• SONO DA MARIARCA 4 STUDIARE. 26/NOV/2025

1 RA 18/02/2018

1*) VENGONO DATE 10 CARTE X 1 A OGNI GIOCATORE CALCOLO LA PROB. DI OTTENERE UNA NAPOLETANA DI DENARI

$$\mathbb{P}(\text{NAPOLETANA}_{\text{DENARI}}) = \frac{\binom{3}{3}\binom{37}{7}}{\binom{40}{10}}$$

2) a) $$\mathbb{P}([\text{NAPOLETANA}_{\text{DENARI}}] \wedge [\text{NAPOLETANA}_{\text{COPPE}}]) = \frac{\binom{6}{6}\binom{34}{4}}{\binom{40}{10}}$$

b) $$\mathbb{P}([\text{NAPOLETANA}_{\text{DENARI}}] \wedge [\text{NAPOLETANA}_{\text{COPPE}}] \wedge [\text{NAPOLETANA}_{\text{BASTONI}}]) = \frac{\binom{9}{9}\binom{31}{1}}{\binom{40}{10}}$$

3) a) $$\mathbb{P}(\text{"TUTTE LE NAP."}) = \frac{\binom{12}{12}\cdots}{} = 0$$

NON E' POSSIBILE INFATTI... DOVREI PRENDERE 12 CARTE IN UNA MANO

b) $$\mathbb{P}(\text{"ALMENO UNA NAP."}) = \frac{\binom{4}{1}\binom{3}{3}\binom{37}{10}}{\binom{40}{10}}$$

4) a) come 1*)

b) $$\mathbb{P}(A_{ND} \mid C_{NB}) = \frac{\binom{3}{3}\binom{3}{3}\binom{34}{7}\binom{24}{7} \Big/ \binom{40}{10}\binom{30}{10}}{\binom{37}{7} \Big/ \binom{40}{10}}$$

5) a) $$\mathbb{P}(\text{"UNA SPECIFICA NAPOLETANA PER UNO"}) = \frac{\binom{12}{12}\binom{28}{7}\binom{21}{7}\binom{14}{7}}{\binom{40}{10}\binom{30}{10}\binom{20}{10}\binom{10}{10}}$$

b) $$\mathbb{P}(\text{"UNA NAP. PER UNO"}) = \left(\frac{4!\binom{12}{12}\binom{28}{7}\binom{21}{7}\binom{14}{7}}{\binom{40}{10}\binom{30}{10}\binom{20}{10}\binom{10}{10}}\right)$$

6 GENNAIO 2019

1^a urna	2 B	4 R
2^a urna	4 B	2 R
3^a urna	6 B	

	1	2
1^a	T	C
2^a	C	T
3^a	C	C

, TT

1) $\mathbb{P}U_i = \begin{cases} \mathbb{P}U_1 = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{2} \\ \mathbb{P}U_2 = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} \\ \mathbb{P}U_3 = \frac{1}{4} \end{cases}$

2) a) $\mathbb{P}(B_1) =$ "USANDO LA FORMULA DELLE PROBABILITÀ TOTALI" $= \mathbb{P}(B_1) = \mathbb{P}(B_1|U_1)\mathbb{P}(U_1) +$

$$\mathbb{P}(B_1|U_2)\mathbb{P}(U_2) + \mathbb{P}(B_1/U_3)\mathbb{P}(U_3) =$$

$$\left(2/6 \cdot 1/2 + 4/6 \cdot 1/4 + 1 \cdot 1/4\right) = 0,58\bar{3}$$

b) $\mathbb{P}(B_2) =$ "USANDO LA FOR..." $=$

$$\mathbb{P}(B_2) = \mathbb{P}(B_2|B_1)\mathbb{P}(B_1) + \mathbb{P}(B_2|\bar{B}_1)\mathbb{P}(\bar{B}_1) =$$

$$\left(1/5 \cdot \frac{1}{2} + 3/5 \cdot \frac{1}{4} + 1/4\right) \cdot 0,58\bar{3} + \left(2/5 \cdot \frac{1}{2} + 4/5 \cdot \frac{1}{4}\right) \cdot (1 - 0,58\bar{3})$$

3) "USANDO LA FORMULA DI BAYES"

$$\mathbb{P}(U_2|B_1) = \frac{\mathbb{P}(B_1|U_2)\mathbb{P}(U_2)}{\mathbb{P}(B_1)} = \frac{1/4 \cdot 2/6}{0,58\bar{3}} \simeq 0,142$$

4) $\mathbb{P}(BBR) = \left(\begin{matrix}\mathbb{P}B_1B_2R_3 + \mathbb{P}B_1R_2B_3 \ldots \\ [\ldots]\end{matrix}\right) = \frac{1}{4}\left(\frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}}\right) +$

$$\frac{1}{2}\left(\frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}}\right) = \frac{1}{4}\left(\frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}}\right) + \frac{1}{2}\left(\binom{4}{2}\binom{2}{1} \Big/ \binom{6}{3}\right)$$

LA PROBABILITÀ DI PRENDERE 2 ELEMENTI DI UN TIPO E 1 ELEMENTO DI UN ALTRO DA UN INSIEME DI ELEMENTI È COME SE INSTANZIO UNA VARIABILE ALEATORIA $X \sim$ Ipergeometrica$(6,3,2)$

5) $$\mathbb{P}(U_i \mid BBR) = \boxed{\frac{\mathbb{P}(BBR \mid U_i)\,\mathbb{P}(U_i)}{\mathbb{P}(BBR)} \quad \text{con } 1 \le i \le 3}$$

$$\mathbb{P}(U_3 \mid BBR) = 0$$

$$\mathbb{P}(U_2 \mid BBR) = \left(\left(\frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}}\right)\cdot\frac{1}{2}\right) \Big/ \left(\frac{1}{4}\left(\frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}}\right) + \frac{1}{2}\left(\frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}}\right)\right)$$

$$\mathbb{P}(U_1 \mid BBR) = \left(\left(\frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}}\right)\cdot\frac{1}{4}\right) \Big/ \left(\frac{1}{4}\left(\frac{\binom{2}{2}\binom{4}{1}}{\binom{6}{3}}\right) + \frac{1}{2}\left(\frac{\binom{4}{2}\binom{2}{1}}{\binom{6}{3}}\right)\right)$$

ESERCIZIO 2 DEL 18 / GEN / 2018

A
1   1,2
2   3,4
3   5,6

B
1   1,2
2   1,2 ; 1,2
3

1) CALCOLA DENSITA' DISCRETA DI X_A a)
VALORE ATTESO DI X_A b)
VARIANZA DI X_A c)

$$\mathbb{P}(X_A = k) = \begin{cases} 1/3 & \text{se } 1 \le k \le 3 \\ 0 & \text{ALTRIMENTI} \end{cases}$$

$$E[X_A] = 1\cdot\frac{1}{3} + 2\cdot\frac{1}{3} + \cancel{3}\cdot\frac{1}{\cancel{3}} = \frac{1}{3} + \frac{2}{3} + \frac{3}{3} = 2$$

$$Var[X_A] = E(X_A^2) - [E(X_A)]^2 = \left(\sum_{i=1}^{3} i^2\,\mathbb{P}(X=i)\right) - 1 = \left(1\cdot\frac{1}{3} + 4\cdot\frac{1}{3} + 9\cdot\frac{1}{3}\right) - 1 =$$
$$= \frac{1}{3} + \frac{4}{3} + \frac{9}{3} = \frac{14}{3}$$

2) RICHIESTA:
$Cov(X_A, X_B)$ (MI ASPETTO ESCA 0 DATO CHE I LANCI SONO INDIPENDENTI)
... allora... ma visto tutto per trovare questa covarianza ho bisogno di $E(X_B)$ dato che $cov(x,y) = E\big[(X - E[X])(Y - E[Y])\big]$

$$E[X_B] = 1\cdot\frac{1}{3} + 2\cdot\left(\frac{1}{3}\cdot\frac{4}{6}\right) + 3\cdot\left(\frac{4}{6}\right)^2 = 2,\bar{1}$$

$$Cov(X_A, X_B) = E[XY] - E[X]E[Y] \quad (\text{SVILUPPANDO}) =$$

$$= \left(\sum_{i,j=1}^{3} i \cdot j \cdot \mathbb{P}(X_A = i) \mathbb{P}(X_B = j) \right) - (2,\bar{1} \cdot 2) = 1 \cdot \frac{1}{3} \cdot \frac{1}{3} + 2 \frac{1}{3}\left(\frac{1}{3}\,\frac{4}{6}\right) + 2\left(\frac{1}{3} \cdot \frac{1}{3}\right) +$$

$$3 \cdot \frac{1}{3}\left(\frac{4}{6}\right)^2 + 3\left(\frac{1}{3}\right)\frac{1}{3} + 6\left(\frac{1}{3}\right)\left(\frac{4}{9}\right)^2 + 6\left(\frac{1}{3}\frac{4}{6}\right)\left(\frac{1}{3}\right) + 9\left(\frac{4}{6}\right)^2\left(\frac{1}{3}\right) + 4\left(\frac{1}{3}\right)\left(\frac{1}{3} \cdot \frac{4}{6}\right)$$

$$-(2,\bar{1} \cdot 2) = 0$$

DATO CHE X e Y SONO INDIPENDENTI $E[X]E[Y] = E[XY] \rightarrow Cov(X_A, X_B) = 0$
DI CONSEGUENZA

$$3)\ \mathbb{P}(X_B = h) = \begin{cases} 1/3 \text{ se } h = 1 & \rightarrow 1/3 \\ 1/3 \cdot 4/6 \text{ se } h = 2 & \rightarrow 2/9 \\ 4/6 \cdot 4/6 \text{ se } h = 3 & \rightarrow \cancel{\frac{16}{36}}\ \cancel{\frac{8}{18}}\ \frac{4}{9} \rightarrow 4/9 \\ 0 \text{ altrimenti} \end{cases}$$

$$4)\ \mathbb{P}(X_A = X_B) = \begin{cases} \mathbb{P}(X_A = 1, X_B = 1) = 1/3 \cdot 1/3 = 1/3^2 = \frac{1}{9} + \\ \mathbb{P}(X_A = 2, X_B = 2) = 1/3 \cdot 2/9 = \frac{2}{27} + \\ \mathbb{P}(X_A = 3, X_B = 3) = 1/3 \cdot 4/9 = \frac{4}{27} = \frac{9}{27} \end{cases}$$

$$5)\ a)\ \mathbb{P}(V) = \mathbb{P}(X_A = X_B) = 9/27$$

b)
$$\mathbb{P}(X_A, X_B = 1 \mid V) = \text{USANDO LA FORMULA DI BAYES} = \frac{1/9 \cdot 1/9}{1/9} = 1/9$$

E DI CONSEGUENZA

$$\mathbb{P}(X_A, X_B = 2 \mid V) = 2/27$$

$$\mathbb{P}(X_A, X_B = 3 \mid V) = 4/27$$

ESERCIZIO 2 DEL 8 FEBBRAIO 2018

1) $p = 2/5$

$$\mathbb{P}(A_1) = (2/5)^2 + (3/5)^2 = \frac{4}{25} + \frac{9}{25} = \frac{13}{25}$$ e così per tutti...

$$\mathbb{P}(A_2) = \frac{13}{25} = \mathbb{P}(A_3)$$

2) $\mathbb{P}(A_1 \cap A_2) =$ DALLA FORMULA DELLE PROBABILITÀ TOTALI ...

$$\mathbb{P}(A_1 \cap A_2) = \mathbb{P}(A_1 \cap A_2 | (T)) \mathbb{P}(T) + \mathbb{P}(A_1 \cap A_2 | C) \mathbb{P}(C) =$$

$$\left(\frac{2}{5}\right) \cdot \left(\frac{2}{5}\right) + \left(\frac{3}{5}\right) \left(\frac{3}{5}\right)$$ quindi:

$$\mathbb{P}(A_1 \cap A_2) = \mathbb{P}(A_1) = \mathbb{P}(A_2) = \mathbb{P}(A_3) \neq \mathbb{P}(A_1)\mathbb{P}(A_2) \Longrightarrow$$

A_1, A_2, A_3 non sono EVENTI INDIPENDENTI

a) $\mathbb{P}(A_1 \cap A_2) = 13/25 \quad \mathbb{P}(A_1)\mathbb{P}(A_2) \; (13/25)^2 \Longrightarrow$

$\mathbb{P}(A_1 \cap A_2) > \mathbb{P}(A_1)\mathbb{P}(A_2)$ QUINDI A_1, A_2 SONO CORRELATI POSITIVAMENTE

b) VEDI (*)

3) $T_i :=$ EVENTO esce testa all'i-esimo lancio

$$\mathbb{P}(T_1 | A_1) = \frac{(\mathbb{P}(T_1 \cap A_1))}{\mathbb{P}(A_1)} = (2/5)^2 \Big/ (2/5)^2 + (3/5)^2 =$$

$$\frac{4}{25} \cdot \frac{25}{13} = 4/13$$

(*) $\mathbb{P}(A_1 \cap A_3) = \mathbb{P}(A_1)\mathbb{P}(A_3) = \left((2/5)^2 + (3/5)^2\right) \cdot \left((2/5)^2 + (3/5)^2\right)$ quindi: INDIPENDENTI E NON CORRELATI.

4) $E[X] = \sum_{i=1}^{3} i \, \mathbb{P}(X = i) = ????$

$$\boxed{\mathbb{P}(A_1 \cap A_2 \cap A_3) = \left(\frac{2}{5}\right)^3 + \left(\frac{3}{5}\right)^3}$$

$$\mathbb{P}(X = i) = \begin{cases} \binom{3}{1} \frac{13}{25} & \text{se } i = 1 \\ \left(\frac{2^2}{5} + \frac{3^2}{5}\right)^2 + 2\left(\frac{13}{25}\right)^2 & \text{se } i = 2 \\ & \text{se } i = 3 \end{cases}$$

$$E[X] = 1\cdot\binom{3}{1}\frac{13}{25} + 2\left(2\left(\frac{2}{5}^2+\frac{3}{5}^2\right)+\frac{13}{25}\right) + 3\left(\frac{2}{5}^3+\frac{3}{5}^3\right)$$

OPPURE... dato che $1A_1, 1A_2, 1A_3$ identicamente distribuiti

$$E[X] = E[1A_1] + E[1A_2] + E[1A_3] = 3\left(\frac{13}{25}\right)$$

5) $Var(X) = E[X^2] - (E[X])^2 =$ ma anche $\sum_{i=1}^{3} Var(1A_i) + 2\sum_{i\neq j}^{3} Cov(1A_i, 1A_j)$ per il motivo citato sopra.

6) a) $E(X|A_1) = ???????$

$$\mathbb{P}(X=k|A_1) = \begin{cases} \frac{13}{25} & \text{se } k=1 \\ \left(\frac{13}{25}\right)^2 + \left(\frac{2}{5}^2+\frac{3}{5}^2\right)^2 & \text{se } k=2 \\ \left(\frac{2}{5}\right)^3+\left(\frac{3}{5}\right)^3 & \text{se } k=3 \end{cases}$$

6 a) FATTO BENE

$$E(X|A_1) = E(1_{A_1}|A_1) + E(1_{A_2}|A_1) + E(1_{A_3}|A_1)$$