HW5

CO21BTECH11004

Que 1) and Que 2) b derivation Handwritten. Change the timestep in code for Q2 c) part.

Que 2): -

(a) 4^{th} order central difference scheme: -

- $i = 0 \rightarrow boundary\ condition$
- $i = 1 \rightarrow \left(\frac{\partial^2 T}{\partial x^2}\right)_1 = \frac{1}{(\Delta x)^2}\left[\frac{10T_0 - 15T_1 - 4T_2 + 14T_3 - 6T_4 + T_5}{12}\right]$
- $i = 2,3,..,nx-3 \rightarrow \left(\frac{\partial^2 T}{\partial x^2}\right)_i = \frac{1}{(\Delta x)^2}\left[\frac{16(T_{i+1}+T_{i-1}) - (T_{i+2}+T_{i-2}) - 30T_i}{12}\right]$
- $i = nx-2 \rightarrow \left(\frac{\partial^2 T}{\partial x^2}\right)_{nx-2} = \left[\frac{10T_{nx-1} - 15T_{nx-2} - 4T_{nx-3} + 14T_{nx-4} - 6T_{nx-5} + T_{nx-6}}{12(\Delta x)^2}\right]$
- $i = nx-1 \rightarrow boundary\ condition$

For $i = 1\ and\ i = nx - 2$, finding 4^{th} order scheme for double derivative requires 5 points as after taking five points we get $O(h^6)$ and for double derivative $O\left(\frac{h^6}{h^2}\right) = O(h^2)$ using one-sided stencils. These formulae found using Taylor series expansion.

In the case of central difference, for $i = 2,3,\ldots,nx-3$ due to symmetry by taking 4 other points, h^5 terms get cancel and we get $O(h^6)$ terms, which for double derivate become $O(h^4)$.

(b) From Von-Neumann stability we get,

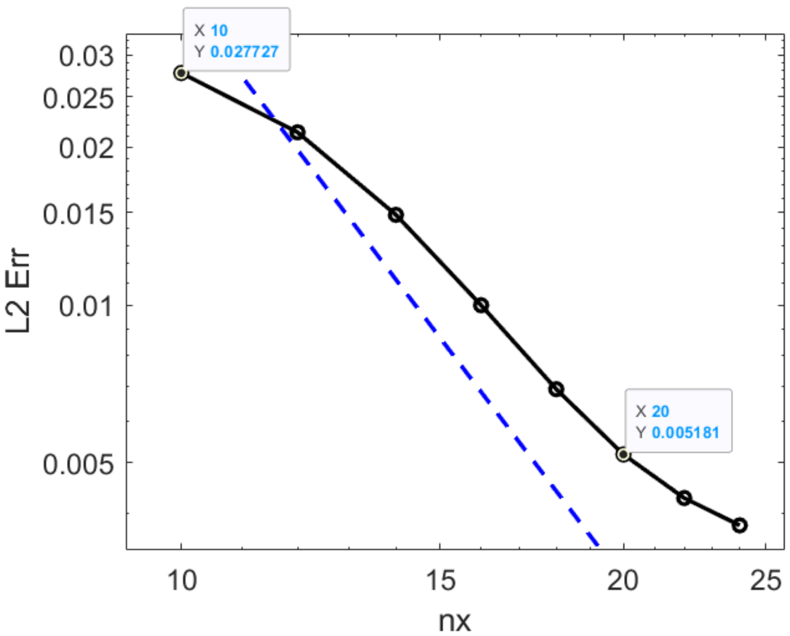
$$r = \frac{\kappa(\Delta t)}{(\Delta x)^2} < \frac{2}{5.33} \sim 0.375$$

- $\Delta t = 1.25 \times 10^{-7}, \Delta x > 0.58 \times 10^{-3}$
  $nx < 1724$, which is true error don't blow up till $nx = 1000$.
- $\Delta t = 3.125 \times 10^{-6}, \Delta x > 2.88 \times 10^{-3}$
  $nx < 347$, which is true solution error start blowing up after $nx = 350$.

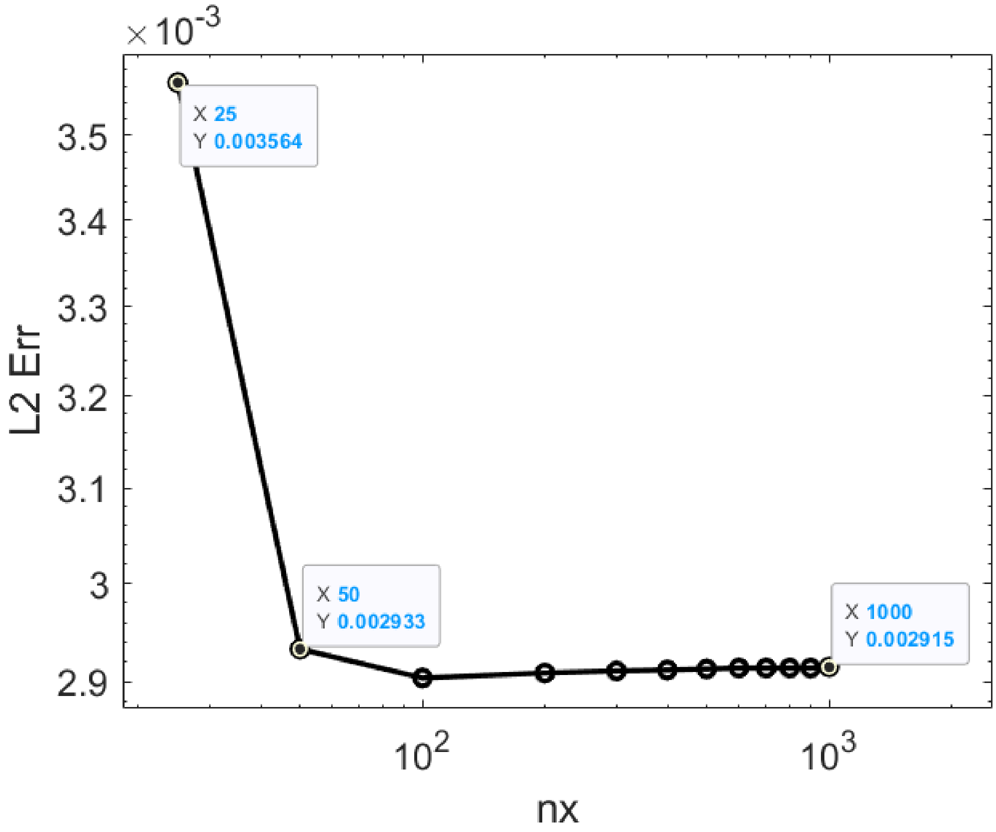
(c)

$$\boldsymbol{For\ part\ (a){:}-\ \Delta t = 1.25 \times 10^{-7}}$$

For small nx, $10 \le nx \le 24$, loglog error graph, error approximately $O(h^4)$.

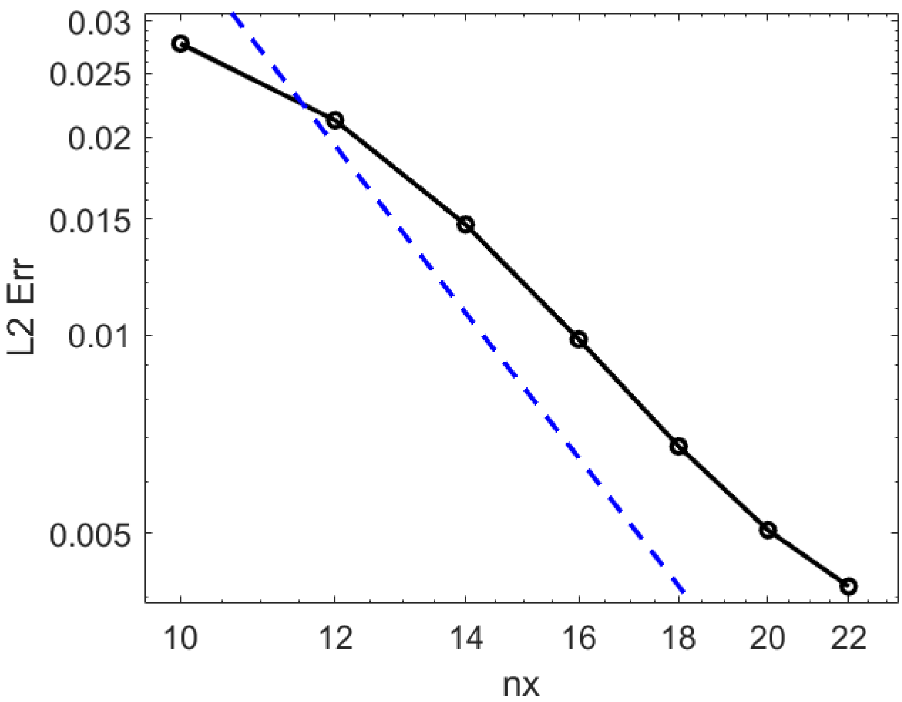


For $20 \le nx \le 1000$, loglog error graph saturates after $nx = 100$ and didn't blow up till $nx = 1000$, which can be seen from stability analysis by which we got, for $nx < 1724$, solution is stable.

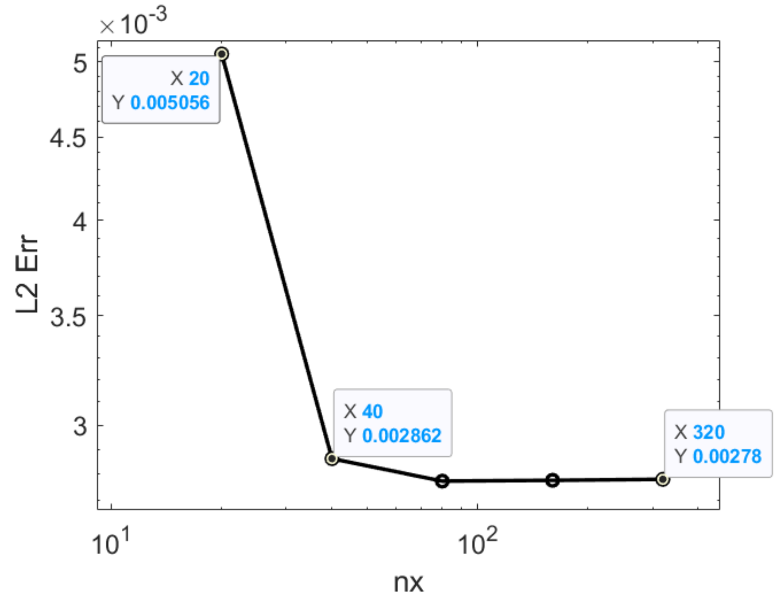


For part (b): $-\,\Delta t = 3.125 \times 10^{-6}$

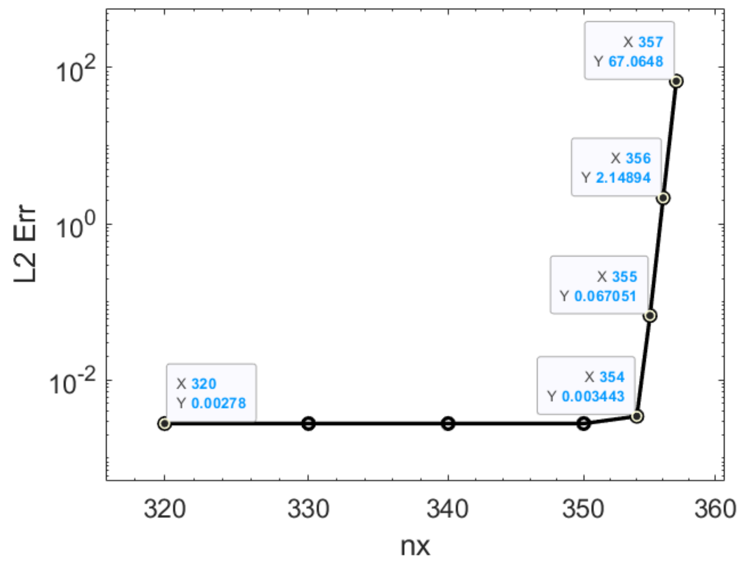
For small nx, $10 \le nx \le 22$, loglog error graph, error approximately $O(h^4)$.



For $20 \leq nx \leq 320$, error saturates.



For $320 \leq nx \leq 360$, error start growing beyond $nx = 350$, which is close to what we got from stability analysis, $nx < 347$ for stable solution.



Combined loglog error graph for both time,