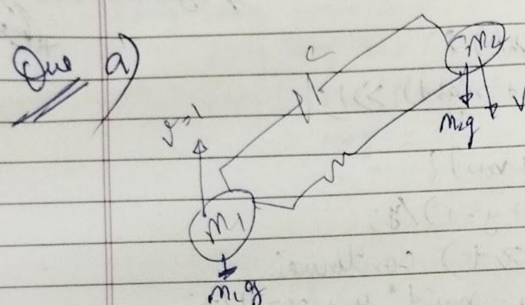
ME3030 Assignment 1
CO21BTECH11004

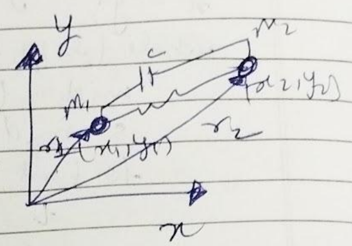
Que 1) Part a

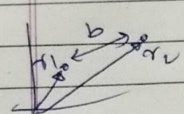
Assignment - 1

Name - Darpan Gaur
Roll No - CO21BTECH11004

Que a)







$\Rightarrow$ $b = r_2 - r_1$, $\quad \hat{b} = \dfrac{r_2 - r_1}{|r_2 - r_1|}$ direction

$|b| = |r_2 - r_1|$

Extension/Compression in spring $= |r_2 - r_1| - l$

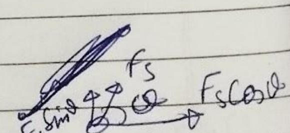
Spring Force $\Rightarrow k(|r_2 - r_1| - l)\hat{b}$

Damping Force $\Rightarrow c(\dot{r}_2 - \dot{r}_1)$

Gravitational Force $= -mg$

Eq $\Rightarrow$ $m\ddot{r} = k(|r_2 - r_1| - l)\hat{b} + c(\dot{r}_2 - \dot{r}_1) - mg$

In x-y plane
$x \rightarrow$ Take x component
$y \rightarrow$ Take y componen



$$\cos\theta = \frac{x_2 - x_1}{\underbrace{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}_{|r_2 - r_1|}}$$

$$\sin\theta = \frac{y_2 - y_1}{\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}}$$

Equations

- $$m_1 \begin{Bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \end{Bmatrix} = \frac{k(|r_2 - r_1| - l)}{|r_2 - r_1|} \begin{Bmatrix} x_2 - x_1 \\ y_2 - y_1 \end{Bmatrix} + c \begin{Bmatrix} \dot{x}_2 - \dot{x}_1 \\ \dot{y}_2 - \dot{y}_1 \end{Bmatrix} - m_1 g \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$$

- $$m_2 \begin{Bmatrix} \ddot{x}_2 \\ \ddot{y}_2 \end{Bmatrix} = -m_2 g \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} - \frac{k(|r_2 - r_1| - l)}{|r_2 - r_1|} \begin{Bmatrix} x_2 - x_1 \\ y_2 - y_1 \end{Bmatrix} - c \begin{Bmatrix} \dot{x}_2 - \dot{x}_1 \\ \dot{y}_2 - \dot{y}_1 \end{Bmatrix}$$

For solving,

put $z_1 = x_1$, $z_2 = \dot{x}_1$, $z_3 = y_1, \ldots$ $z_7 = y_2$, $z_8 = \dot{y}_2$

$$\ddot{x}_1 \quad \dot{z}_2 = \frac{k\, b \times (z_5 - z_1)}{m_1 \sqrt{(z_5 - z_1)^2 + (z_7 - z_3)^2}} + \frac{c}{m}(z_6 - z_2)$$

$$\begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{Bmatrix} = \begin{Bmatrix} z_2 \\ \dfrac{kb(z_5 - z_1)}{m_1 \sqrt{(z_5 - z_1)^2 + (z_7 - z_3)^2}} + \dfrac{c}{m}(z_6 - z_2) \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix}^{i+1} = \begin{Bmatrix} z_1 \\ z_2 \end{Bmatrix}^{i} + \Delta t \begin{Bmatrix} z_2 \\ \dfrac{kb(z_5 - z_1)}{m_1 \sqrt{(z_5 - z_1)^2 + (z_7 - z_3)^2}} + \dfrac{c(z_6 - z_2)}{m_1} \end{Bmatrix}$$

similarly for $\underset{\to y_1}{z_3}$, $z_4 \to \dot{y}_1$

$$\begin{Bmatrix} z_3 \\ z_4 \end{Bmatrix}^{i+1} = \begin{Bmatrix} z_3 \\ z_4 \end{Bmatrix}^{i} + \Delta t \begin{Bmatrix} z_4 \\ \dfrac{kb(z_7 - z_3)}{m_1 \sqrt{(z_5 - z_1)^2 + (z_7 - z_3)^2}} + \dfrac{c}{m_1}(z_8 - z_4) - g \end{Bmatrix}$$

Same for z_5, z_6, z_7, z_8

Solve iteratively

Que 1) Part B

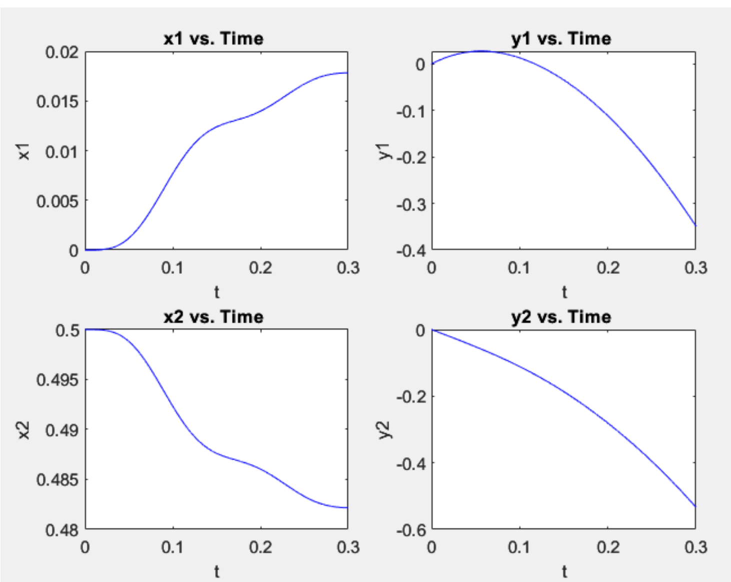
```
% Name          :- Darpan Gaur
% Roll Number   :- CO21BTECH11004
```

```
dt = 0.00001;
t = 0:dt:0.5;

% arrays to store positoin
x1 = zeros(1, length(t));
x2 = zeros(1, length(t));
y1 = zeros(1, length(t));
y2 = zeros(1, length(t));

% arrays to store velocity
x1_1 = zeros(1, length(t));
x2_1 = zeros(1, length(t));
y1_1 = zeros(1, length(t));
y2_1 = zeros(1, length(t));

% set boumdary conditions
x1(1) = 0.0;
x2(1) = 0.5;
y1(1) = 0.0;
y2(1) = 0.0;
x1_1(1) = 0.0;
x2_1(1) = 0.0;
y1_1(1) = 1.0;
y2_1(1) = -1.0;

% constants
l = 0.5;
m1 = 1.0;
m2 = 1.0;
c = 5.0;
k = 1000.0;
g = 9.81;

% array to store Energy
KE = zeros(1, length(t));
S_PE = zeros(1, length(t));
G_PE = zeros(1, length(t));
TE = zeros(1, length(t));

KE(1) = (m1*((x1_1(1))^2 + (y1_1(1))^2) + m2*((x2_1(1))^2 + (y2_1(1))^2))*0.5;
% b = |r2-r1|
b = sqrt((x2(1) - x1(1))^2 + (y2(1) - y1(1))^2);
S_PE(1) = k*(b-l)*(b-l)*0.5;
G_PE(1) = (m1*y1(1) + m2*y2(1))*g;
TE(1) = KE(1) + S_PE(1) + G_PE(1);

for i=1:length(t)-1
```

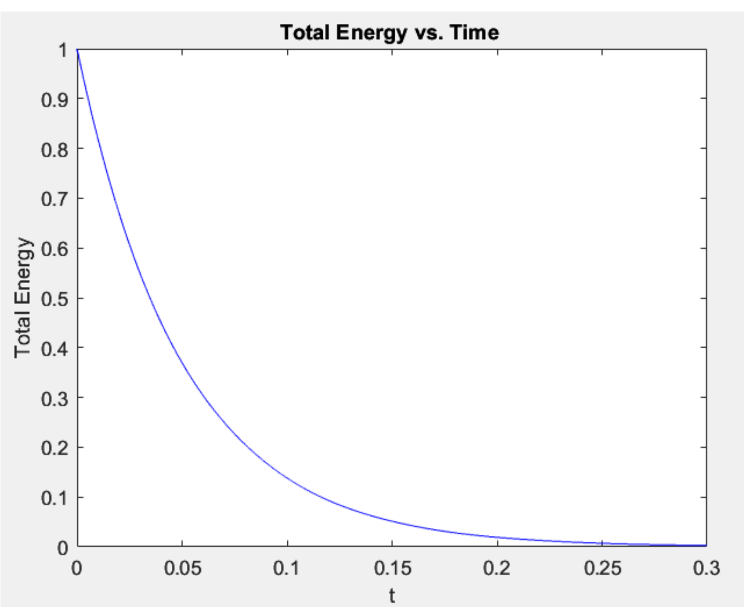
```
   % b = |r2-r1|
   b = sqrt((x2(i) - x1(i))^2 + (y2(i) - y1(i))^2);
   fSpring = k*(b-l);
   fD_x = c*(x2_1(i) - x1_1(i));
   fD_y = c*(y2_1(i) - y1_1(i));

   x1_2 = (fSpring*(x2(i)-x1(i)))/(m1*b) + fD_x/m1;
   x2_2 = -(fSpring*(x2(i)-x1(i)))/(m2*b) - fD_x/m2;
   y1_2 = -g + (fSpring*(y2(i)-y1(i)))/(m1*b) + fD_y/m2;
   y2_2 = -g -(fSpring*(y2(i)-y1(i)))/(m2*b) - fD_y/m2;
   x1_1(i+1) = x1_1(i) + x1_2*dt;
   x2_1(i+1) = x2_1(i) + x2_2*dt;
   y1_1(i+1) = y1_1(i) + y1_2*dt;
   y2_1(i+1) = y2_1(i) + y2_2*dt;
   x1(i+1) = x1(i) + x1_1(i+1)*dt;
   x2(i+1) = x2(i) + x2_1(i+1)*dt;
   y1(i+1) = y1(i) + y1_1(i+1)*dt;
   y2(i+1) = y2(i) + y2_1(i+1)*dt;
   b = sqrt((x2(i+1) - x1(i+1))^2 + (y2(i+1) - y1(i+1))^2);

   KE(i+1) = (m1*((x1_1(i+1))^2 + (y1_1(i+1))^2) + m2*((x2_1(i+1))^2 +
(y2_1(i+1))^2))*0.5;
   S_PE(i+1) = k*(b-l)*(b-l)*0.5;
   G_PE(i+1) = (m1*y1(i+1) + m2*y2(i+1))*g;
   TE(i+1) = KE(i+1) + S_PE(i+1) + G_PE(i+1);
   TE(i+1) = round(TE(i+1), 3);    % for correcting round off error
end

% plot
figure;

subplot(2,2,1);
plot(t, x1,'b');
xlabel('t');
ylabel('x1');
title('x1 vs. Time ');

subplot(2,2,2);
plot(t, y1, 'b');
xlabel('t');
ylabel('y1');
title('y1 vs. Time ');

subplot(2,2,3);
plot(t, x2, 'b');
xlabel('t');
ylabel('x2');
```

```
title('x2 vs. Time ');

subplot(2,2,4);
plot(t, y2, 'b');
xlabel('t');
ylabel('y2');
title('y2 vs. Time ');

figure;

plot(t, TE, 'b');
xlabel('t');
ylabel('Total Energy');
title('Total Energy vs. Time ');
```

Plots

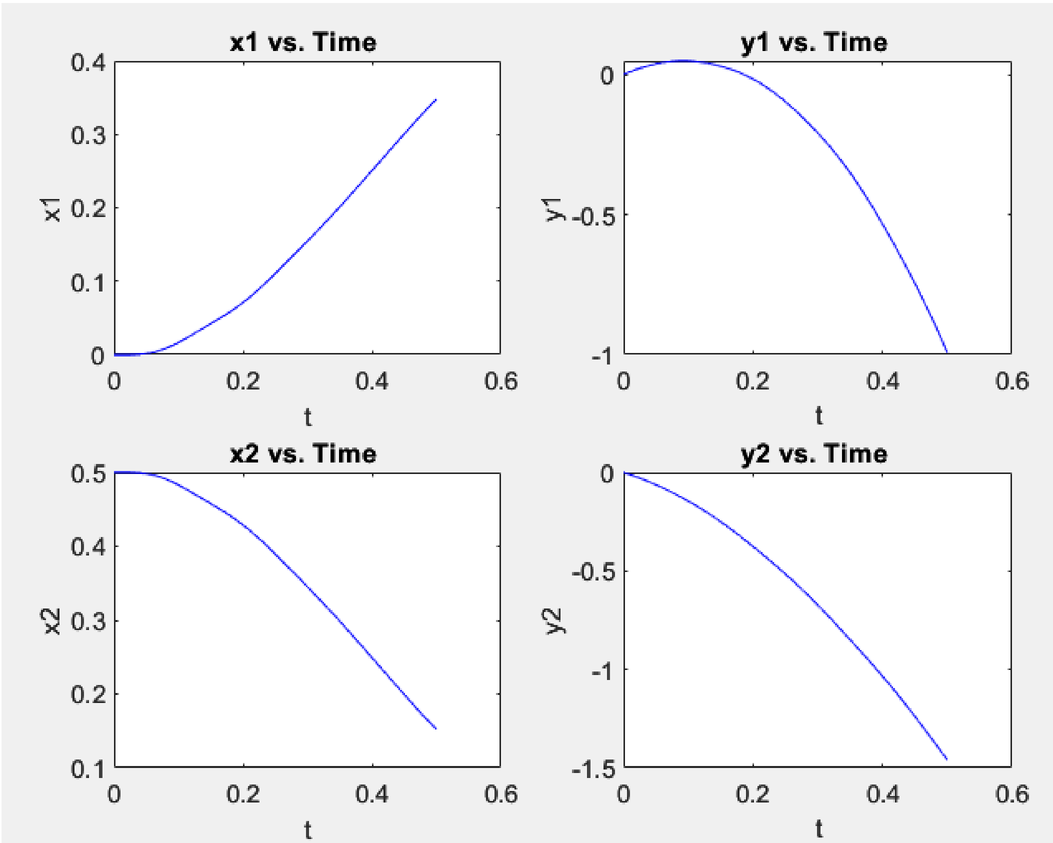


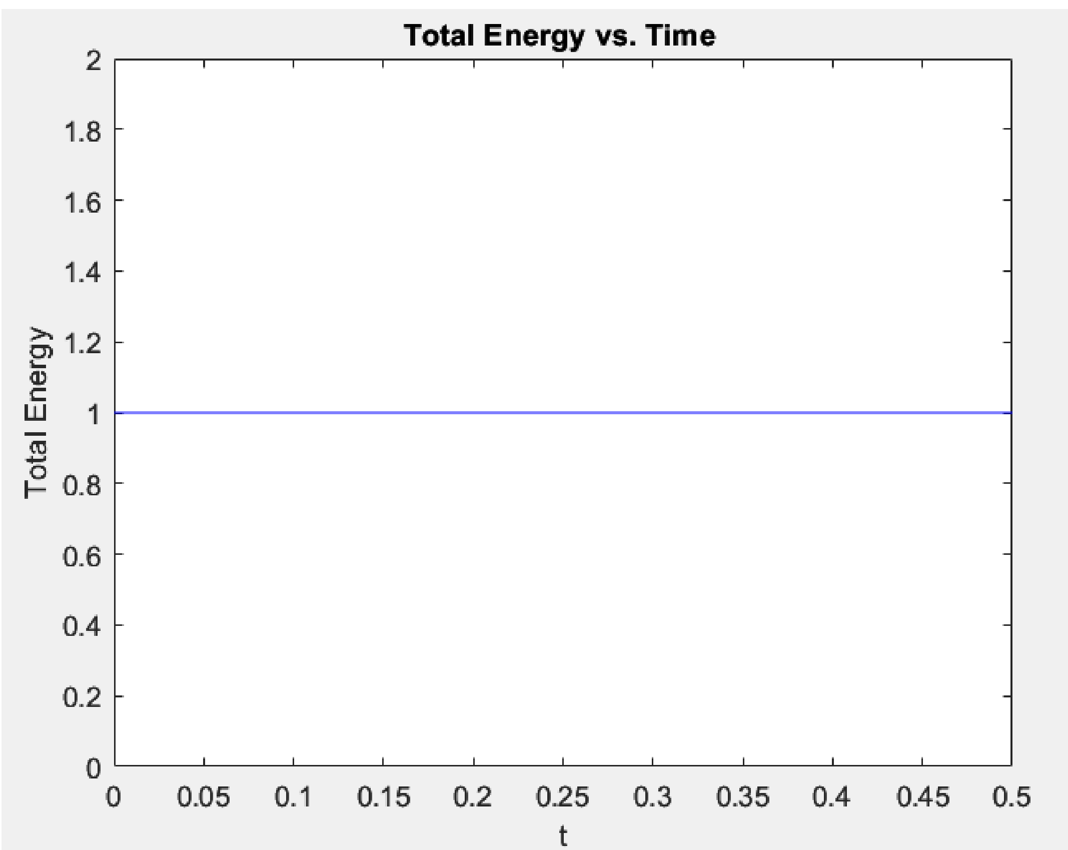
- Plots are prepared with following parameters:
  - c = 5
  - g = 9.81



- Energy decreases as damping is there. So, due to damping energy is lost.

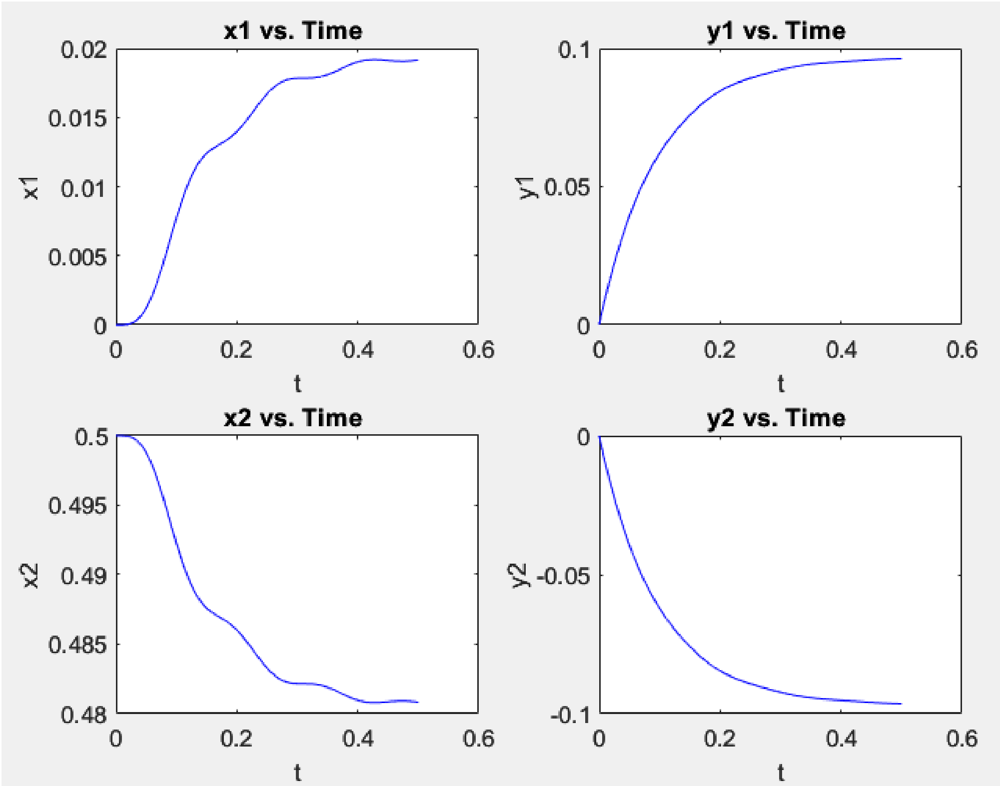
Que 1) Part C

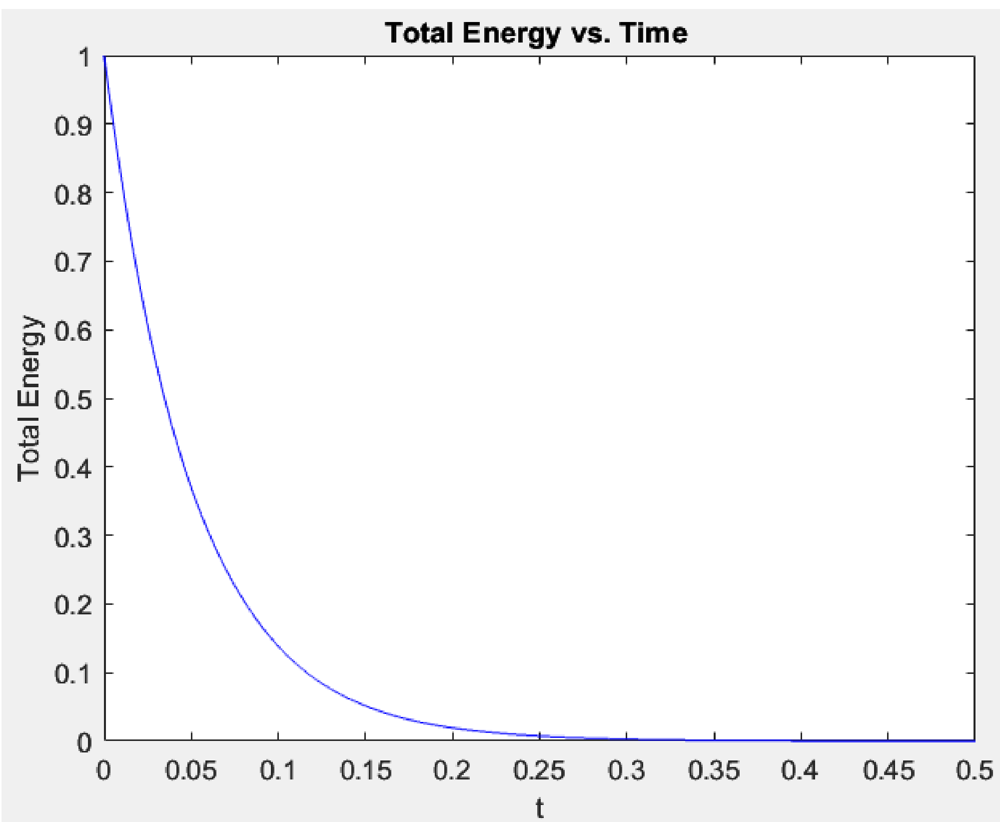




- Put c = 0.0, in the code for this part.
- Since, there is no damping so energy is almost constant.
- Smooth graph is formed, as no damping.

Que 1) Part D





- Put g = 0.0 for this part of the problem in the code.
- When no gravity, and damping present, energy decreases.
- y1 reaches a new maximum value as compared to the case when gravity is present because it tends to decrease the velocity and hence distance converd in y direction.