

Number theory, Talteori 6hp, Kurskod TATA54, Provkod TEN1
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Each problem is worth 3 points. To receive full points, a solution needs to be complete. Indicate which theorems from the textbook that you have used, and include all auxillary calculations.

No aids, no calculators, tables, nor textbooks.

1) Determine all solutions to $180x \equiv 120 \pmod{240}$.

2) Find all solutions to the congruence

$$x^7 + x^3 + x + 1 \equiv 0 \pmod{16}.$$

3) Consider the polynomial $f(t) = t^4 + 2t^2 - 4$. Does f have a zero which is an integer? A zero mod 19? A zero mod 43? Find examples of such zeroes, when possible.

4) Write 41 as a sum of two squares, and then write 205 as a sum of two squares. Finally, write 222 as a sum of four squares.

5) Find the continued fraction expansion of $\sqrt{17}$, then approximate $\sqrt{17}$ with a rational number, with an error less than 0.002.

6) Let f be a multiplicative arithmetical function. If the argument n has prime factorization $n = p_1^{a_1} \cdots p_k^{a_k}$, show that

$$\sum_{d|n} \mu(d)f(d) = (1 - f(p_1)) \cdots (1 - f(p_k)).$$

Use this to show that

$$\sum_{d|n} \frac{\mu(d)}{d} = \frac{\phi(n)}{n}.$$

7) Determine all positive integer solutions to $x^2 + 2y^2 = z^2$.