Theory Assignment 2
CS5280

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Problem 1

$$s = r_1(x)r_3(x)w_3(y)w_2(x)r_4(y)c_2w_4(x)c_4r_5(x)c_3w_5(z)c_5w_1(z)c_1$$

FSR
For a history $s \in \text{FSR}$, there exists a serial history s' such that $\text{LRF}(s) = \text{LRF}(s')$.
Consider $s' = t_5t_3t_1t_2t_4$. Hence, $s' = r_5(x)w_5(z)r_3(x)w_3(y)r_1(x)w_1(x)w_2(x)r_4(y)w_4(x)c_5c_3c_1c_2c_4$.

$$\text{LRF}(s) = \{(t_0, x, t_1), (t_0, x, t_3), (t_3, y, t_4), (t_4, x, t_\infty), (t_3, y, t_\infty), (t_1, z, t_\infty)\}$$
$$\text{LRF}(s') = \{(t_0, x, t_3), (t_0, x, t_1), (t_3, y, t_4), (t_4, x, t_\infty), (t_3, y, t_\infty), (t_1, z, t_\infty)\}$$

As $\text{LRF}(s) = \text{LRF}(s')$, hence $s \in \text{FSR}$.

VSR
For a history $s \in \text{VSR}$, there exists a serial history s' such that $\text{RF}(s) = \text{RF}(s')$.

$$\text{RF}(s) = \{(t_0, x, t_1), (t_0, x, t_3), (t_3, y, t_4), (t_4, x, t_5), (t_4, x, t_\infty), (t_3, y, t_\infty), (t_1, z, t_\infty)\}$$

Proof by contradiction.
Let us assume that $s \in \text{VSR}$. So there exists a serial history s' such that $\text{RF}(s) = \text{RF}(s')$.
In $\text{RF}(s)$: -

- (t_4, x, t_5), so t_4 preceeds t_5 in s', i.e., $t_4 < t_5$.
- (t_3, y, t_4), so t_3 preceeds t_4 in s', i.e., $t_3 < t_4$.
- (t_1, z, t_∞), and both t_1 and t_5 have $w(z)$ operations, so $t_5 preceeds t_1$ in s', i.e., $t_5 < t_1$.
- $(t_0, x, t_3), and (t_0, x, t_1)$, so t_1 and t_3 preceeds $w_2(x)$ hence t_2 in s', i.e., $t_1 < t_2$ and $t_3 < t_2$.

From above restrictinos we get the following order for s': $t_3 < t_4 < t_5 < t_1 < t_2$.
But this is not possible as t_2 at end in s' will change (t_4, x, t_∞) to (t_2, x, t_∞), which is not in $\text{RF}(s)$. Hence contradiction, and $s \notin \text{VSR}$.

CSR
For a history $s \in$ CSR, we need to show that the conflict graph of s is acyclic.
Conflict graph is shown in Figure 1.
As there exist a cycle $t_1 \rightarrow t_2 \rightarrow t_4 \rightarrow t_5 \rightarrow t_1$, therefore conflict graph is cyclic and $s \notin$ CSR.

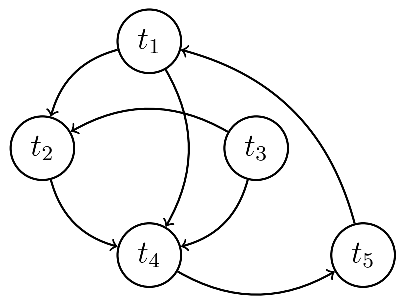

Figure 1: Conflict Graph

Problem 2

$$s = r_1(z)r_3(x)r_2(z)w_1(z)w_1(y)c_1w_2(y)w_2(u)c_2w_3(y)c_3$$

VSR
We know that history s is view serializable if there exists a serial history s' such that $\mathrm{RF}(s) = \mathrm{RF}(s')$. Consider $s' = t_2t_1t_3$. Hence, $s' = r_2(z)w_2(y)w_2(u)r_1(z)w_1(z)w_1(y)r_3(x)w_3(y)c_1c_2c_3$.

$$\mathrm{RF}(s) = \{(t_0, z, t_1), (t_0, x, t_3), (t_0, z, t_2), (t_0, x, t_\infty), (t_3, y, t_\infty), (t_2, u, t_\infty), (t_0, z, t_\infty)\}$$
$$\mathrm{RF}(s') = \{(t_0, z, t_2), (t_0, z, t_1), (t_0, x, t_3), (t_0, x, t_\infty), (t_3, y, t_\infty), (t_2, u, t_\infty), (t_0, z, t_\infty)\}$$

As $\mathrm{op}(s) = \mathrm{op}(s')$ and $\mathrm{RF}(s) = \mathrm{RF}(s')$, hence s is view serializable.

CSR
Conflict graph is shown in Figure 2.
As the conflict graph of s is cyclic, $s \notin$ CSR.

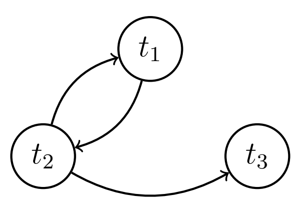

Figure 2: Conflict Graph

As the history $s \in$ VSR and $s \notin$ CSR, hence $s \in$ VSR $-$ CSR.

Problem 3

$$s = r_1(x)w_1(z)r_2(z)w_1(y)c_1r_3(y)w_2(z)c_2w_3(x)w_3(y)c_3$$

Conflict graph is shown in Figure 3.
As the conflict graph of s is acyclic, $s \in$ CSR.

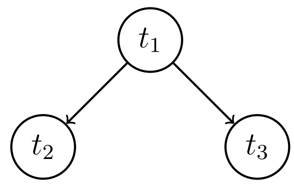

Figure 3: Conflict Graph

The equivalent serial history is $t_1t_2t_3$, and the equivalent serial schedule is
$s' = r_1(x)w_1(z)r_2(z)w_1(y)r_3(y)w_2(z)w_3(x)w_3(y)$.
As the serialization order in s' is same as the actual order in s, hence $s \in$ OCSR also holds.