Constraint (2)

$$r^{\circ}_{O_1} + R(\theta_1)\, r_p{}' = r_p{}^{\circ}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{— (5)}$$

Differentiate

$$\begin{bmatrix} \dot{x}_1 - a\dot{\theta}_1 \sin\theta_1 - b\dot{\theta}_1 \cos\theta_1 \\ \dot{y}_1 + a\dot{\theta}_1 \cos\theta_1 - b\dot{\theta}_1 \sin\theta_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Differentiate

$$\Rightarrow \ddot{x}_1 - a\ddot{\theta}_1 \sin\theta_1 - a\dot{\theta}_1^2 \cos\theta_1 + b\dot{\theta}_2^2 \sin\theta_2 - b\ddot{\theta}_1 \cos\theta_1 = 0$$

$$\Rightarrow \ddot{x}_1 + (-a\sin\theta_1 - b\cos\theta_1)\ddot{\theta}_1 = a\dot{\theta}_1^2 \cos\theta_1 - b\dot{\theta}_1^2 \sin\theta_1 \quad \text{— (6)}$$

$$\Rightarrow \ddot{y}_1 + a\ddot{\theta}_1 \cos\theta_1 - a\dot{\theta}_1^2 \sin\theta_1 - b\dot{\theta}_2^2 \cos\theta_1 - b\ddot{\theta}_2 \sin\theta_1 = 0$$

$$\Rightarrow \ddot{y}_1 + (a\cos\theta_1 - b\sin\theta_1)\ddot{\theta}_1 = a\dot{\theta}_1^2 \sin\theta_1 + b\dot{\theta}_1^2 \cos\theta_1 \quad \text{— (7)}$$

~~So we get~~

Newton Laws of Motion :-

$$q = \begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\theta}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{\theta}_2 \end{bmatrix} \qquad m = \begin{bmatrix} m_1 & & & & & \\ & m_1 & & & & \\ & & J_1 & & & \\ & & & m_2 & & \\ & & & & m_2 & \\ & & & & & J_2 \end{bmatrix} \qquad F = \begin{bmatrix} 0 \\ -m_1 g \\ 0 \\ 0 \\ -m_2 g \\ 0 \end{bmatrix}$$

Equation, we get from above. from (3), & (4)

$$U = \begin{bmatrix} 1 & 0 & a\sin\theta_1 + b\cos\theta_1 & -1 & 0 & a\sin\theta_2 + b\cos\theta_2 \\ 0 & 1 & b\sin\theta_1 - a\cos\theta_1 & 0 & -1 & b\sin\theta_2 - a\cos\theta_2 \\ 1 & 0 & -a\sin\theta_1 - b\cos\theta_1 & 0 & 0 & 0 \\ 0 & 1 & a\cos\theta_1 - b\sin\theta_1 & 0 & 0 & 0 \end{bmatrix}$$

$$v = \begin{bmatrix} \dot{\theta}_1^2 (b\sin\theta_1 - a\cos\theta_1) + \dot{\theta}_1^2 (b\sin\theta_2 - a\cos\theta_2) \\ \dot{\theta}_1^2 (a\cos\theta_1 - b\sin\theta_1) + \dot{\theta}_2^2 (-a\sin\theta_2 - b\cos\theta_2) \\ \dot{\theta}_1^2 (a\cos\theta_1 - b\sin\theta_1) \\ \dot{\theta}_1^2 (b\cos\theta_1 + a\sin\theta_1) \end{bmatrix} \text{ From (6) \& (7)}$$

$$z = [x_1\ y_1\ \theta_1\ x_2\ y_2\ \theta_2\ \dot{x}_1\ \dot{y}_1\ \dot{\theta}_1\ \dot{x}_2\ \dot{y}_2\ \dot{\theta}_2]$$

Assignment

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CO21BTECH11004

Assignment - 5

Darpan Gaur   CO21BTECH11004

Given in que the following:—

$$r^0_{O_1} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} \quad r^0_{O_2} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \quad r^0_p = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad r^1_p = \begin{bmatrix} a \\ b \end{bmatrix}$$

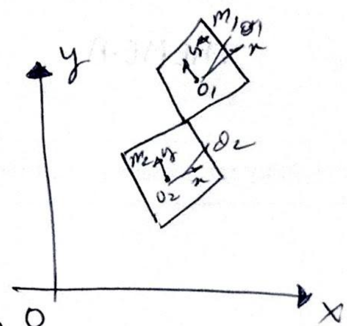
$$r^1_q = \begin{bmatrix} -a \\ -b \end{bmatrix} \quad r^2_q = \begin{bmatrix} a \\ b \end{bmatrix}.$$

Parameters:—

$m_1 = 1,\ m_2 = 2,\ J_1 = 1,\ J_2 = 2,\ g = 10,\ a = 0.2,\ b = 0.2$

Initial Conditions:—

$\theta_1(0) = \pi/2,\ \theta_2(0) = \pi/4$



Constrains (4):—

$$r^0_{O_1} + R(\theta_1) r^1_q = r^0_{O_2} + R(\theta_2) r^2_q$$

$$\Rightarrow \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} + \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} -a \\ -b \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} + \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_1 - a\cos\theta_1 + b\sin\theta_1 \\ y_1 - a\sin\theta_1 - b\cos\theta_1 \end{bmatrix} = \begin{bmatrix} x_2 + a\cos\theta_2 - b\sin\theta_2 \\ y_2 + a\sin\theta_2 + b\cos\theta_2 \end{bmatrix} \quad \text{— (1)}$$

Differentiate

$$\Rightarrow \begin{bmatrix} \dot{x}_1 + a\dot{\theta}_1 \sin\theta_1 + b\dot{\theta}_1 \cos\theta_1 \\ \dot{y}_1 - a\dot{\theta}_1 \cos\theta_1 + b\dot{\theta}_1 \sin\theta_1 \end{bmatrix} = \begin{bmatrix} \dot{x}_2 - a\dot{\theta}_2 \sin\theta_2 - b\dot{\theta}_2 \cos\theta_2 \\ \dot{y}_2 + a\dot{\theta}_2 \cos\theta_2 - b\dot{\theta}_2 \sin\theta_2 \end{bmatrix} \quad \text{— (2)}$$

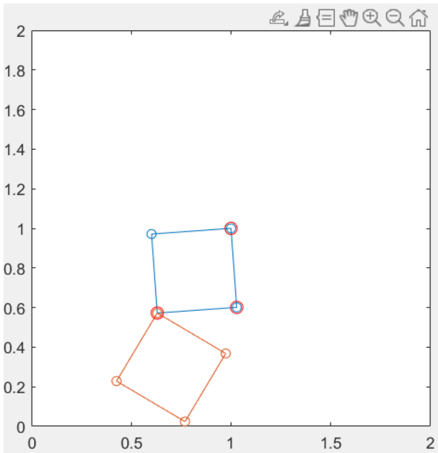
Differentiate

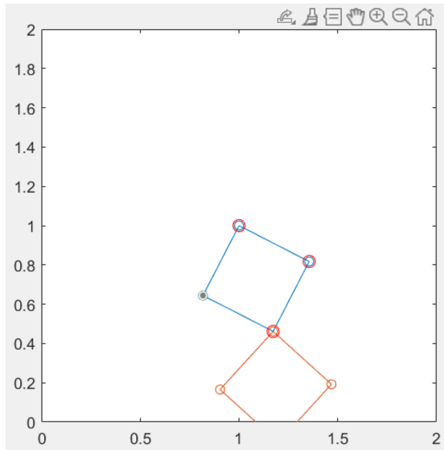
$\Rightarrow$

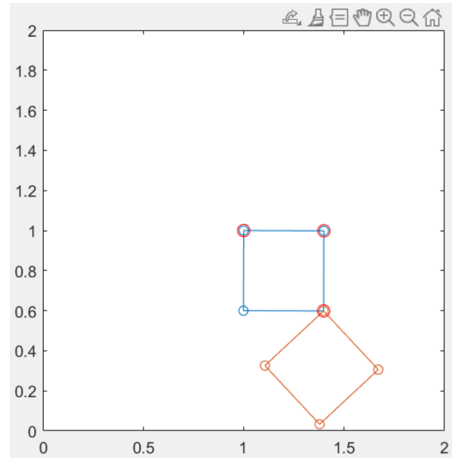
- $\ddot{x}_1 + a\dot{\theta}_1^2 \cos\theta_1 + a\ddot{\theta}_1 \sin\theta_1 + b\dot{\theta}_1^2(-\sin\theta_1) + b\ddot{\theta}_1 \cos\theta_1$
  $= \ddot{x}_2 - a\ddot{\theta}_2 \sin\theta_2 - \dot{\theta}_2^2 \cos\theta_2 - b\dot{\theta}_2^2(-\sin\theta_2) - b\ddot{\theta}_2 \cos\theta_2$ — (3)

- $\ddot{y}_1 + a\dot{\theta}_1^2 \sin\theta_1 - a\ddot{\theta}_1 \cos\theta_1 + b\dot{\theta}_1^2 \cos\theta_1 + b\ddot{\theta}_1 \sin\theta_1$
  $= \ddot{y}_2 + a\ddot{\theta}_2 \cos\theta_2 - a\dot{\theta}_2^2 \sin\theta_2 - b\dot{\theta}_2^2 \sin\theta_2 - b\ddot{\theta}_2 \cos\theta_2$ — (4)

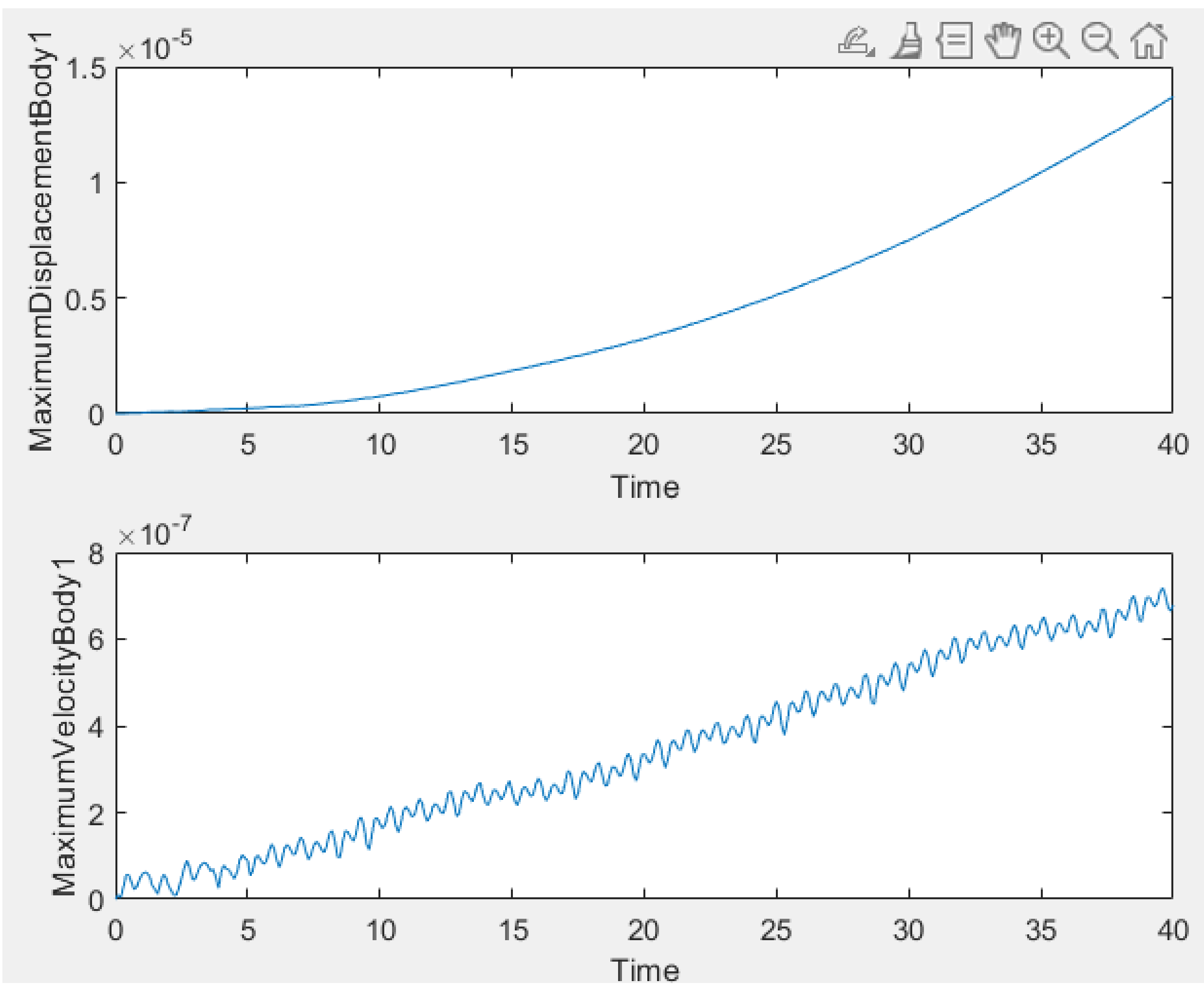
Simulation Images : -







Plot of maximum displacement and velocity: -



Code: -

```
% Name           :- Darpan Gaur
% Roll Number    :- CO21BTECH11004
% Declare global variables
global m1 m2 J1 J2 g a b
% constants
m1 = 1;
m2 = 2;
J1 = 1;
J2 = 2;
g = 10;
a = 0.2;
b = 0.2;
% Set boundary consitions
thetha1Initial = pi/2;
thetha2Initial = pi/4;
rpin = [1 1]';
Rinit1 = [cos(thetha1Initial) -sin(thetha1Initial);
sin(thetha1Initial) cos(thetha1Initial)]; % Initial rotation
matrix for body 1
Rinit2 = [cos(thetha2Initial) -sin(thetha2Initial);
sin(thetha2Initial) cos(thetha2Initial)]; % Initial rotation
matrix for body 2
rcg1 = rpin - Rinit1 * [a b]';
rcg2 = rpin - Rinit2 * [a b]';
init = [rcg1(1) rcg1(2) thetha1Initial rcg2(1) rcg2(2)
thetha2Initial 0 0 0 0 0 0]; % Initial state vector
tspan = 0:0.1:40; % Time span
options = odeset('Reltol', 1e-8, 'AbsTol', 1e-8); % ODE solver
options
% Solve the system of ODEs using ode15s
[t, z] = ode15s(@BES, tspan, init, options);
% Extract states from the solution
xcg1 = z(:, 1);
ycg1 = z(:, 2);
```

```
theta1 = z(:, 3);
xdcg1 = z(:, 7);
ydcg1 = z(:, 8);
thetad1 = z(:, 9);
xcg2 = z(:, 4);
ycg2 = z(:, 5);
theta2 = z(:, 6);
xdcg2 = z(:, 10);
ydcg2 = z(:, 11);
thetad2 = z(:, 12);
% Animation Loop
figure;
for i = 1:length(t)
   % Compute the positions of the four corners of body 1
   rcg1 = [xcg1(i) ycg1(i)]';
   R1 = [cos(theta1(i)) -sin(theta1(i)); sin(theta1(i))
cos(theta1(i))];
   R2 = [cos(theta2(i)) -sin(theta2(i)); sin(theta2(i))
cos(theta2(i))];
   r11 = rcg1 + R1 * [a b]';
   r21 = rcg1 + R1 * [-a b]';
   r31 = rcg1 + R1 * [-a -b]';
   r41 = rcg1 + R1 * [a -b]';
   % Location of P and Q for body 1
   r0P1 = rcg1;
   r1P1 = rcg1 + R1 * [a b]';
   r1Q1 = rcg1 + R1 * [-a -b]';
   r2Q1 = rcg1 + R1 * [a -b]';
   % Location of P and Q for body 2 (fixed at Q for body 1)
   rcg2 = rcg1 + R1*[-a -b]' - R2*[a b]'; % Fixed at Q for body
1
   R2 = [cos(theta2(i)) -sin(theta2(i)); sin(theta2(i))
cos(theta2(i))];
   r12 = rcg2 + R2 * [a b]';
   r22 = rcg2 + R2 * [-a b]';
```

```
   r32 = rcg2 + R2 * [-a -b]';
   r42 = rcg2 + R2 * [a -b]';
   % Plot both bodies and their pinned locations
   plot([r11(1) r21(1) r31(1) r41(1) r11(1)], [r11(2) r21(2)
r31(2) r41(2) r11(2)], 'o-');
   hold on;
   plot([r12(1) r22(1) r32(1) r42(1) r12(1)], [r12(2) r22(2)
r32(2) r42(2) r12(2)], 'o-');
   plot(r1P1(1), r1P1(2), 'ro', 'MarkerSize', 8);  % Pin
location Q for body 1
   plot(r1Q1(1), r1Q1(2), 'ro', 'MarkerSize', 8);
   plot(r2Q1(1), r2Q1(2), 'ro', 'MarkerSize', 8);
   plot(r1Q1(1), r1Q1(2), 'ro', 'MarkerSize', 8);  % Fixed
location Q for body 2
   plot(r1P1(1), r1P1(2), 'ro', 'MarkerSize', 8);  % Pin
location Q for body 2
   hold off;
   axis equal
   xlim([0 2])
   ylim([0 2])
   pause(0.1)
end
% Calculate and plot the maximum displacements for body 1
C1 = zeros(1, length(t));
Cd1 = zeros(1, length(t));
for i = 1:1:length(t)
   xc1 = 1; yc1 = 1;
   xcd1 = 0; ycd1 = 0;

   rcg1 = [xcg1(i) ycg1(i)]'; vcg1 = [xdcg1(i) ydcg1(i)]';

   rc1 = [xc1 yc1]';
   rcd1 = [xcd1 ycd1]';
   R1 = [cos(theta1(i)) -sin(theta1(i)); sin(theta1(i))
cos(theta1(i))];
```

```
   Rd1 = thetad1(i) * [-sin(theta1(i)) -cos(theta1(i));
cos(theta1(i)) -sin(theta1(i))];

   C1(i) = max(abs(rcg1 + R1 * [a b]' - rc1));
   Cd1(i) = max(abs(vcg1 + Rd1 * [a b]' - rcd1));
end
% Plot the results
figure;
subplot(2,1,1);
plot(t, C1)
xlabel('Time')
ylabel('MaximumDisplacementBody1')
subplot(2,1,2);
plot(t, Cd1)
xlabel('Time')
ylabel('MaximumVelocityBody1')
function zdot=BES(t,z)
global m1 m2 J1 J2 a b g A omega
M=diag([m1 m1 J1 m2 m2 J2]);
F=[0 -m1*g 0 0 -m2*g 0]';
theta1=z(3);
theta2=z(6);
theta1d=z(9);
theta2d=z(12);
U=[1 0 a*sin(theta1)+b*cos(theta1) -1 0
a*sin(theta2)+b*cos(theta2);
  0 1 b*sin(theta1)-a*cos(theta1) 0 -1
b*sin(theta2)-a*cos(theta2);
  1 0 -a*sin(theta1)-b*cos(theta1) 0 0 0;
  0 1 a*cos(theta1)-b*sin(theta1) 0 0 0];
v=[theta1d^2*(b*sin(theta1)-a*cos(theta1)) +
theta2d^2*(b*sin(theta2)-a*cos(theta2));
  theta1d^2*(-a*sin(theta1)-b*cos(theta1)) +
theta2d^2*(-a*sin(theta2)-b*cos(theta2));
  theta1d^2*(a*cos(theta1)-b*sin(theta1));
```

```
  theta1d^2*(b*cos(theta1)+a*(sin(theta1)))];
acc=M\F+(M^(-0.5))*pinv(U*(M^(-0.5)))*(v-U*(M\F));
zdot=[z(7) z(8) z(9) z(10) z(11) z(12) acc']';
end
```