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**Probability Distribution**

A probability distribution describes the likelihood of obtaining the possible outcomes of a random variable. One commonly encountered distribution is the uniform distribution.

**Uniform Distribution**

In a uniform distribution, every outcome within a given range has an equal probability of occurring. For example, if we consider a fair six-sided die, each side has a probability of 1/6​ of being rolled.

**Bayesian Inference:**

Bayesian inference is a statistical method used to update beliefs or predictions about a hypothesis as new evidence becomes available. It combines prior knowledge with observed data to form a posterior distribution, which represents the updated beliefs about the hypothesis.

**Sample Problem for Binomial Distribution**

Consider a scenario where we want to estimate the bias of a coin. Represent the bias of the coin with a parameter *p,* where *p* is the probability of obtaining a head (success) when the coin is flipped.

**Prior Knowledge:** Initially, There is no information about the bias of the coin, so assume a uniform prior distribution for *p* between 0 and 1. It means that believe all values of *p* are equally likely before observing any data.

**Likelihood:** Suppose that flip the coin 10 times and observe 8 heads and 2 tails. Use the binomial distribution as likelihood function, which gives the probability of observing the data given a specific value of *p*. In this case, the likelihood function will give the probability of getting 8 heads and 2 tails out of 10 flips for each value of *p*.

**Posterior Probability:** Using Bayesian inference, update the beliefs about *p* based on the observed data and prior distribution. The posterior distribution represents updated beliefs about the bias of the coin after observing the data.

**Bayesian Inference:** From the posterior distribution, calculate summary statistics such as the mean, median, or mode to estimate the most likely value of *p* and quantify the uncertainty around this estimate using credible intervals.

**Generalization**

After observing the data (8 heads and 2 tails out of 10 flips) and combining it with the uniform prior distribution, calculate the posterior distribution for *p*. This posterior distribution tells that the updated beliefs about the bias of the coin, considering the observed data. Use the mean, median, or mode of the posterior distribution as our estimate for *p*, along with credible intervals to quantify uncertainty.

By using Bayesian inference, it enables incorporate prior knowledge, update the beliefs based on observed data, and obtain a more informed estimate of the parameter of interest, in this case, the bias of the coin.

**Sample Problem for Uniform Distribution**

Suppose a teacher wants to estimate the average height of students in a school. Select a random sample of 20 students and measure their heights in centimeters. Using Bayesian inference, the teacher can estimate the population mean height and quantify the uncertainty around this estimate.

1. **Prior Distribution:** Assume a uniform prior distribution for the population mean height, *μ*, ranging from a minimum plausible height to a maximum plausible height. This means we believe all values of *μ* within this range are equally likely before observing any data.
2. **Likelihood Function:** After measuring the heights of the 20 students, the teacher can use a normal distribution as our likelihood function. The normal distribution gives the probability of observing the data (height measurements) given a specific value of *μ* and a known standard deviation *σ*.

**Posterior Distribution:** Using Bayesian inference, update the beliefs about *μ* based on the observed data and the prior distribution. The posterior distribution represents the dated beliefs about the population mean height after observing the data.

**Bayesian Inference:** From the posterior distribution, calculate the summary statistics such as the mean, median, or mode to estimate the most likely value of *μ* and quantify the uncertainty around this estimate using credible intervals.

**Summary**

After measuring the heights of the 20 students and combining the data with the uniform prior distribution, we calculate the posterior distribution for *μ*. This posterior distribution tells us the updated beliefs about the population mean height, considering the observed data. We can use summary statistics such as the mean, median, or mode of the posterior distribution as our estimate for *μ*, along with credible intervals to quantify uncertainty.

**Reference:**

OpenAI ChatGPT. (2024, April 6). Bayesian Inference with Uniform Distribution - Chat

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