Introduction to Fuzzy

Diambil dari materi kuliah oleh:
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Motivation

- The term “fuzzy logic” refers to a logic of approximation.
- Boolean logic assumes that every fact is either entirely true or false.
- Fuzzy logic allows for varying degrees of truth.
- Computers can apply this logic to represent vague and imprecise ideas, such as “hot”, “tall” or “balding”.

History

- The precision of mathematics owes its success in large part to the efforts of Aristotle and the philosophers who preceded him.
- Their efforts led to a concise theory of logic and mathematics.
- The “Law of the Excluded Middle,” states that every proposition must either be True or False.
- There were strong and immediate objections. For example, Heraclitus proposed that things could be simultaneously True and not True.

History

- Plato laid a foundation for what would become fuzzy logic, indicating that there was a third region (beyond True and False) where these opposites “tumbled about.”
- The modern philosophers, Hegel, Marx, and Engels, echoed this sentiment.
- Lukasiewicz proposed a systematic alternative to the bi-valued logic of Aristotle.

History

- In the early 1900's, Lukasiewicz described a three-valued logic. The third value can be translated as the term "possible," and he assigned it a numeric value between True and False.
- Later, he explored four-valued logics, five-valued logics, and declared that in principle there was nothing to prevent the derivation of an infinite-valued logic.

History

- Knuth proposed a three-valued logic similar to Lukasiewicz's.
- He speculated that mathematics would become even more elegant than in traditional bi-valued logic.
- His insight was to use the integral range `[-1, 0 +1]` rather than `[0, 1, 2]`.

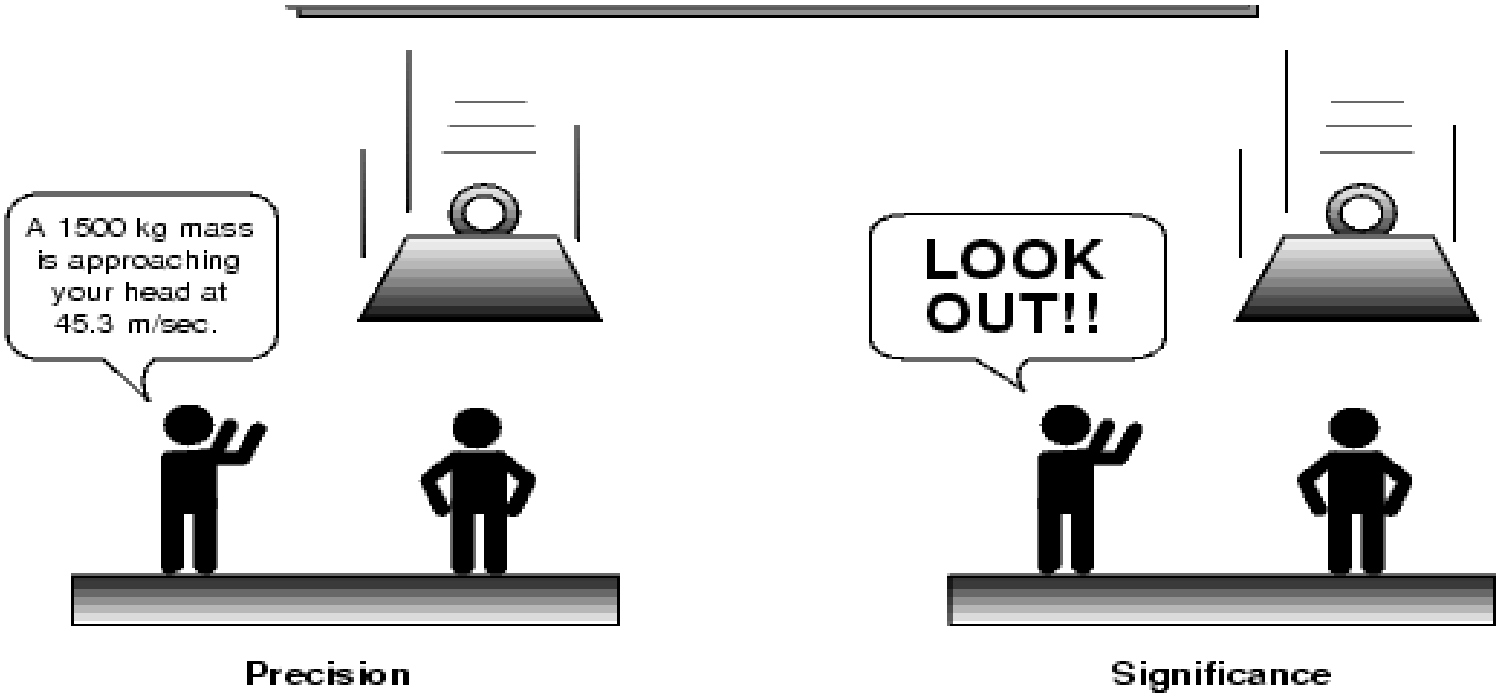
History

- Lotfi Zadeh, at the University of California at Berkeley, first presented fuzzy logic in the mid-1960's.
- Zadeh developed fuzzy logic as a way of processing data. Instead of requiring a data element to be either a member or non-member of a set, he introduced the idea of partial set membership.
- In 1974 Mamdani and Assilian used fuzzy logic to regulate a steam engine.
- In 1985 researchers at Bell laboratories developed the first fuzzy logic chip.

The World is Vague

- Natural language employs many vague and imprecise concepts.
- Translating such statements into more precise language removes some of their semantic value. The statement “Dan has 100,035 hairs on his head” does not explicitly state that he is balding, nor does “Dan’s head hair count is 1.6 standard deviations below the mean head hair count for people of his genetic pool”.
- Suppose Dan were actually only 1.559999999 standard deviations below the mean? How does one determine his genetic pool?

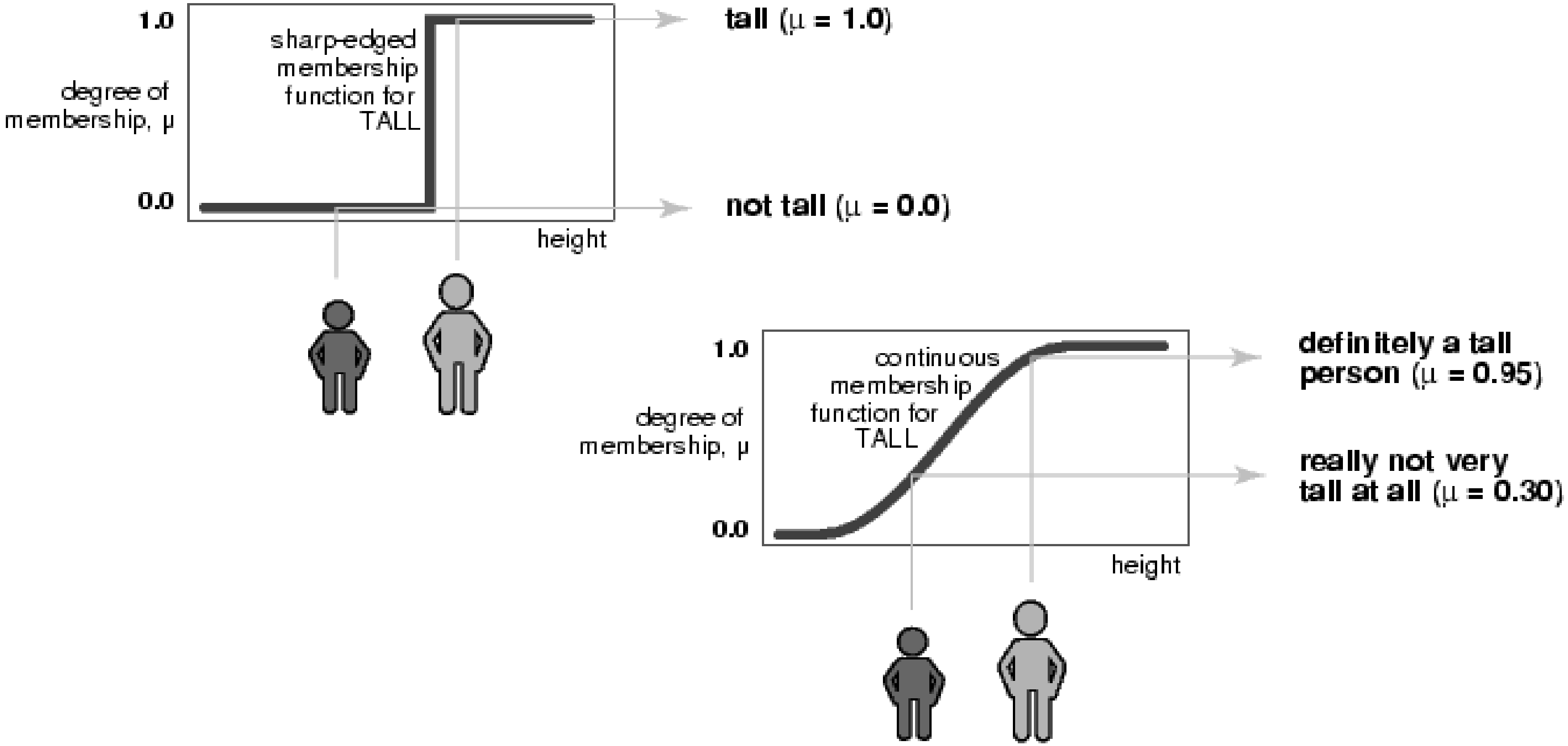
Precision and Significance



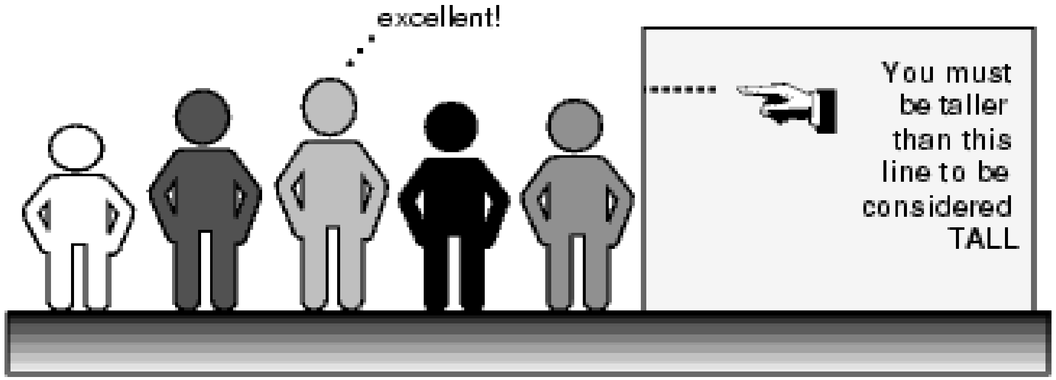
Eliminate the Vague?

- It might be argued that vagueness is an obstacle to clarity of meaning.
- But there does seem to be a loss of expressiveness when statements like, “Dan is balding” are eliminated from the language.
- This is what happens when natural language is translated into classic logic. The loss is not severe for accounting programs or computational mathematics programs, but will appear when the programming task turns to issues of queries and knowledge.

What Is Lost……


1.0
sharp-edged
membership
function for
TALL
degree of
membership, μ
0.0
height
tall (μ = 1.0)
not tall (μ = 0.0)
1.0
continuous
membership
function for
TALL
degree of
membership, μ
0.0
height
definitely a tall
person (μ = 0.95)
really not very
tall at all (μ = 0.30)

Could Be Significant



Experts are Vague

- To design an expert system a major task is to codify the expert's decision-making process.
- In a domain there may be precise, scientific tests and measurements that are used in a "fuzzy", intuitive manner to evaluate results, symptoms, relationships, causes, or remedies.
- While some of the decisions and calculations could be done using traditional logic, fuzzy systems afford a broader, richer field of data and manipulations than do more traditional methods.

I. INTRODUCTION

- Fuzzy sets are sets that have gradations of belonging
  EXAMPLES:
  Green
  BIG
  Near

  2+2=5 for very large 2
- Classical sets, either an element belongs or it does not
  EXAMPLES:
  Set of integers – a real number is an integer or not
  You are either in an airplane or not
  Your bank account has x dollars and y cents
  "There is no such thing as *fuzzy* mathematics" (Bush to Gore during the 2000 campaign)

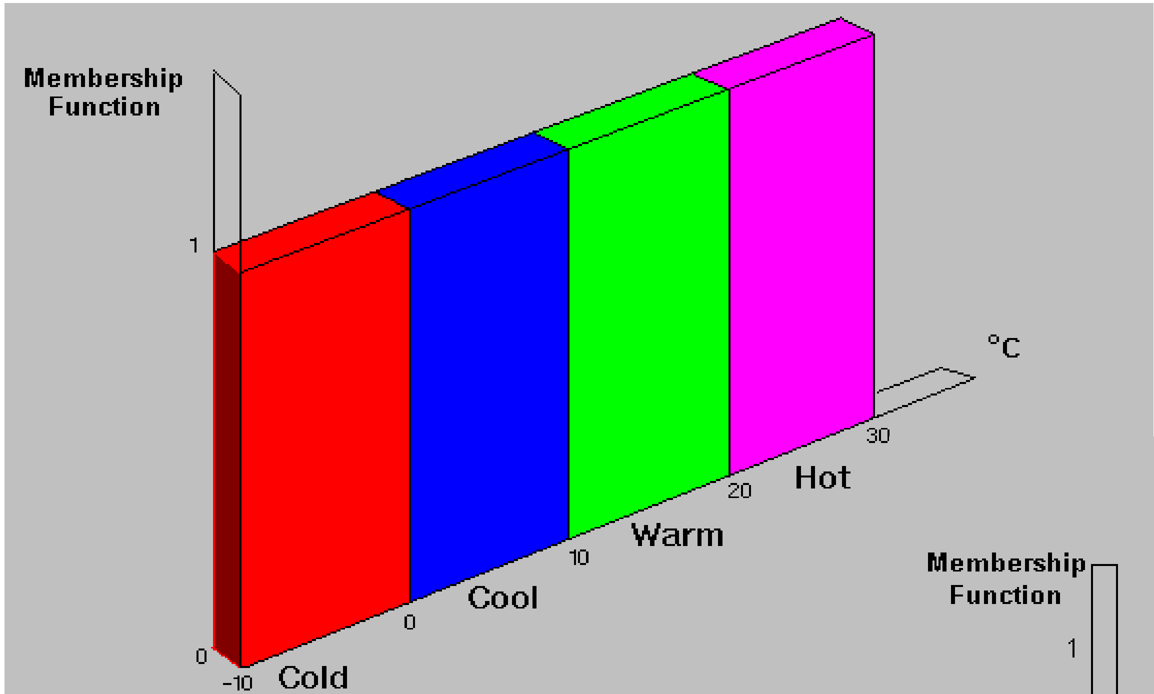
Bivalence

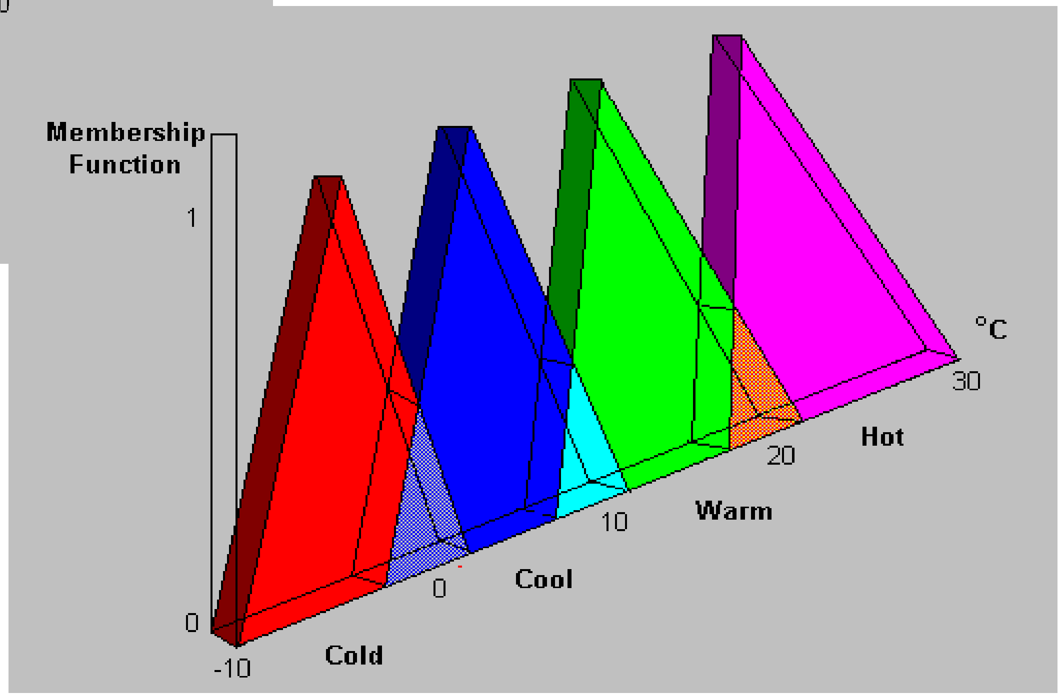
- Boolean logic assumes that every element is either a member or a non-member of a given set (never both). This imposes an inherent restriction on the representation of imprecise concepts.
- For example at 100° F a room is "hot" and at 25° F it is "cold".
- If the room temperature is 75° F, it is much more difficult to classify the temperature as "hot" or "cold".
- Boolean logic does not provide the means to identify an intermediate value.

Introduce Fuzziness

- Fuzzy logic extends Boolean logic to handle the expression of vague concepts.
- To express imprecision quantitatively, a set membership function maps elements to real values between zero and one (inclusive). The value indicates the “degree” to which an element belongs to a set.
- A fuzzy logic representation for the “hotness” of a room, would assign100° F a membership value of one and 25° F a membership value of zero. 75° F would have a membership value between zero and one.

Bivalence and Fuzz





Being Fuzzy

- For fuzzy systems, truth values (fuzzy logic) or membership values (fuzzy sets) are in the range [0.0, 1.0], with 0.0 representing absolute falseness and 1.0 representing absolute truth.
- For example,

"Dan is balding."

If Dan's hair count is 0.3 times the average hair count, we might assign the statement the truth value of 0.8. The statement could be translated into set terminology as:

"Dan is a member of the set of balding people."

This statement would be rendered symbolically with fuzzy sets as:

mBALDING(Dan) = 0.8

where m is the membership function, operating on the fuzzy set of balding people and returns a value between 0.0 and 1.0.

Fuzzy Is Not Probability

- Fuzzy systems and probability operate over the same numeric range.
- The probabilistic approach yields the natural-language statement, “There is an 80% chance that Dan is balding.” The fuzzy terminology corresponds to “Dan's degree of membership within the set of balding people is 0.80.”

Fuzzy Is Not Probability

- The probability view assumes that Dan is or is not balding (the Law of the Excluded Middle) and that we only have an 80% chance of knowing which set he is in.
- Fuzzy supposes that Dan is “more or less” balding, corresponding to the value of 0.80.
- Confidence factors also assume that Dan is or is not balding. The confidence factor simply indicates how confident, how sure, one is that he is in one or the other group.

A. Why fuzzy sets?

- Modeling with uncertainty requires more than probability theory
- There are problems where boundaries are gradual

  EXAMPLES:

  * What is the boundary of the USA? Is the boundary a mathematical curve? What is the area of USA? Is the area a real number? What is the contour/edge of a liver, tumor?
  * Where does a tumor, where is a liver, where is a spleen?
  * What is the habitat of rabbits in 20km radius from here?
  * What is the depth of the ocean 30 km east of Myrtle Beach?
- Example application
  1. **Data reduction** – driving a car, computing with language,
  2. **Control and fuzzy logic**
     * Appliances, automatic gear shifting in a car
     * Subway system in Sendai, Japan (control outperformed humans in giving smoother rides)
     * Elevators, approximation of functions

Temperature control in NASA space shuttles

IF x AND y THEN z is A

IF x IS Y THEN z is A … etc.

If the temperature is hot and increasing very fast then air conditioner fan is (set to) very fast and temperature is (set to) coldest. There are four types of fuzzy propositions we will study later.

3. **Pattern recognition, cluster analysis**

- A bank that issues credit cards wants to discover whether or not the spending pattern indicates that it is stolen or being illegally used prior to a customer reporting it missing
- Given a cat-scan determine the organs and their position; given a satellite imagine, classify the land/cover use
- Given a mobile telephone, send the signal to/from a particular receiver to/from the telephone

4. **Decision making**

- Locate mobile telephone receptors/transmitters to optimally cover a given area
- Locate recycling bins (tempat penyimpanan) to optimally cover UCD
- Position a satellite to cover the most number of mobile phone users
- Deliver sufficient radiation to a tumor to kill the cancerous cells while at that same time sparing healthy cells (maximize dosage up to a limit at the tumor while minimizing dosage at all other cells)
- Design a product in the following way: I want the product to be very light, very strong, last a very long, time and the cost of production is the cheapest.

B. Types of Uncertainty

1. **Deterministic** – the difference between a known real number value and its approximation is a real number (a single number). Here one has **error**. For example, if we know the answer x must be the square root of 2 and we have an approximation y, then the error is exactly, with certainty, x-y (or if you wish, y-x).
2. **Interval** – uncertainty is an interval. For example, measuring , pi, using Archimedes' approach. Two interpretations – unknown value is somewhere in the interval, unknown distribution with known support being the interval [a,b]
3. **Probabilistic** – uncertainty is modeled by a probability distribution function depending on the frequency, uncertainty is frequency
4. **Fuzzy** – uncertainty is modeled by a fuzzy membership function, uncertainty is gradual
5. **Possibilistic** - uncertainty is modeled by a possibility distribution function, generated by nested sets (monotone), uncertainty is information deficiency
6. **Random sets** – uncertainty in the actual probability distribution resulting in a set-valued random variable (the elements of the domain are sets, rather than a singletons) such that the sum of the distribution over the domain set-values over the sets is 1.

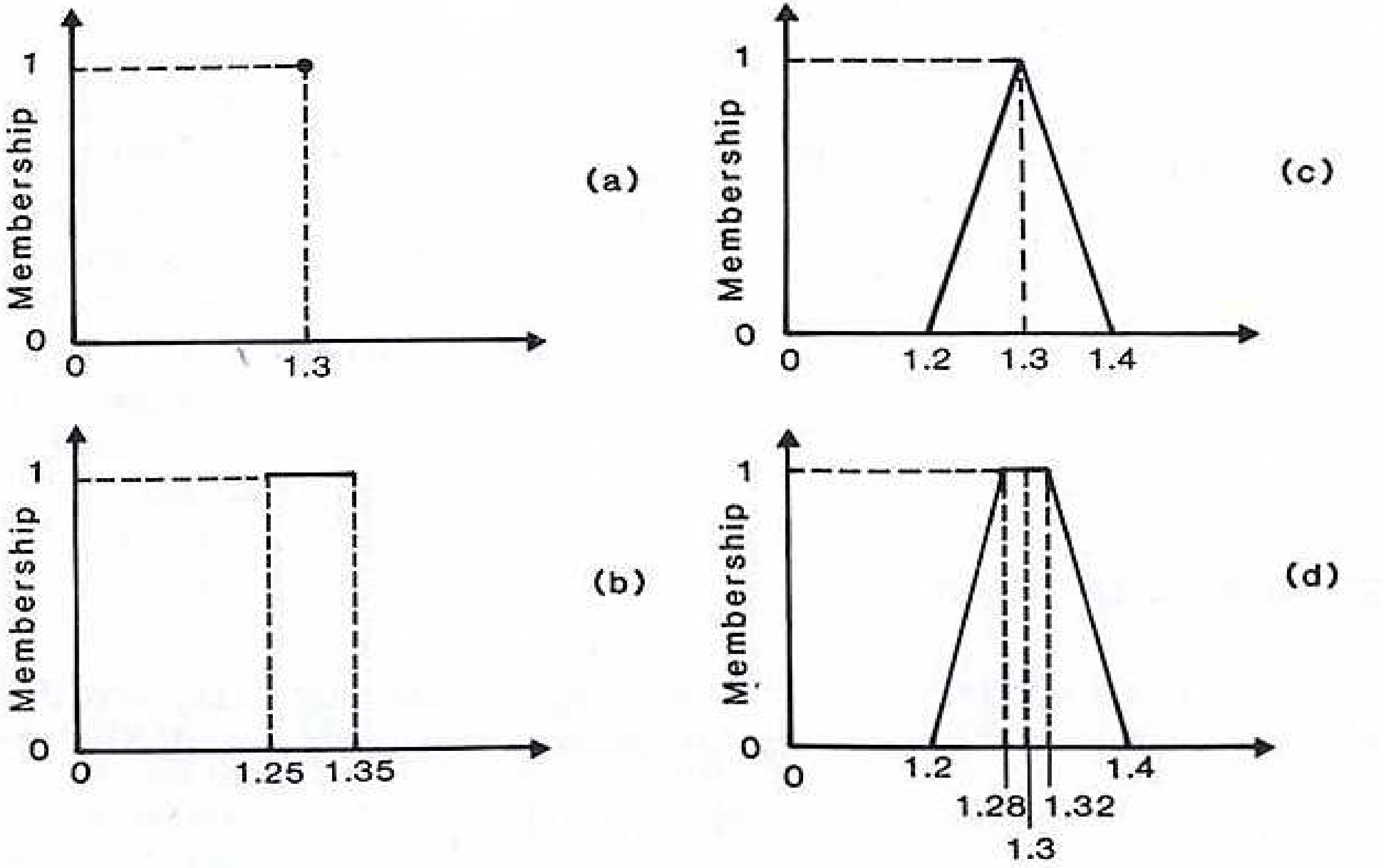
REMARK

A deterministic set of relationships models the ***actual system.***

A fuzzy set of relationships models the ***knowledge about the system*** (not the system itself).

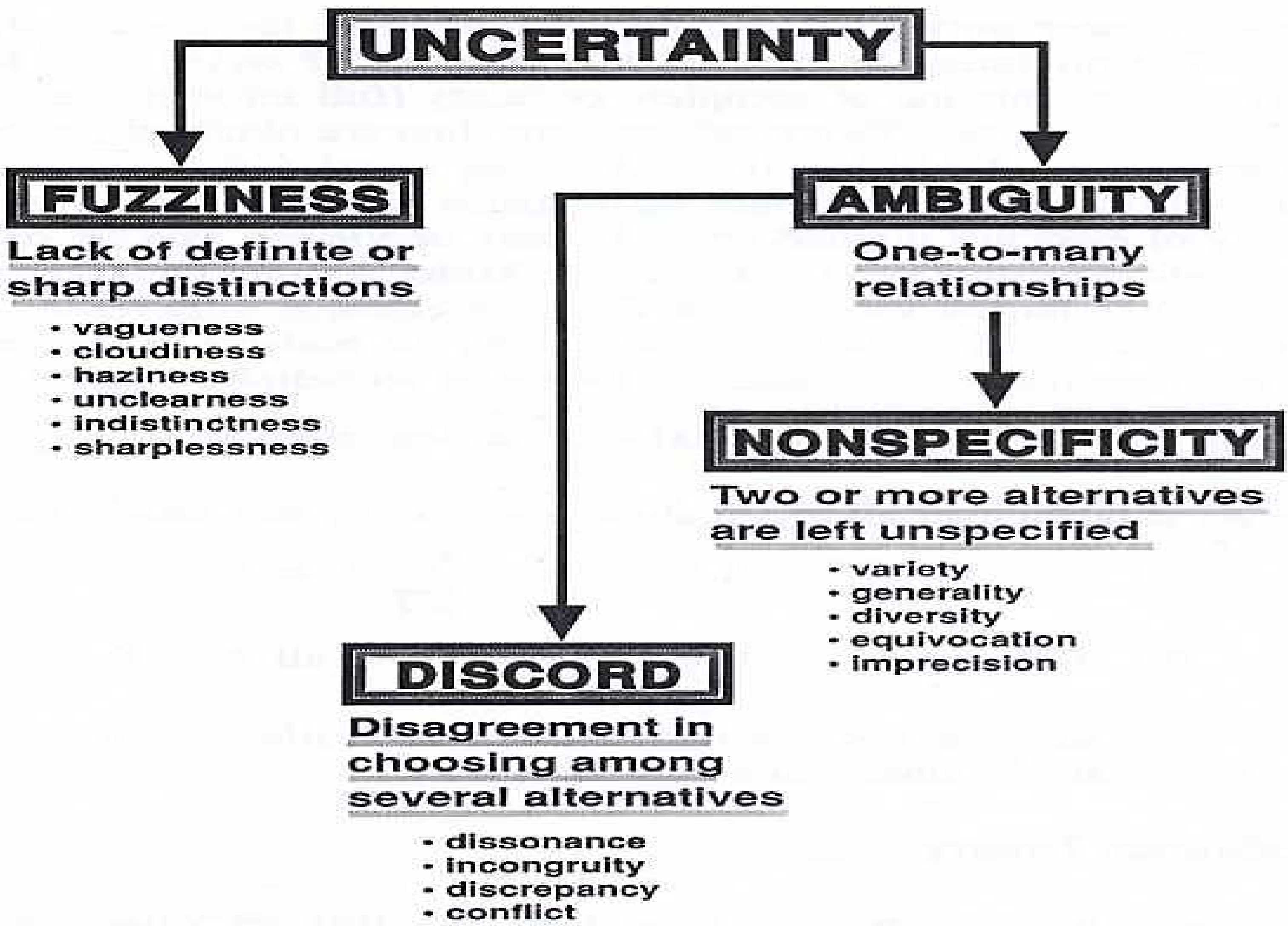
For example, a deterministic model of an automatic shifting mechanism on a car models the actual shifting. A fuzzy set model would model our knowledge of when to shift and into what gear. In the end, there is a “deterministic” decision (shift into a particular gear), but a fuzzy model gathers all the knowledge and then makes a decision. A deterministic model, models the actual system so its output is the decision.

Some types of (uncertainty sets (figure from Klir&Yuan)



A comparison of a real number and a crisp interval with a fuzzy number and a fuzzy interval, respectively.

(figure from Klir&Yuan)



Three basic types of uncertainty.

AMBIGUITY: a one to many relationship; for example, she is tall, he is rich. There are a variety of alternatives

1. **Non-specificity:** Suppose one has a heart blockage and is prescribed a **treatment**. In this case “treatment” is a non-specificity in that it can be an angioplasty, medication, surgery (to name three alternatives)
2. **Dissonance/contradiction:** One physician says to operate and another says go to Tahiti. I would go to Tahiti, of course.

VAGUENESS – lack of sharp distinction or boundaries, our ability to discriminate between different states of an event, undecidability (is a glass half full/empty)

SET THEORY

PROBABILITY

POSSIBILITY THEORY

FUZZY SET THEORY

DEMPSTER/SHAFER THEORY – Random Sets

Example 1: Tejo (Portugal) River

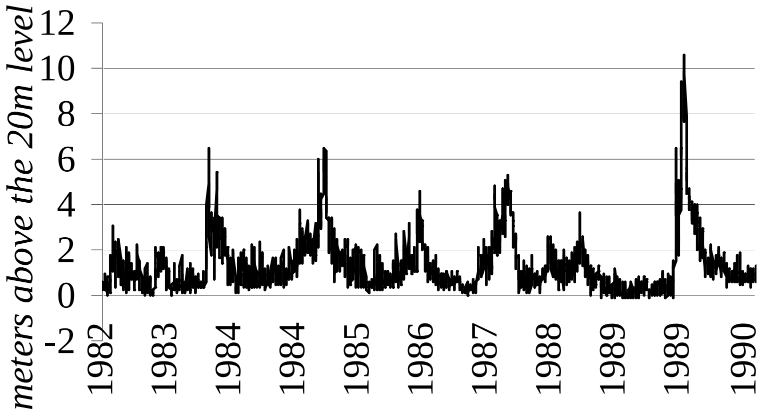
The dimension of water bodies, and consequently their position, is subject to variation over time, especially in regions which are frequently flooded or subject to tidal variations, creating considerable uncertainty in positioning these geographical entities. River Tejo is an example, since frequent floods occur in several places along its bed. The region near the village of Constância, where rivers Tejo and Zezere meet, is such a place.

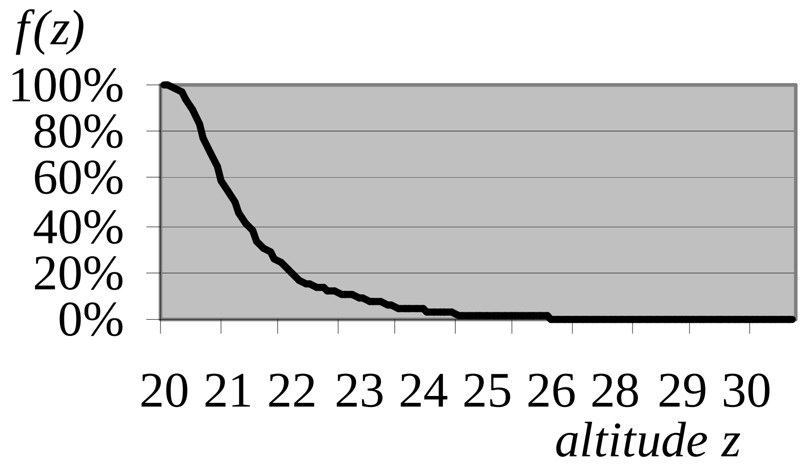
A fuzzy geographical entity corresponding to rivers Tejo and Zezere is considered a fuzzy set. To generate this fuzzy entity, the membership function has to be constructed. This was done using a Digital Elevation Model of the region, created from the contours of the 1:25 000 map of the Army Geographical Institute of Portugal and information regarding the daily means of the river water level registered in the hydrometric station of Almourol, located in the vicinity, from 1982 to 1990. The variation of the water level during these year are on the next slide:

Example 1 (figures from Cidalia Fonte & Lodwick)

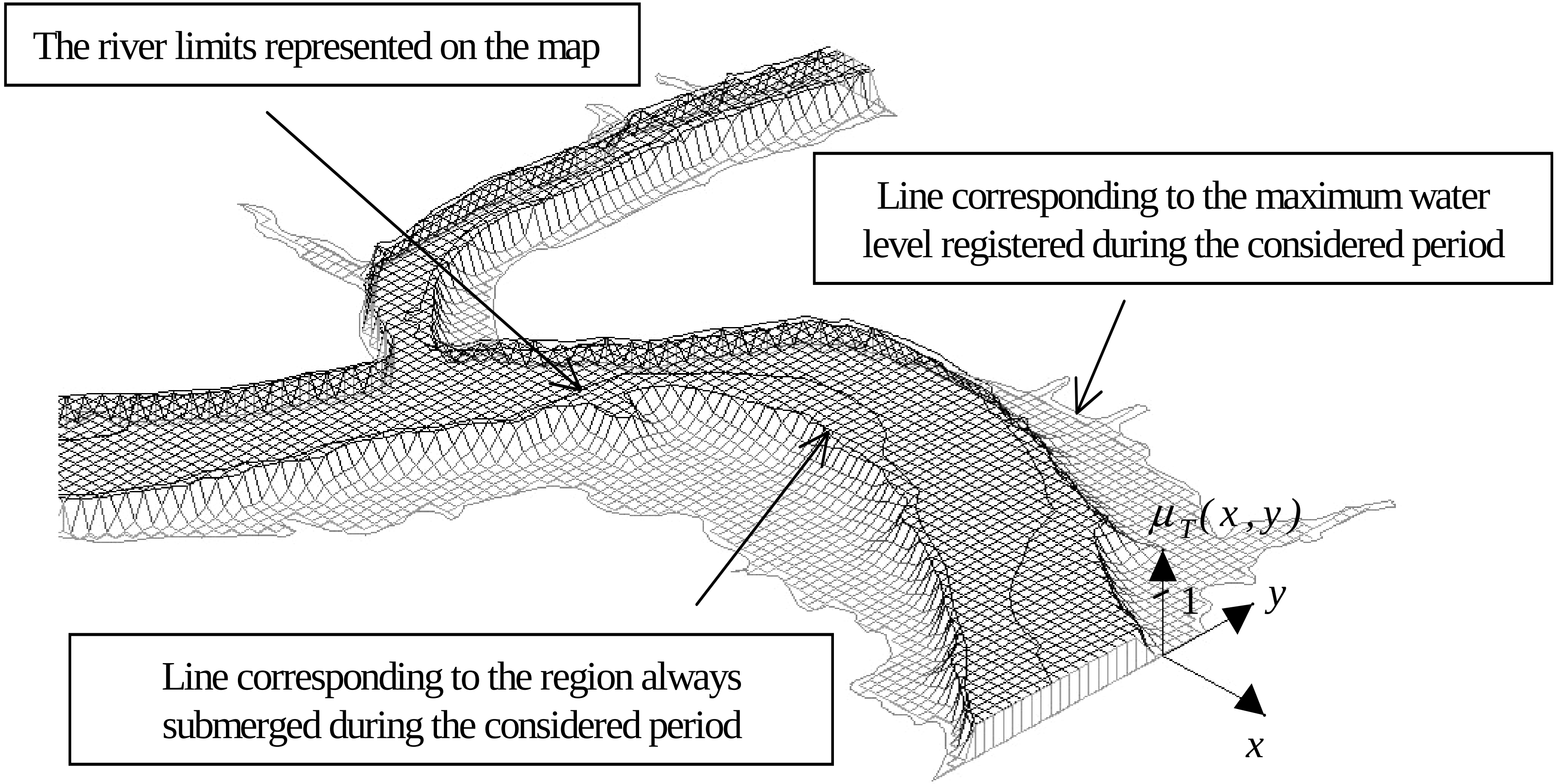
The membership function of points to the fuzzy set is given by:

$$\mu_T(x,y)=\frac{f[z(x,y)]}{100}$$

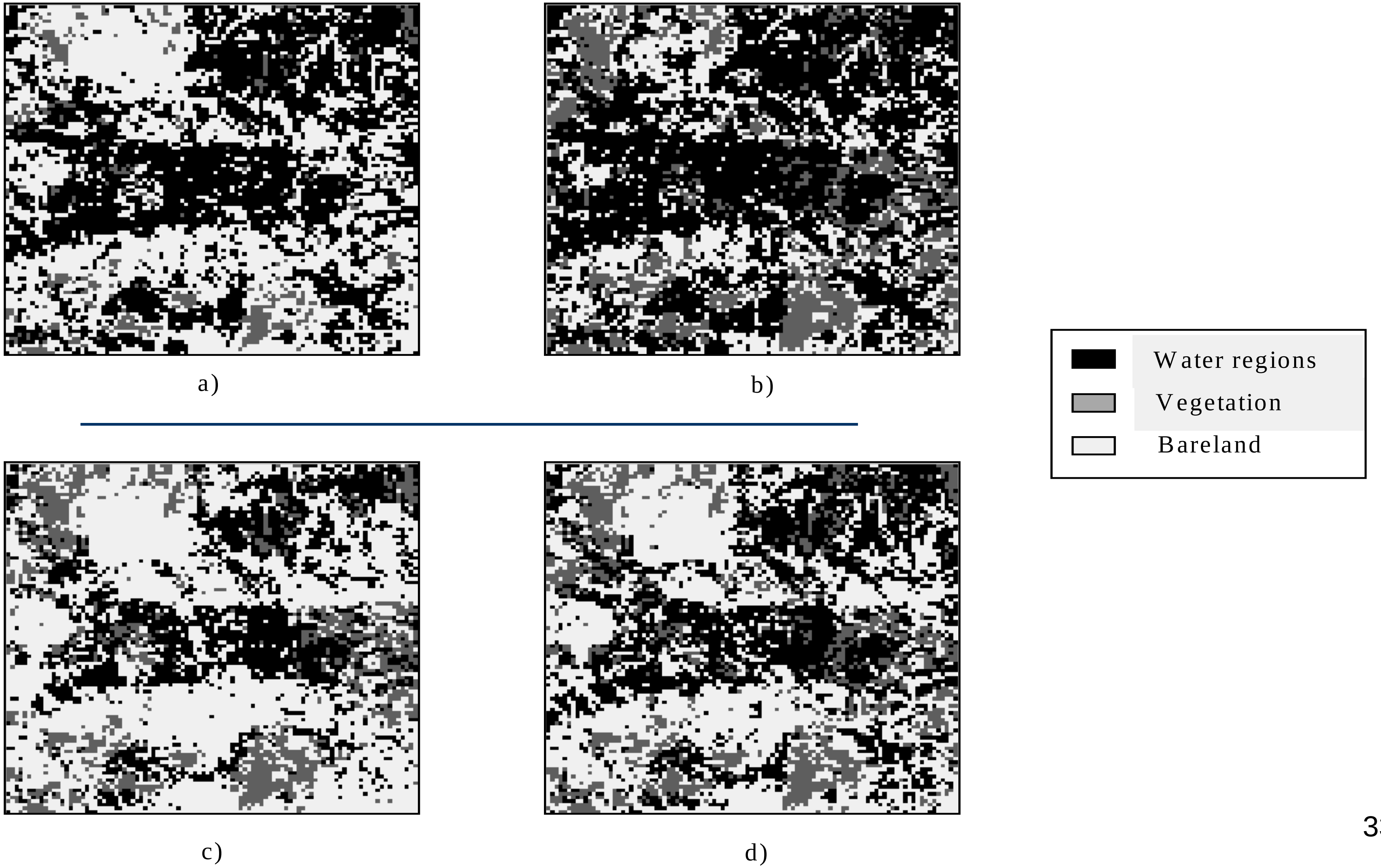




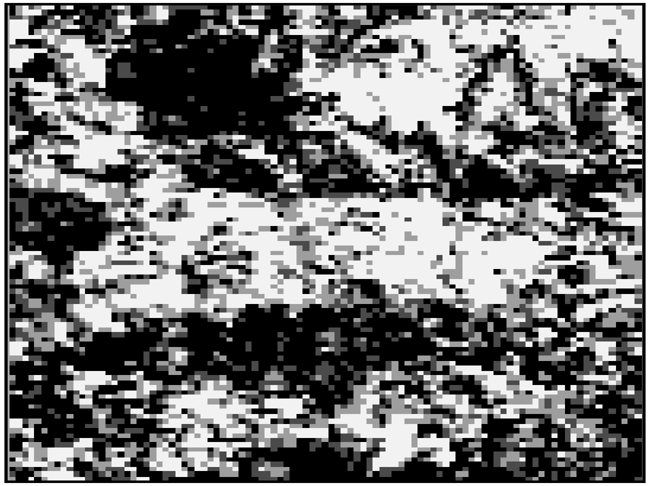
Example 1 (figure from Cidalia Fonte & Lodwick)



Example 2 – Landcover/use
(figures from Cidalia Fonte & Lodwick)

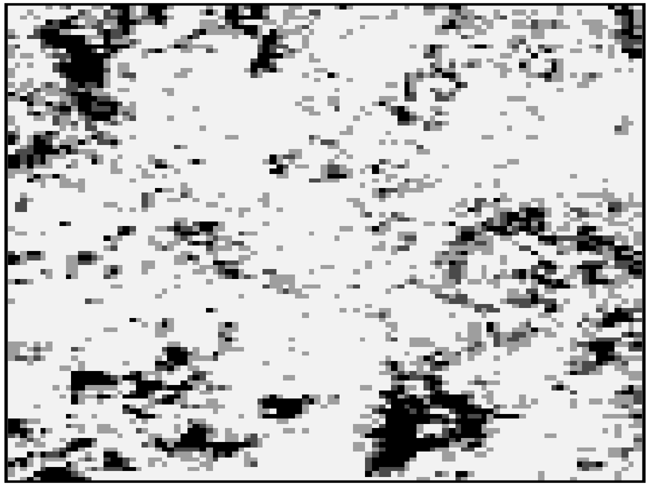


Example 2 – Landcover/use continued

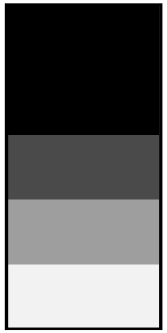


$\mu_{Bareland}(x,y)$

$\mu_{Water\ regions}(x,y)$



$\mu_{Vegetation}(x,y)$



$\mu(x,y) = 1$

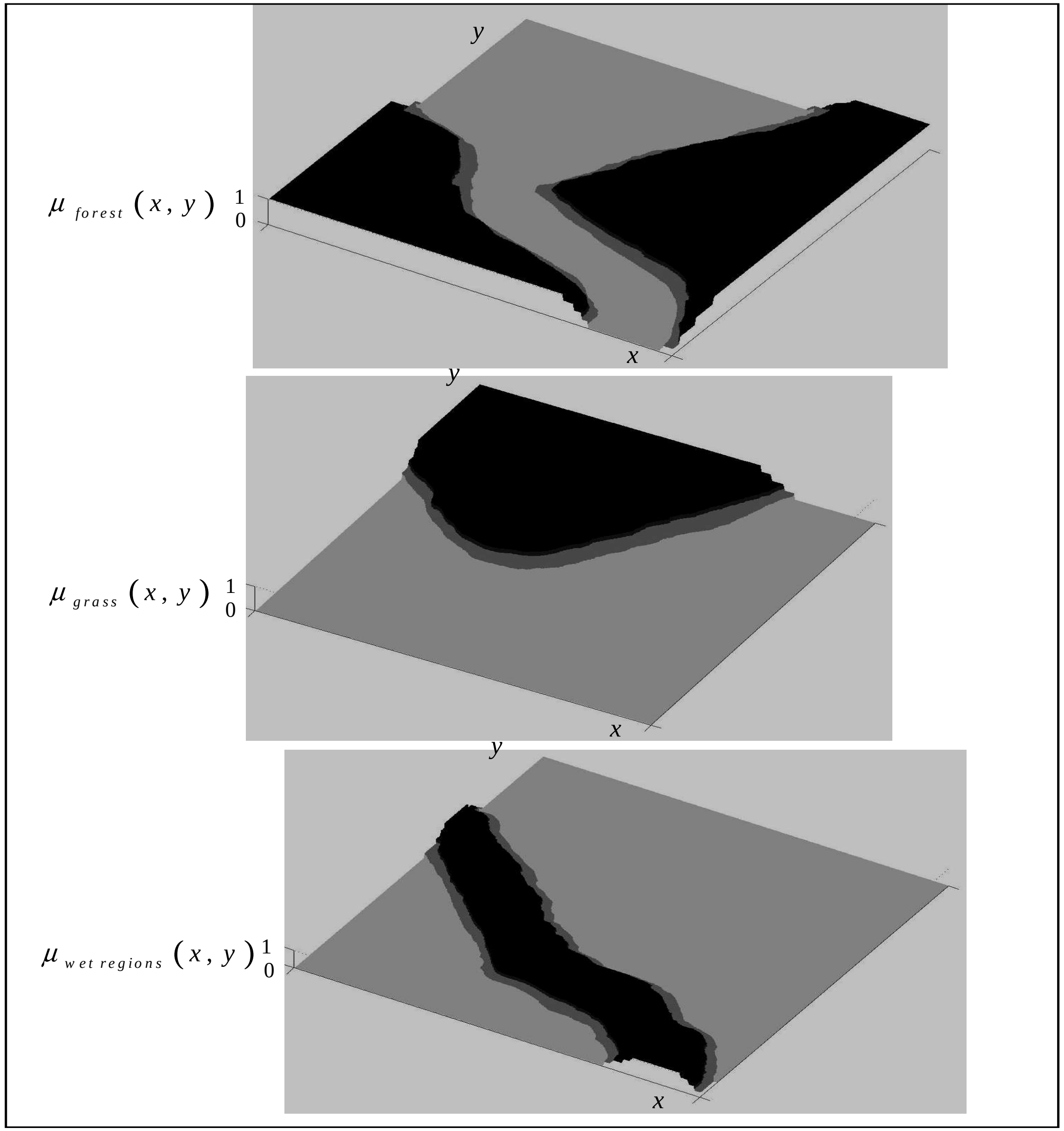
$\mu(x,y) = 0.75$

$\mu(x,y) = 0.5$

$\mu(x,y) = 0.25$

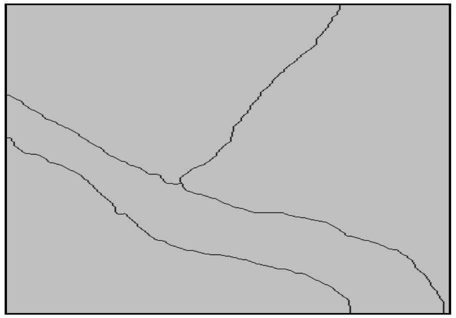
$\mu(x,y) = 0$

Display



a)

b)



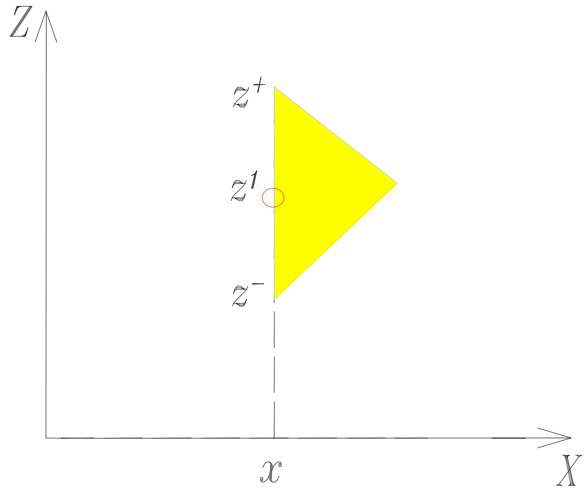
c)

Example 3 – Surface modeling

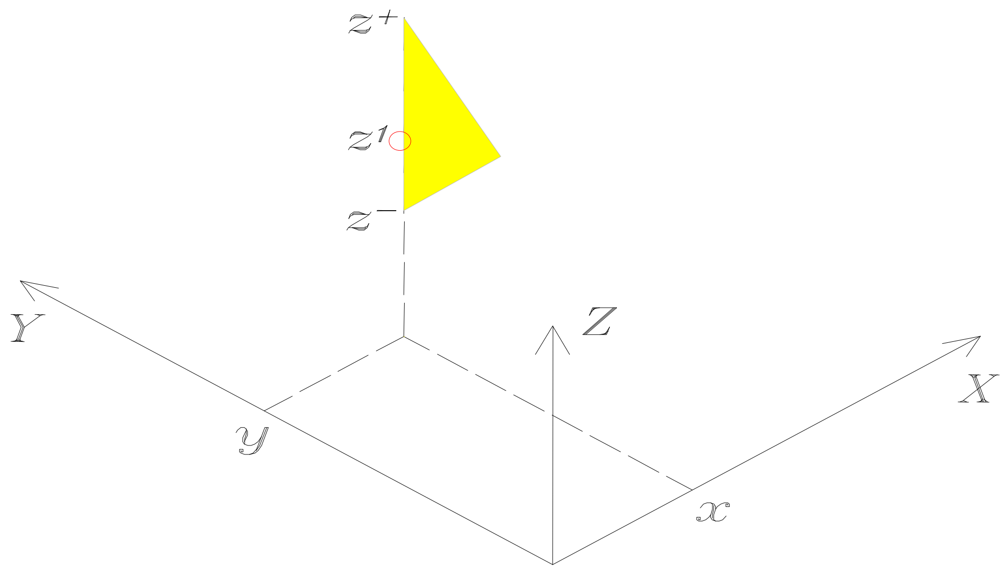
3. Surface models
   - The problem: Given a set of reading of the bottom of the ocean whose values are uncertain, generate a surface that explicitly incorporates this uncertainty mathematically and visually - The approach: Consistent fuzzy surfaces

   - Here with just introduce the associated ideas

Imprecision in Points: Fuzzy Points (figures from Jorge dos Santos)



2D



3D

Transformation of real-valued functions to fuzzy functions

Instead of a real-valued function $z = f(x)$ or $z = f(x, y)$ let's now consider a fuzzy function $\tilde{z} = \tilde{f}(x)$ or $\tilde{z} = \tilde{f}(x, y)$ where every element x or (x,y) is associated with a fuzzy number $\tilde{z}$.

Statement of the Interpolation Problem

Knowing the values $\{\tilde{z}_i\}$ of a fuzzy function over a finite set of points

$\{x_i\}$ or $\{(x_i,y_i)\}$, interpolate over the domain in question to obtain a (nested) set of surfaces that represent the uncertainty in the data.

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Computing surfaces

Given a data set of fuzzy numbers:

$$\tilde{z} = 1-d \quad fuzzy\ triangular = a/b/c$$

$$\tilde{p}(x) = \sum_{i=1}^{N} \tilde{z}_i L_i(x)$$

$$[\tilde{p}(x)]_\alpha = \sum_{i=1}^{N} z_i(\alpha) L_i(x)$$

Computing surfaces – Example

$$\tilde{z}_1 = 0.5/1.5/2, \quad \tilde{z}_2 = 0.75/1/1.5$$

$$L_1(x) = x + 2, \quad L_2(x) = 3x - 1$$

$$x = 1 \Rightarrow L_1(1) = 1 + 2 = 3, \quad L_2(1) = 3*1 - 1 = 2$$

$$\Rightarrow \tilde{p}(1) = 3\tilde{z}_1 + 2\tilde{z}_2 = 1.5/4.5/6 + 1.5/2/3$$

$$= 3/6.5/9$$

$$[\tilde{p}(1)]_{\alpha=0} = [3, 9]$$

$$[\tilde{p}(1)]_{0.5} = [4.75, 7.75]$$

$$[\tilde{p}(1)]_1 = [6.5, 6.5]$$

Consistent Fuzzy Surfaces (curves)

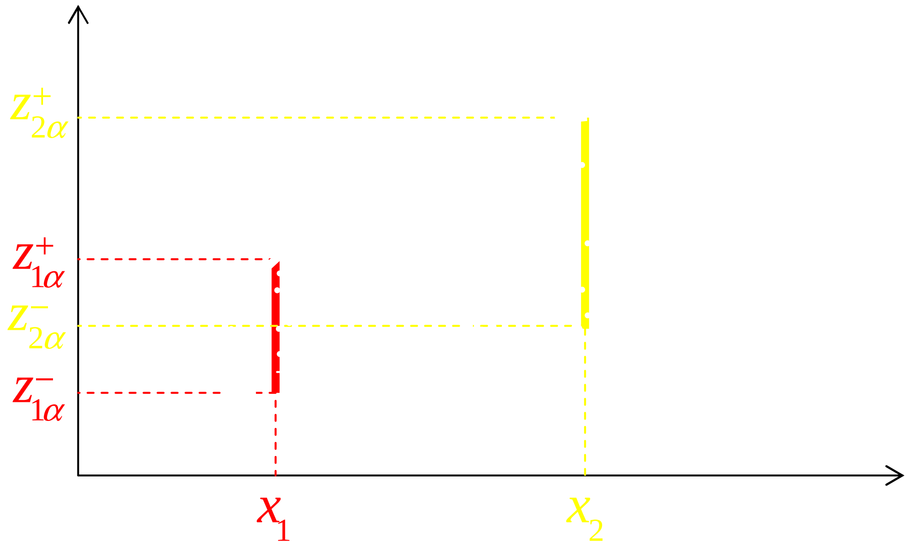
The surfaces (curves) are defined enforcing the following properties:

1. The surfaces are defined analytically via the fuzzy functions; that is, model directly the uncertainty using fuzzy functions $\tilde{z} = \tilde{f}(x)$ or $\tilde{z} = \tilde{f}(x, y)$
2. All fuzzy surfaces maintain the characteristics of the generating method. That is, if splines are being used then all generated fuzzy surfaces have the continuity and smoothness conditions associated with the splines being used.

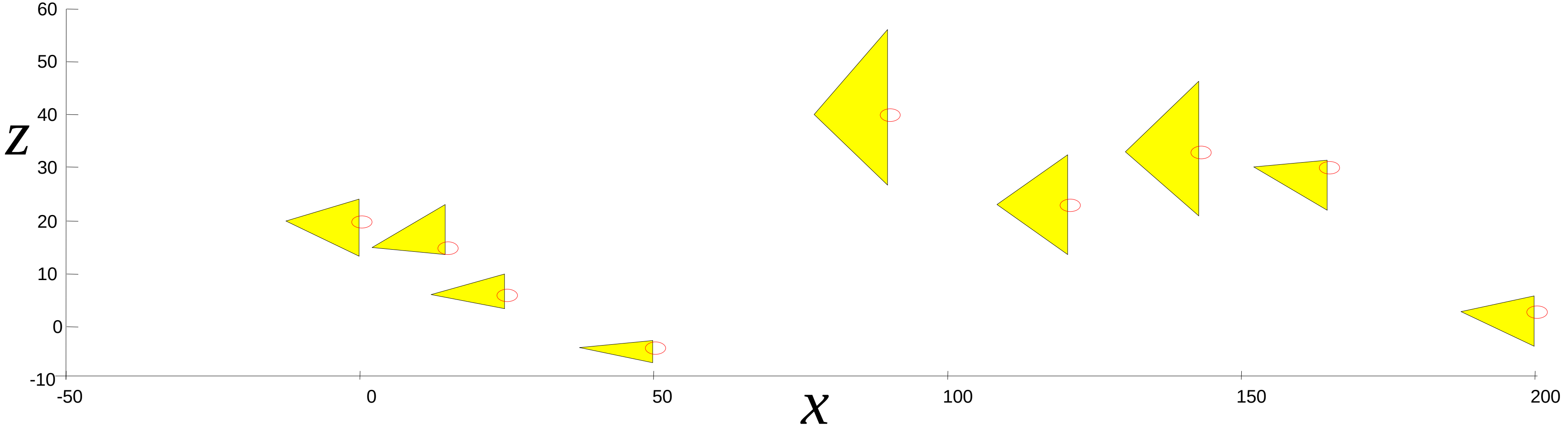
Fuzzy Interpolating Polynomial - $\tilde{p}(x)$ (figure from Jorge dos Santos & Lodwick)

Utilizing alpha-levels to obtain fuzzy polynomials, we have:

$$\left[\tilde{p}(x)\right]_\alpha \equiv \left[p_\alpha^-(x), p_\alpha^+(x)\right] = \left\{z \in R : z = p_d(x),\ d_i \in [z_i]_\alpha\right\}$$

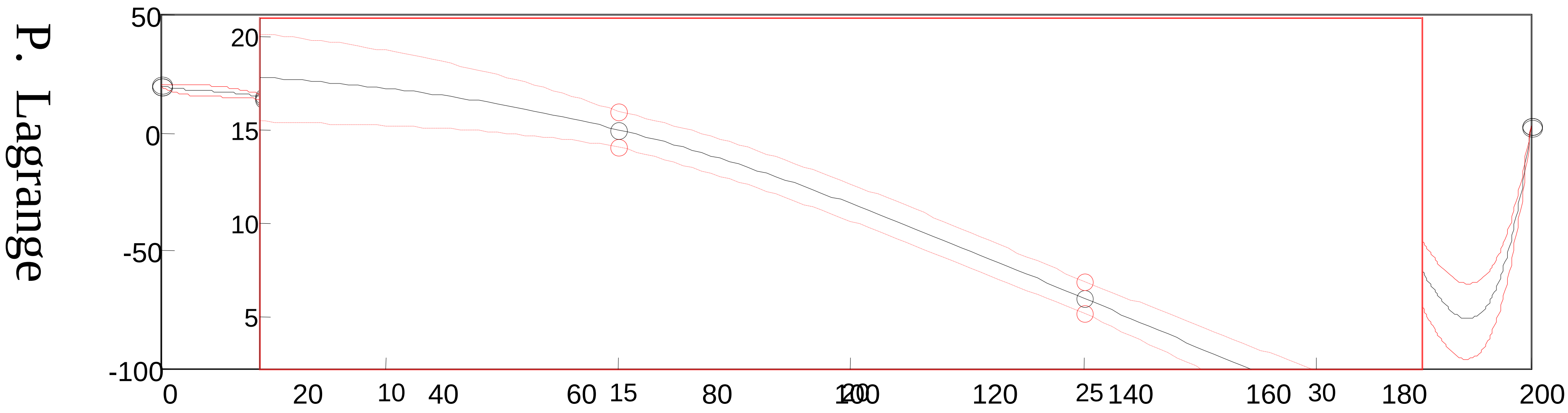


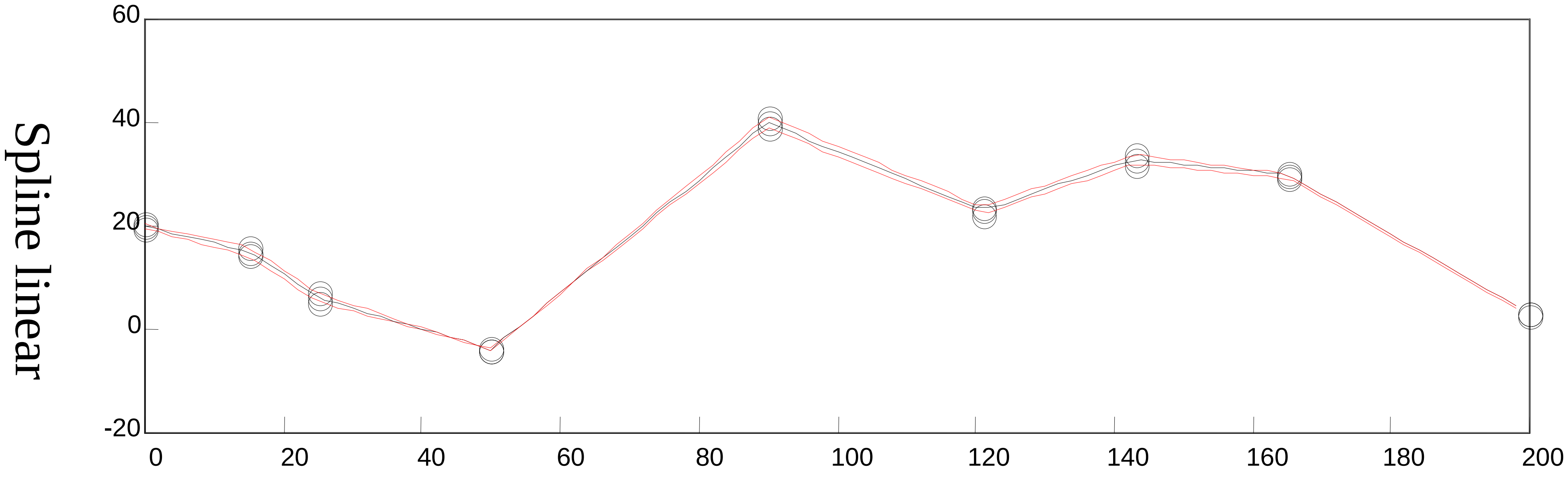
2-D Example (from Jorge dos Santos & Lodwick)



x_i	0	15	25	50	90	121	143	165	200
z_i^-	19.5	14.9	5.8	-3.9	39.0	22.3	32.1	29.4	2.5
z_i^1	20.0	15.0	6.0	-4.0	40.0	23.0	33.0	30.0	3.0
z_i^+	20.3	15.6	6.3	-4.2	41.2	23.7	34.0	30.1	3.2

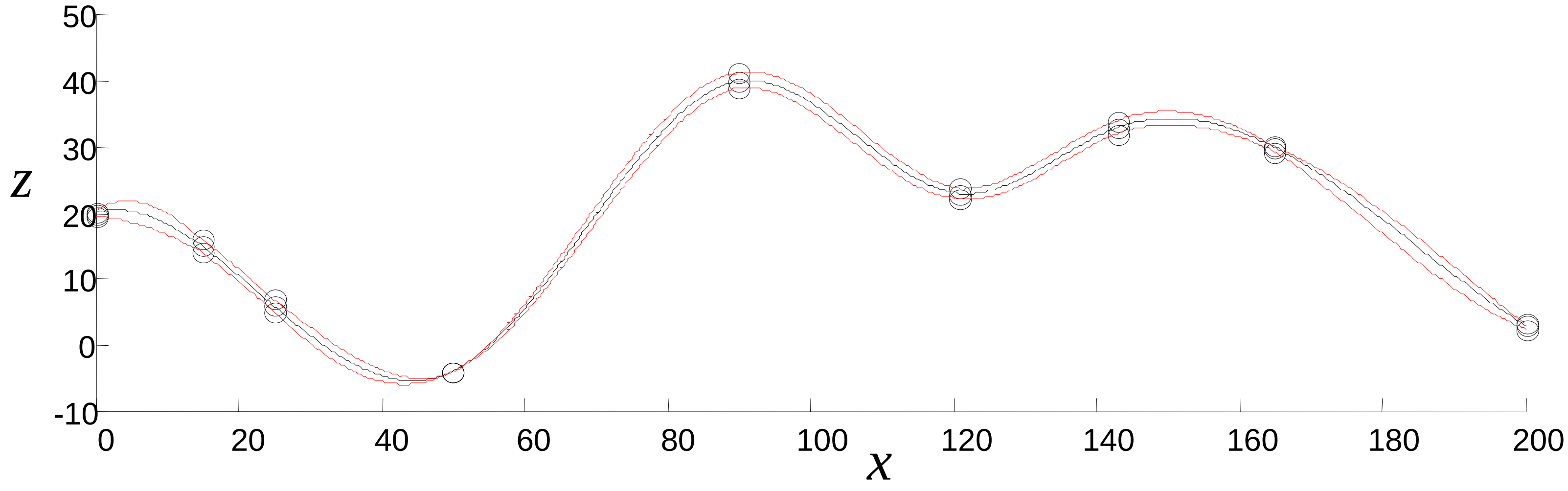
Fuzzy Curves (figures from Jorge dos Santos & Lodwick)



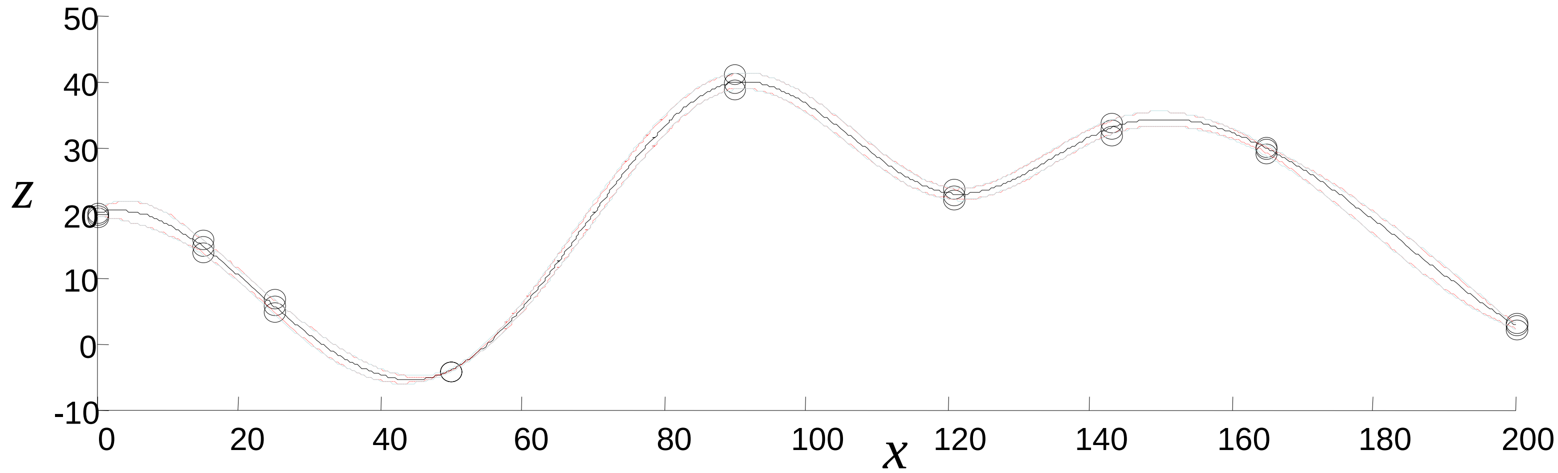


Fuzzy Curves (figures from Jorge dos Santos & Lodwick)

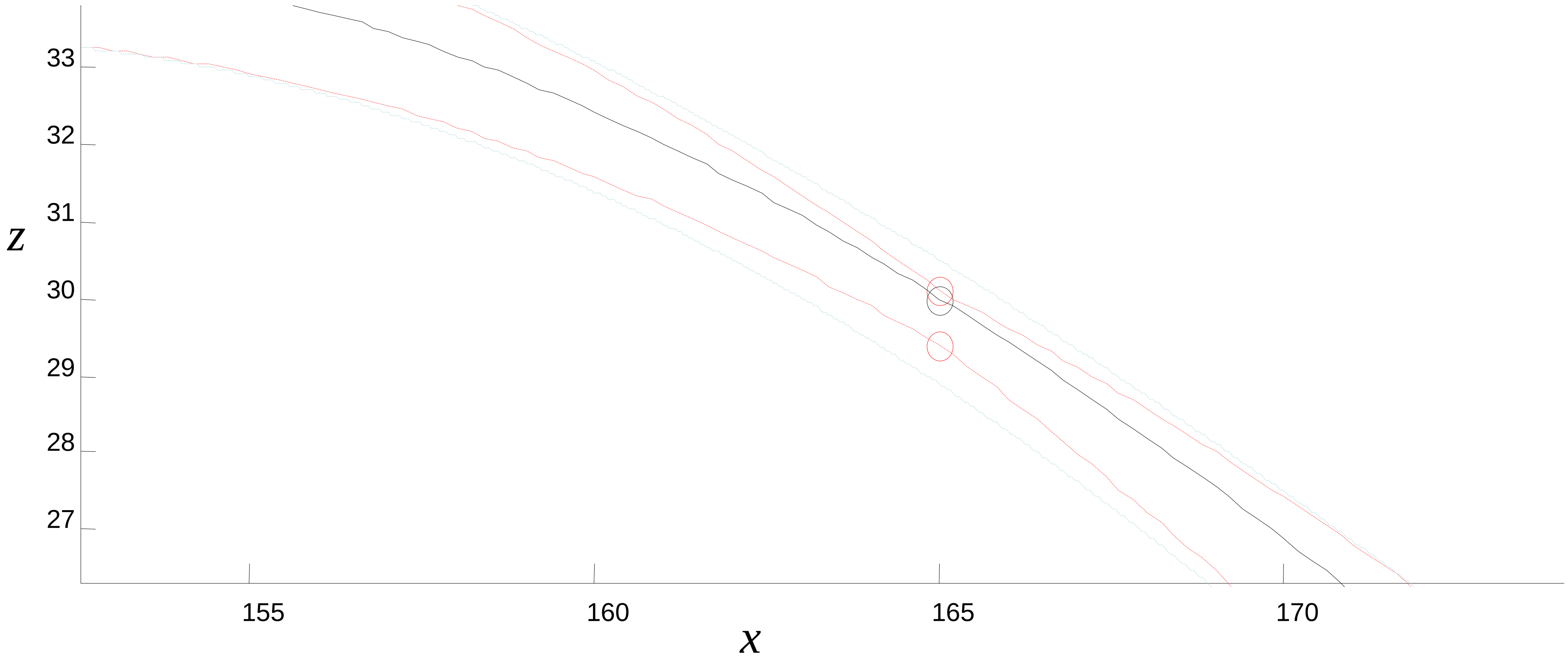
Cubic Spline



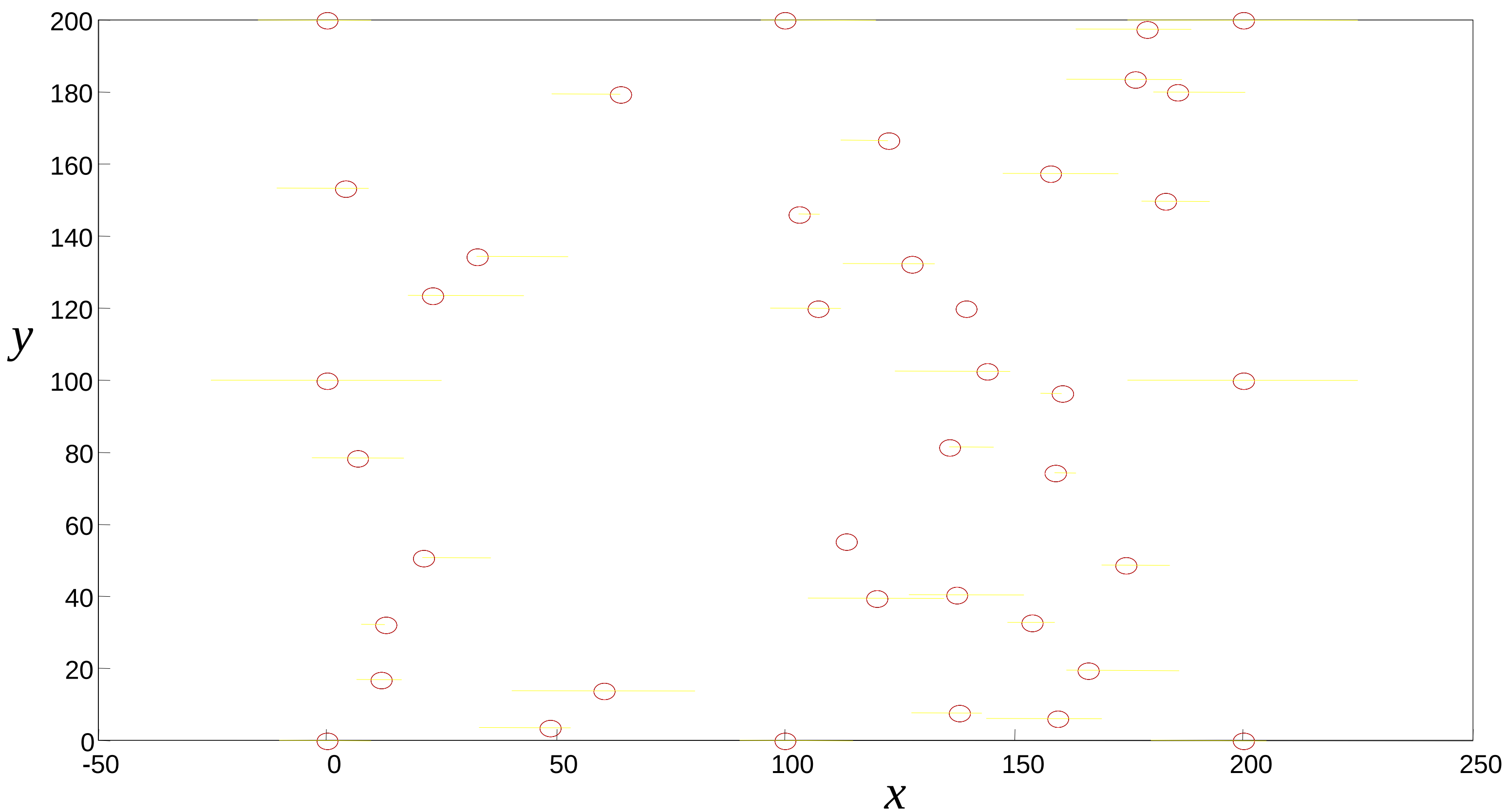
Consistent Cubic Spline



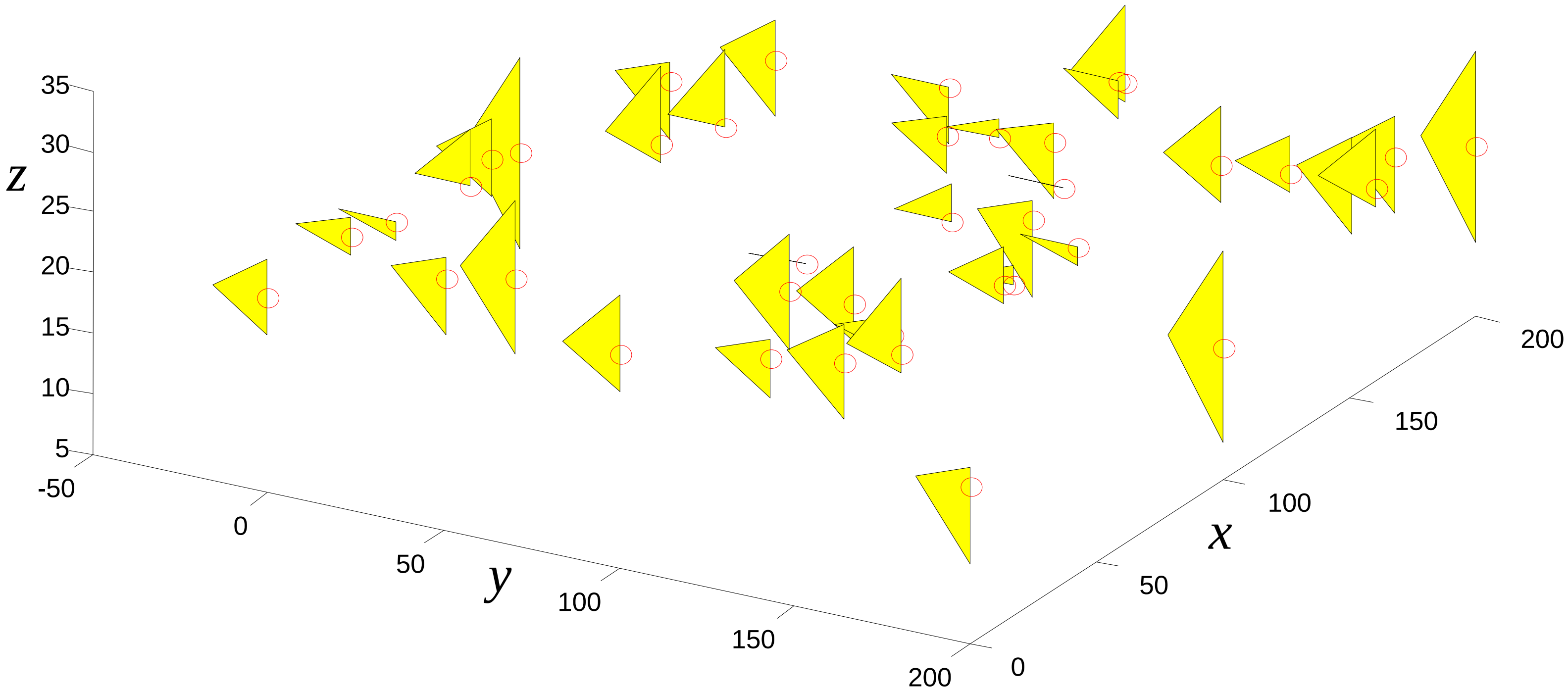
Details of the Consistent Fuzzy Cubic Spline (figures from Jorge dos Santos & Lodwick)



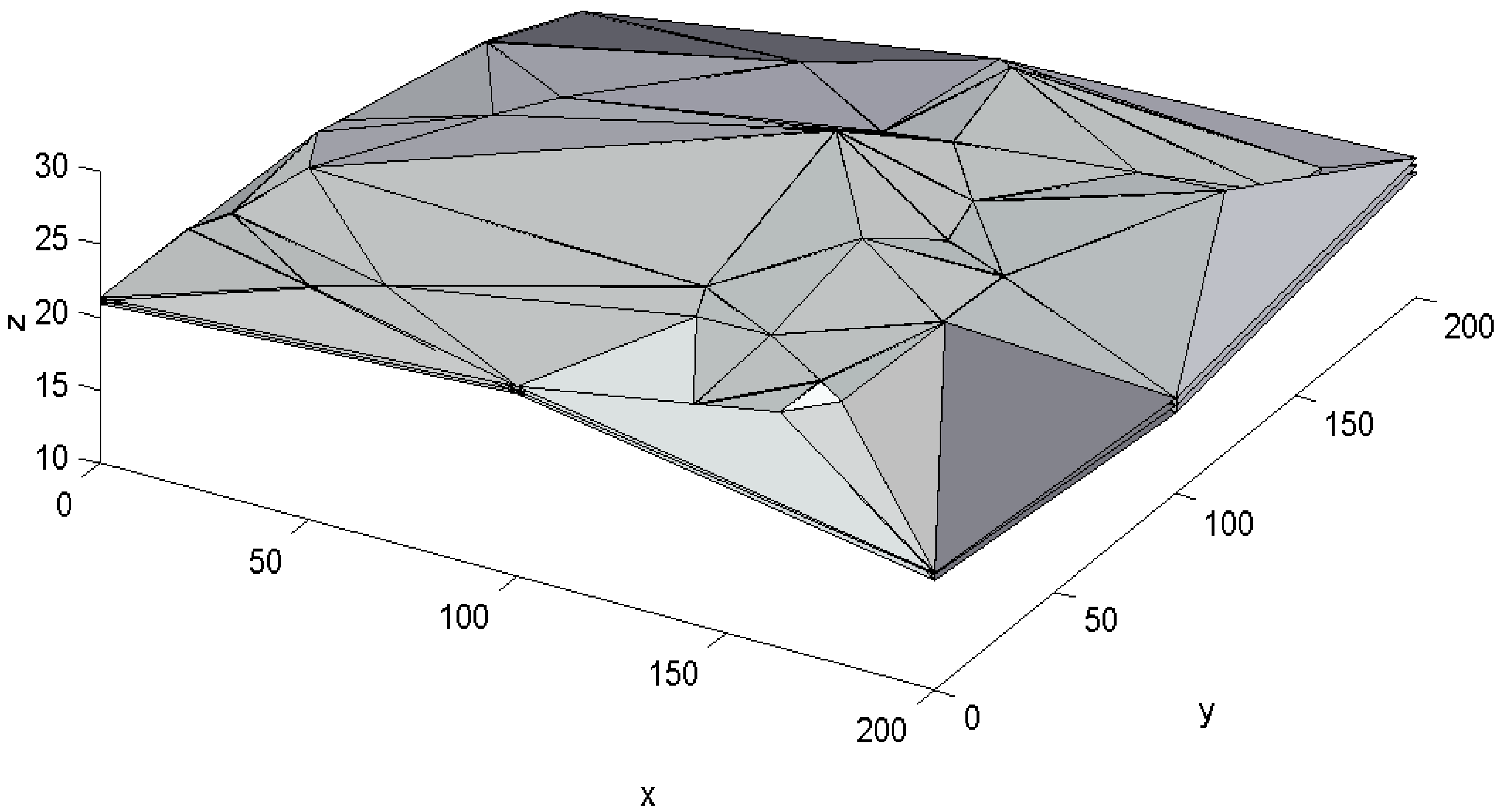
3-D Example (from Jorge dos Santos & Lodwick)



Another Representation/View of the Fuzzy Points (figure from Jorge dos Santos & Lodwick)



Fuzzy Surface via Triangulation (figure from Jorge dos Santos & Lodwick)



Fuzzy Surfaces via Linear Splines
(figure from Jorge dos Santos & Lodwick)

Fuzzy Surfaces via Cubic Splines
(figure from Jorge dos Santos & Lodwick)