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ME3180 Homework 1

Que 1

By construction in fig,

$$\vec{a} = \vec{OP} = \vec{OQ} + \vec{OR} = \vec{a}_x + \vec{a}_y \quad \text{--- (1)}$$

Let $|\vec{a}| = r$

$$\vec{a}_x = r\cos(\theta+\phi)$$
$$= r(\cos\theta\cos\phi - \sin\theta\sin\phi) \quad \text{--- (2)}$$

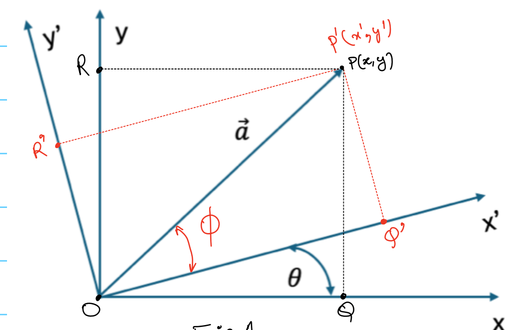


Fig 1

$$\vec{a}_y = r\sin(\theta+\phi)$$
$$= r(\sin\theta\cos\phi + \cos\theta\sin\phi) \quad \text{--- (3)}$$

$$\vec{a}' = \vec{OP}' = \vec{OQ}' + \vec{OR}' = \vec{a}_x' + \vec{a}_y'$$

We know that by rotation of axis there is no change in magnitude $\therefore |\vec{a}| = |\vec{a}'| = r$

$$\Rightarrow \quad \vec{a}_x' = r\cos\phi \quad , \quad \vec{a}_y' = r\sin\phi$$

Substitute in (2) & (3), we get

$$\vec{a}_x = (r\cos\phi)\cos\theta - (r\sin\phi)\sin\theta = \vec{a}_x'\cos\theta - \vec{a}_y'\sin\theta$$

$$\vec{a}_y = (r\cos\phi)\sin\theta + (r\sin\phi)\cos\theta = a_x'\sin\theta + a_y'\cos\theta$$

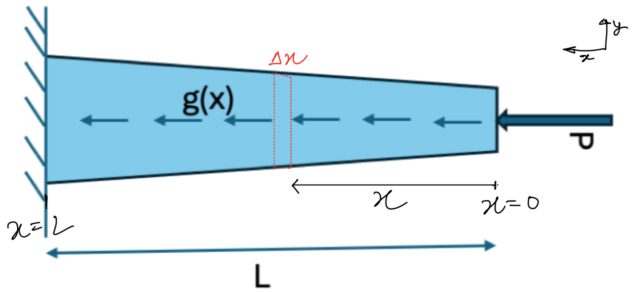
Writing in matrix form,

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} a_x' \\ a_y' \end{bmatrix} = \begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

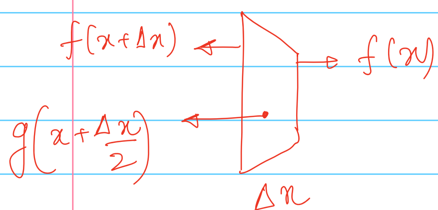
$$\Rightarrow \begin{bmatrix} a_x' \\ a_y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^{-1}\begin{bmatrix} a_x \\ a_y \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} a_x \\ a_y \end{bmatrix}$$

$$\Rightarrow \boxed{a' = \begin{pmatrix} a_x' \\ a_y' \end{pmatrix} = \begin{bmatrix} a_x\cos\theta + a_y\sin\theta \\ -a_x\sin\theta + a_y\cos\theta \end{bmatrix}}$$

Que 2



The differential equation for the bar is obtained from equilibrium of internal force $f(x)$ and external force $g(x)$ acting on body on axial direction.



Equilibrium eq in x direction

$$\Rightarrow f(x+\Delta x) + g\left(x+\frac{\Delta x}{2}\right)\Delta x - f(x) = 0$$

$$\Rightarrow \frac{f(x+\Delta x) - f(x)}{\Delta x} + g\left(x+\frac{\Delta x}{2}\right) = 0$$

Taking limit $\Delta x \to 0$

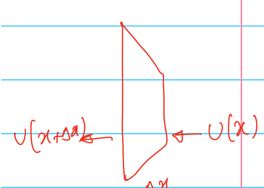
$$\Rightarrow \frac{d}{dx} f(x) + g(x) = 0 \quad \text{—— (1)}$$

Stress $\sigma(x) = \frac{f(x)}{A(x)} \Rightarrow f(x) = \sigma(x)\, A(x)$ —— (2)

Note :- $A(x) = A_1 + \frac{(A_0 - A_1)}{L} x$; $E(x) = E$

By (1) & (2), we get

$$\boxed{\frac{d}{dx}\left(\sigma(x) \cdot A(x)\right) + g(x) = 0}$$



Strain $(\varepsilon(x)) = \frac{\text{Elongation}}{\text{Original length}} = \frac{u(x+\Delta x) - u(x)}{\Delta x} = \frac{du}{dx}$ [As $\Delta x \to 0$]

Hooke's Law $\sigma(x) = E(x)\, \varepsilon(x)$

So eq $\to$ $$\boxed{\frac{d}{dx}\left(A(x) \cdot E(x) \frac{du}{dx}\right) + g(x) = 0}$$

Que 3

$$A = \begin{bmatrix} -1 & 3 & -2 \\ 2 & -4 & 2 \\ 0 & 4 & 1 \end{bmatrix}$$

Adjoint Method

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A)$$

$$|A| = -1\begin{vmatrix} -4 & 2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & -4 \\ 0 & 4 \end{vmatrix}$$

$$= -1(-4-8) - 3(2-0) - 2(8-0) = -10$$

$$\text{Adj}(A) = \begin{bmatrix} +\begin{vmatrix} -4 & 2 \\ 4 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & 2 \\ 0 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & -4 \\ 0 & 4 \end{vmatrix} \\ -\begin{vmatrix} 3 & -2 \\ 4 & 1 \end{vmatrix} & +\begin{vmatrix} -1 & -2 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 3 \\ 0 & 4 \end{vmatrix} \\ +\begin{vmatrix} 3 & -2 \\ -4 & 2 \end{vmatrix} & -\begin{vmatrix} -1 & -2 \\ 2 & 2 \end{vmatrix} & +\begin{vmatrix} -1 & 3 \\ 2 & -4 \end{vmatrix} \end{bmatrix}^T = \begin{bmatrix} (-4-8) & -(2) & (8) \\ -(3+8) & (1) & -(-4) \\ (6-8) & -(-2+4) & (4-6 \end{bmatrix}^T$$

$$= \begin{bmatrix} -12 & -2 & 8 \\ -11 & -1 & 4 \\ -2 & -2 & -2 \end{bmatrix}^T = \begin{bmatrix} -12 & -11 & -2 \\ -2 & -1 & -2 \\ 8 & 4 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \frac{1}{-10}\begin{bmatrix} -12 & -11 & -2 \\ -2 & -1 & -2 \\ 8 & 4 & -2 \end{bmatrix}$$

$$\Rightarrow \boxed{A^{-1} = \begin{bmatrix} 1.2 & 1.1 & 0.2 \\ 0.2 & 0.1 & 0.2 \\ -0.8 & -0.4 & 0.2 \end{bmatrix}}$$

Que 4

$$\begin{bmatrix} -1 & 3 & -2 \\ 2 & -4 & 2 \\ 0 & 4 & 1 \end{bmatrix} = A$$

Augmented Matrix

$$\left(\begin{array}{ccc|ccc} -1 & 3 & -2 & 1 & 0 & 0 \\ 2 & -4 & 2 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_1 \rightarrow -R_1$

$$\Rightarrow \left(\begin{array}{ccc|ccc} 1 & -3 & 2 & -1 & 0 & 0 \\ 2 & -4 & 2 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_2 \rightarrow R_2 - 2R_1$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & -1 & 0 & 0 \\ 0 & 2 & -2 & 2 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_2 \rightarrow \frac{1}{2} R_2$

$$\left(\begin{array}{ccc|ccc} 1 & -3 & 2 & -1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1/2 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right)$$

$R_1 \rightarrow R_1 + 3R_2$

$R_3 \rightarrow R_3 - 4R_2$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & 3/2 & 0 \\ 0 & 1 & -1 & 1 & 1/2 & 0 \\ 0 & 0 & 5 & -4 & -2 & 1 \end{array}\right)$$

$R_3 \rightarrow \frac{1}{5} R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & 3/2 & 0 \\ 0 & 1 & -1 & 1 & 1/2 & 0 \\ 0 & 0 & 1 & -4/5 & -2/5 & 1/5 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & 3/2 & 0 \\ 0 & 1 & -1 & 1 & 1/2 & 0 \\ 0 & 0 & 1 & -4/5 & -2/5 & 1/5 \end{array}\right)$$

$R_1 \rightarrow R_1 + R_3$

$R_2 \rightarrow R_2 + R_3$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 6/5 & 11/5 & 1/5 \\ 0 & 1 & 0 & 1/5 & 1/10 & 1/5 \\ 0 & 0 & 1 & -4/5 & -2/5 & 1/5 \end{array}\right)$$

$$\therefore A^{-1} = \begin{bmatrix} 6/5 & 11/10 & 1/5 \\ 1/5 & 1/10 & 1/5 \\ -4/5 & -2/5 & 1/5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1.2 & 1.1 & 0.2 \\ 0.2 & 0.1 & 0.2 \\ -0.8 & -0.4 & 0.2 \end{bmatrix}$$

Que 6

$$A = \begin{bmatrix} -1 & 3 & -2 \\ 2 & -4 & 2 \\ 0 & 4 & 1 \end{bmatrix}$$

Def^n of eigenvector v corresponding to eigen value λ

$$Av = \lambda v$$

$$\Rightarrow (A - \lambda I)\, v = 0$$

Eq^n has nonzero solutions if and only if

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} -1-\lambda & 3 & -2 \\ 2 & -4-\lambda & 2 \\ 0 & 4 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (-1-\lambda)(-4-\lambda)(1-\lambda) - (1-\lambda)(2.4) - 3(1-\lambda)2 - 2(2.4) = 0$$

$$\Rightarrow -\lambda^3 - 4\lambda^2 + 15\lambda - 10 = 0$$

$$\Rightarrow (\lambda-1)(\lambda^2 + 5\lambda - 10) = 0$$

$$\boxed{\text{Roots} \Rightarrow \lambda = 1, \ \lambda = \frac{-5 \pm \sqrt{65}}{2}}$$

\# For $\lambda = 1$

$$(A - \lambda I)(v) = 0 \Rightarrow \begin{bmatrix} -2 & 3 & -2 \\ 2 & -5 & 2 \\ 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow 0.x_1 + 4.x_2 + 0.x_3 = 0 \Rightarrow x_2 = 0$$

$$2x_1 - 5x_2 + 2x_3 = 0 \Rightarrow x_1 = -x_2$$

$$\Rightarrow \boxed{v = \begin{bmatrix} x \\ 0 \\ -x \end{bmatrix} \quad \text{for } x = 1 \quad v = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}$$

\# Similarly

for $\lambda = \frac{-5+\sqrt{65}}{2}$

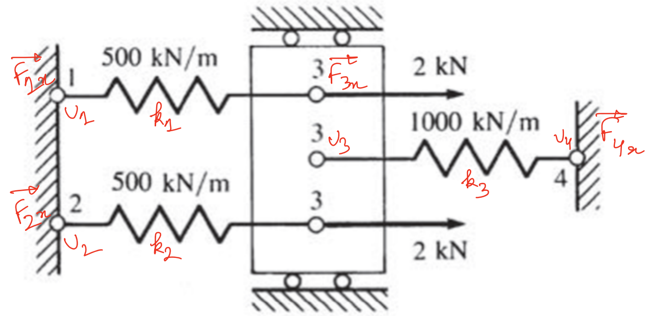
$$\boxed{v = \begin{bmatrix} \frac{-\sqrt{65}+3}{8} \\ \frac{\sqrt{65}-7}{8} \\ 1 \end{bmatrix} x}$$

$\lambda = \frac{-5-\sqrt{65}}{2}$

$$\boxed{v = \begin{bmatrix} \frac{\sqrt{65}+3}{8} \\ \frac{-\sqrt{65}-7}{8} \\ 1 \end{bmatrix} x}$$

Que 7

For spring 1 stiffness eqⁿ:-



$$\begin{bmatrix} f_{1x}^{(1)} \\ f_{3(x)}^{(1)} \end{bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{bmatrix} u_1^{(1)} \\ u_3^{(1)} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} f_{1x}^{(1)} \\ f_{3x}^{(1)} \end{bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix} \quad \text{--- (1)}$$

For spring 2 stiffness eqⁿ:-

$$\begin{bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} u_2^{(2)} \\ u_3^{(2)} \end{bmatrix} \Rightarrow \begin{bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} \text{--- (2)}$$

For spring 3 stiffness eqⁿ:-

$$\begin{bmatrix} f_{3x}^{(3)} \\ f_{4x}^{(3)} \end{bmatrix} = \begin{bmatrix} k_3 & -k_3 \\ -k_3 & k_3 \end{bmatrix} \begin{bmatrix} u_3^{(3)} \\ u_4^{(3)} \end{bmatrix} \Rightarrow \begin{bmatrix} f_{3x}^{(3)} \\ f_{4x}^{(3)} \end{bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix} \text{--- (3)}$$

Combining (1), (2) & (3) we get,

$$\boxed{\begin{bmatrix} F_{1x} \\ F_{2x} \\ F_{3x} \\ F_{4x} \end{bmatrix} = \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}}$$

As points 1, 2 & 4 are fixed $\therefore u_1 = u_2 = u_4 = 0$

Also, external force on 3 = 4kN $\therefore F_{3x} = 4\,kN$

$$\begin{bmatrix} F_{1x} \\ F_{2x} \\ 4 \\ F_{4x} \end{bmatrix} = \begin{bmatrix} 500 & 0 & -500 & 0 \\ 0 & 500 & -500 & 0 \\ -500 & -500 & 2000 & -1000 \\ 0 & 0 & -1000 & 1000 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_3 \\ u_4 \end{bmatrix}$$

$$\Rightarrow 2000\, u_3 = 4 \quad \Rightarrow \boxed{u_3 = \frac{2}{1000} = 0.002\,m}$$

$$\boxed{\begin{aligned} F_{1x} &= -500U_3 = -1\,kN \\ F_{2x} &= -500U_3 = -1\,kN \\ F_{4x} &= -1000U_3 = -2\,kN \end{aligned}}$$

Eq 1

$$\begin{bmatrix} f_{1x}^{(1)} \\ f_{3x}^{(1)} \end{bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{bmatrix} 0 \\ U_3 \end{bmatrix} \Rightarrow \begin{aligned} f_{1x}^{(1)} &= -500U_3 = -1\,kN \\ f_{3x}^{(1)} &= 500U_3 = 1\,kN \end{aligned}$$

Eq 2

$$\begin{bmatrix} f_{2x}^{(2)} \\ f_{3x}^{(2)} \end{bmatrix} = \begin{bmatrix} 500 & -500 \\ -500 & 500 \end{bmatrix} \begin{bmatrix} 0 \\ U_3 \end{bmatrix} \Rightarrow \begin{aligned} f_{2x}^{(2)} &= -500U_3 = -1\,kN \\ f_{3x}^{(2)} &= 500U_3 = 1\,kN \end{aligned}$$

Eq 3

$$\begin{bmatrix} f_{3x}^{(3)} \\ f_{4x}^{(3)} \end{bmatrix} = \begin{bmatrix} 1000 & -1000 \\ -1000 & 1000 \end{bmatrix} \begin{bmatrix} U_3 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} f_{3x}^{(3)} &= 1000U_3 = 2\,kN \\ f_{4x}^{(3)} &= -1000U_3 = -2\,kN \end{aligned}$$

Que 5 Python Code

Que 8 Python Code