Theory Assignment 1
CS5280

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Problem 1

To show that prefix order defined on Σ^* is a partial order, we need to show that it is reflexive, anti-symmetric and transitive.

(a) Reflexivity: $(a, a) \in \Sigma^*$
Let $a \in \Sigma^*$, then a is prefix of itself.
$\implies a = a \quad \implies a \preceq a$
Hence, $(a, a) \in \Sigma^*$, so relation is reflexive.

(b) Anti-symmetry: $(a, b) \in \Sigma^*$ and $(b, a) \in \Sigma^* \implies a = b$
Let $a, b \in \Sigma^*$ such that $(a, b) \in \Sigma^*$ and $(b, a) \in \Sigma^*$
$\implies a$ is prefix of b and b is prefix of a
This can only happen when $a = b$
Hence, $(a, b) \in \Sigma^*$ and $(b, a) \in \Sigma^* \implies a = b$

(c) Transitivity: $(a, b) \in \Sigma^*$ and $(b, c) \in \Sigma^* \implies (a, c) \in \Sigma^*$
Let $a, b, c \in \Sigma^*$ such that a is prefix of b and b is prefix of c
$\implies a \preceq b$ and $b \preceq c$
$\implies a \preceq c$
Hence, $(a, b) \in \Sigma^*$ and $(b, c) \in \Sigma^* \implies (a, c) \in \Sigma^*$

Problem 2

$$s = r_1(x)r_2(y)w_1(y)r_3(z)w_3(z)r_2(x)w_2(z)w_1(x)c_1c_2c_3$$

(a) $H[s]$

$$H[s](x) = H_s(w_1(x)) = f_{1x}(H_s(r_1(x))) = f_{1x}(H_s(w_0(x))) = f_{1x}(f_{0x}())$$

$$H[s](y) = H_s(w_1(y)) = f_{1y}(H_s(r_1(x))) = f_{1y}(H_s(w_0(x))) = f_{1y}(f_{0x}())$$

$$\begin{aligned}
H[s](z) &= H_s(w_2(z)) \\
&= f_{2z}(H_s(r_2(x), H_s(r_2(y)))) \\
&= f_{2z}(H_s(w_0(x)), H_s(w_0(y))) \\
&= f_{2z}(f_{0x}(), f_{0y}())
\end{aligned}$$

(b) RF, LRF

$$RF(s) = \{(t_0, x, t_1), (t_0, y, t_2), (t_0, z, t_3), (t_0, x, t_2), (t_1, x, t_\infty), (t_1, y, t_\infty), (t_2, z, t_\infty)\}$$

$$LRF(s) = \{(t_0, x, t_1), (t_0, y, t_2), (t_0, x, t_2), (t_1, x, t_\infty), (t_1, y, t_\infty), (t_2, z, t_\infty)\}$$

(c) Step Graph Find the step graph of the schedule s in the figure 1.

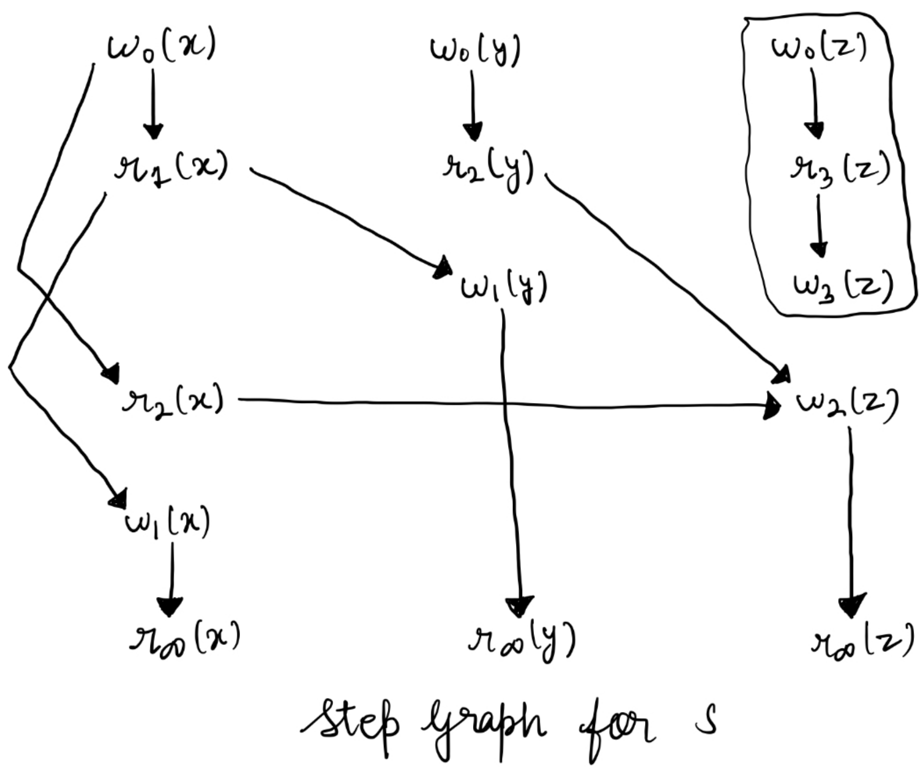

Figure 1: Step Graph

Problem 3

$$s = r_3(z) r_1(y) w_3(z) w_1(y) r_1(x) r_2(y) w_2(y) w_1(x) r_2(x) w_2(x) c_1 c_2 c_3$$

$$s' = r_3(z) w_3(z) r_2(y) w_2(y) r_1(y) w_1(y) r_2(x) w_2(x) r_1(x) w_1(x) c_3 c_2 c_1$$

(a) $H[s]$

$$\begin{aligned}
H(s)[x] &= H_s(w_2(x)) \\
&= f_{2x}(H_s(r_2(x)), H_s(r_2(y))) \\
&= f_{2x}(H_s(w_1(x)), H_s(w_1(y))) \\
&= f_{2x}(f_{1x}(H_s(r_1(x)), H_s(r_1(y))), f_{1y}(H_s(r_1(y))) \\
&= f_{2x}(f_{1x}(f_{0x}(), f_{0y}()), f_{1y}(f_{0y}()))
\end{aligned}$$

$$\begin{aligned}
H(s)[y] &= H_s(w_2(y)) \\
&= f_{2y}(H_s(r_2(y))) \\
&= f_{2y}(H_s(w_1(y))) \\
&= f_{2y}(f_{1y}(H_s(r_1(y)))) \\
&= f_{2y}(f_{1y}(f_{0y}()))
\end{aligned}$$

$$\begin{aligned}
H(s)[z] &= H_s(w_3(z)) \\
&= f_{3z}(H_s(r_3(z))) \\
&= f_{3z}(f_{0z}())
\end{aligned}$$

(b) $H[s']$

$$\begin{aligned}
H(s)[x] &= H_s(w_1(x)) \\
&= f_{1x}(H_s(r_1(x)), H_s(r_1(y))) \\
&= f_{1x}(H_s(w_2(x)), H_s(w_2(y))) \\
&= f_{1x}(f_{2x}(H_s(r_2(x)), H_s(r_2(y))), f_{2y}(H_s(r_2(y)))) \\
&= f_{1x}(f_{2x}(f_{0x}(), f_{0y}()), f_{2y}(f_{0y}()))
\end{aligned}$$

$$\begin{aligned}
H(s)[y] &= = H_s(w_1(y)) \\
&= f_{1y}(H_s(r_1(y))) \\
&= f_{1y}(H_s(w_2(y))) \\
&= f_{1y}(f_{2y}(H_s(r_2(y))) \\
&= f_{1y}(f_{2y}(f_{0y}()))
\end{aligned}$$

$$\begin{aligned}
H(s)[z] &= H_s(w_3(z)) \\
&= f_{3z}(H_s(r_3(z))) \\
&= f_{3z}(f_{0z}())
\end{aligned}$$