Assignment – 1 Initial Value Problem
CO21BTECH11004

Detailed description of method used can be understood by code for each problem.

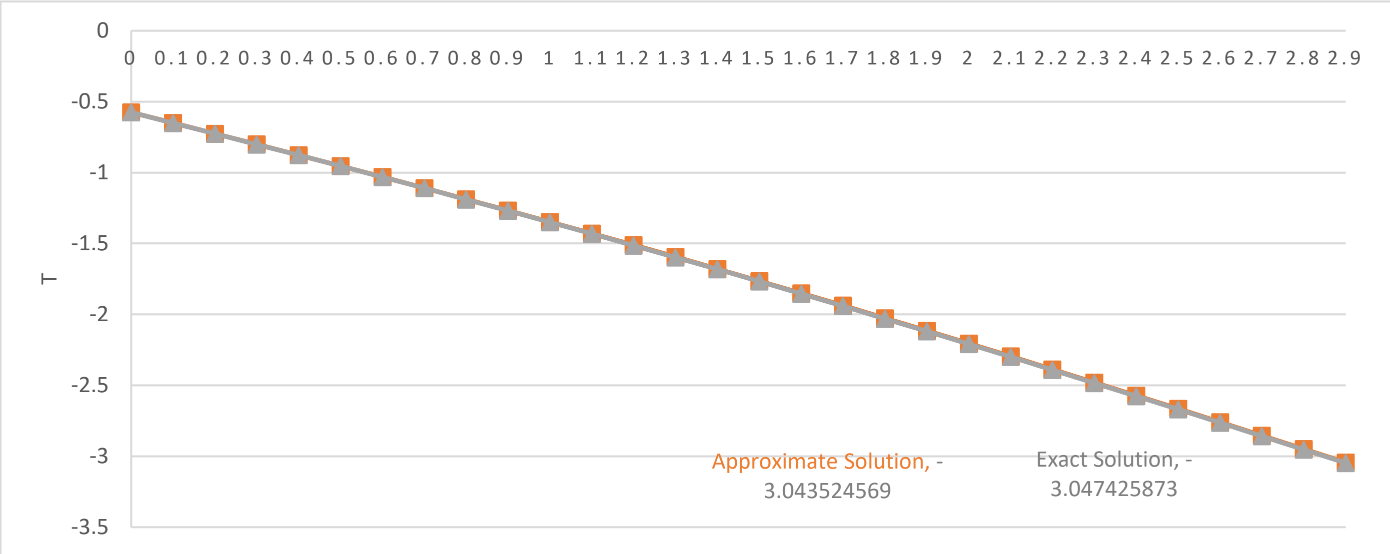
Que1: -

Using Euler's Method: - $x_{i+1} = x_i + h \cdot f(x_i, t_i)$

$$f(x,t) = \frac{dx}{dt} = -(1+t+t^2) - (2t+1)x - x^2$$

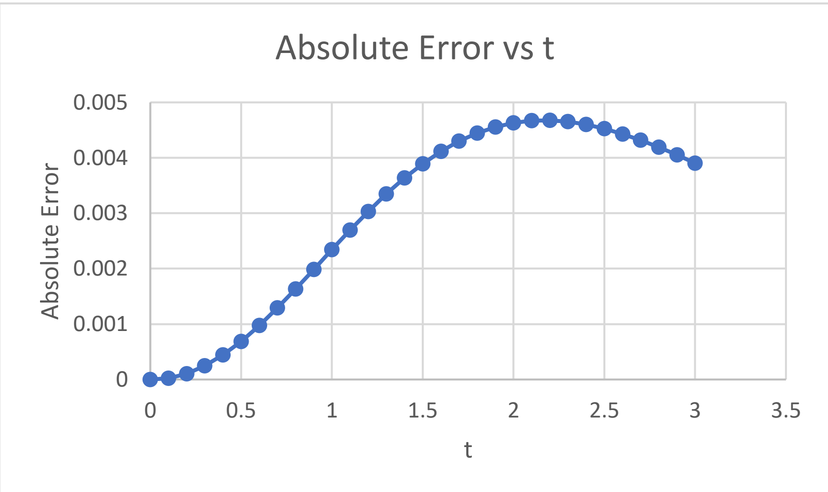
(a) $0 \le t \le 3,\ x(0) = -0.5,\ Exact\ Solution: x(t) = -t - \frac{1}{e^t+1}$

Illustrating using h=0.1



Convergence Rate: - The table show, when step size is halved the error also get reduced by same factor. Euler Method:- O(h)

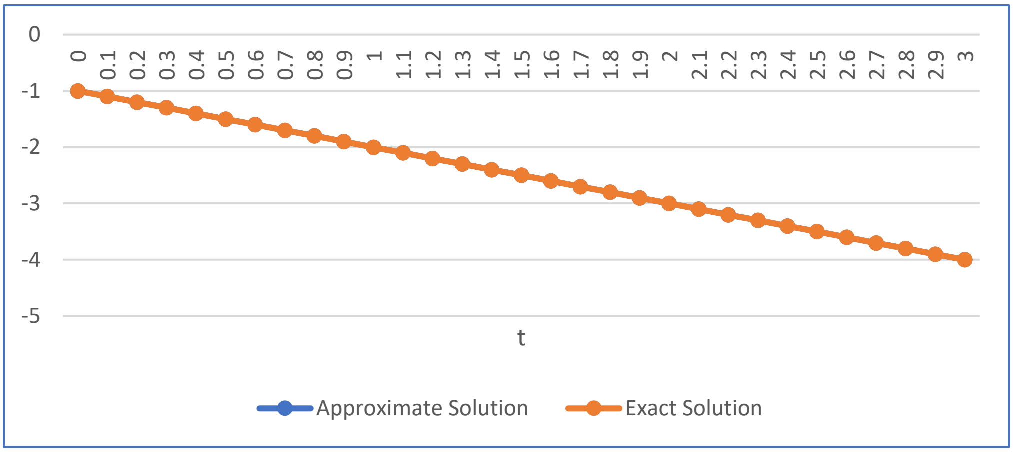
h	Approximate Solution	Exact Solution	Absolute Error	Error Ratio
1/2	-3.027101223	-3.047425873	0.02032465	
1/4	-3.037527739	-3.047425873	0.009898134	2.053381971
1/8	-3.042537482	-3.047425873	0.004888391	2.024824474
1/16	-3.044996269	-3.047425873	0.002429604	2.012011291
1/32	-3.046214651	-3.047425873	0.001211222	2.005911683
1/64	-3.046821149	-3.047425873	0.000604724	2.002933069
1/128	-3.047123732	-3.047425873	0.000302141	2.001460928
1/256	-3.047274858	-3.047425873	0.000151016	2.000729073
1/512	-3.047350379	-3.047425873	7.54941E-05	2.00036419



Error increases as t increase then decrease afterwards towards end.

(b) $0 \le t \le 3,\ x(0) = -1,\ Exact\ Solution: x(t) = -t - 1$

Illustrating using h = 0.1, absolute solution vs t & Exact solution vs t

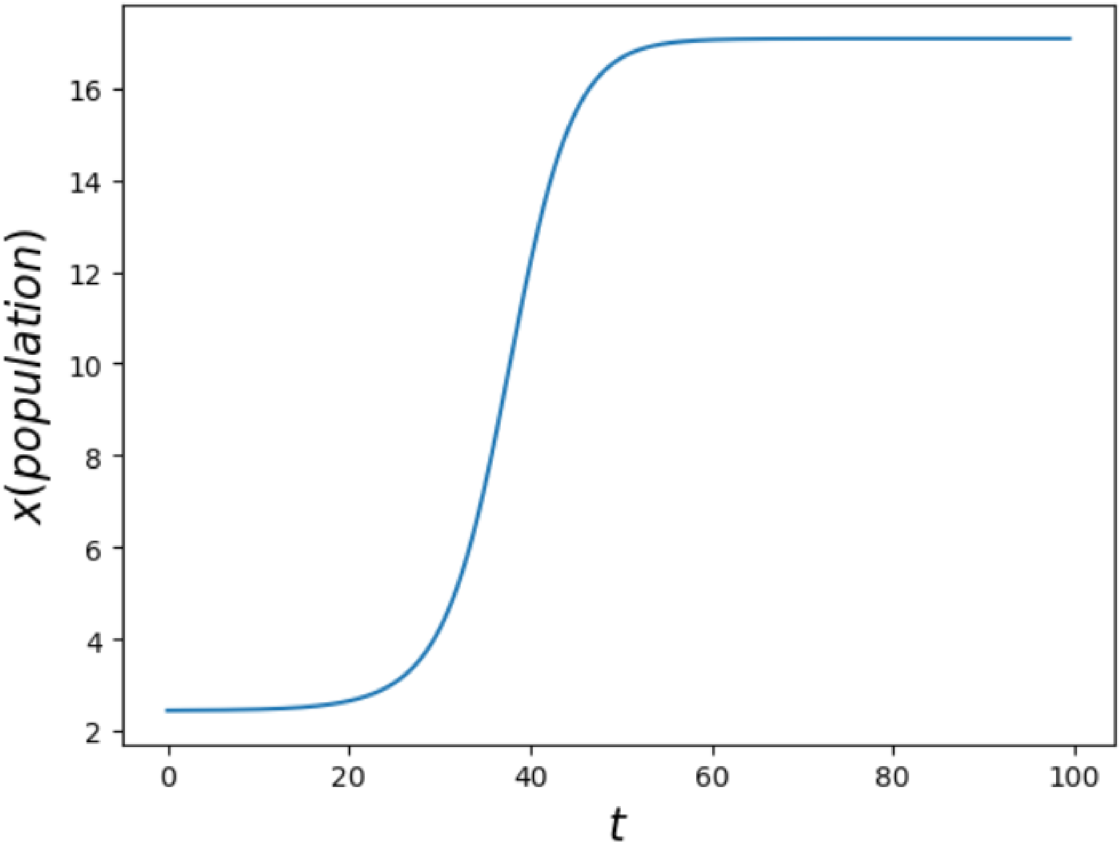


- As the given function is straight line, it is best approximated by Euler's method as compared to function given in part (a).
- Error increase as t (time) increases.
- In this problem also, error changes with the same rate as the step size, so Error: - O(h).

Que 2: - Using Euler's Method: - $x_{i+1} = x_i + h \cdot f(x_i, t_i)$

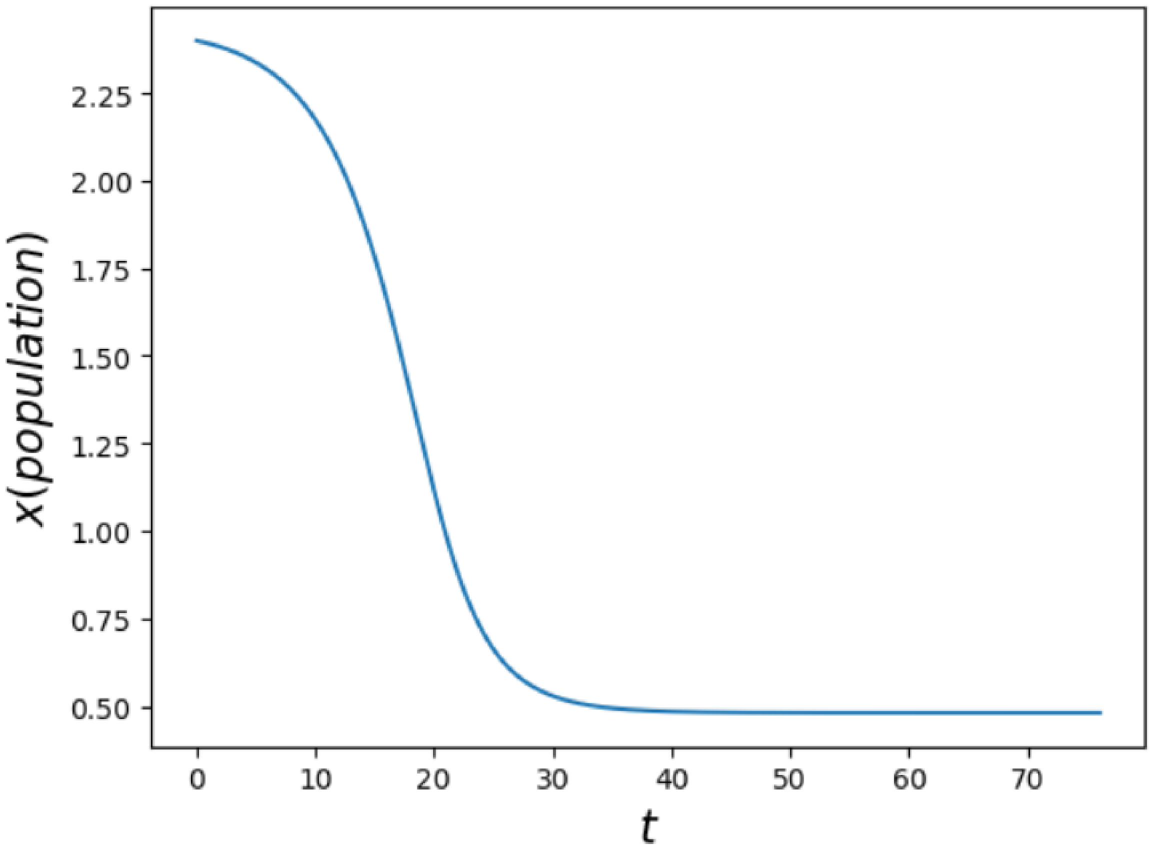
$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{x^2}{(1 + x^2)}$$

(a) Plot between population and t



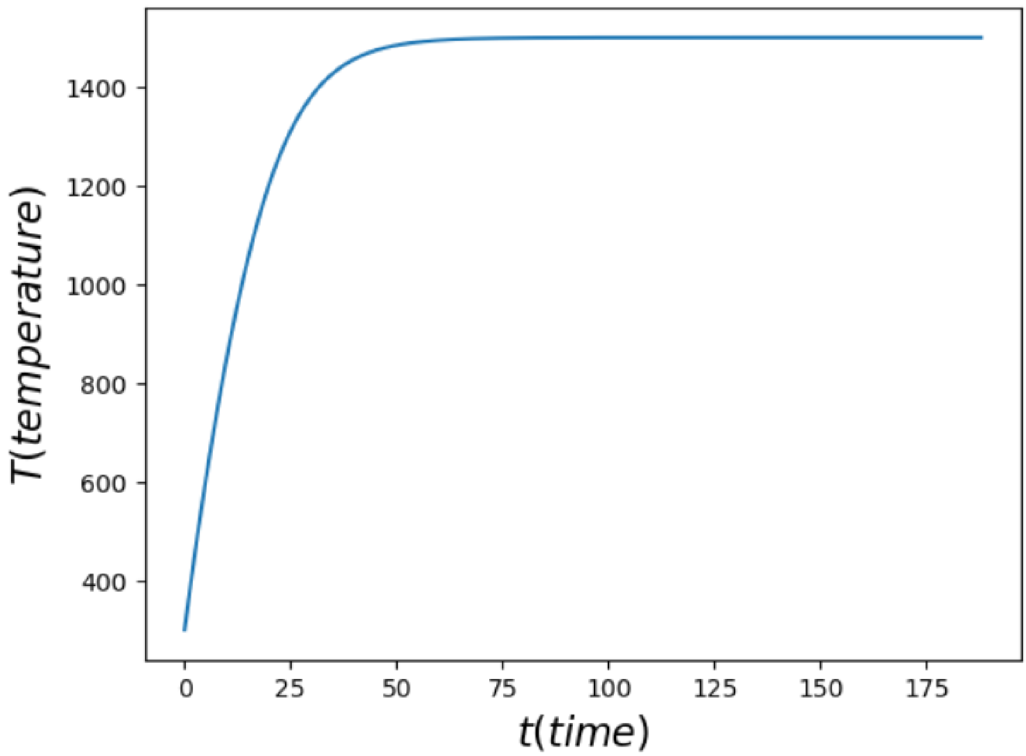
The eventual population level reached from an initial population of 2.44, is ≈ 17.083131

(b) Plot between population and t



The eventual population level reached from an initial population of 2.44, is ≈ 0.480536

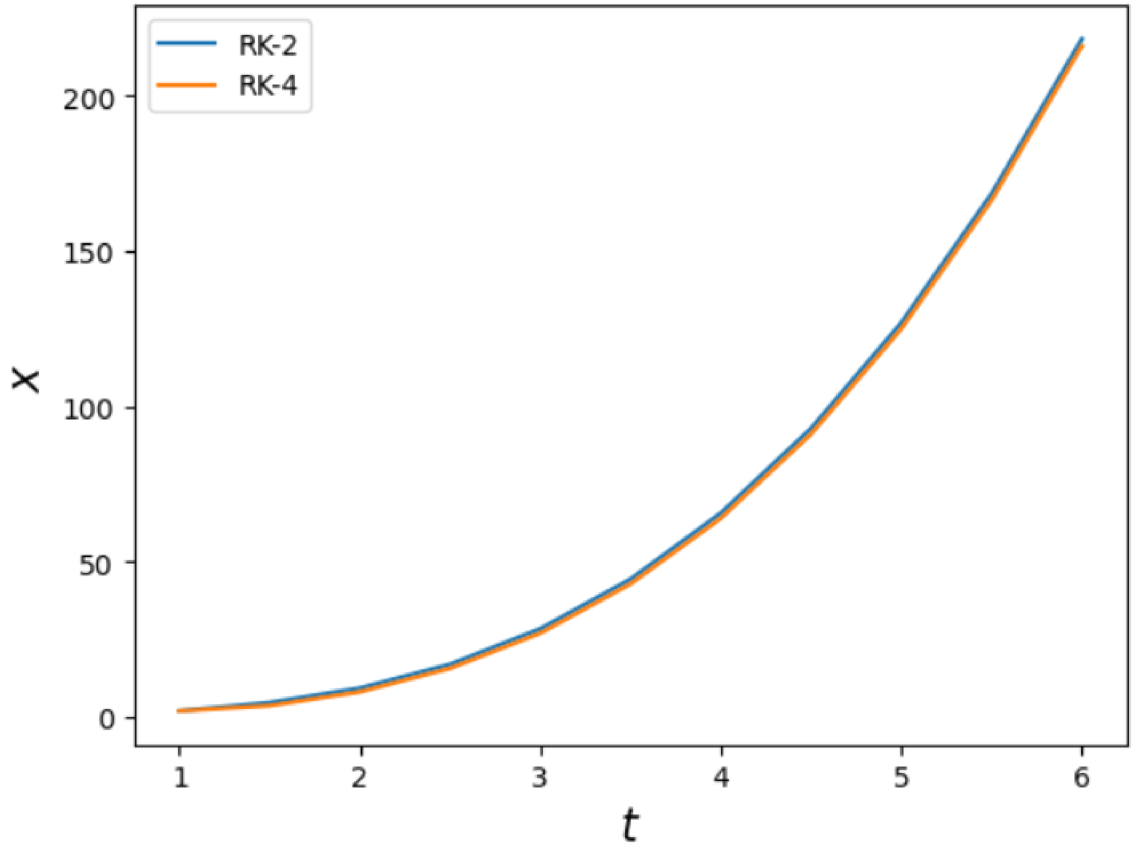
Que 3: - Used RK-4 method to solve the problem.
Plot between temperature of plate suspended in furnace of $T_f = 1500K$ with time.



It takes approximately 187.8788 sec to reach thermal equilibrium.

Que 4: -

RK-2 vs RK-4



Percentage Error at t=6: -

Taking Step Size (h) = 0.01

RK-2: - %age Error = 0.000362284982763%

RK-4: - %age Error = 0.000000000843147%

RK-2

Convergence Rate: - As h (step size) decreases by a factor 2, error decreases by a factor 4. So, in RK-2, Error: - $O(h^2)$

h	Approximate Solution	Exact Solution	Absolute Error	Error Ratio
0.5	218.2986573	216.0007716	2.297885707	
0.25	216.5271101	216.0007716	0.526338517	4.365794319
0.125	216.1272939	216.0007716	0.126522288	4.160045823
0.0625	216.0318138	216.0007716	0.031042223	4.075812771
0.03125	216.0084611	216.0007716	0.007689453	4.036987188
0.015625	216.0026852	216.0007716	0.001913619	4.018277673
0.0078125	216.0012489	216.0007716	0.00047732	4.009086443
0.00390625	216.0008908	216.0007716	0.000119195	4.004530314
0.001953125	216.0008014	216.0007716	2.97819E-05	4.002261921
0.000976563	216.000779	216.0007716	7.44338E-06	4.001130197

Rk-4

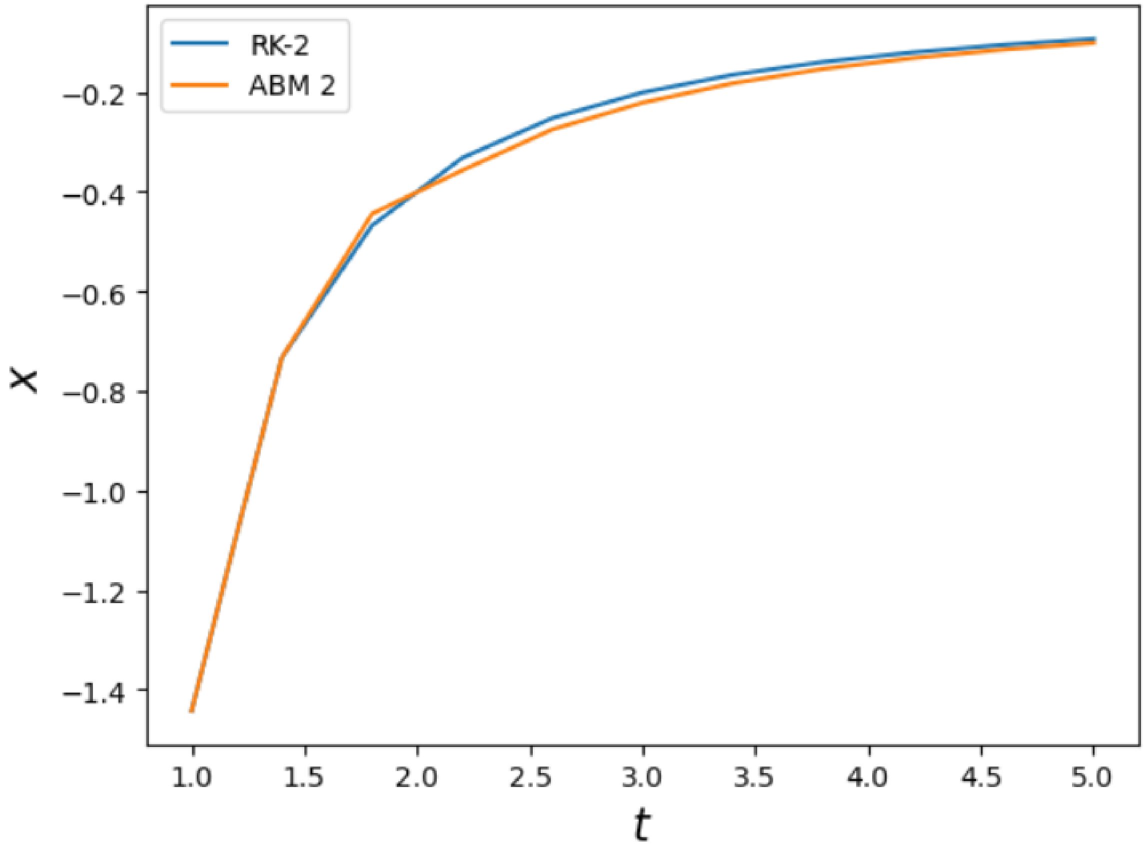
Convergence rate: - As h (step size) decreases by a factor 2, error decreases by a factor 16. So, in RK-4, Error: - $O(h^4)$

h	Approximate Solution	Exact Solution	Absolute Error	Error Ratio
0.5	216.0140297	216.0007716	0.013258093	
0.25	216.0015373	216.0007716	0.000765657	17.31597781
0.125	216.0008176	216.0007716	4.60325E-05	16.63294147
0.0625	216.0007744	216.0007716	2.8217E-06	16.31374865
0.03125	216.0007718	216.0007716	1.74649E-07	16.15638739
0.015625	216.0007716	216.0007716	1.08626E-08	16.07801961
0.0078125	216.0007716	216.0007716	6.77269E-10	16.03884867
0.00390625	216.0007716	216.0007716	4.2129E-11	16.07607586
0.001953125	216.0007716	216.0007716	2.651E-12	15.89173897
0.000976563	216.0007716	216.0007716	1.21E-13	21.90909091

Que 5: -

RK-2 vs ABM2

Plot of Approximate Solution by RK-2 & ABM2 at h (step size) = 0.4



RK-2

Convergence Rate: - As h (step size) decreases by a factor 2, error decreases by a factor 4. So, in RK-2, Error: - $O(h^2)$

t	Approximate Solution	Exact Solution	Absolute Error	Error Ratio
0.5	-0.089342214	-0.086858896	0.002483318	
0.25	-0.089848128	-0.086858896	0.002989231	0.830754628
0.125	-0.087599568	-0.086858896	0.000740671	4.035840115
0.0625	-0.087034569	-0.086858896	0.000175673	4.21620403
0.03125	-0.086901385	-0.086858896	4.24884E-05	4.134598028
0.015625	-0.086869332	-0.086858896	1.04359E-05	4.071364574
0.0078125	-0.086861482	-0.086858896	2.58544E-06	4.036424
0.00390625	-0.08685954	-0.086858896	6.43405E-07	4.018366445

0.001953125	-0.086859057	-0.086858896	1.60481E-07	4.00921816
0.000976563	-0.086858936	-0.086858896	4.00741E-08	4.004617323

ABM-2

Convergence Rate: - Initially error ratio for h = 0.25 and 0.125 is large because h (step size) is small and initial 2 values are guessed using RK-2 then ABM-2.

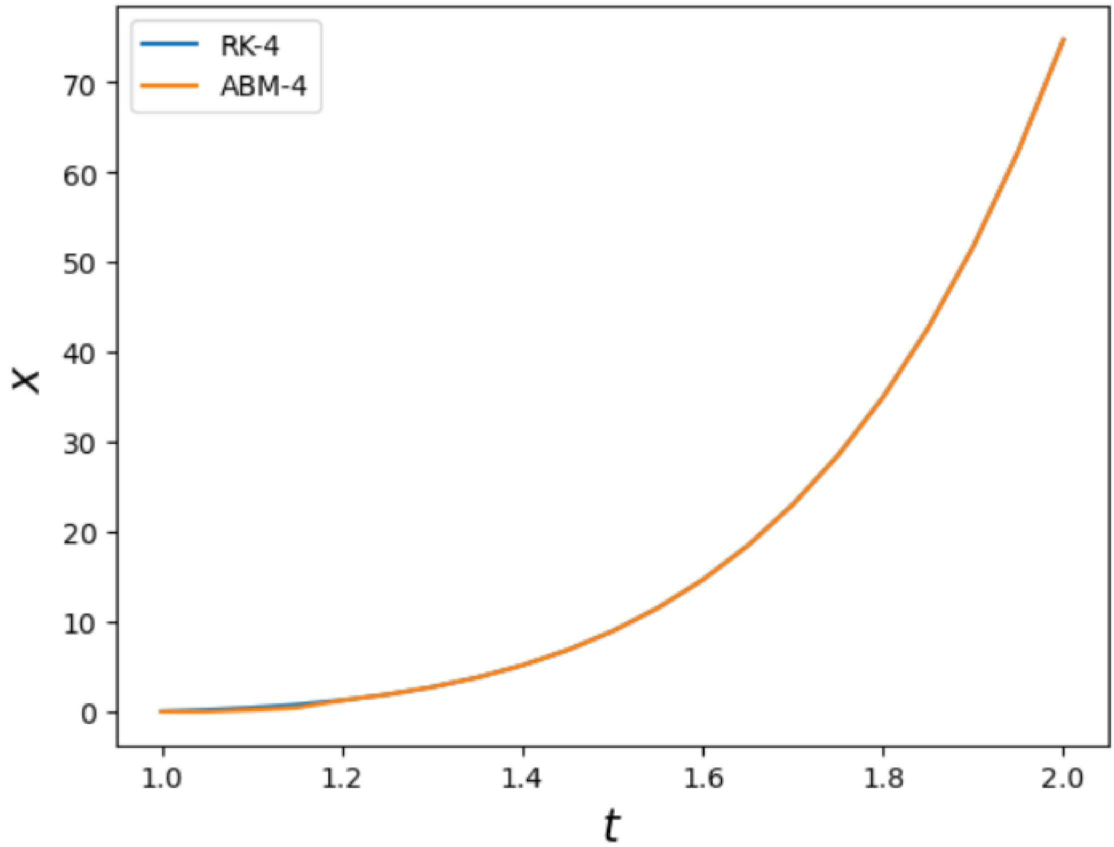
As h (step size) decreases by a factor 2, error decreases by a factor2. So, in AMB-2, Error: - $O(h)$

t	Approximate Solution	Exact Solution	Absolute Error	Error Ratio
0.5	0.082648491	-0.086858896	0.169507387	
0.25	-0.097633532	-0.086858896	0.010774635	15.73207681
0.125	-0.087075736	-0.086858896	0.00021684	49.68943917
0.0625	-0.085443065	-0.086858896	0.001415831	0.153153536
0.03125	-0.08578417	-0.086858896	0.001074726	1.317387405
0.015625	-0.086233712	-0.086858896	0.000625185	1.719054309
0.0078125	-0.086524921	-0.086858896	0.000333975	1.871948454
0.00390625	-0.086686638	-0.086858896	0.000172258	1.938806771
0.001953125	-0.086771459	-0.086858896	8.74372E-05	1.970080201
0.000976563	-0.086814852	-0.086858896	4.40444E-05	1.98520554

Que 6: -

RK-4 vs ABM-4

Plot of Approximate Solution by RK-2 & ABM2 at h (step size) = 0.05



RK-4

Convergence Rate: - As h (step size) decreases by a factor 2, error decreases by a factor 16. So, in RK-4, Error: - $O(h^4)$

T	Approximate Solution	Exact Solution	Absolute Error	Error Ratio
0.125	91057.71023	91059.2983	1.588071034	
0.0625	91059.18338	91059.2983	0.11491264	13.81981161
0.03125	91059.29055	91059.2983	0.007742088	14.84259089
0.015625	91059.29779	91059.2983	0.000502624	15.40334353
0.0078125	91059.29826	91059.2983	3.20203E-05	15.69703728
0.00390625	91059.29829	91059.2983	2.02046E-06	15.84799115
0.001953125	91059.2983	91059.2983	1.26882E-07	15.92399506
0.000976563	91059.2983	91059.2983	8.0508E-09	15.76013327

ABM-4

Convergence Rate: - Initially error ratio for h = 0.0625 and 0.03125 is less because h (step size) is small and initial 4 values are guessed using RK-4 then ABM-2.

As h (step size) decreases by a factor 2, error decreases by a factor 16. So, in AMB-4, Error: - $O(h^4)$

t	Approximate Solution	Exact Solution	Absolute Error	Error Ratio
0.125	91062.32732	91059.2983	3.029019795	
0.0625	91059.72933	91059.2983	0.431036067	7.027300101
0.03125	91059.33682	91059.2983	0.038523096	11.18902974
0.015625	91059.30116	91059.2983	0.002865604	13.44327198
0.0078125	91059.29849	91059.2983	0.000195317	14.67155334
0.00390625	91059.29831	91059.2983	1.2748E-05	15.3213706
0.001953125	91059.2983	91059.2983	8.14292E-07	15.65532401
0.000976563	91059.2983	91059.2983	5.14229E-08	15.83521942
0.000488281	91059.2983	91059.2983	3.09596E-09	16.6096536

Que 7: - Used RK-4 to Approximate the function.

Plot for x from t=0 to t=30. Simulated the first 4 cycles with step size, h = 0.1.

By plot x(t) has a periodic nature.