**Indian Academy of Sciences, Bengaluru**
**Indian National Science Academy, New Delhi**
**The National Academy of Sciences India, Prayagraj**

**SUMMER RESEARCH FELLOWSHIPS — 2023**

# Format for the four-week Report*,^,@

| | | |
|---|---|---|
| Name of the candidate | : | Darpan Bhattacharya |
| Application Registration no. | : | ENGS 302 |
| Date of joining | : | 22.05.2023 |
| Name of the guide | : | Prof. L. Sunil Chandran |
| Guide's institution | : | Indian Institute of Science, Bengaluru |
| Place of stay during the tenure of the fellowship | : | Hostel provided by<br>Guide<br>Own arrangement<br>Other (Specify) IASc Fellows Residency, Jalahalli |

Darpan Bhattacharya
Signature of the candidate
Date: 20.06.2023

Signature of the guide
Date: 21.06.2023

| | | | |
|---|---|---|---|
| | **INSPIRE/KVPY FELLOWSHIP (please fill this box)#** | | |
| 1. | I am currently a recipient of | INSPIRE FELLOWSHIP | ☐ Yes / ☑ No |
| | | KVPY FELLOWSHIP | ☐ Yes / ☑ No |
| | | **If, YES, fill cols. 2, 3 & 4** | |
| 2. | **INSPIRE/KVPY** Fellowship is from ____[month]/____[yr] to ____[month]/____[yr] | | |
| 3. | I receive a monthly fellowship of Rs. ________ from INSPIRE/KVPY towards my living expenses | | |
| 4. | I also receive towards contingencies a sum of Rs. ________ per year | | |
| | **I affirm that the information given above is correct.** | | Darpan Bhattacharya<br>Signature of the candidate |

**IMPORTANT NOTES:**
- * **The four-week report could be between 300 and 350 words.**
- ^ **This format should be the first page of the report and should be stapled with the main report.**
- # **Mandatory to fill this section, this should be filled and signed by you even if you are not an INSPIRE/KVPY Fellow. Otherwise release of fellowship amount will be withheld.**
- @ **The hard copy of the duly signed report should reach the Academy office within 10 days of completing the first month fellowship. If delayed the fellowship amount will not be paid.**

**(For office use only; do not fill/tear)**

| | |
|---|---|
| Candidate's name: | Fellowship amount: |
| Student: Teacher: | Deduction: |
| Guide's name: | TA fare: |
| KVPY Fellow: INSPIRE Fellow: | Amount to be paid: |
| PFMS Unique Code: | A/c holder's name: |
| Others | |

# IASc-INSA-NASI SRFP 2023
## Four-week Report

Darpan Bhattacharya
Application ID: ENGS302
Guide: Prof L. Sunil Chandran, IISc Bengaluru

19 June, 2023

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## Introduction

This is the four-week report of my summer research project under Prof L. Sunil Chandran at Indian Institute of Science, Bengaluru.
I have primarily worked for the past four weeks trying to learn and understand the nature and number of monochromatic perfect matchings in graphs and $n$-uniform hypergraphs, and the main topics that I have learnt and worked on over the past four weeks are summarized below.

## Perfect matchings in graphs

In a graph $G = (V, E)$, a *perfect matching* is a matching $M$ of the graph such that $\forall\, v \in V(G)$, $v$ is a part of some matched edge in $M$.
For a perfectly monochromatic colouring $c$ of $G$, the number of normal colour classes containing at least one perfect matching is represented by $\mu(G, c)$. The *matching index* of $G$, denoted by $\mu(G)$ is defined as $\mu(G) = \max\limits_{c \in \mathcal{C}(G)} \mu(G, c)$, where $\mathcal{C}(G)$ is the set of all perfectly monochromatic colourings of $G$.

In a graph, Bogdanov[3] proved that $\forall\, G$ isomorphic to $K_4$, the number of *disjoint* perfect matchings of $G = 3$, and for all other graphs non-isomorphic to $K_4$, the number of disjoint perfect matchings is at most 2. This proof was used in [2] to theorize that $\forall\, G$ isomorphic to $K_4$, $\mu(G) = 3$, otherwise $\mu(G) \leq 2$.

## Hypergraphs

According to [1], a *hypergraph* is a pair $H = (V, E)$, where $V$ is a finite set whose elements are called *vertices*, and $E$ is a family of subsets of $V$, called *edges*.
A hypergraph $H$ is said to be *$n$-uniform* if $\forall\, e \in E$, $e$ contains precisely $n$ vertices.

A *perfect matching*, $\mathcal{M}$ of a $n$-uniform hypergraph $H = (V, E)$ can be defined as a matching $M$ of $H$ such that $\forall\, v \in V$, $v$ is part of some edge in $M$. It is trivial to note that for a $n$-uniform hypergraph to have a perfect matching, $|V| \equiv 0 \pmod{n}$.

A $n$-uniform hypergraph $H$ is said to be perfectly monochromatic if the following properties hold:

1. Every perfect matching, $\mathcal{M}$ of $H$ is monochromatic.

2. Let $\mathcal{C}$ be set of colour classes such that $\exists$ a perfect matching $\mathcal{M}$ of $H$ such that $\mathcal{M}$ belongs to a colour class $c \in \mathcal{C}$. Then $\forall\, c \in \mathcal{C}$, $c$ must belong to *exactly* 1 perfect matching $\mathcal{M}$ of $H$.

Now, for a perfectly monochromatic hypergraph $H$, we call the dimension $D = |\mathcal{C}|$ of the hypergraph to be the number of colours in it. We are intereseted in finding the bounds on $D$ for a $n$-uniform hypergraph containing $k \cdot n$ vertices.

## Perfect matchings in 3-uniform hypergraphs

Since we are interested in finding the bounds on $D$, we start with trying to find the bounds on $D$ for 3-uniform hypergraphs. We analyzed some particular cases and the results we came up with are:

- For $|V| = 3$, it can be trivially observed that $D = 1$.
- For $|V| = 6$, the upper bound on $D = 10$.
  This can be verified by observing that in a complete 3-uniform hypergraph with $|V| = 6$, there are $\binom{6}{3} = 20$ edges. We can colour each edge and its complement with the same colour, forming a monochromatic perfect matching, and thus the number of monochromatic perfect matchings for a 3-uniform hypergraph with $|V| = 6$ is $20/2 = 10$.
- For $|V| = 9$, the upper bound on $D$ turned out to be 13. I verified it independently by writing a program which found out the value of $D$ by combining randomization and brute force techniques.
- For $|V| = 12$, I developed a partly randomized program to compute the bounds on $D$, and the upper bound from that program turned out to be 10, which surprisingly enough, turns out to be the same value as for $|V| = 6$.

Besides trying to analyze and compute the bounds for $D$ for 3-uniform hypergraphs, I have also tried implementing *Algorithm 1 - To decide whether a non-trivial matching covered graph non-isomorphic to* $K_4$ *is Type* 1 *or Type* 2 in [2], in the process of which I learnt about the Edmonds' blossom algorithm for finding the maximum number of perfect matchings in a graph, and the Edmonds-Karp algorithm for finding the maximum flow in a graph.

## References


[1] N. Alon and J. H. Spencer. *The Probabilistic Method.* Wiley Publishing, 4th edition, 2016.

[2] L. S. Chandran and R. Gajjala. Perfect matchings and quantum physics: Progress on krenn's conjecture, 2022.

[3] I. B. (https://mathoverflow.net/users/17581/ilya bogdanov). Graphs with only disjoint perfect matchings. MathOverflow. URL:https://mathoverflow.net/q/267013 (version: 2017-04-12).