CRAM 1 - Programming Lab

Problem Set 4: Vector Autoregressive (VAR) Models

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Goals

This set of exercises has four goals:

- First, students learn how to set-up and estimate a simple VAR(1) model.
- Second, how to calculate an impulse response function (IRF).
- Third, how to use impulse response functions to analyze economic relationships.
- Fourth, an application to learn about how future bond prices depend on changes in the macro economy.

Submission

The solution to the problem set has to be submitted in form of two files:

- *ps4.py*: Containing the code to reproduce the quantitative results.
- A power point presentation answering the qualitative questions ($\sim$ 1-2 slide per question). Please upload it as a pdf file.

Submit your solution via the Praktomat https://praktomat.cs.kit.edu/cram_2017_WS latest 11:59 a.m. on Wednesday, 15/11/2017. Note, please name your Python file 'ps4.py' to allow for evaluation by the Praktomat and set the status along your code to SOLN.

Vector Autoregressive (VAR) Models

Question 1a: Treasury Yields

Read-in the provided txt-file with the daily Treasury yields and plot them. Describe the time series of the yields and compare the different maturities. What are differences to stock returns?

For the further analysis take the '1 year', '5 year' and '10 year' yield from 1995 onwards.

Question 1b: Macroeconomic Data

Read-in the time series of the unemployment and inflation rate of the US. Plot the data and describe it. What economic events happened during this time period?

Then, join it with the yield subset from the previous questions.

Question 1c: Class 'VAR'

Create a class 'VAR' which allows for the estimation of a VAR(p) model and also returns the covariance matrix of the error terms.

If the OLS assumptions hold and if you have no further constraints for the parameters, you can conveniently run OLS row by row. As a result, you will determine

$$\hat{A}_{ols}, \quad \hat{\epsilon}_{ols}, \quad \hat{\Sigma}_{ols},$$

where $\hat{\Sigma}_{ols}$ is the OLS estimate of the error covariance matrix (see next slide for the model notation).

Question 1d: VAR Estimation

Demean the time series and estimate a VAR(1) model (without a constant) for the selected subset of yields and macro variables:

$$x_t = A \times x_{t-1} + \epsilon_t, \quad \epsilon :\sim i.i.d.(0, \Sigma), \tag{1}$$

where A is the matrix that collects the autoregressive coefficients, $x_t \equiv [unempl_t, infl_t, y_t^{1yr}, y_t^{5yr}, y_t^{10yr}]$ and Σ is the 'empirical' covariance matrix.

Give an economic interpretation to the parameters and t-statistics.

How can you make a forecast based on the estimated model?

Question 1e: Impulse Response Function

Add a new method to your class 'VAR' which calculates the impulse response function for a given number of periods:

$$IRF(j) = A \times IRF(j-1), \quad IRF(0) = C \tag{2}$$

with $C := \text{Chol}(\Sigma)$.

With C being the Cholesky decomposition one can rewrite the statistical errors in terms of fundamental (economic) errors η

$$C\,\eta_t \;=\; \epsilon_t, \tag{3}$$

which leads e.g. for a VAR(1) to

$$x_t = A \times x_{t-1} + C \times \eta_t, \qquad E[\eta_t\,\eta_t'] = I. \tag{4}$$

Question 1f: Economic Analysis

Use the VAR model from Question 1d to analyze the economic relationships between the macroeconomy and the three yields. Therefore, calculate and describe the impulse responses of the different yields to a shock to unemployment over a 5 year period.

How does a change in fundamental economic conditions affect the fixed income market?

Hint: To evaluate the statistical significance use the confidence bounds provided by the 'statsmodels' package.