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HW4

Que 1 a) $T_{ex} = x(1-x)\cos(\pi y)$ → Exact solution

for $k=1$ on domain $L_x = 1$ & $L_y = 0.5$

For $k=1$, heat conductivity equation,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} = 0$$

transforms to,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \dot{q} = 0 \quad \text{— (1)}$$

$$\frac{\partial T_{ex}}{\partial x} = (1-2x)\cos\pi y$$

$$\frac{\partial^2 T_{ex}}{\partial x^2} = -2\cos\pi y$$

$$\frac{\partial T_{ex}}{\partial y} = -\pi(x(1-x))\sin\pi y$$

$$\frac{\partial^2 T_{ex}}{\partial y^2} = -\pi^2(x(1-x))\cos\pi y$$

Put in (1),

$$-2\cos\pi y - \pi^2(x(1-x))\cos\pi y + \dot{q} = 0$$

$$\Rightarrow \boxed{\dot{q} = \cos\pi y\,(2 + \pi^2(x(1-x)))}$$

Que 2 a) $T_{ex} = x(1-x)\cos \pi y \qquad k = A + BT = 1 + 0.8T$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \dot{q} = 0$$

$$\frac{\partial T_{ex}}{\partial x} = (1-2x)\cos \pi y \qquad \frac{\partial T_{ex}}{\partial y} = -\pi(x(1-x))\sin(\pi y)$$

$$\frac{\partial^2 T_{ex}}{\partial x^2} = -2\cos \pi y \qquad \frac{\partial^2 T_{ex}}{\partial y^2} = -\pi^2(x(1-x))\cos(\pi y)$$

$$\frac{\partial k}{\partial x} = 0.8\frac{\partial T_{ex}}{\partial x} \qquad \frac{\partial k}{\partial y} = 0.8\frac{\partial T_{ex}}{\partial y}$$

$$\rightarrow \frac{\partial k}{\partial x}\cdot\frac{\partial T}{\partial x} + k\frac{\partial^2 T}{\partial x^2} + \frac{\partial k}{\partial y}\cdot\frac{\partial^2 T}{\partial y^2} + \dot{q} + k\frac{\partial^2 T}{\partial y^2} = 0$$

$$\Rightarrow k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + 0.8\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right) + \dot{q} = 0$$

$$\Rightarrow \dot{q} = -k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) - 0.8\left(\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right)$$

$$\dot{q} = (1 + 0.8\,x(1-x)\cos \pi y)\left[\cos \pi y\,(2 + \pi^2 x(1-x))\right] -$$

$$0.8\left(((1-2x)\cos \pi y)^2 + (\pi(x(1-x))\sin \pi y)^2\right)$$