.5314	0.3771	0.2621	0.1780	0.1176	0.0754	0.0467	0.0277	0.0156	
16	0	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.3191	0.2333	0.1636	0.1094	
17	0	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.6471	0.5443	0.4415	0.3438	
18	0	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8826	0.8208	0.7447	0.6563	
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
20	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0490	0.0280	0.0152	0.0078	
21	0	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.2338	0.1586	0.1024	0.0625	
22	0	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.5323	0.4199	0.3124	0.2266	
23	0	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.8002	0.7102	0.6083	0.5000	
24	0	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9444	0.9037	0.8471	0.7734	
25	0	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9910	0.9812	0.9643	0.9375	
26	0	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9984	0.9963	0.9922	0.9900	
27	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0319	0.0168	0.0084	0.0039	
28	0	1.0000	0.8131	0.6572	0.5033	0.3671	0.2553	0.1691	0.1064	0.0632	0.0352	
29	0	2.0000	0.9619	0.8948	0.7969	0.6785	0.5518	0.4278	0.3154	0.2201	0.1445	
30	0	3.0000	0.9950	0.9786	0.8862	0.8059	0.7064	0.5941	0.4770	0.3633	0.2444	
31	0	4.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8939	0.8263	0.7396	0.6367	
32	0	5.0000	1.0000	0.9998	0.9988	0.9887	0.9747	0.9502	0.9115	0.8555	0.7733	
33	0	6.0000	1.0000	1.0000	0.9999	0.9987	0.9964	0.9915	0.9819	0.9643	0.9000	
34	0	7.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9983	0.9961	0.9900	
35	0	8.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
36	0	9.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
37	0	10.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
38	0	11.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
39	0	12.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
 If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1-p) \geq n-k)$.

n	k	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	p
11	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0088	0.0036	0.0014	0.0005	
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0057	0.0022	0.0008	0.0002	
13	0	0.5040	0.2242	0.1087	0.0544	0.0274	0.0121	0.0055	0.0023	0.0009	0.0003	
14	0	0.4816	0.1659	0.0843	0.0443	0.0274	0.0158	0.0085	0.0042	0.0016	0.0003	
15	0	0.4552	0.1174	0.0574	0.0317	0.0197	0.0102	0.0053	0.0026	0.0011	0.0003	
16	0	0.4292	0.0973	0.0496	0.0275	0.0153	0.0084	0.0042	0.0021	0.0013	0.0003	
17	0	0.4048	0.0781	0.0378	0.0226	0.0132	0.0071	0.0041	0.0021	0.0013	0.0003	
18	0	0.3816	0.0616	0.0317	0.0202	0.0121	0.0066	0.0040	0.0020	0.0012	0.0003	
19	0	0.3604	0.0492	0.0297	0.0197	0.0112	0.0061	0.0038	0.0019	0.0010	0.0003	
20	0	0.3404	0.0381	0.0202	0.0121	0.0066	0.0038	0.0020	0.0010	0.0005	0.0003	
21	0	0.3216	0.0281	0.0187	0.0102	0.0056	0.0031	0.0016	0.0008	0.0004	0.0003	
22	0	0.3036	0.0202	0.0121	0.0066	0.0038	0.0020	0.0010	0.0005	0.0003	0.0003	
23	0	0.2863	0.0141	0.0087	0.0056	0.0031	0.0016	0.0008	0.0004	0.0003	0.0003	
24	0	0.2702	0.0102	0.0056	0.0031	0.0016	0.0008	0.0004	0.0003	0.0003	0.0003	
25	0	0.2550	0.0071	0.0041	0.0021	0.0011	0.0005	0.0003	0.0002	0.0003	0.0003	
26	0	0.2410	0.0051	0.0028	0.0014	0.0007	0.0004	0.0002	0.0003	0.0003	0.0003	
27	0	0.2280	0.0036	0.0018	0.0009	0.0005	0.0003	0.0002	0.0003	0.0003	0.0003	
28	0	0.2150	0.0024	0.0011	0.0005	0.0003	0.0002	0.0001	0.0003	0.0003	0.0003	
29	0	0.2030	0.0016	0.0007	0.0004	0.0002	0.0001	0.0001	0.0003	0.0003	0.0003	
30	0	0.1910	0.0010	0.0004	0.0002	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
31	0	0.1800	0.0006	0.0003	0.0002	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
32	0	0.1700	0.0004	0.0002	0.0001	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
33	0	0.1600	0.0003	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
34	0	0.1500	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
35	0	0.1400	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
36	0	0.1300	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
37	0	0.1200	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
38	0	0.1100	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	
39	0	0.1000	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0003	0.0003	0.0003	

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1-p) \geq n-k)$.

n	k	0.05	0.10	0.15	0.20	0.25	p	0.30	0.35	0.40	0.45	0.50
14	0	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0024	0.0008	0.0002	0.0001	
1	0.8470	0.5284	0.3567	0.1979	0.1010	0.0475	0.0205	0.0081	0.0009			
2	0.9699	0.8416	0.6479	0.4481	0.2811	0.1608	0.0839	0.0398	0.0170	0.0065		
3	0.9558	0.9559	0.8535	0.6982	0.5213	0.3552	0.2205	0.1243	0.0632	0.0287	0.0123	
4	0.9996	0.9908	0.9533	0.8702	0.7415	0.5842	0.4227	0.2793	0.1672	0.0898	0.0464	
5	1.0000	0.9985	0.9885	0.9561	0.8883	0.7805	0.6405	0.4859	0.3373	0.2120	0.1260	0.0596
6	1.0000	0.9998	0.9978	0.9884	0.9617	0.9067	0.8164	0.6925	0.5461	0.3953	0.2348	0.0245
7	1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.9247	0.8499	0.7414	0.6047	0.4478	0.2902
8	1.0000	1.0000	1.0000	0.9996	0.9978	0.9917	0.9757	0.8811	0.7830	0.6405	0.4743	0.3145
9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9940	0.9825	0.9574	0.9102	0.8011	0.6626
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9961	0.9886	0.9713	0.9177	0.6855
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9978	0.9935	0.9777	0.8338
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9894	0.9283
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9755
14	0	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0016	0.0005	0.0001	0.0000	
1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0142	0.0052	0.0017	0.0005		
2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0617	0.0271	0.0107	0.0037		
3	0.9445	0.9444	0.8227	0.6482	0.4613	0.2969	0.1727	0.0905	0.0424	0.0176		
4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.3519	0.2173	0.1204	0.0592		
5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.5643	0.4032	0.2608	0.1509		
6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.7548	0.6098	0.4522	0.3036		
7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.8868	0.7869	0.6535	0.5000		
8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9578	0.9050	0.8182	0.6964		
9	1.0000	1.0000	1.0000	0.9990	0.9992	0.9963	0.9876	0.9662	0.9231	0.8491		
10	1.0000	1.0000	1.0000	0.9999	0.9993	0.9972	0.9907	0.9745	0.9408	0.8743		
11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9981	0.9937	0.9824	0.8809		
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9963	0.9790		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9998		
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
15	0	0.4401	0.1853	0.0743	0.0281	0.0100	0.0033	0.0010	0.0003	0.0001	0.0000	
1	0.8108	0.5147	0.2839	0.1407	0.0635	0.0261	0.0098	0.0033	0.0010	0.0003		
2	0.9571	0.7892	0.5614	0.3518	0.1971	0.0994	0.0451	0.0183	0.0066	0.0021		
3	0.9360	0.9316	0.7899	0.5981	0.4050	0.2459	0.1339	0.0651	0.0281	0.0106		
4	0.9991	0.9830	0.9209	0.7982	0.6302	0.4499	0.2892	0.1666	0.0853	0.0384		
5	0.9999	0.9967	0.9765	0.9183	0.8103	0.6598	0.4900	0.3288	0.1976	0.1051		
6	1.0000	0.9995	0.9944	0.9733	0.9204	0.8247	0.6881	0.5272	0.3660	0.2272		
7	1.0000	0.9999	0.9930	0.9729	0.9256	0.8406	0.7161	0.5629	0.4018	0.2409		
8	1.0000	1.0000	0.9998	0.9985	0.9795	0.9743	0.9329	0.8579	0.7441	0.5982		
9	1.0000	1.0000	1.0000	0.9998	0.9924	0.9771	0.9417	0.8759	0.7728	0.6454		
10	1.0000	1.0000	1.0000	0.9997	0.9984	0.9938	0.9809	0.9514	0.8949	0.7439		
11	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9851	0.9616	0.8180		
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9965	0.9894	0.8988		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		

Table for Binomial random variable $P(Bin(n, p) \leq k)$ if $p \leq 0.5$.
If $p > 0.5$, then $P(Bin(n, p) \leq k) = P(Bin(n, 1-p) \geq n-k)$.

n	k	0.05	0.10	0.15	0.20	0.25	p	0.30	0.35	0.40	0.45	0.50
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0007	0.0002	0.0000	0.0000	
1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0067	0.0021	0.0006	0.0001		
2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0327	0.0123	0.0041	0.0012		
3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.1028	0.0464	0.0184	0.0064		
4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.2348	0.1260	0.0596	0.0245		
5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5968	0.4197	0.2639	0.1471	0.0717		
6	1.0000	0.9992	0.9623	0.8929	0.7752	0.6188	0.4478	0.2902	0.1662			
7	1.0000	0.9999	0.9883	0.9598	0.8787	0.7872	0.6405	0.4743	0.3145			
8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.9006	0.8011	0.6626	0.5000		
9	1.0000	1.0000	1.0000	0.9999	0.9999	0.9970	0.9894	0.9699	0.9283	0.7597		
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
16	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0003	0.0001	0.0000		
1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0031	0.0008	0.0002	0.0000		
2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0170	0.0055	0.0015	0.0004		
3	0.9836	0.8850	0.6841	0.4551	0.2631	0.1332	0.0591	0.0230	0.0077	0.0022		
4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.1500	0.0696	0.0280	0.0096		
5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.2968	0.1629	0.0777	0.0318		
6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.4812	0.3081	0.1727	0.0835		
7	1.0000	0.9997	0.9959	0.9767	0.9225	0.8180	0.6656	0.4878	0.3169	0.1796		
8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.8145	0.6675	0.4940	0.3238		
9	1.0000	1.0000	0.9999	0.9974	0.9674	0.9125	0.8139	0.6710	0.4917	0.3238		
10	1.0000	1.0000	1.0000	0.9997	0.9797	0.9563	0.9115	0.8159	0.6762	0.4917		
11	1.0000	1.0000	1.0000	0.9995	0.9972	0.9886	0.9648	0.9129	0.8204	0.6822		
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9969	0.9884	0.9658	0.9165	0.7473		
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		

(9.5) Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	μ									
k	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066	0.3679
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725	0.7358
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371	0.9197
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865	0.9810
4	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977	0.9963	0.9956
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997	0.9994
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
k	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
1	0.6890	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
2	0.904	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
3	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
4	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
5	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9834	0.9775
6	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
7	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
k	3.2	3.4	3.6	3.8	4.0	4.2	4.4	4.6	4.8	5.0
0	0.0408	0.0334	0.0273	0.0224	0.0183	0.0150	0.0123	0.0101	0.0082	0.0067
1	0.1712	0.1468	0.1257	0.1074	0.0916	0.0780	0.0663	0.0563	0.0477	0.0404
2	0.3799	0.3397	0.3027	0.2689	0.2381	0.2102	0.1851	0.1626	0.1425	0.1247
3	0.6025	0.5584	0.5152	0.4735	0.4335	0.3954	0.3594	0.3257	0.2942	0.2650
4	0.7806	0.7442	0.7064	0.6678	0.6288	0.5898	0.5512	0.5132	0.4763	0.4405
5	0.8946	0.8705	0.8441	0.8156	0.7851	0.7531	0.7199	0.6858	0.6510	0.6160
6	0.9554	0.9421	0.9267	0.9091	0.8893	0.8675	0.8436	0.8180	0.7908	0.7622
7	0.9832	0.9769	0.9692	0.9559	0.9489	0.9361	0.9214	0.9049	0.8867	0.8666
8	0.9943	0.9917	0.9883	0.9840	0.9786	0.9721	0.9642	0.9549	0.9442	0.9319
9	0.9995	0.9992	0.9987	0.9981	0.9972	0.9959	0.9943	0.9922	0.9896	0.9863
k	5.2	5.4	5.6	5.8	6.0	6.5	7.0	7.5	8.0	8.5
0	0.0055	0.0045	0.0037	0.0030	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002
1	0.0342	0.0289	0.0244	0.0206	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019
2	0.1088	0.0948	0.0824	0.0715	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093
3	0.2381	0.2133	0.1906	0.1700	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301
4	0.4061	0.3733	0.3422	0.3127	0.2851	0.2237	0.1730	0.1321	0.0966	0.0744
5	0.5809	0.5461	0.5119	0.4783	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496
6	0.7324	0.7017	0.6703	0.6384	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562
7	0.8449	0.8149	0.7970	0.7710	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856
8	0.9181	0.9027	0.8857	0.8672	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231
9	0.9603	0.9512	0.9409	0.9292	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530
10	0.9823	0.9775	0.9718	0.9651	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634
11	0.9904	0.9875	0.9841	0.9799	0.9761	0.9467	0.9208	0.8881	0.8487	0.8047
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
k	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550	0.0498
1	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146	0.1991
2	0.6496	0.6227	0.5960	0.5597	0.5348	0.5184	0.4936	0.4695	0.4460	0.4232
3	0.8386	0.8194	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696	0.6472	0.6141
4	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8477	0.8318	0.8153	0.7894
5	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258	0.9161
6	0.9941	0.9925	0.9884	0.9828	0.9794	0.9756	0.9713	0.9665	0.9623	0.9573
7	0.9985	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901	0.9881	0.9850
8	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969	0.9962
9	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993	0.9992
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table for Poisson random variable $P(Po(\mu) \leq k)$.

k	9.0	9.5	10.0	11.0	12.0	13.0	14.0	15.0	16.0	17.0	μ
0	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0012	0.0008	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0062	0.0042	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.0212	0.0149	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000
4	0.0550	0.0403	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0000
5	0.1157	0.0885	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0000
6	0.2068	0.1649	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0000
7	0.3239	0.2687	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0000
8	0.4557	0.3918	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0000
9	0.5874	0.5218	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0000
10	0.7060	0.6453	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0000
11	0.8030	0.7520	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0000
12	0.8758	0.8364	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0000
13	0.9261	0.8981	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.0000
14	0.9585	0.9400	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.0000
15	0.9780	0.9665	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.0000
16	0.9889	0.9823	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.0000
17	0.9947	0.9911	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.0000
18	0.9976	0.9957	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.0000
19	0.9989	0.9980	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.0000
20	0.9996	0.9991	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.0000
21	0.9998	0.9996	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.0000
22	0.9999	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.0000
23	1.0000	0.9999	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.0000
24	1.0000	1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.0000
25	1.0000	1.0000	1.0000	0.9999	0.9997	0.9974	0.9938	0.9869	0.9748	0.9548	0.0000
26	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.0000
27	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.0000
28	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9900	0.0000
29	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9973	0.9947	0.0000
30	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9976	0.0000
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9980	0.0000
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.0000
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.0000
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.0000
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000