

# A Multi-Criteria approach to the selection of maintenance service providers based on Delay Time

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## Abstract

The performance of industrial maintenance in organizations is highly relevant for the survival of companies in a highly competitive market. Therefore, defining the most appropriate service providers that perform maintenance helps reduce total costs and minimize the repair time of components, which can reduce the trade-off between these factors. Thus, the objective of this research is to develop the multi-criteria decision model that uses the Multi-Attribute Utility Theory (MAUT) to select maintenance service providers linked to the concept of Delay Time (DT), this being an approach not yet explored in the literature. Two criteria were addressed separately: unavailability (hours) of the component and cost (\$), later they were aggregated. A numerical application was proposed to validate the developed model. The technique presents itself as a tool to select optimal maintenance service providers with the DT design.

**keywords:** *Maintenance policy; Multi-criteria decision; Service Provider Contract*

# 1 Introduction

The maintenance of a company is expensive, requires dedication and a well-defined plan. As reported by [20], approximately 15% to 40% (an average of 28%) of the total industrial costs are related to maintenance. However, [15, 34] remark that the adequacy of maintenance actions timing reduce the number of incidents due to blueprint failure and will optimize the maintenance function through increased trust in machine and equipment. The maintenance quality is intrinsically linked to a series of factors, new aspects, and the implementation of maintenance in each organization can happen in a relatively simple way [29].

Among these factors, a company may not have the necessary skills to manage the maintenance of its organization. The solution can be centered on outsourcing, as [24] points to the technique as a valuable strategy adopted by managers to achieve organizational goals.[16] arrive at the conception of maintenance and repair contracts, as an agreement between the manufacturer of original equipment, or the contractor and the owner of the equipment or user, knowing that the manufacturer or the contractor will perform, in this case, all actions related to maintenance and repair, at a rate that can be expressed in hours. Maintenance contracts, due to the complexity related to the dynamic environment and the duration of the contract period, significantly impact the attributions and sharing of benefits and risks that exist in a contract [10].

On several occasions, organizations do not know how to effectively measure, determine, and assess the value that outsourced companies bring to their organization, and numerous times, contractors do not comply with the current terms of the contract [36]. Therefore, it is essential to prioritize agreements that minimize the risks that may arise. Recently, a lot of attention has been given to the contracting of maintenance services and the relationship between the client and the contract providers, according to [13].

For [6, 12], when it concerns maintenance, the implementation of the Delay Time (DT) concept in companies that involve periodic monitoring's main problem is the determination of the monitoring interval. To check what is the status of the system, he proposes a policy linked to the decision-making process which aims to analyze more than one aspect - the determination of the inspection interval to carry out monitoring and involves the cost and downtime of the system. [21] explain that most works assume that the replacement of spares occurs soon after the detection of the defect during the inspection in a DT policy, however, this replacement could be delayed. Thus, he develops a replacement model which is 'relaxed' for an extra time and brings attention to the development of new models using this delayed replacement policy.

However, few studies are linked to maintenance agreements when there is an emphasis on a multi-criteria approach [1], and no studies involving the selection of service providers from the Daley Time perspective are found in the literature. The multi-criteria analysis is a powerful tool that helps the decision-making process through mathematical modeling in problems that have multiple objectives that should be achieved simultaneously and was introduced by [14]. The Multi-attribute Utility Theory (MAUT) is used to quantify how attractive the alternatives are [28]. Therefore, this article aims to portray a different approach to studies in the area as it encompasses the decision theory to select maintenance service providers under a DT policy. Two criteria, the Down Time ( $D$ ) and the Costs ( $C$ ) are evaluated separately and aggregated later.

As of the structuring of this work, it has been divided into five sections. In the next (second) section, the literature review on studies on the selection of service providers focusing on multi-criteria models and research on DT is explored. In the third topic, the quantitative axiomatic policy adopted, the justification for choosing the multi-criteria method chosen is introduced, we

demonstrate the construction of the proposed model and a numerical application. The fourth section presents the results and discussions obtained and in the fifth and last section, the conclusion of the research is portrayed.

## 2 Literature Review

### 2.1 Multi-criteria approach and selection of service providers

There is a wide range of multicriteria model types and applications. In MAUT, the model is called the Utility Theory based on [19] who introduce the idea of uncertainty due to the probabilistic connotations in its composition, reaching an expected utility of the utility function. The basic concept of Utility Theory focuses on allocating probabilities  $p_i$  because of reaching the expected value of  $x_i$  [14, 33]. MAUT is applied in cases that involve reliability due to these specific characteristics of the method and other multi-criteria methods are frequently applied in the literature today.

On the other hand, studies with MAUT applications are rarely carried out in the selection of suppliers and providers. [1] reports a study on spare parts suppliers for the planning of maintenance contracts, also using a MAUT approach and two criteria, interruption time ( $IT$ ) and cost ( $C$ ). [26] combined the multiattribute used theory (MAUT) with linear programming (LP) to classify and select the best suppliers by defining the optimal order amount among these selected, finally presenting a numerical application.

Other multi-criteria methods are also explored. *E.g.*, in the busiest periods of operations, reliability and safety are in the design of the service network, in the maintenance and selection of providers from the evaluation of quantitative and qualitative aspects [22]. In another aspect, since it involves multiple alternatives, attributes, and experts, the selection of cloud service providers can be associated with the problem of multicriteria decision-making (MCDM).

The selection of service providers in the logistics segment has also been the agenda of studies in the manufacture of high-cost medical devices in health. [32] highlight that the great challenge is to select service providers amidst the variety in the provision of logistics services, reliability, punctuality, and effectiveness, in addition to using a decision method in the Analytical Hierarchical Process (AHP) with a combination of the weighted density of cluster analysis. For validation, the authors perform an application in a case study in a company of a health sector manufacturer and stress that this application can be expanded to the selection of providers in other segments.

### 2.2 Delay Time

Delay Time (DT) presents as one of the precursor's concepts [6], describing it as the interval between the arrival of the defect until the moment when the system fails. Normally, the parameter of  $h$  is assigned to this interval and  $u$  is the time of arrival of the defect, thus, if an inspection takes place exactly during this stage, the replacement or repair of the item is carried out.

Other classic studies have been discussed over time by [8], [2], and [7]. Currently, there is a wide range of studies addressing DT, as in integration between regional and global navigation satellite systems with time deviation [5]; studies of the effect of instability on the combustion of non-premixed methane-oxygen flames using a coaxial injector [3]; and stability analysis in the automatic steering system with a systematic method [39].

Still exploring the DT: [12], [40], [4], [41], [31] and others. As demonstrated, DT is widely discussed in the literature. Although several studies explore the perspective of provider selection

and DT, but the literature has not yet explored the combination of both studies with a perspective of the (MAUT).

### 3 Adopted Policy, Multicriteria Method and Application

#### 3.1 Adopted Policy

The Policy chosen to be adopted is linked to a single component for the model that involves the DT, assuming a perfect inspection and an infinite time horizon. The three possible states of the system are considered: defective when the system is operating but not in its ideal state, failure, when it is inoperative due to the arrival of the failure, and good, when the system is considered normal functional.

It should be noted that the first part of the generalized model in this work had [37] as its precursor and Weibull probability distribution, which was chosen due to its  $\beta$ -shaped parameters ( $\beta_d$  for defect and  $\beta_f$  for failure) and  $\eta$ -scaled parameters ( $\eta_d$  for defect and  $\eta_f$  for failure) suitable for modeling maintenance problems due to their great flexibility and have a probability density function *pdf* highlighted by Equation (1) and an accumulated probability density function *adf* in Equation (2), according to [17]; and therefore, this model was incorporated into the Multiattribute Utility Theory, as can be seen in the next topic. The variable  $x$  will be associated with the time horizon for our model.

$$f(x) = \frac{\beta}{\eta} \left( \frac{x}{\eta} \right)^{\beta-1} \exp \left[ - \left( \frac{x}{\eta} \right)^{\beta} \right], \text{ to } x > 0, \quad (1)$$

$$F(x) = 1 - \exp \left[ - \left( \frac{x}{\eta} \right)^{\beta} \right]. \quad (2)$$

For the construction of the model, it is essential to know the variables that will be part of the development. The time variable  $u$  represents the arrival of the defect, with a probability density function  $g(u)$  and an accumulated probability function  $G(u)$ . The DT  $h$  interval is also represented by a *pdf*  $f(h)$  and *adf*  $F(h)$ .

The  $i$ -th inspection is completed at  $t_i$  and the inspection intervals are constant and will be set to  $T$ . The item is replaced if a defect is identified in the inspection, although the failure has not yet arrived.

The  $P_b(t_{i-1}, t_i)$  is the probability of failure to renew between  $(t_{i-1}, t_i)$ .  $P_m(t_i)$  is the probability of preemptive replacement due to detection of a defect ( $t_i$ ).

The costs are defined by  $C_p$ ,  $C_f$  e  $C_i$ , respectively representing the cost of preventive maintenance, failure, and inspection. While the times are given by  $T_p$ ,  $T_f$  e  $T_i$ , respectively, the time to carry out preventive maintenance, time spent on failure, and inspection time.

An item cycle is expected to contain an outage (Down Time)  $ED(T)$  and a cost  $EC(T)$ , in a given  $T$ . And the life cycle expectation  $EL(T)$ , in this same  $T$ . The  $C(T)$  is the cost in an infinite time horizon as a function of  $T$ , and  $D(T)$  is its respective unavailability.

#### 3.2 Choice of Multicriteria Method

The chosen method to build the model revolves around the MAUT. Such a method was deemed necessary mainly because of its particularities in portraying the uncertainties of the maintenance

contract and the probabilities arising from the attributes, mostly because it involves the chance that an event will occur, in this case, in intervals that can lead to failure and defect [1, 23]. Therefore, MAUT was considered an appropriate method to portray this type of problem in maintenance management.

### 3.3 The Model Construction

For the construction of the model, it initially demonstrates the modeling of the DT that subsequently its resulting functions are incorporated into MAUT.

The probability of the failure (3) to happen between  $(t_{i-1}, t_i)$ , will be the integral of  $g(u)$  multiplied by  $F(t_i - u)$ , since the failure is assumed to occur between  $t_{i-1} < u < t_i$ :

$$P_b(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} g(u)F(t_i - u)du. \quad (3)$$

Knowing that *pdf* and *adf* of a *Weibull* are described by (1) and (2), respectively, and substituting in (3), we have (4):

$$P_b(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} \left[ \frac{\beta_d}{\eta_d} \left( \frac{u}{\eta_d} \right)^{\beta_d-1} \exp \left[ - \left( \frac{u}{\eta_d} \right)^{\beta_d} \right] \right] \left[ 1 - \exp \left[ - \left( \frac{t_i - u}{\eta_f} \right)^{\beta_f} \right] \right] du. \quad (4)$$

The probability of preemptive replacement due (5) to detection of a defect in  $t_i$  will be the integral of  $g(u)$  multiplied by  $1 - F(t_{i-1} - u)$ :

$$P_m(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} g(u)[F(t_{i-1} - u)]du. \quad (5)$$

Substituting (1) and (2) into (5), as they are respectively a *pdf* and an *adf*, you will reach (6):

$$P_m(t_{i-1}, t_i) = \int_{t_{i-1}}^{t_i} \left[ \frac{\beta_d}{\eta_d} \left( \frac{u}{\eta_d} \right)^{\beta_d-1} \exp \left[ - \left( \frac{u}{\eta_d} \right)^{\beta_d} \right] \right] \left[ 1 - \left[ 1 - \exp \left[ - \left( \frac{t_i - u}{\eta_f} \right)^{\beta_f} \right] \right] \right] du. \quad (6)$$

The expression of the expected cost of the renewal cycle (7) will depend on the sum of inspections before the one being carried out, multiplied by the cost related to the inspection, which will be added to the failure cost, all multiplied by  $P_b(t_{i-1}, t_i)$  plus  $P_m(t_i)$  multiplied by the  $i$ -th inspection, times the inspection cost, plus the failure cost. And the unavailability (8) is given in a similar way however the costs are replaced by the times and are described by, respectively:

$$EC(T) = \sum_{i=1}^{\infty} [(i-1)C_i + C_f]P_b(t_{i-1}, t_i) + (iC_i + C_p).P_m(t_i), \quad (7)$$

$$ED(T) = \sum_{i=1}^{\infty} [(i-1)T_i + T_f]P_b(t_{i-1}, t_i) + (iT_i + T_p).P_m(t_i). \quad (8)$$

The life cycle equation is given by [9]:

$$EL(T) = \sum_{i=1}^{\infty} \left[ \int_{t_{i-1}}^{t_i} \int_0^{t_{i-1}} (u+h)g(u)f(h)dhdu + t_i \cdot P_m(t_i) \right]. \quad (9)$$

Thus, the cost per unit of time (10) can be arrived at with the  $EC(T)/EL(T)$ :

$$C(T) = EC(T)/EL(T). \quad (10)$$

Analogously, however, for the downtime, which is precisely (1 - availability) per unit of time,  $ED(T)/EL(T)$ , we have (11):

$$D(T) = ED(T)/EL(T). \quad (11)$$

Equations (10) and (11) are fundamental to aggregate the individual utility function of each one and about the global utility function, from MAUT.

In the MAUT, it is known that it is plausible to first verify the individual utility and then incorporate both, to arrive at the real value of each of the alternatives/actions, which in our application will be the selection of service providers in maintenance.

[14] and [25] obtain the individual and aggregate utility obtained by the respective equations, (12) and (13).

$$U(\vartheta) = \exp(-A_i \vartheta). \quad (12)$$

As these are two attributes, and considering additive independence between them, the utility in MAUT is given by:

$$U(\vartheta_1, \vartheta_2) = K_1 \exp(-A_1 \vartheta_1) + K_2 \exp(-A_2 \vartheta_2). \quad (13)$$

where  $K_1$  e  $K_2$  are scale constants, and in this situation,  $K_1 + K_2 = 1$  and  $A_i$  are the parameters for each  $\vartheta$  is the attributes  $i$ . According to [1], in MAUT and Bayesian Decision Theory, utility is maximized by (14):

$$\max_a \sum_m U(m, a). \quad (14)$$

Knowing that  $a$  represents the action and for this case,  $m$  represents the state of nature, with the factors not being controllable. With the respective attributes, it is finally possible to arrive at the utility of each provider, applying (10) and (11) in (13):

$$U(D(T), C(T)) = K_1 \exp(-A_1 D(T)) + K_2 \exp(-A_2 C(T)). \quad (15)$$

### 3.4 Numerical Application

It is worth noting that the numerical application was hypothetical, as each company will have its parameters and because of the unavailability of real data. To verify the model behavior with a quantitative approach in a maintenance service provider selection simulation.

In this application, you want to select a maintenance service provider through an Inspection Policy using the concept of DT, in which you have three possible candidates, each with its particularities concerning the time of carrying out preventive maintenance, action time to repair the fault

and time to complete the inspection. The same is true for costs, which are the costs of carrying out preventive maintenance, the cost of a faulty repair, and the cost of the inspection.

The model portrays all probabilities as a Weibull distribution for the  $f_{dp}$  and  $f_{da}$ , knowing that the  $f_{dp}$  Weibull distribution is derived from the  $f_{da}$  Weibull. The  $\beta$ -shaped parameters arrival time of defect will assume values of 2.5 and 3.5, just like the failure arrival time, assigning all possible combinations between the  $\beta$ s defect and failure arrival time, and the scale parameter  $\eta$  will be assigned a value of 20 for the fault arrival time and 25 for the fault arrival time. It should be noted that the model is a generalist and only by submitting values consistent with the real context, concerning the disparity between the variables, it will be possible to reach the expected results.

The policy of each provider is generally treated in the contracting through a contract value, but contracts are often based on payment by several activities, in this case, the values of each activity under the provider's responsibility are considered.

On the other hand, it is in the interest of the contracting party that the actions of the provider can have positive repercussions in terms of unavailability, for that, the times that are spent on average in each activity are extremely important to determine the final unavailability.

It is observed that a hypothesis adopted is that the provider follows the optimal policy, for this, 10,000 Monte Carlo simulations were carried out to reach the optimal times ( $T$ ) for each of the providers involved. That is the policy that obtains the lowest cost values. For the scenarios, the Monte Carlo simulation reached the following results, and the respective values for the other variables were adopted:

\* Provider 1: The costs will be of  $C_p$  \$300.00 per unit,  $C_f$  will be of \$1,000.00 per unit and  $C_i$  is of \$100.00 per unit with the respective timings of  $T_p$  equals 1 hour,  $T_f$  with 8 hours and  $T_i$  representing 0.400 hours with  $T = 6.992$  hours.

\* Provider 2: The costs will be  $C_p$  \$320.00 per unit,  $C_f$  will be \$1,200.00 per unit and  $C_i$  is of \$150.00 per unit with the respective timings of  $T_p$  equals 0.700 hours,  $T_f$  with 7.000 hours, and  $T_i$  representing 0.320 hours with  $T = 8.989$  hours.

\* Provider 3: The costs will be  $C_p$  \$350.00 per unit,  $C_f$  will be of \$1,200.00 per unit and  $C_i$  is of \$100.00 per unit with the respective timings of  $T_p$  of value 0.600 hours,  $T_f$  with 7.000 hours and  $T_i$  representing 0.400 hours, with  $T = 9.988$  hours.

It is observed that higher costs were intentionally placed to perform a task that is done more efficiently in the execution of the activity. And maintainers adopt lower-cost policies according to their competencies.

Regarding the parameters involving the theory with a multicriteria approach from the perspective of the MAUT method, the scale constants  $k_1$  and  $k_2$  are assigned, the respective values of 0.55 and 0.45, respecting the axiomatic theory of MAUT with  $k_1 + k_2 = 1$  and with the presence of additive independence, which facilitates obtaining the total utility. And the constants  $A_1 = 0.01$  and  $A_2 = 0.03$ .

## 4 Results obtained and discussions

To obtain the results, the software  $\textcircled{R}$ MathCad was used as support and tabulated in  $\textcircled{R}$ Excel. For the first and third providers, their cost (expressed in dollars) was relatively close for any combination of  $\beta$ , whether a defect or failure, with average values, evidenced \$27.44 and \$28.52, respectively for Providers 1 and 2. The second provider suggests higher costs (approximate average value of \$37.85), as shown in Figure 1-(a).

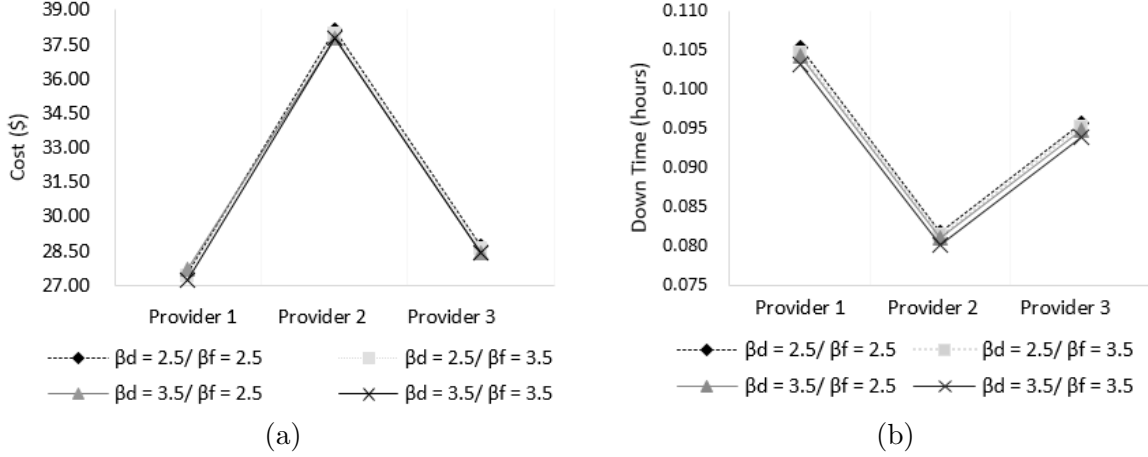


Figure 1: (a) and (b) - The image denotes the values for the cost function (\$) and Down Time (hours), respectively in (a) and (b), for each service provider of maintenance as a function of the simulated  $\beta$  parameters.

Maintenance contracts cover the costs related to this practice in a pre-defined time window, there is a fixed service fee involving the expected costs during the term of the contract, which must guarantee the profitability of the components and vary according to the characteristics inherent to the equipment [11].

When entering the unavailability analysis, the best performance for  $\beta$ 's failure and defect combination, both with values of 3.5 for Provider 2, at the lowest value, was of approximately 0.080 hours. For this provider, the average value was slightly higher than this value (See Figure 1-(b)).

The unavailability of the equipment depends on several factors, such as how the component is used, maintenance periods and policy, aging, and beyond the effects of nature (rain, temperature, and others). In the case of aging, the unavailability must be observed with caution due to the complexity of modeling increasing unavailability [35]. The loss of availability influences the drop in production, accumulation of raw materials, increase in fixed costs, idleness of operators, and can lead to the loss of customers. Additionally, good planning management and control of maintenance are essential to avoid these factors [27].

After analyzing the cost and unavailability, it was possible to calculate their respective individual utilities. From Figure 2-(a), the second provider presented the lowest utilities about cost, an average value of (0.685) for the combinations of the  $\beta$  parameters, which negatively affects the decision maker's choice, because the closer to 1 the utility, the better it will be that alternative. While Providers 1 and 3 had their values close to 0.750 to 0.760, because of presenting the lowest costs as already reported. The utility of unavailability followed a certain pattern, for Providers 1 and 3 with 0.997 utility and Provider 2 with a value of 0.998, highlighted in Figure 2-(b).

At this point, it is possible to calculate the aggregate utility  $U(C, D)$ , as highlighted in Figure 3, showing their respective values and classification. It can be noted that no matter the values of  $\beta$ , the final classification is always the same for these chosen parameters, with remarkably similar values. Thus, Provider 1 is in the first place, followed by Providers 3 and 2.

Misconceptions in the perception of MAUT-based decision systems negatively affect the use of the method and the decision space is given by the relationships between attributes [30]. However, the integration of MAUT with other methods, in our case, DT, helps to accelerate probabilistic



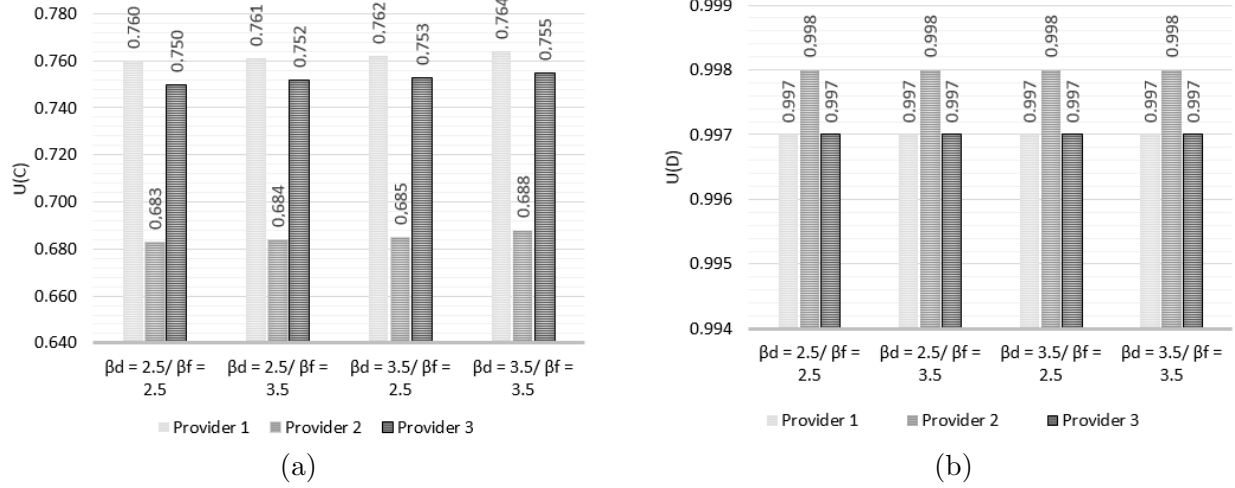


Figure 2: (a) and (b) - The image suggests the values of individual utilities as a function of cost and unavailability, respectively in (a) and (b), for each maintenance service provider concerning to  $\beta$  parameters.

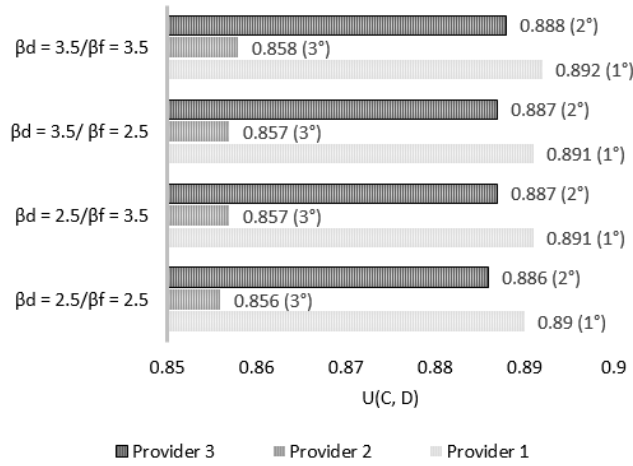


Figure 3: The aggregated utility allows you to arrive at the total value and ranking of suppliers for each combination of parameters  $\beta$ .

calculations to identify the best selection options [18]. An ideal service provider enables companies to focus on organizational strategy [38].

The evidence suggests that in the case of utility, cardinality is precisely in the difference of values and not in the absolute value of utilities. Arbitrarily choose one of the hypothetical situations, at a point in which both  $\beta$ s assume values of 2.5 for the failure and the defect, as the classification is always the same in these simulations with values of the aggregate utility quite like each other regardless of the type of  $\beta$  combination. Moreover, it was possible to calculate the difference between the first and the second place (0.004) and between the second and the third place (0.033), according to Table 1.

No model exactly addresses the selection of providers that provide maintenance services with

Ranking	Providers	Difference
1 <sup>o</sup> - 2 <sup>o</sup>	P1 - P3	0,004
2 <sup>o</sup> - 3 <sup>o</sup>	P3 - P2	0,033

Table 1: It is possible to verify the classification of service providers in the simulation that both  $\beta$ 's are equal to 2.5 and obtain the cardinality in the last column between the differences in aggregate utilities, where P1, P2 and P3, respectively represent Providers 1, 2 and 3.

a DT policy. This study can be approximated by [1] because it presents spare parts suppliers in maintenance contracts, the method used is MAUT, Weibull distribution, and two criteria ( $IT$  and  $C$ ). Even some parameters are similar ( $A_1 = 0.05$ ,  $A_2 = 0.01$ , and  $\beta = 2.5$ ), by varying these parameters and the others, to verify the behavior of the model in a fictitious way. Finally, regarding the interruption time considered by [1], it only considers the time after the equipment stops, on the other hand, we were able to capture in one stage before the equipment collapses, which results in better higher utilities.

[12] addresses another approximate study that reports on the two criteria  $D$ ,  $C$  and which also used MAUT, but does not focus on maintenance contracts and even so it reaches satisfactory results applying the model in electric power distribution.

## 5 Conclusion

It can be observed that the multicriteria decision model with a MAUT approach has proven appropriate for the selection of maintenance services providers in a system that has the DT in its industrial policy, directly impacting the quality of the provided service. The model considered two criteria/attributes as being cost and equipment unavailability, aware that the lower these numbers the better for the organization's competitiveness since it will save the company's capital and increase the equipment use efficiency. The results show how robust the model is through variations of defect and failure since a Weibull-based probability of distribution was adopted.

There is a need to investigate the benefits and effects of service provider management on aspects of total costs, variability in service delivery to identify the most suitable providers and service charges. Thus, it is of interest to carry out a practical application of the developed model to measure these real gains that the developed model brings to an organization.

From the perspective of future work, it is mandatory to incorporate more criteria and applications with similar criteria to compare the results obtained. Another factor to be evaluated later is the modification of parameters and in different probability distributions.

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## References

- [1] A. T. Almeida. Multicriteria decision making on maintenance: Spares and contracts planning. *European Journal of Operational Research*, pages 235–241, 2001. doi: [10.1016/S0377-2217\(00\)00220-4](https://doi.org/10.1016/S0377-2217(00)00220-4).

- [2] T. Aven and B. Bergman. Optimal replacement times a general set-up. *Journal of Applied Probability*, 23:432–442, 1986.
- [3] J. Bae, S. Jeong, and Y. Yoon. Effect of delay time on the combustion instability in a single-element combustor. *Acta Astronautica*, 178:783–792, 2020. doi: [10.1016/j.actaastro.2020.10.004](https://doi.org/10.1016/j.actaastro.2020.10.004).
- [4] R. J. I. Basten and J. K. Ryan. The value of maintenance delay flexibility for improved spare parts inventory management. *European Journal of Operational Research*, 278(3):646–657, 2019. doi: [10.1016/j.ejor.2019.04.023](https://doi.org/10.1016/j.ejor.2019.04.023).
- [5] X. Cao, F. Shen, S. Zhang, and J. Li. Time delay bias between the second and third generation of beidou navigation satellite system and its effect on precise point positioning. *Measurement*, page 108346, 2020. doi: [10.1016/j.measurement.2020.108346](https://doi.org/10.1016/j.measurement.2020.108346).
- [6] A. H. Christer. Modelling inspection policies for building maintenance. *Journal of the Operational Research Society*, page 723–732, 1982.
- [7] A. H. Christer and S. K. Lee. Modelling ship operational reliability over a mission under regular inspections. *Journal of the Operational Research Society*, 48(7):688–699, 1997.
- [8] A. H. Christer and D. F. Redmond. A recent mathematical development in maintenance theory. *IMA Journal of Management Mathematics*, 2(2):97–108, 1998.
- [9] A. H. Christer and W. M. Waller. Delay time model so industrial inspection maintenance problems. *Journal of the Operational Research Society*, 35(5):401–406, 1984.
- [10] Q. Deng, L. Zhang, Q. Cui, and X. Jiang. A simulation-based decision model for designing contract period in building energy performance contracting. *Building and Environment*, pages 71–80, 2014. doi: [10.1016/j.buildenv.2013.09.010](https://doi.org/10.1016/j.buildenv.2013.09.010).
- [11] L. Deprez, K. Antonio, and R. Boute. Pricing service maintenance contracts using predictive analytics. *European Journal of Operational Research*, 290(2):530–545, 2021. doi: [10.1016/j.ejor.2020.08.022](https://doi.org/10.1016/j.ejor.2020.08.022).
- [12] R. J. P. Ferreira, A. T. Almeida, and C. A. A. Cavalcante. A multi-criteria decision model to determine inspection intervals of condition monitoring based on delay time analysis. *Reliability Engineering and System Safety*, page 905–912, 2009. doi: [10.1016/j.ress.2008.10.001](https://doi.org/10.1016/j.ress.2008.10.001).
- [13] T. Jin, Z. Tian, and M. Xie. A game-theoretical approach for optimizing maintenance spares and service capacity in performance contracting. *Int. J. Production Economics*, page 31–43, 2015. doi: [10.1016/j.ijpe.2014.11.010](https://doi.org/10.1016/j.ijpe.2014.11.010).
- [14] R. L. Keeney and H. Raiffa. Decisions with multiple objectives: Preferences and value trade-offs. *Wiley: New York*, 1979.
- [15] G. Liu, S. Chen, H. Jin, and S. Lui. Optimum opportunistic maintenance schedule incorporating delay time theory with imperfect maintenance. *Reliability Engineering & System Safety*, 213:107668, 2021. doi: [10.1016/j.ress.2021.107668](https://doi.org/10.1016/j.ress.2021.107668).

- [16] D. Lugtigheid, D. A. K. S. Jardine, and X. Jiang. Optimizing the performance of a repairable system under a maintenance and repair contract. *Quality and Reliability Engineering International*, 23:943–960, 2007.
- [17] D. Montgomery. Design and analysis of experiment. In *7th Ed. New York: John Wiley Sons Inc*, 2009.
- [18] E. O. B. Nara, D. C. Sordi, J. L. Schaefer, J. N. C. Schreiber, M. A. Sellitto, and J. C. Furtado. Prioritization of ohs key performance indicators that affecting business competitiveness – a demonstration based on maut and neural networks. *Safety Science*, 118:826–834, 2019. doi: [10.1016/j.ssci.2019.06.017](https://doi.org/10.1016/j.ssci.2019.06.017).
- [19] J. V. Neumann and O. Morgenstern. Theory of games and economic behavior. In *Princeton University Press, Princeton*, 1947.
- [20] K. Odolinski and H. E. Boysen. Railway line capacity utilization and its impact on maintenance costs. *Journal of Rail Transport Planning & Management*, 9:22–33, 2019. doi:[10.1016/j.jrtpm.2018.12.001](https://doi.org/10.1016/j.jrtpm.2018.12.001).
- [21] C. D. Oosterom, D. C. Elwany, and G. J. Houtum. Optimal policies for a delay time model with postponed replacement. *European Journal of Operational Research*, page 186–197, 2014. doi: [10.1016/j.ejor.2013.06.038](https://doi.org/10.1016/j.ejor.2013.06.038).
- [22] W. Ren, K. Wu, Q. Gu, and Y. Hu. Intelligent decision making for service providers selection in maintenance service network: An adaptive fuzzy-neuro approach. *Knowledge-Based Systems*, page 105263, 2019. doi: [10.1016/j.knosys.2019.105263](https://doi.org/10.1016/j.knosys.2019.105263).
- [23] P. H. Ritchken and C. S. Tapiero. Warranty design under buyer and seller risk aversion. *Naval Research Logistic Quarterly*, 33(4):657–671, 1986.
- [24] S. A. Ross. The economic theory of agency: The principal’s problem. *The American Economic Review*, 63(2):134–139, 1973.
- [25] B. Roy. Multicriteria methodology for decision aiding. In *Kluwer Academic Publishers*, 1996.
- [26] A. Sanayei, S. M. Farid, M. R. Abdi, and A. Mohaghar. An integrated group decision-making process for supplier selection and order allocation using multi-attribute utility theory and linear programming. *Journal of the Franklin Institute*, 345(7):731–747, 2008. doi: [10.1016/j.jfranklin.2008.03.005](https://doi.org/10.1016/j.jfranklin.2008.03.005).
- [27] A. M. Sanches, L. J. Souza, S. E. G. Costa, and E. P. A. Lima. Proposal for the support of demand required from production through the alignment of production planning and control strategies and maintenance planning and control: An analytical approach. *Procedia Manufacturing*, 39:868–876, 2019. doi: [10.1016/j.promfg.2020.01.409](https://doi.org/10.1016/j.promfg.2020.01.409).
- [28] R. K. Sarin. Multi-attribute utility theory. In *Gass S.I., Fu M.C. Encyclopedia of Operations Research and Management Science, Boston, MA*. Springer, 2013.
- [29] F. A. Scarf and C. A. V. Cavalcante. Modelling quality in replacement and inspection maintenance. *Int. J. Production Economics*, page 372–381, 2012. doi: [10.1016/j.ijpe.2011.08.011](https://doi.org/10.1016/j.ijpe.2011.08.011).

- [30] M. Scholz, M. Franz, and O. Hinz. Effects of decision space information on maut-based systems that support purchase decision processes. *Decision Support Systems*, 97:43–57, 2017. doi: [10.1016/j.dss.2017.03.004](https://doi.org/10.1016/j.dss.2017.03.004).
- [31] F. Sun, X. Liao, and J. Kurths. Mean-square consensus for heterogeneous multi-agent systems with probabilistic time delay. *Information Sciences*, 543:112–124, 2021. doi: [10.1016/j.ins.2020.07.021](https://doi.org/10.1016/j.ins.2020.07.021).
- [32] L. Tu, L. Zhang, and X. Cao. Logistics service provider selection decision making for healthcare industry based on a novel weighted density-based hierarchical clustering. *Advanced Engineering Informatics*, 48:101301, 2021. doi: [10.1016/j.aei.2021.101301](https://doi.org/10.1016/j.aei.2021.101301).
- [33] P. Vincke. Multicriteria decision-aid. In *New York: John Wiley*, 1992.
- [34] W. Wang. An overview of the recent advances in delay-time-based maintenance modelling. *Reliability Engineering and System Safety*, page 165–178, 2012. doi: [10.1016/j.ress.2012.04.004](https://doi.org/10.1016/j.ress.2012.04.004).
- [35] J. A. M. V. D. Weide and M. D. Pandey. A stochastic alternating renewal process model for unavailability analysis of standby safety equipment. *Reliability Engineering System Safety*, 139:97–104, 2015. doi: [10.1016/j.ress.2015.03.005](https://doi.org/10.1016/j.ress.2015.03.005).
- [36] J. M. White. Contracted services. *Security Risk Assessment*, pages 171–181, 2014.
- [37] C. U. I. Xi. Delay time modelling and software development. In *Doctoral dissertation, University of Salford*, 2002.
- [38] M. Yazdani, A. Mohammed, C. Bai, and A. A. Labib. A novel hesitant-fuzzy-based group decision approach for outsourcing risk. *Expert Systems with Applications*, 5:115517, 2021. doi: [10.1016/j.eswa.2021.115517](https://doi.org/10.1016/j.eswa.2021.115517).
- [39] Q. Ye, R. Wang, Y. Cai, and M. Chadli. The stability and accuracy analysis of automatic steering system with time delay. *ISA Transactions*, 14:278–286, 2020. doi: [10.1016/j.isatra.2020.04.004](https://doi.org/10.1016/j.isatra.2020.04.004).
- [40] X. Zhao, Z. He, and M. Xie. Optimal condition-based maintenance policy with delay for systems subject to competing failures under continuous monitoring. *Computers Industrial Engineering*, 124:535–544, 2018. doi: [10.1016/j.cie.2018.08.006](https://doi.org/10.1016/j.cie.2018.08.006).
- [41] J. Zhu, Q. Cheng, H. Fan, Z. Han, R. Hou, and C. T. Li. Transient delay-period activity of agranular insular cortex controls working memory maintenance in learning novel tasks. *Neuron*, 105(5):934–946.e5, 2020. doi: [10.1016/j.neuron.2019.12.008](https://doi.org/10.1016/j.neuron.2019.12.008).