Meteor Madness

Contents

1 Datasets to consider

1.1 NASA CNEO API

This will be handy. Information about asteroids, closest approach (normal and minimum), relative velocity, max velocity, time of closest approach etc. is provided.

- API Key: JPL CAD API
- Dataset : Click Here

1.2 U.S. Geological Survey (USGS)

USGS will help in getting the geographical dataset. Relevant data from USGS.

- Elevation Models
- Earthquake Data
- API Documentation for Earthquake Data

- Tsunami Data

2 Building the asteroid impact simulation.

It's important to understand how to actually simulate the asteroid impact from real physics. The problem says to use Keplerian orbital elements, hence we will not base this on Einstein's general relativity.

2.1 The Algorithm (Important)

2.1.1 Initialize Parameters

- Calculate asteroid mass m from its density and radius.
- Set initial position $r = r_0$ and velocity $v = v_0$.
- Initialize time $t = 0$.
- Set T_{max} to a certain value.

2.1.2 While $t < T_{max}$ and $||r|| > R_{earth}$

- Compute acceleration due to earth's gravity. (Look at (1)).
- Apply RK4 to update position and velocity.

Listing 1: RK4 step for asteroid simulation

```
    # RK4 update for asteroid position and velocity
    k1_r = v * dt
    k1_v = a(r) * dt

    k2_r = (v + 0.5 * k1_v) * dt
    k2_v = a(r + 0.5 * k1_r) * dt

    k3_r = (v + 0.5 * k2_v) * dt
    k3_v = a(r + 0.5 * k2_r) * dt

    k4_r = (v + k3_v) * dt
    k4_v = a(r + k3_r) * dt

    r_next = r + (k1_r + 2*k2_r + 2*k3_r + k4_r) / 6
    v_next = v + (k1_v + 2*k2_v + 2*k3_v + k4_v) / 6
```

- Store r_{next} in the trajectory array.
- Increment time $t \to t + dt$

2.1.3 Check $r < R_{earth}$

If true :

- Impact time $t_{impact} = t$
- Impact coordinates : converting cartesian to lat/lon.
- Impact speed $v_{impact} = ||v||$

2.2 The Physics

Newton's equation of motion (Earth gravity only) for the asteroid is

$$\ddot{\mathbf{r}} = \mathbf{a}(\mathbf{r}) = -G\,\frac{M_\oplus}{\|\mathbf{r}\|^3}\,\mathbf{r}, \tag{1}$$

where

$$G = 6.67430 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$$

is the gravitational constant,

$$M_\oplus \approx 5.972 \times 10^{24}\ \mathrm{kg}$$

is the Earth's mass, and

$$\|\mathbf{r}\| = \sqrt{x^2 + y^2 + z^2}$$

is the Euclidean norm of the position vector, i.e. the distance from the Earth's center.

This second-order ordinary differential equation can be rewritten as a first-order system by defining the state vector

$$\mathbf{y} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix},$$

with

$$\dot{\mathbf{r}} = \mathbf{v}, \qquad \dot{\mathbf{v}} = \mathbf{a}(\mathbf{r}).$$

It's alright if you don't understand this. What we must understand is the core: we are solving the differential equation we stated above. For this we apply RK4 or the Runge-Kutta Method of approximating the solution.

2.3 Runge–Kutta 4 (RK4) Integration

RK4 allows us to approximate the solution of first-order differential equations of the form

$$\dot{\mathbf{y}} = f(\mathbf{y}, t),$$

where $\mathbf{y}$ is the state vector containing position $\mathbf{r}$ and velocity $\mathbf{v}$.

State vector:

$$\mathbf{y} = \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}, \quad f(\mathbf{y}) = \begin{bmatrix} \mathbf{v} \\ \mathbf{a}(\mathbf{r}) \end{bmatrix}.$$

RK4 step: Given the state $\mathbf{y}_n$ at time t_n, the state at $t_{n+1} = t_n + \Delta t$ is approximated by

$$\mathbf{y}_{n+1} = \mathbf{y}_n + \frac{1}{6}\left(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4\right),$$

where

$$\begin{aligned}
\mathbf{k}_1 &= \Delta t\, f(\mathbf{y}_n), \\
\mathbf{k}_2 &= \Delta t\, f\left(\mathbf{y}_n + \frac{\mathbf{k}_1}{2}\right), \\
\mathbf{k}_3 &= \Delta t\, f\left(\mathbf{y}_n + \frac{\mathbf{k}_2}{2}\right), \\
\mathbf{k}_4 &= \Delta t\, f\left(\mathbf{y}_n + \mathbf{k}_3\right).
\end{aligned}$$

—

In simple words:

- For small time step, we observe at where the asteroid is and it's velocity.

- We calculate four “guesses” of how position and velocity change (k_1, k_2, k_3, k_4) using the acceleration due to gravity.
- We then combine those four guesses to get a really good estimate of the new position and velocity, the weighted average.
- Repition is done over and over until the asteroid hits Earth or misses.

3 Impact Energy Calculations

3.1 TNT Unit

Trinitrotulene (TNT) is a yellow organic compound that was used as a standard military explosive in the 20th century. It has a consistent well-measured energy release per mass of 4.184 MJ/Kg. Hence, a conversion from TNT to Joule is given as
1 Ton (t) TNT = $4.184 \cdot 10^9$ Joule.
1 Kiloton (Kt) TNT = $4.184 \cdot 10^{12}$ Joule.
1 Megaton (Mt) TNT = $4.184 \cdot 10^{15}$ Joule. (Standard)

3.2 Destructive Capability of a Falling Asteroid

The potential destructive power of an asteroid is calculated by the Kinetic Energy of the falling meteor.

$$E = \frac{2}{3}\pi\rho r^3 v^2 \tag{2}$$

where ρ is the density, r is the radius and v is the velocity of the asteroid. This is to be converted into a Megaton TNT unit.

$$E_{Mt} = \frac{\frac{2}{3}\pi\rho r^3 v^2}{4.184 \cdot 10^{15}} \tag{3}$$

4 Seismic + Tsunami Effects

4.1 Estimating the seismic magnitude of earthquake.

From eqn (2), we can use a simple Joule → Richter conversion. The estimated Richter magnitude M of an asteroid impact can be calculated using the empirical formula:

$$M = \frac{2}{3}\log_{10}\left(\frac{E}{E_0}\right) + 3.2 \tag{4}$$

where:

- M = estimated Richter magnitude of the ”impact earthquake”
- E = kinetic energy of the asteroid (in Joules), calculated as $E = \frac{1}{2}mv^2$
- $E_0 = 10^{4.4}$ J (or sometimes 10^{15} J depending on the reference)
- 3.2 = empirical scaling constant derived from studies of large impacts

Example:
If $E = 10^{18}$ J and $E_0 = 10^{15}$ J, then:

$$M = \frac{2}{3} \log_{10} \left(\frac{10^{18}}{10^{15}} \right) + 3.2 = \frac{2}{3} \cdot 3 + 3.2 = 5.2 \tag{5}$$

This corresponds roughly to a magnitude 5.2 earthquake.

4.2 Estimating wave size (If asteroid hits the water)

We need to check whether the asteroid hits water or the ocean using the relevant DEM data.

I'm still researching on how to get the wave height. This section is not complete and will need further revamping.

5 Mitigation / Deflection Strategies

DART was a NASA space mission where the goal was to deflect the asteroid Didymos and it's moonlet Dimorphos from its regular orbit through kinetic impact. They launched the spacecraft on November 23, 2021, and it crashed into the asteroid on September 26, 2022.

This event can provide key help for us in the context of asteroid deflection. Of course, this is still yet to be researched, which i will do after I figure out the coding part, which is the big hurdle for now.