CRAM 1 - Programming Lab

Problem Set 2: Modeling of Equity Returns

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Goals

These real world problems teach you

- How to characterize and model ‘real world’ equity returns
- How to simulate data given the ‘true’ data generating process
- How to implement the OLS approach in an object-oriented way
- How to estimate the parameters of a linear model
- How to forecast returns based on an AR(1) model

Submission

The solution to the problem set has to be submitted in form of two files:

- *ps2.py*: Containing the code to reproduce the quantitative results.
- A power point presentation answering the qualitative questions ($\sim$ 1-2 slide per question). Please upload it as a pdf file.

Submit your solution via the Praktomat https://praktomat.cs.kit.edu/cram_2017_WS latest 11:59 a.m. on Wednesday, 01/11/2017. Note, please name your Python file 'ps2.py' to allow for evaluation by the Praktomat and set the status along your code to SOLN.

Modeling of Equity Returns

Question 1a: Equity Index Returns

Read-in the daily price data of the Euro Stoxx 50 and calculate the log returns (in percentage terms):

$$\ln\left(\frac{price_t}{price_{t-1}}\right). \tag{1}$$

Plot the time series of the prices and returns and describe them. Would you characterize the return time series as 'homoscedastic' or 'heteroscedastic'?
Then plot a histogram of the return series and compare it with a normal distribution. Is such a Gaussian distribution a good modelling choice for stock returns?

Question 1b: Jarque-Bera Test

Implement a function that calculates the Jarque-Bera test for a given data set and returns the t-statistic and p-value. The test statistic is defined as:

$$JB = n\left(\frac{S^2}{6} + \frac{(K-3)^2}{24}\right) \tag{2}$$

with n being the number of observations and S (K) being the sample skewness (kurtosis). The null hypothesis is a joint hypothesis of the skewness being zero and the excess kurtosis being zero.

Calculate the skewness and kurtosis for the ESTOXX 50 returns and apply the Jarque-Bera test. Are stock returns Gaussian?

Question 1c: Simulating Returns

Use a random number generator to generate the following time series:

- Gaussian White Noise (GWN) $\sim N(0,1)$,
- AR(1) with $\phi_0 = 0$, $\phi_1 = 0.8$ and $\sigma = 1$.
  An AR(1) is defined as:

$$x_t = \phi_0 + \phi_1 * x_{t-1} + \sigma * \epsilon_t, \ \epsilon_t \sim N(0,1). \tag{3}$$

Describe the two resulting (return) series and calculate the higher moments. Apply the Jarque-Bera test to test for normality.

Question 1d: Ordinary Least Squares (OLS)

Implement the *function* ‘run_OLS()’ of the *class* ‘OLS’ which performs a parameter estimation based on the ordinary least squares approach:

$$\hat{\beta}_{ols} := (X'X)^{-1}X'Y, \quad \hat{\epsilon}_{ols} := Y - X\hat{\beta}_{ols},$$

$$\hat{\sigma}^2_{\epsilon,ols} := \frac{1}{T-p}\,\hat{\epsilon}'_{ols}\,\hat{\epsilon}_{ols}, \quad v\hat{a}r_{ols}[\beta] := \hat{\sigma}^2_{\epsilon,ols} \times (X'X)^{-1},$$

$$s.\hat{e}._{ols}[\beta_i] := \sqrt{[v\hat{a}r_{ols}[\beta]]_{[i,i]}}\ for\ i \in [0, 1, ..., p], \quad \hat{t}_{ols}[\beta_i] := \frac{[\hat{\beta}_{ols}]_{[i,1]}}{s.\hat{e}._{ols}[\beta_i]},$$

$$Adj.\ \hat{R}^2_{ols} := 1 - \frac{\hat{\sigma}^2_{\epsilon,ols}}{\sigma^2_y},\ where\ \sigma^2_y := \frac{\sum_{t=1}^{T}(y_t - \bar{y})^2}{T-1}.$$

Question 1e: Estimate AR(1)

Regress the ESTOXX 50 return/ GWN and the AR(1) time series on its first lag:

$r_t = \beta_0 + \beta_1 * r_{t-1} + \epsilon_t.$

Report the beta estimates, the t-stats and the adj. R^2 and interpret them. Double check the results with the *statsmodels* OLS package.

Question 1f: Forecasting

Assume, returns follow an AR(1) with the estimated (AR(1)) coefficients from Question 1e. Implement two functions that give you the conditional *j-step* ahead forecast and variance:

$$E_t[r_{t+j}] = c \times \sum_{i=0}^{j-1} \phi^i + \phi^j \times r_t,$$

$$Var_t(r_{t+j}) = \sigma^2 \times \sum_{k=0}^{j-1} \phi^{2k}.$$

Use these functions to calculate the 1 and 2 step ahead forecast and it's uncertainty.