ME30180: Finite Element Method

Homework-3

Due Date: September 10, 2024

1. Consider a four-node cubic element in one dimension. The element length is 3 with nodes being equally spaced.

   (a) Construct the element shape functions.

   (b) Find the displacement field, $u(x)$, in the element when

   $$\boldsymbol{d}^e = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = 10^{-3} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 4 \end{bmatrix}$$

   (c) Evaluate the B^e matrix and find the strain for the above displacement field.

   (d) Plot the displacement $u(x)$ and strain $\epsilon(x)$.

   (e) Find the strain field when the nodal displacements are $\boldsymbol{d}^e = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. Why is this result expected?

2. Consider a five-node element in one dimension. The element length is 4, with node 1 at $x = 2$, and the remaining nodes are equally spaced along the x-axis.

   (a) Construct the element shape functions.

   (b) The temperatures at the nodes are given by $T_1 = 3°C; T_2 = 1°C; T3 = 0°C; T_4 = -1°C; T_5 = 2°C$. Find the temperature field at $x = 3.5$ using shape functions constructed in (a).

3. Use Gauss quadrature to obtain exact values for the following integrals. Verify by analytical integration:

   (a) $\int_0^4 (x^2 + 1)dx$

   (b) $\int_{-1}^1 (\zeta^4 + 2\zeta^2)d\zeta$

   (c) $\int_{-1}^1 \frac{\zeta}{(\zeta^2+1)} dx$

   (d) $\int_{-1}^1 cos^2(\pi\zeta)d\zeta$

   (e) $\int_{-1}^1 (3\zeta^3 + 2)d\zeta$

   (f) Write a code that performs Gauss integration. Check your manual calculations against the code.

4. Construct the shape functions for the five-node triangular element shown in Figure below, which has quadratic shape functions along two sides and linear shape functions along the third. Ensure all shape functions for all nodes are linear between nodes 1 and 2. Use triangular coordinates and express your answer in terms of triangular coordinates.

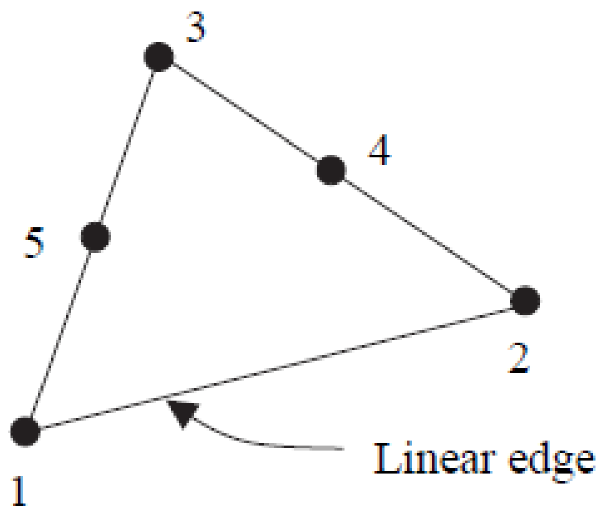

Figure 1: Five-node triangular element.

5. Develop the finite element equations for heat conduction with surface convection. The strong form, in this case, is given by

$$\kappa A\left(\frac{d^2T}{dx^2}\right) = \beta h\left(T - T_\infty\right), \quad 0 \le x \le l$$

in which κ, h, A, β and, T_∞ is moment, are constants. $\beta = 2\pi r$ is the perimeter of the fin.

6. Consider a three-node quadratic element in one dimension with unequally spaced nodes (Figure below)

Figure 2: Linear element.

(a) Obtain the B^e matrix.

(b) Consider an element with $x_1 = 0$, $x_2 = 0.25$ and $x_3 = 1$. Evaluate strain e in terms of u_2 and $u_3(u_1 = 0)$, and check what happens when ζ approaches 0.

(c) If you evaluate $\boldsymbol{K}^e$ by one-point quadrature using $\boldsymbol{B}^{e^T}\boldsymbol{E}^e\boldsymbol{A}^e\boldsymbol{D}\boldsymbol{B}^e$ for same coordinates as in (b) and constrain node 1 (i.e., $u_1 = 0$), is $\boldsymbol{K}^e$ invertible?

(d) If $u(x)$ in part (b) is given by $1/2x^2$ at the nodes, does $\epsilon = x$?

7. Determine the Jacobian matrix for the $(x, y) - (\zeta, \eta)$ transformation for the element shown in Figure below. Also, find the area of the triangle using the Jacobian.

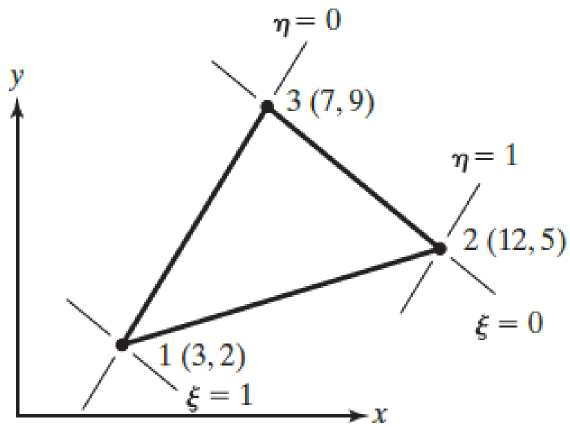

Figure 3: Triangular element.

8. The nodal coordinates of a triangular element are shown in the Figure below. At the interior point P, the x-coordinate is 3.3 and $N_1 = 0.3$. Determine N_2, N_3, and the y-coordinate at point P.

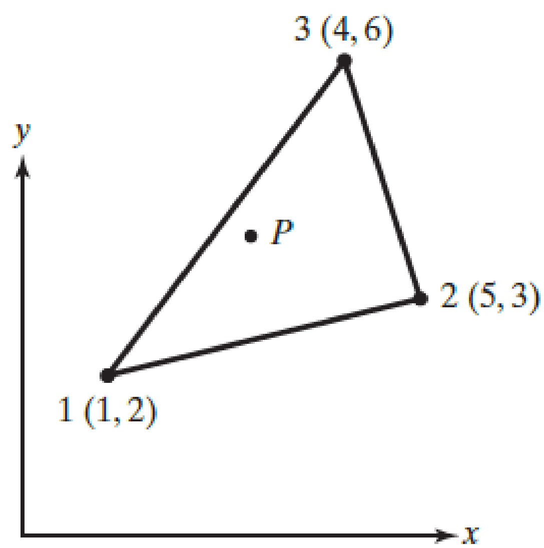

Figure 4: Triangular element.