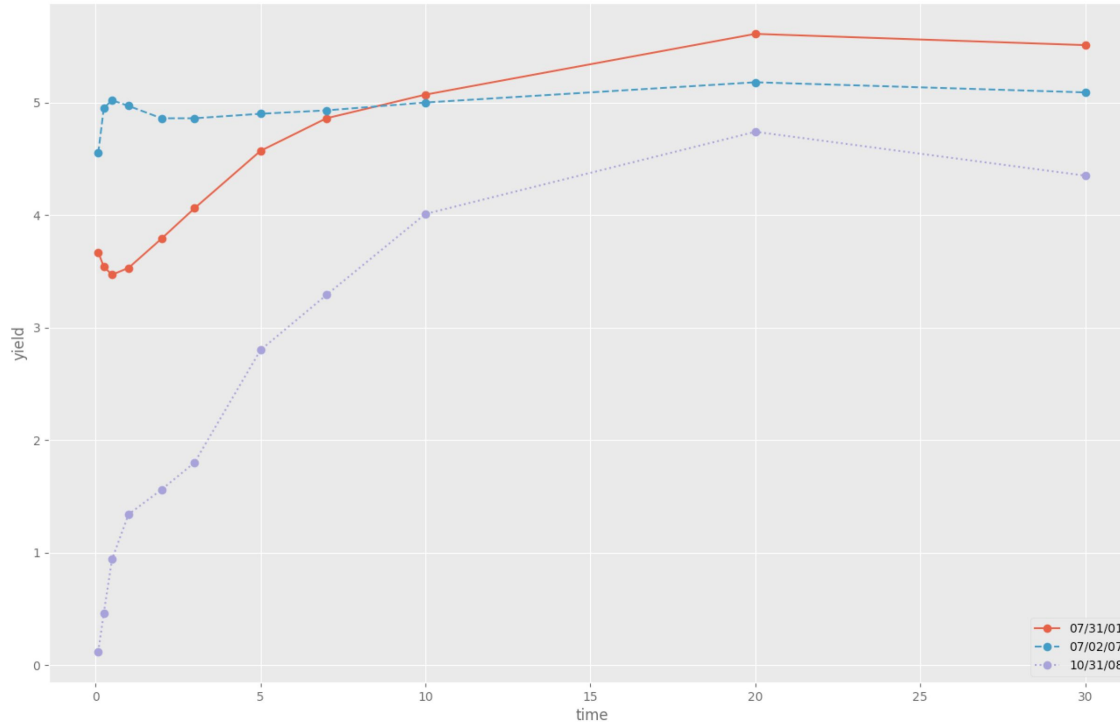


CRAM1

Problem Set 5

Jonas Redfoot and Fabio Iron

1a: treasury yield curves



2008: steep upward slope -> high uncertainty due to financial crisis

2007: (close to) no slope in for maturity > 1 year, for < 1 year upward slope

2001: downward slope for < 6 month then upward slow

For all yield curves: 30 year yield is lower than 20 year yield

1b: PCA Intuition

- Data is transformed so that the statistical factors are uncorrelated
- $Y = X * E_x$ whereby E_x the matrix of eigenvectors of $\text{Cov}(X)$
- Usually the eigenvectors in E_x are ordered according in ascending order of their corresponding eigenvalues
- The transformation is chosen in such way that each principal component in the order explains a maximal amount of variance in the data

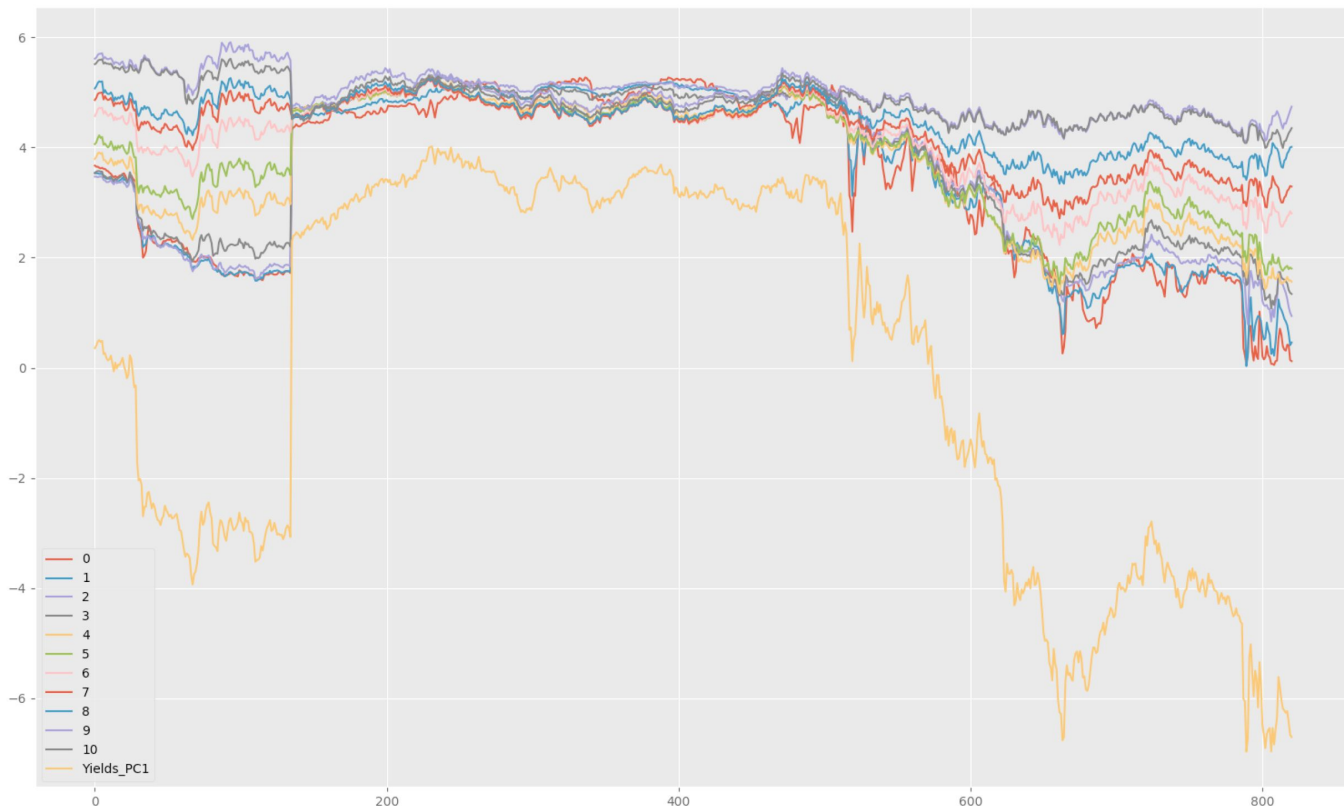
→ PCA orthogonalizes the data so that the maximal amount of variance is explained by each PC in the order -> Usually just a fraction of the PCs capture nearly 100% of the variance in data -> dimensionality reduction

13: Importance of the PCs

```
Variance Explained: [ 0.928 0.062 0.008 0.002 0. 0. 0. 0. 0. 0. 0. ]  
Agg Variance Explained: [ 0.928 0.99 0.997 0.999 0.999 1. 1. 1. 1. 1. 1. ]
```

- 2 PCs explain 99% of the variance -> I would use just the first two components

1d: Principal Components of Yields



PC#1:

- on average lower than the yields (-> downwards shifted)
- Positively correlated with the yield curves

1d: Principal Components of Yields

Correlation matrix:

	0	1	2	3	4	5 \
0	1.000000	0.992274	0.980586	0.974837	0.943761	0.910039
1	0.992274	1.000000	0.995143	0.990757	0.959826	0.924269
2	0.980586	0.995143	1.000000	0.997001	0.963748	0.924868
3	0.974837	0.990757	0.997001	1.000000	0.979884	0.948334
4	0.943761	0.959826	0.963748	0.979884	1.000000	0.991700
5	0.910039	0.924269	0.924868	0.948334	0.991700	1.000000
6	0.827853	0.838397	0.832623	0.864946	0.942311	0.975436
7	0.734527	0.741194	0.729865	0.768078	0.869992	0.922169
8	0.641692	0.648847	0.637259	0.679618	0.797190	0.861675
9	0.256500	0.249254	0.221731	0.270859	0.430274	0.533190
10	0.276857	0.265329	0.232635	0.277096	0.426721	0.525575
Yields_PC1	0.981521	0.990371	0.988389	0.993831	0.988143	0.967185

	6	7	8	9	10	Yields_PC1
0	0.827853	0.734527	0.641692	0.256500	0.276857	0.981521
1	0.838397	0.741194	0.648847	0.249254	0.265329	0.990371
2	0.832623	0.729865	0.637259	0.221731	0.232635	0.988389
3	0.864946	0.768078	0.679618	0.270859	0.277096	0.993831
4	0.942311	0.869992	0.797190	0.430274	0.426721	0.988143
5	0.975436	0.922169	0.861675	0.533190	0.525575	0.967185
6	1.000000	0.984161	0.949609	0.700157	0.684056	0.902241
7	0.984161	1.000000	0.987884	0.811746	0.790748	0.819309
8	0.949609	0.987884	1.000000	0.879523	0.853961	0.737426
9	0.700157	0.811746	0.879523	1.000000	0.983053	0.355724
10	0.684056	0.790748	0.853961	0.983053	1.000000	0.363069
Yields_PC1	0.902241	0.819309	0.737426	0.355724	0.363069	1.000000

PC#1:

- on average lower than the yields (-> downwards shifted)
- Positively correlated with the yield curves
- Stronger correlation with short-term bond yields

1f: Forecasting the Yield Curve

- Forecast the dynamic of the whole cross-section of a panel with just three PCs
- **How would you adjust your bond pf?**

